SUSY-QCD corrections to Neutralino (Co)annihilation and their Impact on the Relic Density

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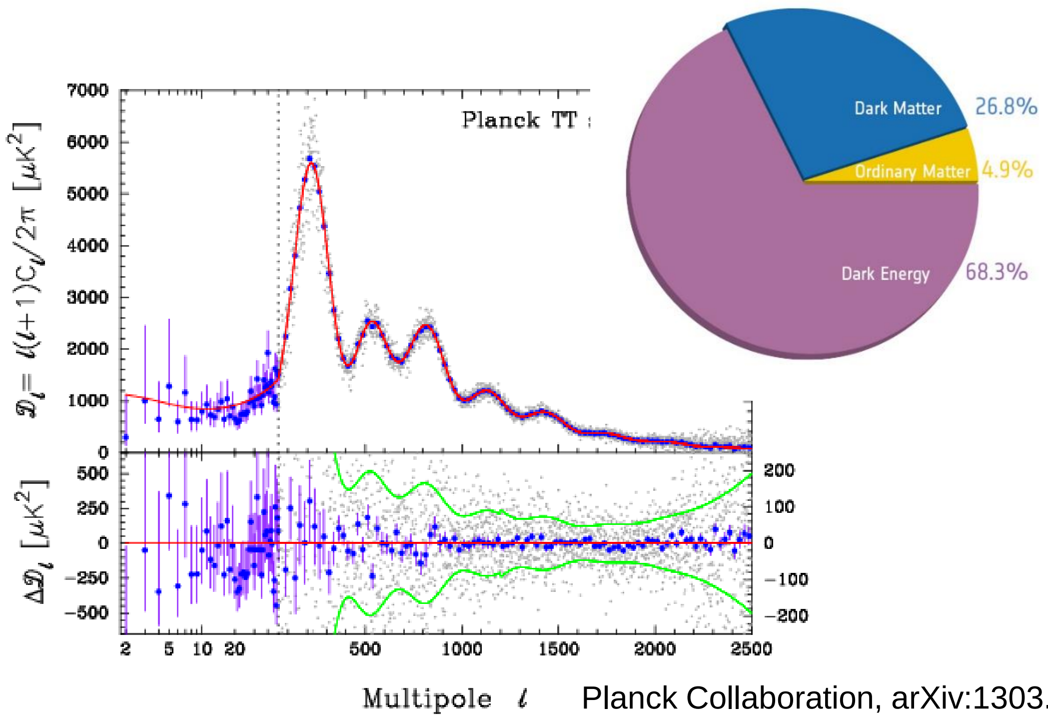
in collaboration with B. Herrmann, M. Klasen and K. Kovarik

J. Harz, B. Herrmann, M. Klasen, K. Kovařík and Q. Le Boulc'h,
Phys. Rev. D 87: 054031 (2013), arXiv:1212.5241 [hep-ph]
J. Harz, B. Herrmann, M. Klasen, K. Kovařík,
in preparation

9. June 2014
UK DM meeting, Durham

CMB Measurement

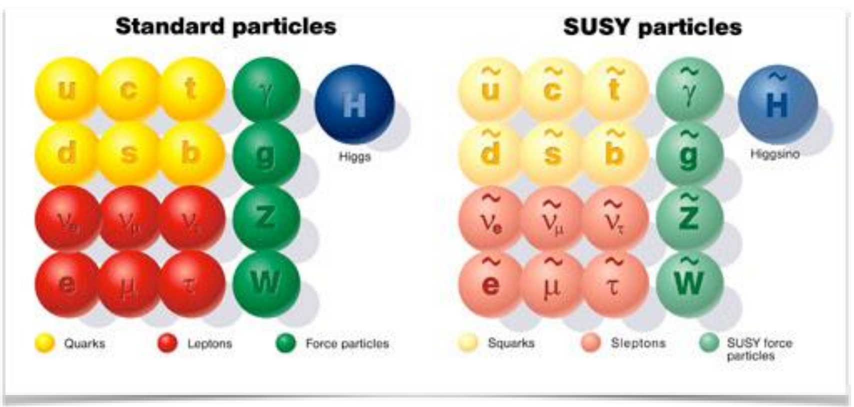


Planck Collaboration, arXiv:1303.5076

- precise determination of relic density by PLANCK to

$$\Omega_{\mathrm{CDM}} = 0.1199 \pm 0.0027$$

Particle Physics Theory



Minimal Supersymmetric Standard Model (MSSM)

- lightest neutralino $\tilde{\chi}_1^0$ is a good cold DM candidate
- theoretical prediction of relic density possible

$$\Omega_{\mathrm{CDM\,theoret.}}$$

⇩ ⇩

Need of precise theoretical prediction to match experimental precision

Theoretical Prediction of Dark Matter Relic Density

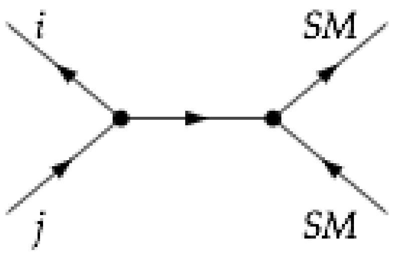
- number density of DM in the early universe can be described by the Boltzmann equation

$$\dot{n} + 3Hn = -\langle \sigma_{\mathrm{eff}\, v} \rangle (n^2 - n_{\mathrm{eq}}^2)$$

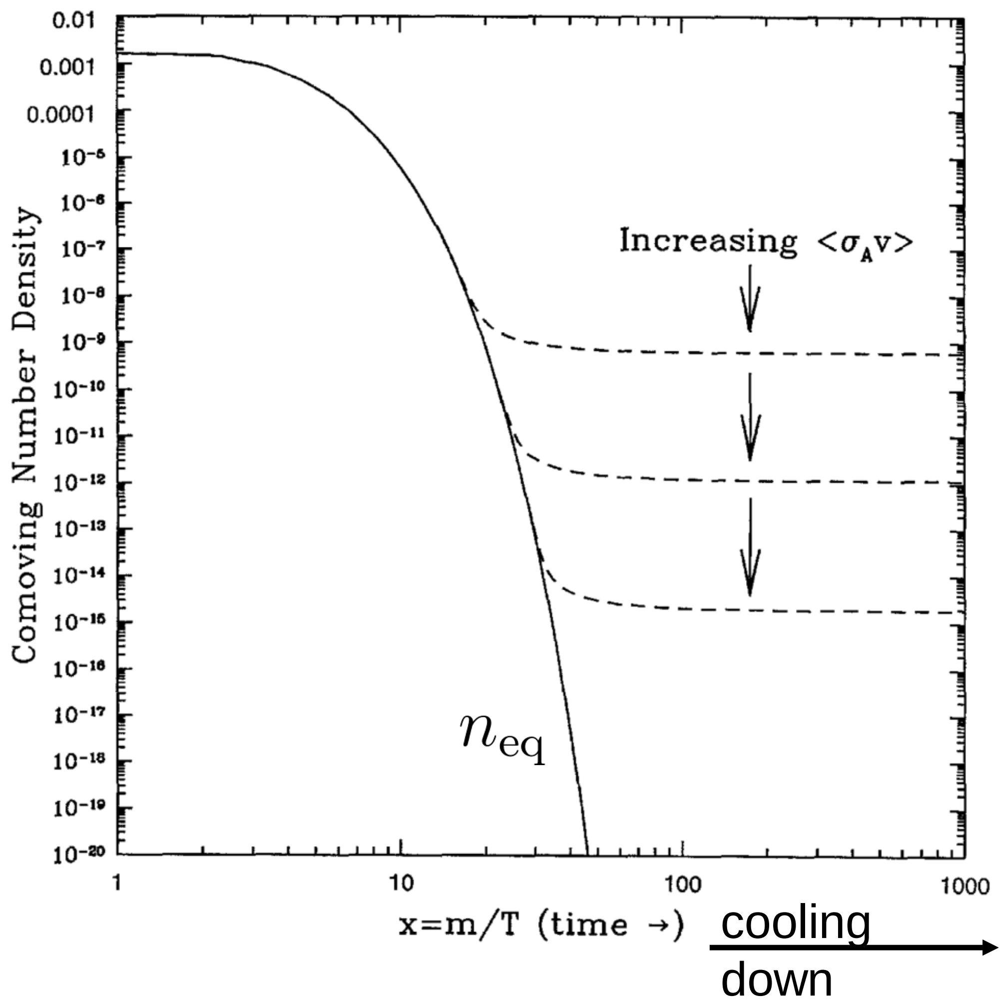
- relic density can be determined by solving the Boltzmann equation

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\mathrm{eff}} v \rangle}$$

particle physics



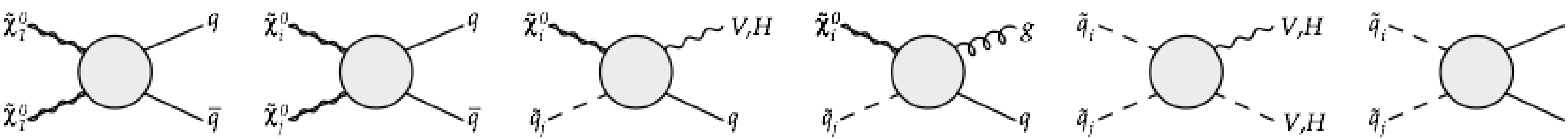
Jungman, Kamionkowski, Griest, Phys. Reports 267 (1995)



$$\langle \sigma_{\mathrm{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}}$$

with

$$\frac{n_i^{eq}}{n^{eq}} \propto \exp^{\frac{-(m_i - m_\chi)}{T}} = \exp^{\frac{-(m_i - m_\chi)}{x \;\; m_\chi}}$$



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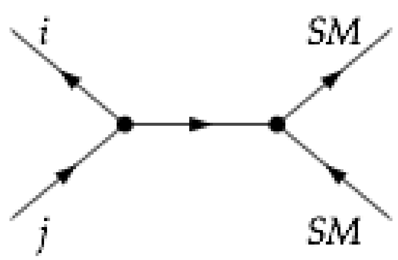
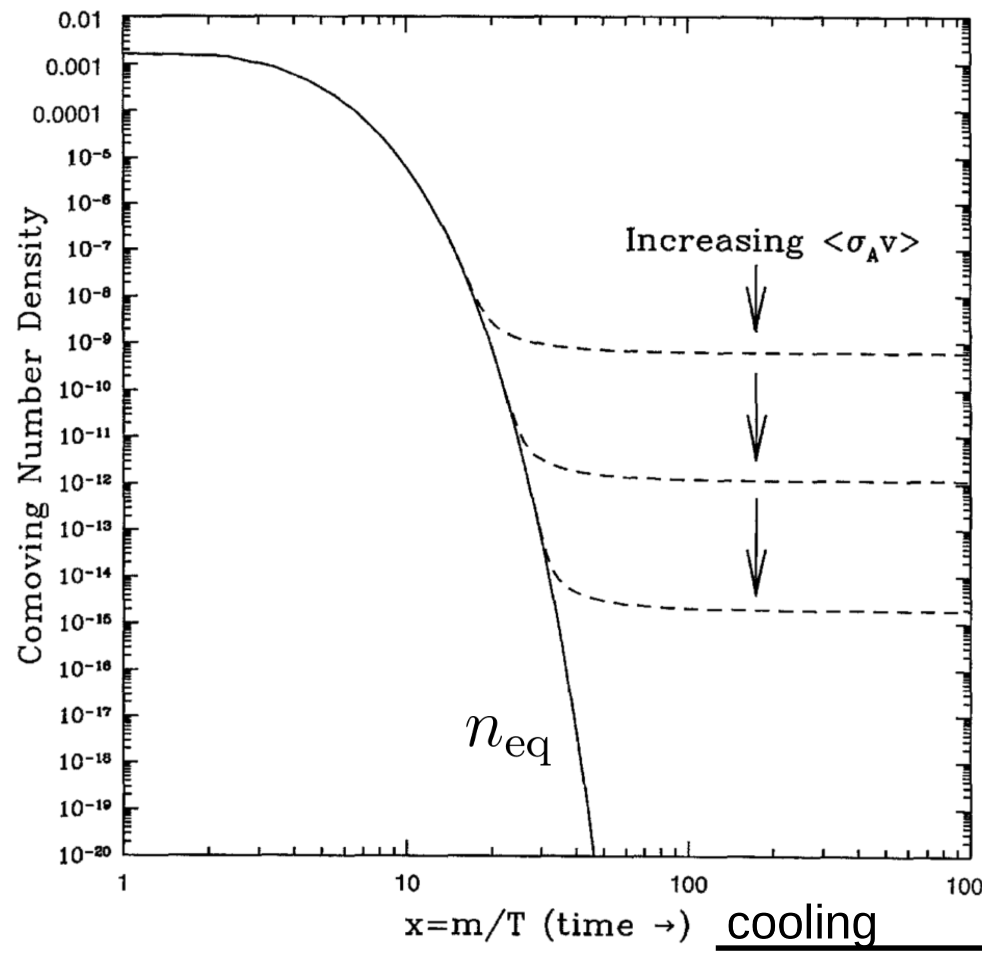
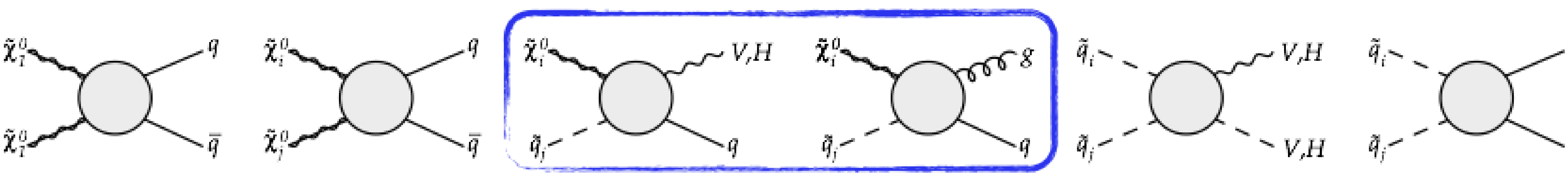




cooling down

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq}}{n^{eq}} \frac{n_j^{eq}}{n^{eq}} \quad \text{with} \quad \frac{n_i^{eq}}{n^{eq}} \propto \exp^{\frac{-(m_i - m_\chi)}{T}} = \exp^{\frac{-(m_i - m_\chi)}{x \; m_\chi}}$$

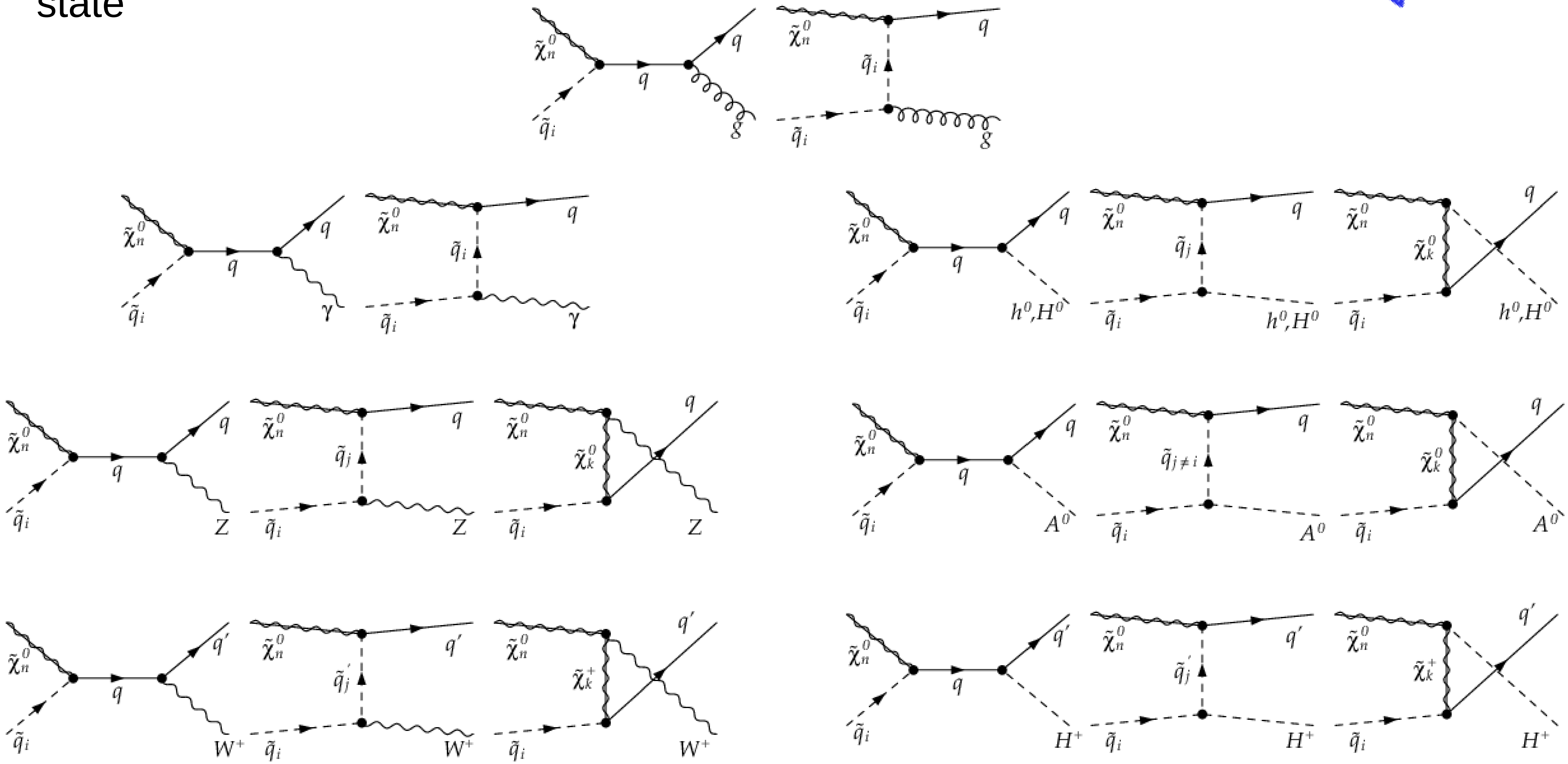
$\tilde{\chi}_1^0 \tilde{t}_1$ Coannihilation



- 8 different final states have to be considered
- gluon, Higgs and electroweak vector bosons in the final state

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma_{\mathrm{eff}} v \rangle}$$

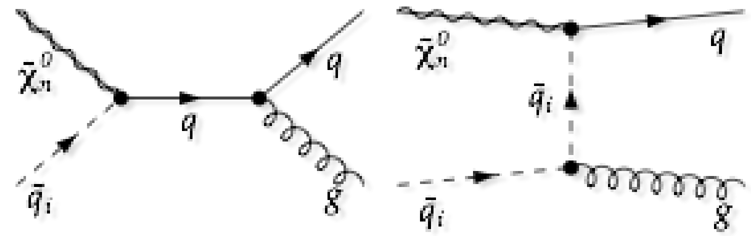
particle physics



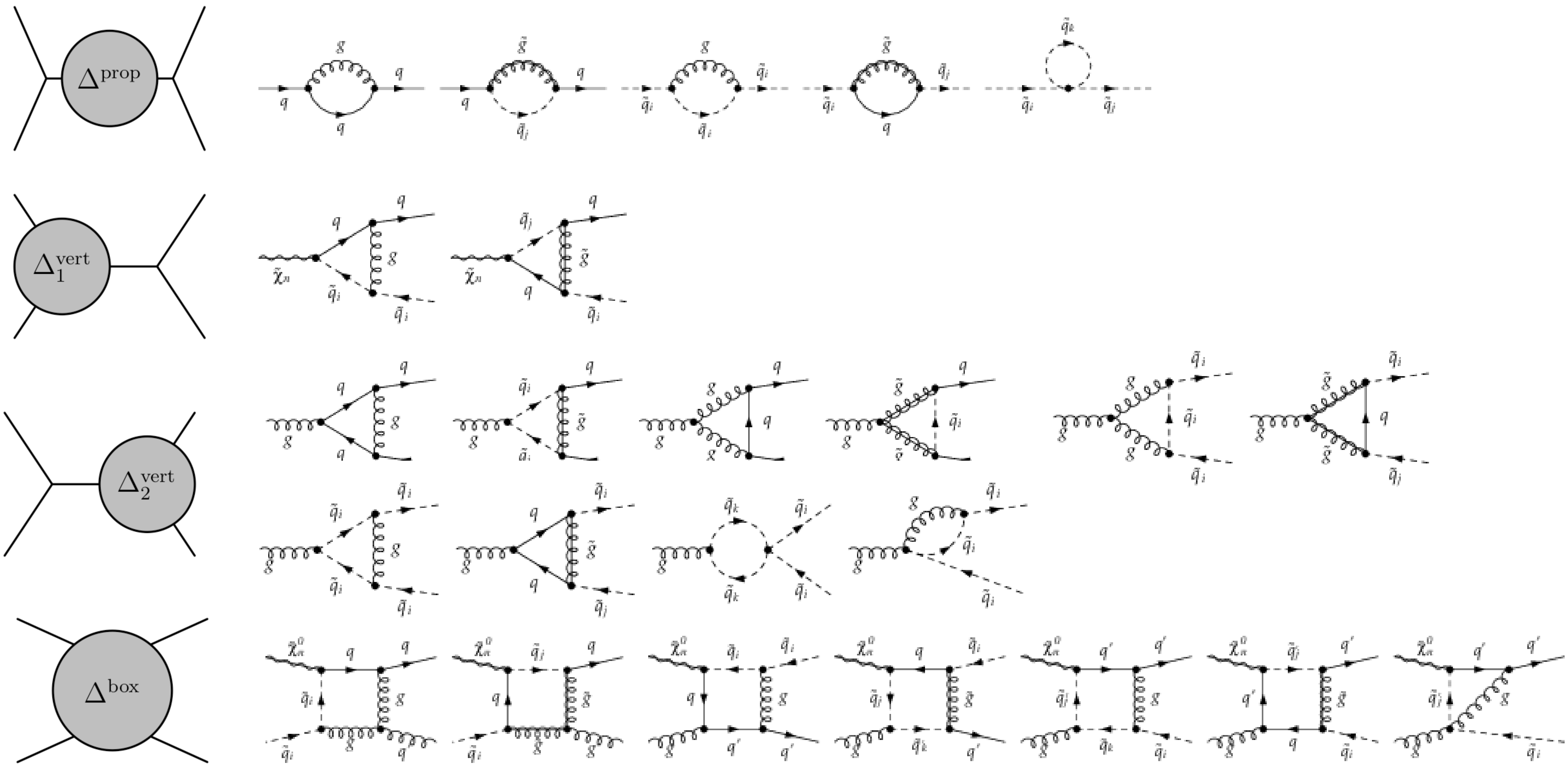
calculating loop corrections to increase theoretical precision

… and at one-loop level including SUSY-QCD

- tree level with gluon in the final state



- NLO corrections give rise to propagator corrections, vertex corrections and box diagrams



… and at one-loop level including SUSY-QCD

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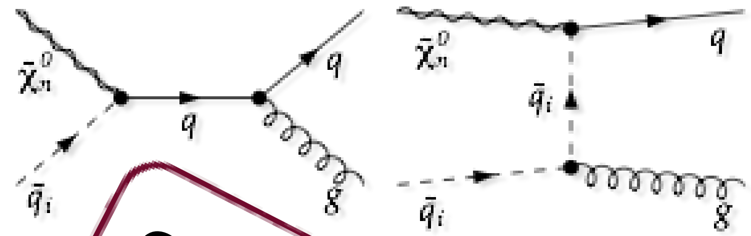
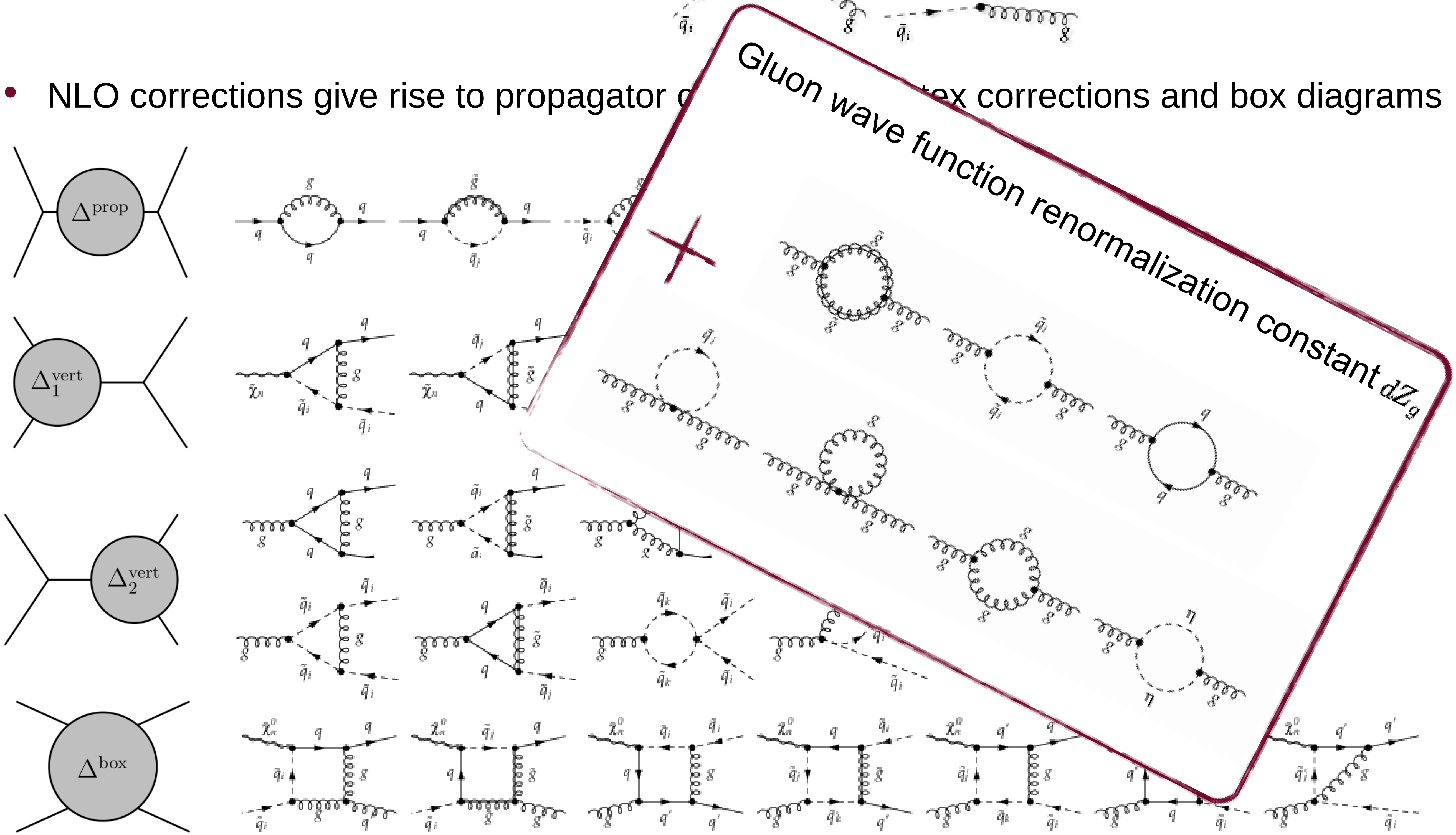
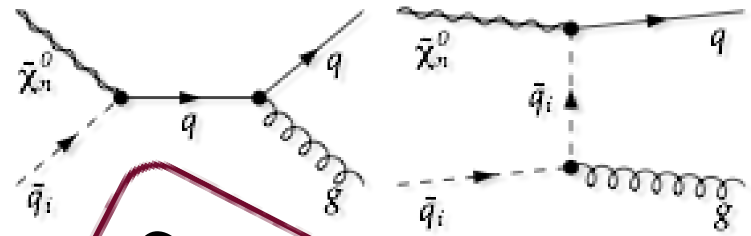


- NLO corrections give rise to propagator c… …tex corrections and box diagrams



- tree level with gluon in the final state



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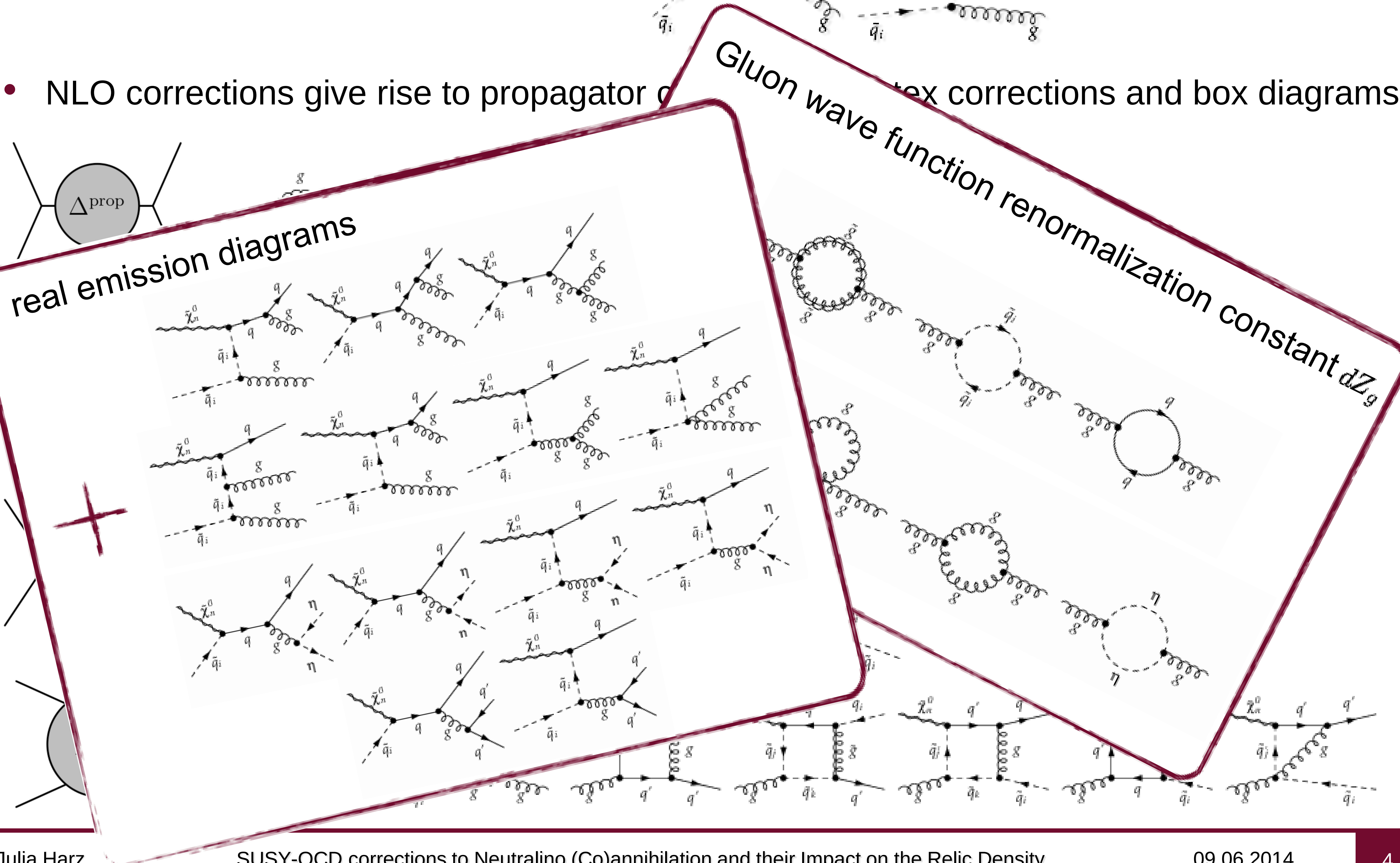
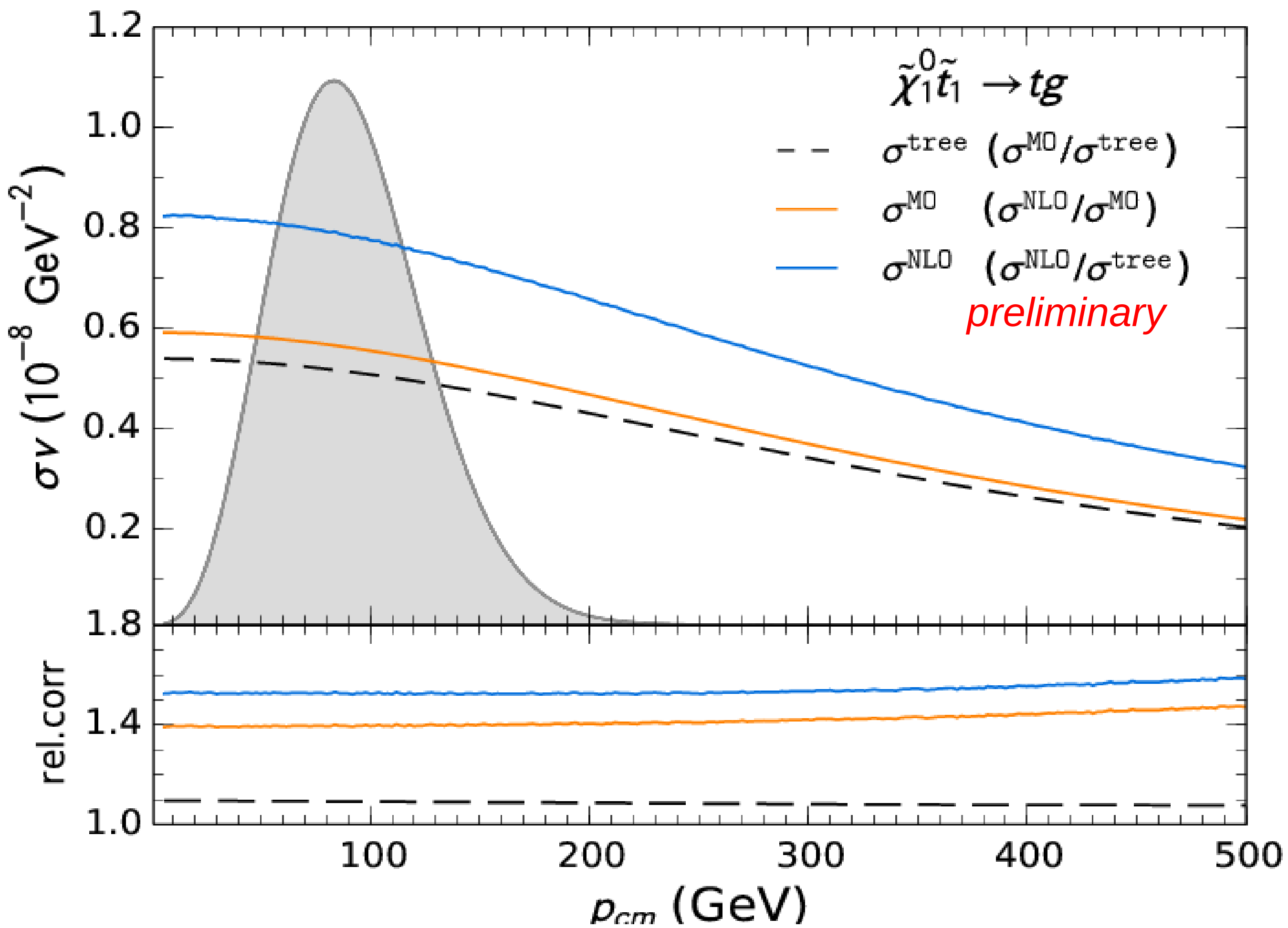
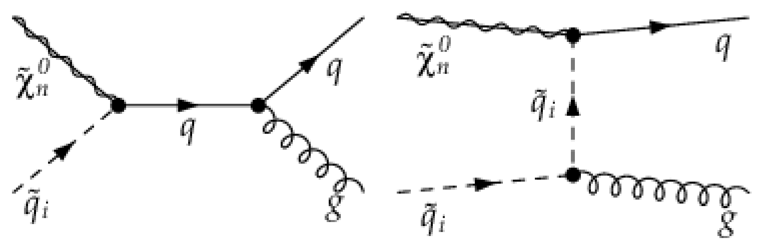


Comparison of tree-level, one-loop and MicrOMEGAs result





- micrOMEGAs uses effective tree-level cross section

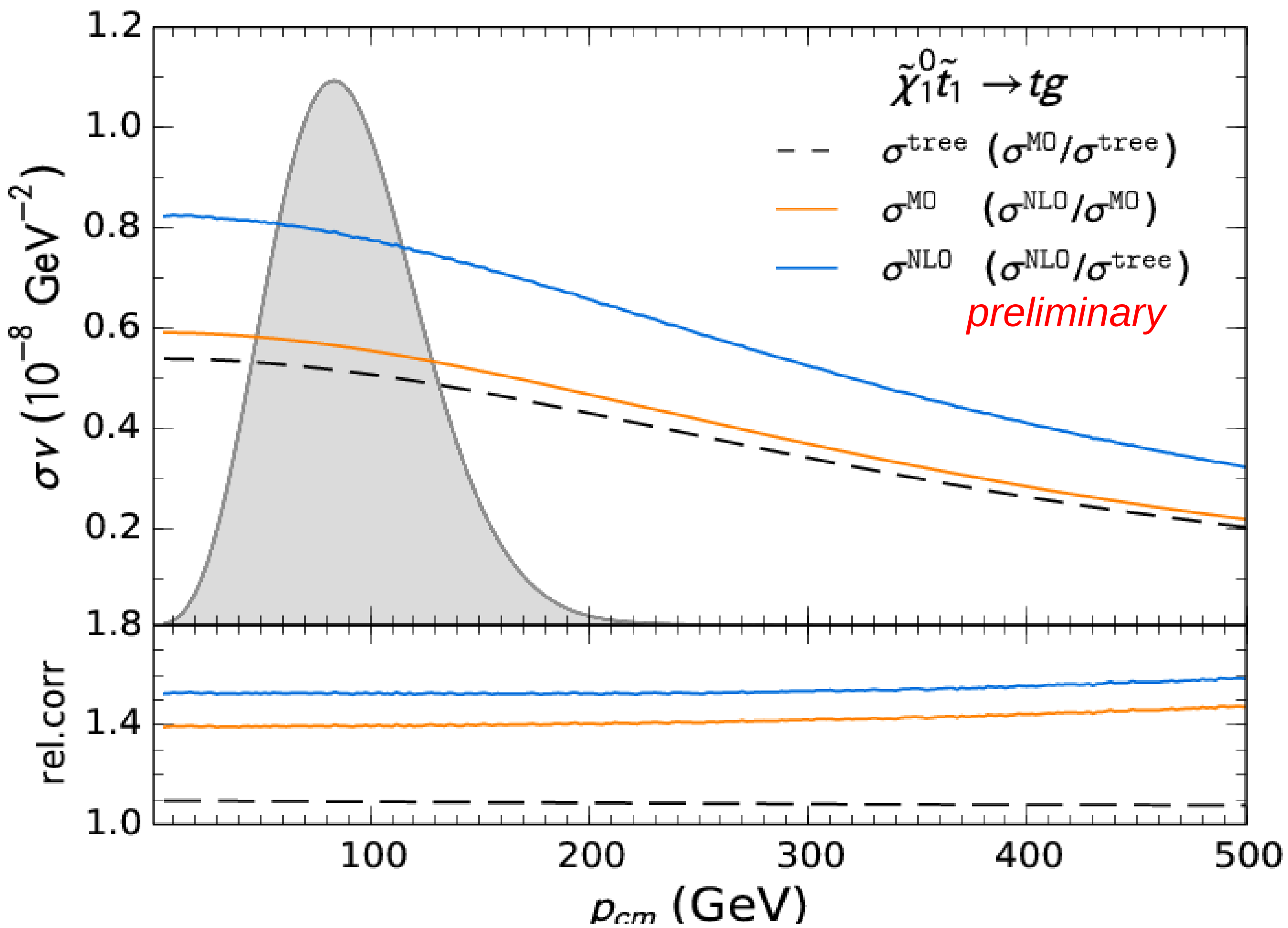
Belanger et al.
Comput.Phys.Commun.182:842-856 (2011)
arXiv:1004.1092 [hep-ph]

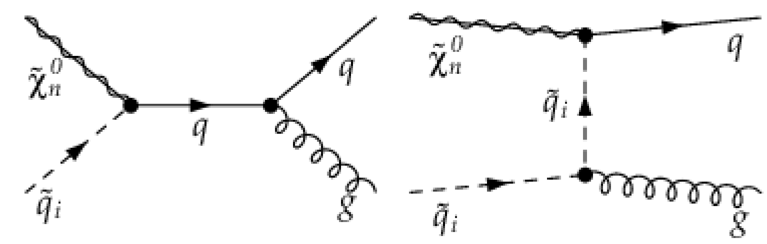
- SUSY-QCD corrections lead to a relative correction of up to 40 % on cross section with respect to default micrOMEGAs

$\Omega_\chi h^2$	$\tilde{\chi}_1^0\tilde{t}_1 \to tg$	$\tilde{\chi}_1^0\tilde{t}_1 \to th^0$	$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to t\bar{t}$	$\tilde{\chi}_1^0\tilde{t}_1 \to bW^+$	$\sum$
0.1135	22.8%	22.7%	14.9%	10.4%	70.8%

M_1	M_2	M_3	$M_{\tilde{q}_{1,2}}$	$M_{\tilde{q}_3}$	$M_{\tilde{u}_3}$	$M_{\tilde{\ell}}$	T_t	m_A	μ	$\tan\beta$	$m_{\tilde{\chi}_1^0}$	$m_{\tilde{t}_1}$	m_{h^0}
335.0	1954.1	1945.	3215.1	1578.0	609.2	3263.9	2704.1	948.8	2929.8	5.8	338.3	375.6	121.94

Comparison of tree-level, one-loop and MicrOMEGAs result





- micrOMEGAs (Belanger et al. Comput.Phys.Commun.182:842-856 (2011) arXiv:1004.1092 [hep-ph]) uses effective tree-level cross section
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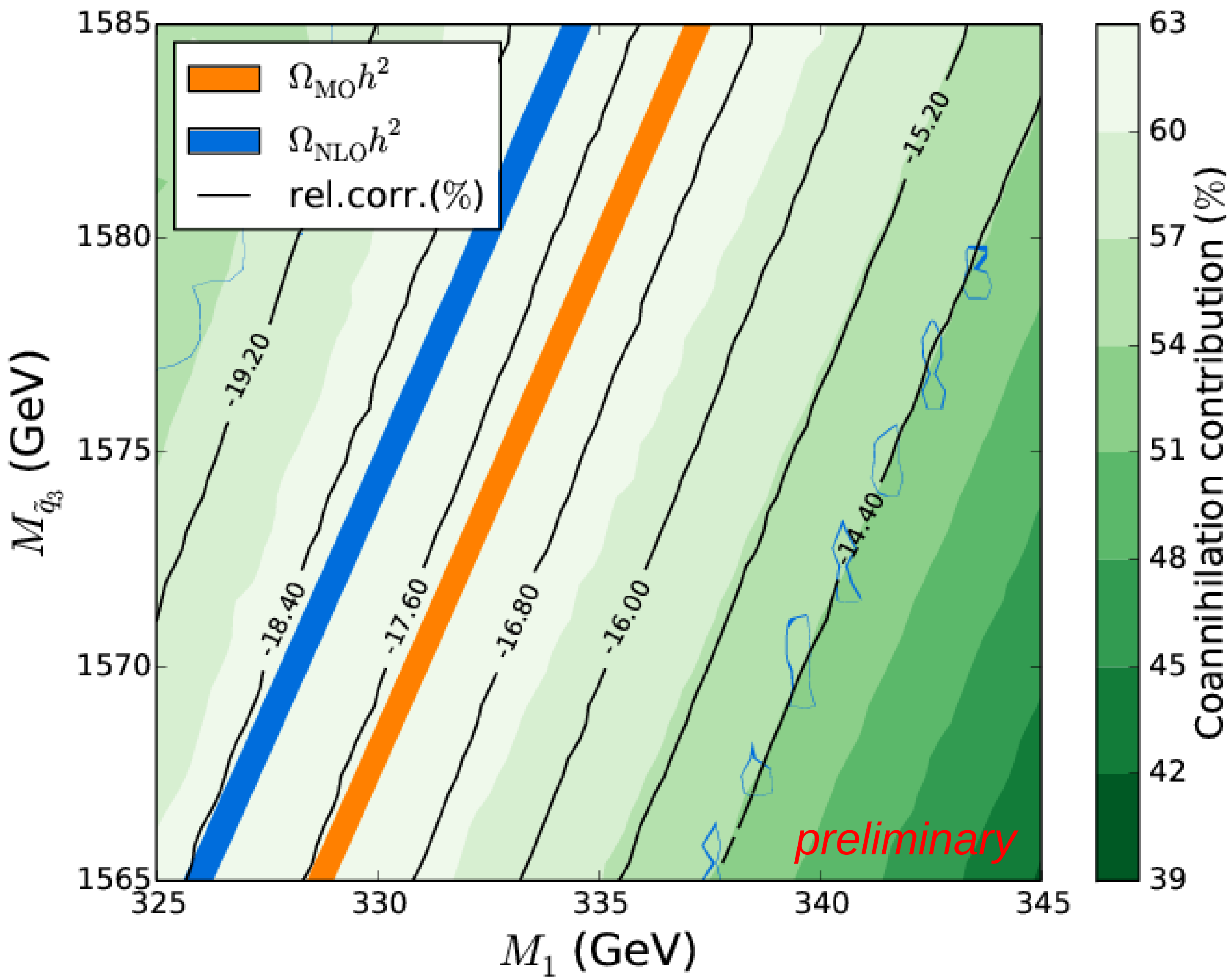
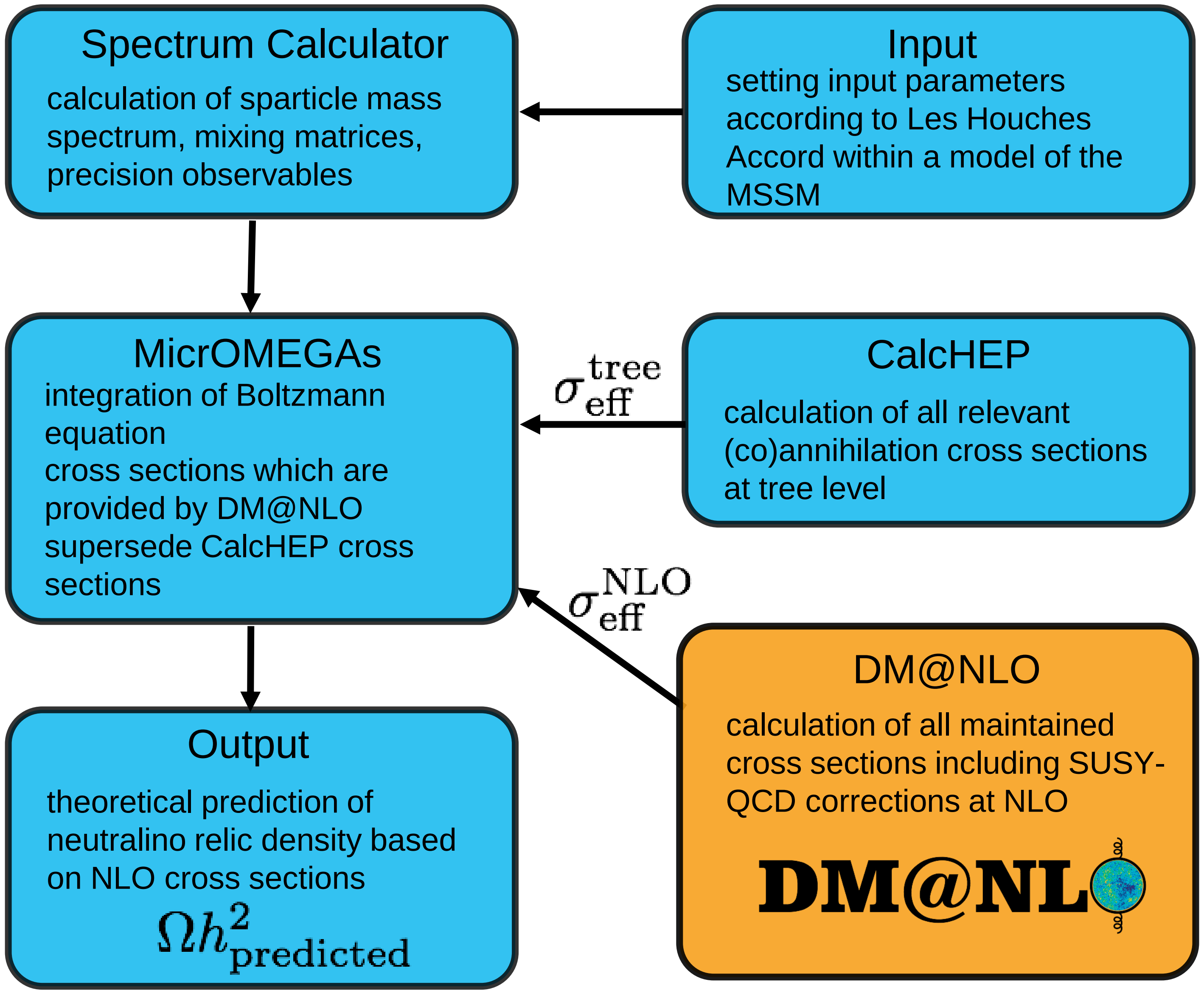
⇨ NLO calculation crucial for an uncertainty estimation of the theoretical cross section!

Impact of all NLO-corrected channels on the relic density

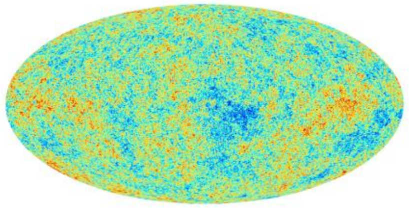


⇨ large relative corrections on the relic density with respect to default MicrOMEGAs (~ 20%)

- current public tools: calculation based on (max.) effective tree level
- DM@NLO provides (co)annihilation processes including $O(\alpha_s)$ SUSY-QCD corrections
- DM@NLO will be publically available as Fortran library
- interface to MicrOMEGAs (link to DarkSUSY in progress)
- easy to use, ability to perform broad parameter scans

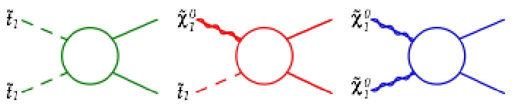


- precise determination of dark matter relic density by PLANCK

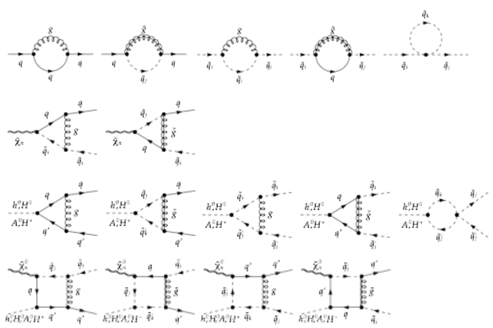


$$\Omega_{\mathrm{CDM}} = 0.1199 \pm 0.0027$$

- need of a reduction of uncertainties in the theoretical prediction

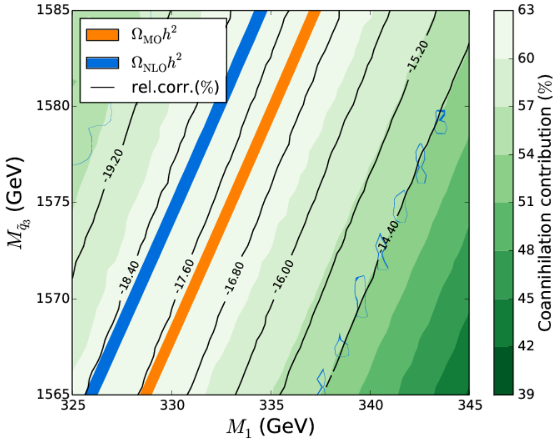


- calculation of (co)annihilation cross section at full next-to-leading order including SUSY-QCD



- e.g. neutralino-stop coannihilation can be important in order to obtain the relic density in the right ballpark

- impact larger than current experimental uncertainties



DM@NLO

http://dmnlo.hepforge.org/

Backup

Uncertainties Entering the Calculation

Arising from cosmology

- choice of cosmological model
  Hamann, Hannestad, et al. , Phys. Rev. D (2007)
- variation in hubble expansion rate
  Arbey, Mahmoudi, Phys. Lett. B (2008)
- effective degrees of freedom of the universe
  Hindmarsh, Philipsen, Phys. Rev. D (2005)

Arising from particle physics

- three-body processes
  Yaguna, Phys. Rev. D (2010)
- determination of mass parameters
  Allanach, Kraml, Porod, JHEP (2003)
  Allanch, Belanger, JHEP (2004)
  Belanger, Kraml, Porod, Phys. Rev. D (2005)
- precision of (co)annihilation cross section σ_{eff}

DM@NLO

Precision data from CMB measurements

PLANCK: ~ 2% uncertainty

- huge variety of processes are contributing to the relic density

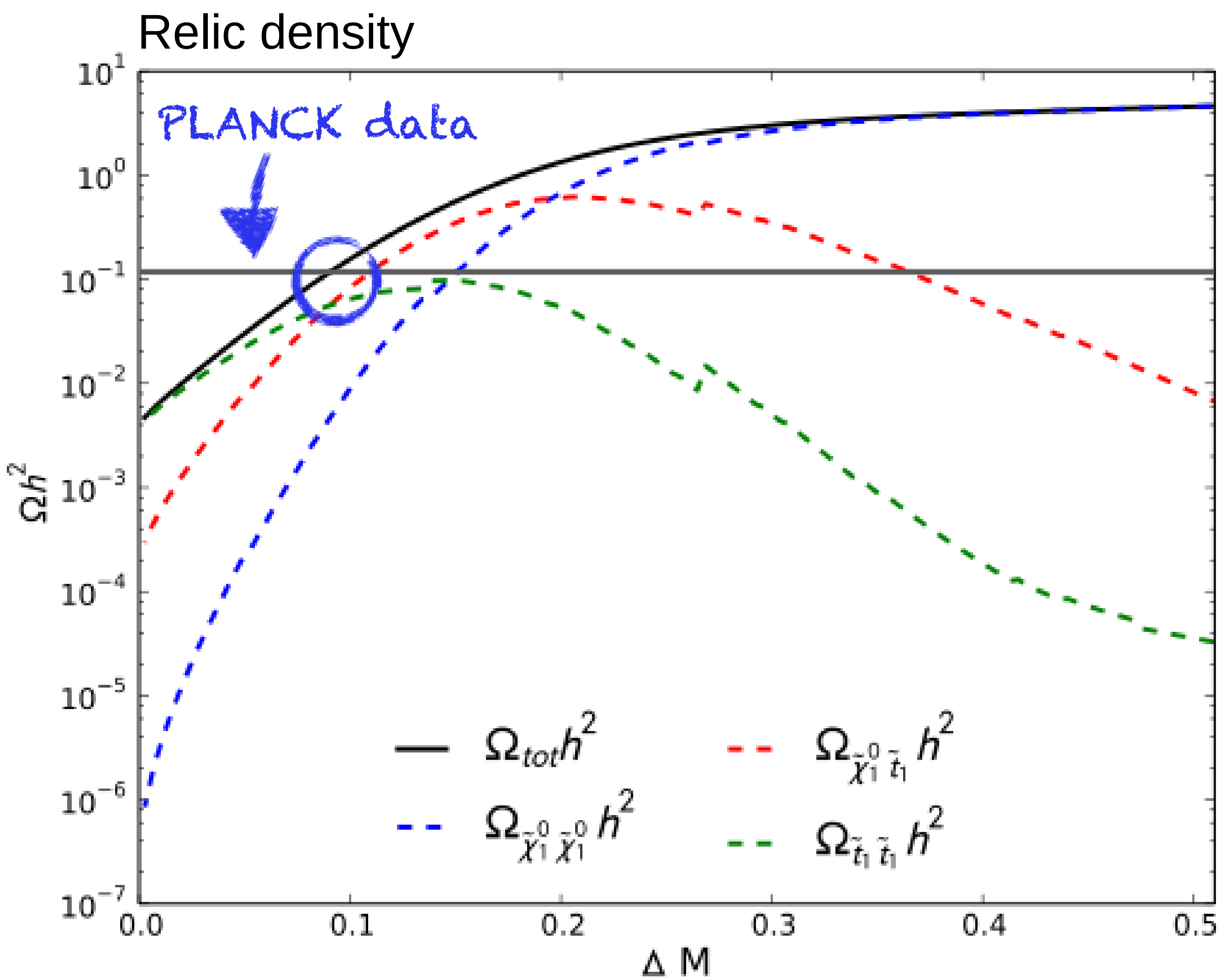
$$\dot{n} + 3Hn = -\langle\sigma_{\text{eff}}v\rangle(n^2 - n_{\text{eq}}^2)$$

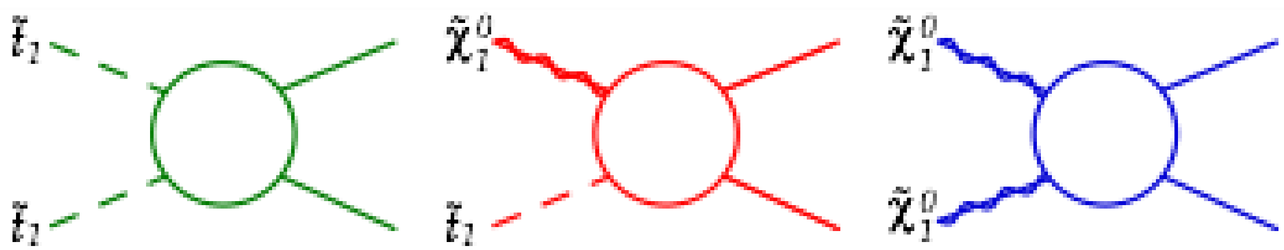
$$\langle\sigma_{\text{eff}}v\rangle = \sum_{ij}\langle\sigma_{ij}v_{ij}\rangle\frac{n_i^{eq}}{n^{eq}}\frac{n_j^{eq}}{n^{eq}}$$

$$\frac{n_i^{eq}}{n^{eq}} \propto \exp^{\frac{-(m_i-m_\chi)}{T}} = \exp^{\frac{-(m_i-m_\chi)}{x\ m_\chi}}$$

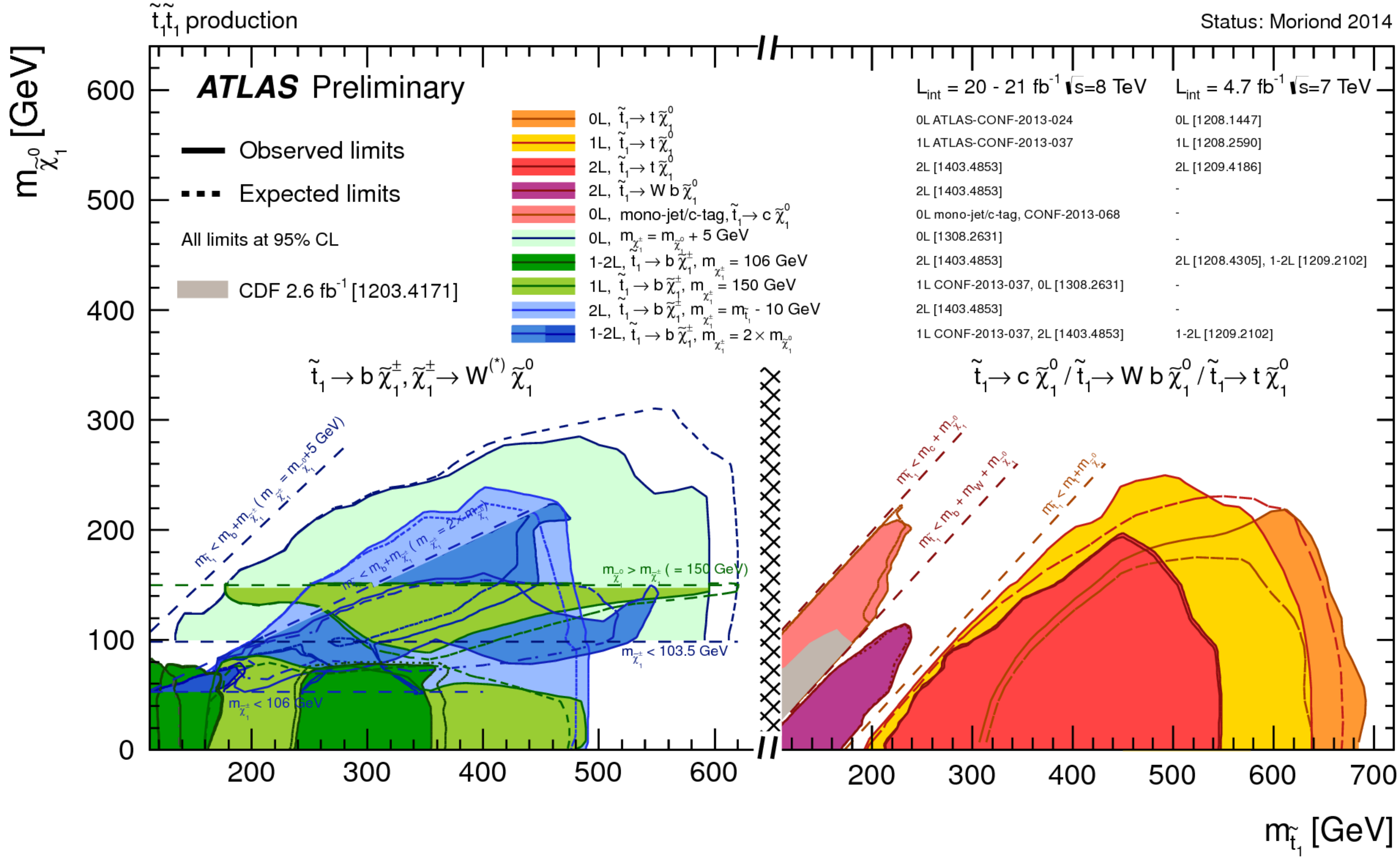
- our case:
  assuming lightest stop being the NLSP

$$\Delta M = \frac{m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0}}{m_{\tilde{\chi}_1^0}}$$





⇨ right admixture of neutralino-stop coannihilation and stop-stop annihilation processes can be important to get the right relic abundance



Neutralino-Stop coannihilation strip very hard to probe

… and a 126 GeV Higgs …?

- Higgs mass corrected by dominant one-loop corrections

$$m_{h_0}^2 \approx m_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\frac{M_{\mathrm{SUSY}}^2}{m_t^2} + \frac{X_t^2}{M_{\mathrm{SUSY}}^2}\left(1 - \frac{X_t^2}{12 M_{\mathrm{SUSY}}^2}\right)\right]$$

with $X_t = A_t - \mu/\tan\beta$ and $M_{\mathrm{SUSY}} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$

- Stop mixing matrix

$$\begin{pmatrix} m_{\tilde{t}_1}^2 & 0 \\ 0 & m_{\tilde{t}_2}^2 \end{pmatrix} = U^{\tilde{q}} \begin{pmatrix} M_{\tilde{Q}}^2 + m_t^2 + (I_q^{3L} - e_q \sin_W^2)\cos 2\beta m_Z^2 & m_t X_t \\ m_t X_t & M_{\tilde{U}}^2 + m_t^2 + e_q \sin_W^2 \cos 2\beta m_Z^2 \end{pmatrix} (U^{\tilde{q}})^\dagger$$

maximal contribution from stop mixing for $|X_t| \approx \sqrt{6}/M_{\mathrm{SUSY}}$

⇨ Neutralino-stop coannihilation interesting for collider phenomenology as well as for relic density

Treatment of divergences

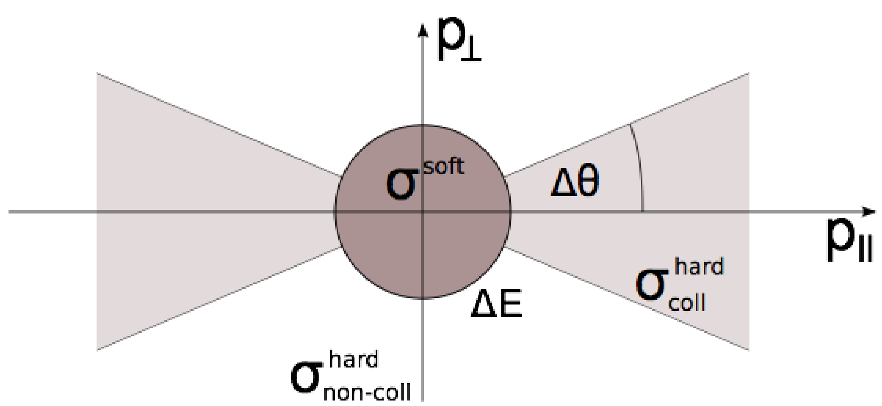
$$\sigma^{\text{NLO}} = \int_{2\to 2} d\sigma^{\text{virtual}} + \int_{2\to 3} d\sigma^{\text{real}} = \text{finite}$$

- UV divergences: hybrid on-shell / $\overline{\text{DR}}$ renormalisation scheme

$m_{\tilde{t}_1}^{\text{OS}}, m_{\tilde{b}_1}^{\text{OS}}, m_{\tilde{b}_2}^{\text{OS}} \quad A_t^{\overline{\text{DR}}}, A_b^{\overline{\text{DR}}} \quad m_t^{\text{OS}}, m_b^{\overline{\text{DR}}}$ ⟹ $m_{\tilde{t}_2}, \theta_{\tilde{t}}, \theta_{\tilde{b}}$

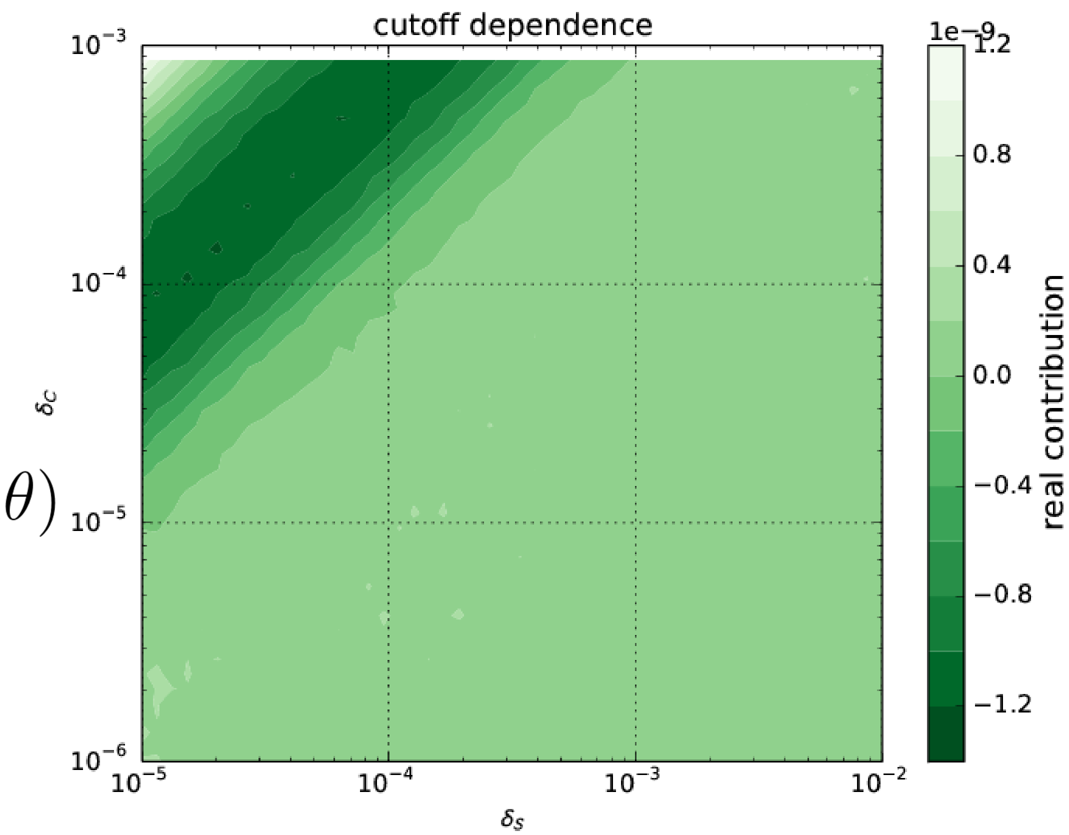
independent parameters — dependent parameters

- IR divergences: 2-cutoff phase space slicing



$$\sigma^{real} = \sigma^{\text{soft}}(\Delta E) + \sigma^{\text{hard}}_{\text{coll}}(\Delta E, \Delta\theta) + \sigma^{\text{hard}}_{\text{non-coll}}(\Delta E, \Delta\theta)$$

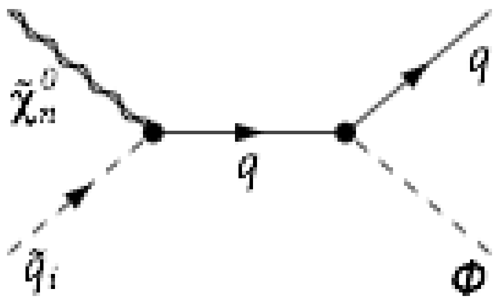
eikonal approximation — hard-collinear approximation — pure $2 \longrightarrow 3$ processes



Renormalization

aim: Renormalization scheme which is valid over a wide parameter space for all (co)annihilation processes within DM@NLO

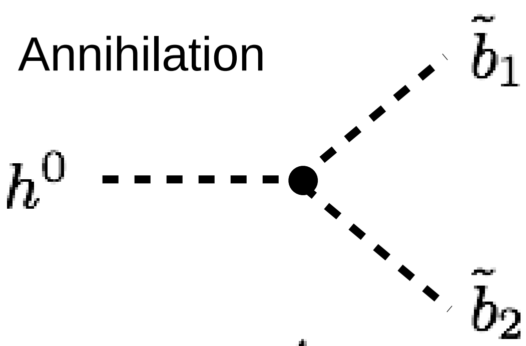
- relevant parameters: $m_{\tilde{t}_1}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, A_b, A_t, m_t, m_b, m_{\tilde{t}_2}, \theta_{\tilde{t}}, \theta_{\tilde{b}}$



choice: hybrid on-shell / $\overline{\text{DR}}$ renormalization scheme:

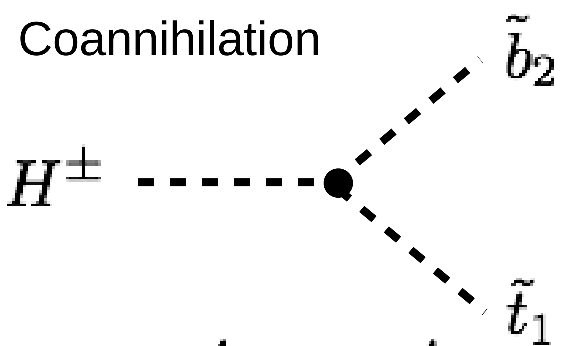
- input parameters: $m^{\text{OS}}_{\tilde{t}_1}, m^{\text{OS}}_{\tilde{b}_1}, m^{\text{OS}}_{\tilde{b}_2}, A^{\overline{\text{DR}}}_b, A^{\overline{\text{DR}}}_t, m^{\text{OS}}_t, m^{\overline{\text{DR}}}_b$

Annihilation



$\sim m_b, A_b$

Coannihilation



$\sim m_b, A_b, m_t, A_t$

Renormalization (cont.)

- input parameters: $m_{\tilde{t}_1}^{\mathrm{OS}}, m_{\tilde{b}_1}^{\mathrm{OS}}, m_{\tilde{b}_2}^{\mathrm{OS}}, A_b^{\overline{\mathrm{DR}}}, A_t^{\overline{\mathrm{DR}}}, m_t^{\mathrm{OS}}, m_b^{\overline{\mathrm{DR}}}$

  mass extraction:

$$m_b^{\overline{\mathrm{MS}},\,\mathrm{SM}}(m_b) \xrightarrow[\text{running}]{\text{SM NNLO}} m_b^{\overline{\mathrm{MS}},\,\mathrm{SM}}(Q) \xrightarrow{\text{conversion}} m_b^{\overline{\mathrm{DR}},\,\mathrm{SM}}(Q) \xrightarrow[\text{corrections}]{\text{threshold}} m_b^{\overline{\mathrm{DR}},\,\mathrm{MSSM}}(Q)$$

Baer, Ferrandis, et al., Phys. Rev. D 66 (2002)

- dependent parameters: $m_{\tilde{t}_2}, \theta_{\tilde{t}}, \theta_{\tilde{b}}$

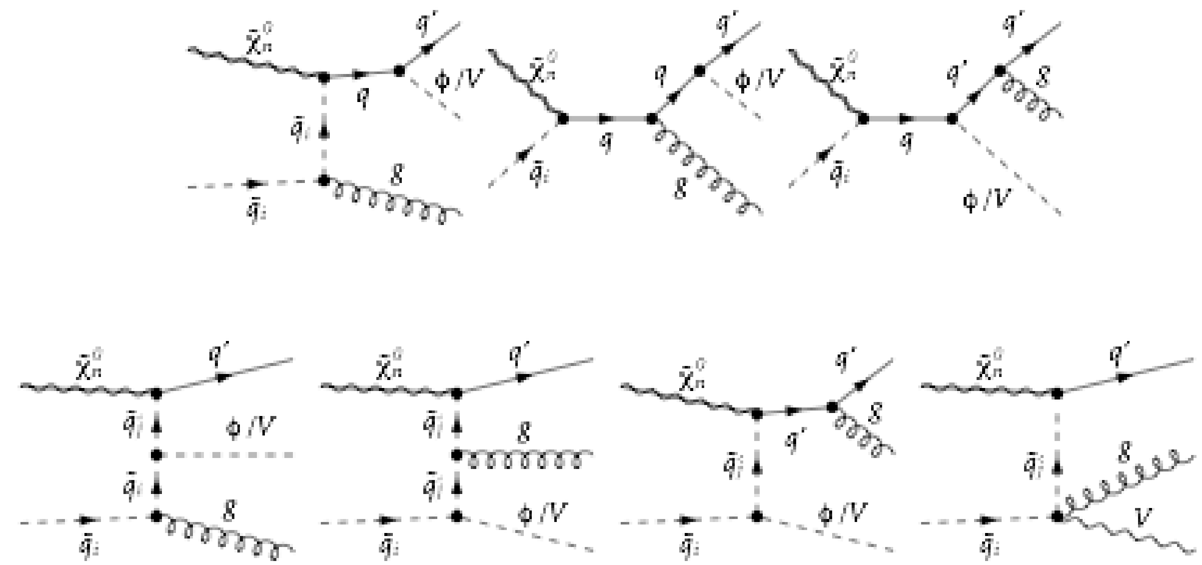
$$\begin{pmatrix} m_{\tilde{q}_1}^2 & 0 \\ 0 & m_{\tilde{q}_2}^2 \end{pmatrix} = U^{\tilde{q}} \begin{pmatrix} M_{\tilde{Q}}^2 + (I_q^{3L} - e_q\, s_W^2)\cos 2\beta\, m_Z^2 + m_q^2 & m_q\left(A_q - \mu\,(\tan\beta)^{-2I_q^{3L}}\right) \\ m_q\left(A_q - \mu\,(\tan\beta)^{-2I_q^{3L}}\right) & M_{\{\tilde{U},\,\tilde{D}\}}^2 + e_q\, s_W^2 \cos 2\beta\, m_Z^2 + m_q^2 \end{pmatrix} (U^{\tilde{q}})^\dagger$$

- choice of $\theta_{\tilde{t}}, \theta_{\tilde{b}}$ such that $\delta\theta_{\tilde{q}}$ remains stable

$$\delta\theta_{\tilde{q}} \propto \frac{1}{(U_{21}^{\tilde{q}} U_{12}^{\tilde{q}} + U_{11}^{\tilde{q}} U_{22}^{\tilde{q}})}$$

⇨ UV finite calculation obtained

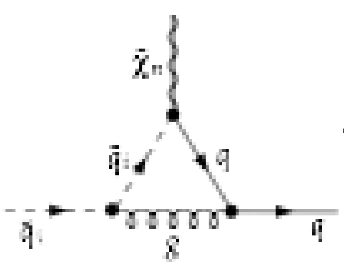
Real Corrections



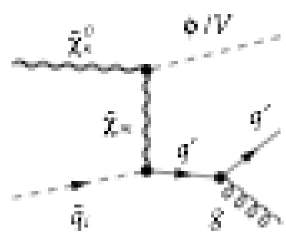
- Kinoshita-Lee-Nauenberg theorem:

$$\sigma^{\mathrm{NLO}} = \int_{2\to 2} d\sigma^{\mathrm{virtual}} + \int_{2\to 3} d\sigma^{\mathrm{real}} = \mathrm{finite}$$

- example: infrared divergent vertex correction



$+ \int_{2\to 3}$



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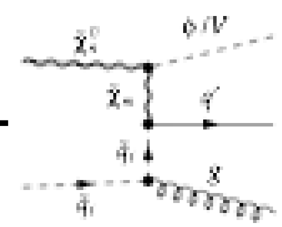
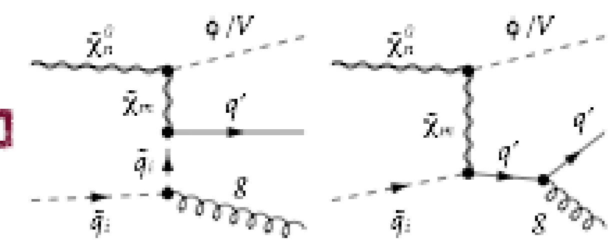
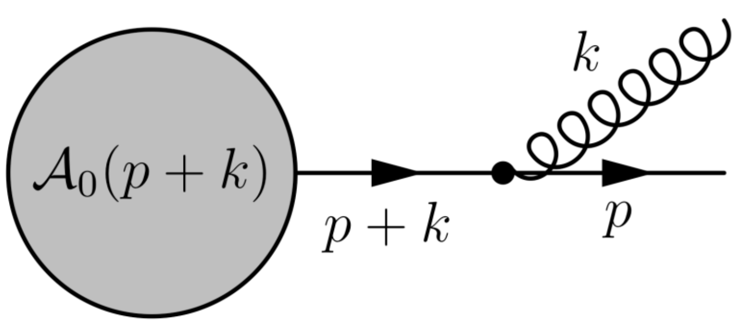


= finite



- soft and collinear divergences

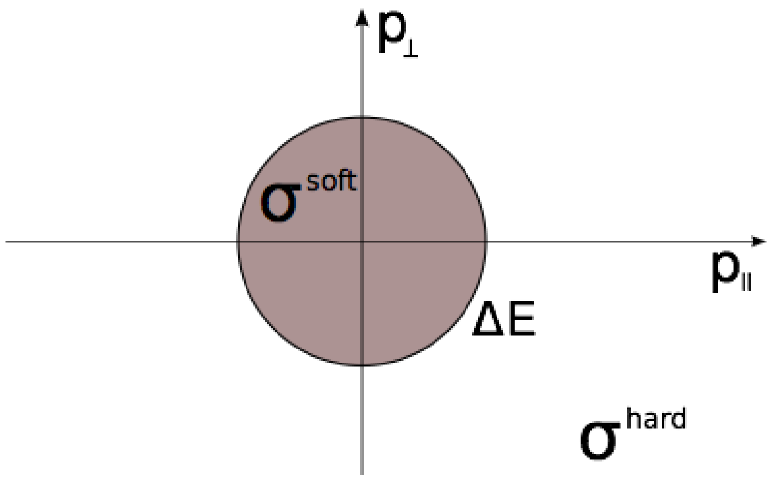


$$\frac{1}{(p+k)^2 - m^2} = \frac{1}{2\,p\cdot k} = \frac{1}{\omega(E_p - |\vec{p}|\cos\theta)} \xrightarrow[m_p=0\ \wedge\ \theta\to 0]{\omega\to 0\ \vee} \infty$$

only soft, no collinear divergences in case of the Higgs final state

Phase Space Slicing (one cutoff)

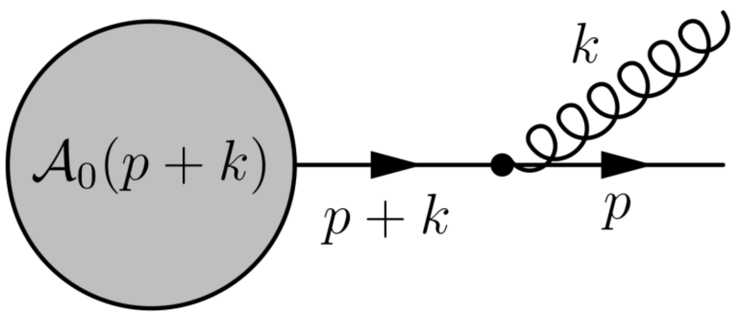
- phase space is devided in soft and hard part by cut off ΔE



$$\sigma^{\text{real}} = \sigma^{\text{soft}}(\Delta E) + \sigma^{\text{hard}}(\Delta E)$$

- use eikonal approximation in soft limit

$$\mathcal{M} = \mathcal{A}_0(p+k)\frac{i(\not{p}+\not{k}+m)}{(p+k)^2-m^2}(-ig_sT^a\gamma^\mu)\overline{u}(p)\varepsilon^*_\mu(k)$$



$$\mathcal{M} = \mathcal{A}_0(p)\overline{u}(p)\frac{p\cdot\varepsilon^*}{p\cdot k}(g_sT^a) \qquad \text{with} \qquad k^\mu \to 0$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{soft}} = -\left(\frac{d\sigma}{d\Omega}\right)_0 \times \frac{g_s^2 C_F \mu^{4-D}}{8\pi^3}\int_{|\vec{k}|\le\Delta E}\frac{d^{D-1}k}{(2\pi)^{D-4}}\frac{1}{2E_k}\left[-\frac{2p_1\cdot p_2}{(p_1\cdot k)(k\cdot p_2)}\right]$$

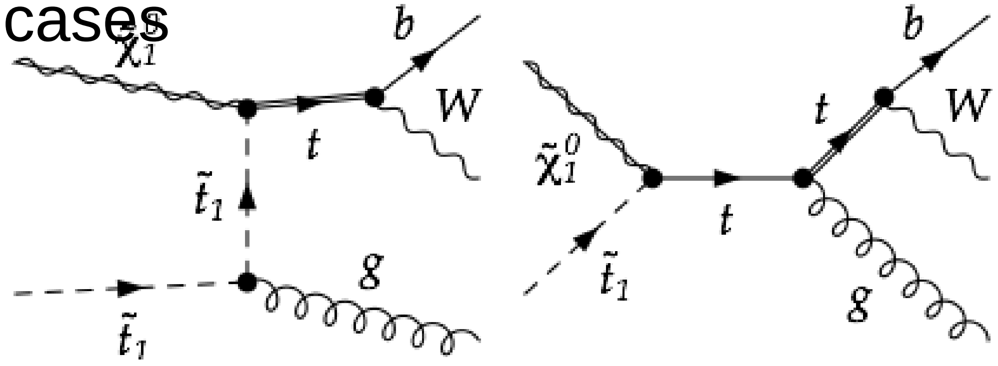
Veltmann, `t Hooft, Nuclear Physics B 153 (1979)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{soft}} = \left(\frac{d\sigma}{d\Omega}\right)_0 \times \frac{-g_s^2 C_F}{8\pi^2}\left[-\frac{1}{\epsilon}+\dots\right]$$

⇨ a finite total cross section is achieved

Intermediate On-Shell States

- unphysical enhancement of real corrections for two cases



- with $m_t > m_b + m_W$ an intermediate on-shell state can occur as soon as $\sqrt{s} > m_t$
- similar to the already considered tree level process $\tilde{\chi}_1^0 \tilde{t}_1 \to tg$ which further decays to $t \to bW^\pm$ ⇨ double-counting
- local on-shell subtractions (DS) introduces kind of counterterm, "Prospino" scheme

Beenakker, Nuclear Physics B 492 (1997)

$$|\mathcal{M}|^2 = |\mathcal{M}_{res}|^2 - |\mathcal{M}_{res}^{sub}|^2 + 2Re(\mathcal{M}_{res}^* \mathcal{M}_{rem}) + |\mathcal{M}_{rem}|^2$$

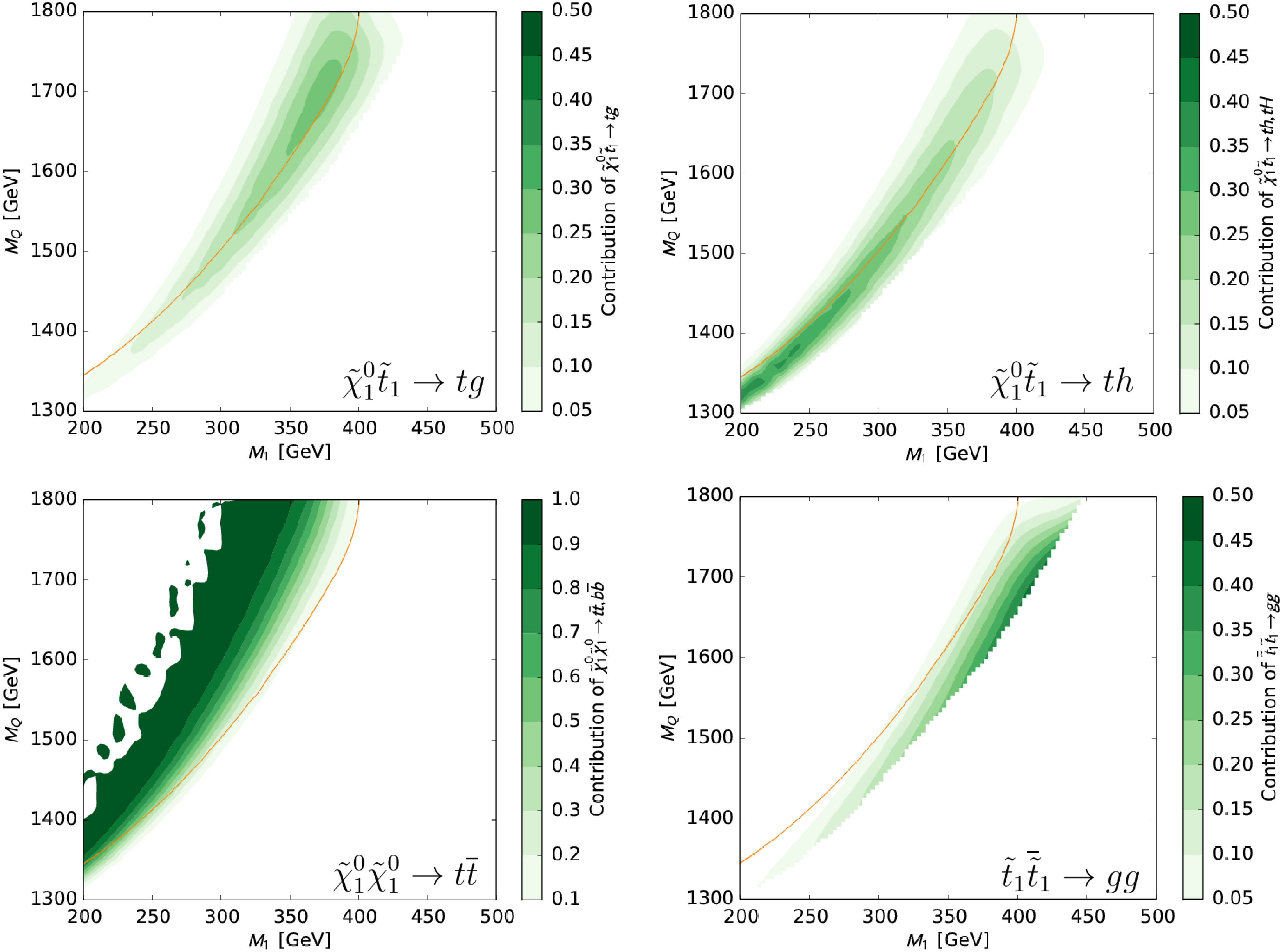
counterterm consists of Breit-Wigner weighted on-shell squared matrix element

$$|\mathcal{M}_{res}^{sub}|^2 = \frac{m_t^2 \Gamma_t^2}{(p_t^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} |\mathcal{M}_{res}|^2_{p_t^2 = m_t^2}$$

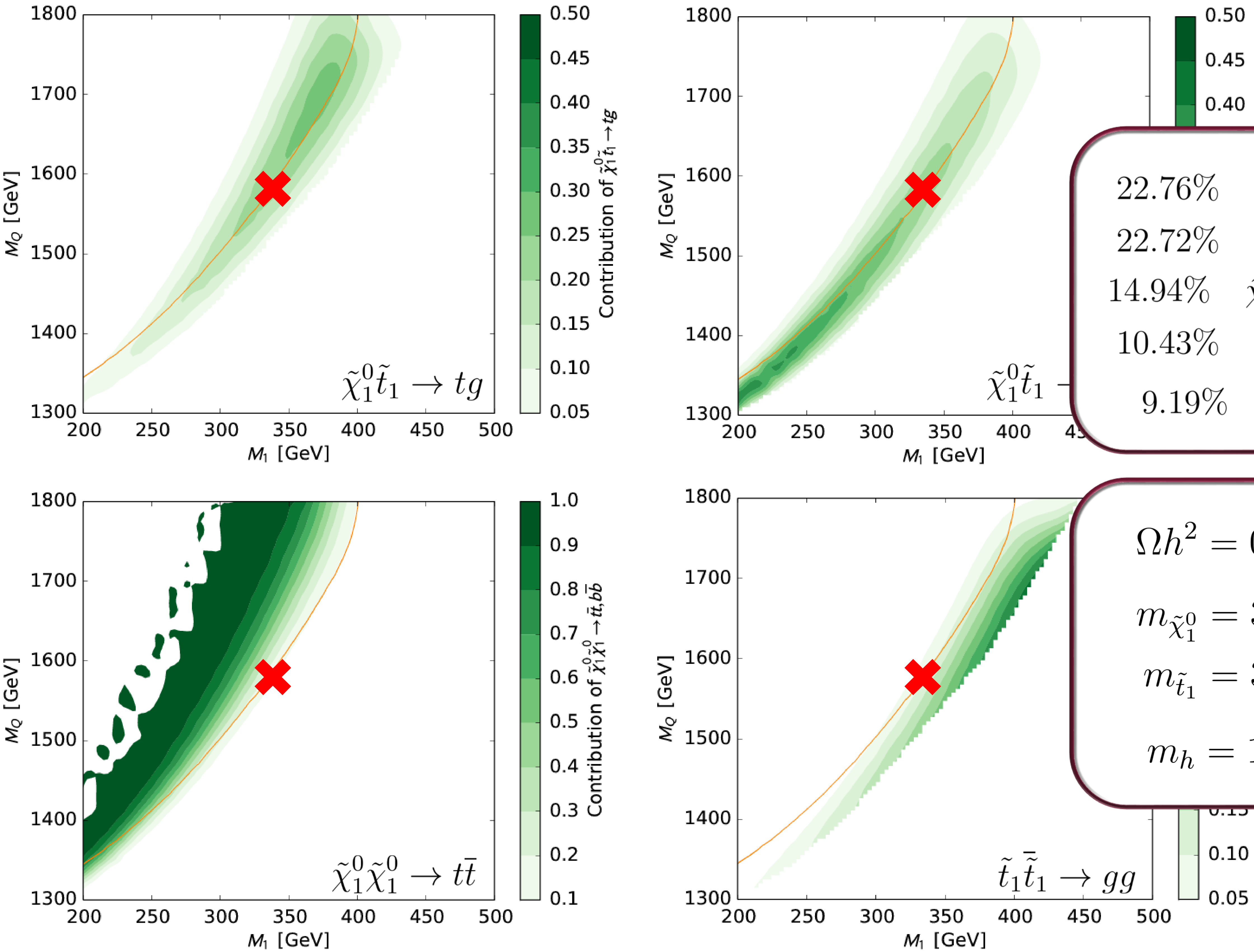
resonant propagators are regularized by Breit-Wigner propagator $\frac{1}{p^2 - m^2} \longrightarrow \frac{1}{p^2 - m^2 + im\Gamma}$

⇨ consistent, width independent, gauge invariant treatment retaining interference terms

Interplay of different (co)annihilating Channels



Interplay of different (co)annihilating Channels



22.76%	$\tilde{\chi}_1^0 \tilde{t}_1 \to tg$
22.72%	$\tilde{\chi}_1^0 \tilde{t}_1 \to th$
14.94%	$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to t\bar{t}$
10.43%	$\tilde{\chi}_1^0 \tilde{t}_1 \to bW^+$
9.19%	$\tilde{t}_1 \bar{\tilde{t}}_1 \to gg$

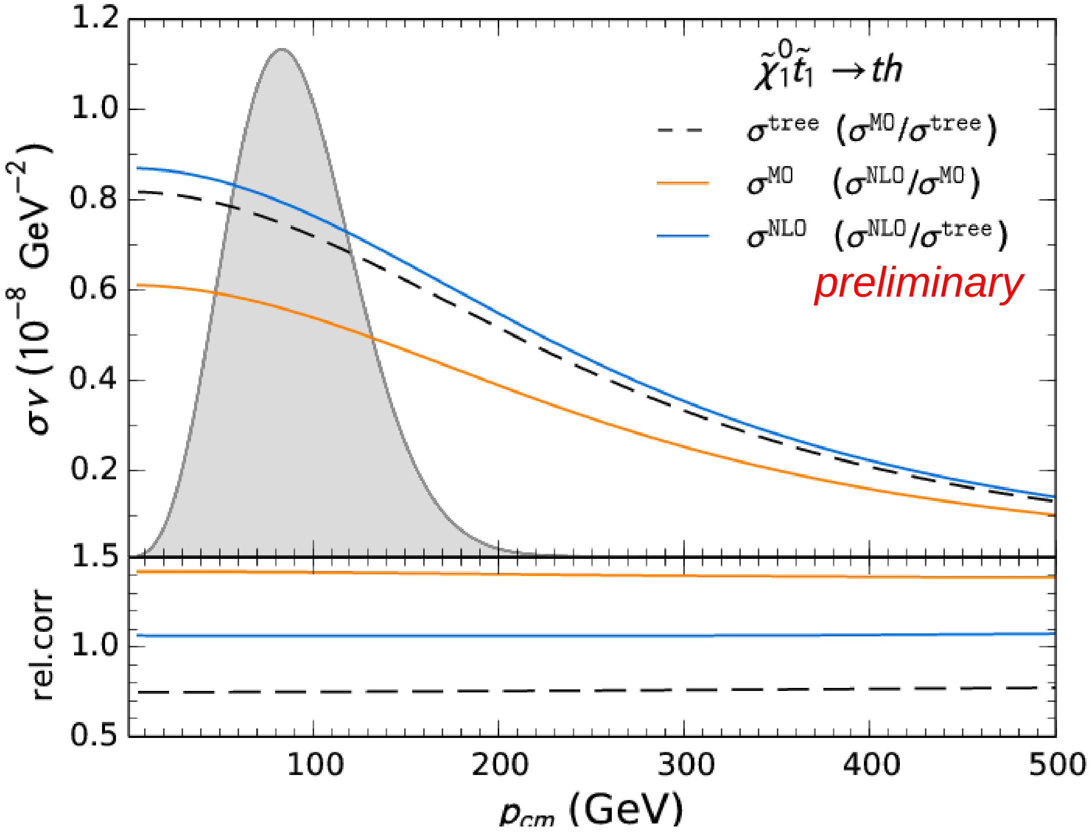
$$\Omega h^2 = 0.1135$$

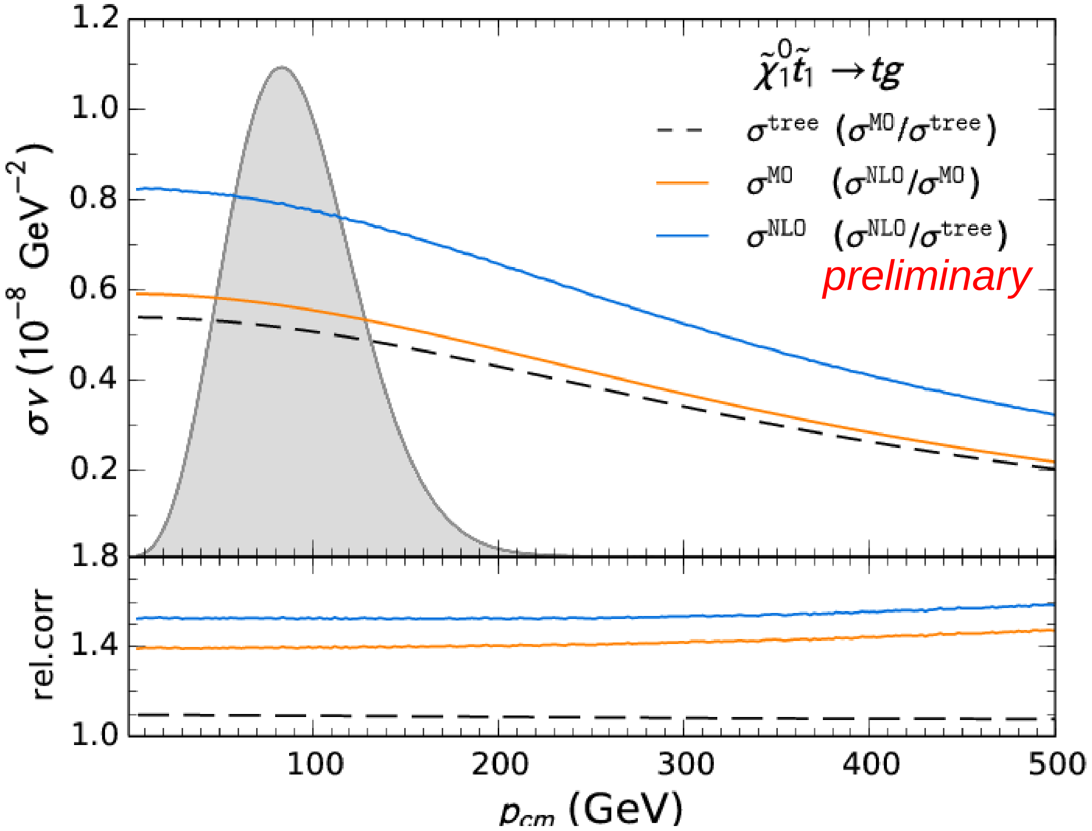
$$m_{\tilde{\chi}_1^0} = 338.3 \text{ GeV}$$

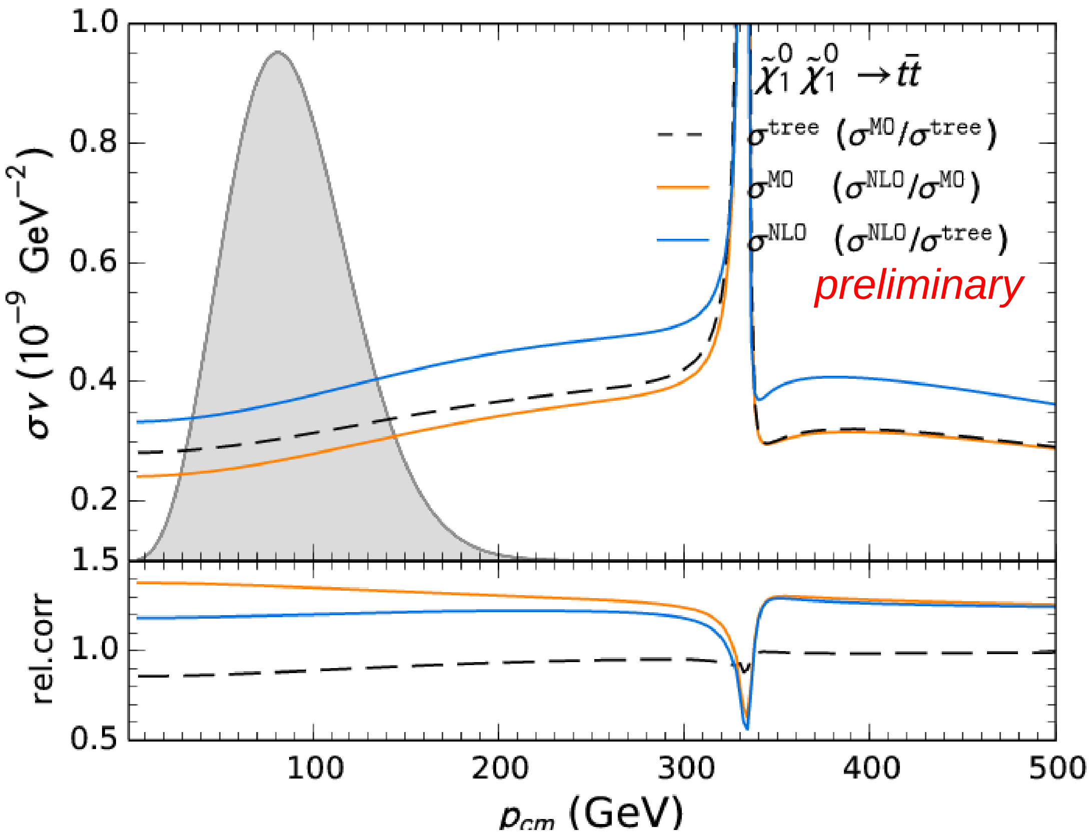
$$m_{\tilde{t}_1} = 375.6 \text{ GeV}$$

$$m_h = 121.94 \text{ GeV}$$

Impact on the (Co)annihilation Cross Section







- micrOMEGAs — Belanger et al. Comput.Phys.Commun.182:842-856 (2011) arXiv:1004.1092 [hep-ph]

  uses effective tree-level cross section

- SUSY-QCD corrections lead to a relative correction of up to 40 % on cross section with respect to default micrOMEGAs

Scale Variation

$$\frac{1}{2}\mu_R < \mu < 2\mu_R$$