Correlation functions and amplitudes to five points in N=4 SYM

Paul Heslop

Durham University

Integrable Structures in Scattering Amplitudes: IPPP Durham

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based on work with:
Eden, Korchemsky, Sokatchev (arxiv:1108.3557,1201.5329)
Ambrosio, Eden, Goddard, Taylor (arXiv:1312.1163)
Chicherin, Doobary, Eden, Korchemsky, Mason, Pollock, Sokatchev, in progress

Outline

Four-point correlation functions in planar $\mathcal{N} = 4$ SYM

- Summarise recent progress over last two or three years

Amplitudes in planar $\mathcal{N} = 4$ SYM

- Obtain 5 point amplitude integrand from 4 point correlator

Five (and higher) point correlator (details in Eden's talk): Twistor/ Grassmannian / Amplituhedron-like approach to correlators in planar $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM? Prototype gauge theory

$SU(N)$ 4d gauge theory, 't Hooft coupling 'a':

- Gauge field
- 6 massless scalar fields (adjoint rep) ϕ_{AB}
- 4 massless fermions (adjoint rep)

- Finite, conformally invariant ("a" a freely tunable parameter)

The simplest $d = 4$ quantum field theory.

Starting point in our quest to properly understand 4d QFT.

- AdS/CFT correspondence

Correlators

(Correlation functions of gauge invariant operators)

- *Gauge invariant operators:* gauge invariant products (ie traces) of the fundamental fields
- Simplest operator trϕ^2 (ϕ one of the scalars)
- The simplest *non-trivial* correlation function is

$$G_4 := \langle \mathcal{O}(x_1)\bar{\mathcal{O}}(x_2)\mathcal{O}(x_3)\bar{\mathcal{O}}(x_4)\rangle \qquad \mathcal{O} = \mathrm{Tr}(\phi_{12}\phi_{12})$$

- $\mathcal{O} \in$ stress energy supermultiplet. We consider correlators of all operators in this multiplet.
- onshell Lagrangian $\in$ same supermultiplet

Why are they interesting?

AdS/CFT

Supergravity/String theory on $AdS_5 \times S^5 \quad = \quad \mathcal{N}$=4 super Yang-Mills

- Correlation functions of gauge invariant operators in SYM $\leftrightarrow$ string scattering in *AdS*
- strong coupling ("a"$\to \infty$) correlator computed from supergravity action [’d Hoker Freedman, Arutyunov Frolov]
- much studied at weak coupling ($a \to 0$) (1- and 2-loops [Eden Schubert Sokatchev]).
  Many attempts 8 or 9 years ago at 3-loops, abandoned until recently...
- Contain data about anomalous dimensions of operators and 3 point functions (integrability) via OPE
- Big Bonus of last 3 years Correlators contain all scattering amplitudes (more later)

Four-point correlator

- Hidden (permutation) symmetry uniquely fixes the four-point planar correlator/amplitude to 3 loops [Eden Korchemsky Sokatchev PH]
- Fixes 4 loops planar to 3 constants
- 5 loops planar to 7 constants
- 6 loops planar to 36 constants
- All constants fixed uniquely by examining divergences (non-planar, few remaining constants)
- (Alternatively fixed from the four-point amplitude)

Hidden symmetry

Superconformal symmetry implies

$$\langle \mathcal{O}_{\Lambda_1}\mathcal{O}_{\Lambda_2}\mathcal{O}_{\Lambda_3}\mathcal{O}_{\Lambda_4}\rangle = \langle \mathcal{O}_{\Lambda_1}\mathcal{O}_{\Lambda_2}\mathcal{O}_{\Lambda_3}\mathcal{O}_{\Lambda_4}\rangle_{\text{tree}} + \mathcal{I}_{\Lambda_1\Lambda_2\Lambda_3\Lambda_4}(x_i) \times f(x_i;a)$$

- Any four-point correlator is given in terms of a single function of x_i [Eden Schubert Sokatchev]
- integrand $f(x_i;a) = \sum_{\ell=1}^{\infty} \frac{a^\ell}{\ell!} \int d^4x_5 \dots d^4x_{4+\ell}\, f^{(\ell)}(x_1,\dots,x_{4+\ell})$

Hidden symmetry:

$$f^{(\ell)}(x_1,\dots x_{4+\ell}) = f^{(\ell)}(x_{\sigma_1},\dots x_{\sigma_{4+\ell}}) \qquad\qquad \forall\, \sigma \in S_{4+\ell}$$

- NB, the symmetry mixes external variables $x_1,\dots x_4$ with integration variables $x_5 \dots x_{4+\ell}$

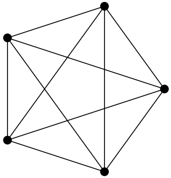
1-, 2- and 3-loop integrands

- Entire correlator defined (perturbatively) via $f^{(\ell)}$
  - conformal weight 4 at each point
  - permutation invariant
  - No double poles (from OPE)
- Naively equivalent to: degree (valency) 4 graphs on $4+\ell$ points

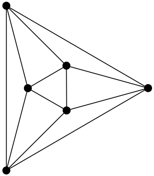
$$\text{graph edge} = \frac{1}{x_{ij}^2}$$

- (But: we are also allowed numerator lines $\Rightarrow$ degree ≥ 4 graphs).
- Don't need to label graph since we sum over permutations $\Rightarrow$ sum over all possible ways of labelling

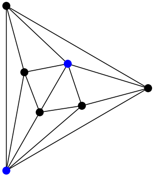
$$f^{(1)} = \frac{1}{\prod_{1 \le i < j \le 5} x_{ij}^2}$$



$$f^{(2)} = \frac{x_{12}^2 x_{34}^2 x_{56}^2 \; + \; S_6 \text{ perms}}{\prod_{1 \le i < j \le 6} x_{ij}^2}$$

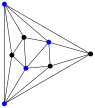
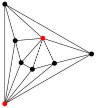
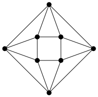


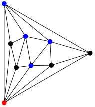
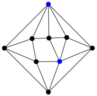
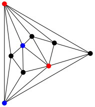
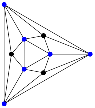
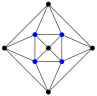
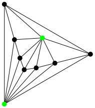
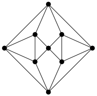
$$f^{(3)} = \frac{(x_{12}^4)(x_{34}^2 x_{45}^2 x_{56}^2 x_{67}^2 x_{73}^2) \; + \; S_7 \text{ perms}}{\prod_{1 \le i < j \le 7} x_{ij}^2}$$



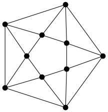
Unique (planar) possibilities

Four- and five-loops

$f^{(4)} =$  $+$  $-$ 

$f^{(5)} =$  $-$  $+$  $+$  $-$  $+$  $+$ 

- Very compact writing
- All come with coefficients 1,-1
- From 6 loops we start to see integrands with the coefficient 2 (and also 0), the first being:

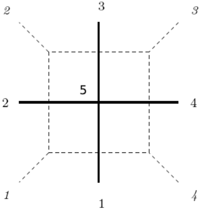


Integrals

* skip to amplitudes

- So much for the integrand. What about the integrals?
- f-graphs treat all points (external and internal) the same
- In order to do the integrals we need to distinguish external and internal points
- inequivalent choices of four external points give different integrals.
- 1 loop only one choice $f^{(1)} = \dfrac{1}{\prod_{1 \leq i < j \leq 5} x_{ij}^2} =$

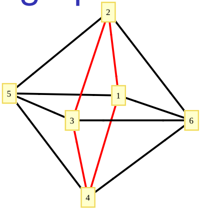
$$\frac{1}{\prod_{1 \leq i < j \leq 4} x_{ij}^2} \times \frac{1}{x_{15}^2 x_{25}^2 x_{35}^2 x_{45}^2} =$$



$$:= g_{1234}$$

- The massive (offshell) box function

From f-graphs to integrals at 2 loops



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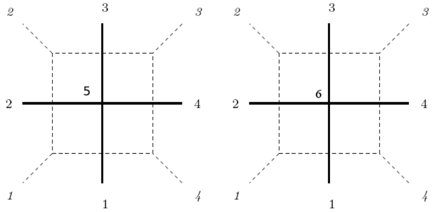

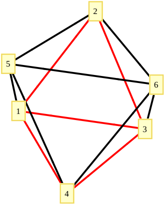

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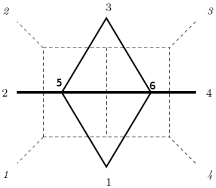
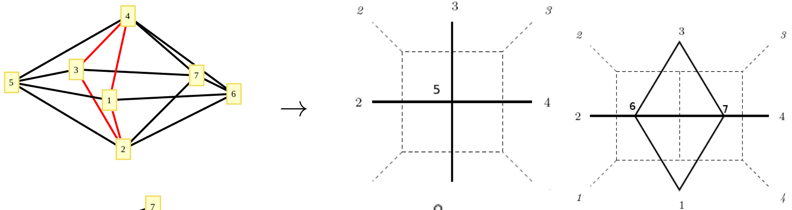
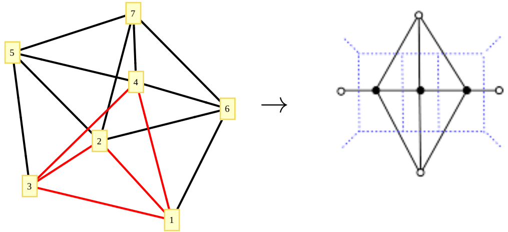


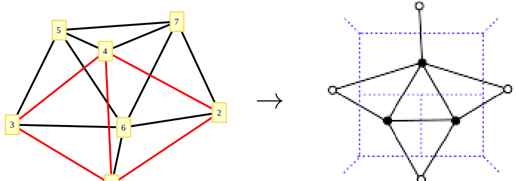
$:= h_{13;24}$

- Procedure: Take any four points in the f-graph
- Ignore all lines between these vertices to obtain the integrands
- (one loop box)^2 + (two loop ladder)

3 loop integrals

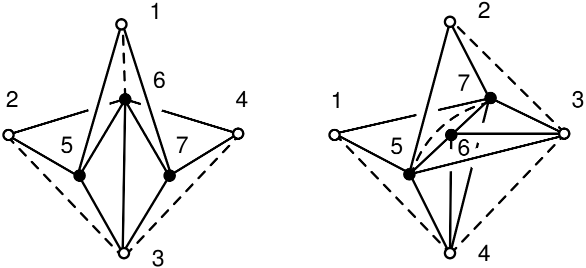






Integrals

- All these integrals are completely known analytically: massive ladder integrals
  [Davydychev Ussyukina]
- Two new three-loop integrals surviving away from the light-like limit. The dashed lines between adjacent external points indicate that these integrals vanish in the limit.
- Not known analytically previously (no corresponding momentum space graph)



$E_{12;34}$ $H_{12;34}$

Method of computation

[Drummond Duhr Eden Pennington Smirnov PH]

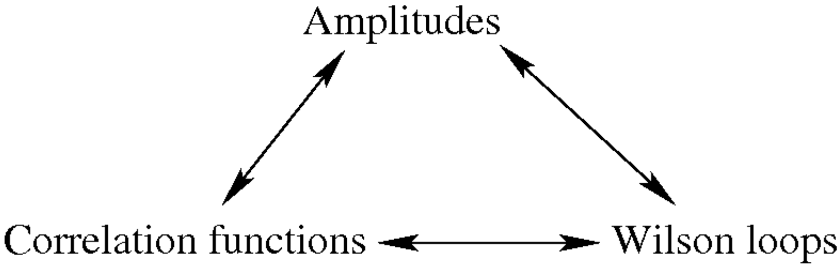
Method/ assumptions:

- The integrals are of the form: $\sum$ rational*(pure polylogs)
- The rational pieces can be computed via leading singularities
- Pure polyog term has a symbol with letters $x, \bar{x}, 1-x, 1-\bar{x}, x-\bar{x}$
- Finally we are able to compute the integral in various limits $u \to 0, 1, \infty$
- match with ansatz: can completely fix the answer

Uses several techniques available only in the last few years arising out of the amplitude context.
(Schnetz and Panzer study similar integrals also at 4 loops.)

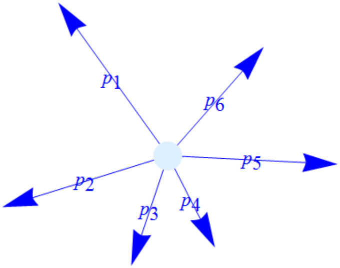
Relation to amplitudes

- triality between three objects in $\mathcal{N} = 4$ SYM

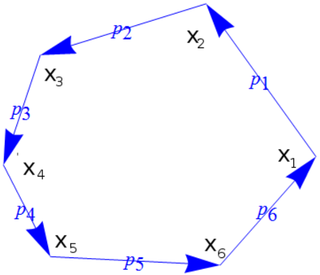


[Alday Maldacena, Drummond Henn Korchemsky Sokatchev, Brandhuber Travaglini PH, Mason Skinner, Caron-Huot, Alday Eden Korchemsky Maldacena Sokatchev, Eden Korchemsky Sokatchev PH, Adamo Bullimore Mason Skinner, ...]

Triality



$$\frac{\text{Full planar superamplitude}}{\text{MHV tree}}$$



super Wilson loop (vev)

$$\lim_{x_{i\,i+1}^2 \to 0} \frac{\langle \mathcal{T}(x_1, \rho_1, y_1)\mathcal{T}(x_2, \rho_2, y_2)\ldots\mathcal{T}(x_n, \rho_n, y_n)\rangle}{\langle \mathcal{T}(x_1, \rho_1, y_1)\mathcal{T}(x_2, \rho_2, y_2)\ldots\mathcal{T}(x_n, \rho_n, y_n)\rangle_{\text{tree}}}$$

Full super-correlation function (ys completely factorise)

skip superspace

Superspaces: superamplitudes

- Use Nair's $\mathcal{N}$=4 on-shell superspace, all particles $\rightarrow$ superparticle

super-particle

$$\Phi(p,\eta) = G^+(p) + \eta\psi + \eta^2\phi(p) + \eta^3\bar{\psi}(p) + \eta^4 G^-(p)$$

- All amplitudes $\rightarrow$ superamplitudes

$$A(x_i) \rightarrow \mathcal{A}(x_i, \eta_i)$$

super-amplitude structure: $\mathcal{A}(x_i, \eta_i) =$

$$\begin{aligned}&[\eta^8] A_{MHV} + [\eta^{12}] A_{NMHV} + [\eta^{16}] A_{NNMHV} + \ldots + [\eta^{4(n-2)}] A_{\overline{MHV}}\\ =&A^{\text{tree}}_{MHV}\left(\hat{A}_{MHV} + [\eta^4]\hat{A}_{NMHV} + [\eta^8]\hat{A}_{NNMHV} + \ldots + [\eta^{4(n-4)}]\hat{A}_{\overline{MHV}}\right)\end{aligned}$$

Superspace: correlation functions

Similar expansion for correlation functions:

energy momentum supermultiplet

$$\mathcal{T}(x,\rho,\bar{\rho}=0,y)=\mathcal{O}(x,y)+\ldots+\rho^4\mathcal{L}(x),$$

- correlation function of $\mathcal{T}$s: ρ-expansion organised in powers of ρ^{4k}

Similar superspace expansion to the superamplitude

$$\begin{aligned} G_n|_{\bar{\rho}=0} &:= \langle \mathcal{T}(1)\mathcal{T}(2)\ldots\mathcal{T}(n)\rangle \\ &= G_{n;0}+\left[\rho^4\right]G_{n;1}+\left[\rho^8\right]G_{n;2}+\cdots+\left[\rho^{4(n-4)}\right]G_{n;n-4} \end{aligned}$$

- $G_{n;n-4}$ is known (at least for $n \leq 11$ given by $f^{(n-4)}$). Very little known about the others.

Superamplitude/ supercorrelation function duality

Superduality

$$\lim_{x^2_{i\,i+1}\to 0} \frac{\langle \mathcal{T}(1)\dots\mathcal{T}(n)\rangle}{\langle \mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{\text{tree}}_{n;0}}(x,\rho,\bar\rho=0,y) \;=\; \Big(\frac{\mathcal{A}_n}{\mathcal{A}^{\text{tree}}_{n;\text{MHV}}}(x,\eta)\Big)^2$$

- duality works at the level of the integrand...
- Amplitude written in terms of dual/region momenta
- $p_i = x_i - x_{i+1}$
- NB both sides are expansions both in superspace variables and in the coupling (loop expansion).
- Both sides start $1 + \dots$.

Four-point and five-point duality

- **4-points:** $\langle \mathcal{T}(1)\ldots\mathcal{T}(4)\rangle = G_{4;0}$ $\qquad \dfrac{\mathcal{A}_4}{\mathcal{A}^{\text{tree}}_{4;\text{MHV}}} := \mathcal{M}_4$
- **5-points:**
  $\langle \mathcal{T}(1)\ldots\mathcal{T}(5)\rangle = G_{5;0} + G_{5;1}$ $\qquad \dfrac{\mathcal{A}_5}{\mathcal{A}^{\text{tree}}_{5;\text{MHV}}} := \mathcal{M}_5 + \dfrac{\mathcal{A}^{\text{tree}}_{5;\text{NMHV}}}{\mathcal{A}^{\text{tree}}_{5;\text{MHV}}}\overline{\mathcal{M}}_5$
- Expanding out the square in the superduality:

Four points

$$\lim_{4\textit{pointlightlike}} G_4/G_4^{\text{tree}} = (\mathcal{M}_4)^2$$

Five points

$$\lim_{5\textit{pointlightlike}} G_{5;0}/G_{5;0}^{\text{tree}} = (\mathcal{M}_5)^2 \qquad \lim_{5\textit{pointlightlike}} G_{5;1} = 2\frac{\mathcal{A}^{\text{tree}}_{5;\text{NMHV}}}{\mathcal{A}^{\text{tree}}_{5;\text{MHV}}}\mathcal{M}_5\overline{\mathcal{M}}_5$$

Functions $\mathcal{M}$ depend on bosonic variables and the 'tHooft coupling a only (but no η dependence.)

Integrands = correlators with Lagrangian insertions

- Loop corrections $\Rightarrow$ Lagrangian insertions.

1 loop correlator

$$\langle \mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{(1)} = \int d^4x_0\,\langle \mathcal{L}(x_0)\mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{(0)}$$
$$= \int d^4x_0 d^4\rho_0 \langle \mathcal{T}(0)\mathcal{T}(1)\dots\mathcal{T}(n)\rangle^{(0)}|_{\rho_i=0}$$

- so the *Born-level* $(n+1)$-point correlator with the (chiral part of the on-shell) Lagrangian inserted at new point x_0 defines the 1 loop integrand
- ℓ-loops $\Rightarrow$ ℓ Lagrangian insertions $\Rightarrow$ $n+\ell$-point tree correlator
- NB parameter space of amplitudes n, k, ℓ for correlators we only need n, k.

Correlator amplitude duality at 4,5 points

- So $G^{(\ell)}_{5;1} = G^{(\ell+1)}_{4;0}$ at the integrand level
- Both four-point and five-point amplitudes are given in terms of the same objects: f-graphs

- external factor $\times \lim_{\substack{x^2_{i\,i+1}\to 0 \\ (\text{mod } 4)}} \int d^4x_5 \ldots d^4x_{4+\ell} \frac{f^{(\ell)}}{\ell!} := F_4^{(\ell)} = (\mathcal{M}_4)^2$
- external factor $\times \lim_{\substack{x^2_{i\,i+1}\to 0 \\ (i=1\ldots 5)}} \int d^4x_6 \ldots d^4x_{5+\ell} \frac{f^{(\ell+1)}}{\ell!} := F_5^{(\ell)} = \mathcal{M}_5 \overline{\mathcal{M}}_5$

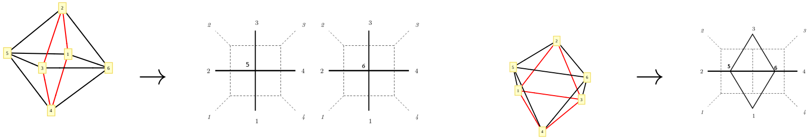
Amplitude information from $f^{(2)}$ (octahedron)

- Having expanded in fermionic variables, we now expand in the coupling a: $\mathcal{M}_n := 1 + a\mathcal{M}_n^{(1)} + a^2\mathcal{M}_n^{(2)} + a^3\mathcal{M}_n^{(3)} + \dots$

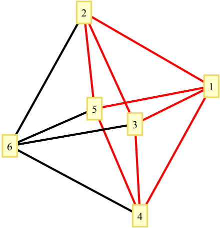
Octahedron $f^{(2)} \rightarrow F_4^{(2)}, F_5^{(1)}$

$$F_4^{(2)} = 2\mathcal{M}_4^{(2)} + \left(\mathcal{M}_4^{(1)}\right)^2 \qquad\qquad F_5^{(1)} = \mathcal{M}_5^{(1)} + \overline{\mathcal{M}}_5^{(1)}$$

Graphically at four points, recall:



Graphically at **five**-points (one loop):



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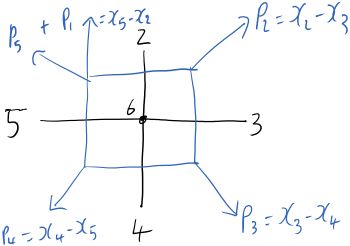
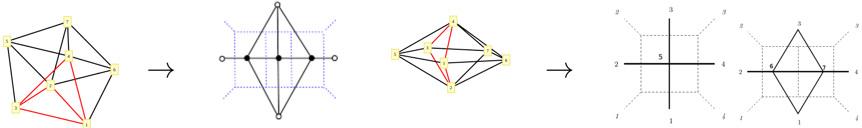
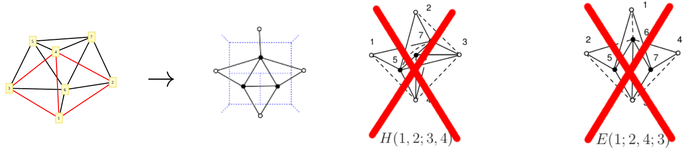


- Summing all permutations gives the sum over 1 mass box functions = parity even 1-loop 5-point amplitude [Bern Dixon Dunbar Kosower 1994].
- Also a well known parity odd part $O(\epsilon)$ but important eg in BDS
- To see this let's consider the next order...

Four-points, 3 loop (from $f^{(3)}$)

We have $F_4^{(3)} = 2\mathcal{M}_4^{(3)} + \mathcal{M}_4^{(1)}\mathcal{M}_4^{(2)}$

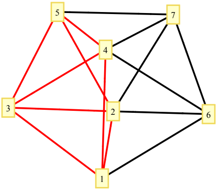




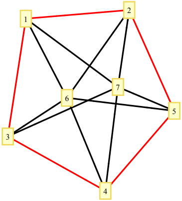
- Last two topologies absent for the amplitude: vanish in limit
- Graphically: the four external points we pick need to be connected consecutively: four-cycle.
- Four-cycle splits the planar graph into two pieces. Correspond to product terms.

Five-points, 2 loop and parity odd 1 loop (from $f^{(3)}$)

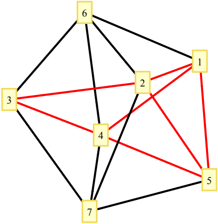
- We have $F_5^{(2)} = \mathcal{M}_5^{(2)} + \overline{\mathcal{M}}_5^{(2)} + \mathcal{M}_5^{(1)}\overline{\mathcal{M}}_5^{(1)}$
- If the 5-cycle splits the f-graph in two it contributes to $\mathcal{M}_5^{(1)}\overline{\mathcal{M}}_5^{(1)}$ otherwise to $\mathcal{M}_5^{(2)} + \overline{\mathcal{M}}_5^{(2)}$



2 loop ladder



pentagon^2



box$\times$ box

- We now have two equations ($\mathcal{M}_5^{(1)} + \overline{\mathcal{M}}_5^{(1)} =$ sum over boxes and $\mathcal{M}_5^{(1)}\overline{\mathcal{M}}_5^{(1)} =$ products) for two unknowns $\mathcal{M}_5^{(1)}$ and $\overline{\mathcal{M}}_5^{(1)}$,

The equation is quadratic and has solution

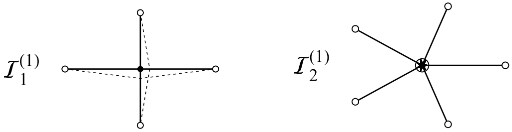
$$M_5^{(1)} = \frac{1}{2}\left(F_5^{(1)} \pm \sqrt{(F_5^{(1)})^2 - 4F^{(2)}_{5,\text{products}}}\right)$$

- One can check that this simplifies very nicely to:

$$M_5^{(1)} = \frac{1}{2}\left(\mathcal{I}_1^{(1)} + \mathcal{I}_2^{(1)}\right) \; .$$

$$\mathcal{I}_1^{(1)} = \text{cyc}\left[\frac{x_{13}^2 x_{25}^2}{x_{16}^2 x_{26}^2 x_{36}^2 x_{56}^2}\right] \quad \mathcal{I}_2^{(1)} = \text{cyc}\left[\frac{i\epsilon_{123456}}{x_{16}^2 x_{26}^2 x_{36}^2 x_{46}^2 x_{56}^2}\right]$$

- The terms are displayed graphically as



- The starred vertex v indicates a factor $i\epsilon_{12345v}$.

M_4 as $6d$ Embedding space

- Extremely useful to view M_4 as set of null 6 vectors in $\mathbb{P}^5$ with $(2,4)$ signature
- conformal group $SO(2,4)$ acts linearly
- invariant tensors are δ_{IJ} and (parity odd) ϵ_{ABCDEF} (like Lorentz invariants)
- for conformally invariant quantities $x_{ij}^2 \to X_i.X_j$
- $\epsilon_{123456} := \epsilon_{ABCDEF} X_1^A X_2^B X_3^C X_4^D X_5^E X_6^F$

$$\begin{aligned} &\epsilon_{123456} \times \epsilon_{123457} \\ =&\mathrm{cyc}\left(2x_{67}^2 x_{13}^2 x_{24}^2 x_{35}^2 x_{14}^2 x_{25}^2 + x_{13}^4 x_{24}^2 x_{25}^2 x_{46}^2 x_{57}^2 - x_{13}^4 x_{24}^4 x_{56}^2 x_{57}^2 - x_{13}^2 x_{14}^2 x_{24}^2 x_{25}^2 x_{36}^2 x_{57}^2\right) \end{aligned}$$

= - argument of square root.

Story continues to higher loops (from $f^{(3)}$ and $f^{(4)}$)

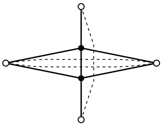
$$F^{(\ell)} = M_5^{(\ell)} + \overline{M}_5^{(\ell)} + \sum_{m=1}^{\ell-1} M_5^{(m)} \overline{M}_5^{(\ell-m)}$$

$$F^{(\ell+1)} = M_5^{(\ell+1)} + \overline{M}_5^{(\ell+1)} + M_5^{(\ell)} \overline{M}_5^{(1)} + M_5^{(1)} \overline{M}_5^{(\ell)} + \sum_{m=2}^{\ell-1} M_5^{(m)} \overline{M}_5^{(\ell-m+1)}$$

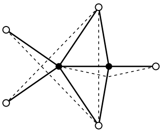
The full two-loop amplitude is

$$M_5^{(2)} = \frac{1}{2 \times 2!} \left(\mathcal{I}_1^{(2)} + \mathcal{I}_2^{(2)} + \mathcal{I}_3^{(2)} \right)$$

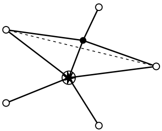
$\mathcal{I}_1^{(2)}$



$\mathcal{I}_2^{(2)}$



$\mathcal{I}_3^{(2)}$



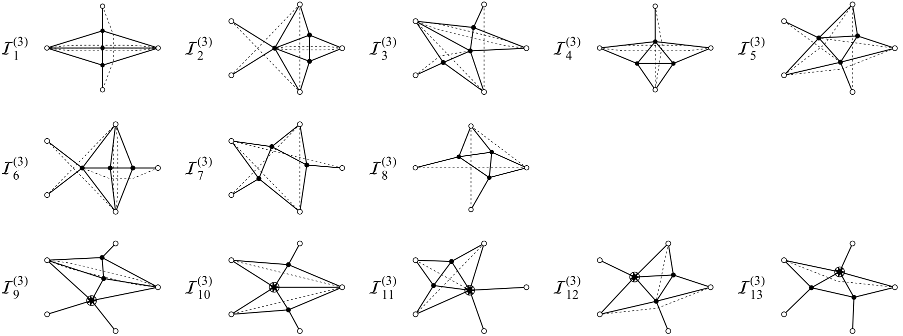
(To help find the result we have conjectured that the only parity odd object is ϵ_{12345v} (Never get two internal variables in an ϵ.)

...and higher loops (from $f^{(4)}$ and $f^{(5)}$)

The full three-loop amplitude is

$$M_5^{(3)} = \frac{1}{2.3!} \int d^4x_6 d^4x_7 d^4x_8 \left(\sum_{i=1}^{13} c_i \mathcal{I}_i^{(3)} \right)$$

$$c_1 = \cdots = c_6 = c_9 = \ldots c_{12} = 1 \qquad c_7 = c_8 = c_{13} = -1$$



...and higher loops

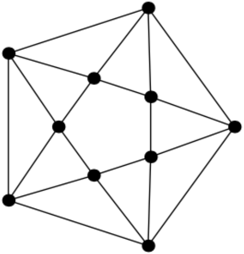
- We have up to $f^{(7)}$ and thus we have $\mathcal{M}_5^{(5)}$ completely and $\mathcal{M}_5^{(6)}$ (parity even part).

- In progress: determine parity even in terms of parity odd

- Understand how cancellation of non-planar graphs works

- Understand how this determines correlator coefficients (extension of rung rule - just consistency determines everything up to $f^{(5)}$)

(NB But still not the intriguing $f^{(6)}$ graph occurring with coefficient 2)



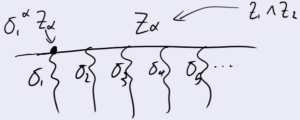
Twistor approach to correlators

Twistor Wilson loop [Bullimore Mason Skinner]

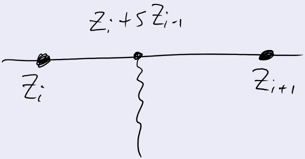
Compute using twistor Feynman rules

$$\Delta_*(Z_1, Z_2) = \int_{\mathbb{CP}^2} \frac{D^2 c}{c_1 c_2 c_3}\, \delta^{4|4}(c_1 Z_1 + c_2 Z_2 + c_3 Z_*) =$$

$$\int d^{4|8}Z \int \frac{(d^2\sigma_1\, d^2\sigma_2 \cdots)}{\sigma_1\cdot\sigma_2\ \ \sigma_2\cdot\sigma_3 \cdots}\ (\text{Internal vertex}) =$$



$$\int \frac{ds}{s}\ \ (\text{External vertex}) =$$



Key point for correlator application

- Internal vertex corresponds to insertion of the action as always
- Action = $g^2 \int_\Gamma d^4x d^8\theta \log\det \mathcal{D}|_x = g^2 d^4x d^4\rho\, \mathcal{T}(x,\rho,\bar\rho,Y)$
- So $\mathcal{T} = Y^{IJ}Y^{KL}\int d^4\theta_{IJ,KL} \log\det\mathcal{D}|_x$
- $\langle \mathcal{T}_1 \dots \mathcal{T}_n\rangle = \left(\prod_{i=1}^n Y_i^{IJ}Y_i^{KL}\int d^4\theta_{i;IJ,KL}\right) \times \sum$ n-point vacuum (ie no external Wilson loop) twistor diagrams (stripped of their $d^{4|8}Z$ integrations.)
- Chiral and Invariant under superconformal. (CF analytic superspace, analytic and covariant).
- External data (x_i, θ_i) are the insertion "vertices/lines".
- Checked agreement explicitly for 5-point correlator and 6-point correlator, ρ^4 component.
- NB. Works like this only for $\mathcal{T}$ correlators. For higher charge correlators will need covariants again.

Twistor diagrams for the correlator

Two of the 6-point ρ^4 twistor diagrams

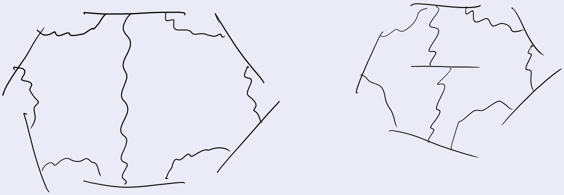


figure for talks Page 1

- Lightlike limit $\rightarrow$ amplitude is automatic diagram by diagram
- Twistor rules imply that an n-point correlator of odd degree θ^{4k} has $n + k$ propagators. (Here $n = 6$ $k = 1$)
- Checked: agrees with Feynman computation (with Doobary)
- Miracle is that this is independent of Z^*

Grassmannians from twistor rules

[Mason Skinner, Arkani-Hamed Cachazo Cheung Kaplan]

- Close relation between twistor rules and Grassmannians for amplitudes.
- k propagators n external twistor points $\to Gr(k,n)$ Grassmannian
- $\displaystyle\int \frac{d^{k\times(n-k)}C}{(C_1\dots C_k)\dots(C_n\dots C_{k-1})}\delta^{4k|4k}(\sum_i C_i^a \mathcal{Z}_i)$
- eg 5 point NMHV ($k=1$) $\displaystyle\int \frac{d^4C}{(C_1)\dots(C_5)}\delta^{4|4}(\sum_i C_i \mathcal{Z}_i)$

Grassmannian form for maximally nilpotent correlators

- Twistor rules for correlators ($n+k$ propagators ending on n twistor lines).
- Suggests $Gr(n+k,2n)$ Grassmanian form.

Eg 5-point θ^4 correlator (corresponding to 4-point 1 loop)

$$\int d^{24}C\,\delta^{24|24}(C^a{}_{\alpha i}\mathcal{Z}^{\alpha i})\frac{1}{\prod_{i<j<k}(ijk)}$$

$T^a{}_{\alpha i} \in Gr(6,10)$

$(ijk) := C^a{}_{i1}C^b{}_{i2}C^c{}_{j1}C^d{}_{j2}C^e{}_{k1}C^f{}_{k1}\epsilon_{acdef}$

- Higher point maximally nilpotent ($\theta^{4(n-k)}$) correlators can be defined similarly. Integration measure more complicated.

Amplituhedron form

- Grassmannian formulae can be converted into "amplituhedron formulae" (Cells of the amplituhedron) [Arkani-Hamed Trnka].
- Superdeltafunctions $\rightarrow$ purely bosonic delta functions.
  - Introduce $4 \times k$ Grassman odd variables $\phi^{I}{}_{\underline{a}}$
  - Supertwistor $\mathcal{Z}_i = (Z_i|\chi_i) \in P^{3|4} \rightarrow Z_{i\underline{A}} = (Z_{iA}, \chi_{iI}\phi^{I}_{\underline{a}}) \in P^{3+k}$. $(a, \underline{a} = 1 \ldots k,\ \underline{A} = 1 \ldots (4+k))$
  - Further define $Y^{a}{}_{\underline{A}} = \sum_i C^a_i Z_{i\underline{A}}$
  - $\delta^{4k|4k}(\sum_i C^a_i \mathcal{Z}_i) \rightarrow \int d^{4k}\phi^{I}{}_{\underline{a}}\, \delta^{k\times(4+k)}(Y^{a}{}_{\underline{A}}; Y^{a}_{0}{}_{\underline{A}})$
  - $Y^{a}_{0}{}_{\underline{A}} = (0_{k\times 4}, 1_{k\times k})$

Eg Five-point amplitude

$$\int d^4\phi \int \frac{\langle Y d^4 Y\rangle\langle 12345\rangle^4}{\langle Y1234\rangle\langle Y2345\rangle\langle Y3451\rangle\langle Y4512\rangle\langle Y5123\rangle} \times \delta^4(Y_{\underline{A}}; Y_{0\,\underline{A}})$$

Towards the "correlahedron"

- Similar manipulations can be performed on correlators

Eg five-point $k = 1$ correlator

$$\int (\prod_{a=1}^{6} d^4\phi_a) \int d^{24}Y \, \frac{1}{\prod_{i<j}\langle Y X_i X_j\rangle} \langle X_1 X_2 \dots X_5\rangle^4 \delta^{6\times 10}(Y; Y_0)$$

$$X_i^{\underline{AB}} := Z_{i\alpha}{}^{\underline{A}} Z_i^{\alpha \underline{B}} \qquad \underline{A}, \underline{B} = 1 \dots 10$$

- Again, the rest of the maximally nilpotent $k = n - 4$ correlators can be written similarly
- Simply rewrite $f^{(\ell)}$ via $x_{ij}^2 \to \langle Y X_i X_j\rangle$

- So we can rewrite correlators in an amplituhedron-like form
- We can now do the same for the 6 point NMHV correlator also (see Eden talk)
- But the real beauty of the amplituhedron is that the amplitude is uniquely defined by the unique differential form in the space $Gr(k, k+4)$ with log divergences on the boundaries $\langle Yi(i+1)j(j+1)\rangle = 0$.
- Simple analogue $Gr(1, 3)$. n-point amplituhedron = polygon in two-dimensions. Y= points inside the polygon. Boundary $\langle Yi(i+1)\rangle = 0$.
- Loops can also be incorporated to the amplituhedron (but in a slightly different form.)
- Correlahedron should look like the loop sector of the amplituhedron.
- Can it then also then be defined as the (unique?) form on $Gr(n+k, n+k+4)$ with log divergences on the boundaries $\langle YX_iX_j\rangle = 0$ (which no longer form a closed geometry?)
- Must reproduce the loop level $(\text{amplitude})^2$ in all possible m-point lightlike limits

Conclusions

- We have found the analytic correlator up to three loops. Anomalous dimensions and three-point functions can be extracted. Integrability?
- General method for attacking other integrals
- Integrand level we have four-point correlator up to 7 loops (using four-point amplitude)
- Conversely use correlator to obtain 5-point amplitude (up to 5 loops or 6 loops parity even)

$$\begin{array}{c}\text{Four-point amplitude}\\ \Downarrow \\ \text{Four-point correlator}\\ \Downarrow \\ \text{Five-point amplitude}\end{array}$$

- Higher-point MHV amplitude from four-point correlator (disentangle mixing from $(\mathrm{NMHV})^2$??)

Conclusions (cont):

- 5-point correlator? (Work in progress - hard but we now have a good handle of this both from superconformal representation theory (see Eden's talk) and Twistor field theory)
- Amplituhedron/ positive Grassmanians/onshell diagrams: nice new insights concerning the amplitude integrand.
- We have seen how a single object contains many different amplitude integrands in different limits
- Generalisation of amplituhedron type ideas to the correlator....?

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