Flavour Physics and Lattice QCD

Chris Sachrajda

School of Physics and Astronomy
University of Southampton
Southampton SO17 1BJ
UK

(RBC-UKQCD and FLAG Collaborations)

UK Annual Theory Meeting
Durham
December 16th – 18th 2013

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The 31st Annual International Symposium on Lattice Field Theory, held last summer, had 510 participants and the following parallel sessions:

1. [QCD at] Non-zero temperature and density
2. Theoretical developments
3. Standard Model parameters and renormalization
4. Algorithms and machines
5. Vacuum structure and confinement
6. Physics beyond the standard model
7. Hadron spectroscopy and interactions
8. Hadron Structure
9. Weak decays and matrix elements
10. Chiral symmetry
11. Applications beyond QCD
12. Coding efforts

- In this talk I focus on lattice computations contributing to precision flavour physics.

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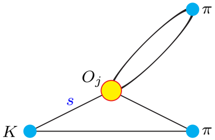
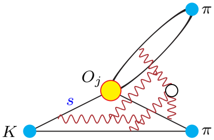
- (Precision) Flavour physics is a key tool in exploring the limits of the Standard Model of Particle Physics and in searches for new physics.
- It is complementary to high-energy experiments (most notably the LHC).
  - If, as expected/hoped the LHC experiments discover new elementary particles BSM, then precision flavour physics will be necessary to understand the underlying framework.
  - The discovery potential of precision flavour physics should also not be underestimated.
  - Precision flavour physics requires control of hadronic effects for which lattice QCD simulations are essential.



means



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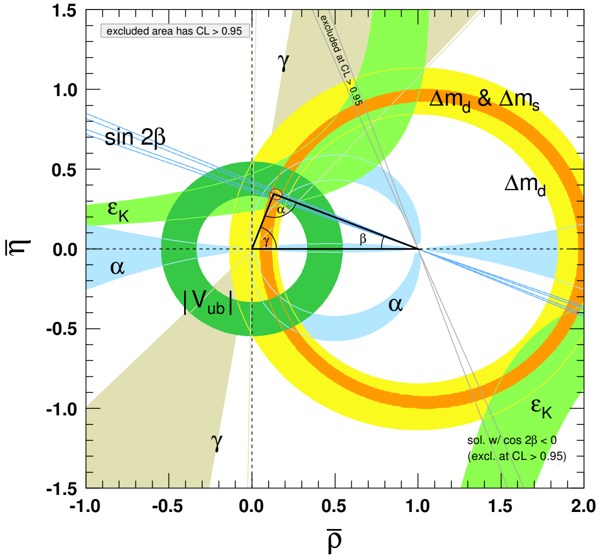
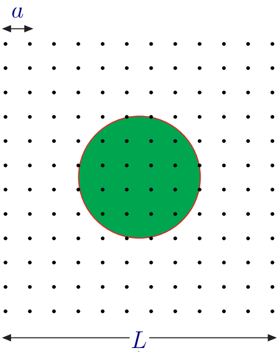
J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012)



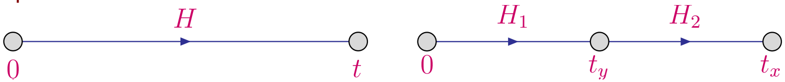


- Lattice phenomenology starts with the evaluation of correlation functions of the form:

$$\langle 0|\,O(x_1,x_2,\cdots,x_n)\,|0\rangle \;=\; \frac{1}{Z}\int [dA_\mu]\,[d\psi]\,[d\bar\psi]\,e^{-S}\,O(x_1,x_2,\cdots,x_n)\,,$$

where $O(x_1,x_2,\cdots,x_n)$ is a multilocal operator composed of quark and gluon fields and Z is the partition function.

- The physics which can be studied depends on the choice of the multilocal operator O.



- The functional integral is performed by discretising space-time and using Monte-Carlo Integration.

Outline of Talk

1. Introduction
   - 1a Introducing the Flavour Physics Lattice Averaging Group (FLAG)
2. (Selected) Physics of light flavours
   - 2a *Standard Quantities*: quark masses, B_K, V_{us} and V_{ud}.
   - 2b Isospin Breaking Effects.
   - 2c Mini-puzzles: g_A and $\langle x \rangle_{u-d}$.
3. Extending the range of lattice calculations:
   - 3a $K \to \pi\pi$ Decays
   - 3b Prospects for the calculation of Δm_K and rare kaon decay amplitudes.
4. Heavy quark physics
   - 4a B-physics
   - 4b D-physics
5. Summary and Conclusions

1a Flavour Physics Lattice Averaging Group (FLAG)

- Most of the compilations in this talk are taken from the current results of the FLAG collaboration: "Review of lattice results concerning low energy particle physics,"
  S. Aoki, Y. Aoki, C. Bernard, T. Blum, G. Colangelo, M. Della Morte, S. Dürr, A. El Khadra, H. Fukaya, A. Jüttner, R. Horsley, T. Kaneko, J. Laiho, L. Lellouch, H. Leutwyler, V. Lubicz, E. Lunghi, S. Necco, T. Onogi, C. Pena, C. Sachrajda, J. Shigemitsu, S. Simula, S. Sharpe, R. Sommer, R. Van de Water, A. Vladikas, U. Wenger, H. Wittig. arXiv:1310.8555, (255 pages!)

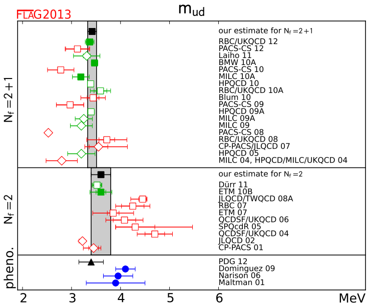
- This is an extension and continuation of the work of the Flavianet Lattice Averaging Group:
  G. Colangelo, S. Durr, A. Juttner, L. Lellouch, H. Leutwyler, V. Lubicz, S. Necco, C. T. Sachrajda, S. Simula, A. Vladikas, U. Wenger, H. Wittig arXiv:1011.4408

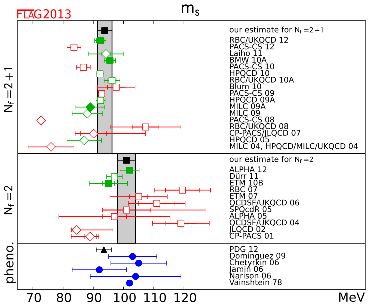
- Motivation - to present to the wider community an *average* of lattice results for important quantities obtained after a critical expert review.
- Danger - original papers (particularly those which pioneer new techniques) do not get cited appropriately by the community.
- The closing date for arXiv:1310.8555 was April 30th 2012 (currently being updated to Nov 30th 2013).

2. Selected results from the light-meson physics (FLAG)

Quark Masses: From $N_f = 2+1$ simulations, $\overline{\text{MS}}$ at $\mu = 2$ GeV

$$\frac{m_u + m_d}{2} = (3.42 \pm 0.09)\,\text{MeV} \qquad m_s = (93.8 \pm 2.4)\,\text{MeV} \qquad \frac{m_s}{m_{ud}} = 27.5 \pm 0.4$$





- Filled green points used in the averages.
- Diamonds = Pert. Normalization; Squares = Non-Pert. Renormalization.
- Precision is now such that electromagnetic and other isospin breaking effects should be included. (I will briefly return to this.)
- This is necessary for determinations of m_u and m_d separately.

- From $N_f = 2+1$ simulations, $\overline{\mathrm{MS}}$ at $\mu =$2 GeV

$$\frac{m_u+m_d}{2} = (3.42\pm 0.09)\,\mathrm{MeV} \qquad m_s = (93.8\pm 2.4)\,\mathrm{MeV} \qquad \frac{m_s}{m_{ud}} = 27.5\pm 0.4$$

arXiv:1310.8555

- 2002 PDG Review (earliest one to which I contributed)

$$\frac{m_u+m_d}{2} = (4.2\pm 1.0)\,\mathrm{MeV} \qquad m_s = (105\pm 25)\,\mathrm{MeV}$$

K. Hagiwara et al. (PDG), Phys. Rev. D. **66** 010001 (2002)

- I did add the phrase “It should be noted that recent results from simulations with two flavors of sea quarks suggest that the light-quark masses may be in the lower parts of the ranges quoted above $\cdots$".

2b Isospin Breaking (IB) Effects

- 1% precision in QCD simulations $\Rightarrow$ IB effects ($m_u \neq m_d$ and electromagnetism) must be included. See e.g. A.Portelli, arXiv 1307.6056
- Until recently all the calculations were performed in the isospin limit without electromagnetism, but now the explicit calculations of IB-effects are included.
- For the evaluation of m_u and m_d separately, some results have been obtained by combining isospin-symmetric computations with continuum phenomenology.
- Defining $\Delta_\pi \equiv m^2_{\pi^+} - m^2_{\pi^0}$ and $\Delta^\gamma_P \equiv m^2_P - \hat{m}^2_P$ ($\hat{m}_p$ is the mass of P in QCD alone), violations of Dashen's Theorem can be parametrized:

$$\Delta^\gamma_{K^+} - \Delta^\gamma_{K^0} - \Delta^\gamma_{\pi^+} + \Delta^\gamma_{\pi^0} = \varepsilon\,\Delta_\pi$$

  (ε does not parametrize all IB effects.)

- Some groups take ε from phenomenological studies (e.g. MILC), others introduce quenched electromagnetism as a $U(1)$ degree of freedom (e.g. RBC, BMW) or by performing perturbation theory in α (RM123):

$$\varepsilon = 0.7(3), \quad m_u = 2.16(9)(7)\,\text{MeV}, \quad m_d = 4.68(14)(7)\,\text{MeV}, \quad \frac{m_u}{m_d} = 0.46(2)(2)$$

$\overline{\text{MS}}$ masses at 2 GeV, FLAG estimates

- Evaluating IB effects will (continue to) be a major activity in the coming period.

$$\varepsilon_K = \frac{A(K_L\to(\pi\pi)_{I=0})}{A(K_S\to(\pi\pi)_{I=0})} = e^{i\phi_\varepsilon}\sin\phi_\varepsilon\left[\frac{\mathrm{Im}\langle\bar K^0\,|H_W^{\Delta S=2}\,|K^0\rangle}{\Delta m_K}+\text{L.D. effects}\right]$$

where

Buras, Guadagnoli, arXiv:0805.3887

$$\begin{aligned}
|\varepsilon_K| &= 2.228(11)\times 10^{-3}\\
\phi_\varepsilon &= \arctan\frac{\Delta m_K}{\Delta\Gamma_K/2} = 43.52(5)^\circ\\
\Delta m_K &= m_{K_L}-m_{K_S} = 3.4839(59)\times 10^{-12}\,\text{MeV}\\
\Delta\Gamma_K &= \Gamma_S-\Gamma_L = 7.3382(33)\times 10^{-15}\,\text{GeV}.
\end{aligned}$$

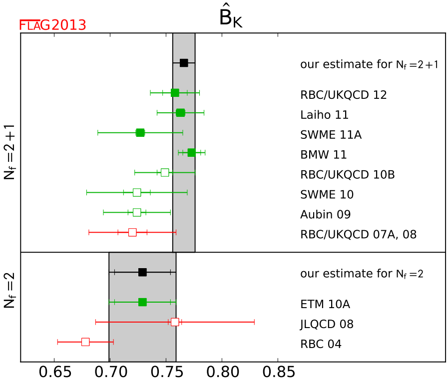
- It is conventional to present the short-distance contribution in terms of the B_K parameter:

$$\langle\bar K^0\,|H_W^{\Delta S=2}\,|K^0\rangle \propto \langle\bar K^0\,|(\bar s\gamma^\mu(1-\gamma^5)d)\,(\bar s\gamma_\mu(1-\gamma^5)d)|K^0\rangle \equiv \frac{8}{3}f_K^2 m_K^2 B_K(\mu)\,.$$

- Lattice calculations of B_K have been performed since the mid 1980s. The precision is now such that the $O(5\%)$ long-distance (LD) effects have to be considered.

Buras, Guadagnoli, Isidori arXiv:1002.3612

Results for B_K (FLAG)



- FLAG-2 quote from simulations with $N_f = 2+1$:

$$\hat{B}_K = 0.766(10) \text{ corresponding to } B_K^{\overline{\rm MS}}(2\,\text{GeV}) = 0.560(7)\,.$$

- The FLAG-1 result was $\hat{B}_K = 0.738(20)$ and in 1993 I quoted a summary $\hat{B}_K = 0.8(2)$. M. Lusignoli, L. Maiani, G. Martinelli and L. Reina, Nucl.Phys. B369 (1992) 139
- The dominant contribution to $\varepsilon_K \propto |V_{cb}|^4$ and PDG(2012) quote $|V_{cb}| = (40.9 \pm 1.1) \times 10^{-3}$ error on B_K is no longer the dominant one.

- Many of the quantities computed in lattice QCD require renormalisation and the traditional way of dividing responsibilities is:

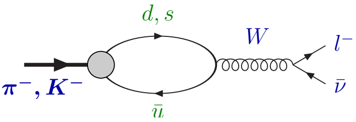
$$\text{Physics} \;=\; \underset{\substack{\uparrow \\ \text{Perturbative} \\ \text{QCD}}}{C} \;\times\; \underset{\substack{\uparrow \\ \text{Lattice} \\ \text{QCD}}}{\langle f\,|\,O\,|\,i\rangle}$$

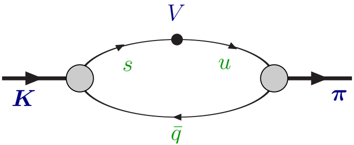
- The two factors have to be calculated in the same renormalisation scheme.
- It is possible and indeed standard practice, to perform the renormalisation *non-perturbatively* (RI-MoM, RI-SMoM, Schrödinger Functional, $\cdots$).
- It is not possible however, to perform simulations in $4+2\varepsilon$ dimensions and to compute matrix elements in the $\overline{\mathrm{MS}}$ scheme $\Rightarrow$ we introduce an intermediate scheme which we can simulate:

$$\begin{array}{c}\text{bare}\\ \text{lattice}\\ \text{operators}\end{array} \longrightarrow \textbf{Intermediate Scheme} \longleftrightarrow \begin{array}{c}\text{operators}\\ \text{renormalized}\\ \text{in } \overline{\mathrm{MS}} \text{ scheme}\end{array}$$

- The, necessarily perturbative, matching between the intermediate scheme and $\overline{\mathrm{MS}}$ (as well as the precision with $C^{\overline{\mathrm{MS}}}$ is calculated) can be a very significant contribution to the systematic error.

  J.Gracey, M.Gorbahn and S.Jager, L.Almeida and C.Sturm, HPQCD + Karlsruhe Group, $\ldots$ have and are working to decrease this source of error.





- For leptonic decays of a pseudoscalar meson P, all QCD effects are contained in a single constant, f_P, the (leptonic) decay constant:

$$\langle 0 | \bar{s}(\bar{d}) \gamma^\mu \gamma^5 u | P(p) \rangle \equiv i f_P \, p^\mu .$$

- For $K_{\ell 3}$ decays QCD effects are contained in form factors e.g. for $B \to \pi$ decays:

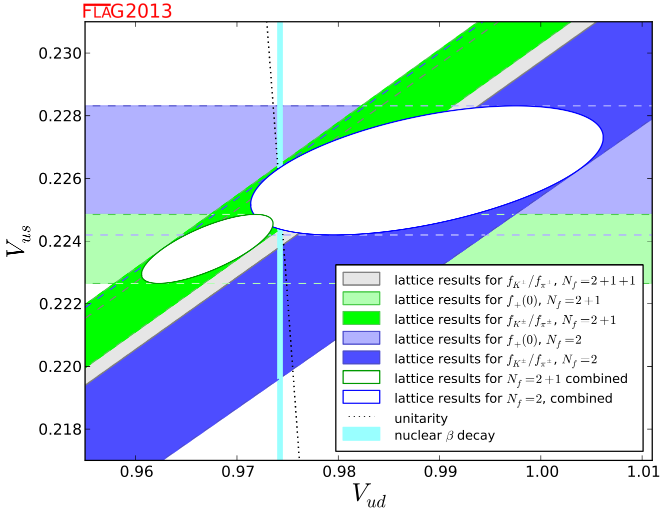
$$\langle \pi(p_\pi) | \bar{s} \gamma_\mu u | K(p_K) \rangle = f_0(q^2) \frac{m_K^2 - m_\pi^2}{q^2} q_\mu + f_+(q^2) \left[(p_\pi + p_K)_\mu - \frac{m_K^2 - m_\pi^2}{q^2} q_\mu \right]$$

  where $q \equiv p_B - p_\pi$.

- We start with the very precise experimental data:

$$\frac{|V_{us}|}{|V_{ud}|} \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2758(5) \qquad \text{and} \qquad |V_{us}| f_+(0) = 0.2163(5) .$$

  The job of the lattice community is therefore to calculate f_K / f_π and $f_+(0)$.


FLAG2013
0.230
0.228
0.226
0.224
0.222
0.220
0.218
V_{us}
0.96
0.97
0.98
0.99
1.00
1.01
V_{ud}
lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2+1+1$
lattice results for $f_+(0)$, $N_f = 2+1$
lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2+1$
lattice results for $f_+(0)$, $N_f = 2$
lattice results for $f_{K^\pm}/f_{\pi^\pm}$, $N_f = 2$
lattice results for $N_f = 2+1$ combined
lattice results for $N_f = 2$, combined
unitarity
nuclear β decay

V_{ud} and V_{us} (FLAG)

$(N_f = 2+1)$

- For leptonic decays:

$$\frac{f_{K^\pm}}{f_{\pi^\pm}} = 1.192(5) \quad \Rightarrow \quad \frac{|V_{us}|}{|V_{ud}|} = 0.2314(11)\,.$$

- For $K_{\ell 3}$ decays,

$$f_+(0) = 0.9667(23)(33) \quad \Rightarrow \quad |V_{us}| = 0.2238(7)(8)\,.$$

- Unitarity of the first row of the CKM Matrix (check on quark-lepton universality of the effective G_F):

$$\begin{aligned}
|V_u|^2 \;&\equiv\; |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \;\simeq\; |V_{ud}|^2 + |V_{us}|^2 \\
&=\; 0.985(13) \qquad (N_f = 2+1 \text{ results for } f_K/f_\pi \text{ and } f_+(0)) \\
&=\; 0.9992(6) \qquad (N_f = 2+1 \text{ results for } f_+(0) \text{ and } |V_{ud}| = 0.97425(22)\,)
\end{aligned}$$

J.C.Hardy & I.S.Towner, arXiv:0812.1202

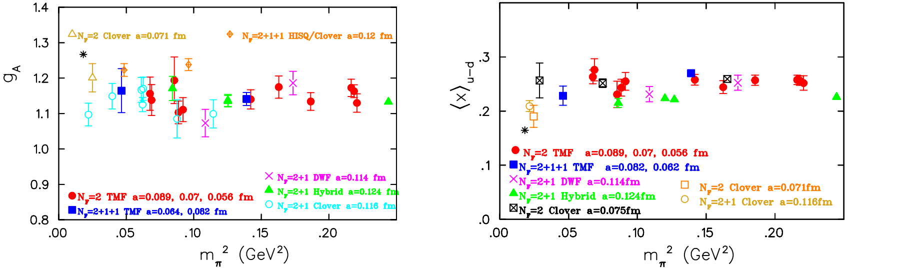
$$= \; 1.0000(6) \qquad (N_f = 2+1 \text{ results for } f_K/f_\pi \text{ and } |V_{ud}| = 0.97425(22))$$

- There is very little room for any discrepancy of $|V_u|^2 = 1$.

2b A couple of mini(?)-puzzles

- Is everything completely under control in computations of standard quantities?

  Almost, but there are a few puzzles, particularly in baryon physics.



Courtesy of C.Alexandrou; ETMC, arXiv:1303.5979

- The lattice results are in good agreement with each other, but as $m_\pi \to m_\pi^{\text{phys}}$ g_A appears to be low and $\langle x\rangle_{u-d}$ is perhaps too high?
- The leading candidates for this discrepancy are finite-volume effects (or perhaps contamination by excited states or finite-T effects).
- Much work is being done to understand the systematic errors in these quantities.

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- The RBC-UKQCD collaboration has performed the first calculation of the $K \to (\pi\pi)_{I=2}$ amplitude A_2: RBC-UKQCD, T.Blum et al., arXiv:1111.1699, arXiv:1206.5142

$$\begin{aligned} \mathrm{Re}A_2 &= (1.381 \pm 0.046_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}})\, 10^{-8}\,\mathrm{GeV} \\ \mathrm{Im}A_2 &= -(6.54 \pm 0.46_{\mathrm{stat}} \pm 1.20_{\mathrm{syst}})\, 10^{-13}\,\mathrm{GeV}. \end{aligned}$$

- The result for Re A_2 agrees well with the experimental value of $1.479(4) \times 10^{-8}$ GeV obtained from K^+ decays .
- Im A_2 is unknown so that our result provides its first direct determination.
- Combining our result for Im A_2 with the experimental results for Re A_2, Re $A_0 = 3.3201(18) \cdot 10^{-7}$ GeV and ε'/ε we obtain:

$$\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} = -1.61(19)_{\mathrm{stat}}(20)_{\mathrm{syst}} \times 10^{-4}\,.$$

(Of course, we wish to confirm this directly.)

$$\begin{array}{cccccc} \dfrac{\mathrm{Im}A_0}{\mathrm{Re}A_0} & = & \dfrac{\mathrm{Im}A_2}{\mathrm{Re}A_2} & - & \dfrac{\sqrt{2}|\varepsilon|}{\omega}\dfrac{\varepsilon'}{\varepsilon} \\ -1.61(19)_{\mathrm{stat}}(20)_{\mathrm{syst}} \times 10^{-4} & = & -4.42(31)_{\mathrm{stat}}(89)_{\mathrm{syst}} \times 10^{-5} & - & 1.16(18) \times 10^{-4}\,. \end{array}$$

$$\begin{aligned} \mathrm{Re}A_2 &= (1.381 \pm 0.046_{\rm stat} \pm 0.258_{\rm syst})\,10^{-8}\,\mathrm{GeV} \\ \mathrm{Im}A_2 &= -(6.54 \pm 0.46_{\rm stat} \pm 1.20_{\rm syst})\,10^{-13}\,\mathrm{GeV}. \end{aligned}$$

RBC-UKQCD, T.Blum et al., arXiv:1111.1699, arXiv:1206.5142

- Currently the error is dominated by *lattice artefacts*, since the calculation was performed at a single, rather coarse, lattice spacing.
  - Preliminary results at two finer lattice spacings were presented at Lattice 2013 $\Rightarrow$ this uncertainty will be reduced very significantly.

Volume	a^{-1}	Re A_2	Im A_2
48^3	1.73(1) GeV	1.411(22)$\times 10^{-8}$ GeV	-6.40(11)$\times 10^{-13}$ GeV
64^3	2.30(4) GeV	1.398(17)$\times 10^{-8}$ GeV	-6.438(74) $\times 10^{-13}$ GeV

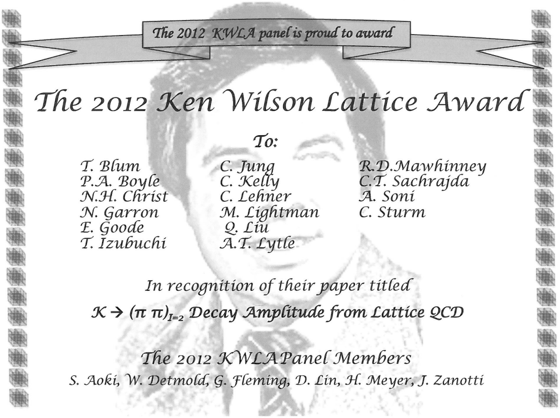
RBC-UKQCD, T.Janowski et al., arXiv:1311.3944

  - Only statistical errors are shown here.
  - The discretization of QCD is different in these new simulations, so it is not easy to combine the results with the first calculation on a courser lattice (a^{-1}=1.375(9) GeV).
  - Analysis is currently being completed and the publication is being prepared.

$$\begin{aligned} \mathrm{Re}A_2 &= (1.381 \pm 0.046_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}})\, 10^{-8}\,\mathrm{GeV} \\ \mathrm{Im}A_2 &= -(6.54 \pm 0.46_{\mathrm{stat}} \pm 1.20_{\mathrm{syst}})\, 10^{-13}\,\mathrm{GeV}. \end{aligned}$$

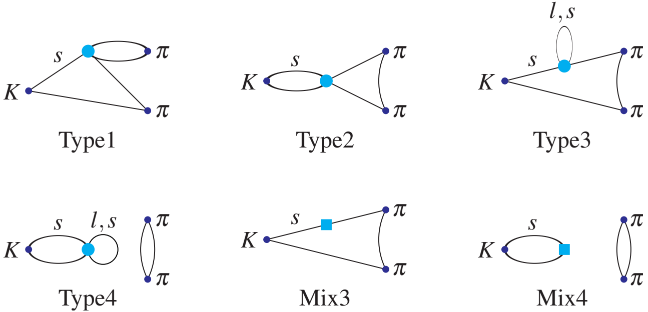
RBC-UKQCD, T.Blum et al., arXiv:1111.1699, arXiv:1206.5142

- For two-pion states, control of finite-volume and rescattering effects becomes particularly important. M.Lüscher, $\cdots$
  - For hadronic B-decays, for which inelastic intermediate states are important, we do not even know how to formulate a possible computation.
- In a finite volume, with the kaon at rest, the two-pion spectrum is discrete, with the ground state corresponding to both pions at rest.
  - $E_{\pi\pi} = m_K$ requires an excited state (possible in principle, but difficult).
  - To avoid this, we use the Wigner-Eckart theorem, relating $K^+ \to \pi^+\pi^0$ to $K^+ \to \pi^+\pi^+$ matrix elements and impose antiperiodic boundary conditions on the u quark and periodic for the remaining fields.
  - In particular with our choice of volume, the energy of the state $|\pi^+(\pi/L, \pi/L, 0)\, \pi^+(-\pi/L, -\pi/L, 0)\rangle \simeq m_K$.



- For our calculation of A_2 we received the 2012 Ken Wilson Lattice award at Lattice 2012.
- Criteria: The paper must be important research beyond the existing state of the art. $\cdots$

- The calculation is much more difficult for the $K \to (\pi\pi)_{I=0}$ amplitude A_0:
  - The presence of disconnected diagrams:



  - The efficient evaluation of disconnected diagrams is a major area of research in the lattice community.
  - Breaking I-invariance by different boundary conditions for u and d quarks fatal. Even without interactions, $|\pi^+\pi^-\rangle$ and $|\pi^0\pi^0\rangle$ have different energies.

- RBC-UKQCD have computed A_0 with the two pions at rest and with unphysical masses, finding e.g. arXiv:1106.2714, Qi Liu Columbia Un.Thesis

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 9.1 \pm 2.1 \qquad 877\,\mathrm{MeV\ kaon\ decaying\ into\ two\ 422\,MeV\ pions}$$
$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 12.0 \pm 1.7 \qquad 662\,\mathrm{MeV\ kaon\ decaying\ into\ two\ 329\,MeV\ pions}$$

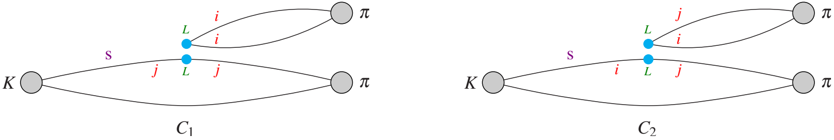
- Whilst both these results are obtained at unphysical kinematics and are different from the physical value of 22.5, it is nevertheless interesting to understand the origin of these enhancements.
- 99% of the contribution to the real part of A_0 and A_2 come from the matrix elements of the current-current operators.
- For a calculation of ε'/ε at physical kinematics, RBC-UKQCD are developing G-parity boundary conditions (estimate timescale $\sim$ 2 years).

RBC-UKQCD Collaboration, arXiv:1212.1474

- Re A_2 is dominated by a simple operator:

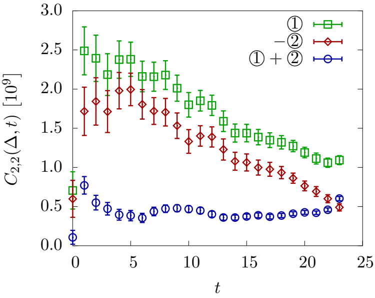
$$O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \{(\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L\} + (\bar{s}^i u^i)_L (\bar{u}^j d^j)_L$$

and two diagrams:

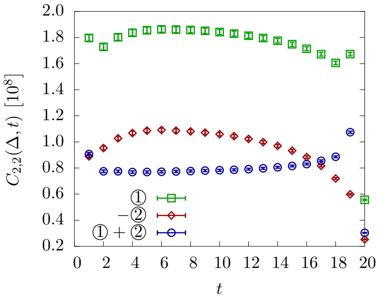


- Re A_2 is proportional to $C_1 + C_2$.
- The contribution to Re A_0 from Q_2 is proportional to $2C_1 - C_2$ and that from Q_1 is proportional to $C_1 - 2C_2$ with the same overall sign.
- Colour counting might suggest that $C_2 \simeq \frac{1}{3} C_1$.
- We find instead that $C_2 \approx -C_1$ so that A_2 is significantly suppressed!
- **We believe that the strong suppression of Re A_2 and the (less-strong) enhancement of Re A_0 is a major factor in the $\Delta I = 1/2$ rule.**

Evidence for the Suppression of Re A_2



Physical Kinematics



$m_\pi \simeq 330$ MeV at threshold.

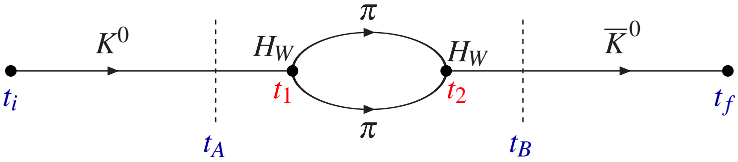
- Notation ⓘ $\equiv C_i,\ i=1,2$.
- Of course before claiming a quantitative understanding of the $\Delta I = 1/2$ rule we need to compute Re A_0 at physical kinematics and reproduce the experimental value of 22.5.
- Much early phenomenology was based on the vacuum insertion approach; although the qualitative picture we find had been suggested by Bardeen, Buras and Gerard in 1987.

3b Long Distance Effects in Kaon Physics

- RBC-UKQCD collaboration is developing the theoretical and computational framework for long-distance effects in $\Delta m_K = m_{K_L} - m_{K_S}$ and in rare-kaon decays.
- This requires the evaluation of correlation functions of the form:

$$\int d^4x \int d^4y \, \langle h_2 \, | \, T\{O_1(x)\, O_2(y)\} \, | \, h_1 \rangle \,,$$

where $h_{1,2}$ are hadrons and $O_{1,2}$ are composite local operators. For example in Δm_K the correlation function can be represented as:

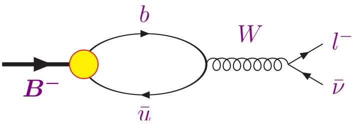


and exploratory results have been presented in N.H.Christ et al., arXiv:1212.5931.

  - The use of $T = t_B - t_A$ dependence to isolate the relevant contribution has had to be understood.
  - Finite-volume effects have had to be understood.
  - GIM mechanism has to be invoked to cancel quadratic UV divergences.
- For rare-kaon decays, such as $K_S \to \pi^0 \ell^+ \ell^-$, $O_1 = H_W$ and $O_2 = J_{\rm em}$.
- This will continue to be be a major area of our research in the coming period.

- The b-quark is light-enough to be produced copiously and heavy enough to have a huge number of possible decay channels.
- In addition to the lattice systematics already discussed, we now have to deal with the fact that $m_b a \gtrsim 1$.
- Most approaches rely on effective theories and invest a considerable effort in matching the effective theory to QCD.
  - Heavy Quark Effective Theory (expansion in $\frac{\Lambda_{\mathrm{QCD}}}{m_B}$).
  - Nonrelativistic QCD (expansion in the quark's velocity).
  - Relativistic Heavy Quarks ("Fermilab Approach" and extensions).

  A. El Khadra, A. Kronfeld and P. Mackenzie, hep-lat/9604004
- Some groups also extrapolate results from the charm to the bottom region, using scaling laws where applicable and possibly using results in the static limit.
- There are far fewer calculations in heavy-quark physics, so less opportunity to check for consistency of different approaches.
  This is not a criticism of those who have done the calculations but of those of us who have not!
- Unfortunately we do not know (yet?) how to compute non-leptonic B-decays ($B \to \pi\pi$, $B \to \pi K$ etc).



All QCD effects are contained in a single constant, f_B, the B-meson's (leptonic) decay constant:

$$\langle 0|\,\bar{b}\gamma^\mu\gamma^5 u\,|B(p)\rangle \equiv i f_B\, p^\mu\,.$$

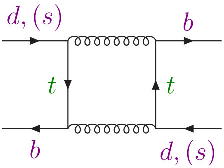
- The FLAG compilation for $N_f = 2+1$ simulations:

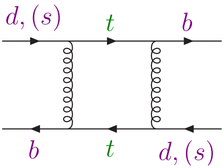
$$f_B = (190.5 \pm 4.2)\,\mathrm{MeV} \quad f_{B_s} = (227.7 \pm 4.5)\,\mathrm{MeV} \quad \frac{f_{B_s}}{f_B} = 1.202 \pm 0.022\,.$$

- CTS at EPS 1993: $f_B = 180 \pm 40\,\mathrm{MeV}$ and "Lattice simulations typically give a result 10-20% larger for B_s and D_s than for f_B and f_D".
- (In principle at least) Experimental measurements of B$(B \to \tau\nu_\tau)$ by Belle and BaBar + Lattice determinations of $f_B \Rightarrow V_{ub}$. However there are still $\sim 2\sigma$ experimental questions:

$$\begin{array}{lll}
\text{PDG 2012} & B(B^+ \to \tau^+\nu_\tau) = (1.65 \pm 0.34) \times 10^{-4} & \\
\Rightarrow & |V_{ub}| = (5.07 \pm 0.52 \pm 0.11) \times 10^{-3} & (N_f = 2+1) \\
\text{Belle (arXiv : 1208.4678)} & B(B^+ \to \tau^+\nu_\tau) = (0.72^{+0.27}_{-0.25} \pm 0.11) \times 10^{-4} & \\
\Rightarrow & |V_{ub}| = (3.35 \pm 0.65 \pm 0.07) \times 10^{-3} & (N_f = 2+1)
\end{array}$$

- At present this is not the most competitive way to determine V_{ub}.





- For the SU(3)-breaking parameter ξ, FLAG take the result of the FNAL/MILC collaboration as currently the best result: FNAL/MILC, arXiv:1205.7013

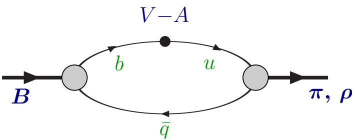
$$\xi^2 \equiv \frac{\langle \bar{B}_s^0 \,|\, (\bar{b}\gamma^\mu(1-\gamma^5)s)(\bar{b}\gamma_\mu(1-\gamma^5)s)\,|B_s^0\rangle}{\langle \bar{B}_d^0 \,|\, (\bar{b}\gamma^\mu(1-\gamma^5)d)(\bar{b}\gamma_\mu(1-\gamma^5)d)\,|B_d^0\rangle} = 1.268(63)\,.$$

- Combining this result with experimental values of Δm_d and $\Delta m_s \Rightarrow$

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.216 \pm 0.011\,.$$

FNAL/MILC, arXiv:1205.7013

- For generic BSM theories, there are 5 $\Delta B = 2$ operators (and 5 $\Delta S = 2$ operators for neutral kaon mixing) whose matrix elements can be computed in a similar way.



QCD effects are contained in form factors e.g. for $B \to \pi$ decays:

$$\langle \pi(p_\pi) \,|\bar{b}\gamma_\mu u|\, B(p_B) \rangle = f_0(q^2)\,\frac{m_B^2 - m_\pi^2}{q^2}\,q_\mu + f_+(q^2)\left[(p_\pi + p_B)_\mu - \frac{m_B^2 - m_\pi^2}{q^2}\,q_\mu\right]$$

where $q \equiv p_B - p_\pi$.

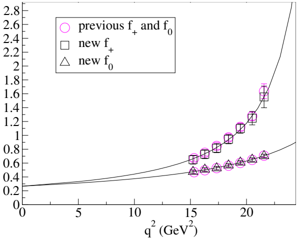
- For B-decays, in order to avoid lattice artefacts, the momentum of the π or ρ is limited $\Rightarrow$ get results only at large values of q^2.
- Thus V_{ub} can only be obtained directly by combining the lattice results with a subset of the experimental data:

$$\Delta\zeta(q_1^2, q_2^2) = \frac{1}{|V_{ub}|^2}\int_{q_1^2}^{q_2^2} dq^2\,\frac{d\Gamma}{dq^2}\,.$$

- The lattice results can be combined with theoretically motivated parametrisations for the form factors, including perhaps constrains from analyticity and other general properties of field theory, to extend the range of the predictions. (Not discussed here.)

The (peer-reviewed) published values for the form factors are relatively old:

Collaboration	Reference	$\Delta\zeta$ ps^{-1}
FNAL/MILC	arXiv:0811.3640	$2.21^{+0.47}_{-0.42}$
HPQCD	hep-lat/0601021	2.07(41)(39)



HPQCD

- The two collaborations use overlapping sets of rooted staggered ensembles, but different treatments of the heavy quarks (HPQCD use NRQCD and FNAL/MILC use the FNAL approach). Assuming (conservatively) a 100% correlation FLAG quote

$$\Delta\zeta(16\,\mathrm{GeV}^2, q^2_{\mathrm{max}}) = 2.16(50)\,\mathrm{ps}^{-1}$$

- FLAG, perform a detailed analysis, finding a preferred parametrization and quote

$$\begin{array}{rl} \text{Lattice + BaBar} & |V_{ub}| = 3.37(21)\times 10^{-3} \\ \text{Lattice + Belle} & |V_{ub}| = 3.47(22)\times 10^{-3}. \end{array}$$

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$$\begin{array}{rl} \text{Lattice + BaBar} & |V_{ub}| = 3.37(21) \times 10^{-3} \\ \text{Lattice + Belle} & |V_{ub}| = 3.47(22) \times 10^{-3}. \end{array}$$

- Assuming (not assuming) unitarity PDG quote $|V_{ub}| = 3.51^{+0.15}_{-0.14} \times 10^{-3}$ ($|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3}$).
- The issue is the tension with the inclusive determination $|V_{ub}| = (4.41 \pm 0.15^{+0.15}_{-0.19}) \times 10^{-3}$. This has very different systematics and cannot be studied in lattice simulations.
- The evaluation of $f_+(q^2)$ and $f_0(q^2)$ and the subsequent determination of V_{ub} is clearly a major priority for lattice simulations and several collaborations have made conference presentations about their current work:

Collaboration	Approach	Reference
Alpha	HQET	arXiv:1210.3478, arXiv:1211.6327
HPQCD	NRQCD	arXiv:1210.6992
RBC/UKQCD	Relativistic Heavy Quarks	arXiv:1211.0956
FNAL/MILC	Fermilab	arXiv:1211.1390

- We can look forward to many new results for the form factors.

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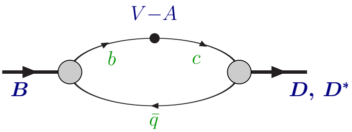


- V_{cb} is known more precisely that V_{ub}, because the experimental cuts are at higher energies where the OPE is more reliable and heavy-quark symmetry $\Rightarrow$ the form factors are close to 1.
- V_{cb} is not known well enough however! (For example $\varepsilon_K \propto |V_{cb}|^4$.)
- Work in this area has been led by the FNAL/MILC collaboration, using the Fermilab approach and staggered fermions. arXiv:0808.2519, arXiv:1011.2166
- Defining $\omega = v_B \cdot v_{D^*}$,

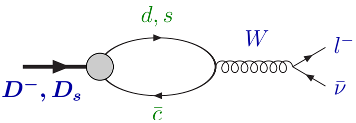
$$\frac{d\Gamma_{B^- \to D^{0*}\ell\bar{\nu}}}{d\omega} \propto |V_{cb}|^2 |\mathscr{F}(\omega)|^2\,,$$

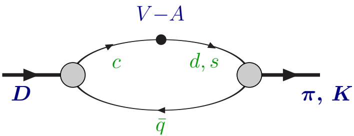
and at zero recoil FNAL/MILC find

$$\mathscr{F}(1) = 0.9017(51)(156) \Rightarrow |V_{cb}| = 39.55(72)(50) \times 10^{-3}\,.$$

FLAG

- For inclusive $B \to X_c \ell \nu$ decays, PDG quote $|V_{cb}|^{\text{incl}} = (41.9 \pm 0.7) \times 10^{-3}$.
- New lattice calculations of $\mathscr{F}(\omega)$ are being performed $\Rightarrow$ improved determination of $|V_{cb}|$ in the next 1-2 years.



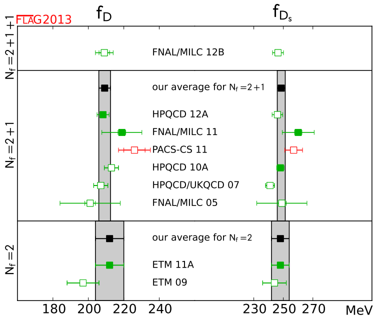


- The charm quark is somewhere in between *light* and *heavy*, i.e. typically $am_c \lesssim 1$.
- Some calculations are performed with the light-quark actions (increasingly this is the case) and some using one of the heavy approaches.
- FLAG $N_f = 2+1$ averages:

$$f_D = (209.2 \pm 3.3)\,\mathrm{MeV}, \quad f_{D_s} = (248.6 \pm 2.7)\,\mathrm{MeV}, \quad \frac{f_{D_s}}{f_D} = 1.187 \pm 0.012\,.$$

$$f_+^{D\pi}(0) = 0.666(29) \qquad f_+^{DK}(0) = 0.747(19)$$

- Leptonic decays $\Rightarrow |V_{cd}| = 0.2218(35)(95)$ and $|V_{cs}| = 1.018(11)(21)$.
- Semileptonic decays $\Rightarrow |V_{cd}| = 0.2192(95)(45)$ and $|V_{cs}| = 0.9746(248)(67)$.



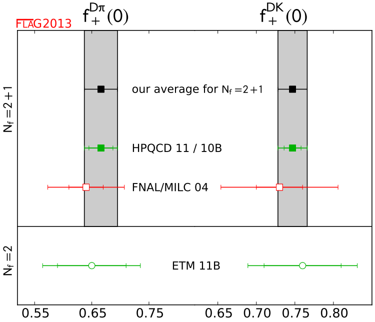
- In 2007-8 there was some excitement concerning f_{D_s}:

$$f_{D_s} = (277\pm9)\,\text{MeV} \qquad \text{compilation of experiments, Dobrescu \& Kronfeld, arXiv:0803.0512}$$

$$f_{D_s} = (241\pm3)\,\text{MeV} \qquad \text{HPQCD arXiv:0706.1726}$$

- This HPQCD result was updated to (248.0 ± 2.5) MeV in arXiv:1008.4018
  and the experimental average has come down e.g. from the B-factories
  $f_{D_s} = (257.2\pm4.5)$ MeV. A.Zupanc, arXiv:1301.7218

Results for semileptonic form factors (FLAG)



- Far fewer computations than for the corresponding quantities in light-quark physics.
- A number of new calculations under way.

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Light-quark physics

Quantity	$N_f = 2+1+1$	$N_f = 2+1$	$N_f = 2$
m_s (MeV)		93.8(2.4)	101(3)
m_{ud} (MeV)		3.42(9)	3.6(2)
m_s/m_{ud}		27.5(4)	28.1(1.2)
m_u (MeV)		4.68(14)(7)	4.8(15)(17)
m_d (MeV)		2.16(9)(7)	2.40(15)(17)
m_u/m_d		0.46(2)(2)	0.50(2)(3)
$f_+^{K\pi}(0)$		0.9667(23)(33)	0.9560(57)(62)
f_{K^+}/f_{π^+}	1.195(3)(4)	1.192(5)	1.205(6)(17)
f_K (MeV)		156.3(0.8)	158.1(2.5)
f_π (MeV)		130.2(1.4)	
$\Sigma^{\frac{1}{3}}$ (MeV)		265(17)	270(7)
f_π/f	1.0760(28)	1.0620(34)	1.0733(73)
$\bar{l}_3$	3.70(27)	2.77(1.27)	3.45(26)
$\bar{l}_4$	4.67(10)	3.95(35)	4.59(26)
$\hat{B}_K$		0.766(10)	0.729(25)(17)
$B_K^{\overline{\mathrm{MS}}}$(2 GeV)		0.560(7)	0.533(18)(12)

Heavy-quark physics

Quantity	$N_f = 2+1+1$	$N_f = 2+1$	$N_f = 2$
f_D (MeV)		209.2(3.3)	212(8)
f_{D_s} (MeV)		248.6(2.7)	248(6)
f_{D_s}/f_D		1.187(12)	1.17(5)
$f_+^{D\pi}(0)$		0.666(29)	
$f_+^{DK}(0)$		0.747(19)	
f_B (MeV)		190.5(4.2)	197(10)
f_{B_s} (MeV)		227.7(4.5)	234(6)
f_{B_s}/f_B		1.202(22)	1.19(5)
$f_B\sqrt{\hat{B}_B}$ (MeV)		216(15)	
$f_{B_s}\sqrt{\hat{B}_{B_s}}$ (MeV)		266(18)	
$\hat{B}$		1.27(10)	
$\hat{B}_s$		1.33(10)	
ξ		1.268(63)	
$\hat{B}_{B_s}/\hat{B}_B$		1.06(11)	

Heavy-quark physics (cont.)

Quantity	$N_f = 2+1+1$	$N_f = 2+1$	$N_f = 2$
$\Delta\zeta^{B\pi}$ (ps)$^{-1}$		2.16(50)	
$f_+^{B\pi}(q^2):\ a_0^{\rm BCL}$		0.453(33)	
$a_1^{\rm BCL}$		-0.43(33)	
$a_2^{\rm BCL}$		0.9(3.9)	
$\mathscr{F}^{B\to D^*}(1)$		0.9017(51)(156)	
$R(D)$		0.316(12)(7)	

Kenneth Wilson
1936-2013
(1982 Nobel Laureate and father of lattice QCD)

- We have now reached the era of $O(1\%)$ precision for many important quantities.
  - Improvement in precision will continue.
  - Isospin breaking effects, including electromagnetism, will increasingly be included.
  - There will be a major expansion of results in heavy-quark physics in the near future, generating competition among different approaches.
  - Technology for calculating *disconnected diagrams*, necessary e.g. for flavour singlets, will be fully developed.
- The range of computed quantities will continue to be extended (e.g. ε'/ε, rare-kaon decay amplitudes, $\cdots$.)
- Lattice QCD has matured into the quantitative ab-initio method for computing non-perturbative strong-interaction effects in an increasing range of processes.

FNAL/MILC Collaboration have presented the first calculation of $R(D)$ which requires the evaluation of the scalar form factor:

$$R(D) = \frac{B(B \to D\tau\nu_\tau)}{B(B \to D\ell\nu_\ell)} = 0.316(12)(7)$$

arXiv:1206.4992

to be compared to the BaBar result

$$R(D) = \frac{B(B \to D\tau\nu_\tau)}{B(B \to D\ell\nu_\ell)} = 0.440(58)(42)\,.$$

arXiv:1205.5442

- The CKMfitter and UTfit collaborations perform global unitarity triangle analyses.
- There is enough data to remove a quantity from the inputs and to make a prediction for the missing quantity e.g.

Observable	Input value	SM prediction	Pull
$\varepsilon_K \times 10^3$	2.23 ± 0.01	1.96 ± 0.20	1.4
Δm_s ps^{-1}	17.69 ± 0.08	18.0 ± 1.3	< 1
$\lvert V_{cb}\rvert \times 10^3$	41.0 ± 1.0	42.3 ± 0.9	< 1
$\lvert V_{ub}\rvert \times 10^3$	3.82 ± 0.56	3.62 ± 0.14	< 1
$\mathrm{Br}(\mathrm{B}\tau\nu) \times 10^4$	1.67 ± 0.30	0.82 ± 0.08	2.7
$\sin 2\beta$	0.68 ± 0.02	0.81 ± 0.05	2.4
α	$91^\circ \pm 6^\circ$	$88^\circ \pm 4^\circ$	< 1
γ	$76^\circ \pm 11^\circ$	$68^\circ \pm 3^\circ$	< 1

C.Tarantino, "Lattice flavor physics with an eye to SuperB,"
PoS LATTICE **2012**, 012 (2012).