Automatic evaluation of UV and R2 terms for beyond the Standard Model Lagrangians

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Plan

- Introduction
- Rational terms
- UV counterterms
- NLOCT
- Validation
- Perspectives and conclusion



Loop computation

$$\mathcal{A}^{1-loop} = \sum_{i} d_{i} \operatorname{Box}_{i} + \sum_{i} c_{i} \operatorname{Triangle}_{i} + \sum_{i} b_{i} \operatorname{Bubble}_{i} + \sum_{i} a_{i} \operatorname{Tadpole}_{i} + R$$

- Box, Triangle, Bubble and Tadpole are known scalar integrals
- Loop computation = find the coefficients
 - Unitarity
 - Multiple cuts
 - Tensor reduction (OPP)



Introduction

- Goal : Automate the one-loop computation for BSM models
- Required ingredients :
 - Tree-level vertices
 R2 vertices (OPP)

Missing

- UV counterterm vertices
- Solution : UFO at NLO

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\mathbf{R}_2

 $\bar{A}(\bar{q}) = \frac{1}{(2\pi)^4} \int d^d \bar{q} \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \dots \bar{D}_{m-1}},$

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$$



$$R_{2} \equiv \lim_{\epsilon \to 0} \frac{1}{\left(2\pi\right)^{4}} \int d^{d}\overline{q} \frac{\tilde{N}\left(\tilde{q}, q, \epsilon\right)}{\overline{D}_{0}\overline{D}_{1}\dots\overline{D}_{m-1}}$$

Finite set of vertices that can be computed once for all

R₂ example



$$\bar{Q}_{1} = \bar{q} + p_{1} = Q_{1} + \tilde{q}$$
$$\bar{Q}_{2} = \bar{q} + p_{2} = Q_{2} + \tilde{q}$$
$$\bar{D}_{0} = \bar{q}^{2}$$
$$\bar{D}_{1} = (\bar{q} + p_{1})^{2}$$
$$\bar{D}_{2} = (\bar{q} + p_{2})^{2}$$
't

't Hooft Veltman scheme $\overline{\eta}^{\overline{\mu}\,\overline{\nu}}\overline{\eta}_{\overline{\mu}\,\overline{\nu}} = d,$ $\overline{\gamma}^{\overline{\mu}}\overline{\gamma}_{\overline{\mu}} = d\mathbb{1},$

 $\bar{N}(\bar{q}) \equiv e^{3} \left\{ \bar{\gamma}_{\bar{\beta}} \left(\bar{Q}_{1} + m_{e} \right) \gamma_{\mu} \left(\bar{Q}_{2} + m_{e} \right) \bar{\gamma}^{\bar{\beta}} \right\}$ $= e^{3} \left\{ \gamma_{\beta} (Q_{1} + m_{e}) \gamma_{\mu} (Q_{2} + m_{e}) \gamma^{\beta} - \epsilon \left(Q_{1} - m_{e} \right) \gamma_{\mu} (Q_{2} - m_{e}) + \epsilon \tilde{q}^{2} \gamma_{\mu} - \tilde{q}^{2} \gamma_{\beta} \gamma_{\mu} \gamma^{\beta} \right\}$

RI

Due to the \mathcal{E} dimensional parts of the denominators

Like for the 4 dimensional part but with a different set of integrals

$$\int d^n \bar{q} \frac{\tilde{q}^2}{\bar{D}_i \bar{D}_j} = -\frac{i\pi^2}{2} \left[m_i^2 + m_j^2 - \frac{(p_i - p_j)^2}{3} \right] + \mathcal{O}(\epsilon) ,$$

$$\int d^n \bar{q} \frac{\tilde{q}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon) ,$$

$$\int d^n \bar{q} \frac{\tilde{q}^4}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\epsilon) .$$

Only R = R_1 + R_2 is gauge invariant Check

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UV

 $\bar{A}(\bar{q}) = \frac{1}{(2\pi)^4} \int d^d \bar{q} \frac{N(\bar{q})}{\bar{D}_0 \bar{D}_1 \dots \bar{D}_{m-1}} = K \frac{1}{\epsilon} + \mathcal{O}\left(\epsilon^0\right)$



Finite set of vertices that can be computed once for all

Renormalization



Internal parameters are renormalised by replacing the external parameters in their expressions

Renormalization conditions

On-shell scheme (or complex mass scheme):

Renormalized mass = Physical mass

Two-point function vanishes on-shell (No external bubbles)



 $i\delta_{ij} (\not p - m_i) + i \left[f_{ij}^L (p^2) \not p \gamma_- + f_{ij}^R (p^2) \not p \gamma_+ + f_{ij}^{SL} (p^2) \gamma_- + f_{ij}^{SR} (p^2) \gamma_+ \right]$

$$\tilde{\mathfrak{K}}\left[f_{ij}^{L}\left(p^{2}\right)m_{i}+f_{ij}^{SR}\left(p^{2}\right)\right]\Big|_{p^{2}=m_{i}^{2}}=0$$
$$\tilde{\mathfrak{K}}\left[f_{ij}^{R}\left(p^{2}\right)m_{i}+f_{ij}^{SL}\left(p^{2}\right)\right]\Big|_{p^{2}=m^{2}}=0$$

 $\tilde{\mathcal{X}}\left[2m_{i}\frac{\partial}{\partial p^{2}}\left[\left(f_{ii}^{L}\left(p^{2}\right)+f_{ii}^{R}\left(p^{2}\right)\right)m_{i}+f_{ii}^{SL}\left(p^{2}\right)+f_{ii}^{SR}\left(p^{2}\right)\right]+f_{ii}^{L}\left(p^{2}\right)+f_{ii}^{R}\left(p^{2}\right)\right]\right|_{p^{2}=m_{i}^{2}}=0$

Similar for the vectors and scalars

Renormalization conditions
Jero momentum scheme available for the gauge couplings

$$\Gamma_{FFV}^{\mu}(p_{1},p_{2}) = igT^{a}\delta_{f_{1},f_{2}} \left[\gamma^{\mu} \left(\frac{\delta g}{g} + \frac{1}{2} \delta Z_{VV} + \frac{1}{2} \delta Z_{FF}^{R} + \frac{1}{2} \delta Z_{FF}^{L} + \frac{g'_{V}}{2g} \delta Z_{V'V} \right) + \gamma^{\mu}\gamma_{5} \left(\frac{1}{2} \delta Z_{FF}^{R} - \frac{1}{2} \delta Z_{FF}^{L} + \frac{g'_{A}}{2g} \delta Z_{V'V} \right) + \left(\gamma^{\mu}h^{V}(k^{2}) + \gamma^{\mu}\gamma_{5}h^{A}(k^{2}) + \frac{(p_{1}-p_{2})^{\mu}}{2m}h^{S}(k^{2}) + \frac{k_{\mu}}{2m}h^{P}(k^{2}) \right) \right]$$

$$\frac{\delta g}{g} + \frac{1}{2} \delta Z_{VV} + \frac{1}{2} \delta Z_{FF}^{R} + \frac{1}{2} \delta Z_{FF}^{L} + \frac{g'_{A}}{2g} \delta Z_{V'V} + h^{V}(0) + h^{S}(0) = 0$$

$$\frac{1}{2} \delta Z_{FF}^{R} - \frac{1}{2} \delta Z_{FF}^{R} + \frac{g'_{A}}{2g} \delta Z_{V'V} + h^{A}(0) = 0.$$
By gauge invariance
$$\frac{\delta g}{g} + \frac{1}{2} \delta Z_{VV} + \frac{g'_{V}}{2g} \delta Z_{V'V} + \frac{g'_{A}}{2g} \delta Z_{V'V} = 0$$

$$MS \text{ scheme for everything else (option for all)}$$

6'

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How does it work?



How does it work?

FeynRules :

...

Lren = OnShellRenormalization[LSM , OCDOnly ->True]; WriteFeynArtsOutput[Lren , Output -> "SMrenoL", GenericFile -> False]

FeynArts / NLOCT :

WriteCT["SMrenoL/SMrenoL" [Lorentz", Output-> "SMQCDreno", QCDoniy -> True]

FeynRules : ... Get["SMQCDreno.nlo"]; WriteUFO[LSM , UVCounterterms -> UV\$vertlist , R2Vertices -> R2\$vertlist]

model.nlo

Model information (FR+FeynArts model/generic files)

R2\$vertlist = {
{{anti[u], 1}, {u, 2}}, ((-I/12)*gs^2*
IndexDelta[Index[Colour, Ext[1]], Index[Colour, Ext[2]]]*IPL[{u, G}]*
(TensDot[SlashedP[2], ProjM][Index[Spin, Ext[1]], Index[Spin, Ext[2]]] +
TensDot[SlashedP[2], ProjP][Index[Spin, Ext[1]], Index[Spin, Ext[2]]]))/Pi^2},

~FeynRules syntaxe

UV\$vertlist (ε is FR\$Eps)

}

```
FR$InteractionOrderPerturbativeExpansion = {{QCD, 1}, {QED, 0}};
NLOCT$assumptions
QCDOnly
WriteCT[...,Assumptions->{...}]
```



Restrictions/Assumptions

- Renormalizable Lagrangian, maximum dimension of the operators is 4
- Feynman Gauge
- $\{\gamma_{\mu}, \gamma_5\} = 0$
- 't Hooft-Veltman scheme
- On-shell scheme for the masses and wave functions
- MS by default for everything else (zero-momentum possible for fermion gauge boson interaction)

NLOCT

- Amplitudes from EFT :No discard diagrams, Higher diagrams like ghost boxes adjacencies, list of amplitudes
- Compute terms at the generic level

$$\vec{c} \cdot \vec{L} = \sum_i c_i L_i$$

• Feynman parameters

propagators>4

- Remove terms with an odd or too low rank
- Dirac algebra

More than one fermion chain

Remove too high dimension

• Gather loop momentum

$$\begin{split} q^{\mu}q^{\nu}q^{\rho}q^{\sigma} & \to \quad q^{4}\frac{1}{d(d+2)}\left(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\rho\nu}\right) \\ q^{\mu}q^{\nu} & \to \quad q^{2}\frac{1}{d}\eta^{\mu\nu}. \end{split}$$



Replace momentum integrals

- Integrate over the Feynman parameters (but for the two-point UV finite terms)
- Replace masses and couplings by their values for each field insertion
 C. Degrande

NLOCT

- Perform the color algebra for triplets and octets
- Write the renormalization conditions (fix p²) End R₂
- Do the integration over the feynman parameters for the UV-finite parts

$$b_0(p^2, m_1, m_2) \equiv \int_0^1 dx \log\left(\frac{p^2(x-1)x + x(m_1^2 - m_2^2) + m_2^2 - i\epsilon_p}{\mu^2}\right)$$
$$b_0(0, 0, 0) = \frac{1}{\overline{\epsilon}}$$

- Solve the renormalization conditions
- Replace the counterterms by their values in the CT vertices
 Merge R2EFT with NLOCT



• UV divergent part of the vertex is the opposite of the loop amplitude

$$-i\delta^{a_1a_2}\delta Z_{gg} \left(p_1^{\mu_2}p_2^{\mu_1} - p_1 \cdot p_2\eta^{\mu_1\mu_2}\right) = -A^{UV}\frac{1}{\overline{\epsilon}_{UV}} - i\delta^{a_1a_2}\delta Z_{gg}^{UVfin} \left(p_1^{\mu_2}p_2^{\mu_1} - p_1 \cdot p_2\eta^{\mu_1\mu_2}\right)$$

- Renormalization condition are solved for the UV finite part only
- Advantage
 - 2HDM ymb=4.7, MB=0
 - EFT : no need for the operators remove by EOM

Real/Complex masses



$$\log\left[m_1^2 - p^2\right]\Big|_{p^2 = m^2} \qquad \qquad \mathsf{Faster!}$$

All cases are kept unless the users put some assumptions

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R2:Validation

- tested* on the SM (QCD:P. Draggiotis et al. +QED:M.V. Garzelli et al)
- tested* on MSSM (QCD:H.-S. Shao,Y.-J. Zhang) : test the Majorana

*Analytic comparison of the expressions

UV Validation

- SM QCD : tested* (W. Beenakker, S. Dittmaier, M. Kramer, B. Plumper)
- SM EW : tested* (expressions given by H.-S. Shao from A. Denner)

*Analytic comparison of the expressions

Tests in event generators

- aMC@NLO
- The SM QCD has been tested by V. Hirschi (Comparison with the built-in version)
- The MSSM QCD and SM EW are tested by H. S. Shao and V. Hirschi
- 2HDM QCD is currently tested ($p p > S, H^+ t$)

- gauge invariance
- pole cancelation
- Anomalous top



=== Finite === ML5 default Process Stored ML5 opt ML5 opt Relative diff. Result -1.2565695610e+01 -1.2565705416e+01 -1.2565696276e+01 3.9018817097e-07 Pass d d~ > w+ w- g === Born === Process Stored ML5 opt ML5 opt ML5 default Relative diff. Result $d d \sim w + w - g$ [.85]83]852]e-06 [.85]83]852]e-06 [.85]83]852]e-06 8.06]723]4]]e-15 Pass === Single pole === ML5 default Process Stored ML5 opt ML5 opt Relative diff. Result d d~ > w+ w- g -1.9397426502e+01 -1.9397426502e+01 -1.9397426504e+01 5.5894073017e-11 Pass === Double pole === Stored ML5 opt ML5 opt ML5 default Process Relative diff. Result d d~ > w+ w- g -5.66666666667e+00 -5.66666666667e+00 -5.666666666667e+00 3.0015206007e-14 Pass === Summary === I/I passed, 0/I failed=== Finite === Stored MadLoop v4 ML5 opt ML5 default Process Relative diff. Result d~d>agg -5.3971186943e+01 -5.3971193753e+01 -5.3971189940e+01 6.3091071914e-08 Pass === Born === Stored MadLoop v4 ML5 opt ML5 default Relative diff. Result Process 6.4168774056e-05 6.4168764370e-05 6.4168764370e-05 7.5467680882e-08 Pass d~d>agg === Single pole === ML5 default Process Stored MadLoop v4 ML5 opt Relative diff. Result -3.7439549398e+01 -3.7439549398e+01 -3.7439549397e+01 6.8122965983e-12 Pass d~d>agg === Double pole === Stored MadLoop v4 ML5 opt ML5 default Process Relative diff. Result d~d>agg === Summary === I/I passed, 0/I failed=== Finite ===

 Process
 Stored MadLoop v4
 ML5 opt
 ML5 default
 Relative diff.
 Result

 d~ d > z g g
 -5.3769573669e+01
 -5.3769573347e+01
 -5.3769566412e+01
 6.7475496780e-08
 Pass

SM tests

=== Born ===		
Process	Stored MadLoop v4 ML5 opt ML5 default	Relative diff. Result
d∼ d > z g g	3.1531233900e-04 3.1531235770e-04 3.1531235770e	-04 2.9654886777e-08 Pass
00		
=== Single pole ===		
Process	Stored MadLoop v4 ML5 opt ML5 default	Relative diff. Result
d~ d > z g g	-3.7464897007e+01 -3.7464897007e+01 -3.7464897007	e+01 4.2333025503e-12 Pass
=== Double pole ===		
Process	Stored MadLoop v4 ML5 opt ML5 default	Relative diff. Result
d~ d > z g g	-8.666666666667e+00 -8.666666666667e+00 -8.666666666666	e+00 2.1316282073e-14 Pass
=== Summary ===		
	<pre>I/I passed, 0/I failed=== Finite ===</pre>	
Process	Stored MadLoop v4 ML5 opt ML5 default	Relative diff. Result
d~ d > z z g	-5.9990384275e+00 -5.9990511729e+00 -5.9990379587	e+00 1.1013604745e-06 Pass
D		
Process	Stored MadLoop v4 ML5 opt ML5 default	Relative diff. Result
d~ d > z z g	2.2616997126e-06 2.2617000449e-06 2.2617000449e	-06 /.3450366526e-08 Pass
Process	Stored Madl oop v4 MI 5 opt MI 5 default	Polativo diff Posult
$d \sim d > 7.7 \sigma$	$5469597040_{2}+01 = 15469597040_{2}+01 = 15469597040_{2}$	a+01 = 5226666708a = 1
u - u - z z g	-1:5407507070707070707070707070707070707070	e 01 1.5220000700e-11 1 ass
=== Double pole ===		
Process	Stored Madl oop v4 MI 5 opt MI 5 default	Relative diff Result
$d \sim d > 7.7 \sigma$	-5 6666666667e+00 -5 66666666667e+00 -5 6666666666	e+00 2 6645352591e-15 Pass
=== Summary ===		
I/I passed, 0/I failed=== Finite ===		
Process	Stored MadLoop v4 ML5 opt ML5 default	Relative diff. Result
g g > h t t~	2.9740187004e+01 2.9740187005e+01 2.9740187036e	+01 5.3265970697e-10 Pass

SM tests

=== Born === Stored MadLoop v4 ML5 opt ML5 default Relative diff. Process Result 1.1079653971e-07 1.1079653974e-07 1.1079653974e-07 1.3190849004e-10 Pass g g > h t t~ === Single pole === Stored MadLoop v4 ML5 opt ML5 default Process Relative diff. Result -7.0825709000e+00 -7.0825709000e+00 -7.0825709000e+00 5.0901237085e-13 Pass gg > htt~=== Double pole === ML5 default Stored MadLoop v4 ML5 opt Process Relative diff. Result -6.00000000e+00 -6.00000000e+00 -6.00000000e+00 1.7023419711e-15 Pass gg > htt~=== Summary === I/I passed, 0/I failed=== Finite === Stored MadLoop v4 ML5 opt ML5 default Process Relative diff. Result 3.6409017466e+01 3.6409021125e+01 3.6409021117e+01 5.0242920154e-08 Pass gg>ztt~ === Born === ML5 default Stored MadLoop v4 ML5 opt Process Relative diff. Result 7.0723041711e-07 7.0723046101e-07 7.0723046101e-07 3.1039274206e-08 Pass g g > z t t~ === Single pole === Stored MadLoop v4 ML5 opt ML5 default Relative diff. Process Result -7.1948086812e+00 -7.1948086773e+00 -7.1948086773e+00 2.7349789963e-10 Pass gg>ztt~ === Double pole === Stored MadLoop v4 ML5 opt ML5 default Relative diff. Result Process -6.00000000e+00 -6.00000000e+00 -6.00000000e+00 2.5165055225e-15 Pass gg>ztt~ === Summary === I/I passed, 0/I failed=== Finite === Process Stored ML5 opt ML5 opt ML5 default Relative diff. Result -1.2565695610e+01 -1.2565705416e+01 -1.2565696276e+01 3.9018817097e-07 Pass $d d \sim > w + w - g$

SM tests

=== Born === ML5 default Process Stored ML5 opt ML5 opt Relative diff. Result d d~ > w+ w- g 1.8518318521e-06 1.8518318521e-06 1.8518318521e-06 8.0617231411e-15 Pass === Single pole === Process Stored ML5 opt ML5 opt ML5 default Relative diff. Result -1.9397426502e+01 -1.9397426502e+01 -1.9397426504e+01 5.5894073017e-11 Pass d d~ > w+ w- g === Double pole === ML5 default Relative diff. Process Stored ML5 opt ML5 opt Result === Summary === I/I passed, 0/I failed=== Finite === Process Stored ML5 opt ML5 opt ML5 default Relative diff. Result $d \sim d > a g g$ -1.1504816412e+01 -1.1504816557e+01 -1.1504815497e+01 4.6089385415e-08 Pass === Born === Stored ML5 opt ML5 opt ML5 default Process Relative diff. Result $d \sim d > a g g$ 2.3138920858e-06 2.3138920858e-06 2.3138920858e-06 4.3012538015e-15 Pass === Single pole === Stored ML5 opt ML5 opt ML5 default Relative diff. Result Process -2.8637049838e+01 -2.8637049838e+01 -2.8637049838e+01 1.5718407645e-13 Pass d~d>agg === Double pole === ML5 default Process Stored ML5 opt ML5 opt Relative diff. Result d~d>agg === Summary === 1/1 passed, 0/1 failed=== Finite === Stored ML5 opt ML5 opt ML5 default Relative diff. Result Process d~d>zgg -1.0306105482e+01 -1.0306105654e+01 -1.0306102645e+01 1.4600800434e-07 Pass

+7/3

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Perspectives

- Phenomenology
- 2HDM
 - Charged Higgs
 - Higgs pair production,...
- Anomalous top (FCNC from dimension-six operators)
 - single top, ...
- MSSM
 - Pre-SUSY simplified model



Conclusion

- Automatic BSM@NLO
 - renormalizable
 - Feynman gauge
- Next version



- EFT
- Any gauge
- other renormalization scheme (EW)
- With the help of the FeynRules and Madgraph_aMC@NLO teams