Recent developments in top physics at hadron colliders

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Based on many papers with:

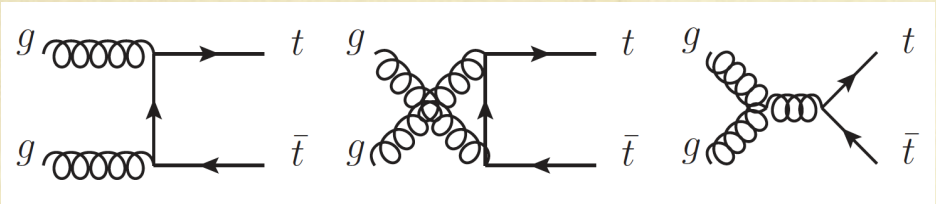
Barnreuther, Cacciari, Czakon, Fiedler, Mangano, Nason, Rojo, Sterman, Sung

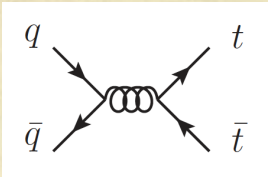
Content of the talk

- Few words about the historic developments
- Why is top production of interest (pheno)?
- How hard of a problem top production is?
  - Analytical properties
  - IR singularities
  - Gauge theory amplitudes
- Computing the NNLO: the methods.
- Precision applications at the LHC: what do we learn about SM and bSM?
- Outlook: the future of precision phenomenology.

Introduction to top production

In this talk I'll consider the process of top-pair production at hadron colliders



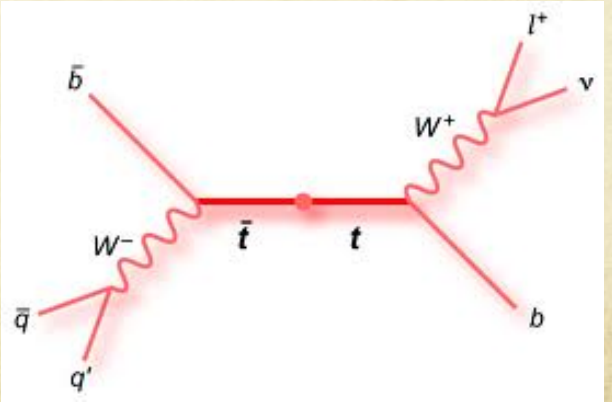


- The contributing partonic channels, and their relative contribution at LHC/Tevatron:

	TeVatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
gg	15.4%	84.8%	86.2%	90.2%
$qg + \bar{q}g$	-1.7%	-1.6%	-1.1%	0.5%
qq	86.3%	16.8%	14.9%	9.3%

- Top quarks decay very fast, so we never observe them directly. <u>They do not form bound states</u>.

- Will ignore their decay in this talk, and will consider them as stable particles (as if they are reconstructed in each event from their decay products – not true in reality).



In this talk I'll focus exclusively on the total inclusive x-section:

NOTE: differential distributions are well understood at NLO.
The total x-section is the first step into NNLO.

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\rm max}} d\beta\, \Phi_{ij}(\beta, \mu_F^2)\, \hat{\sigma}_{ij}(\beta, m^2, \mu_F^2, \mu_R^2)$$

Partonic fluxes
(derived from PDF's)

$$\Phi_{ij}(\beta, \mu_F^2) = \frac{2\beta}{1-\beta^2}\, \mathcal{L}_{ij}\left(\frac{1-\beta_{\rm max}^2}{1-\beta^2}, \mu_F^2\right)$$

$$\mathcal{L}_{ij}(x, \mu_F^2) = x\,(f_i \otimes f_j)\,(x, \mu_F^2)$$

Partonic x-section
(perturbative)

$$\hat{\sigma}_{ij}(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma_{ij}^{(0)} + \alpha_S \sigma_{ij}^{(1)} + \alpha_S^2 \sigma_{ij}^{(2)} + \mathcal{O}(\alpha_S^3)\right)$$

The partonic x-section depends on a single variable

$$\beta = \sqrt{1-\rho}, \text{ with } \rho \equiv 4m^2/s$$

$$0 < \rho \leq 1$$

- ✓ Point β = 0 (absolute threshold)
- ✓ Point β = 1 (high energy limit, i.e. m=0)

Historic prospective

- ✓ Early NLO QCD results (inclusive, semi-inclusive)

  Nason, Dawson, Ellis '88
  Beenakker et al '89

- ✓ Nowadays: *the industry* of the NLO revolution, thanks to advances in NLO technology

  Bern, Dixon, Dunbar, Kosower `94
  Britto, Cachazo, Feng `04
  Ossola, Papadopoulos, Pittau `07
  Giele, Kunszt, Melnikov `08
  aMC@NLO

- ✓ Complete understanding at NLO:

  Bernreuther, Brandenburg, Si, Uwer
  Melnikov, Schulze
  Bevilacqua, Czakon, van Hameren, Papadopoulos, Wore
  Denner, Dittmaier, Kallweit, Pozzorini

- ✓ 1990's: the rise of the soft gluon resummation at NLL

  Kidonakis, Sterman '97
  Bonciani, Catani, Mangano, Nason `98

- ✓ NNLL resummation developed (and approximate NNLO approaches)

  Beneke, Falgari, Schwinn '09
  Czakon, Mitov, Sterman `09
  Beneke, Czakon, Falgari, Mitov, Schwinn `09
  Ahrens, Ferroglia, Neubert, Pecjak, Yang `10-`11

- ✓ Electroweak effects at NLO known (small ~ 1.5%)

  Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth `93
  Hollik, Kollar `07
  Bernreuther, Fuecker, Si '05
  Kuhn, Scharf, Uwer '07

Main features of top-pair production

Top-pair production is completely understood within NLO/NNLL QCD

Main features:

- ✓ Large NLO QCD corrections
- ✓ Total theory uncertainty at (NLO+resummation)~10%
- ✓ Important for Higgs and bSM physics (M. Peskin: "*BSM Hides beneath Top*")
- ✓ Experimental improvements down to 5% (at LHC)
- ✓ Current LHC data agrees well with SM theory
- ✓ Tevatron data generally agrees too.

The notable exception: Forward-backward asymmetry from Tevatron.

Conclusion: "further scrutiny is needed"

Calculation of the total inclusive x-section tT @ NNLO during the last year

- Published qQ → tt +X

  Bärnreuther, Czakon, Mitov `12

- Published all fermionic reactions (qq,qq',qQ')

  Czakon, Mitov `12

- Published gq

  Czakon, Mitov `12

- Published gg

  Czakon, Fiedler, Mitov '13

Now the top pair total x-section is known numerically at NNLO in QCD

No (other) approximations of any kind

- First hadron collider calculation at NNLO with more than 2 colored partons.
- First NNLO hadron collider calculation with massive fermions.

- How to appreciate the complexity of the process?
- Let's look at the NLO result which is analytically known

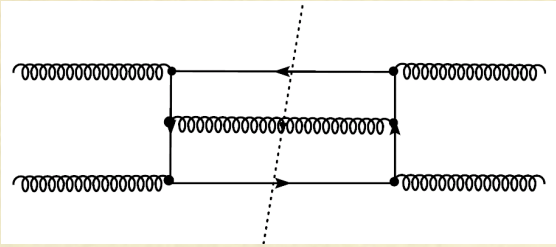
Based on: Czakon, Mitov arXiv:0811.4119

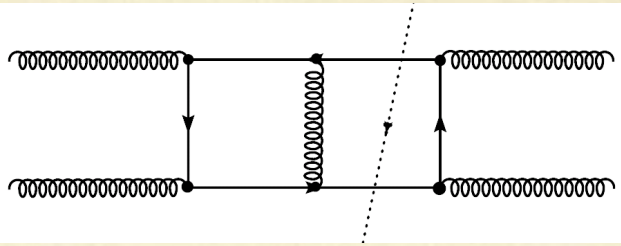
Recall, the NLO x-section first computed numerically

Nason, Dawson, Ellis '88
Beenakker, Kuijf, van Neerven, Smith, '89
Bernreuther, Brandenburg, Si, Uwer '04

Our strategy for the analytic computation:

- Treat Real and Virtual integrations on equal footing — Anastasiou, Melnikov '01
- Use IBP identities — Chetirkyn, Tkachov '81; Laporta '01





+ crossed

The NLO x-section has, approximately, the complexity of a 2-loop massive box

Our approach (it was a good approach):

- identify the possible <u>physical</u> singularities. There are 3 of them:
  - ✓ $m^2 \to 0$ (physical endpoint singularity),
  - ✓ $4m^2=s$ (physical endpoint singularity – partonic threshold),
  - ✓ $|m| \to \infty$ (unphysical singularity).
- change variables to map them to x=(-1,0,1)

$$\frac{m^2}{s} = \frac{x}{(1+x)^2} \qquad x = \frac{1-\sqrt{1-4\frac{m^2}{s}}}{1+\sqrt{1-4\frac{m^2}{s}}}$$

- one expects HPL's only.

- ✓ The whole x-section is mapped into 37 master integrals (real+virtual),
- ✓ We observe unexpected thing:
  - ➢ Few of the most complicated integrals (cross-box like) have additional singularities ("pseudothresholds")
- ✓ Their presence is expected in scattering amplitudes; but we have here a physical cross-section.
- ✓ We see them as additional singularities in the differential equations of the master integrals in the following points.

$s = m^2$; $s = - m^2$; $s = - 4m^2$; $s = -16m^2$
(in addition to $s = 4m^2$ and $m^2=0$).

- ✓ They are outside the physical region, so no numerical problems,
- ✓ The problem is technical: no pure HPL solutions.

✓ The results for the qq and gq reactions in terms of simple polylogs

✓ The gg reaction involves 4 special functions

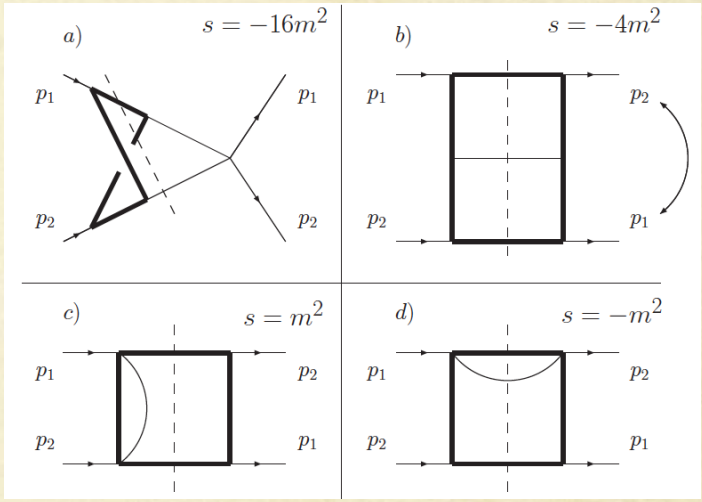
$$
\begin{aligned}
F_1(x) &= -\int_x^1 dz\, \frac{(2z+1)\,(H(-1,0,z)+H(0,-1,z)-H(0,0,z))}{2\,(z^2+z+1)} \\
F_2(x) &= -\int_x^1 dz\, \frac{(2z+3)\,\left(12\,H(-1,0,z)-6\,H(0,0,z)+\pi^2\right)}{4\,(z^2+3z+1)}\,, \\
F_3(x) &= +\int_x^1 dz\, \frac{5(z-1)\,\left(12\,H(-1,0,z)-6\,H(0,0,z)+\pi^2\right)}{8z\sqrt{z^2+6z+1}}\,.
\end{aligned}
$$

$$F_4(x) = \int_\rho^1 d\tau\, I_4(\rho,\tau)$$

$$
\begin{aligned}
I_4(\rho,\tau) = \frac{45\rho}{32\pi\tau}\log\left(\frac{1-\sqrt{1-\tau}}{1+\sqrt{1-\tau}}\right)\Bigg(&\frac{\left((\rho^2+1)\,K(\sqrt{-4\rho})-(\rho-1)E(\sqrt{-4\rho})\right)K\left(\frac{1}{\sqrt{4\tau+1}}\right)}{\sqrt{4\tau+1}} \\
&+\frac{\left((-4\rho^2+3\rho+1)\,E\left(\frac{1}{\sqrt{4\rho+1}}\right)+(3\rho^2-3\rho-2)\,K\left(\frac{1}{\sqrt{4\rho+1}}\right)\right)K(\sqrt{-4\tau})}{\sqrt{4\rho+1}}\Bigg)\,.
\end{aligned}
$$

Elliptic functions of I and II kind

- The structure of the solution is such that it does not allow iterative solution.
- Clear example where it is important to know what the class of solutions is
- Reached beyond where the symbols are useful?
- I am unaware of other example of observable with such unphysical singularities.



Our conclusion: pursue a numerical approach for NNLO

Before the exact NNLO was computed, we knew:

- NNLO in threshold region and soft-gluon resummation at NNLL
- singularities of massive 2-loop gauge theory amplitudes

Soft-gluon resummation at hadron colliders
(and top production in particular)

What is soft-gluon resummation?

- ✓ The effect is mostly driven by kinematics:

Sterman '87
Catani, Trentadue '89

  - ✓ the system is in a corner of phase space where only soft gluons can be emitted
  - ✓ multiple emissions from semi-classical (eikonal) partons
  - ✓ Low scales -> large coupling.
  - ✓ Soft resummation is an alternative expansion not in "fixed coupling" but in "fixed Log"

- ✓ "Easy" for "standard" processes: Higgs, Drell-Yan, DIS, e^+e^-

Key: the number of hard colored partons < 4

- ✓ Harder for top production (there are color correlations for n>=4)

Non-trivial color algebra in this case.

- ✓ NLL resummation for top developed
  - ✓ For total inclusive
  - ✓ For differential

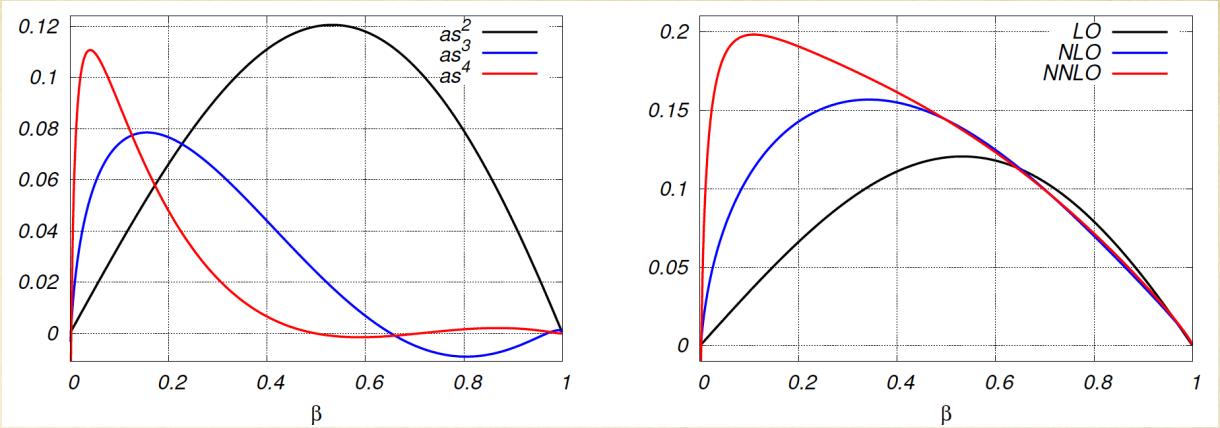
Bonciani, Catani, Mangano, Nason `98
Sterman, Kidonakis, Oderda `96-`98

"Patch" an observable in any kinematical region where usual perturbative expansion breaks down

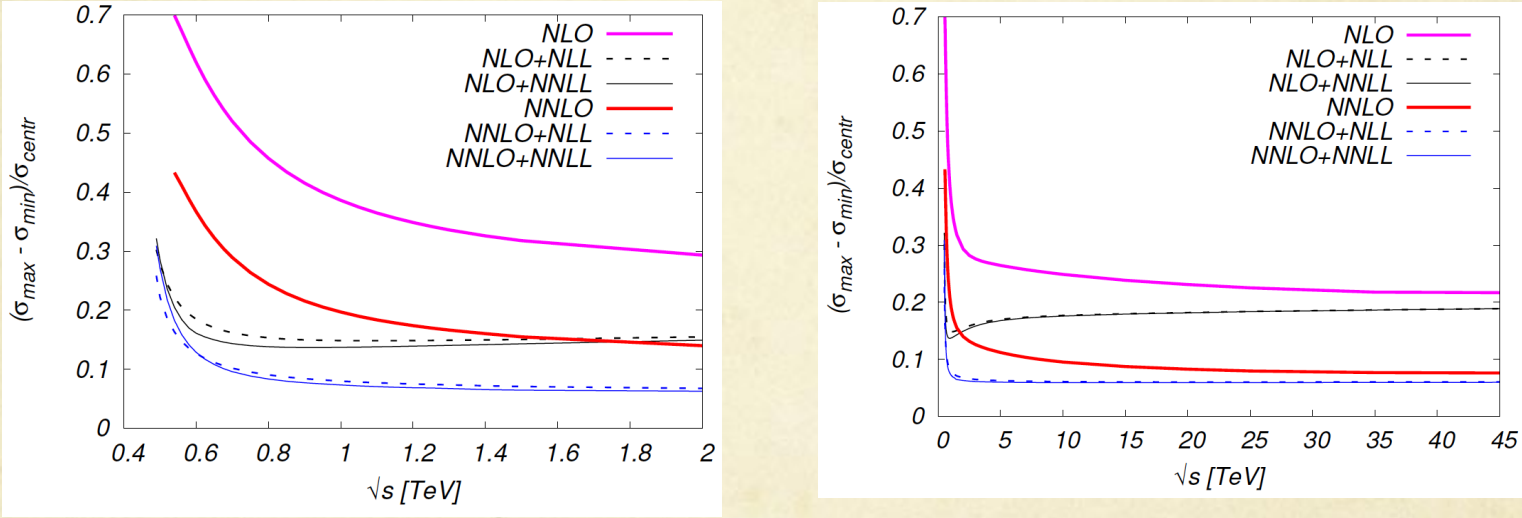
Soft-gluon resummation: an example

Partonic x-section's growth close to threshold (qq reaction):

The expansion there is not converging
Resummation needed



$$\hat{\sigma}(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma^{(0)} + \alpha_S\sigma^{(1)} + \alpha_S^2\sigma^{(2)} + \ldots\right) \equiv \frac{\alpha_S^2}{m^2}\left(f_{\alpha_S^2} + f_{\alpha_S^3} + f_{\alpha_S^4} + \ldots\right)$$



Update of: Cacciari, Czakon, Mangano, Mitov, Nason '11

The resummed results are better close to threshold, as expected.

The top cross-section: NNLL resummation

Factorization of the partonic cross-section close to threshold:

Kidonakis, Sterman '97
Czakon, Mitov, Sterman '09

$$\omega_P\left(N,\hat{\eta},\frac{M^2}{\mu^2},\frac{m^2}{\mu^2},\alpha_s(\mu^2)\right) = J_1(N,\alpha_s(\mu^2))\ldots J_k(N,M/\mu,m/\mu,\alpha_s(\mu^2))$$
$$\times\,\mathrm{Tr}\left[\mathbf{H}^P\left(\frac{M^2}{\mu^2},\frac{m^2}{\mu^2},\alpha_s(\mu^2)\right)\mathbf{S}^P\left(\frac{N^2\mu^2}{M^2},\frac{M^2}{m^2},\alpha_s(\mu^2)\right)\right]+\mathcal{O}(1/N)$$

N – the usual Mellin dual to the kinematical variable that defines the threshold kinematics:

$$\sigma(N) = \int_0^1 dz z^{N-1}\sigma(z)$$

$z = Q^2/s$ ← Drell-Yan

$z = 4m^2/s$ ← t-tbar total X-section

$z = M^2_{t\bar{t}}/s$ ← t-tbar – pair invariant mass

J's – jet functions (different from the ones in amplitudes)

S,H – Soft/Hard functions. Also different.

The top cross-section: NNLL resummation

Here is the result for the Soft function:

$$
\begin{aligned}
\mathbf{S}\left(\frac{N^2\mu^2}{M^2}, \beta_i\cdot\beta_j, \alpha_s(\mu^2)\right)\Bigg|_{\mu=M} &= \overline{\mathcal{P}}\exp\left\{-\int_{M/\bar{N}}^{M}\frac{d\mu'}{\mu'}\mathbf{\Gamma}_S^\dagger\left(\beta_i\cdot\beta_j, \alpha_s\left(\mu'^2\right)\right)\right\} \\
&\quad\times\mathbf{S}\left(1, \beta_i\cdot\beta_j, \alpha_s\left(M^2/\bar{N}^2\right)\right) \\
&\quad\times\mathcal{P}\exp\left\{-\int_{M/\bar{N}}^{M}\frac{d\mu'}{\mu'}\mathbf{\Gamma}_S\left(\beta_i\cdot\beta_j, \alpha_s\left(\mu'^2\right)\right)\right\} \\
&= \overline{\mathcal{P}}\exp\left\{\int_0^1 dx\frac{x^{N-1}-1}{1-x}\,\mathbf{\Gamma}_S^\dagger\left(\beta_i\cdot\beta_j, \alpha_s\left((1-x)^2M^2\right)\right)\right\} \\
&\quad\times\mathbf{S}\left(1, \beta_i\cdot\beta_j, \alpha_s\left(M^2/N^2\right)\right) \\
&\quad\times\mathcal{P}\exp\left\{\int_0^1 dx\frac{x^{N-1}-1}{1-x}\,\mathbf{\Gamma}_S\left(\beta_i\cdot\beta_j, \alpha_s\left((1-x)^2M^2\right)\right)\right\}
\end{aligned}
$$

Note: the Soft function satisfies RGE with the same anomalous dimension matrix as the Soft function of the underlying amplitude!

Therefore: knowing the singularities of an amplitude, allows resummation of soft logs in observables!

Singularities of Massive Gauge Theory Amplitudes

Amplitudes: the basics

- Gauge theory amplitudes: UV renormalized, S-matrix elements
- The amplitudes are not observables:
  - UV renormalized gauge amplitudes are not finite due to IR singularities.
  - Assume they are regulated dimensionally $d=4-2\varepsilon$

Some prior general results

✓ Explicit expression for the IR poles of any one-loop amplitude derived

Catani, Dittmaier, Trocsanyi '00

✓ The small mass limit is proportional to the massless amplitude

Mitov, Moch '06
Becher, Melnikov '07

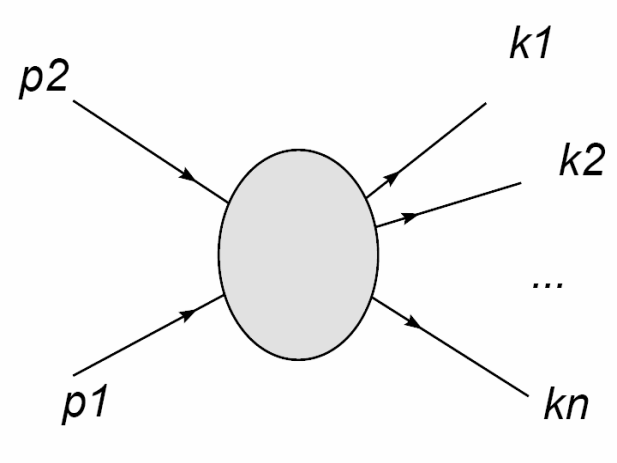
Note: predicts not just the poles but the finite parts too (for $m \to 0$)!

Factorization: "divide and conquer"

Structure of amplitudes becomes transparent thanks to factorization th.

$$M_I(\varepsilon, \mu_R, s_{ij}, m_i) = J(\varepsilon, \mu_R, \mu_F, m_i) \cdot S_{IJ}(\varepsilon, \mu_R, \mu_F, s_{ij}, m_i) \cdot H_J(\varepsilon, \mu_R, \mu_F, s_{ij}, m_i)$$

Note: applicable to both massive and massless cases



I,J – color indexes.

J(…) – "jet" function. Absorbs all the collinear enhancement.

S(…) – "soft" function. All soft non-collinear contributions.

H(…) – "hard" function. Insensitive to IR.

Factorization: the Jet function

$$M_I(Q,m,\epsilon) = J(m,\epsilon)\cdot S_{IJ}(Q,m,\epsilon)\cdot H_J(Q,m)$$

For an amplitude with n-external legs, J(...) is of the form:

$$J(m,\epsilon) = \prod_{i=1}^{n} J_i(m,\epsilon)$$

i.e. we associate a jet factor to each external leg.

Some obvious properties:
- Color singlets,
- Process independent; i.e. do not depend on the hard scale Q.

J_i not unique (only up to sub-leading soft terms).

A natural scheme: J_i = square root of the space-like QCD formfactor.

Sterman and Tejeda-Yeomans '02

Scheme works in both the massless and the massive cases.

The massive form-factor's exponentiation known through 2 loops

Mitov, Moch '06

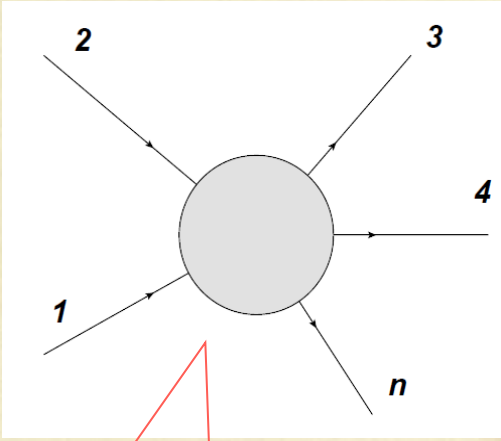
Factorization: the Soft function

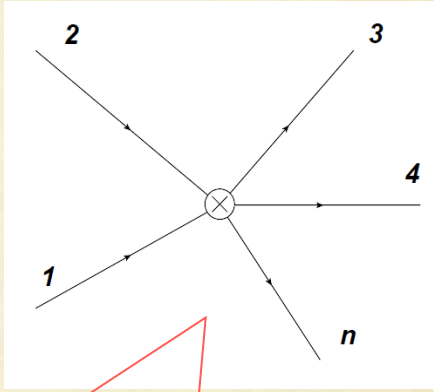
$$M_I(Q,m,\epsilon) = J(m,\epsilon)\cdot S_{IJ}(Q,m,\epsilon)\cdot H_J(Q,m)$$

Soft function is the most non-trivial element
(recall: it contains only soft poles).

But we know that the soft limit is reproduced by the eikonal approximation.

➔ Extract S(…) from the eikonalized amplitude:





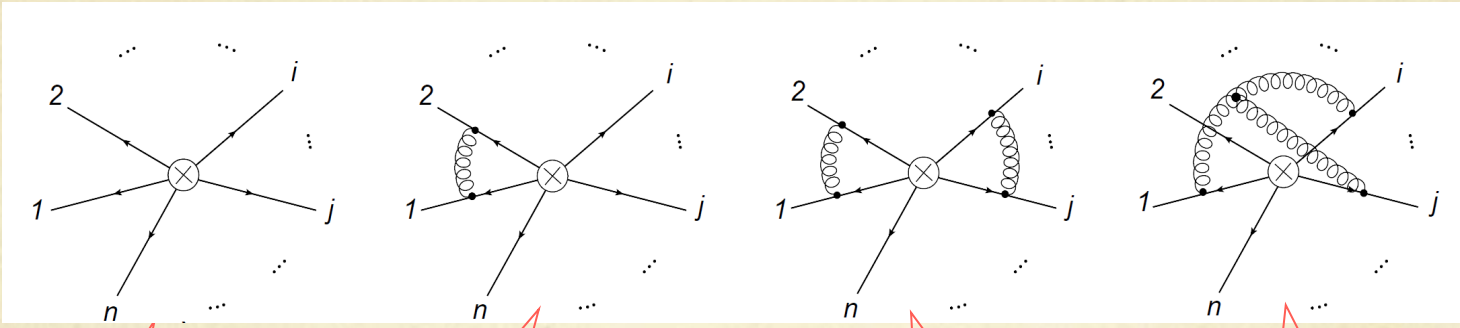
The LO amplitude M(…)

The eikonal version of the amplitude.
(the blob is replaced by an effective n-point vertex)

Factorization: the Soft function

Calculation of the eikonal amplitude:

consider all soft exchanges between the external (hard) partons



LO

1-loop

2-loops

2-loops

The fixed order expansion of the soft function takes the form:

$$
\begin{aligned}
S^{(1)}_{IJ}(\varepsilon, s_{ij}, m_i) &= \frac{1}{\varepsilon}\Gamma^{(1)}_{IJ}(s_{ij}, m_i) + O(\varepsilon^0)\,, \\
S^{(2)}_{IJ}(\varepsilon, s_{ij}, m_i) &= -\frac{\beta_0}{4\varepsilon^2}\Gamma^{(1)}_{IJ}(s_{ij}, m_i) + \frac{1}{2}\left(S^{(1)}_{IJ}(\varepsilon, s_{ij}, m_i)\right)^2 + \frac{1}{\varepsilon}\Gamma^{(2)}_{IJ}(s_{ij}, m_i) + O(\varepsilon^0)\,.
\end{aligned}
$$

... as follows from the usual RG equation:

$$
\left(\mu\frac{\partial}{\partial\mu} + \beta(g,\varepsilon)\frac{\partial}{\partial g}\right) S_{IJ}(\varepsilon, s_{ij}, m_i) = -\Gamma_{IK}(\varepsilon, s_{ij}, m_i)\; S_{KJ}(\varepsilon, s_{ij}, m_i)
$$

➔ All information about S(...) is contained in the anomal's dimension matrix Γ_{IJ}

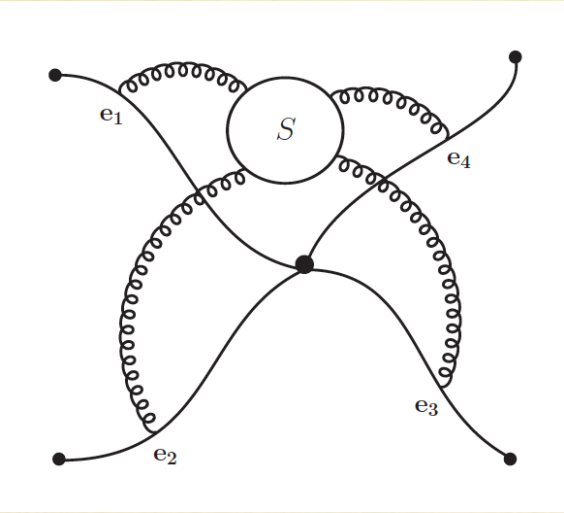
Factorization: the Soft function

How to define and compute these diagrams?

These diagrams are known as "webs". Developed initially for color-singlet vertices.

Gatheral '83
Frenkel and J. C. Taylor '84
Sterman '81

General case now formulated, too



Mitov, Sterman, Sung '10
Gardi, Laenen, Stavenga, White '10

- ✓ The two-loop case is completely solved in QCD (massless and massive cases).
- ✓ Partial results at three loops.

Gardi et al
Becher, Neubert

the Soft function at 1 loop

Here is the result for the anomalous dim. matrix at one loop

$$\mathbf{\Gamma}_S^{(1)} = \underbrace{\frac{1}{2}\sum_{(i\neq j)=1}^{n} T_i\cdot T_j \ln\left(-\frac{\mu^2}{\sigma_{ij}}\right)}_{\text{The massless case}} + \underbrace{\frac{1}{2}\sum_{(i\neq j)\in\mathcal{N}_m} T_i\cdot T_j \left[\ln\left(1+x_{ij}^2\right) + \frac{2x_{ij}^2}{1-x_{ij}^2}\ln(x_{ij})\right]}_{\text{O(m) corrections in the massive case}}$$

where:

- all masses are taken equal,
- written for space-like kinematics (everything is real).

$$\frac{m^2}{s_{ij}} = -\frac{x_{ij}}{(1-x_{ij})^2} \quad , \quad x_{ij} = \frac{\sqrt{1-\frac{4m^2}{s_{ij}}}-1}{\sqrt{1-\frac{4m^2}{s_{ij}}}+1}$$

$$s_{ij} = (p_i+p_j)^2 \text{ and } \sigma_{ij} = 2p_i\cdot p_j = s_{ij} - m_i^2 - m_j^2$$

The Soft function at 2 loops

The simplest approach is the following. Start with the Ansatz:

$$\mathbf{\Gamma}_S^{(2)} = \underbrace{\frac{1}{2}\sum_{(i\neq j)=1}^{n} T_i\cdot T_j\,\frac{K}{2}\,\ln\left(-\frac{\mu^2}{\sigma_{ij}}\right)}_{\text{Reproduces the massless case}} + \underbrace{\frac{1}{2}\sum_{(i\neq j)\in\mathcal{N}_m} T_i\cdot T_j\,P_{ij}^{(2)}}_{\text{Parametrizes the O(m) corrections to the massless case}} + 3E\text{ terms}$$

Then note: the function $P^{(2)}_{ij}$ depends on (i,j) only through s_{ij}

➔ $P^{(2)}_{ij} = P^{(2)}(s_{ij})$

This single function can be extracted from the known n=2 amplitude: the massive two-loop QCD formfactor.

Bernreuther, Bonciani, Gehrmann, Heinesch, Leineweber, Mastrolia, Remiddi `04
Gluza, Mitov, Moch, Riemann `09

The Soft function at 2 loops

The complete result for the 2E reads:

$$P^{(2)} = \frac{K}{2} P^{(1)} + P^{(2),\mathrm{m}}$$

$$\begin{aligned} P^{(2),\mathrm{m}}(x) &= \frac{C_A}{(1-x^2)^2} \Bigg\{ -\frac{(1+x^2)^2}{2} \mathrm{Li}_3(x^2) + \left(\frac{(1+x^2)^2}{2} \ln(x) - \frac{1-x^4}{2} \right) \mathrm{Li}_2(x^2) \\ &+ \frac{x^2(1+x^2)}{3} \ln^3(x) + x^2(1-x^2) \ln^2(x) \\ &+ \left(-(1-x^4) \ln\left(1-x^2\right) + x^2(1+x^2)\zeta_2 \right) \ln(x) + x^2(1-x^2)\zeta_2 + 2x^2\zeta_3 \Bigg\}, \end{aligned}$$

This term breaks the simple relation $\Gamma^{(2)}_{S_\mathrm{f}} = \frac{K}{2} \Gamma^{(1)}_{S_\mathrm{f}}$ from the massless case!

Aybot, Dixon, Sterman '06

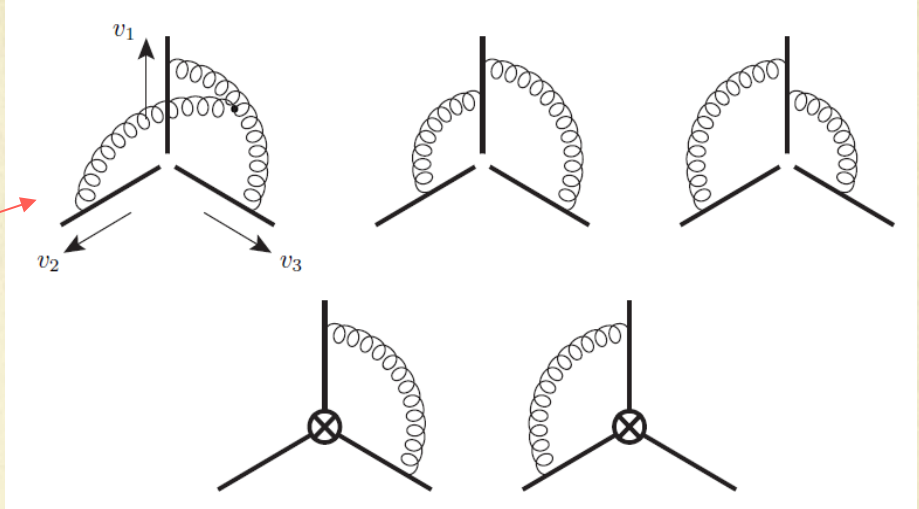
Above result derived by 3 different groups:

Kidonakis '09
Becher, Neubert '09
Czakon, Mitov, Sterman '09

Kidonakis derived the massive eikonal formfactor;
Becher, Neubert used old results of Korchemsky, Radushkin

The Soft function at 2 loops. The 3E diagrams.

The types of contributing diagrams:



The analytical result is very simple:

Ferroglia, Neubert, Pecjak, Yang '09

$$F^{(3g)} \sim \sum_{ijk} \epsilon^{ijk} \ln^2(x_{ij})\, r(x_{ik})$$

where:

$$r(x) = -\frac{1+x^2}{1-x^2}\ln(x)$$

Recall:

it vanishes in the massless case, which makes the relation $\Gamma^{(2)}_{S_f} = \frac{K}{2}\Gamma^{(1)}_{S_f}$ possible.

Aybat, Dixon and Sterman '06

Massive gauge amplitudes: Summary

- The results I presented can be used to predict the poles of any massive 2-loop amplitude with:
  - *n* external colored particles (plus arbitrary number of colorless ones),
  - arbitrary values of the masses (usefull for SUSY).

- Results checked in the 2-loop amplitudes:

$$\langle M^{(2)}|M^{(0)}\rangle(q\bar{q} \to Q\overline{Q})$$
$$\langle M^{(2)}|M^{(0)}\rangle(gg \to Q\overline{Q})$$

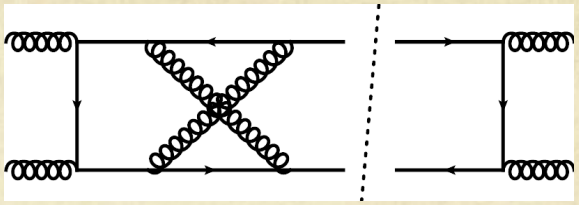
- Needed in jet subtractions with massive particles at 2-loops
- Input for NNLL resummation
- Next frontier: 3-loop anomalous dimension matrix
- Application of webs to N=4 SUSY

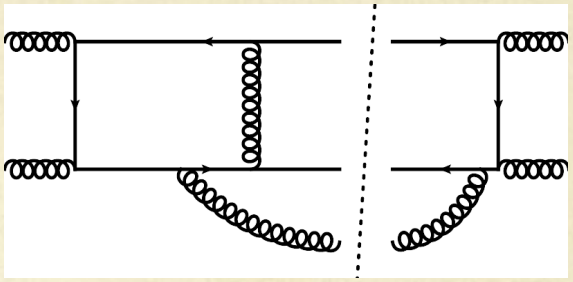
Calculation of the top-pair x-section at NNLO

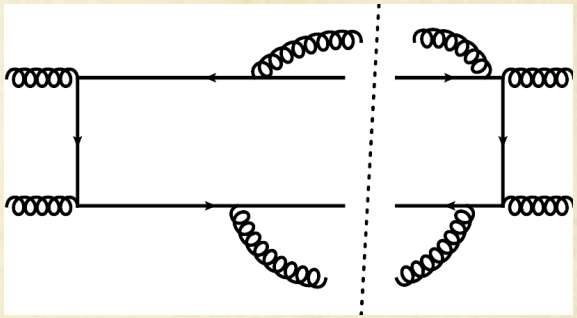
What's needed for NNLO?

There are 3 principle contributions:

- ✓ 2-loop virtual corrections (V-V)
- ✓ 1-loop virtual with one extra parton (R-V)
- ✓ 2 extra emitted partons at tree level (R-R)







And 2 secondary contributions:

- ✓ Collinear subtraction for the initial state
- ✓ One-loop squared amplitudes (analytic)

Known, in principle. Done numerically.

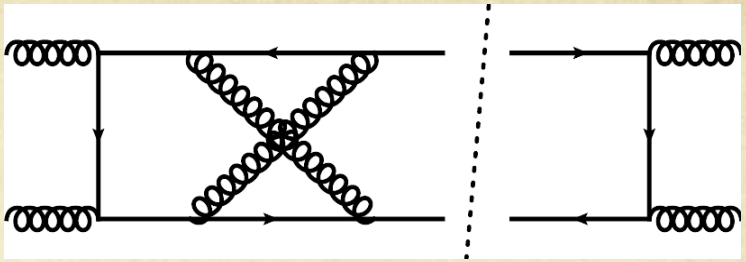
Korner, Merebashvili, Rogal `07
Anastasiou, Mert-Aybot `08

Weinzierl `11

May be avoided?

What's needed for NNLO? V-V

The two-loop amplitude gg → QQ:

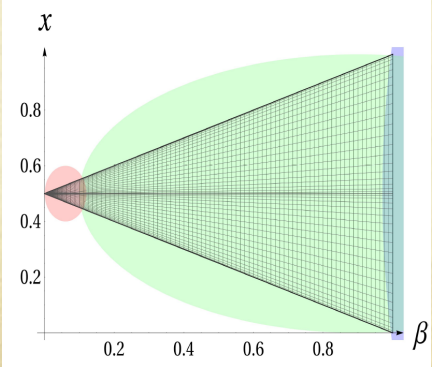


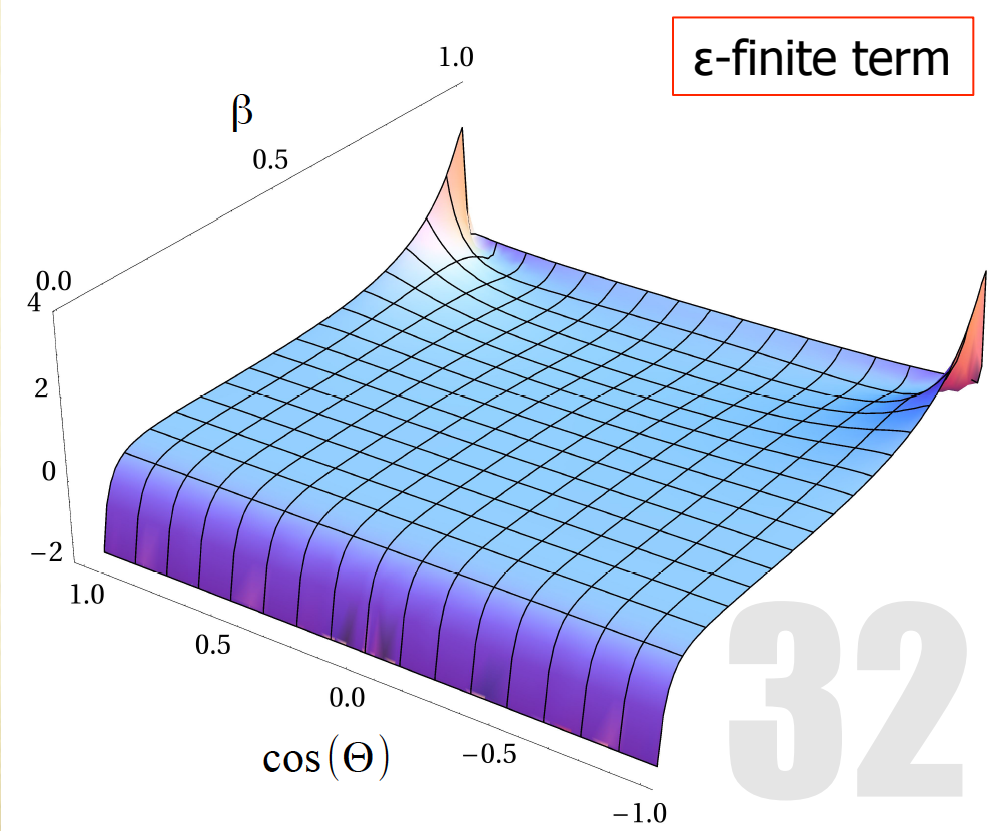
- ✓ Computed numerically
  Bärnreuther, Czakon, Fiedler `13
- ✓ (method similar to qq → QQ)
  Czakon `07
- ✓ Number of color structures known analytically
  Bonciani, Ferroglia, Gehrmann, von Manteuffel, Studerus
- ✓ High-energy limit and poles known analytically

System of 422 masters of 2 variables

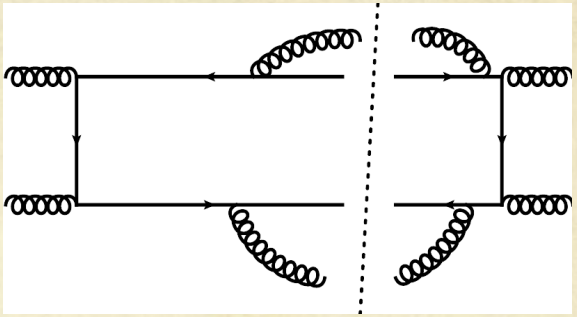
$$x \equiv \frac{m^2 - \hat{t}}{\hat{s}} = \frac{1}{2}\left(1 - \beta \cos(\Theta)\right)$$

Integrated numerically





What's needed for NNLO? R-R



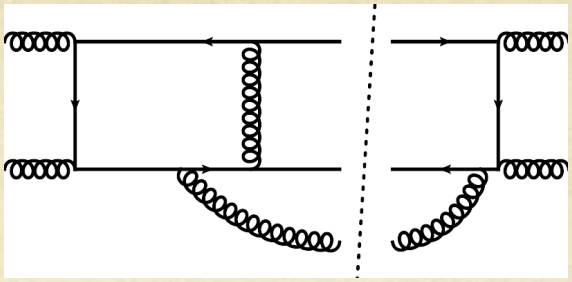
- ✓ A wonderful result By M. Czakon

Czakon `10-11

- ✓ The method is general (also to other processes, differential kinematics, etc).
- ✓ Explicit contribution to the total cross-section given.
- ✓ Just been verified in an extremely non-trivial problem.
- ✓ Applied to other processes too (H+j)

Boughezal, Caola, Melnikov, Petriello, Schulze '13

What's needed for NNLO? R-V



✓ Counterterms all known (i.e. all singular limits)

Bern, Del Duca, Kilgore, Schmidt '98-99
Catani, Grazzini '00
Bierenbaum, Czakon, Mitov '11

The finite piece of the one loop amplitude computed with a private code of Stefan Dittmaier.

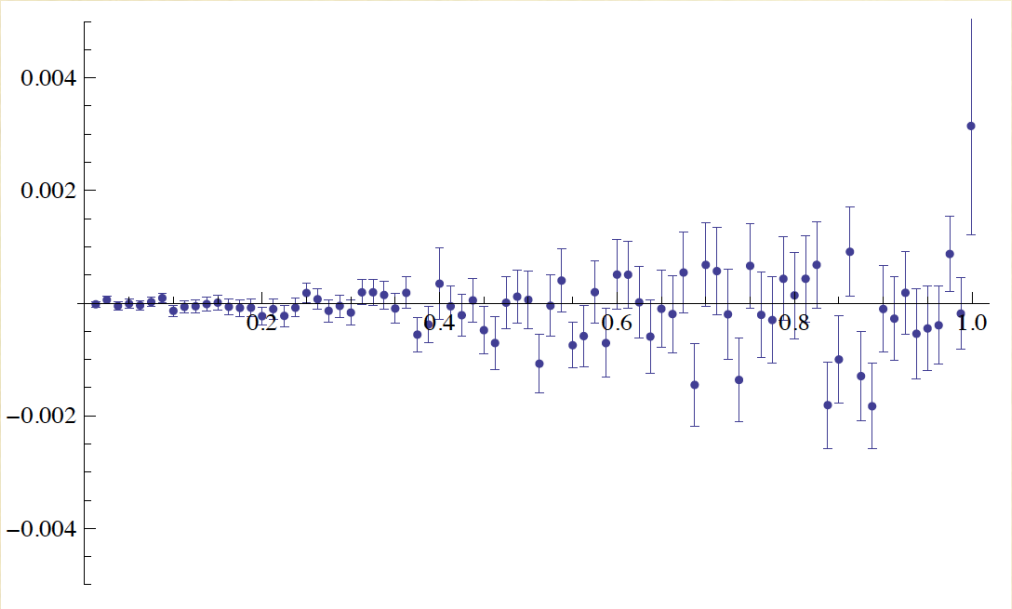
Extremely fast code!

A great help!

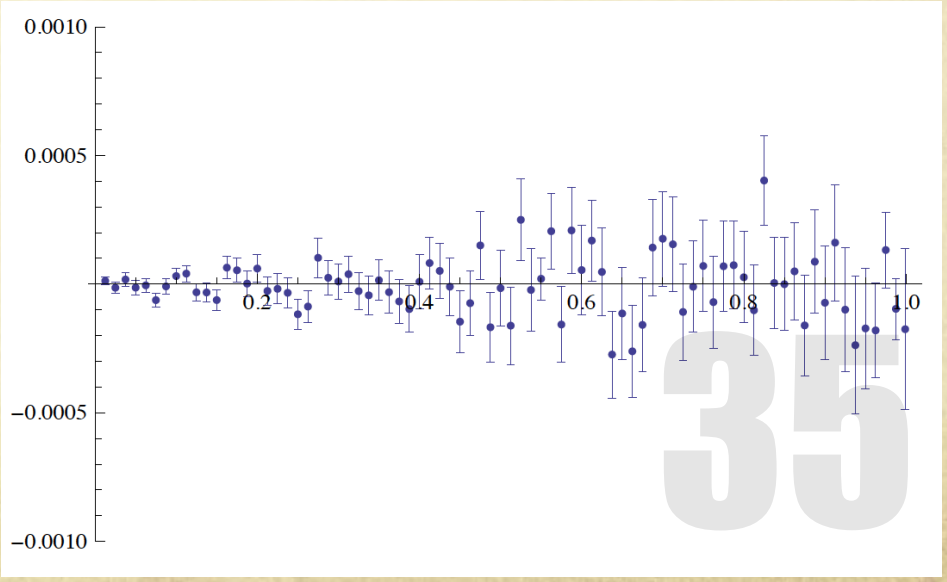
Many thanks!

A note on the calculation

✓ Will only show the cancellation of the deepest singularity 1/ε in gg-> tt:



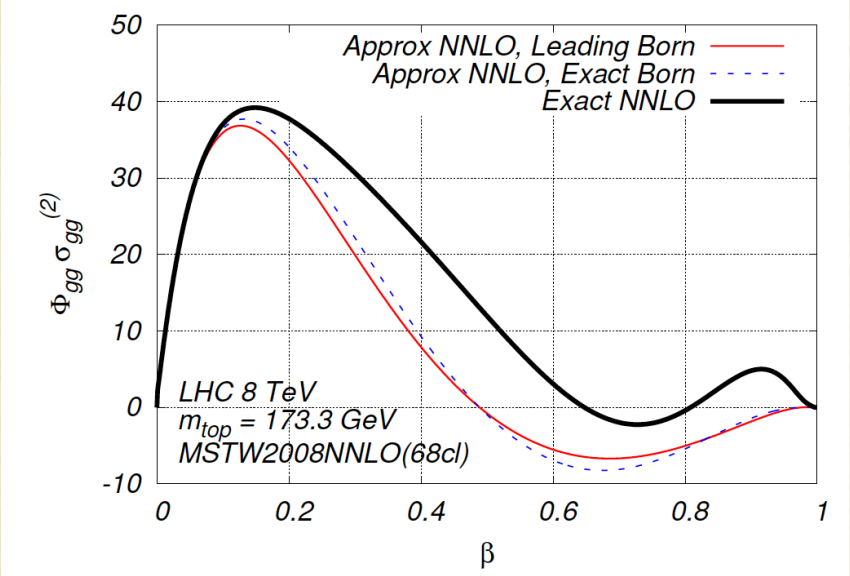
✓ And for $1/\varepsilon^2$ in gg-> tt:



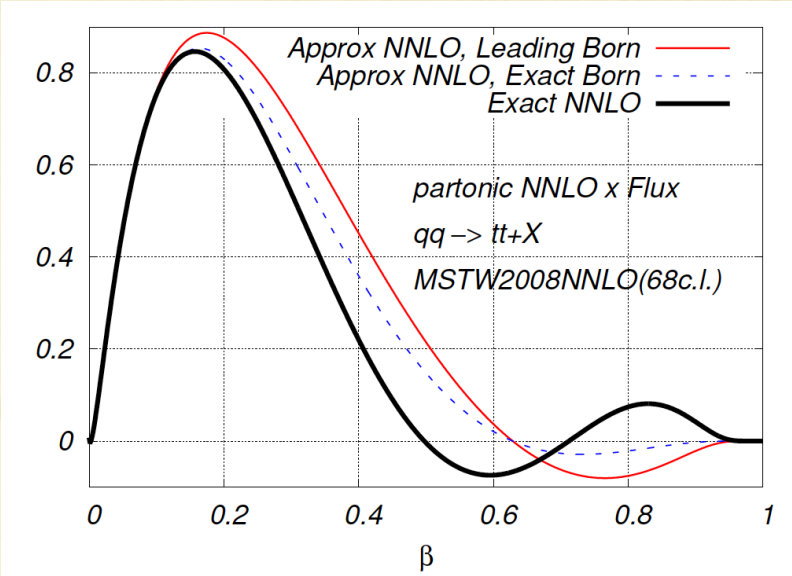
Parton level results

Partonic NNLO cross-sections, convoluted with LHC/Tevatron partonic fluxes

Czakon, Fiedler, Mitov `13



Bärnreuther, Czakon, Mitov `12



Note the agreement between the exact result and the threshold approximation
Derived from soft-gluon resummation + bound state effects

- The exact result is computed numerically, in 80 points on the interval 0<beta<1

Results @ parton level: gg -> ttbar +X

Partonic cross-section through NNLO:

$$\sigma_{ij}\left(\beta,\frac{\mu^2}{m^2}\right)=\frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)}+\alpha_S\left[\sigma_{ij}^{(1)}+L\,\sigma_{ij}^{(1,1)}\right]+\alpha_S^2\left[\sigma_{ij}^{(2)}+L\,\sigma_{ij}^{(2,1)}+L^2\sigma_{ij}^{(2,2)}\right]+\mathcal{O}(\alpha_S^3)\right\},$$

The NNLO term:

$$\sigma_{gg}^{(2)}(\beta)=F_0(\beta)+F_1(\beta)N_L+F_2(\beta)N_L^2$$

Numeric

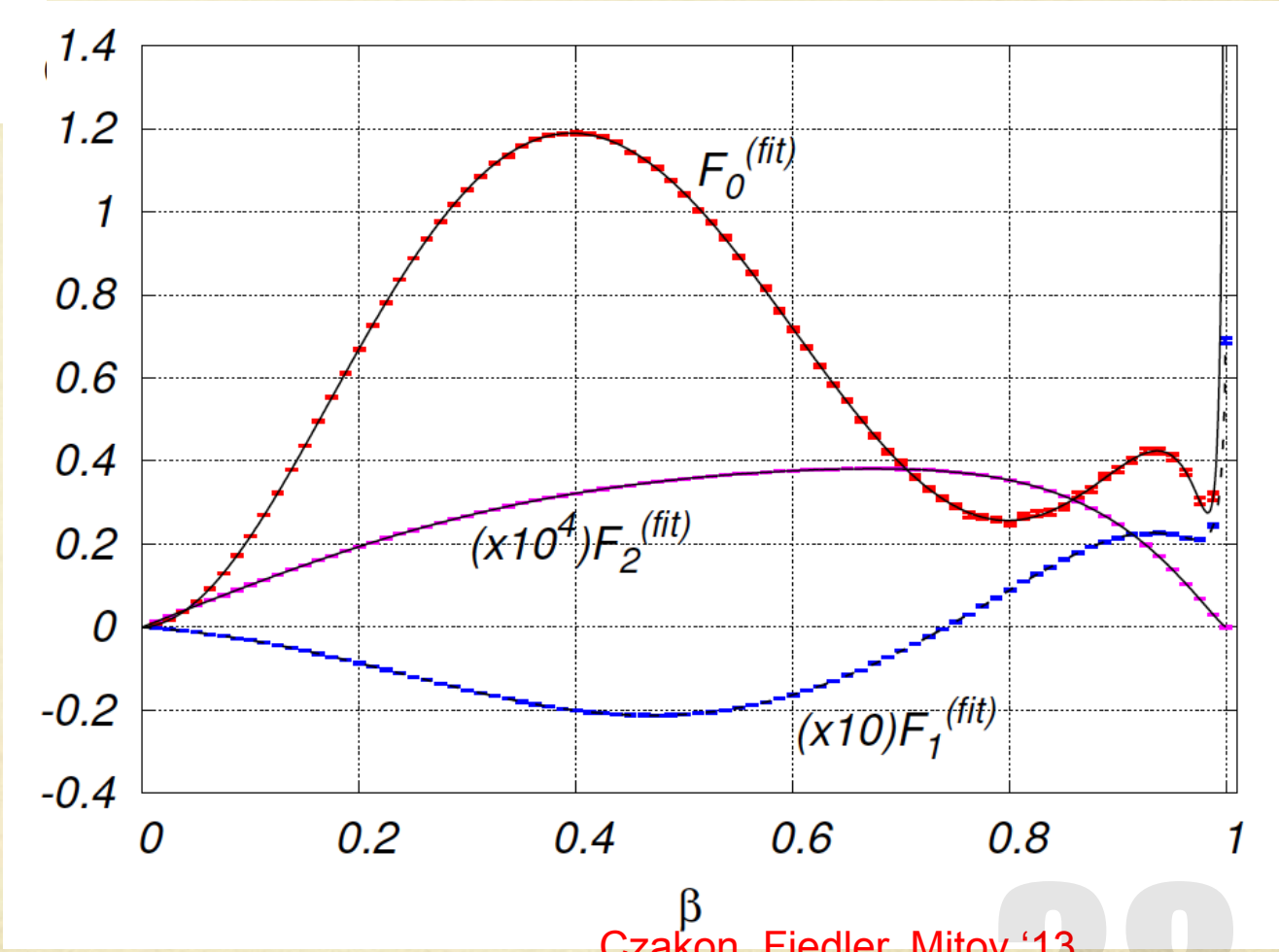
$$F_i\equiv F_i^{(\beta)}+F_i^{(\mathrm{fit})},\ i=0,1,2$$

The known threshold approximation

Beneke, Czakon, Falgari, Mitov, Schwinn `09

Notable features:

- ✓ Small numerical errors
- ✓ Agrees with limits

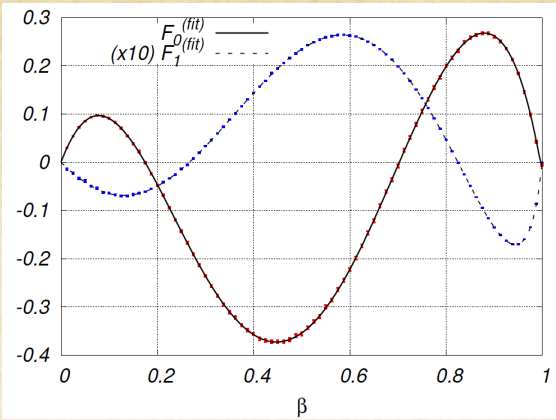


Czakon, Fiedler, Mitov '13

Results @ parton level:
The all-fermionic reactions

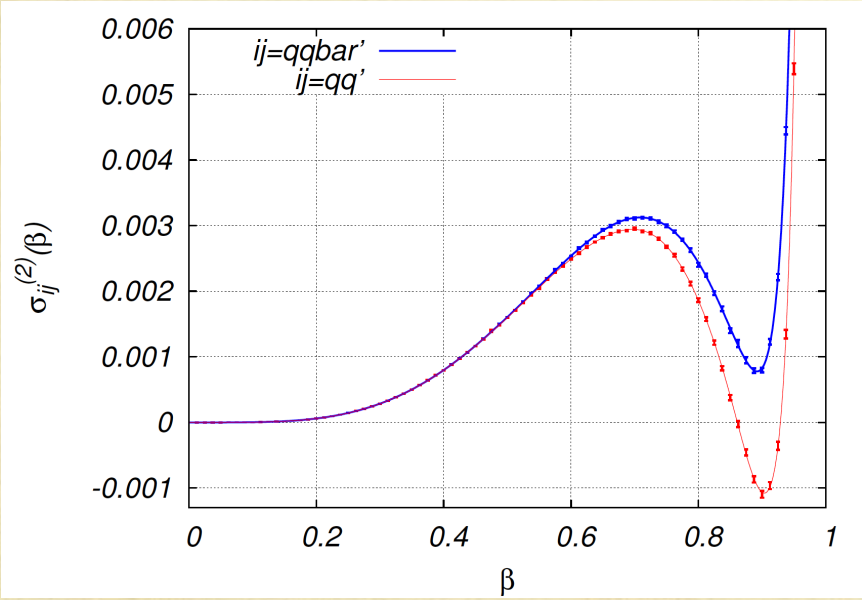
Czakon, Mitov '12

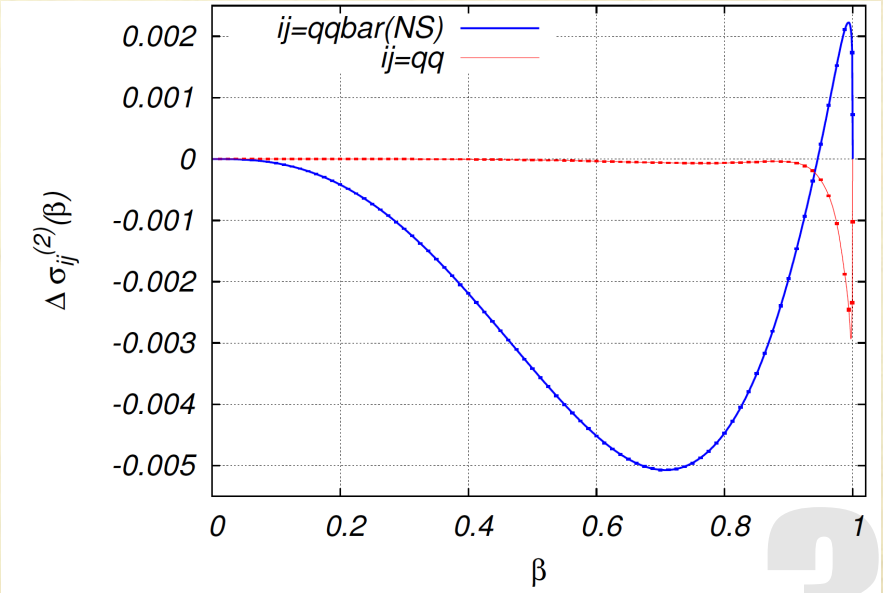
$$q\bar{q} \to t\bar{t} + q\bar{q}\big|_{\rm NS}\,,$$
$$q\bar{q}' \to t\bar{t} + q\bar{q}'\,,$$
$$qq' \to t\bar{t} + qq'\,,$$
$$qq \to t\bar{t} + qq\,.$$



P. Bärnreuther et al arXiv:1204.5201

These partonic cross-sections are very small.
Compare to the ones involving qqbar!

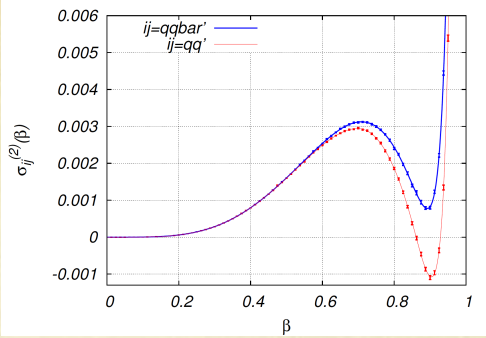


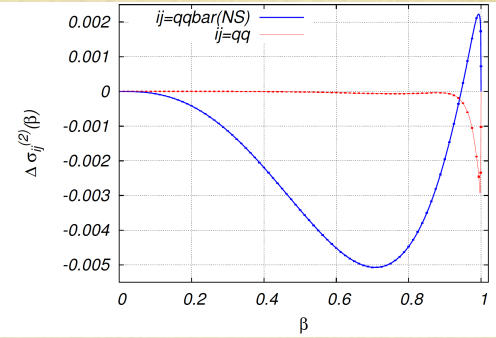


✧ Had to compute up to beta=0.9999 to get the high-energy behavior right.

Results @ parton level:
The all-fermionic reactions

$$q\bar{q} \to t\bar{t} + q\bar{q}\big|_{\rm NS}\,,$$
$$q\bar{q}' \to t\bar{t} + q\bar{q}'\,,$$
$$qq' \to t\bar{t} + qq'\,,$$
$$qq \to t\bar{t} + qq\,.$$





The interesting feature: high-energy logarithmic rise:

$$\sigma^{(2)}_{f_1 f_2 \to t\bar{t} f_1 f_2}\Big|_{\rho\to 0} \approx c_1 \ln(\rho) + c_0 + \mathcal{O}(\rho) \qquad \rho = \frac{4m_t^2}{s}$$

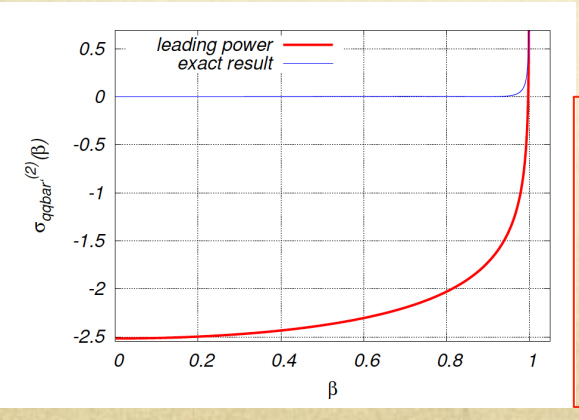
$$c_1 = -0.4768323995789214$$

Known analytically

Ball, Ellis `01

$$c_0 \text{ (from Eqs. (6.3, 6.4))} = \begin{cases} -2.5173 & \text{from} \quad \sigma^{(2)}_{q\bar{q}'} \\ -2.5186 & \text{from} \quad \sigma^{(2)}_{qq'} \end{cases}$$

- Direct extraction from the fits. 5% uncertainty. Czakon, Mitov '12
- Agrees with independent prediction. 50% uncertainty. Moch, Uwer, Vogt '12



High-energy expansion non-convergent.

Applies only to the high-energy limit.

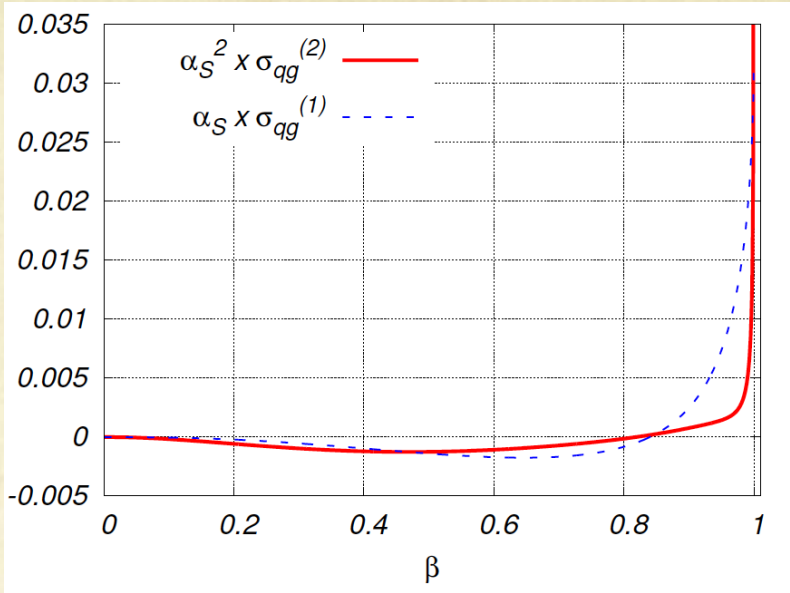
	Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
$\Delta\sigma_{q\bar{q},(\rm NS)}$ [pb]	-0.0020	-0.0097	-0.0124	-0.0299
$\sigma_{q\bar{q},(\rm NS)}$ [pb]	-0.0009	-0.0001	0.0021	0.0464
$\sigma_{\rm all}$ [pb]	0.0003	0.0970	0.1504	0.7885
$\sigma_{\rm tot}$ [pb]	7.0056	154.779	220.761	852.177

Czakon, Mitov '12

Results @ parton level:qg -> ttbar +X

Czakon, Mitov `12

- ✓ Correction about -1% (Tev and LHC).
- ✓ Notable decrease of scale dependence at LHC.
- ✓ NNLO large compared to NLO.



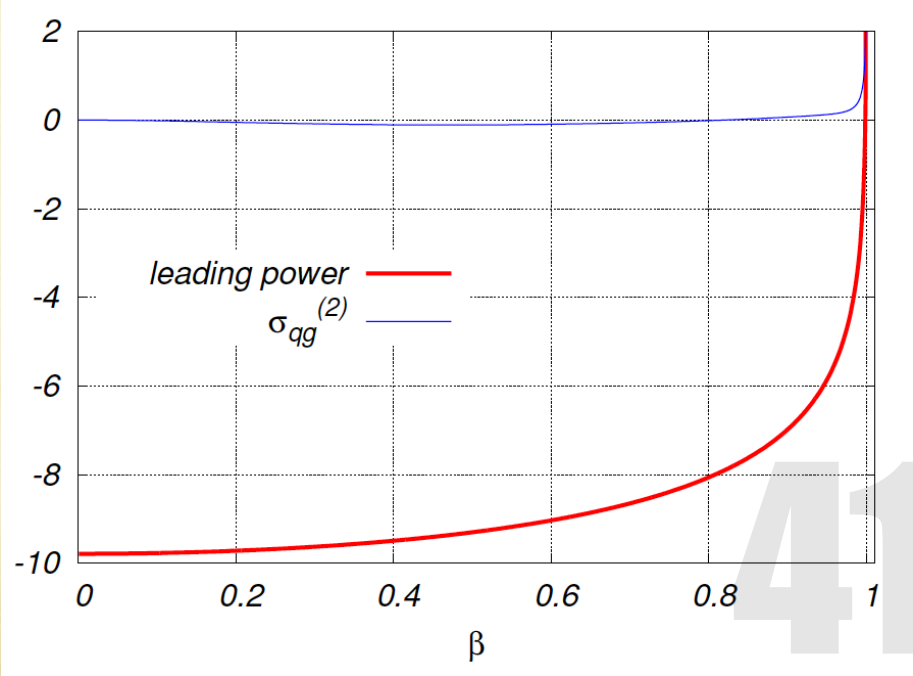
- ✓ High-energy log-limit correct

Ball, Ellis `01

- ✓ Agree for the constant with

Moch, Uwer, Vogt '12

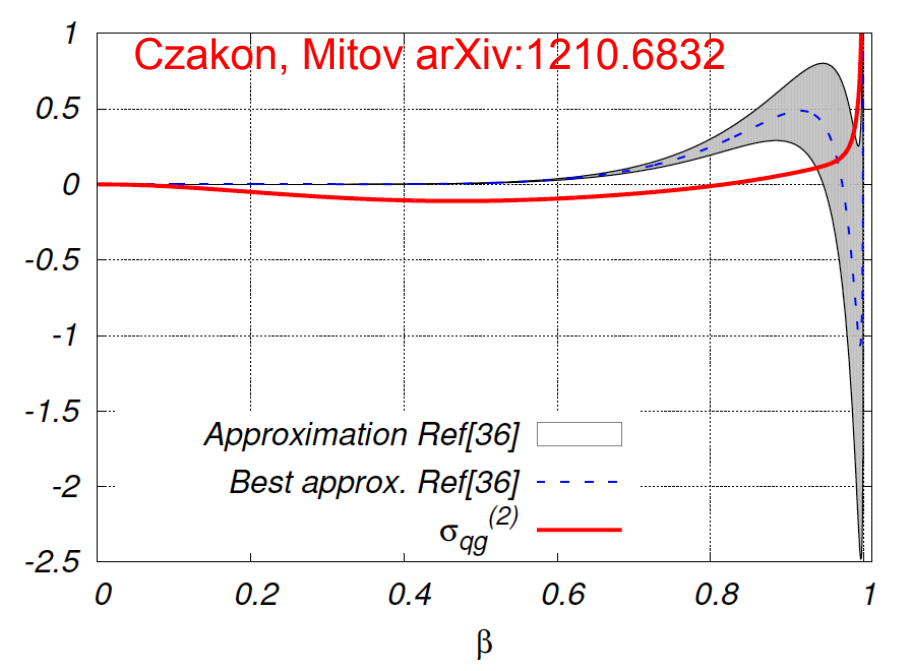
- ✓ The limit itself plays no Pheno role



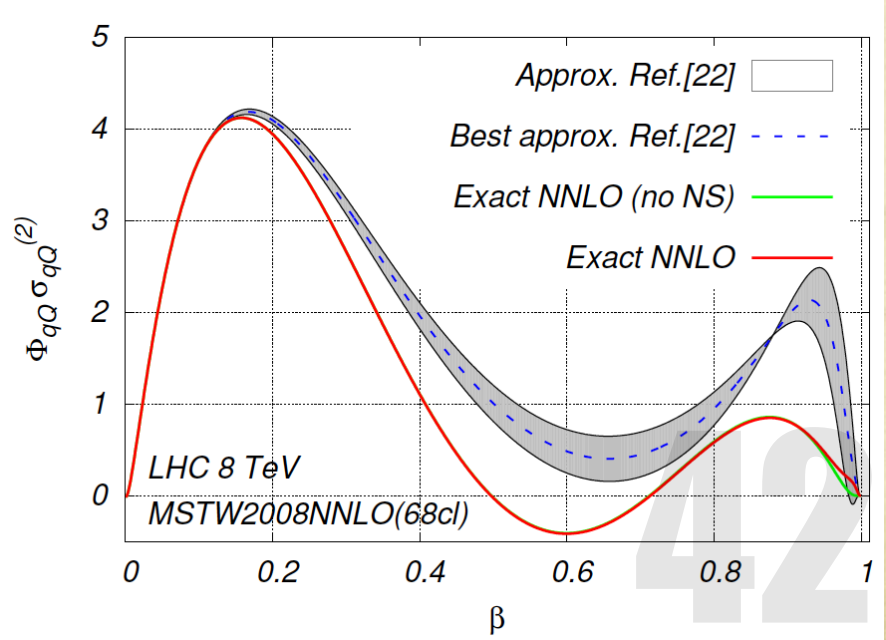
Checking the high-energy limit approximation

- ✓ It was suggested to use the high-energy limit of the X-section to predict it everywhere:

Moch, Uwer, Vogt '12



- ✓ MUV approximation dramatically deviates from the exact gq NNLO result
- ✓ Leads to large difference for the x-section O(5%) from gq alone !
- ✓ Similar deviation for qq->tT+X (flux included)



Precision phenomenological applications

Prediction at NNLO+ resummation (NNLL)

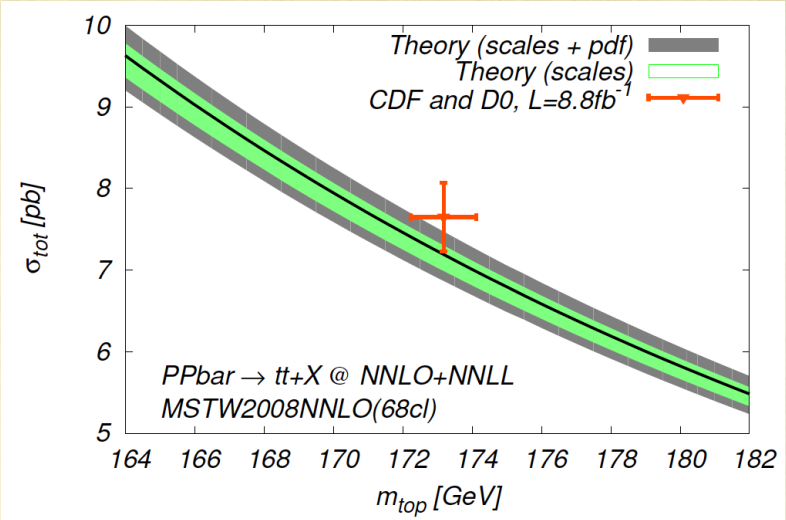
Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.164	$^{+0.110(1.5\%)}_{-0.200(2.8\%)}$	$^{+0.169(2.4\%)}_{-0.122(1.7\%)}$
LHC 7 TeV	172.0	$^{+4.4(2.6\%)}_{-5.8(3.4\%)}$	$^{+4.7(2.7\%)}_{-4.8(2.8\%)}$
LHC 8 TeV	245.8	$^{+6.2(2.5\%)}_{-8.4(3.4\%)}$	$^{+6.2(2.5\%)}_{-6.4(2.6\%)}$
LHC 14 TeV	953.6	$^{+22.7(2.4\%)}_{-33.9(3.6\%)}$	$^{+16.2(1.7\%)}_{-17.8(1.9\%)}$

Pure NNLO

Collider	$\sigma_{\rm tot}$ [pb]	scales [pb]	pdf [pb]
Tevatron	7.009	$^{+0.259(3.7\%)}_{-0.374(5.3\%)}$	$^{+0.169(2.4\%)}_{-0.121(1.7\%)}$
LHC 7 TeV	167.0	$^{+6.7(4.0\%)}_{-10.7(6.4\%)}$	$^{+4.6(2.8\%)}_{-4.7(2.8\%)}$
LHC 8 TeV	239.1	$^{+9.2(3.9\%)}_{-14.8(6.2\%)}$	$^{+6.1(2.5\%)}_{-6.2(2.6\%)}$
LHC 14 TeV	933.0	$^{+31.8(3.4\%)}_{-51.0(5.5\%)}$	$^{+16.1(1.7\%)}_{-17.6(1.9\%)}$

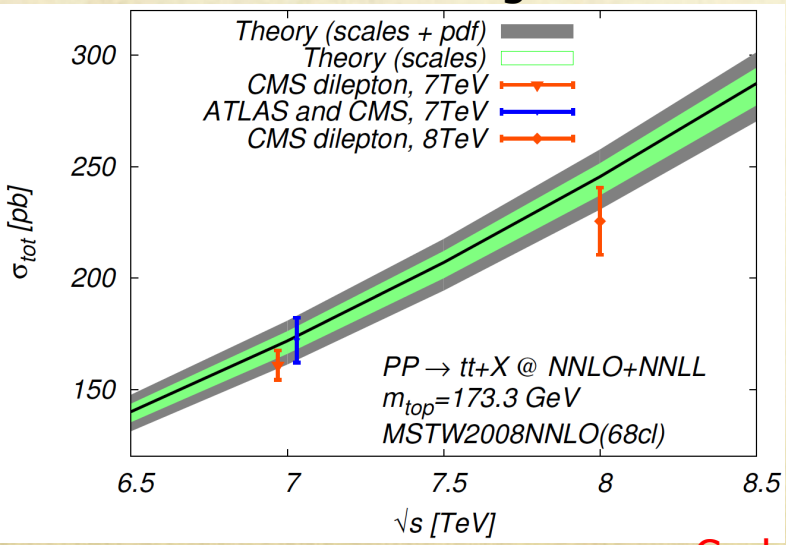
Czakon, Fiedler, Mitov '13

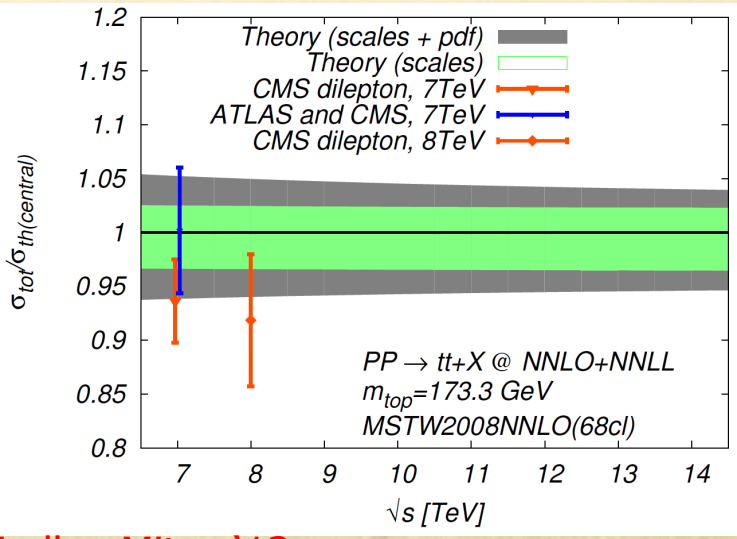
Good agreement with Tevatron measurements



- ✓ Independent F/R scales
- ✓ MSTW2008NNLO
- ✓ mt=173.3

Good agreement with LHC measurements

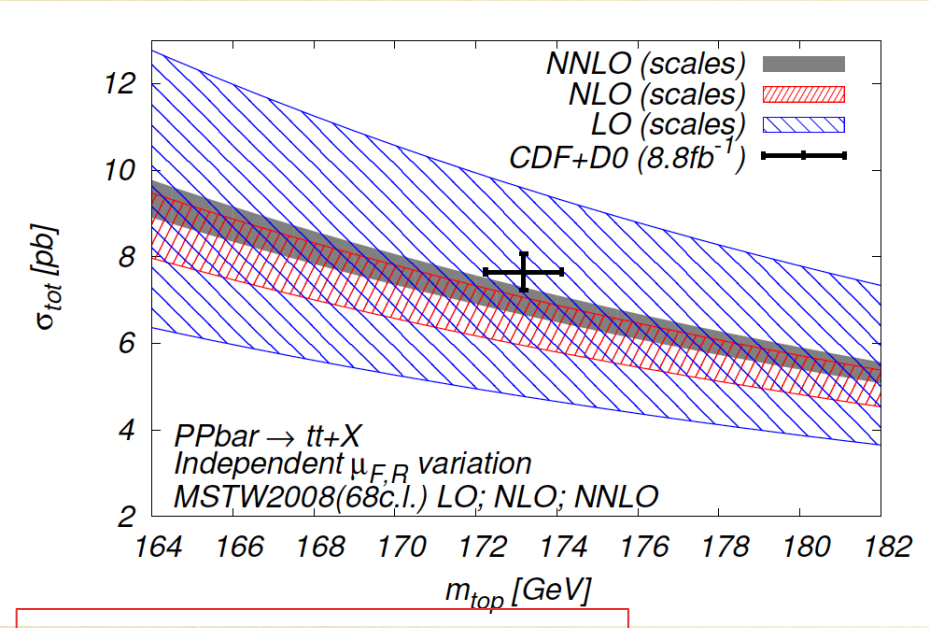




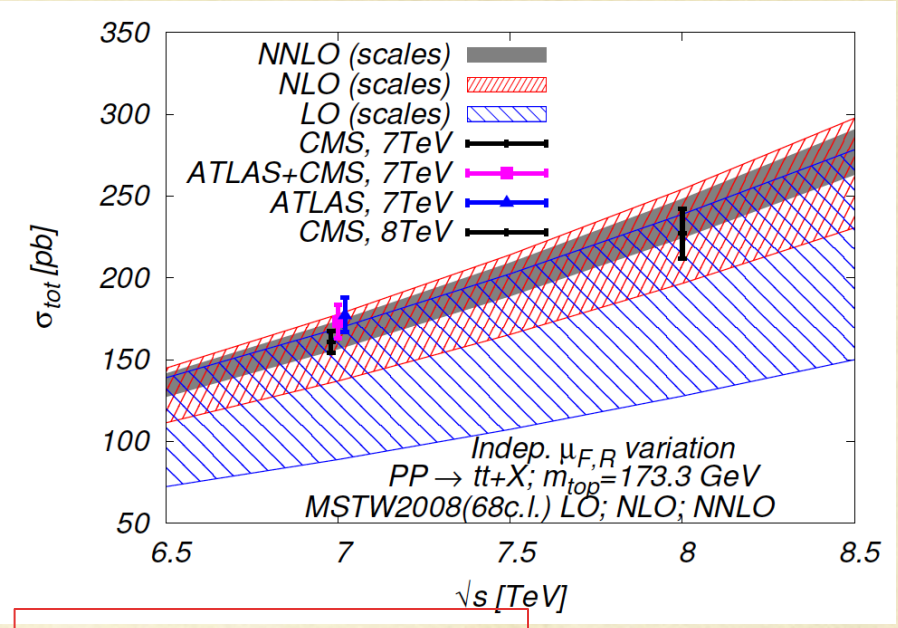
Czakon, Fiedler, Mitov '13

Good perturbative convergence

✓ Independent F/R scales variation



Scale variation @ Tevatron



Scale variation @ LHC

✓ Good overlap of various orders (LO, NLO, NNLO).

✓ Suggests the (restricted) independent scale variation is a good estimate of missing higher order terms!

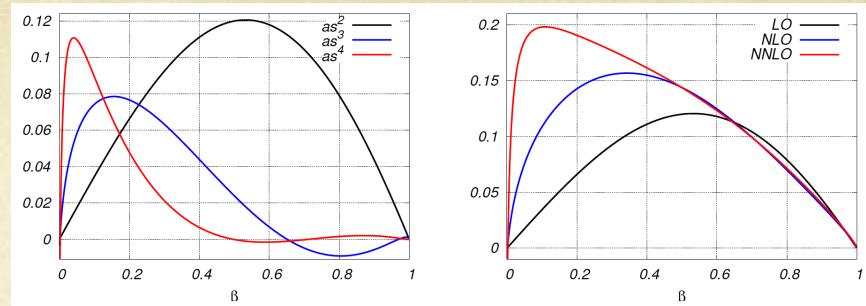
This is very important: good control over the perturbative corrections justifies less-conservative overall error estimate, i.e. more predictive theory (see next 2 slides).

For more detailed comparison, including soft-gluon resummation, see arXiv 1305.3892

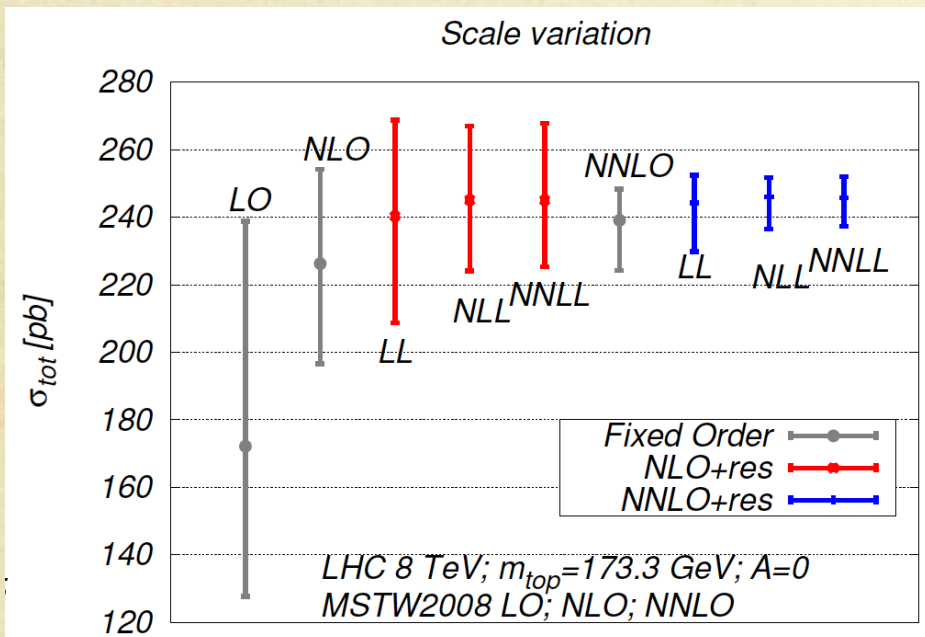
Quantifying soft-gluon resummation

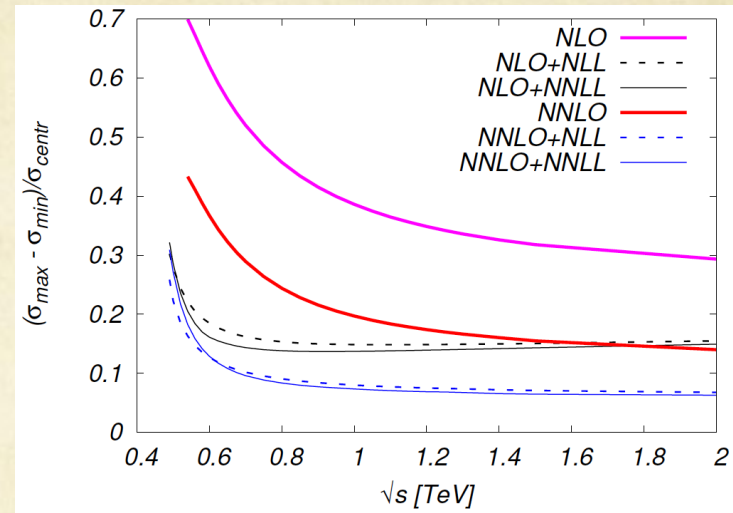
Partonic x-section's growth
close to threshold (qq reaction):

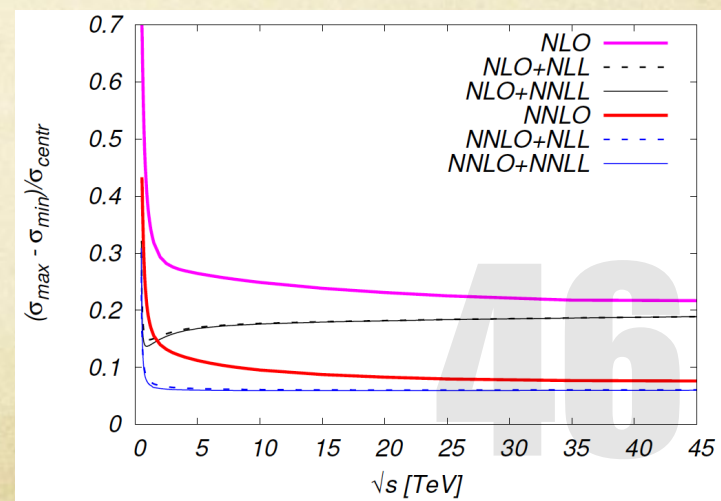
The expansion there is not converging
Resummation needed



$$\hat{\sigma}(\beta) = \frac{\alpha_S^2}{m^2}\left(\sigma^{(0)} + \alpha_S\sigma^{(1)} + \alpha_S^2\sigma^{(2)} + \ldots\right) \equiv \frac{\alpha_S^2}{m^2}\left(f_{\alpha_S^2} + f_{\alpha_S^3} + f_{\alpha_S^4} + \ldots\right)$$







The resummed results are better, as expected.

Update of: Cacciari, Czakon, Mangano, Mitov, Nason '11

LHC: general features at NNLO+NNLL

Czakon, Fiedler, Mitov '13

Czakon, Mangano, Mitov, Rojo '13

- ✓ We have reached a point of saturation: uncertainties due to
  - ✓ scales (i.e. missing yet-higher order corrections) ~ 3%
  - ✓ pdf (at 68%cl) ~ 2-3%
  - ✓ $alpha_S$ (parametric) ~ 1.5%
  - ✓ m_{top} (parametric) ~ 3%

→ All are of similar size!

- ✓ Soft gluon resummation makes a difference: scale uncertainty 5% → 3%

- ✓ The total uncertainty tends to decrease when increasing the LHC energy

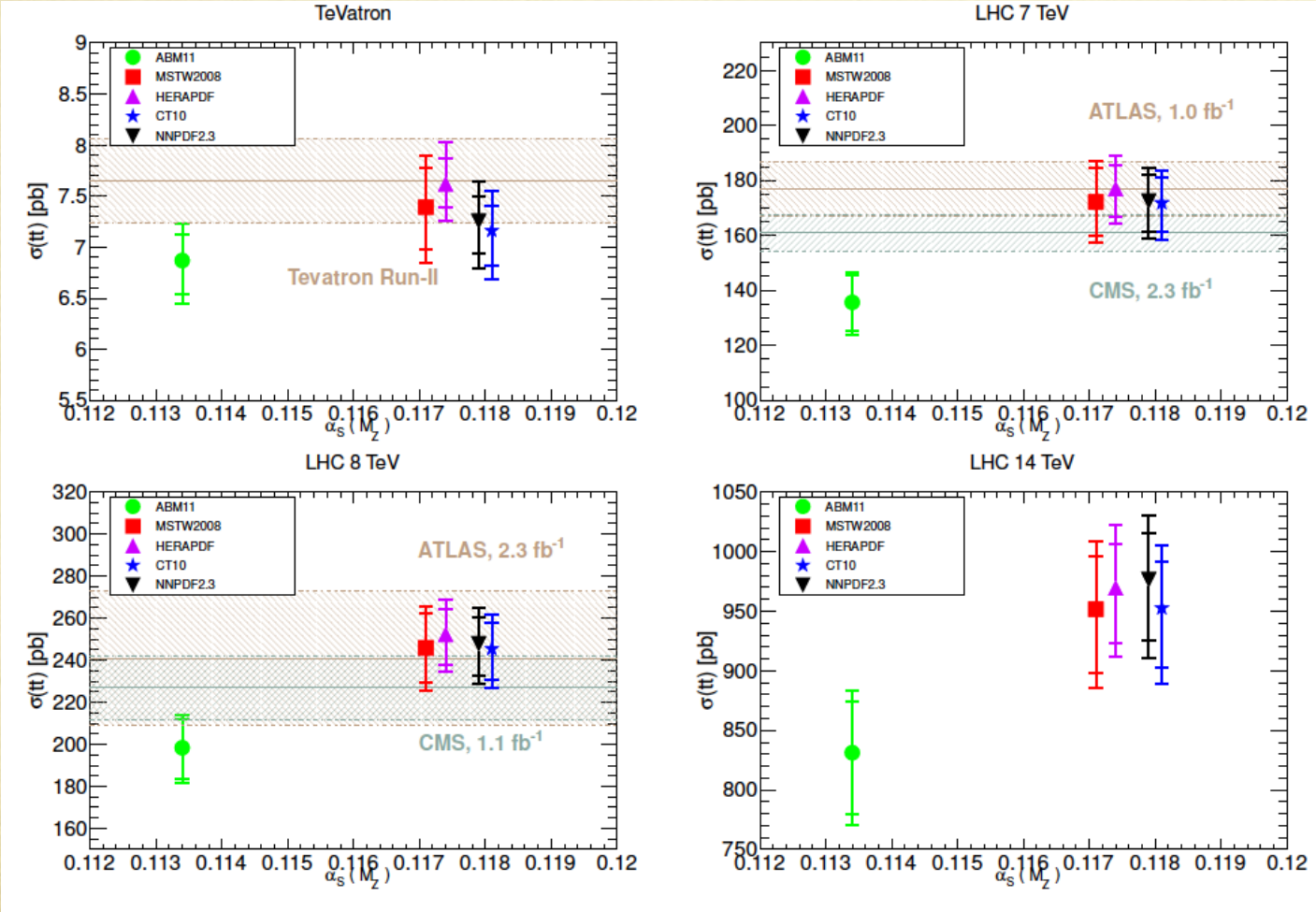
Application to PDF's

Czakon, Mangano, Mitov, Rojo `13

How existing pdf sets fare when compared to existing data?

Most conservative theory uncertainty:

Scales + pdf + as + mtop



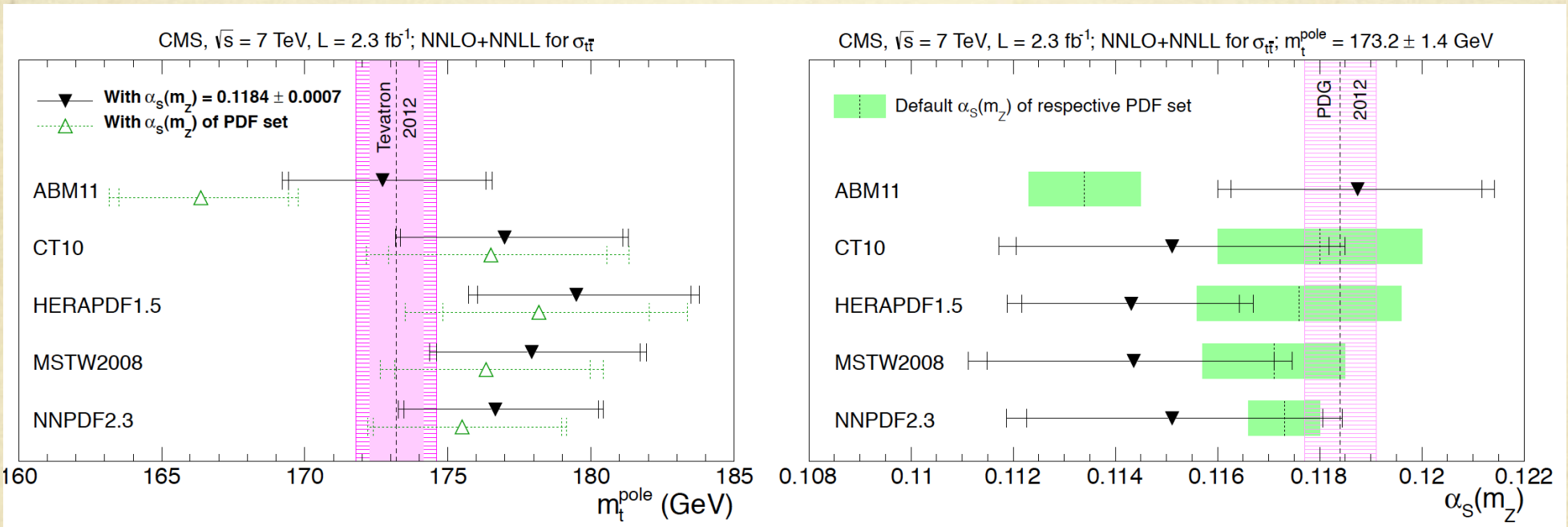
Excellent agreement between almost all pdf sets

alpha$_S$ and m$_{TOP}$ extraction from top data (CMS)

How existing pdf sets fare when compared to existing data?

Excellent agreement between almost all pdf sets

S. Naumann-Emme (CMS) Arxiv:1402.0709



- Results are consistent with world averages, although slight tendency can be seen.
- ABM11 returns value of alpha$_S$ that is incompatible with their assumed value.

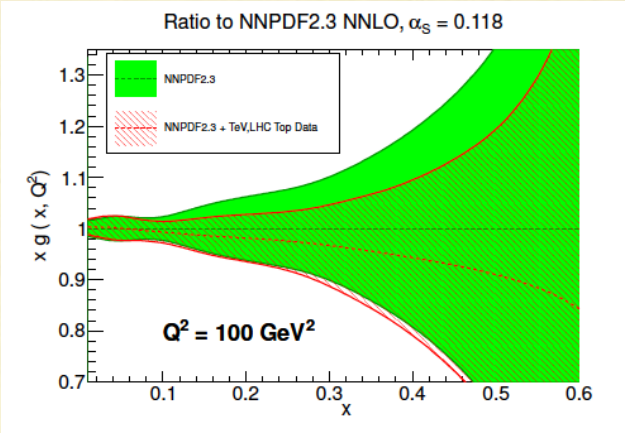
Application to PDF's

✓ tT offers for the first time a direct NNLO handle to the gluon pdf (at hadron colliders)

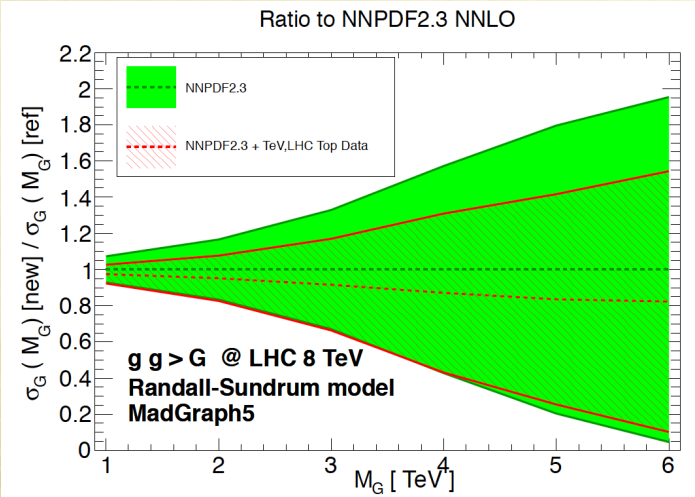
✓ implications to many processes at the LHC: Higgs and bSM production at large masses

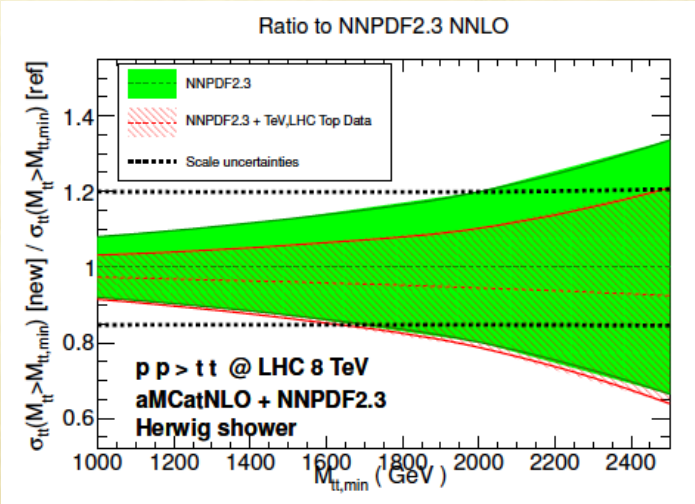
One can use the 5 available (Tevatron/LHC) data-points to improve gluon pdf

"Old" and "new" gluon pdf at large x:



... and PDF uncertainty due to "old" vs. "new" gluon pdf: Czakon, Mangano, Mitov, Rojo '13





Application to bSM searches: stealthy stop

- ✓ Scenario: stop → top + missing energy
  - ✓ m_stop small: just above the top mass.
  - ✓ Stop mass < 225 GeV is allowed by current data
  - ✓ Usual wisdom: the stop signal hides in the top background

- ✓ The idea: use the top x-section to derive a bound on the stop mass. <u>Assumptions</u>:
  - ✓ Same experimental signature as pure tops
  - ✓ the measured x-section is a sum of top + stop
  - ✓ Use precise predictions for stop production @ NLO+NLL

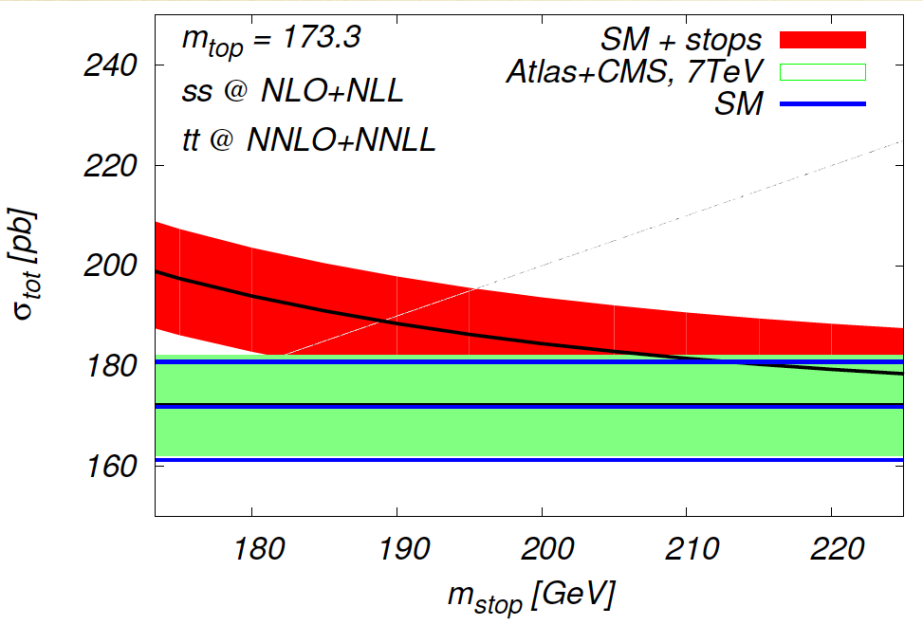
Krämer, Kulesza, van der Leeuw, Mangano, Padhi, Plehn, Portell `12

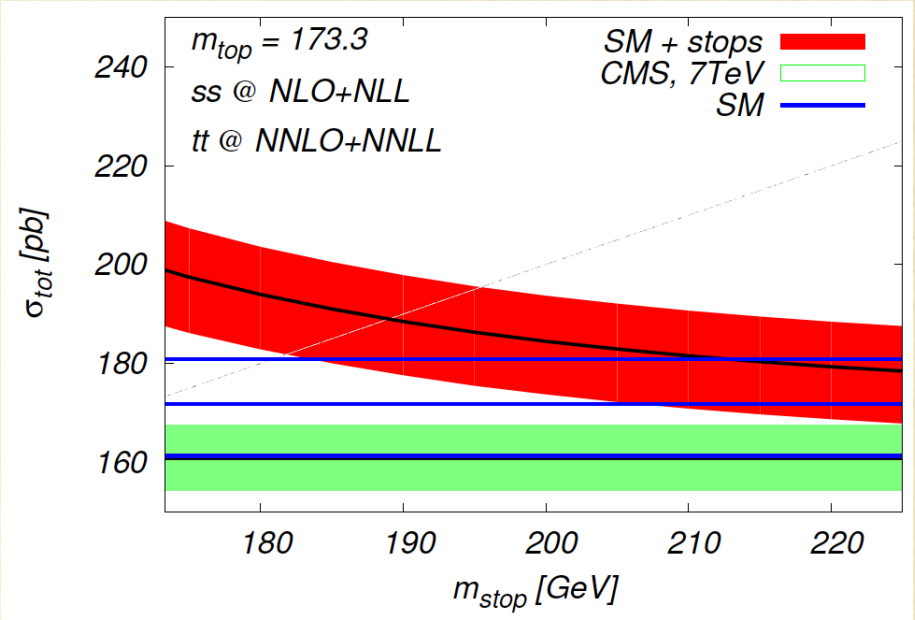
  - ✓ Total theory uncertainty: add SM and SUSY ones in quadrature.

Applications to the bSM searches: stealth stop

✓ Predictions

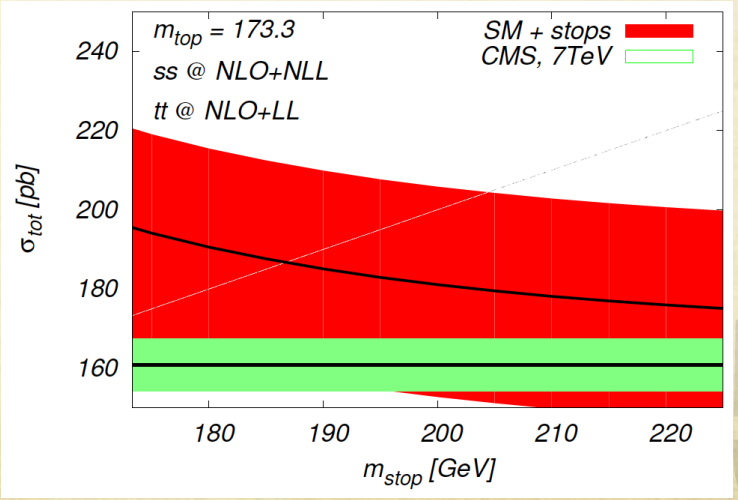
Preliminary





Wonder why limits were not imposed before?

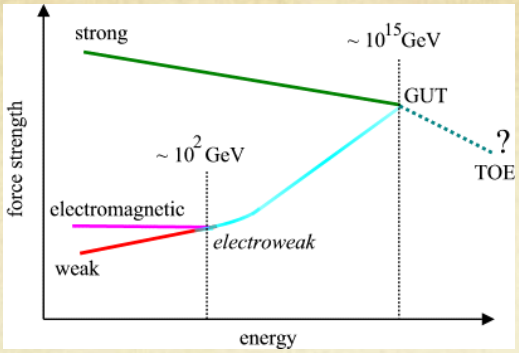
Here is the result with "NLO+shower" accuracy :



Improved NNLO accuracy makes all the difference

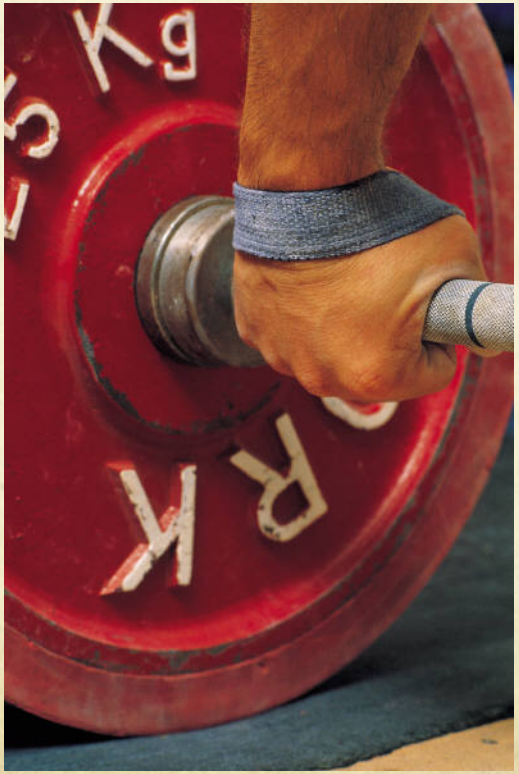
Where is the New Physics?

The desert …



How can we tell if it is a desert or a jungle?

Hey, top mass measurement might help!



Top quark mass

Places where the top mass is crucial:

Bezrukov, Shaposhnikov '07-'08

- Higgs-inflation

Assume non-minimal coupling to gravity:

$$\mathcal{L}_h = -|\partial H|^2 + \mu^2 H^\dagger H - \lambda (H^\dagger H)^2 + \xi H^\dagger H \mathcal{R}$$

Then: Higgs = inflaton provided:

1) $10^3 < \xi < 10^4$

2) $$m_h > 125.7\,\mathrm{GeV} + 3.8\,\mathrm{GeV}\left(\frac{m_t - 171\,\mathrm{GeV}}{2\,\mathrm{GeV}}\right) - 1.4\,\mathrm{GeV}\left(\frac{\alpha_s(m_Z) - 0.1176}{0.0020}\right) \pm \delta$$

3) $m_h \lesssim 190\,\mathrm{GeV}$

- Theory remains perturbative at high energy,
- Has been criticized for inconsistent inflation.

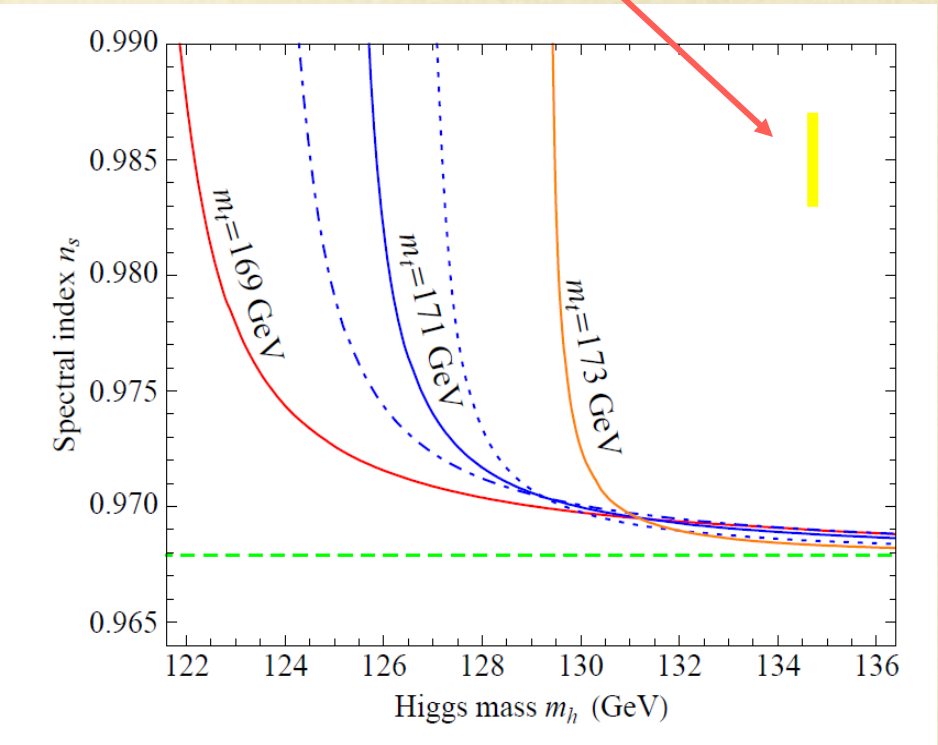
Top quark mass

Results from PLANK (past expectation – not the actual result)

- Higgs-inflation

Bezrukov, Shaposhnikov '07-'08

Provided it works ☺
the model is very predictive!



De Simone, Hertzbergy, Wilczek arXiv:0812.4946v2

Figure 1: The spectral index n_s as a function of the Higgs mass m_h for a range of light Higgs masses. The 3 curves correspond to 3 different values of the top mass: $m_t = 169\,\text{GeV}$ (red curve), $m_t = 171\,\text{GeV}$ (blue curve), and $m_t = 173\,\text{GeV}$ (orange curve). The solid curves are for $\alpha_s(m_Z) = 0.1176$, while for $m_t = 171\,\text{GeV}$ (blue curve) we have have also indicated the 2-sigma spread in $\alpha_s(m_Z) = 0.1176 \pm 0.0020$, where the dotted (dot-dashed) curve corresponds to smaller (larger) α_s. The horizontal dashed green curve, with $n_s \simeq 0.968$, is the classical result. The yellow rectangle indicates the expected accuracy of PLANCK in measuring n_s ($\Delta n_s \approx 0.004$) and the LHC in measuring m_h ($\Delta m_h \approx 0.2\,\text{GeV}$). In this plot we have set $N_e = 60$.

Yet another application of the top mass:

The fate of the Universe might depend on 1 GeV in M_{top}!

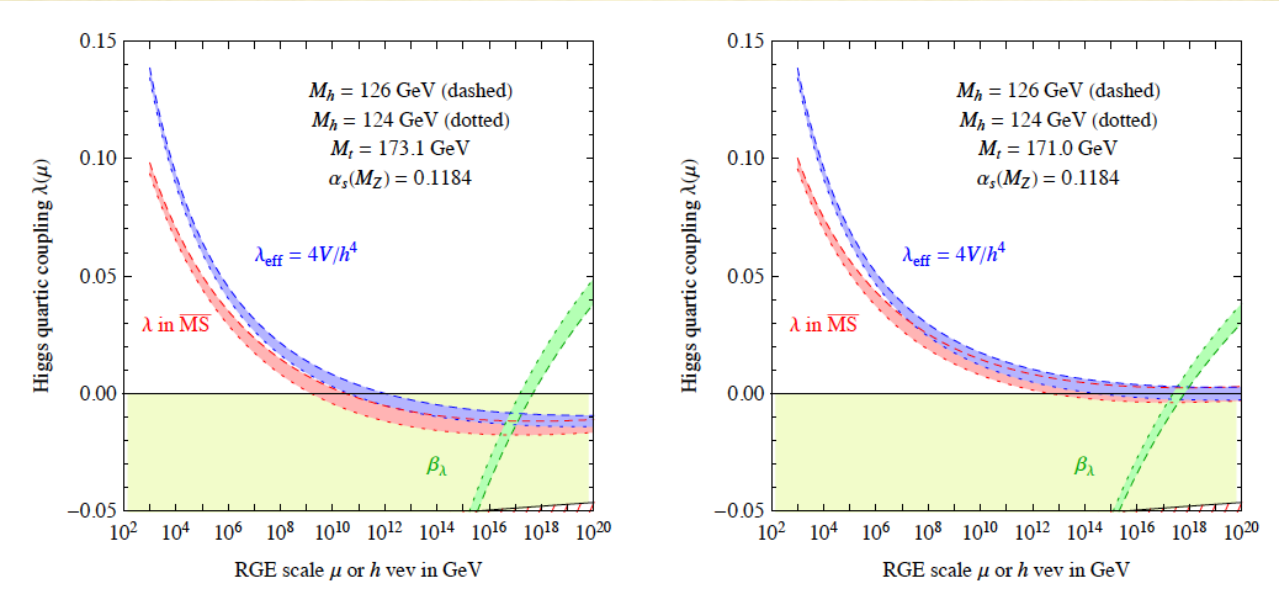
Higgs mass and vacuum stability in the Standard Model at NNLO.

Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia '12

Vacuum stability condition:

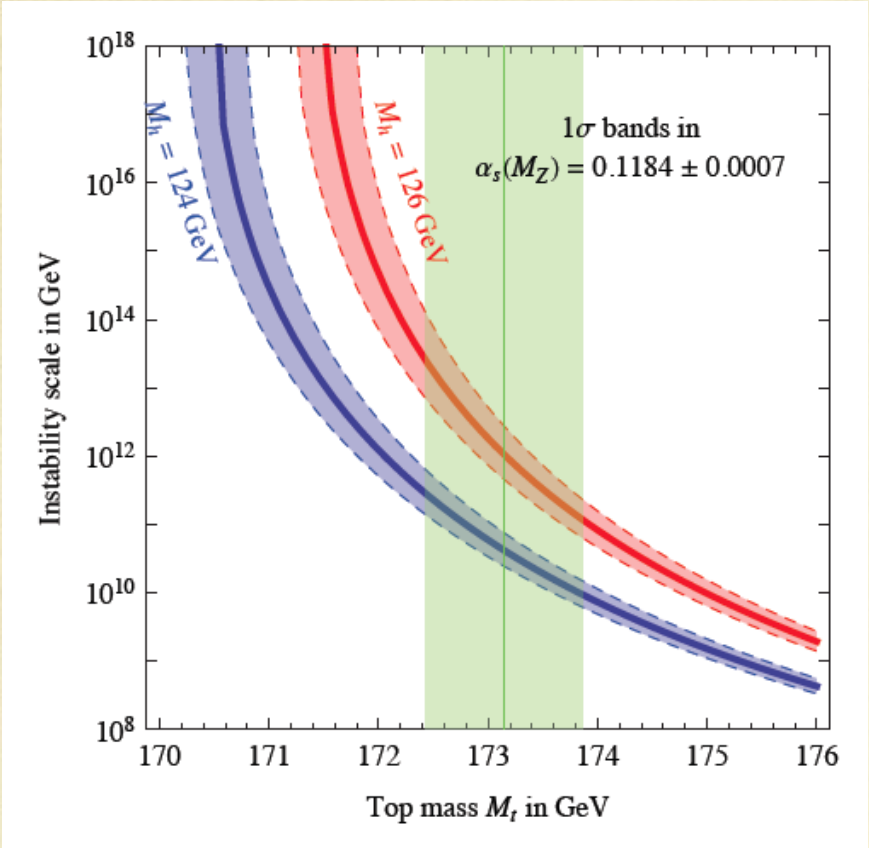
$$V_{\rm eff} = -\frac{m^2}{2}h^2 + \frac{\lambda}{4}h^4 + \Delta V$$

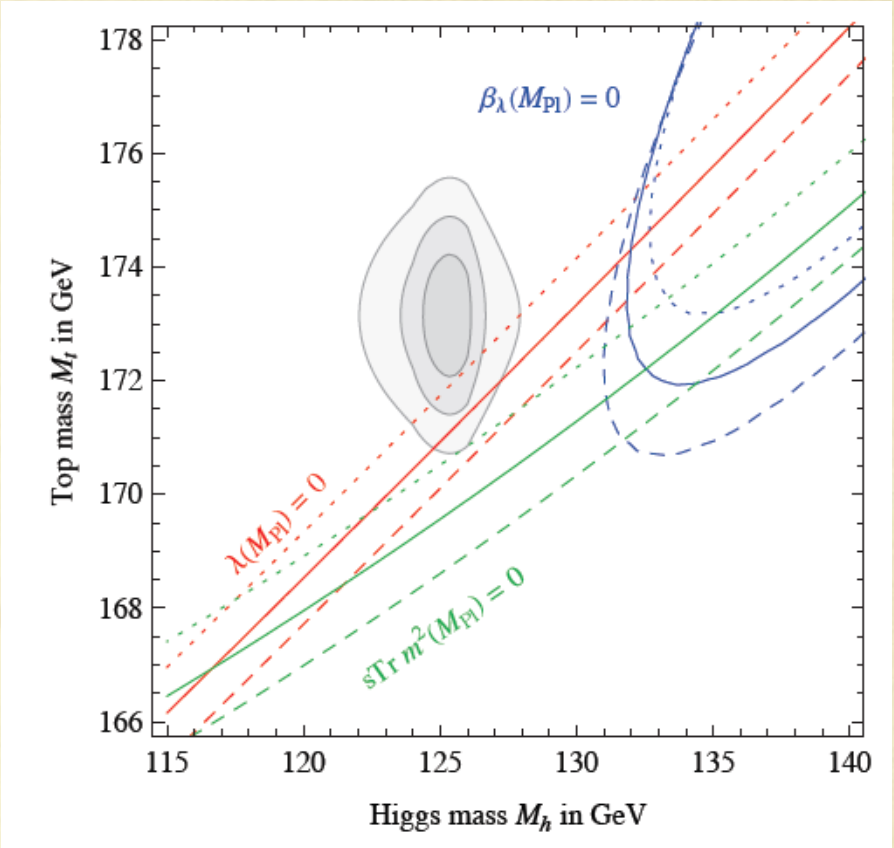
Quantum corrections (included)



Higgs mass and vacuum stability in the Standard Model at NNLO

Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice, Isidori, Strumia '12





Possible implication:

For the right values of the SM parameters (and we are right there) SM might survive the Desert.

✓ Currently a big push for better understanding of the top mass. Precision is crucial here…

Top quark mass: some thoughts

- ✓ The apparent sensitivity to m_{top} requires convincing m_{top} determination (but not for EW fits)
- ✓ What do I mean by convincing?
  - ✓ m_{top} is not an observable; cannot be measured directly.
  - ✓ It is extracted indirectly, through the sensitivity of observables to m_{top}

$$\sigma^{\rm exp}(\{Q\}) = \sigma^{\rm th}(m_t, \{Q\})$$

  - ✓ The implication: the "determined" value of m_{top} is as sensitive to theoretical modeling as it is to the measurement itself

- ✓ A worry: can there be an additional systematic O(1 GeV) shift in m_{top} ?

- ✓ The measured mass is close to the pole mass (it decays …)
- ✓ One needs to go beyond the usual MC's to achieve theoretical control
- ✓ Lots of activity (past and ongoing). A big up-to-date review:

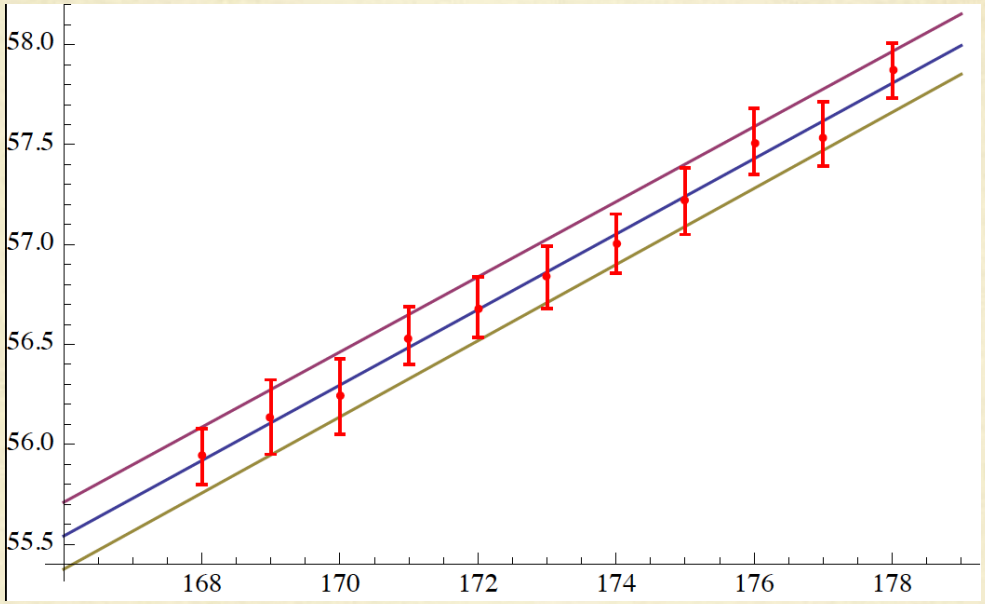
Juste, Mantry, Mitov, Penin, Skands, Varnes, Vos, Wimpenny '13

Top mass from leptonic distributions

- ✓ An example of an orthogonal approach (in NLO QCD)

Work with Frixione, Frederix

Measured Parameter



Predicted top mass

From this distribution, with zero exp error, we can extract m_{top} with error of 0.85 GeV

- ✓ One day, at NNLO, this can be improved.
- ✓ 8 TeV seems better than 14 TeV.

Summary and Conclusions

- Total x-section for tT production now known in full NNLO
- Result of a number of theoretical innovations
- Small scale uncertainty (2.2% Tevatron, 3% LHC). Similar to uncertainties from pdf, α_S, M_{top}
- Important phenomenology
  - Constrain and improve PDF's
  - Searches for new physics
  - Very high-precision test of SM (given exp is already at 5% !). Good agreement.

Future tasks

- This is the beginning of a new stage in precision phenomenology
  - Differential top production, with decays (NWA). A_{FB} to appear soon.
  - Any process can be computed (subject to CPU) given 2-loop amplitudes exist
  - H+1jet was already computed (expect related Z,W+jet) at NNLO

    Boughezal, Caola, Melnikov, Petriello, Schulze '13
  - Full dijet @ NNLO will become available too

    Currie, Gehrmann-De Ridder, Gehrmann, Glover, Pires '13
  - WW, etc.