The Positive Orthogonal Grassmannian and Amplitudes in ABJM

Congkao Wen

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with Yu-tin Huang and Dan Xie

Queen Mary University of London

Integrable Structures in Scattering Amplitudes, IPPP Durham

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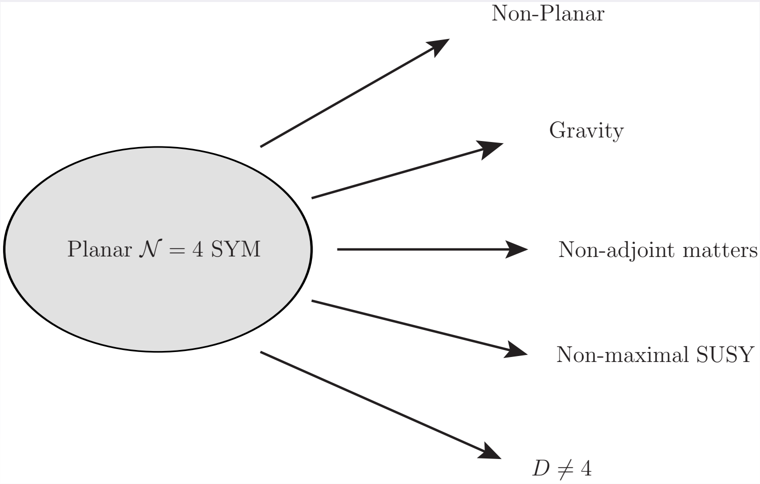
Where do we go from planar $\mathcal{N} = 4$ SYM?



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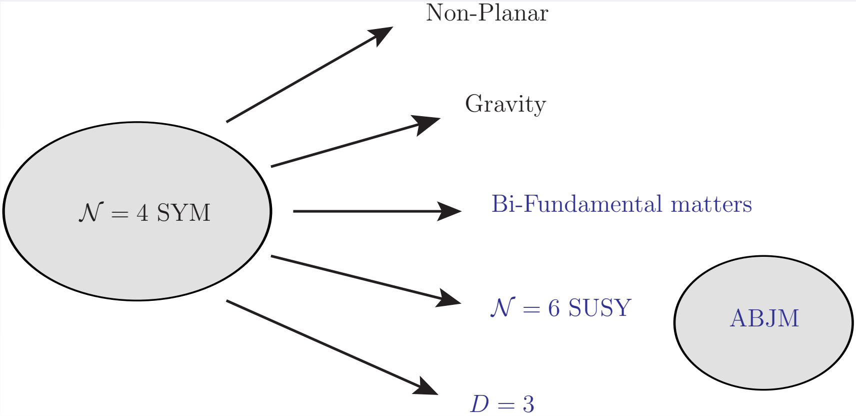

$\mathcal{N} = 4$ SYM v.s. ABJM

- SU(2|2, 4) dual conformal symmetry $\rightarrow$ Yes
  SU(2|2, 4) $\rightarrow$ OSp(6|4)

- (super)Wilson Loop/Correlation dual $\rightarrow$ Not yet

- Uniform transcendentality $\rightarrow$ Yes

  Test: Six-point at two loops, four-point at three loops

- Leading singularities given by residues of $Gr(k, n) \rightarrow$ Yes
  $Gr(k, n) \rightarrow OG(k, 2k)$,
  Test to Eight points; contour of Grassmannian integral was not clear

- Amplitudes as On-shell diagrams, and the connection to $Gr_+(k, n)$??

Lightning review on scattering amplitudes in ABJM

- ABJM theory is a 3D $\mathcal{N}=6$ Chern-Simons matter theory: Besides gauge fields $A_\mu, \hat{A}_\mu$, it has four complex scalars ϕ ($\bar{\phi}$) and fermions $\psi(\bar{\psi})$, transforming as $(N,\bar{N})((\bar{N},N))$ under $U(N)_k \times U(N)_{-k}$ gauge group. [Aharony, Bergman, Jafferis and Maldacena, 08']
- Dual to M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ or IIA string theory on $AdS_4 \times \mathbb{CP}_3$ when $k \to \infty$.

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- On-shell fields in terms of $\mathcal{N}=3$ superspace: [Agarwal, Beisert and McLoughlin, 08']

1, Matter fields

$$\Phi_a^{\bar{b}} = \phi^4 + \eta^A\psi_A + \frac{1}{2!}\epsilon_{ABC}\eta^A\eta^B\phi^C + \frac{1}{3!}\epsilon_{ABC}\eta^A\eta^B\eta^C\psi_4$$

$$\bar{\Phi}_{\bar{a}}^{b} = \bar{\psi}^4 + \eta^A\bar{\phi}_A + \frac{1}{2!}\epsilon_{ABC}\eta^A\eta^B\bar{\psi}^C + \frac{1}{3!}\epsilon_{ABC}\eta^A\eta^B\eta^C\bar{\phi}_4$$

2, Gauge fields have no physical degree of freedom.

Lightning review on scattering amplitudes in ABJM

- Scattering amplitudes in ABJM: Only amplitudes with **even number** of external legs are non-vanishing.

$$A(\Phi, \bar{\Phi}, \ldots, \Phi, \bar{\Phi})$$

or

$$A(\bar{\Phi}, \Phi, \ldots, \bar{\Phi}, \Phi)$$

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- Simplest example and building block: four-point amplitude at tree-level [Agarwal, Beisert and McLoughlin, 08']

$$A(\bar{1}, 2, \bar{3}, 4) = \frac{\delta^3(\sum_i \lambda_i^\alpha \lambda_i^\beta)\delta^6(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1\,2\rangle\langle 2\,3\rangle}$$

Spinor formalism in 3D

$$p_{\alpha\beta} \xrightarrow{\text{massless}} \lambda_\alpha\lambda_\beta\,,\ \langle i\,j\rangle := \ \epsilon^{\alpha\beta}\lambda_\alpha^i\lambda_\beta^j\,,\ (p_i + p_j)^2 = \langle i\,j\rangle^2$$

Grassmannian and Scattering amplitudes

Grassmannian and Scattering amplitudes

- Grassmannian $Gr(k,n)$: k-planes in n-dimensional vector space,

$$\begin{pmatrix} c_{11} & c_{12} & \dots & c_{1\,n-1} & c_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{k1} & c_{k2} & \dots & c_{k\,n-1} & c_{kn} \end{pmatrix}$$

mod $Gl(k)$ symmetry.

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  mod $Gl(k)$ symmetry.

- Orthogonal Grassmannian OG: k-planes in n-dimensional vector space equipped with a symmetric bi-linear Q^{ij}, with constraint

$$Q^{ij} c_{\alpha i} c_{\beta j} = 0$$

  Often one chooses $Q^{ij} = \delta^{ij}$.

Grassmannian and Scattering amplitudes

Grassmannian and Scattering amplitudes

- Positive Grassmannian $Gr_+(k,n)$: Ordered minors are Positive

$$(i_1\, i_2\, \ldots\, i_k) > 0 \quad \text{for} \quad i_1 < i_2 \ldots < i_k$$

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- Positive Orthogonal Grassmannian OG_+: cannot be defined for Euclidean signature

$$Q^{ij} = \delta^{ij} = (+\, +\, \ldots\, +\, +)$$

Instead we choose splitting signature to have the definition of OG_+

$$Q^{ij} = (+\, -\, \ldots\, +\, -)$$

It is a natural choice for scattering amplitudes in ABJM.

Grassmannian and Scattering amplitudes

Grassmannian and Scattering amplitudes

- Tree-level as well as all-loop leading singularities of $\mathcal{N}=4$ SYM are given by [Arkani-hamed, Cachazo, Chueng and Kaplan, 09']

$$\frac{1}{\mathrm{vol}[Gl(k)]}\oint\frac{d^{k\times n}C_{\alpha a}}{(1)(2)\cdots(n-1)(n)}\prod_{\alpha=1}^{k}\delta^{4|4}(C_{\alpha a}\mathcal{W}_a)$$

where $\mathcal{W}:=(\tilde{\mu},\tilde{\lambda},\eta)$ is the external data, with $\tilde{\mu}$ related to λ by Fourier transformation.

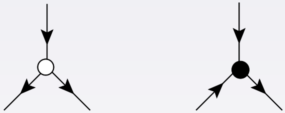
- Tree-level as well as all-loop leading singularities of ABJM theory are given by [Lee, 10']

$$\frac{1}{\mathrm{vol}[Gl(k)]}\oint\frac{d^{k\times 2k}C_{\alpha a}\,\delta(C\cdot C^T)}{(1)(2)\cdots(k)}\prod_{\alpha=1}^{k}\delta^{2|3}(C_{\alpha a}\mathcal{W}_a)$$

where $\mathcal{W}:=(\lambda,\eta)$

On-shell diagrams and Grassmannian in $\mathcal{N} = 4$ SYM

- On-shell diagrams in $\mathcal{N} = 4$ SYM built up by two possible three-point amplitudes: [Arkani-hamed, Bourjaily, Cachazo, Goncharov, Postnikov and Trnka, 12']



The same object appeared in supersymmetric quiver gauge theories: BFT!

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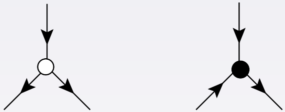

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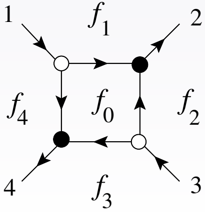
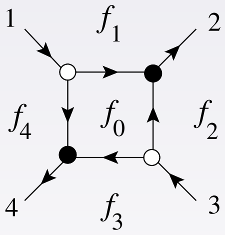
- Example: four-point tree-level amplitude



On-shell diagrams and Grassmannian in $\mathcal{N} = 4$ SYM

- From on-shell diagrams to Grassmannian



with $\prod_{i=0}^{n} f_i = 1$, and the corresponding Grassmannian

$$\begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 1 & 1/f_1 & 0 & -f_4 \\ 0 & f_2 & 1 & 1/f_3 \end{array} \right] \end{array}$$

It is a $Gr(2,4)_+$ after mod $GL(2)$. Positive region: $0 < f_i < \infty$

On-shell diagrams and Grassmannian in $\mathcal{N} = 4$ SYM

- Each on-shell diagram associates with a volume form.

  For leading singularities:

$$\int \frac{df_1}{f_1} \wedge \frac{df_2}{f_2} \wedge \ldots \wedge \frac{df_m}{f_m} \Leftrightarrow \frac{1}{\mathrm{vol}[Gl(k)]} \oint \frac{d^{k\times n} C_{\alpha a}}{(1)(2)\cdots(n-1)(n)}$$

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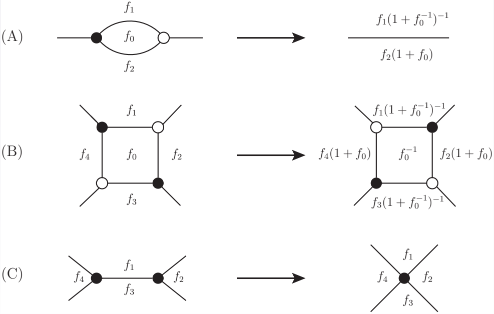
- On-shell diagram are equivalent under following moves

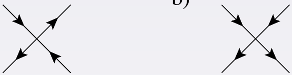


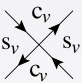
- Reduced diagrams can be characterized by permutations.

On-shell diagrams and Grassmannian in ABJM

- On-shell diagrams in ABJM built by four-point amplitude

a) b)





- Grassmannian (with $Q^{ij} = (+,-,+,-)$)

$$C_A = \begin{pmatrix} 1 & \cos\theta & 0 & -\sin\theta \\ 0 & \sin\theta & 1 & \cos\theta \end{pmatrix}, \quad C_B = \begin{pmatrix} 1 & 0 & -\cot\theta & -\csc\theta \\ 0 & 1 & \csc\theta & \cot\theta \end{pmatrix}$$

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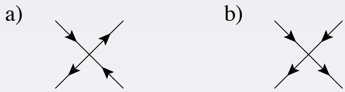

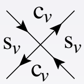

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- Volume form with logarithmic singularity at the boundary of positive region $(0, \frac{\pi}{2}) \Rightarrow$

$$\int d\log\tan(\theta) \Leftrightarrow \frac{1}{\text{vol}[Gl(2)]} \oint \frac{d^{2\times 4} C_{\alpha a}\, \delta(C\cdot C^T)}{(1)(2)} \Big|_{\text{one branch}}$$

Amplitudes as summing over two branches

- $k = 2$, namely four-point amplitude

$$C_+ = \begin{pmatrix} 1 & \cos\theta & 0 & -\sin\theta \\ 0 & \sin\theta & 1 & \cos\theta \end{pmatrix},$$

$$C_- = \begin{pmatrix} 1 & \cos\theta & 0 & \sin\theta \\ 0 & \sin\theta & 1 & -\cos\theta \end{pmatrix}.$$

$$\delta(C_+ \cdot \lambda) \Rightarrow \left\{ \begin{array}{c} \lambda_1 + \cos\theta\lambda_2 - \sin\theta\lambda_4 = 0 \\ \lambda_2 + \sin\theta\lambda_1 + \cos\theta\lambda_4 = 0 \end{array} \right\} \Rightarrow \langle 12 \rangle = \langle 34 \rangle$$

$$\delta(C_- \cdot \lambda) \Rightarrow \left\{ \begin{array}{c} \lambda_1 + \cos\theta\lambda_2 + \sin\theta\lambda_4 = 0 \\ \lambda_2 + \sin\theta\lambda_1 - \cos\theta\lambda_4 = 0 \end{array} \right\} \Rightarrow \langle 12 \rangle = -\langle 34 \rangle$$

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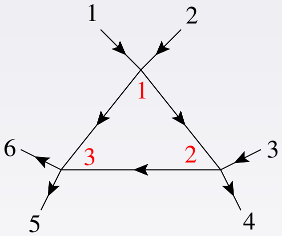
- Precisely corresponding to two-branch solutions of four-point kinematics

$$(p_1+p_2)^2 = (p_3+p_4)^2 \Rightarrow \langle 12\rangle^2 = \langle 34\rangle^2 .$$

On-shell diagrams and Grassmannian in ABJM

- Higher-point diagrams in ABJM



associated with

$$C = \begin{pmatrix} 1 & 0 & 0 & c_{1,4} & c_{1,5} & c_{1,6} \\ 0 & 1 & 0 & c_{2,4} & c_{2,5} & c_{2,6} \\ 0 & 0 & 1 & c_{3,4} & c_{3,5} & c_{3,6} \end{pmatrix},$$

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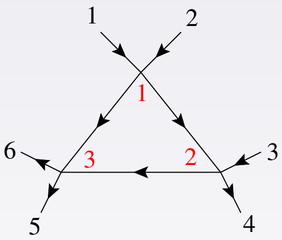

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Decompose into two-dimensional rotations

$$\begin{pmatrix} -\cot_1 & -\csc_1 & 0 \\ \csc_1 & \cot_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -\csc_2 & 0 & -\csc_2 \\ 0 & 1 & 0 \\ \csc_2 & 0 & \cot_2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -\csc_3 & -\csc_3 \\ 0 & \csc_3 & \cot_3 \end{pmatrix}.$$

From on-shell diagrams to amplitudes: BCFW

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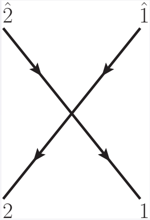
- Four-point vertex as BCFW bridge: [Branhuber, Travaglini, C.W., 12'] [Arkani-hamed, Bourjaily, Cachazo, Goncharov, Postnikov and Trnka, 12']



$$\Rightarrow \delta(C\cdot\lambda) \Rightarrow \begin{cases} \lambda_{\hat{1}} = -\cot\theta\lambda_1 - \csc\theta\lambda_2 \\ \lambda_{\hat{2}} = \csc\theta\lambda_1 + \cot\theta\lambda_2 \end{cases}$$

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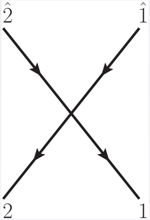

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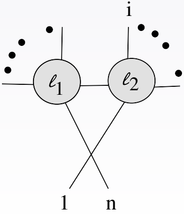
- BCFW recursion relations of amplitudes in ABJM

$$\mathcal{A}_n^{\ell} = \sum_{\ell_1+\ell_2=\ell} \sum_{i=4}^{n-2}$$



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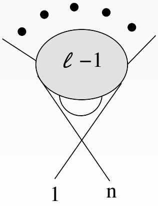

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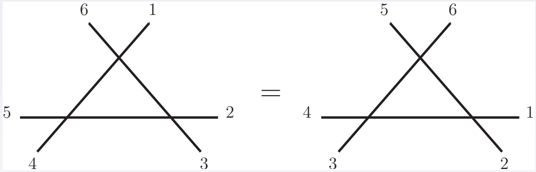
- Six-point amplitude at tree-level



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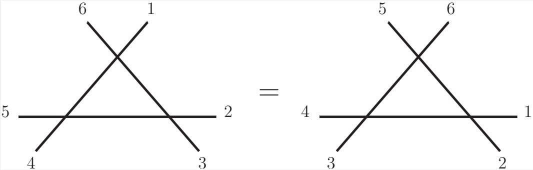
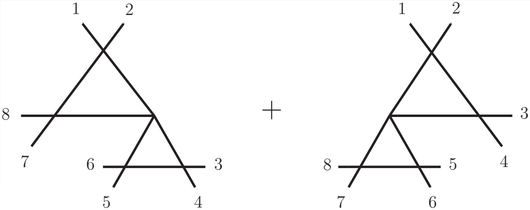


- Eight-point amplitude at tree-level



From on-shell diagrams to amplitudes: BCFW

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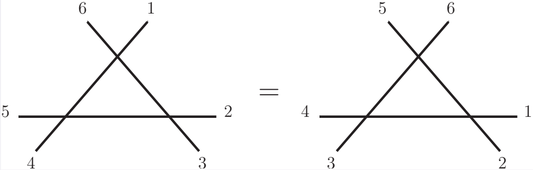

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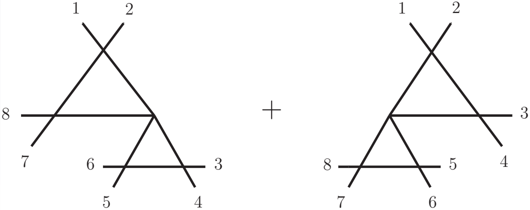
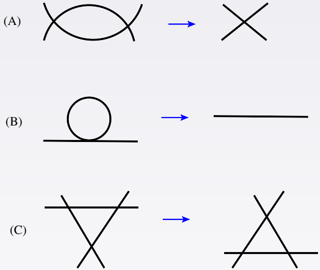


- The results have manifest cyclic symmetry $i \to i + 2$

Equivalence moves and permutation

- Equivalence moves



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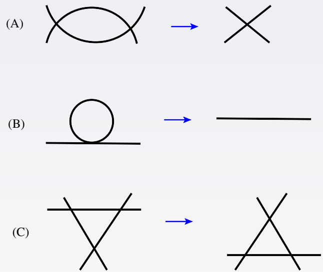
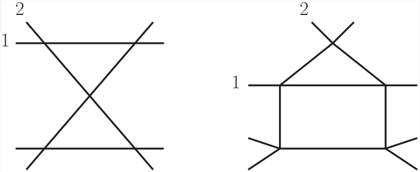


- Example:



Equivalence moves and permutation

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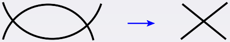
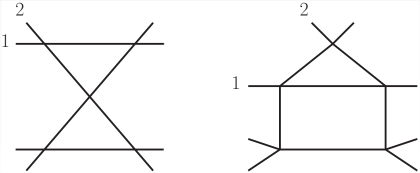
(A)



(B)

(C)

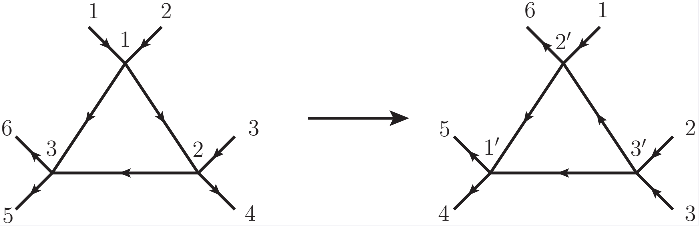
- Example:



- Permutation: $\sigma_{\mathrm{L}} = \sigma_{\mathrm{R}} = [1,4][2,6][3,7][5,8]$.

Triangle move and Tetrahedron equations

- The triangle move



$$(\theta_1, \theta_2, \theta_3) \to (\theta'_1, \theta'_2, \theta'_3) =$$

$$\left(-\mathrm{ArcCos}\left(\frac{c_1 c_2 s_3}{c_1 + s_2 c_3}\right), -\mathrm{ArcSin}\left(\frac{s_1 s_2 s_3}{s_2 + c_1 c_3}\right), -\mathrm{ArcCos}\left(\frac{s_1 c_2 c_3}{c_3 + c_1 s_2}\right)\right)$$

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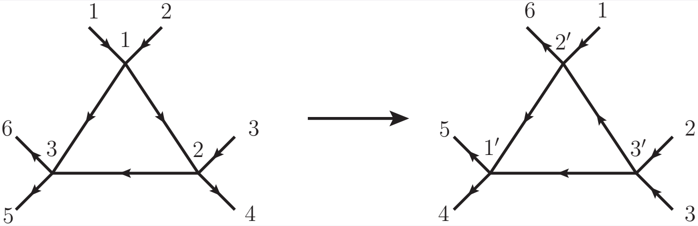


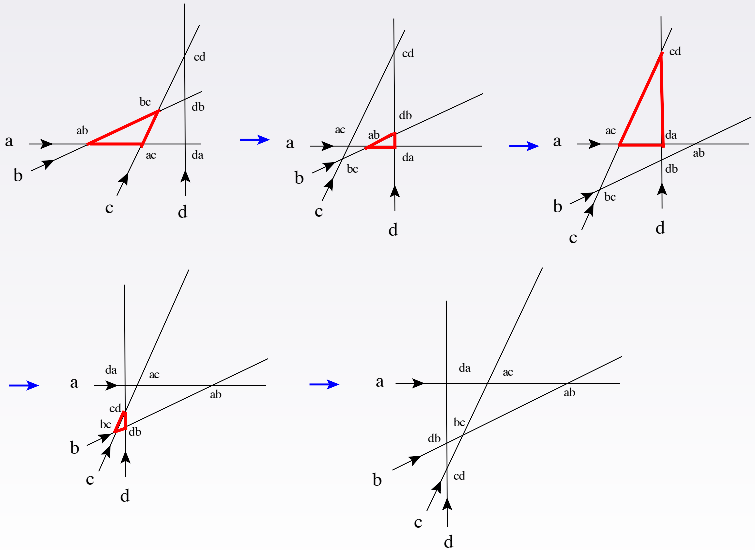
$$(\theta_1, \theta_2, \theta_3) \to (\theta_1', \theta_2', \theta_3') =$$

$$\left(-\mathrm{ArcCos}\left(\frac{c_1 c_2 s_3}{c_1 + s_2 c_3}\right), -\mathrm{ArcSin}\left(\frac{s_1 s_2 s_3}{s_2 + c_1 c_3}\right), -\mathrm{ArcCos}\left(\frac{s_1 c_2 c_3}{c_3 + c_1 s_2}\right)\right)$$

- Tetrahedron equation [Zamolodchikov, 80']

$$R_{1,2,3} R_{1,4,5} R_{2,4,6} R_{3,5,6} = R_{3,5,6} R_{2,4,6} R_{1,4,5} R_{1,2,3}$$

Triangle move and Tetrahedron equations



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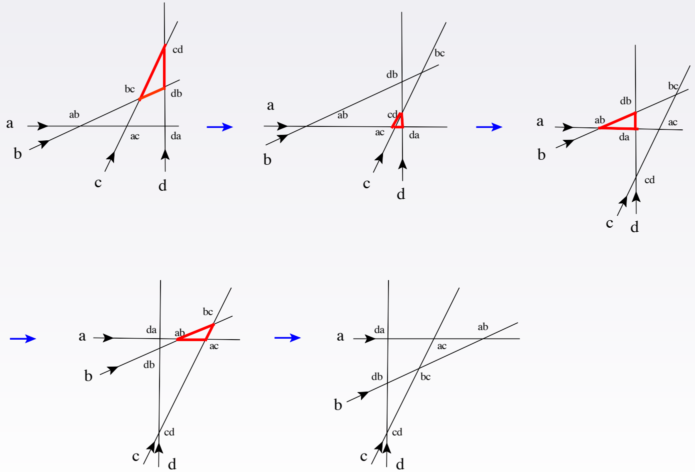
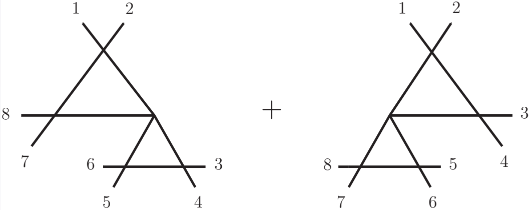
Triangle move and Tetrahedron equations

Another way of achieving the same final configuration



From permutation to integration contour

- Eight-point amplitude



The corresponding permutation paths

$$\sigma_1 = [1,5][2,7][3,6][4,8], \quad \sigma_2 = [1,4][2,6][3,7][5,8]\,,$$

From permutation to integration contour

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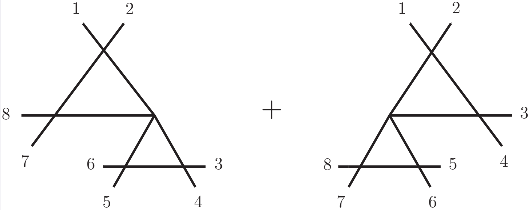


The corresponding permutation paths

$$\sigma_1 = [1,5][2,7][3,6][4,8], \quad \sigma_2 = [1,4][2,6][3,7][5,8]\,,$$

- $\sigma_1, \sigma_2 \Rightarrow$ relations among eight vectors of $OG(4,8)$

$$(\vec{c}_1, \vec{c}_2, \vec{c}_3, \vec{c}_4, \vec{c}_5, \vec{c}_6, \vec{c}_7, \vec{c}_8)$$

$[3,6] \Rightarrow \vec{c}_3$ is spanned by $\vec{c}_4, \vec{c}_5, \vec{c}_6 \Rightarrow$ minor $(3456) = 0$
$[1,4] \Rightarrow \vec{c}_1$ is spanned by $\vec{c}_2, \vec{c}_3, \vec{c}_4 \Rightarrow$ minor $(1234) = 0$

From permutation to integration contour

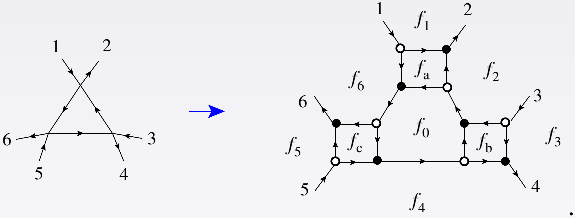
- In terms of Grassmannian contour integral, Eight-point tree amplitude is given by

$$A_8 = \frac{1}{\mathrm{vol}[Gl(4)]} \oint_{(1)=0+(3)=0} \frac{d^{4\times 8} C_{\alpha a}\, \delta(C \cdot C^T)}{(1)(2)(3)(4)} \prod_{\alpha=1}^{k} \delta^{2|3}(C_{\alpha a} \mathcal{W}_a)$$

- The same analysis can be extended to higher points.

ABJM embedding in $\mathcal{N} = 4$ SYM

- Blow up the four-point vertex



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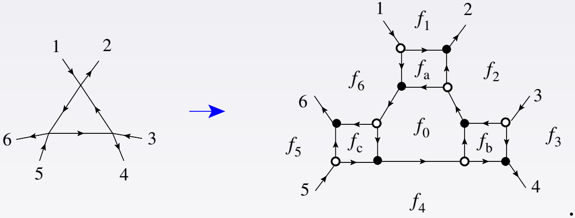

- Permutation can be deduced from that of the bipartite graph.

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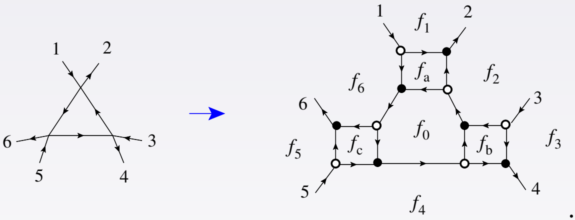

- Permutation can be deduced from that of the bipartite graph.
- Equivalence moves of bipartite ⇒ Equivalence moves of ABJM

ABJM embedding in $\mathcal{N} = 4$ SYM

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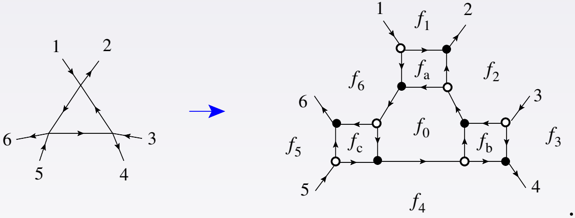
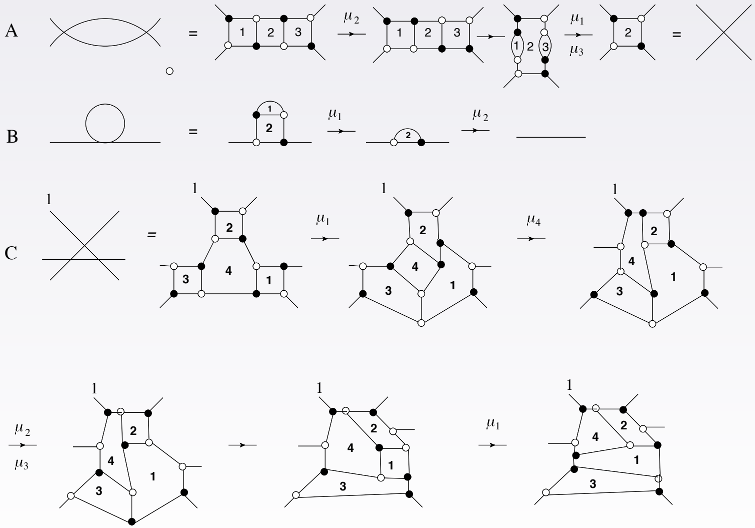


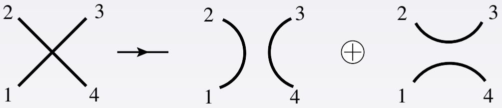
- Permutation can be deduced from that of the bipartite graph.
- Equivalence moves of bipartite $\Rightarrow$ Equivalence moves of ABJM
- Orthogonal conditions: choosing proper face variables.

ABJM embedding in $\mathcal{N} = 4$ SYM



Singularities of on-shell diagrams

- Singularities (boundaries) of a diagram can be read off via the opening of the four-point vertices:



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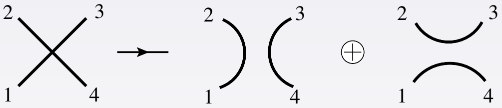

- Each on-shell diagram is an Eulerian poset:

$$\sum_{i=0}^{n}(-1)^{i}N_{i}=1,$$

where N_i is the number of dimension-i boundaries.

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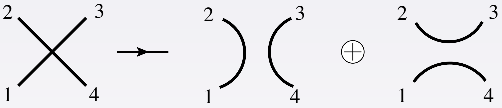

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$$\sum_{i=0}^{n} (-1)^i N_i = 1,$$

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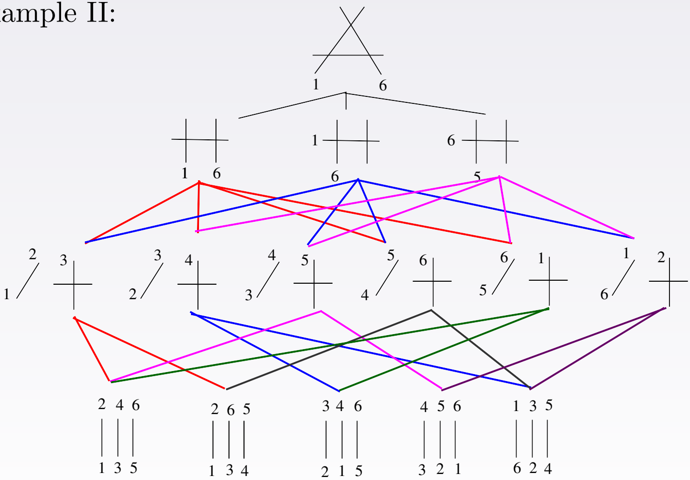
- For the above example: $2 - 1 = 1$

Singularities of on-shell diagrams

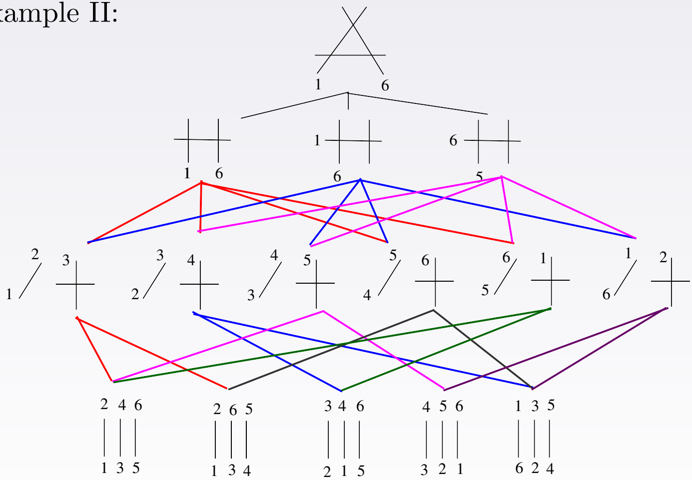
- Example II:



Eulerian poset: $5 - 6 + 3 - 1 = 1$, and

Singularities of on-shell diagrams

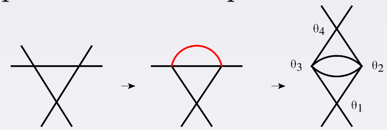
- Example II:



Eulerian poset: $5-6+3-1=1$, and $\partial^2=0$

dLog form of loop amplitudes

- Four-point amplitude at one loop



dLog form of loop amplitudes

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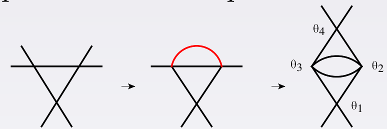
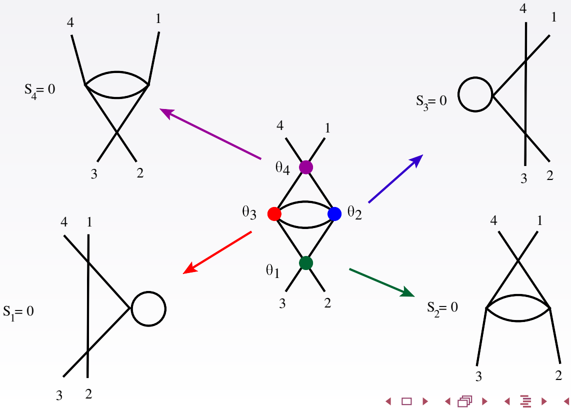
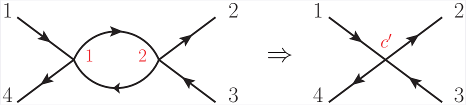


- All physical singularities



dLog form of loop amplitudes

- Compute the integrand by Bubble reduction:



$$\text{Grassmannian}: \begin{pmatrix} 1 & \frac{c_1c_2}{1+s_1s_2} & 0 & -\frac{s_1+s_2}{1+s_1s_2} \\ 0 & \frac{s_1+s_2}{1+s_1s_2} & 1 & \frac{c_1c_2}{1+s_1s_2} \end{pmatrix} = \begin{pmatrix} 1 & c' & 0 & -s' \\ 0 & s' & 1 & c' \end{pmatrix}$$

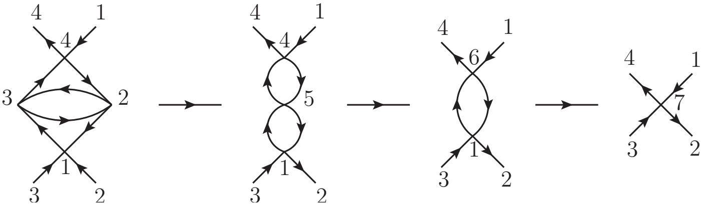
$$\text{Volume form}: (1+s_1s_2)d\log\tan_1 \wedge d\log\tan_2 = d\log\frac{\tan_1}{\tan_2}\wedge d\log\tan'$$

- The appearance of Jacobian $(1+s_1s_2)$ is because the mismatch of boson and fermion in

$$\delta^{2|3}(C\cdot\mathcal{W})$$

dLog form of loop amplitudes

- Bubble reduction of one-loop four-point amplitude



dLog form of loop amplitudes

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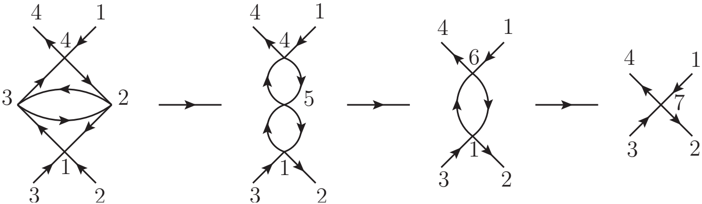

- The result of the reduction

$$A_4^{\text{loop}} = A_4^{\text{tree}} \int d\log(\frac{\tan_2}{\tan_3}) \wedge d\log(\frac{\tan_4}{\tan_5}) \wedge d\log(\frac{\tan_6}{\tan_1})$$

dLog form of loop amplitudes

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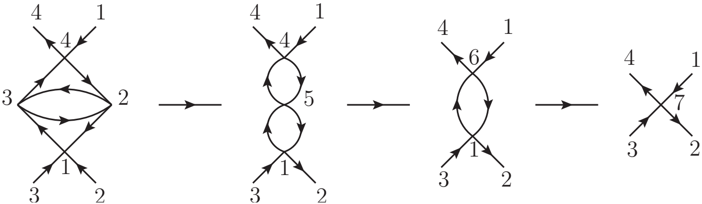

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$$A_4^{\text{loop}} = A_4^{\text{tree}} \int d\log\left(\frac{\tan_2}{\tan_3}\right) \wedge d\log\left(\frac{\tan_4}{\tan_5}\right) \wedge d\log\left(\frac{\tan_6}{\tan_1}\right)$$

- In momentum space

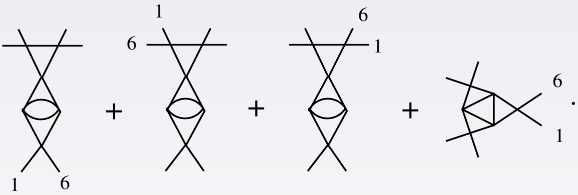
$$\int d^3\ell \frac{\ell^2 \epsilon_{\mu\nu\rho} p_1^\mu p_2^\nu p_4^\rho + s_{12}\epsilon_{\mu\nu\rho}\ell^\mu p_1^\nu p_4^\rho}{\ell^2(\ell-p_1)^2(\ell-p_1-p_2)^2(\ell+p_4)^2}$$

$$= \int d\log\frac{\ell^2}{(\ell-p_1)^2} d\log\frac{(\ell-p_1)^2}{(\ell-p_1-p_2)^2} d\log\frac{(\ell-p_1-p_2)^2}{(\ell+p_4)^2}$$

dLog form of loop amplitudes

- Six-point at one loop

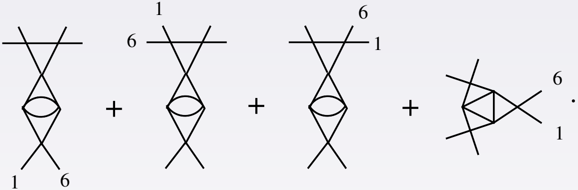
$$A_6^{1-\text{loop}} =$$



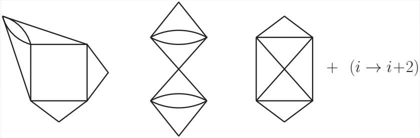
dLog form of loop amplitudes

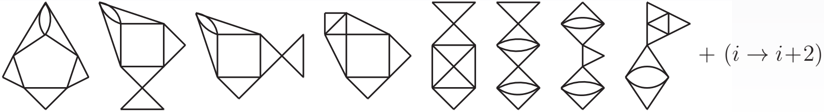
- Six-point at one loop

$$A_6^{1-\text{loop}} =$$



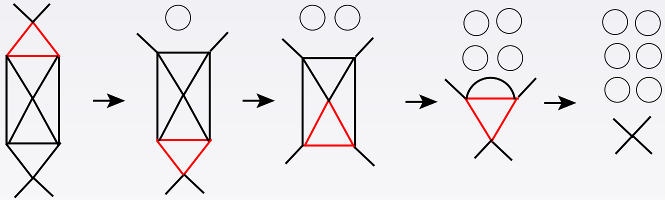
- Four-point and six-point at two loops





dLog form of loop amplitudes

- One can always do bubble-reductions for four- and six-point amplitudes at all loops



dLog form of loop amplitudes

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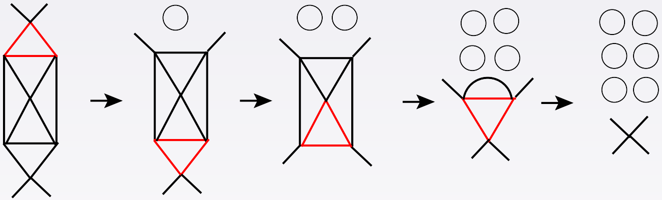

- Four and six-point amplitudes at L loops should be given by uniform transcendental L functions: confirmed by explicit computations [Bianchi et al, 12'], [Beisert et al,12'], [Brandhuber, Travaglini, C.W.,12'], [Chen, Huang^2, Caron-Huot], [Bianchi,Leoni, 14']

Summary and open questions

Summary and open questions

- Positive Grassmannian/on-shell diagrams for the Scattering amplitudes in ABJM theory: Tetrahedron equations, Eulerian poset, Amplitudes have manifest cyclic symmetry, dLog forms for loop amplitudes ... ...
- Applications to lower(non) supersymmetric theories, and/or Non-planar theories?
- How to integrate dLog forms directly?
- Amplituhedron for ABJM?
- Scattering amplitudes v.s. Quiver gauge theories??

Thank You!