QCD at the LHC



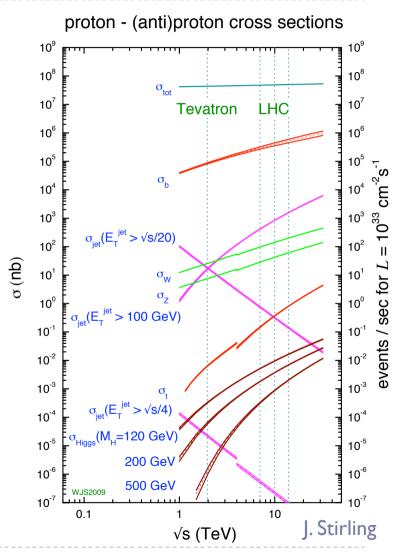
Thomas Gehrmann

Universität Zürich

Annual Theory Meeting, Durham, December 2013

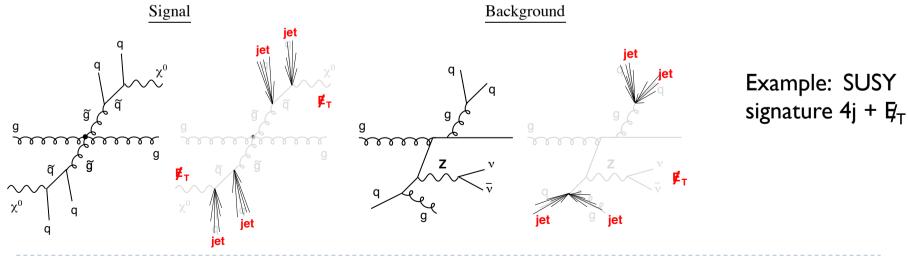
Benchmark processes at LHC

- Large production rates for Standard Model processes
 - jets
 - top quark pairs
 - vector bosons
- Allow precision measurements
 - masses
 - couplings
 - parton distributions
- Require precise theory



Multi-particle production at LHC

- ▶ LHC brings new frontiers in energy and luminosity
- ▶ Production of short-lived heavy states (Higgs, top, SUSY...)
 - detected through their decay products
- ▶ Search for new effects in multi-particle final states
- Need precise predictions for hard scattering processes

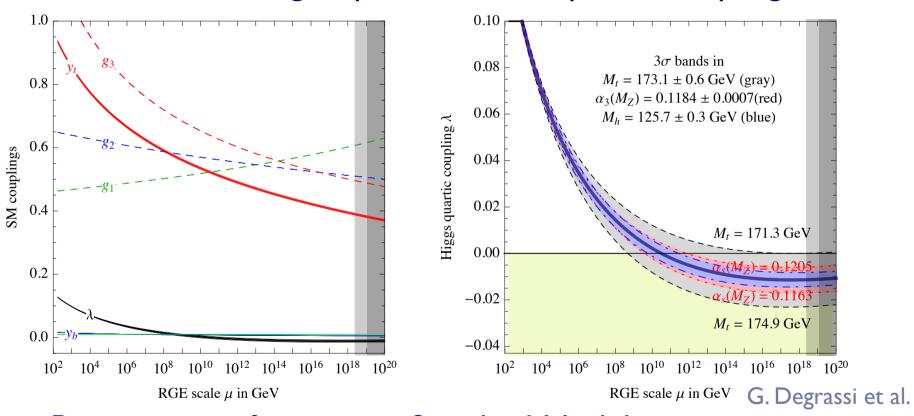


The case for precision

- ▶ Implications of Higgs boson discovery at ATLAS and CMS
 - Higgs mechanism established
 - Higgs boson mass measured: m_H = 125.7 ± 0.3 GeV
 - Standard Model of particle physics complete
- Beyond the Standard Model
 - Planck mass sets fundamental limit: M_p ≈ 10¹⁹ GeV
 - Internal consistency of Standard Model
 - Hierarchy problem
 - Extrapolation to high energies
- Stability of the Higgs potential

Stability of the Higgs potential

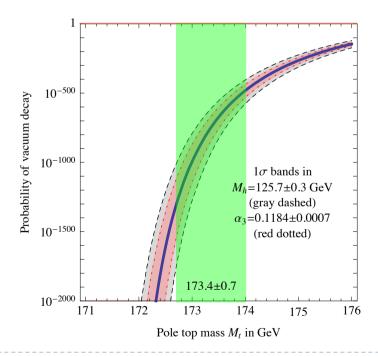
▶ Renormalization group evolution of quartic coupling

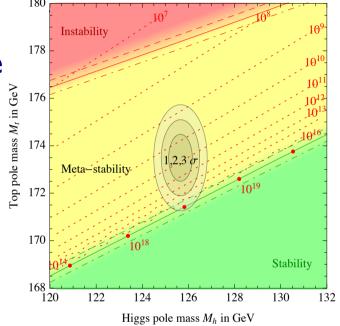


Propagation of errors on Standard Model parameters

Stability of the Higgs potential

- Determines vacuum stability
- Current data indicate metastable state
- Precision on parameters and for RGE evolution and matching crucial





F. Bezrukov, M. Kalmykov, B. Kniehl, M. Shaposhnikov;
J. Elias-Miro et al, G. Degrassi et al.;
F. Jegerlehner

QCD: precision physics at LHC

- ▶ NLO: methods, results, directions
- ▶ Parton showers, resummation, matching
- ▶ NNLO: precision QCD
- ▶ Interpreting LHC data
- Precision frontier: aims and ideas

▶ NLO: methods, results, directions

NLO multi-particle production

▶ Why NLO?

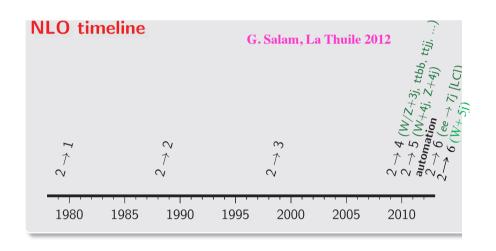
- reduce scale uncertainty of LO theory prediction
- reliable normalization and shape
- accounts for effects of extra radiation
- jet algorithm dependence

Typical observations

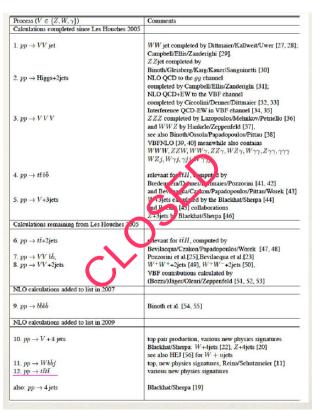
- sizable NLO corrections
- corrections not constant, but kinematics-dependent
- remaining uncertainty at NLO typically 10-20%

NLO multi-parton production

► Enormous progress in getting NLO predictions for $2\rightarrow (4,5,6!)$ processes over the last years



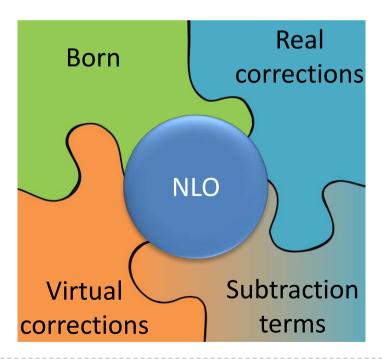
- Made possible by
 - Improved techniques for loop amplitudes
 - Crucial: a high level of automation



K. Melnikov, MITP, 2013

NLO automation

- Well-defined interfaces (Binoth Les Houches accord)
 - combine different ingredients from different codes
- One-loop amplitudes
 - BlackHat (Z. Bern, L. Dixon, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre, K. Ozeren)
 - ▶ GoSam (G. Cullen, N. Greiner, G. Heinrich, G. Luisoni, P. Mastrolia, G. Ossola, T. Reiter, F. Tramontano)
 - DenLoops (F. Cascioli, P. Maierhöfer, S. Pozzorini)
 - NJet (S. Badger, B. Biedermann, P. Uwer, V. Yundin)
 - ► MadLoop/aMC@NLO (R. Frederix et al.)
 - ► CutTools (G. Ossola, C. Papadopoulos, R. Pittau)
- Real radiation, subtraction terms and phase space (infrastructure)
 - ▶ Sherpa (F. Kraus et al.)
 - ► Madgraph/MadEvent (F. Maltoni et al.)
 - ► HelacNLO (G. Bevilacqua, C. Papadopoulos et al.)
 - ► MCFM (J. Campbell, K. Ellis, C. Williams)



Automation in NLO computations

Impressive list of results:

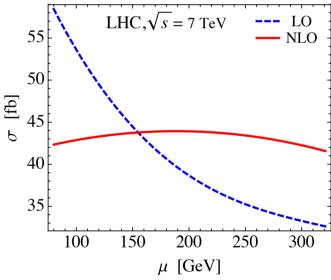
- multiple jets (up to 4) (Blackhat + Sherpa; Njet)
- gauge boson and up to 5 jets (Blackhat + Sherpa)
- two gauge bosons with up to 2 jets (T. Melia et al.; VBFNLO: F. Campanario, M. Kerner, L.D. Ninh, D. Zeppenfeld; GoSam + MadEvent)
- Three gauge bosons (VBFNLO: G. Bozzi, F. Campanario, C. Englert, M. Rauch, D. Zeppenfeld)
- Top quarks with jets (up to 2) (A. Denner, S. Dittmaier, S. Kallweit, S. Pozzorini; G. Bevilacqua, M. Czakon, C. Papadopoulos, M. Worek)
- Top quarks with a gauge boson (A. Lazopoulos, K. Melnikov, F. Petriello; K. Melnikov, M. Schulze, A. Scharf; HelacNLO: A. Kardos, Z. Trocsanyi, C. Papadopoulos; MCFM: J. Campbell, K. Ellis)
- Higgs with a top quark pair and one jet (GoSam + Sherpa + MadEvent: H. van Deurzen, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro)
- Higgs and up to 3 jets (GoSam + Sherpa + Madevent: G. Cullen, H. van Deurzen, N. Greiner, G. Luisoni, P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro, F. Tramontano)
- Address rich phenomenology with few examples

W⁺W⁻+2 jet production at NLO

- ▶ Background to BSM searches and for H→WW decay
- Interplay with electroweak vector boson fusion process (VBFNLO: D. Zeppenfeld et al.)
- Two NLO QCD calculations completed recently (T. Melia, K. Melnikov, R. Rontsch, G. Zanderighi; N. Greiner, G. Heinrich, P. Mastrolia, G. Ossola, T. Reiter, F. Tramontano)
- Including W-boson decays:

$$pp \rightarrow W^+(\rightarrow \nu_e e^+)W^-(\rightarrow \mu^-\bar{\nu}_\mu)jj$$

• Scale variation : Use $\mu = \mu_F = \mu_{R}$.

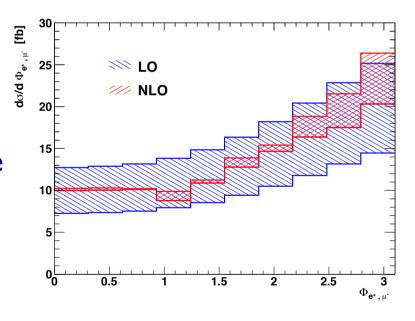


Observe: NLO corrections stabilize scale dependence

W⁺W⁻+2 jet production at NLO

- lacktriangle Distribution in the lepton opening angle Φ_{e^+,μ^-}
- Vary $\mu = \mu_F = \mu_R$ in $M_W < \mu < 4 M_W$

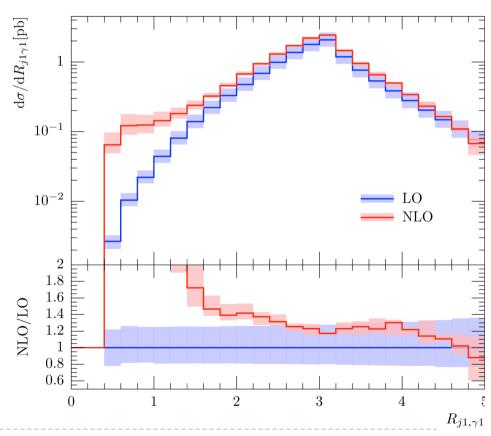
- NLO predictions within LO uncertainty band
- Relevant for designing cuts for the determination of HWW coupling
 - QCD process: peaked at π
 - Higgs signal: peaked at 0



YY + 2 jet production at NLO

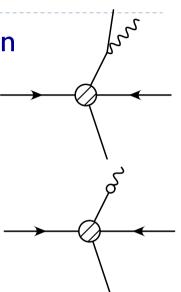
- Diphoton-plus-two-jet important Higgs background
- ► Currently determined from sideband data
- Insufficient for multiple differential measurements
- ► NLO: GoSam+MadEvent (G. Heinrich, N. Greiner, TG)
- ▶ Also: Blackhat+Sherpa
- Photon isolation:dynamical cone (S. Frixione)

$$E_{\text{had,max}}(r_{\gamma}) = \epsilon p_T^{\gamma} \left(\frac{1 - \cos r_{\gamma}}{1 - \cos R}\right)^n$$



Photon production mechanisms

- Direct process: photon produced in hard interaction
 - perturbatively calculable
 - collinear quark-photon contributions present
- Fragmentation of parton into photon:
 - described by a non-perturbative parton-to-photon fragmentation function
 - absorbs collinear singularities from direct process
 - requires non-perturbative input
- Fixed cone isolation (used in experiment)
 - both processes contribute
 - fragmentation contributions reduced but not eliminated
- Smooth cone isolation (preferred by theorists)
 - no collinear nor fragmentation contributions
- Ongoing discussion (Les Houches 2013)

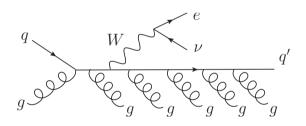


W+5 jets at NLO

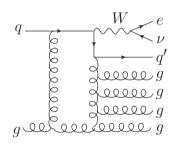
- ▶ First 2→6 NLO calculation at a hadron collider
- Using Blackhat + Sherpa

(Z. Bern, L. Dixon, F. Febres Cordero, S. Höche, H. Ita, D. Kosower, D. Maitre, K. Ozeren)

- Blackhat: virtual one-loop corrections using on-shell methods
- Sherpa: real emission, subtraction, phase space integration



Example diagram for real emission $(2\rightarrow 8)$ at tree level



Example diagram for virtual emission $(2\rightarrow7)$ at one-loop (octogon)

- Computation at the actual frontier of NLO complexity
 - Considered impossible until few years ago

W+5 jets at NLO

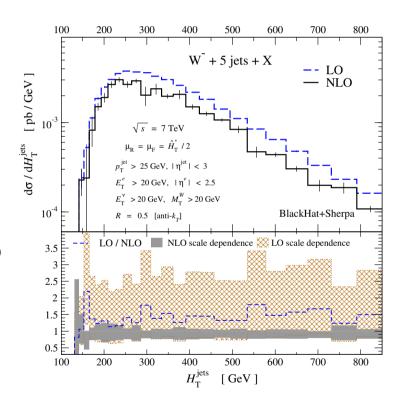
- ▶ Distribution in H_T^{jets} (sum of jet transverse energies)
 - Dynamical scale choice

$$\mu_R = \mu_F = \hat{H}_T'/2$$

$$\hat{H}_T' - \sum_{m} m + E^W$$

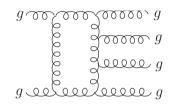
$$\hat{H}_{\mathrm{T}}' \equiv \sum_{m} p_{\mathrm{T}}^{m} + E_{\mathrm{T}}^{W}$$

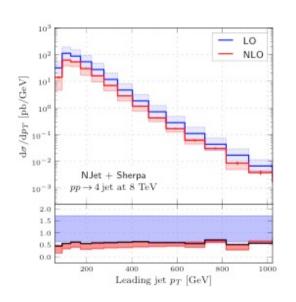
- > scale variation $\mu/2 \dots 2\mu$
- Observe:
 - Scale dependence reduced at NLO
 - ratio NLO/LO constant over full kinematical range
- NLO helps to motivate the scale choice



pp→4 jets at NLO

- ▶ Two calculations using on-shell methods for loop amplitudes
 - Blackhat+Sherpa (Z. Bern, L. Dixon, F. Febres Cordero,
 S. Höche, H. Ita, D. Kosower, D. Maitre, K. Ozeren)
 - NJET+Sherpa (S. Badger, B. Biedermann, P. Uwer, V. Yundin)



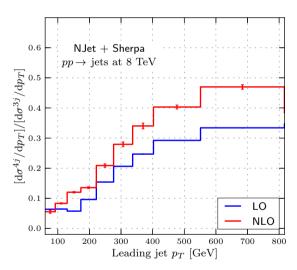


NLO prediction with central scale $\hat{H}_T/2$

Dynamical scale:

$$\mu_R = \mu_F = \mu = \hat{H}_T/2$$

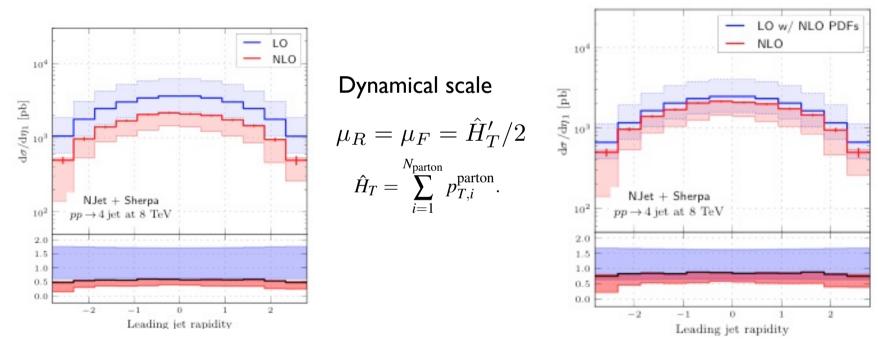
$$\hat{H}_T = \sum_{i=1}^{N_{\mathrm{parton}}} p_{T,i}^{\mathrm{parton}}.$$



4-to-3 jet ratio increases at NLO

pp →4 jets at NLO

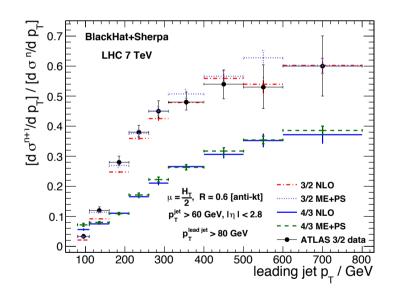
- ▶ To disentangle NLO effects from parton distributions and genuine NLO corrections from hard scattering process
 - Use NLO partons for both NLO and LO predictions



▶ LO with NLO partons closer to full NLO than pure LO

Jet ratios at NLO

- Systematic uncertainties (th. and exp.) cancel in ratios
 - Predictions more reliable
 - Can be used in data-driven background estimation
- Jet ratio as function of leading jet p_T
 - NLO and parton shower both agree with data for large p_T
 - Parton shower (multiple emission) better at low p_T
 - □ Large uncertainty on parton shower not shown



Observe: 3/2 ratio below the data at small p_T

▶ Parton showers, resummation, matching

Fixed order versus parton shower

Fixed order calculations

- Expansion in powers of the coupling constant
- Correctly describes hard radiation pattern
- Final states are described by single hard particles
- NLO: up to two particles in a jet, NNLO: up to three...
- Soft radiation poorly described

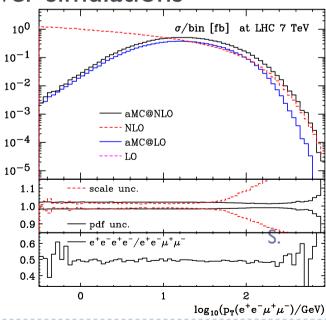
Parton shower

- Exponentiates multiple soft radiation (leading logarithms)
- Describes multi-particle dynamics and jet substructure
- Allows generation of full events (interface to hadronization)
- Basis of multi-purpose generators (SHERPA, HERWIG, PYTHIA)
- Fails to account for hard emissions
- ▶ Ideally: combine virtues of both approaches

Merging of fixed order and parton shower

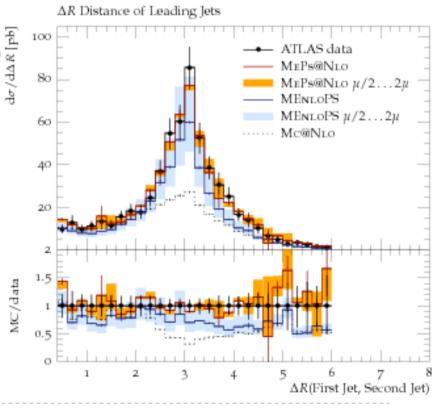
Merging multiplicities

- Combine fixed-order matrix elements at different multiplicity with vetoed shower
- Leading order prescriptions: CKKW (S. Catani, F. Krauss, R. Kuhn, B. Webber) and MLM (M. Mangano)
- Has become standard for parton shower simulations
- Merging NLO with parton shower
 - Combine fixed-multiplicity NLO calculation with parton shower
 - Accomplished for many processes (MC@NLO: S. Frixione, B. Webber; POWHEG: P. Nason, C. Oleari et al.)
 - ► Automation: aMC@NLO (R. Frederix, Frixione, V. Hirschi, F. Maltioni, R. Pittau, P. Torrielli)



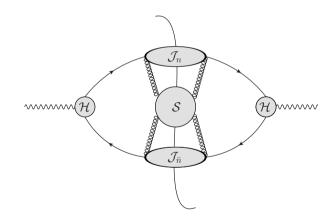
Merging of fixed order and parton shower

- ▶ Combining NLO computations for different multiplicities and interfacing with parton showers (proof-of-principle)
 - SHERPA (S. Höche, F. Krauss, M. Schönherr, F. Siegert)
 - MINLO (K. Hamilton, P. Nason, C. Oleari, G. Zanderighi)
 - **UNLOPS** (L. Lönnblad, S. Prestel)
 - **FxFx** (S. Frixione, R. Frederix)
- Yields combined event samples
- Improves especially jet-jet correlations
- Work in progress



Resummation

- Parton shower: leading logarithmic accuracy (LL)
- ▶ Resummation of higher-order logarithms
 - ▶ NLL: largely automated (CAESAR: A. Banfi, G. Salam, G. Zanderighi)
 - NNLL and beyond: process-by-process calculations
- Methods
 - Laplace-space resummation (CSS: J. Collins, D. Soper, G. Sterman)
 - Soft-collinear effective theory (SCET: C. Bauer, S. Fleming, D. Pirjol, I. Rothstein, I. Stewart; M. Beneke, A. Chapovsky, M. Diehl, T. Feldmann)
 - Systematic extension beyond NLL



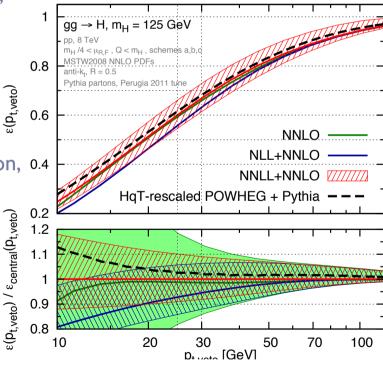
Resummation

▶ Recent NNLL results

- ► Higgs boson p_T distribution (D. de Florian, G. Ferrera, M. Grazzini, D. Tommasini; V. Ahrens, T. Becher, M. Neubert, L.L. Yang)
- Jet veto cross sections (A. Banfi, P.F. Monni, G. Salam, G. Zanderighi; T. Becher, M. Neubert)
- ▶ Jet-p_T distributions in Higgs events (I. Stewart, F. Tackmann, J. Walsh, S. Zuberi)
- P. Falgari, S. Klein, C. Schwinn, M. Cacciari, M. Czakon, 0.4 M. Mangano, A. Mitov, P. Nason, V. Ahrens et al.)

Impact of NNLL

- extended range of theory prediction
- reduction of scale uncertainty



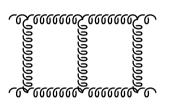
▶ NNLO: towards precision QCD

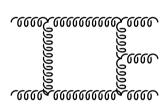
NNLO observables at hadron colliders

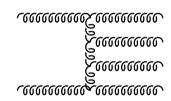
- NNLO predictions
 - expected to have a per-cent level accuracy
 - yielding first reliable estimate of theoretical uncertainty
- ▶ For processes measured to few per cent accuracy
 - jet production
 - vector boson (+jet) production
 - top quark pair production
- For processes with potentially large perturbative corrections
 - New channels and/or phase space regions open up
 - Higgs or vector boson production

NNLO calculations

- ▶ Require three principal ingredients (here: pp \rightarrow 2j)
 - two-loop matrix elements
 - explicit infrared poles from loop integral
 - known for all massless 2 → 2 processes
 - one-loop matrix elements
 - explicit infrared poles from loop integral
 - and implicit poles from single real emission
 - usually known from NLO calculations
 - tree-level matrix elements
 - implicit poles from double real emission
 - known from LO calculations
- Infrared poles cancel in the sum
- ▶ Challenge: combine contributions into parton-level generator
 - Need a method to extract implicit infrared poles







Real radiation at NNLO: methods

Sector decomposition

(T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello)

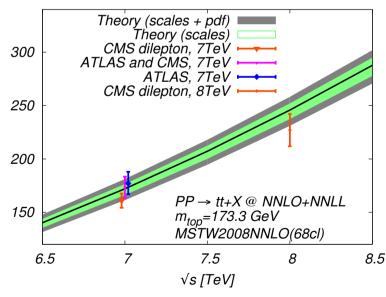
- ▶ pp → H, pp → V, including decays (C.Anastasiou, K. Melnikov, F. Petriello; S. Bühler, F. Herzog, A. Lazopoulos, R. Müller)
- Sector-improved subtraction schemes

(M. Czakon; R. Boughezal, K. Melinkov, F. Petriello)

- ▶ pp → tt (M. Czakon, P. Fiedler, A. Mitov)
- ightharpoonup pp ightharpoonup (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
- ▶ q_T-subtraction (S. Catani, M. Grazzini)
 - ▶ pp \rightarrow H, pp \rightarrow V, pp \rightarrow γ γ , pp \rightarrow VH (S. Catani, L. Cieri, D. de Florian, G. Ferrera M. Grazzini, F.Tramontano)
- ▶ Antenna subtraction (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
 - $ightharpoonup e^+e^-
 ightarrow 3j$ (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG; S. Weinzierl)
 - ▶ pp → 2j (A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)

Top quark pair production at LHC

- ▶ Large production cross section at the LHC (~250pb at 8TeV)
 - lacktriangle Expected experimental error of ~5% for $\sigma_{tar{t}}$
 - NLO+NLL predictions yield an uncertainty of ~10%
- NNLO accuracy of theory needed
- ► Calculation for the total cross section completed (M. Czakon, P. Fiedler, A. Mitov)
 - From a purely numerical code
 - based on sector-improved subtraction
 - numerical cancellation of infrared poles
 - Observe: theoretical and experimental uncertainties comparable (% level)
- Differential distributions in progress



Higgs+jet production at the LHC

- Essential to establish the properties of the newly discovered Higgs boson
- ▶ Experiments select events according to number of jets
 - Different backgrounds for different jet multiplicities
 - ► H+0jet and inclusive H production known at NNLO (C.Anastasiou, K. Melnikov, F. Petriello; S.Catani, M. Grazini)
 - ▶ H+Ijet and H+2jet known at NLO
 - ▶ H+0jet and H+1jet samples of comparable sizes
- NNLO for H+ljet needed
 - luons-only total cross section completed (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
 - Full calculation and differential distributions in progress

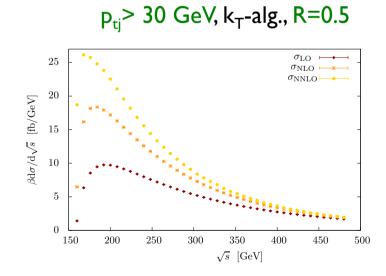
Higgs+jet production at NNLO

- First results for H+jet total cross section (gluons only) (R. Boughezal, F. Caola, K. Melnikov, F. Petriello, M. Schulze)
 - using a purely numerical code
 - Based on sector-improved subtraction
 - numerical cancellation of infrared singularities
 - cross section multiplied by gluon luminosity

$$\beta \frac{\mathrm{d}\sigma_{\mathrm{had}}}{\mathrm{d}\sqrt{s}} = \beta \frac{\mathrm{d}\sigma(s, \alpha_s, \mu_R, \mu_F)}{d\sqrt{s}} \times \mathcal{L}\left(\frac{s}{s_{\mathrm{had}}}, \mu_F\right),$$

with
$$eta=\sqrt{1-rac{E_{th}^2}{s}}, \quad E_{th}pprox 158 GeV$$

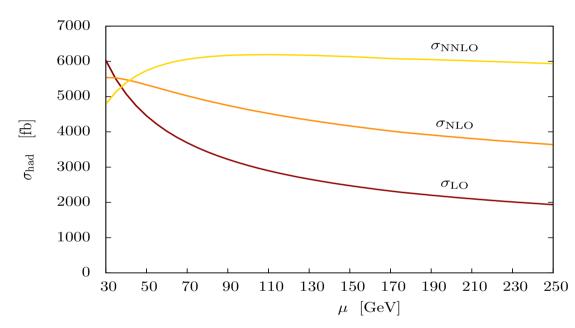
 Observe large NNLO effects close to partonic threshold region



Higgs+jet production at NNLO

Scale dependence of the integrated total cross section

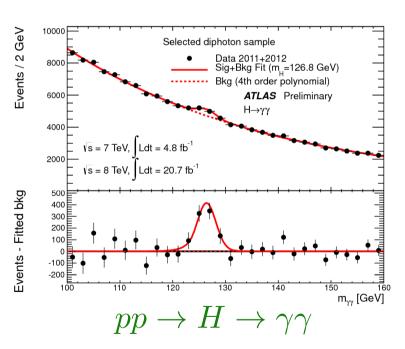
 $\mu = \mu_F = \mu_R$

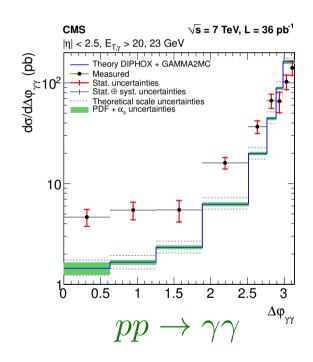


- Considerable stabilization at NNLO
- ▶ Corrections smallest for $\mu = M_H/2$ as in inclusive case

Di-photon production at the LHC

- ▶ Di-photon production: irreducible background for H $\rightarrow \gamma \gamma$
 - > at present determined from sideband data fits
- Discrepancy between NLO theory and data in some distributions





Require precise theoretical predictions (NNLO)

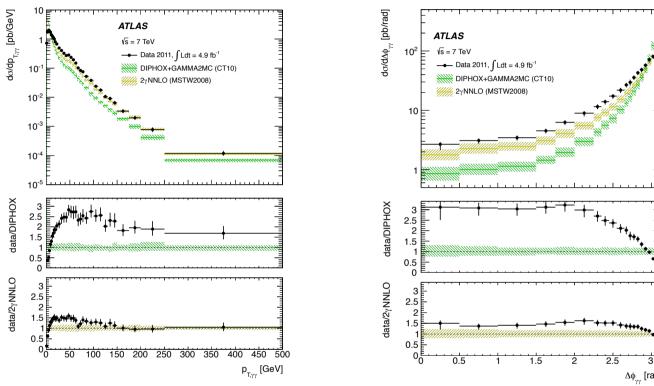
Di-photon production at the LHC

- New NNLO calculation: 2γNNLO
 - (S. Catani, L. Cieri, D. de Florian, G. Ferrera, M. Grazzini)
 - parton-level event generator, based on q_T-subtraction
 - ☐ Analytic cancellation of infrared poles
 - using a smooth isolation criterion to define photons
 - ▶ includes all $O(\alpha_s^2)$ corrections to direct photon production pp $\rightarrow \gamma \gamma$
 - First fully consistent inclusion of the Box contribution nn



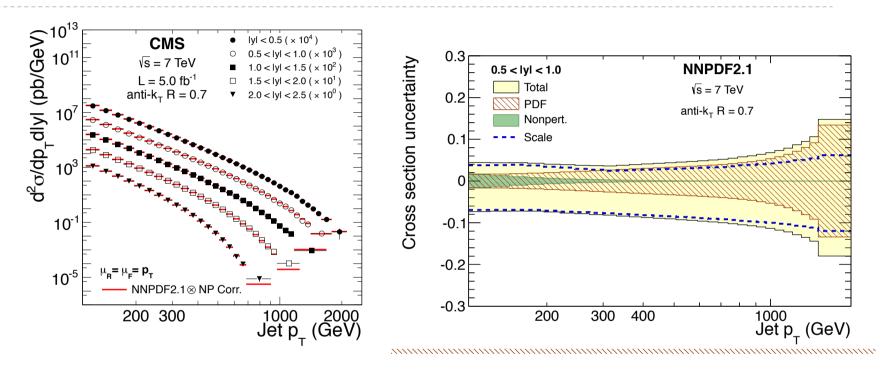
- comparable size to $O(\alpha_s^0)$, qq luminosity $O(\alpha_s^2)$, gluon luminosity
 - Box also included in NLO-type codes (DIPHOX+gamma2MC, MCFM) (T. Binoth, J.P. Guillet, E. Pilon, M. Werlen; Z. Bern, L. Dixon, C. Schmidt; J. Campbell et al.)

ATLAS di-photon results



- Inclusion of NNLO corrections resolves discrepancy between NLO-type prediction and data
 - Despite the use of slightly different cone isolation criteria

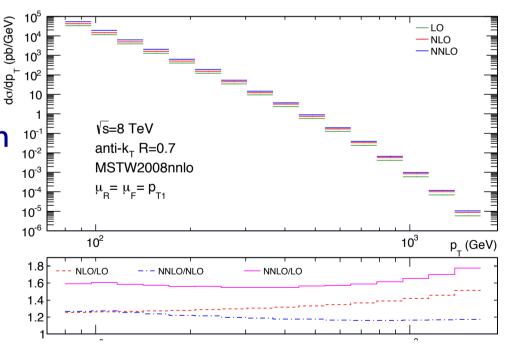
Jet cross sections at LHC



- Jet data can be used to constrain parton distributions
- Scale and PDF uncertainties on NLO prediction of comparable size
- Need improved theory (NNLO)

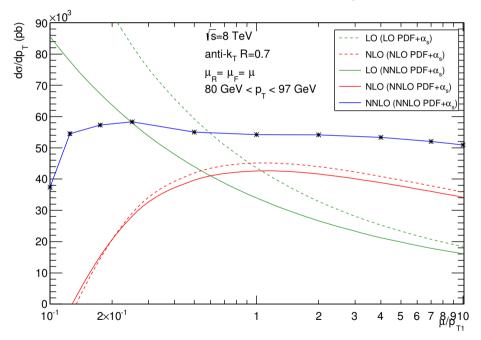
pp → 2jets at NNLO

- ▶ First results at NNLO available
 - ▶ gg → gg subprocess
 (J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
 - Developed a new parton-level event generator NNLOJET
 - using antenna subtraction
 - analytic cancellation of infrared poles
- ▶ Inclusive jet p_T distribution
 - NNLO/NLO differential K-factor flat over the whole p_T range



pp → 2jets at NNLO

- ▶ Inclusive jet p_T distribution: scale dependence (gluons only)
 - (J. Currie, A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
 - Dynamical scale choice: leading jet p_T

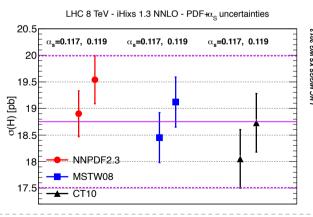


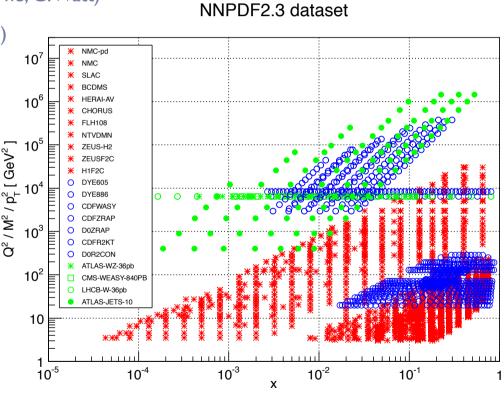
Stabilization at NNLO

▶ Precision physics with LHC data

Parton distributions from the LHC

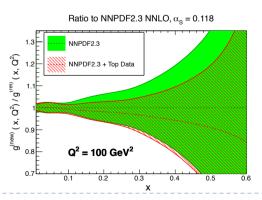
- ▶ Parton distributions determined from global fit to data
 - MSTW (A. Martin, J. Stirling, R. Thorne, G. Watt)
 - ▶ ABM (S. Alekhin, J. Blümlein, S. Moch)
 - ► CTEQ (J. Huston et al.)
 - NNPDF (R. Ball, S. Forte et al.)
 - ▶ JR (P. Jimenez-Delgado, E. Reya)
- Enter all predictions of cross sections

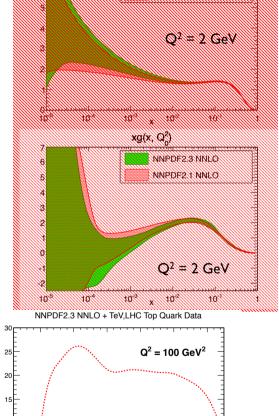




Parton distributions from the LHC

- ▶ LHC data starting to impact (NNPDF)
 - ▶ High-E_T jets
 - Vector bosons
 - Low-x at LHCb
- ▶ Top cross section
 - Dominated by gluon fusion
 - ► Total cross section at NNLO included in fit (M. Czakon, M. Mangano, A. Mitov, J. Rojo)
 - Improved knowledge at large x





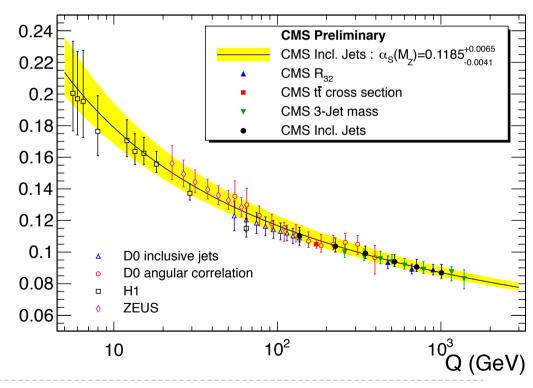
KECK COS)

NNPDF23 NNLO NNPDF21 NNLO

Reduction of PDF error (%)

Strong coupling constant at the LHC

- Inclusive jet production
 - \triangleright Correlation between α , and gluon distribution
- Jet ratios
 - ▶ R_{3/2} particularly sensitive
- Event shapes
 - Classical observable at e⁺e⁻ colliders
 - Under development for hadron colliders,
 e.g. 3-jet mass
- Precision theory-limited
 - NNLO needed



▶ Precision frontier: aims and ideas

Towards NNLO automation

- ▶ Methods for real radiation at NNLO becoming mature
 - ▶ q_T subtraction
 - Sector-improved schemes
 - Antenna subtraction
- Issues
 - Automation of code generation
 - Numerical efficiency and stability

Towards NNLO automation

- Virtual two-loop amplitudes: analytically process-by-process
 - Current stockpile
 - ightharpoonup pp ightharpoonup (C. Anastasiou, N. Glover, C. Oleari, M. Tejeda-Yeomans; Z. Bern, L. Dixon, A. De Freitas)
 - ightharpoonup pp ightharpoonup (L. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG)
 - ▶ pp \rightarrow V+ γ (L.Tancredi, E.Weihs, TG)
 - ▶ $pp \rightarrow H+j$ (N. Glover, M. Jaquier, A. Koukoutsakis, TG)
 - ▶ pp → tt (P. Bärnreuther, M. Czakon, P. Fiedler; R. Bonciani, A. Ferroglia, A. von Manteuffel, C. Studerus, TG)
 - In progress
 - ▶ pp → VV (L. Tancredi, E. Weihs, TG; J. Henn, V. Smirnov)
- ▶ Research directions: towards different masses and $2 \rightarrow 3$
 - ▶ Semi-numerical approaches (P. Bärnreuther, M. Czakon, P. Fiedler)
 - Classification of integral basis (H. Johansson, D. Kosower, K. Larsen)
 - Unitarity-based methods (P. Mastrolia, E. Mirabella, G. Ossola, T. Peraro)

NNLO and beyond: techniques

- Seemingly simple task: check equality of two expressions
 - Becomes very tricky if complicated functions involved
 - e.g. Abel relation (1855)

$$\ln(1-x)\ln(1-y) = \text{Li}_2\left(\frac{x}{1-y}\right) + \text{Li}_2\left(\frac{y}{1-x}\right) - \text{Li}_2(x) - \text{Li}_2(y) - \text{Li}_2\left(\frac{xy}{(1-x)(1-y)}\right)$$

- Systematic procedure for iterated rational integrals
 - Symbol and coproduct (A. Goncharov, M. Spradlin, A. Volovich, C. Vergu; C. Duhr)
 - ▶ Often allows huge simplifications (many pages → few lines)
- starts to get used for loop integrals
 - simplification
 - analytical continuation
 - automated derivation of relations

Beyond NNLO: observables

- ► Hadronic R-ratio in e⁺e⁻
 - Most precise QCD observable in Z and τ decays
 - **Known to O**(α_s^4) (P. Baikov, K. Chetyrkin, H. Kühn, J. Rittinger)
 - Produces most precise $\alpha_s(M_Z) = 0.1198 \pm 0.0015$
- Gluon-fusion Higgs cross section at hadron colliders
 - Large NLO and NNLO corrections
 - ▶ Ultimate precision on Higgs couplings may require N³LO
 - Ingredients
 - ► Three-loop vertex functions (P. Baikov, K. Chetyrkin, A. Smirnov, V. Smirnov, M. Steinhauser; N. Glover, T. Huber, N. Ikizlerli, C. Studerus, TG)
 - Counterterms and lower-order expansions (C.Anastasiou, S. Bühler, C. Duhr, F. Herzog; M. Höschele, J. Hoff, A. Pak, M. Steinhauser, T. Ueda)
 - ▶ Triple real radiation (C.Anastasiou, C. Duhr, F. Dulat, B. Mistlberger)
 - ▶ Interplay of real and virtual corrections at N³LO (C. Duhr et al.)
 - Major work in progress

Instead of a summary: Outlook

Where do we stand?

Witnessed an NLO revolution

- Previously unthinkable NLO multi-particle calculations now feasible due to technological breakthroughs
- High-level of automation
- Standarization of interfaces: combine different codes (providers)
- Interface to experiment (codes, ntuples, histograms,..)?

NLO and parton showers

Matching of individual processes (MC@NLO, POWHEG)

Substantial progress on NNLO calculations

- Several different methods available
- Calculations on process-by-process basis
- Codes typically require HPC infrastructure

Future Directions

- NLO+PS as new standard for event generation
 - Fully automated public codes
 - Consistent matching to parton shower
 - Matching of different multiplicities at NLO
 - Monte Carlo with NLO-accurate event samples
- NNLO automation
 - Uncover analytical structures to organize calculation of real and virtual corrections
 - Develop standard interfaces
 - Interface to experiment ?
- Beyond NNLO
 - ▶ N³LO precision for benchmark processes

- Progress on precision physics on many frontiers
- ▶ Be prepared for exciting times ahead with the LHC

