YETI 2015
Introduction to Cosmology

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1. The general picture, evolution of the universe: assumptions and evidence supporting them.
2. Dark Energy - Dark Matter - Modified Gravity
3. Inflation.

Jan 7th 2015

Durham

1. The Big Bang – (1sec → today)

The cosmological principle -- isotropy and homogeneity on large scales

Test 1

- **The expansion of the Universe**

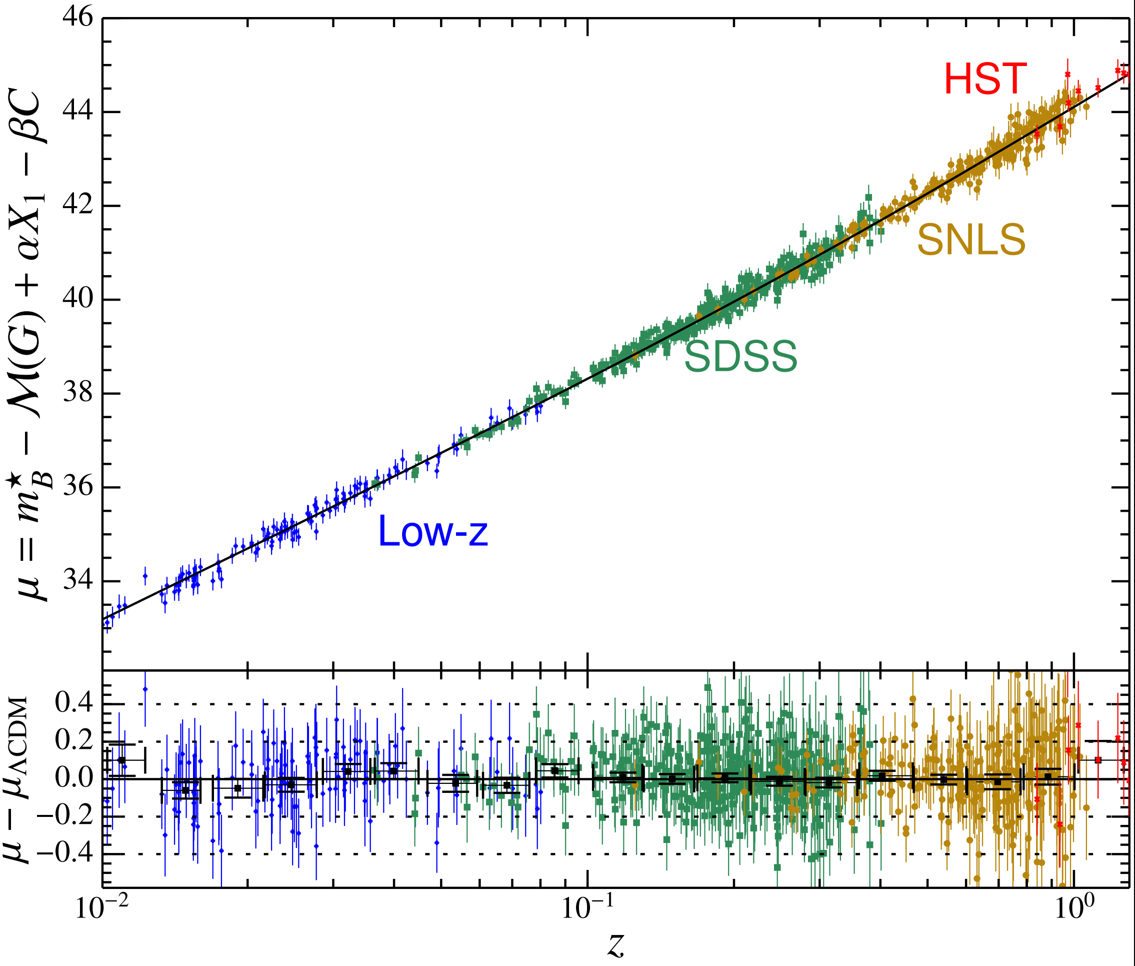
v=H$_0$d

H$_0$=73.8±2.4 km s^{-1} Mpc^{-1}

(Riess et al, 2011)

H$_0$=68.5±1.27 km s^{-1} Mpc^{-1}

(Betoule et al, 2014)



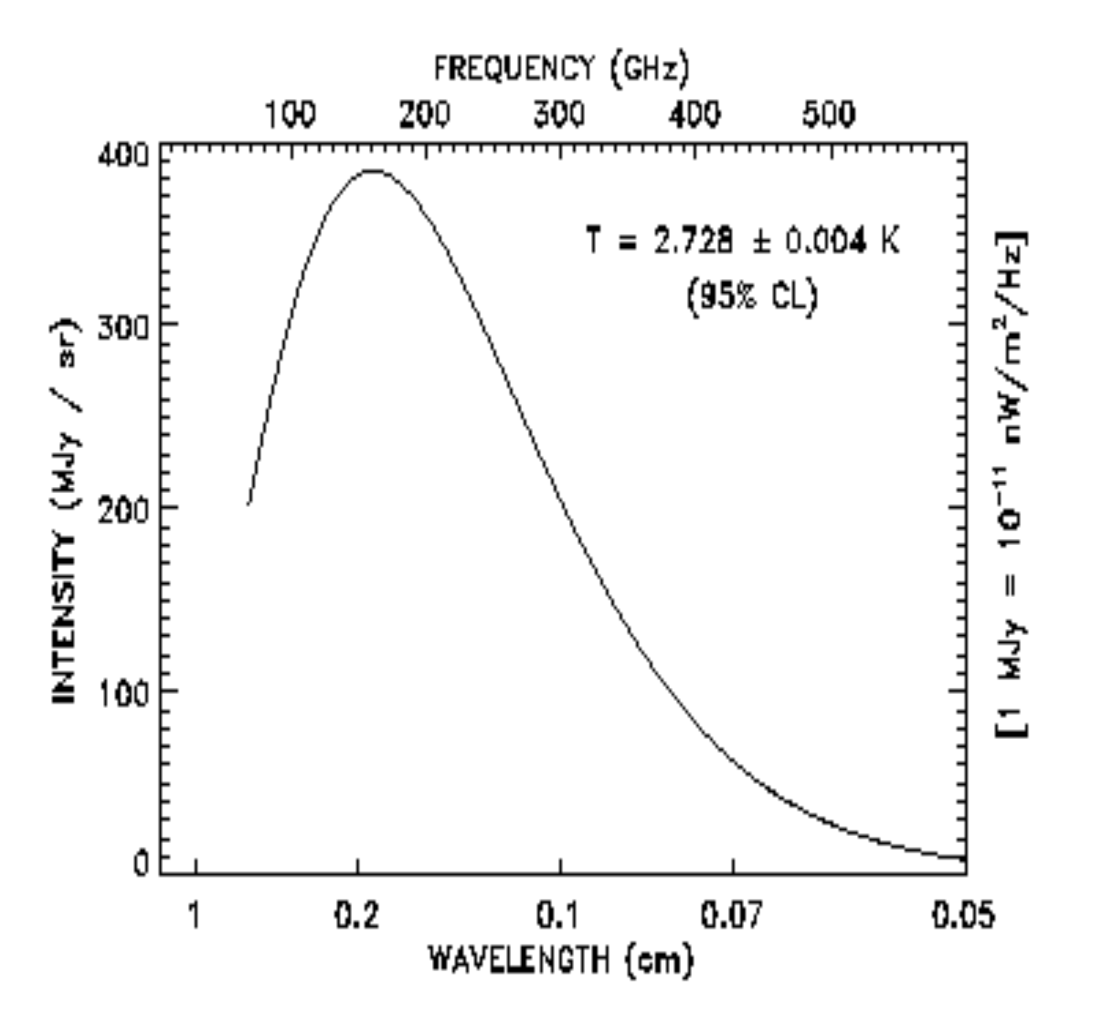
Betoule et al 2014

Distant galaxies receding with vel proportional to distance away.

Relative distance at different times measured by scale factor a(t) with

$$\text{Redshift } 1 + z = \frac{a_0}{a} \qquad H = \frac{\dot{a}}{a}$$

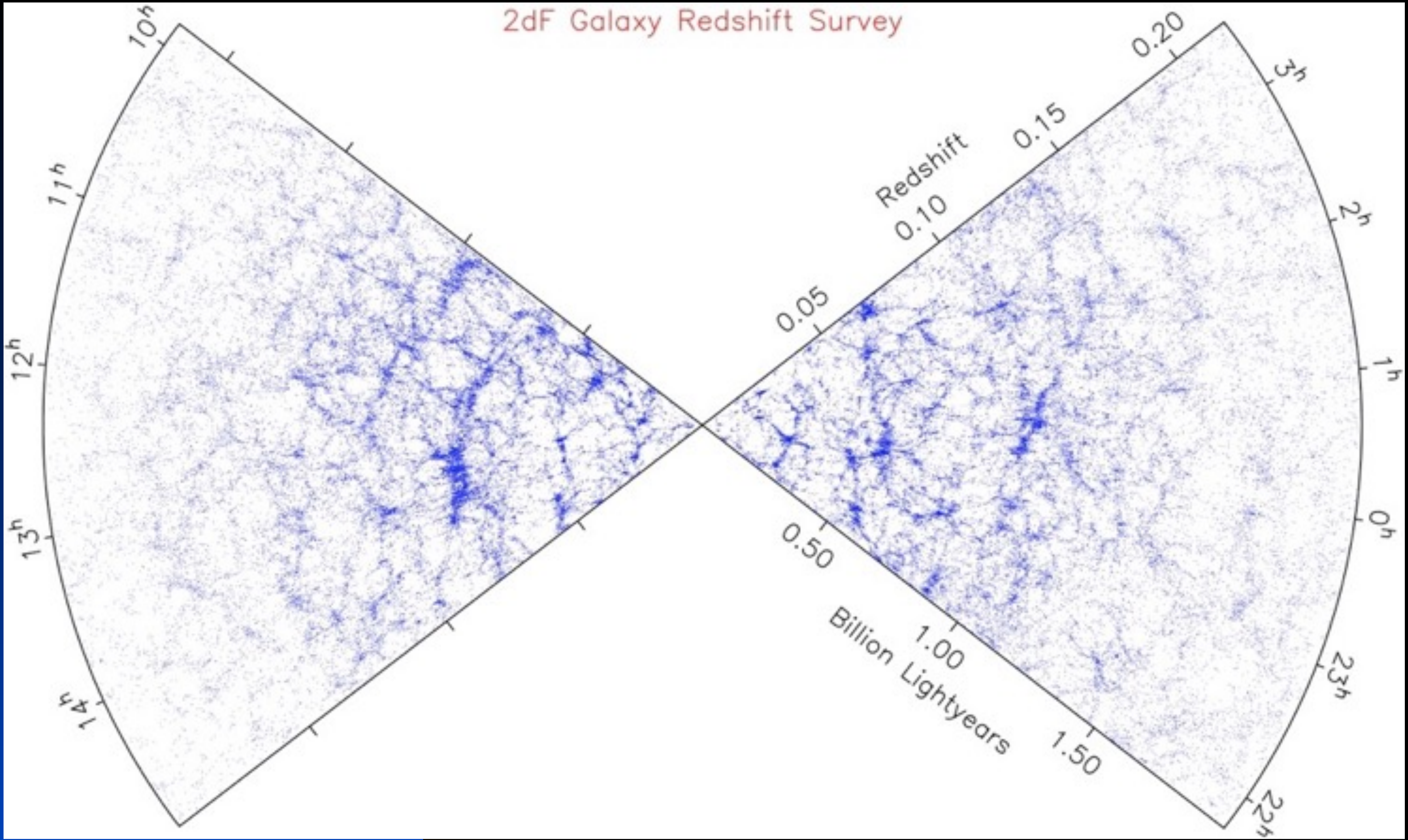
The Big Bang – (1sec → today)



Test 2

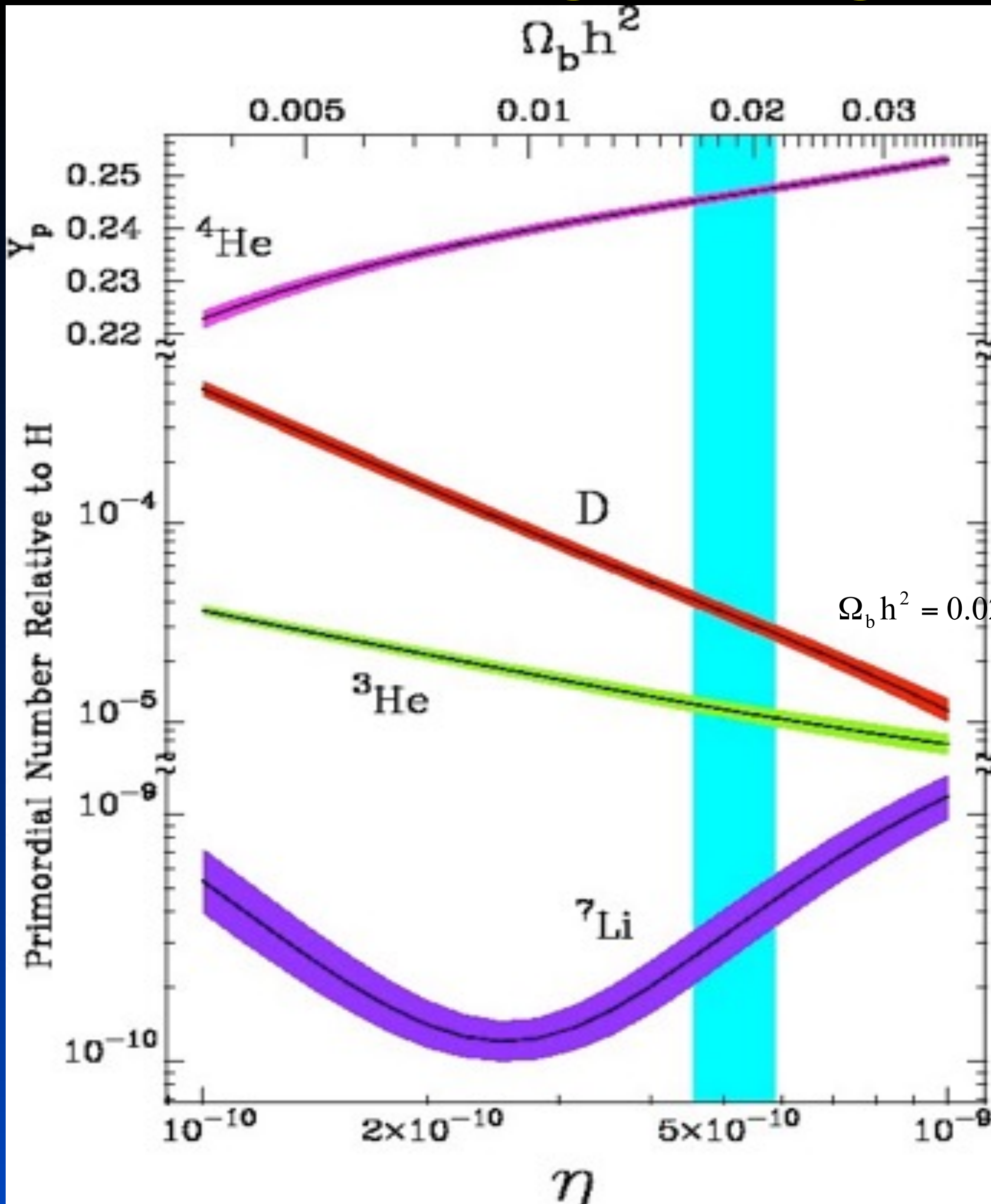
- **The existence and spectrum of the CMBR**
- **T_0=2.728 ± 0.004 K**
- Evidence of isotropy -- detected by COBE to such incredible precision in 1992
- Nobel prize for John Mather 2006

2dF Durham Prize Winning Galaxy Redshift Survey



Homogeneous on large scales?

The Big Bang – (1sec → today)



Test 3

- **The abundance of light elements in the Universe.**
- **Most of the visible matter just hydrogen and helium.**

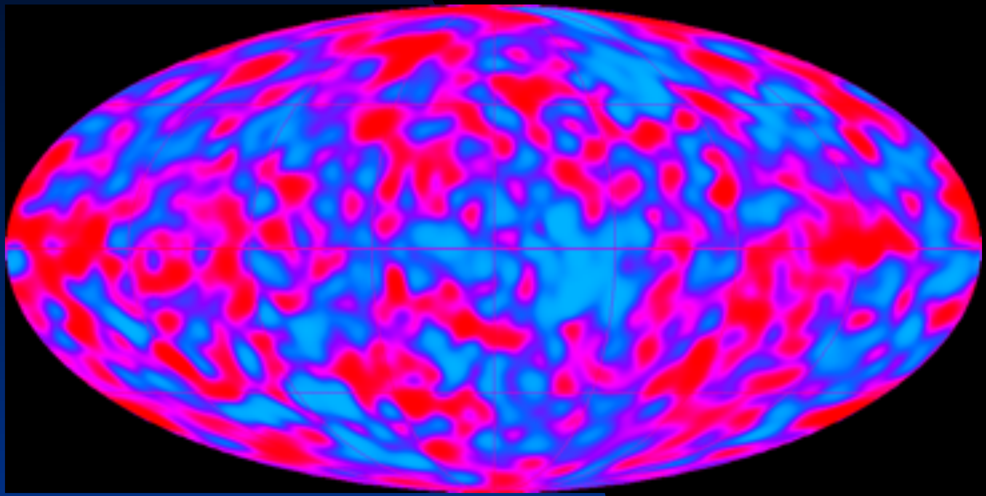
$$\Omega_b h^2 = 0.02207 \pm 0.00033 \ (68\% \ \mathrm{CL})$$

Planck 2013

The Big Bang – (1sec → today)

Test 4

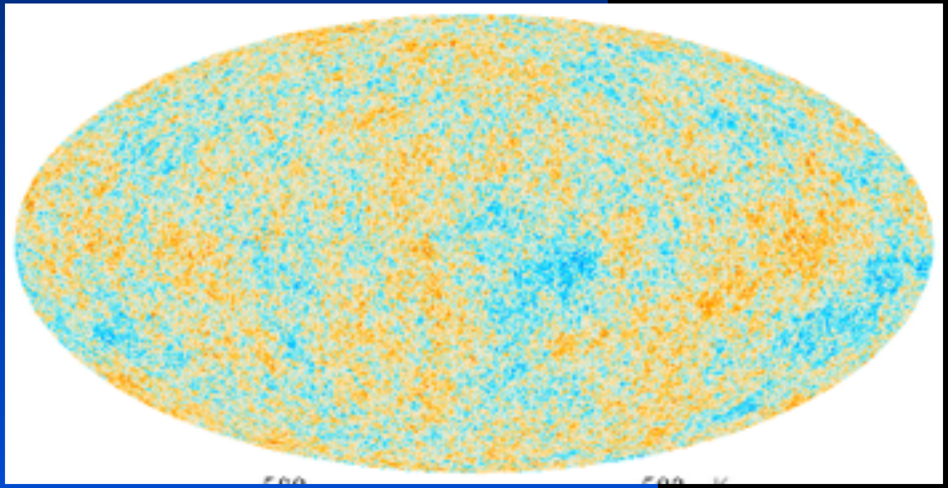
- Given the irregularities seen in the CMBR, the development of structure can be explained through gravitational collapse.



COBE - 1992, 2006

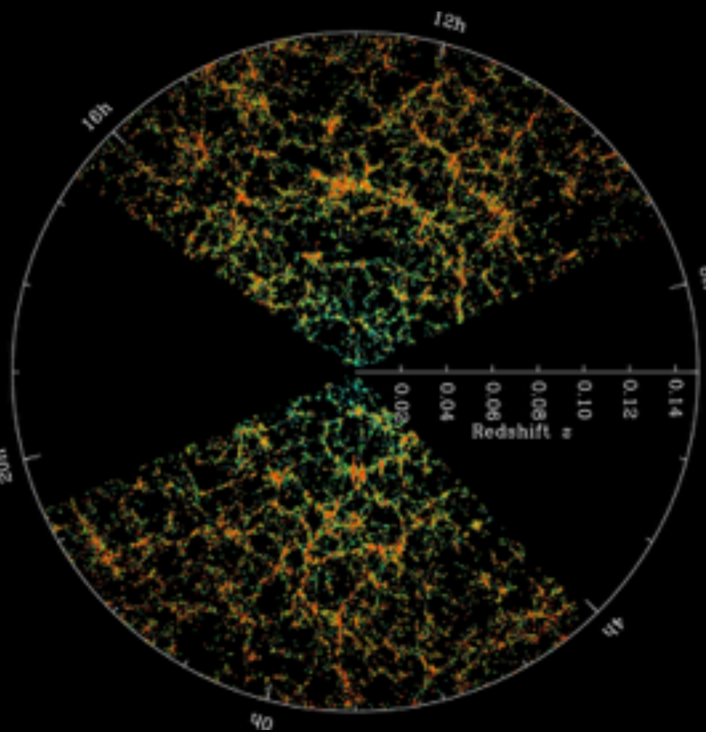
Nobel prize for

George Smoot



PLANCK-2013

SDSS



The key equations

Einstein GR: $$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Geometry — Matter — Cosm const - could be matter or geometry

Relates curvature of spacetime to the matter distribution and its dynamics.

Require metric tensor $g_{\mu\nu}$ from which all curvatures derived indep of matter:

Invariant separation of two spacetime points (μ,ν=0,1,2,3): $$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$$

Einstein tensor $G_{\mu\nu}$ -- function of $g_{\mu\nu}$ and its derivatives.
Energy momentum tensor $T_{\mu\nu}$ -- function of matter fields present.
For most cosmological substances can use perfect fluid representation for which we write

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + p g_{\mu\nu}$$

U^μ : fluid four vel = (1,0,0,0) - because comoving in the cosmological rest frame.
(ρ,p) : energy density and pressure of fluid in its rest frame

$$T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$$

Reminder of curvatures

Christoffel symbols:

$$\Gamma^{\mu}_{\nu\sigma} = \frac{1}{2} g^{\mu\lambda} (g_{\sigma\lambda,\nu} + g_{\nu\lambda,\sigma} - g_{\sigma\nu,\lambda})$$

Riemann's curvature tensor:

$$R^{\mu}_{\nu\sigma\gamma} = \Gamma^{\mu}_{\nu\gamma,\sigma} - \Gamma^{\mu}_{\nu\sigma,\gamma} + \Gamma^{\mu}_{\alpha\sigma}\Gamma^{\alpha}_{\gamma\nu} - \Gamma^{\mu}_{\alpha\gamma}\Gamma^{\alpha}_{\sigma\nu}$$

Ricci tensor:

$$R_{\mu\nu} = R^{\sigma}_{\mu\nu\sigma}$$

Ricci scalar:

$$R = R^{\mu}_{\mu}$$

Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Not needed here

Cosmology - isotropic and homogeneous FRW metric

Copernican Principle: We are in no special place. Since universe appears isotropic around us, this implies the universe is isotropic about every point. Such a universe is also homogeneous.

Line element $$ds^2 = -dt^2 + a^2(t)dx^2$$

$$dx^2 = \frac{1}{1-kr^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

t -- proper time measured by comoving (i.e. const spatial coord) observer. a(t) -- scale factor: k- curvature of spatial sections: k=0 (flat universe), k=-1 (hyperbolic universe), k=+1 (spherical universe)

Aside for those familiar with this stuff -- not chosen a normalisation such that a_0=1. We are not free to do that and simultaneously choose |k|=1. Can do so in the k=0 flat case.

Intro Conformal time : τ(t) $\tau(t) \equiv \int^t \frac{dt'}{a(t')}$

Implies useful simplification : $ds^2 = a^2(\tau)(-d\tau^2 + dx^2)$

Hubble parameter :
(often called Hubble constant)

$$H(t) \equiv \frac{\dot{a}}{a}$$

Hubble parameter relates velocity of recession of distant galaxies from us to their separation from us

$$v = H(t)r$$

$$d = ax$$

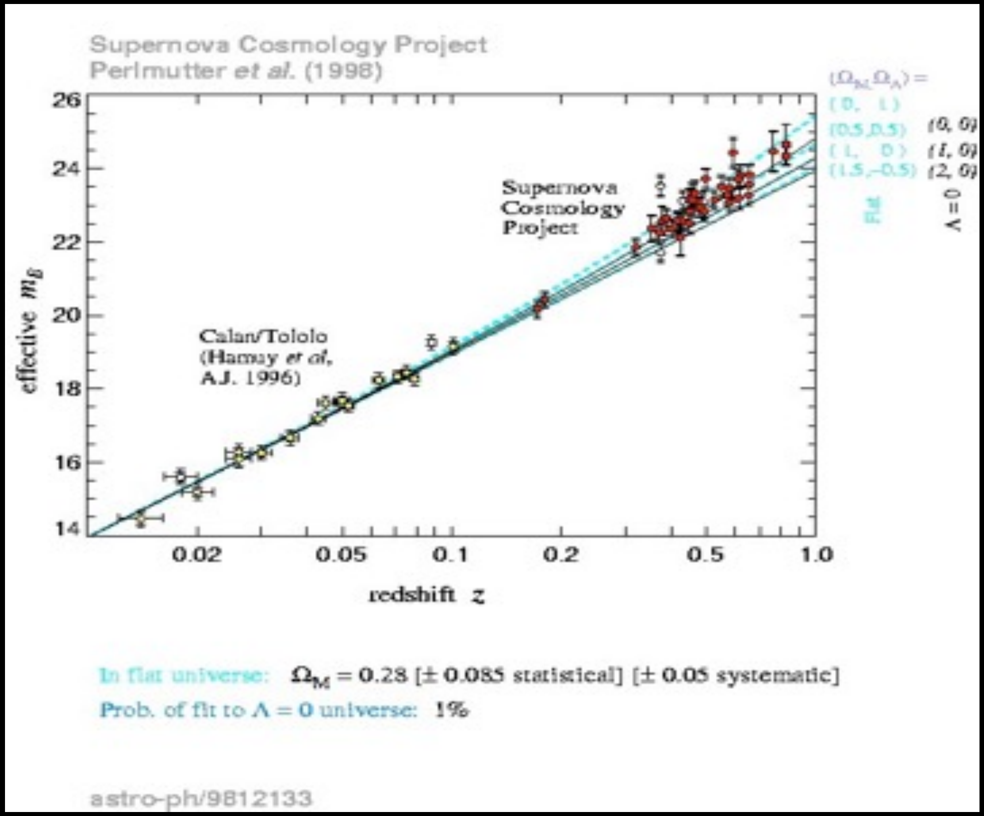
$$\dot{d} = \dot{a}x + a\dot{x}$$

$$\dot{d} = Hd + a\dot{x}$$

$$\dot{d} = v + a\dot{x}$$

Hubble flow

peculiar velocity



$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$ applied to cosmology

Friedmann:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter, $\rho(t)=\Sigma_i \rho_i$ -- sum of all matter contributions, rad, dust, scalar fields ...

Energy density $\rho(t)$: Pressure $p(t)$

Related through: $p = w\rho$

Eqn of state parameters: w=1/3 – Rad dom: w=0 – Mat dom: w=-1– Vac dom

Eqns (Λ=0):

Friedmann + Fluid energy conservation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\nabla^\mu T_{\mu\nu} = 0$$

Combine Friedmann and fluid equation to obtain Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G\left(\rho + 3p\right) --- Accn$$

$$If \;\; \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

Inflation condition -- more later

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} \quad ; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

$$\mathrm{RD} \quad : \quad w = \frac{1}{3} : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-4} \quad ; \; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{2}}$$

$$\mathrm{MD} \quad : \quad w = 0 : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3} \quad ; \; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

$$\mathrm{VD} \quad : \quad w = -1 : \rho(t) = \rho_0 \;\; ; \; a(t) \propto e^{Ht}$$

A neat equation

$$\rho_c(t) \equiv \frac{3H^2}{8\pi G} \quad ; \quad \Omega(t) \equiv \frac{\rho}{\rho_c}$$

$$\Omega > 1 \leftrightarrow k = +1$$

$$\Omega = 1 \leftrightarrow k = 0$$

$$\Omega < 1 \leftrightarrow k = -1$$

Friedmann eqn

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

Ω_m - baryons, dark matter, neutrinos, electrons, radiation ...

Ω_Λ - dark energy ; Ω_k - spatial curvature

$$\rho_c(t_0) \equiv 1.88h^2 * 10^{-29}\ \mathrm{g\,cm}^{-3}$$

Critical density

Bounds on H(z) -- Komatsu et al 2010 - (WMAP7+BAO+SN)

$$\mathbf{H^2(z) = H_0^2\left(\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_{de}\exp\left(3\int_0^z \frac{1+w(z')}{1+z'}dz'\right)\right)}$$

(Expansion rate) -- H_0=70.4 ± 1.3 km/s/Mpc

(radiation) -- $\Omega_r = (8.5 \pm 0.3) \times 10^{-5}$

(baryons) -- $\Omega_b = 0.0456 \pm 0.0016$

(dark matter) -- $\Omega_c = 0.227 \pm 0.014$

(curvature) -- $\Omega_k < 0.08$ (95%CL)

(dark energy) -- $\Omega_{de} = 0.728 \pm 0.015$ -- Implying univ accelerating today

(de eqn of state) -- $1+w = 0.001 \pm 0.057$ -- looks like a cosm const.

If allow variation of form : $w(z) = w_0 + w' z/(1+z)$ then
$w_0 = -0.93 \pm 0.12$ and $w' = -0.38 \pm 0.65$ (68% CL)

Important because distance measurements often rely on assumptions made about the background cosmology.

Planck constraints 2014 - not very different !

BASE ΛCDM MODEL

Parameter	TT	TT,TE,EE
$\Omega_b h^2$	0.02222±0.00023	0.02224±0.00015
$\Omega_c h^2$	0.1199±0.0022	0.1199±0.0014
$100\theta_*$	1.04086±0.00048	1.04073±0.00032
τ	0.078±0.019	0.079±0.017
n_s	0.9652±0.0062	0.9639±0.0047
H_0	67.3±1.0	67.6 ± 0.6 (+BAO)
Ω_m	0.316±0.014	0.316±0.009
σ_8	0.830±0.015	0.831±0.013
z_{re}	9.9±1.9	10.7±1.7

…but beware there are still low level systematics in the polarization spectra

preliminary

Planck consortium 2014 - preliminary

How old are we?

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3}G\rho - \frac{k}{a^2}$$

$$where\ \rho = \rho_m + \rho_r + \rho_\Lambda$$

$$t = \int \frac{da}{\dot{a}} = \int \frac{da}{aH}$$

$$t_0 = H_0^{-1}\int_0^1 \frac{x\,dx}{\left[\Omega_{m0}x + \Omega_{r0} + \Omega_{\Lambda 0}x^4 + (1-\Omega_0)x^2\right]^{1/2}}$$

$$where\ \Omega_0 = \Omega_{m0} + \Omega_{r0} + \Omega_{\Lambda 0}$$

$$Today: H_0^{-1} = 9.8\times 10^9\ h^{-1}\ years; h = 0.7$$

H_0^{-1} – –Hubble time

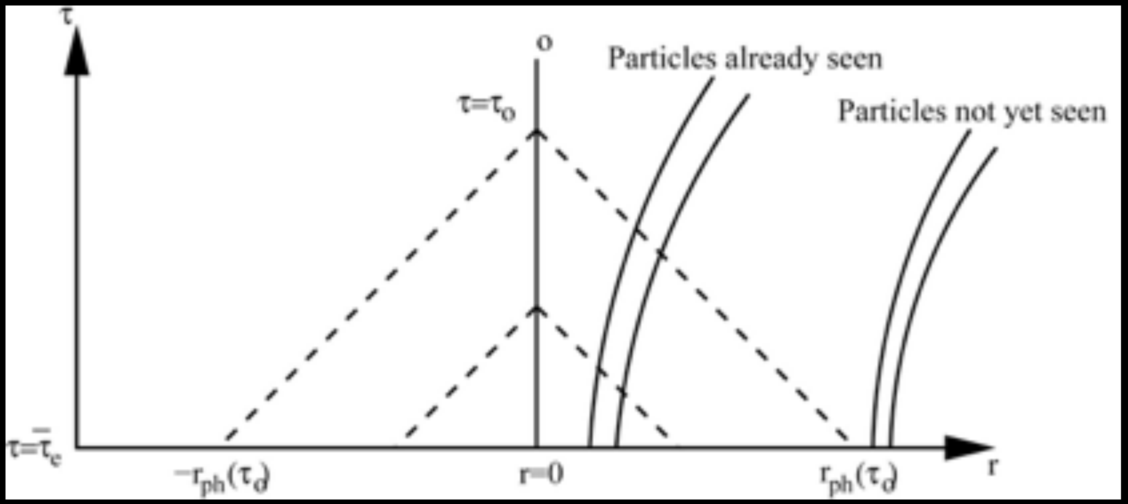
Useful estimate for age of universe

Ω_{m0}	Ω_{r0}	$\Omega_{\Lambda 0}$	t_0
1	0	0	9.4 Gyr
0.3	10^{-5}	0.7	13.4 Gyr
Open			
0.2	10^{-5}	0.2	12.4 Gyr
0.2	10^{-5}	0.6	13.96 Gyr
Closed			
0.3	10^{-5}	0.8	13.96 Gyr
0.4	10^{-5}	0.9	13.6 Gyr

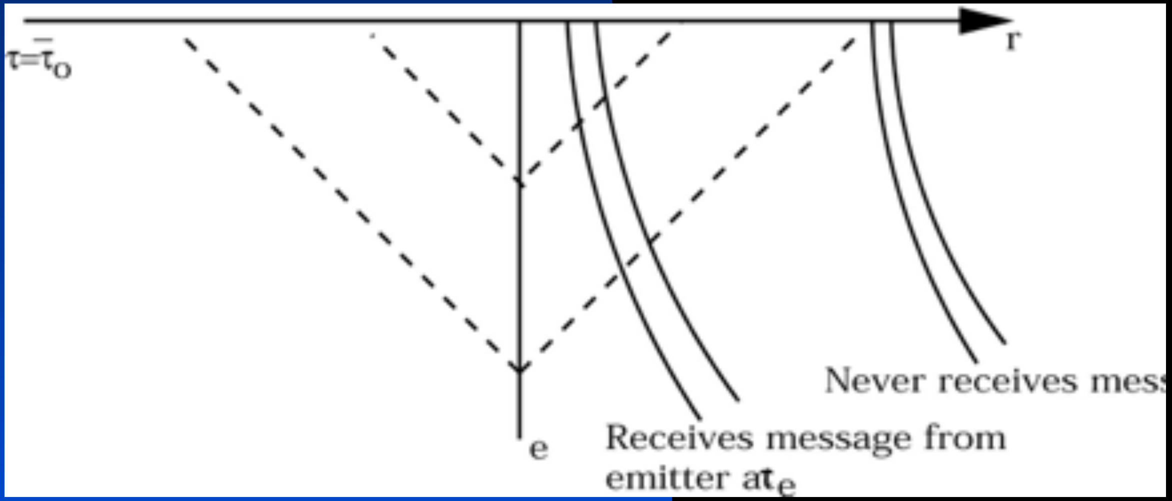
Horizons -- crucial concept in cosmology

a) *Particle horizon:* is the proper distance at time t that light could have travelled since the big bang (i.e. at which *a=0*). It is given by

$$d_p(t) = a(t) \int_0^t \frac{dt'}{a(t')}$$



b) *Event horizon: is* the proper distance at time t that light will be able to travel in the future:



$$d_{\mathrm{EH}}(t) = a(t) \int_t^\infty \frac{dt'}{a(t')}$$

Trodden and Carroll 03

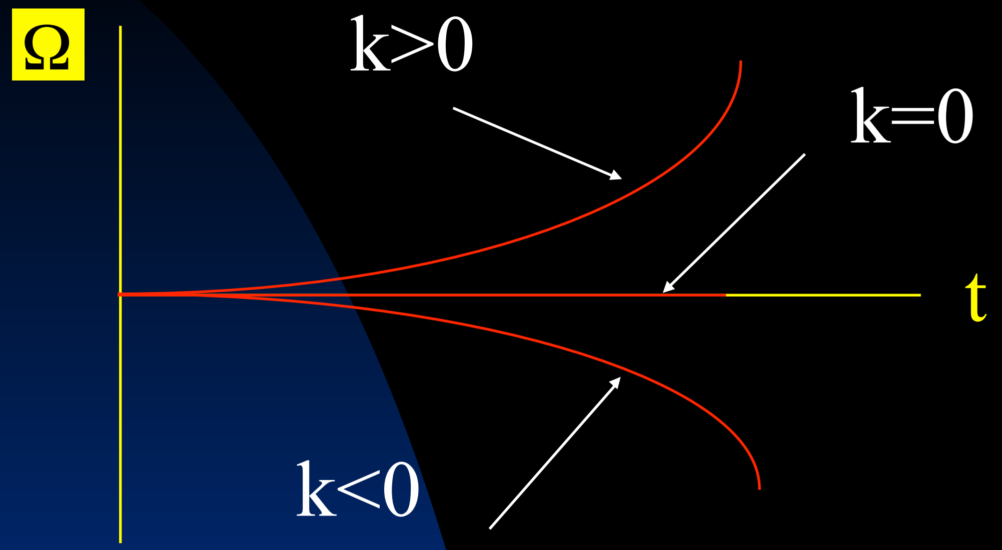
History of the Universe

10	10	10	QG/String epoch (?)
			Inflation begins (?)
10	10	10	Electroweak tran
1 GeV	10	10	Quark-Hadron tran
1 MeV	1 sec	10	Nucleosynthesis
1 eV	10	10	Matter-rad equality
	10	3.10	Decoupling → microwave bgd.
10	10	3K	Present epoch

The Big Bang – issues.

- Flatness problem – observed almost spatially flat cosmology requires fine tuning of initial conditions.
- Horizon problem -- isotropic distribution of CMB over whole sky appears to involve regions that were not in causal contact when CMB produced. How come it is so smooth?
- Monopole problem - where are all the massive defects which should be produced during GUT scale phase transitions.
- Relative abundance of matter – does not predict ratio baryons: radiation: dark matter.
- Origin of the Universe – simply assumes expanding initial conditions.
- Origin of structure in the Universe from initial conditions homogeneous and isotropic.
- The cosmological constant problem.

Flatness problem



Today: $\Omega < 1.1$

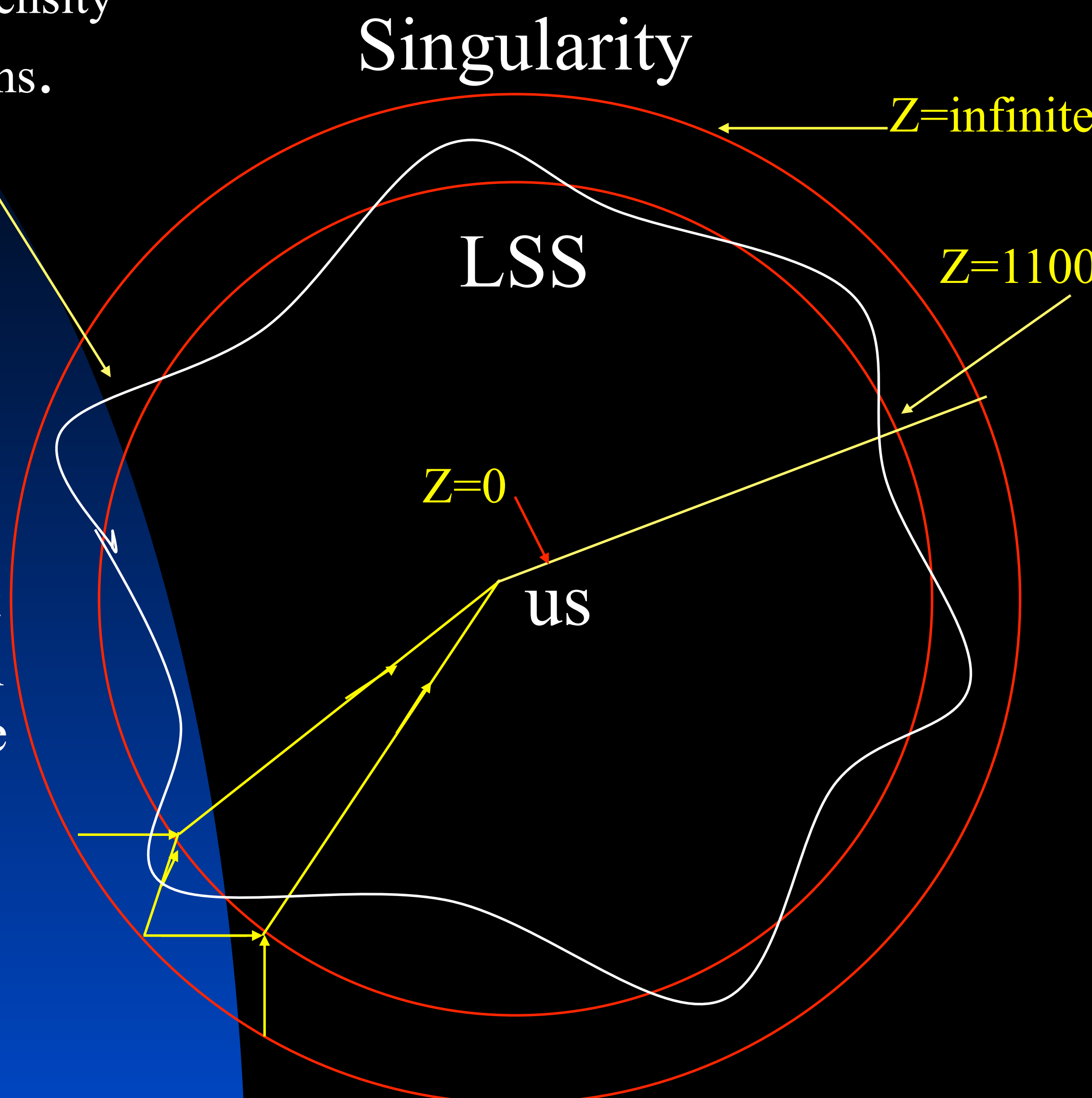
Why?

$$\Omega^{-1} = 1 - \frac{3k}{\kappa^2 \rho a^2} \propto \begin{cases} a & \text{mat} \\ a^2 & \text{rad} \\ a^{-2} & \Lambda \end{cases}$$

$$|\Omega(1\mathrm{s}) - 1| = O(10^{-16})$$

Horizon problem

Primordial density fluctuations.



CMB photons emitted from opp sides of sky are in thermal equilibrium at same temp – but no time for them to interact before photons were emitted because of finite horizon size.

CMBR last interacted at 1+Z = 1100

300,000 yrs after big bang

Hubble radius was 2 degrees, 200 Mpc

LSS thickness – 15Mpc

Any region separated by > 2 deg – causally separated at decoupling.

Monopole problem

Monopoles are generic prediction of GUT type models.

They are massive stable objects, like domain walls and cosmic strings and many moduli fields.

They scale like cold dark matter, so in the early universe would rapidly come to dominate the energy density.

Must find a mechanism to dilute them or avoid forming them.

Some of the big questions in cosmology today

a) What is dark matter? -- 25% of the energy density

b) What is dark energy? -- 70% of the energy density. Does dark energy interact with other stuff in the universe?

c) Is dark energy really a new energy form or does the accelerating universe signal a modification of our theory of gravity?

d) What is the origin of the density perturbations, giving rise to structures?

e) Where is the cosmological gravitational wave background?

f) Are the fluctuations described by Gaussian statistics? If there are deviations from Gaussianity, where do they come from?

g) How many dimensions are there? Why do we observe only three spatial dimensions?

h) Was there really a big bang (i.e. a spacetime singularity)? If not, what was there before?

A bit of thermodynamics - remember your stat mech

Gas -weakly interacting in kinetic eqm. Distribution function for particle species x, physical momentum p

$$f_x(p) = \frac{1}{e^{\frac{E_x - \mu_x}{T}} \pm 1}$$

- sign bosons, + sign fermions, μ chemical pot, T-temp: $E_x^2 = p^2 + m_x^2$

Include internal dof: i.e. spin by g_x (photons have g=2, neutrinos g=1)

number density: $$n_x = \frac{g_x}{(2\pi)^3} \int f_x(p) d^3p$$

energy density: $$\rho_x = \frac{g_x}{(2\pi)^3} \int E_x(p) f_x(p) d^3p$$

pressure: $$p_x = \frac{g_x}{(2\pi)^3} \int \frac{|p|^2}{3E_x(p)} f_x(p) d^3p$$

Non-Rel limit : m>>T

$$n_x \simeq g_x \left(\frac{m_x T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_x}{T}}$$

$$\rho_x \simeq m_x n_x \qquad p_x \simeq T n_x$$

Rel limit : m<<T -- BE and FD

$$n_x^{BE} = \frac{\zeta(3)}{\pi^2} g_x T^3 \qquad n_x^{FD} \simeq \frac{3}{4} n_x^{BE}$$

$$\rho_x^{BE} \simeq \frac{\pi^2}{30} g_x T^4 \qquad \rho_x^{FD} \simeq \frac{7}{8} \rho_x^{BE}$$

$$\zeta(3) = 1.202...$$

Friedmann eqn in early universe during rad dom: $\rho_{\rm rad} = \rho_{BE} + \rho_{FD} = \frac{\pi^2}{30} g_{\rm eff}(T) T^4$

Temp high so all particle species in therm eqm: for std model particles T>1TeV. Total num of dof for fermions (90), gauge and Higgs (28) so:

$$g_{\rm eff}(T = 1TeV) = 106.75$$

If the interaction rate between particles becomes smaller than the expansion rate, then those particles have a smaller temp than the photons (temp T) but might be relativistic. So, intro specific temp for each relativistic species.

$$g_{\rm eff}(T) = \sum_{i=\rm bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{j=\rm fermions} g_j \left(\frac{T_j}{T}\right)^4$$

Hence: $H = 0.33 \sqrt{g_{\rm eff}} \frac{T^2}{m_{\rm Pl}}$ and $t = 1.52 \frac{m_{\rm Pl}}{\sqrt{g_{\rm eff}} T^2}$

Kinetic Equilibrium - characterised by T - particles exchange energy, energy density constant:

$$X_1 + X_2 \leftrightarrow X_1 + X_2$$

Chemical Equilibrium - characterised by μ - species can change number, number density constant:

$$X_1 + X_2 \leftrightarrow X_3 + X_4 \quad \text{with} \quad \mu_1 + \mu_2 = \mu_3 + \mu_4$$

Equilibrium condition: interaction rate happens faster than the expansion rate of the universe. $\Gamma > H$

Now: $\Gamma = n < \sigma v >$ ← Thermal Ave

Number density

Cross section

Ave vel

Ex: Neutrino decoupling: $\gamma\gamma \leftrightarrow e^+e^- \quad \nu e \leftrightarrow \nu e \quad \nu\bar{\nu} \leftrightarrow \nu\bar{\nu}$

Cross section: $\sigma \simeq G_F^2 T^2 \rightarrow \Gamma \simeq G_F^2 T^5$

Hence: $$\frac{\Gamma}{H} = \left(\frac{T}{1\mathrm{MeV}}\right)^3$$

So for T>1 MeV, neutrinos in thermal eqm with photons, but below 1MeV, interaction rate too low to maintain eqm with photon plasma.

Decoupling: - departure from Kinetic Equilibrium

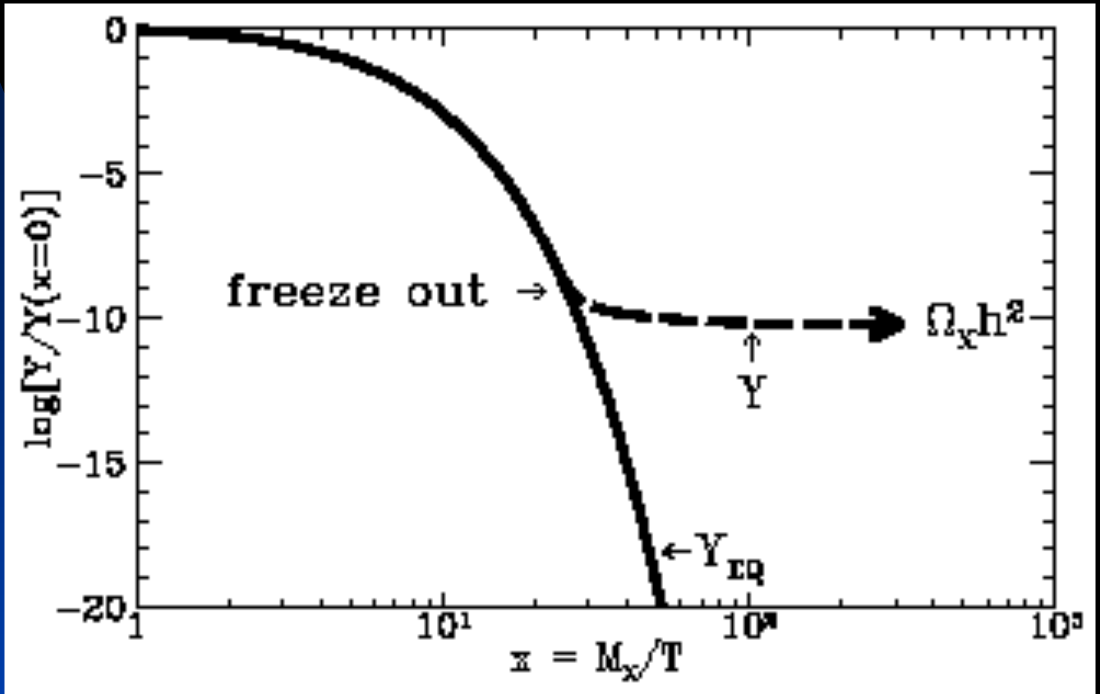
Freeze out: - departure from Chemical Equilibrium

Estimate decoupling or freeze out temp by Γ=H:

$$n < \sigma v > \simeq \sqrt{g_{\mathrm{eff}}}\frac{T^2}{m_{\mathrm{Pl}}}$$

Note that for neutrinos with m<1 MeV, we have m<T hence relativistic. Such particles which are relativistic at freeze-out are hot-dark-matter candidates.

Weakly interacting particles tend to have m/T ~ 20, so non-relativistic particles and cold dark matter candidates.



Taken from http://nedwww.ipac.caltech.edu/level5/Kolb/Kolb5_1.html

Y - ratio of number density to entropy density

Turns out cold dark matter needed for structure formation. Doesn't match observations if it is hot.

Dark matter candidates: $\Omega_c h^2 = 0.1198 \pm 0.0026 \ (68\% \ \mathrm{CL})$

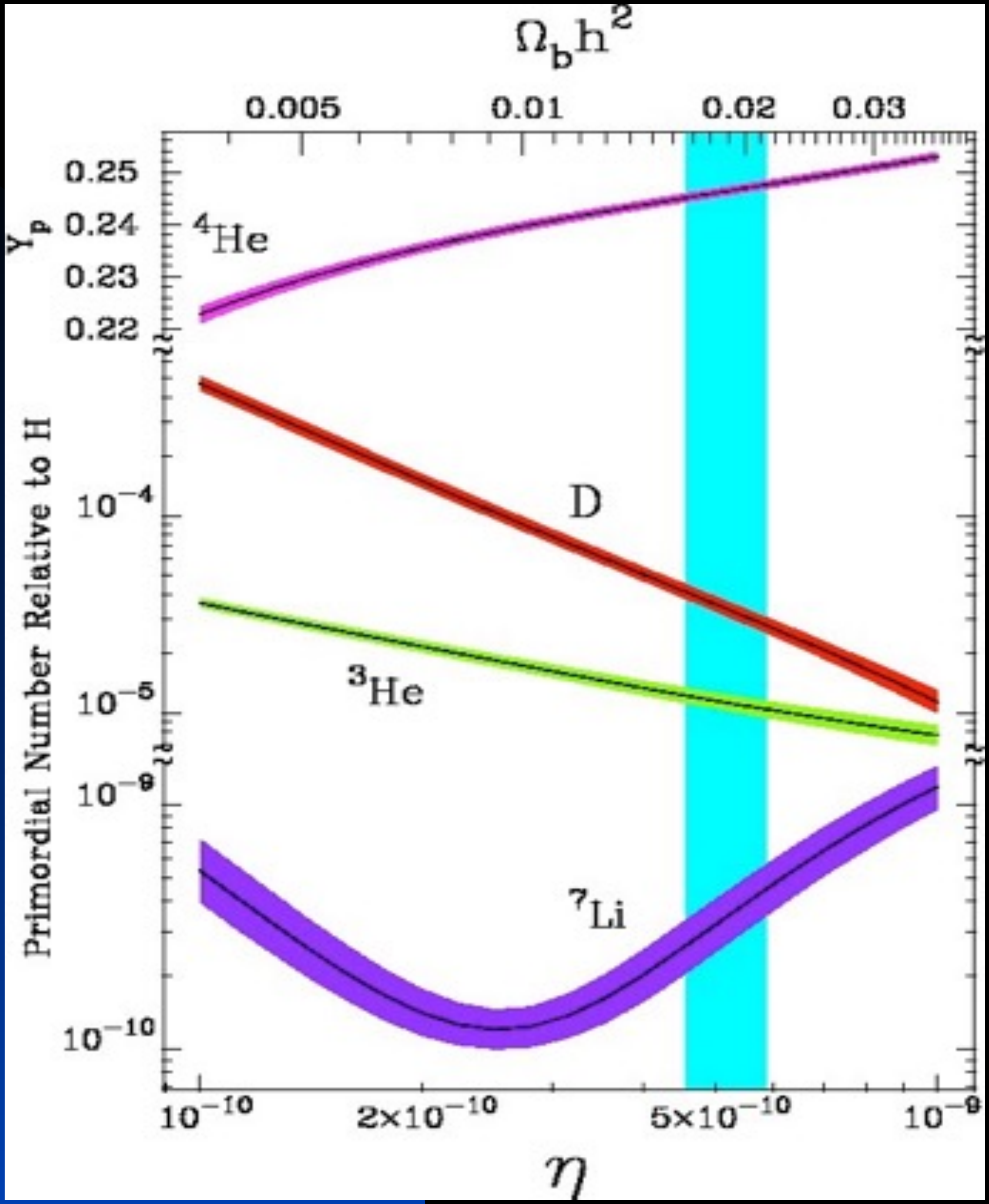
(Planck 2013)

* Axion (solves CP problem of QCD), Axinos
* Neutrino – known to have mass, cannot be dominant dark matter.
* Neutralino – lightest supersymmetric particle.
* Gravitinos, Q-balls, WIMP-zillas…
* Kaluza-Klein dark matter
* Black holes
* …

Big Bang Nucleosynthesis -- formation of the lightest nuclei

If the temperature is low enough, protons and neutrons can bind together to produce elements such as ^{4}He, D, ^{7}Li. For this to happen, the temperature must drop below about 1 MeV.

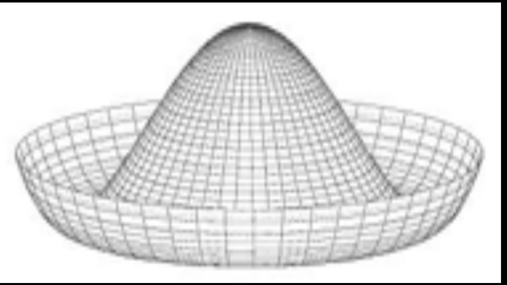
- Binding starts at T below the binding energy of the nuclei.
- During BBN the light elements are produced (in particular 3He, 4He, D, ^{7}Li). Heavier elements are created in stars at a much later time.
- Can predict the abundances as a function of the energy density in baryons-- a great success of the Hot Big Bang



Regarded as great success of HBB but actually questions over the predictions and how they match observations, especially ^{7}Li, which appears to be larger than predicted.

$$\Omega_b h^2 = 0.02207 \pm 0.00033 \ (68\% \ \mathrm{CL})$$ (Planck 2013)

Phase Transitions in the Early Universe -- could be vital!
Spontaneous symmetry breaking : Higgs, topological defects,
Finite temp effective potential:



$$V_T(\phi) = \left(-\frac{1}{2}m^2 + \frac{\lambda}{8}T^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4 + K$$

$$T > \frac{2m}{\sqrt{\lambda}} \quad \text{then} \quad m_{\text{eff}} > 0 \quad \text{and} \quad <\phi> = 0$$ symmetry restored

$$T < \frac{2m}{\sqrt{\lambda}} \quad \text{then} \quad m_{\text{eff}} < 0 \quad \text{and} \quad <\phi> \neq 0$$ symmetry broken

Example: GUT phase transition, Electroweak PT, QCD PT
Formation of topological defects such as cosmic strings, domain walls, monopoles, textures ...

I owe a great deal to cosmic strings -- they are neat and through cosmic superstrings could provide the first observational evidence for string theory.

Unfortunately they are very very shy !

Weighing the Universe

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

$1. \Omega_m$

a. Cluster baryon abundance using X-ray measurements of intracluster gas, or SZ measurements.

b. Weak grav lensing and large scale peculiar velocities.

c. Large scale structure distribution.

d. Numerical simulations of cluster formation.

e. Cosmic Microwave Background Anisotropies

$$\Omega_m = 0.314 \pm 0.020 \ (68\% \ \mathrm{CL})$$

(Planck 2013)

$\Omega_m << 1$

$H_0 = 67.4 \pm 1.4$ km s^{-1} Mpc^{-1}

2. Ω_b

BBN → $\Omega_b h^2 = 0.02207 \pm 0.00033 \ (68\% \ \mathrm{CL})$

Majority of baryonic matter dark.

$\boxed{\Omega_b << \Omega_m}$ → **Require Dark matte !!**

Candidates: WIMPS (Neutralinos, Kaluza Klein Particles, Universal Extra Dimensions...)

Axinos, Axions, Axion-like light bosons, Sterile neutrinos, Q-balls, WIMPzillas, Elementary Black Holes...

Search for them is on:

1. Direct detection -- 20 expts worldwide

2. Indirect detection -- i.e. Bullet Cluster !

3. LHC -- i.e. missing momentum and energy

WIMP Direct Detection 2013

Lines

Blue: LUX

Red: XENON100

Light blue: SIMPLE

Brown: Xenon10

Dark Yellow: Edelweiss

Green: CDMS

Dark green: CDMSlite

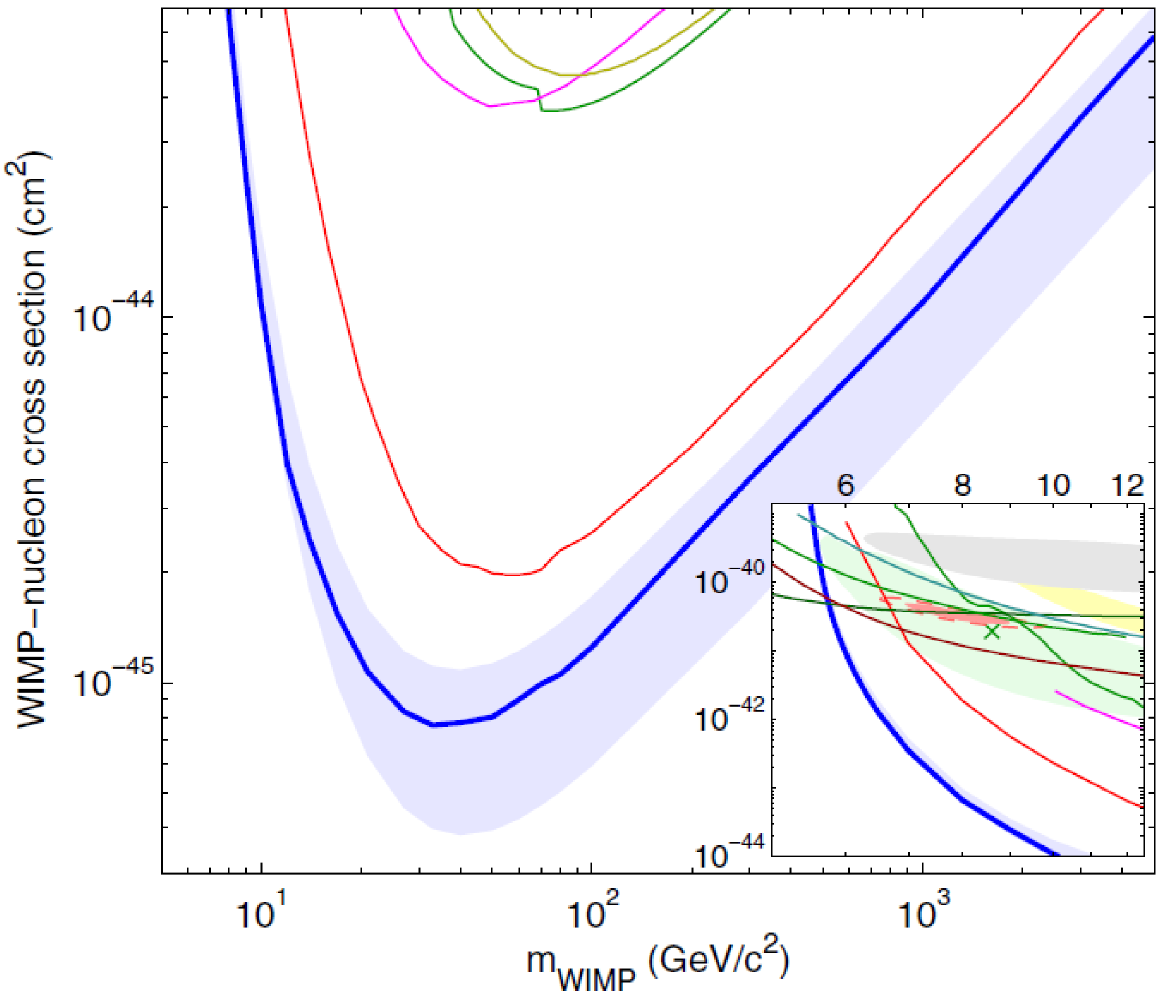
Regions

Red: CoGeNT

Green: CDMS II Si

Yellow: CRESST II

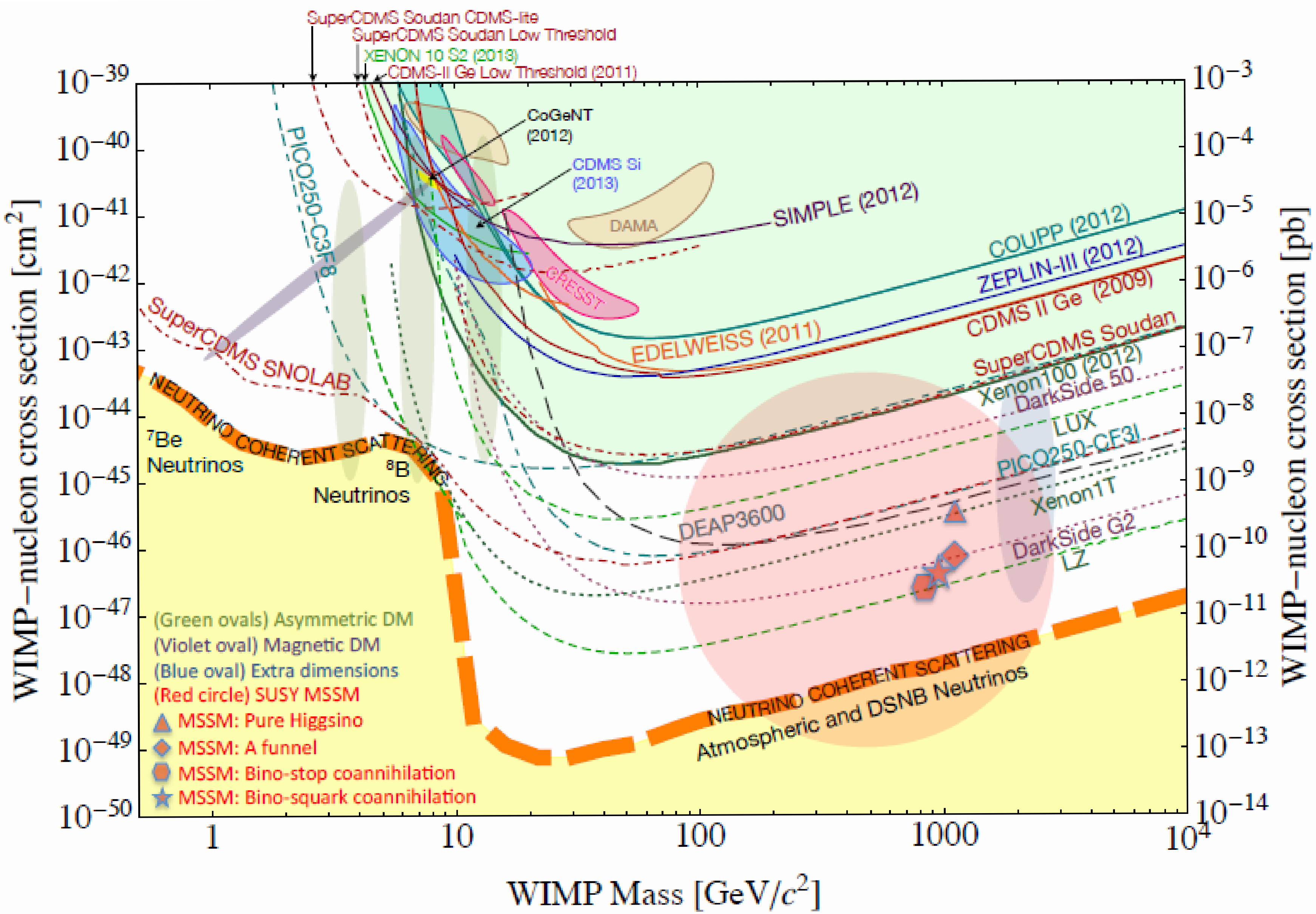
Grey: DAMA/LIBRA



Lux collaboration 2014

c/o: Clare Burrage

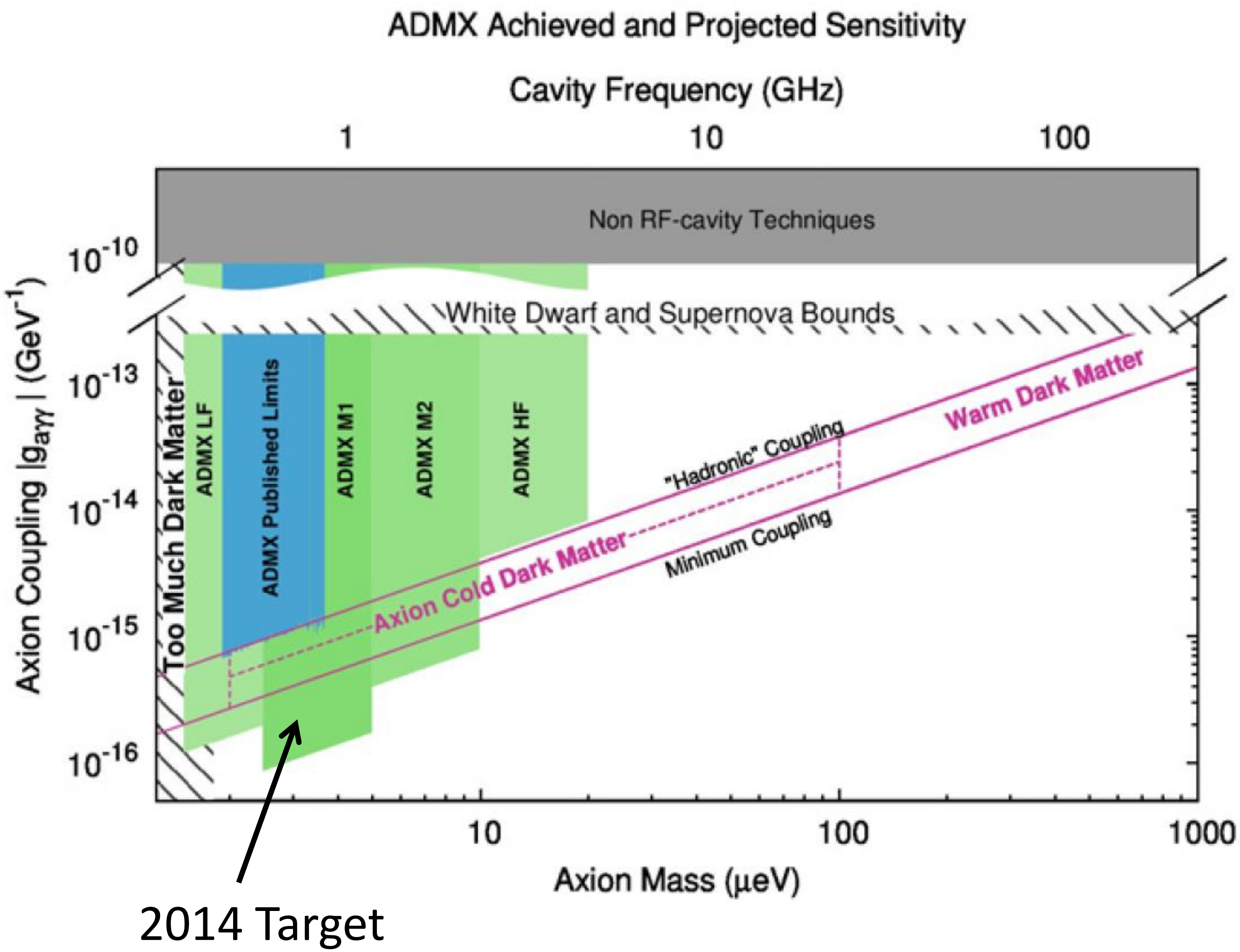
The future of WIMP direct detection



Bauer et al. 2013.

c/o: Clare Burrage

Axion Direct Detection

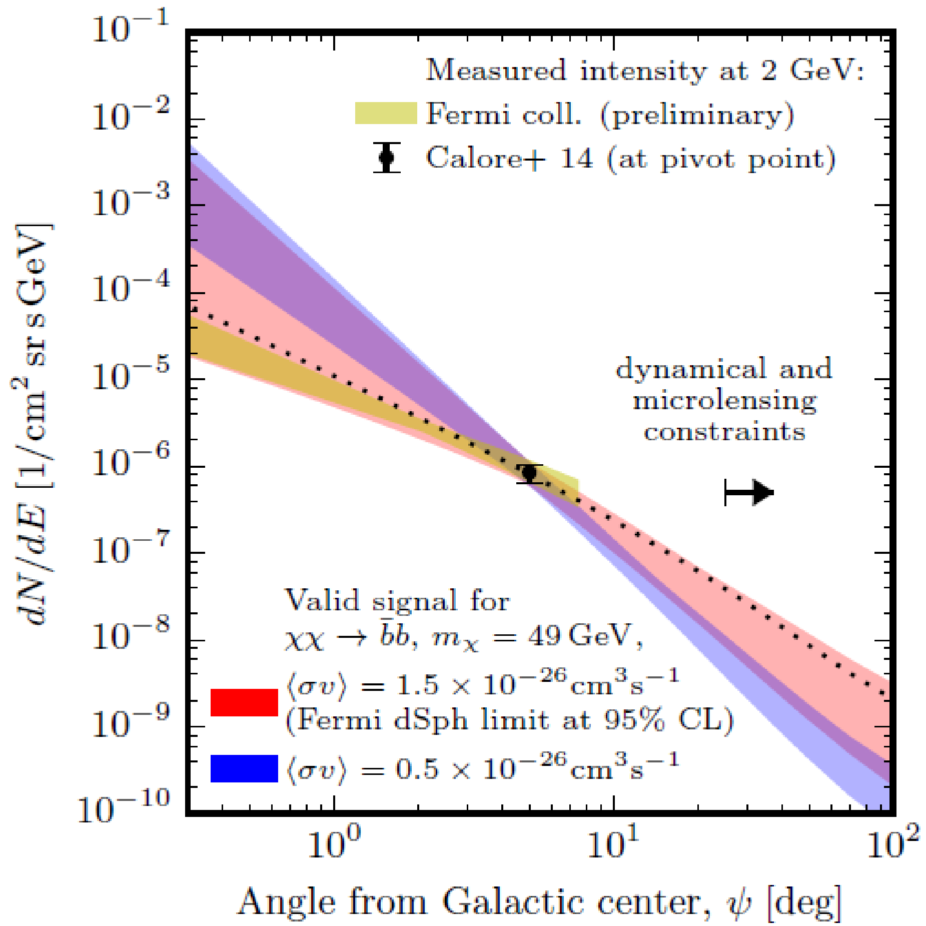


Rybka. 2014.

c/o: Clare Burrage

Emission from Dwarf Spheriodal Galaxies

Uncertainties on amount of DM in galactic centre and in dwarfs (DM dominated)



Calore et al 2014

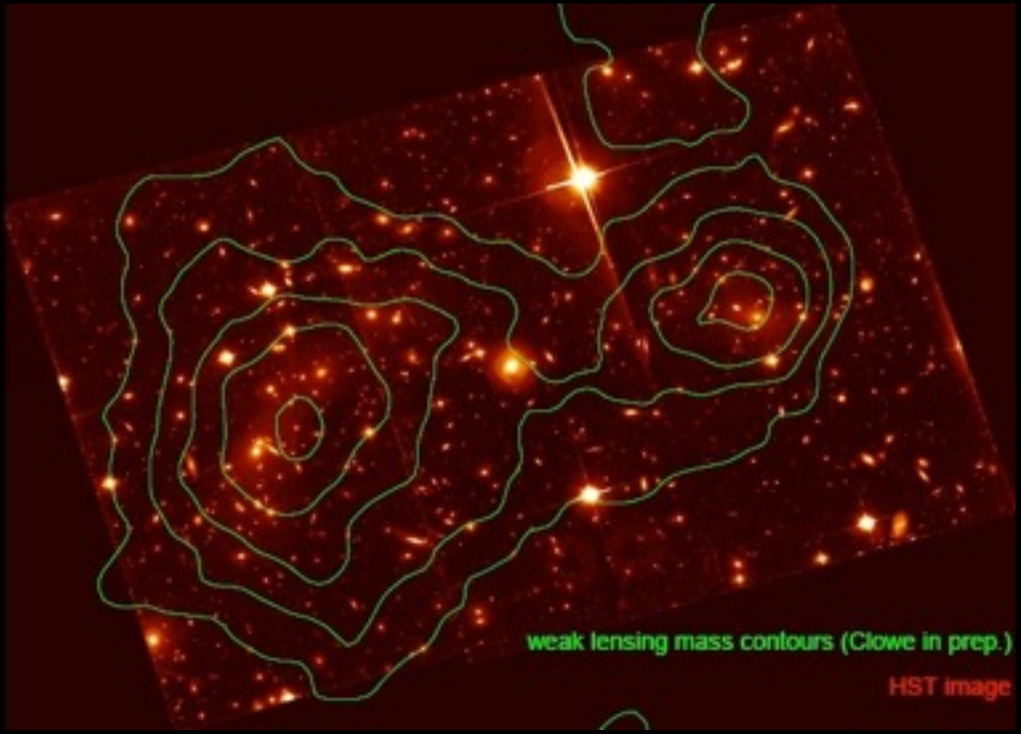
c/o: Clare Burrage

Indirect evidence for Dark Matter -- Bullet Cluster

Two clusters of galaxies colliding.

Dark matter in each passes straight through and doesn't interact -- seen through weak lensing in right image.

Ordinary matter in each interacts in collision and heats up -- seen through infra red image on left.



Clowe et al 2006

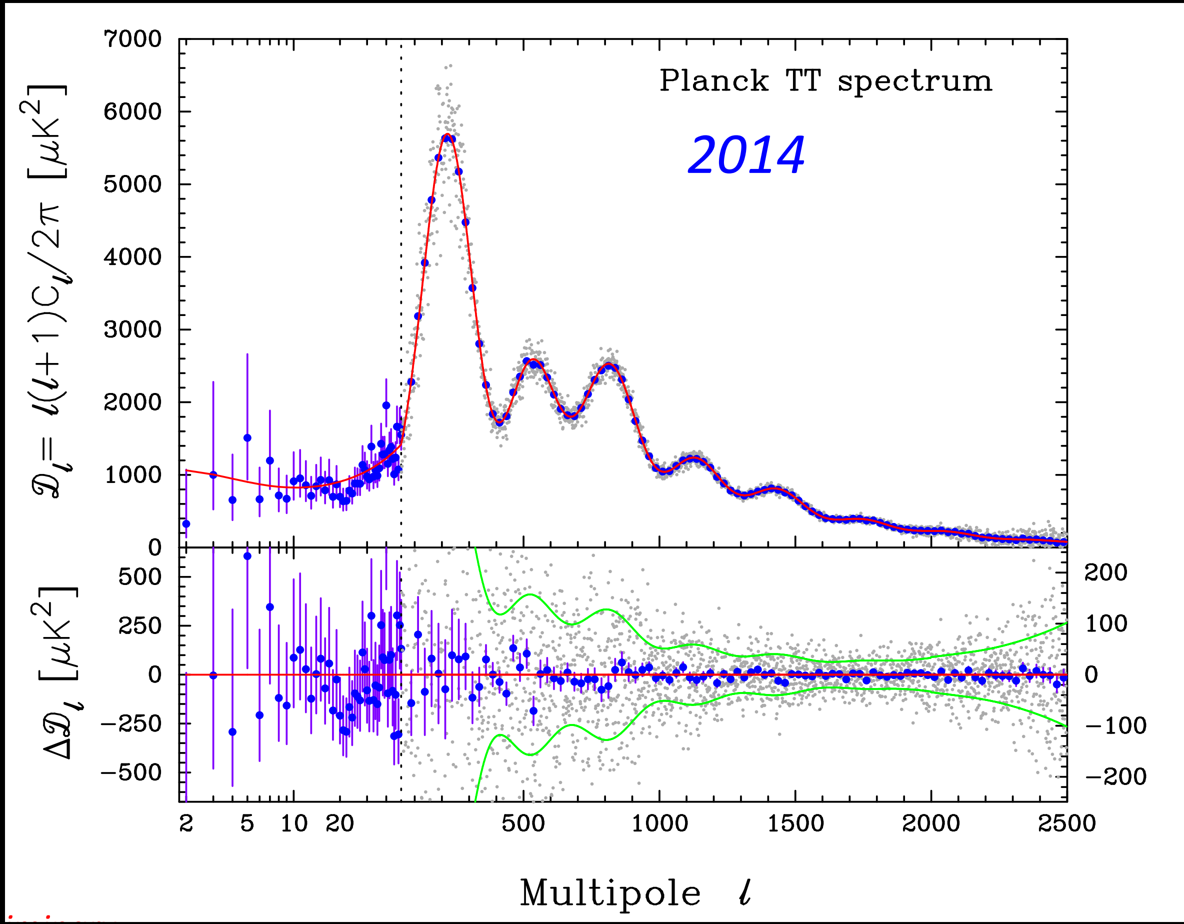
Evidence for Dark Energy?

Enter CMBR:

$$3.\ \Omega_0 = \Omega_m + \Omega_\Lambda$$

Provides clue. 1st angular peak in power spectrum.

$$l_{\text{peak}} \approx \frac{220}{\sqrt{\Omega_0}}$$



Planck consortium 2014 - preliminary

$$\Omega_k = 0.000 \pm 0.005\ (95\%\ \text{CL})$$

Planck + Lensing+ BAO consortium 2014 - preliminary

Dark Energy

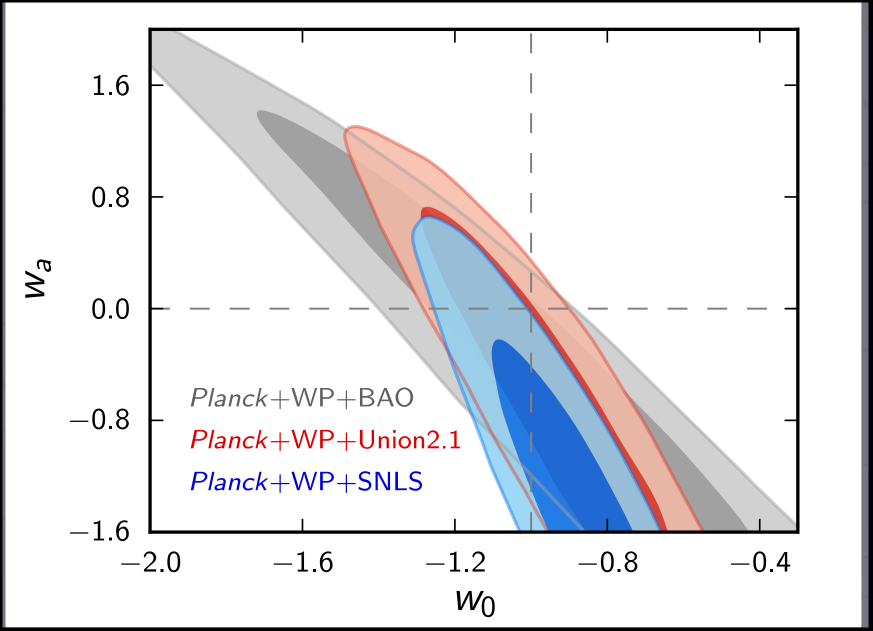
Parameterise eos: $$w(a) \equiv \frac{p}{\rho} = w_0 + (1-a)w_a$$

Planck alone weak constraints on DE because of degeneracy of w with H_0:

Break with other probes including lensing, SN, BAO ...

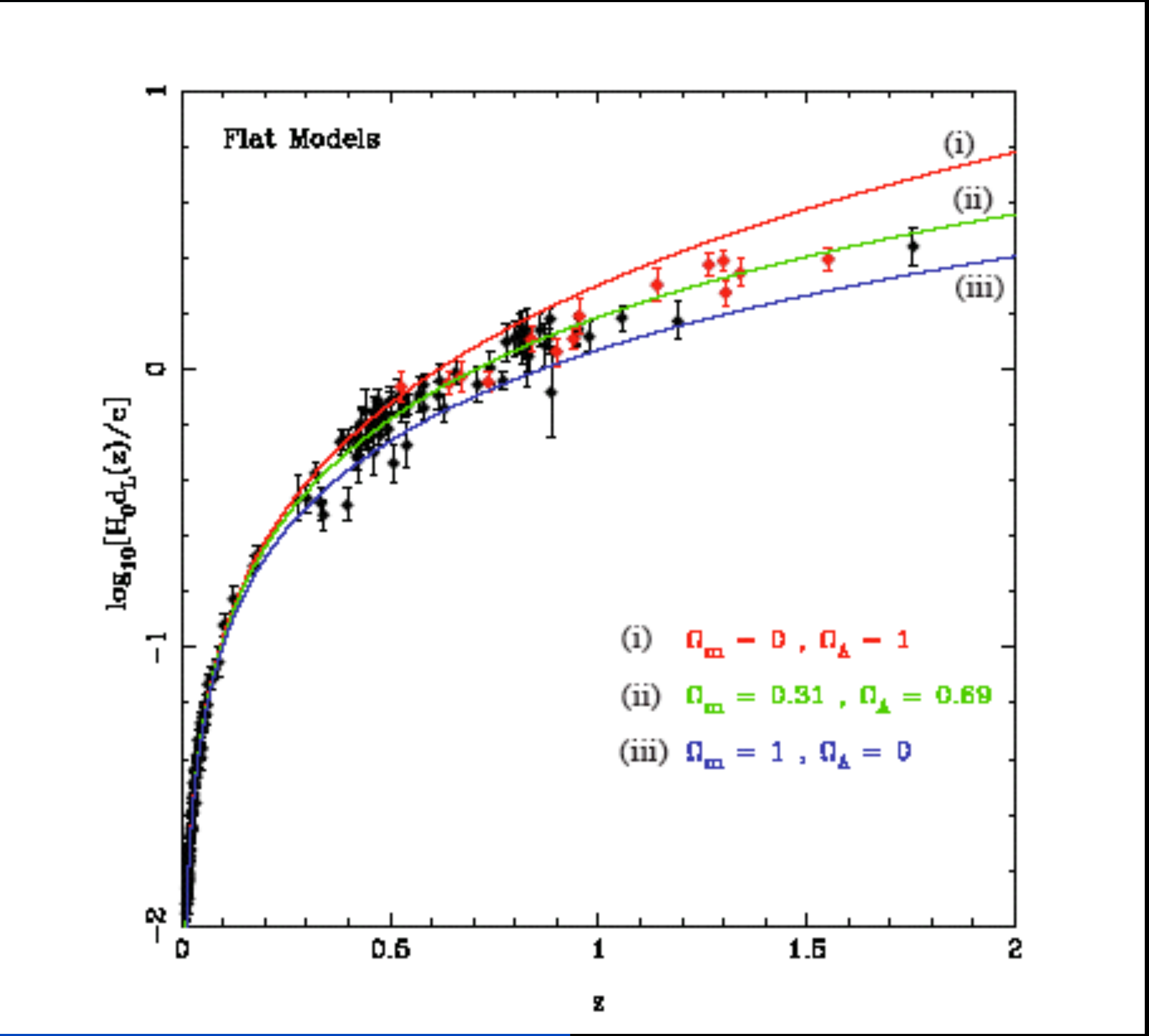
Example - if assume $w_a = 0$

$$w = -1.13 \pm 0.24 \quad (95\%, \textit{Planck} + \mathrm{WP} + \mathrm{BAO})$$
$$w = -1.09 \pm 0.17 \quad (95\%, \textit{Planck} + \mathrm{WP} + \mathrm{Union2.1})$$
$$w = -1.13^{+0.13}_{-0.14} \quad (95\%, \textit{Planck} + \mathrm{WP} + \mathrm{SNLS}),$$
$$w = -1.24^{+0.18}_{-0.19} \quad (95\%, \textit{Planck} + \mathrm{WP} + \mathrm{HST})$$



How should we parametrise w(a) ?

Type Ia Luminosity distance v z [Reiss et al 2004]



Flat model
Black dots -- Gold data set
Red dots -- HST

$$(i)\Omega_m = 0,\ \Omega_\Lambda = 1 \quad (ii)\Omega_m = 0.31,\ \Omega_\Lambda = 0.69 \quad (iii)\Omega_m = 1,\ \Omega_\Lambda = 0$$

Coincidence problem – why now?

Recall: $\frac{\ddot{a}}{a} \geq 0 <-> = (\rho + 3p) \leq 0$

If: $\rho_x = \rho_x^0 \, a^{-3(1+w_x)}$

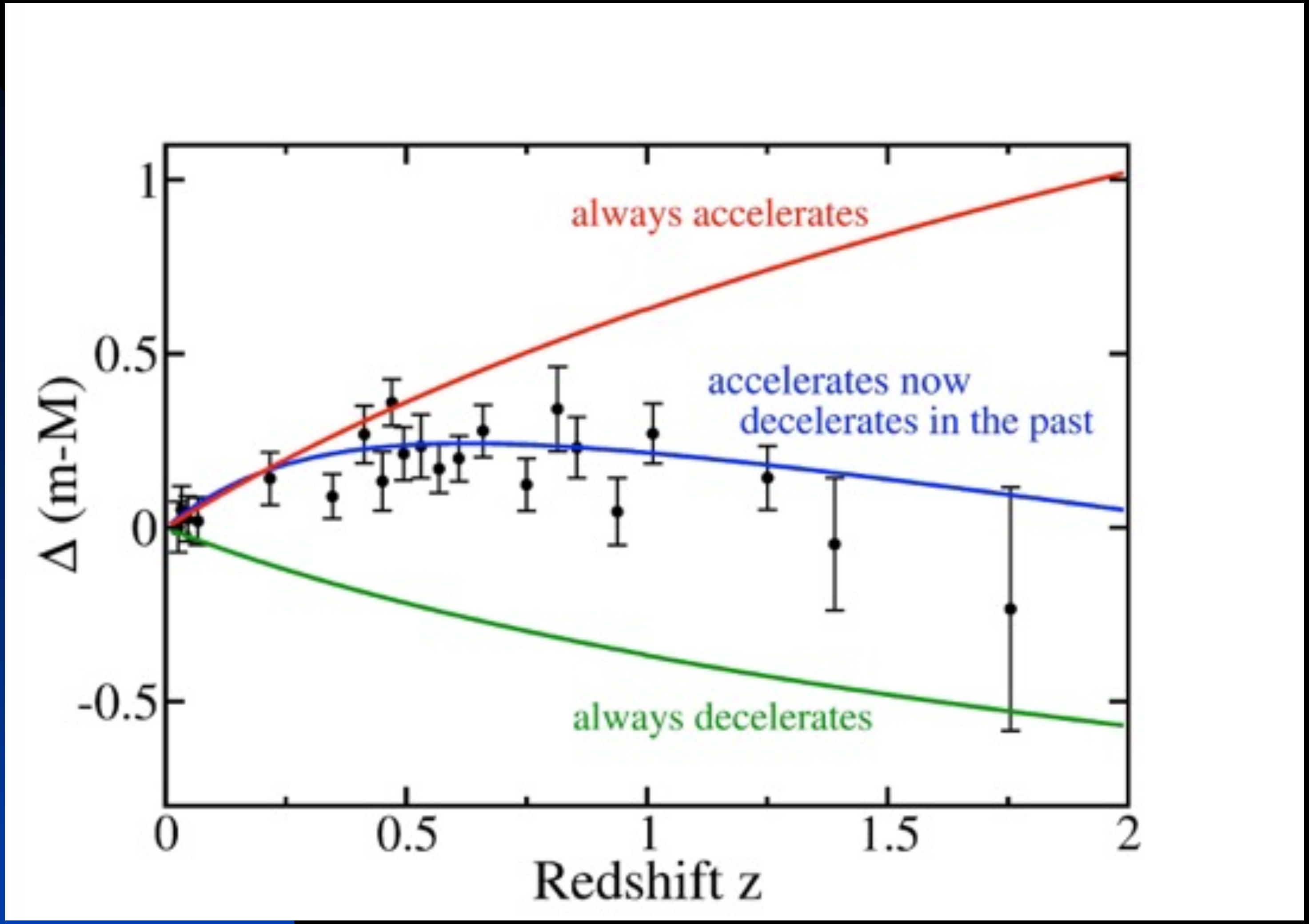
Universe dom by dark energy at: $z_x = \left(\frac{\Omega_x}{\Omega_m}\right)^{\frac{1}{3w_x}} - 1$

$$\left(\frac{\Omega_x}{\Omega_m}\right) = \frac{7}{3} \rightarrow z_x = 0.5,\ 0.3 \text{ for } w_x = -\frac{2}{3},\ -1$$

Univ accelerates at: $z_a = \left(-(1+3w_x)\frac{\Omega_x}{\Omega_m}\right)^{\frac{-1}{3w_x}} - 1$

$$z_a = 0.7,\ 0.5 \text{ for } w_x = -\frac{2}{3},\ -1$$

The acceleration has not been forever -- pinning down the turnover will provide a very useful piece of information.



What is making the Universe accelerate?

Dark energy -- a weird form of energy that exists in empty space and pervades the universe -- also known as vacuum energy or cosmological constant.

Smoothly distributed, doesn't cluster.

Constant density or very slowly varying

Doesn't interact with ordinary matter -- only with gravity

Big problem though. When you estimate how much you expect there to be, from the Quantum world, the observed amount is far less than expected.

Theoretical prediction = 10^{120} times observation

The problem with the cosmological constant

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Einstein (1917) -- static universe with dust

Not easy to get rid of it, once universe found to be expanding.

Anything that contributes to energy density of vacuum acts like a cosmological constant

$$< T_{\mu\nu} > = < \rho > g_{\mu\nu}$$

Lorentz inv

$$\lambda_{eff} = \lambda + 8\pi G < \rho >$$

or

$$\rho_V = \lambda_{eff}/8\pi G$$

Effective cosm const

Effective vac energy

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - < \rho > | \leq \frac{3H_0^2}{8\pi G}$$

Age

Flat

Non-vac matter

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - <\rho>| \leq \frac{3H_0^2}{8\pi G}$$

Hence: $$\lambda_{eff} \leq H_0^2 \; or |\rho_V| \leq 10^{-29} gcm^{-3} \simeq 10^{-47} GeV^4$$

Problem: expect $<\rho>$ of empty space to be much larger. Consider summing zero-point energies ($\hbar\omega/2$) of all normal modes of some field of mass m up to wave number cut off $\Lambda >> m$:

$$<\rho> = \int_0^{\Lambda} \frac{4\pi k^2 dk}{2(2\pi)^3} \sqrt{k^2 + m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

For many fields (i.e. leptons, quarks, gauge fields etc...):

$$<\rho> = \frac{1}{2} \sum_{\text{fields}} g_i \int_0^{\Lambda_i} \sqrt{k^2 + m^2} \frac{d^3k}{(2\pi)^3} \simeq \sum_{\text{fields}} \frac{g_i \Lambda_i^4}{16\pi^2}$$

where g_i are the dof of the field (+ for bosons, - for fermions).

Imagine just one field contributed an energy density $\rho_{cr} \sim (10^{-3}\ \text{eV})^4$. Implies the cut-off scale $\Lambda < 0.01$ eV -- well below scales we understand the physics of.

Planck scale: $\Lambda \simeq (8\pi G)^{-1/2} \rightarrow <\rho> \simeq 2 \times 10^{71} GeV^4$

But: $|\rho_V| = |<\rho> + \lambda/8\pi G| \leq 2 \times 10^{-47} GeV^4$

Must cancel to better than 118 decimal places.

Even at QCD scale require 41 decimal places!

Very unlikely a classical contribution to the vacuum energy density will cancel this quantum contribution to such high precision

Not all is lost -- what if there is a symmetry present to reduce it? Supersymmetry does that. Every boson has an equal mass SUSY fermion partner and vice-versa, so their contributions to $<\rho>$ cancel.

However, SUSY seems broken today - no SUSY partners have been observed, so they must be much heavier than their standard model partners. If SUSY broken at scale M, expect $<\rho> \sim M^4$ because of breakdown of cancellations. Current bounds suggest M~1TeV which leads to a discrepancy of 60 orders of magnitude as opposed to 118 !

Still a problem of course -- is there some unknown mechanism perhaps from quantum gravity that will make the vacuum energy vanish ?

Different approaches to Dark Energy include amongst many:

A true cosmological constant -- but why this value?

Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.

Modifications of Einstein gravity leading to acceleration today.

Anthropic arguments.

Perhaps GR but Universe is inhomogeneous.

Hiding the cosmological constant -- its there all the time but just doesn't gravitate

Yet to be proposed ...

String - theory -- where are the realistic models?

`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

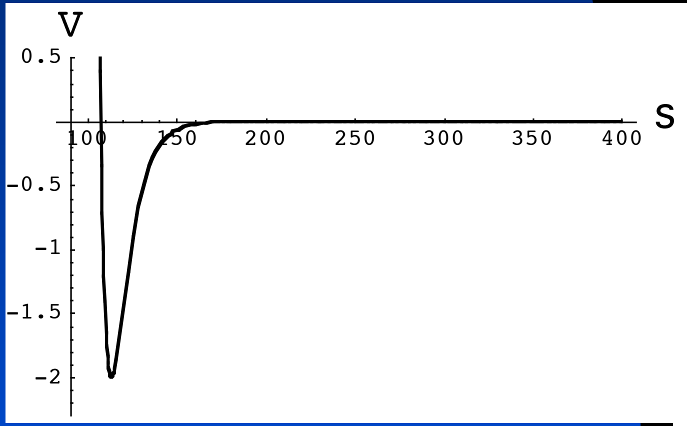
Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent scalar fields (radion)

2. Brane world set up require uplifting terms to achieve de Sitter vacua hence accn

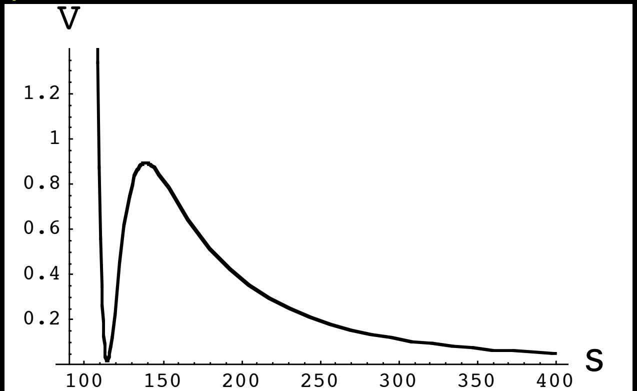
Example of stabilised scenario: Metastable de Sitter string vacua in TypeIIB string theory, based on stable highly warped IIB compactifications with NS and RR three-form fluxes. [Kachru, Kallosh, Linde and Trivedi 2003]

Metastable minima arises from adding positive energy of anti-D3 brane in warped Calabi-Yau space.



AdS minimum

$$V_{\rm KKLT} = V_{\rm AdS} + \frac{D}{\sigma^2}$$



Metastable dS minimum

The String Landscape approach

Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes. Assumes natural AdS vacuum uplifted to de Sitter vacuum through additional fluxes !

Many many vacua ~ 10^{500} !

Typical separation ~ $10^{-500}\ \Lambda_{pl}$

Assume randomly distributed, tunnelling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}\ \Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

Landscape gives a realisation of the multiverse picture.

There isn't one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

This sounds like bad news, we will rely on anthropic arguments to explain it through introducing the correct measures and establishing peaks in probability distributions.

Or perhaps, it isn't a cosmological constant, but a new field such as Quintessence which will eventually drive us to a unique vacuum with zero vacuum energy -- that too has problems, such as fifth force constraints, as we will see.

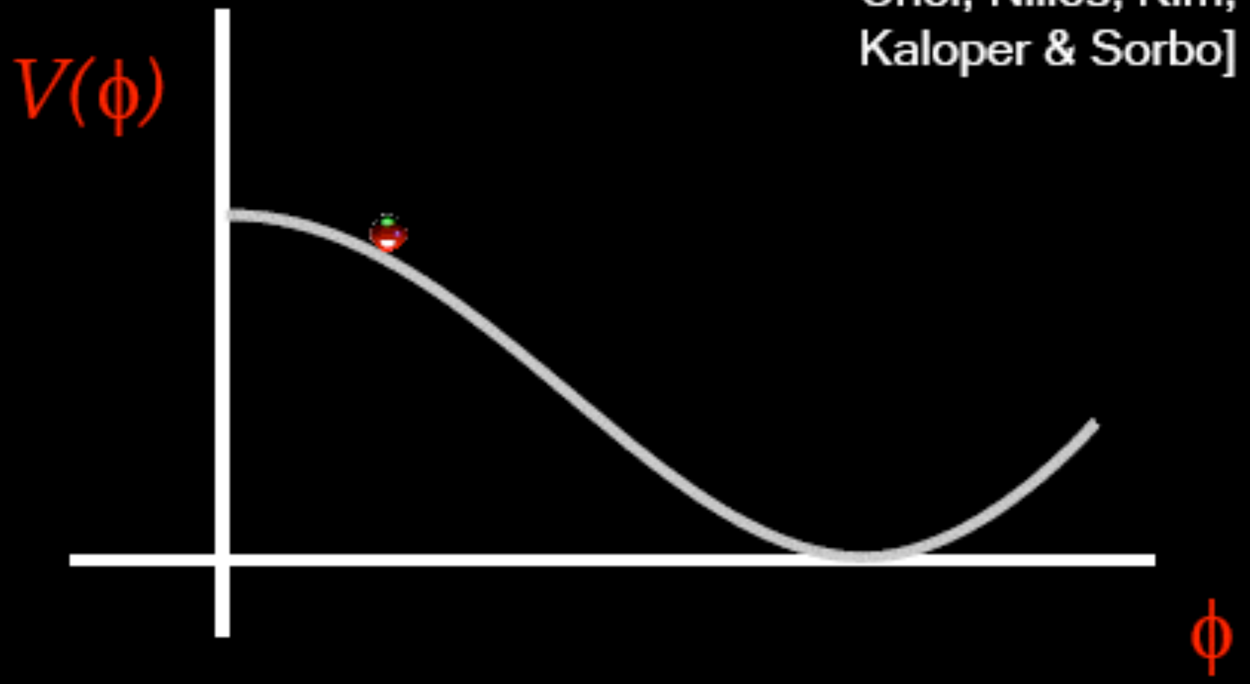
Particle physics inspired models?

Pseudo-Goldstone Bosons -- approx sym $\phi \rightarrow \phi$ + const.

Leads to naturally small masses, naturally small couplings

[Hill, Freiman, et al; Choi; Nilles; Kim; Kaloper & Sorbo]

Barbieri et al



$$V(\phi) = \lambda^4 (1 + \cos(\phi / F_a))$$

Axions could be useful for strong CP problem, dark matter and dark energy.

Axions could be useful for strong CP problem, dark matter and dark energy.

Strong CP problem intro axion : $m_a = \frac{\Lambda^2_{\rm QCD}}{F_a}; \; F_a \; - \;$ decay constant

PQ axion ruled out but invisible axion still allowed:

$$10^9 \text{ GeV} \leq F_a \leq 10^{12} \text{ GeV}$$

Sun stability CDM constraint

String theory has lots of antisymmetric tensor fields in 10d, hence many light axion candidates.

Can have $F_a \sim 10^{17}$-10^{18} GeV

Quintessential axion -- dark energy candidate [Kim & Nilles].

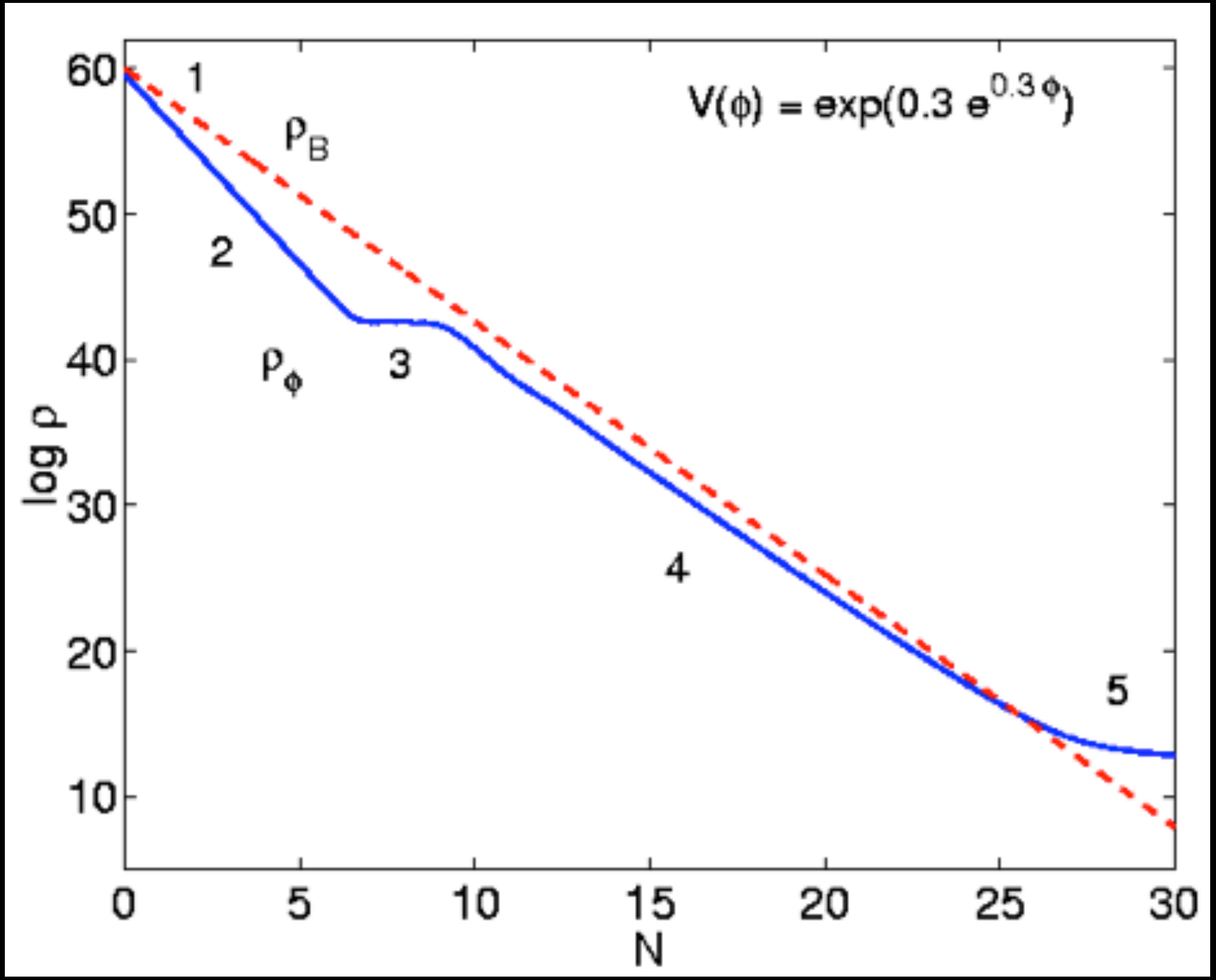
Requires $F_a \sim 10^{18}$ GeV which can give:

$$E_{\rm vac} = (10^{-3} \text{ eV})^4 \rightarrow m_{\rm axion} \sim 10^{-33} \text{ eV}$$

Because axion is pseudoscalar -- mass is protected, hence avoids fifth force constraints

Slowly rolling scalar fields
Quintessence - Generic behaviour

1. **PE → KE**
2. **KE dom scalar field energy den.**
3. **Const field.**
4. **Attractor solution: almost const ratio KE/PE.**
5. **PE dom.**



Nunes

Attractors make initial conditions less important

1. Chameleon fields [Khoury and Weltman (2003) …]

Non-minimal coupling of scalar to matter in order to avoid fifth force type constraints on Quintessence models: the effective mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

2. K-essence [Armendariz-Picon et al …]

Scalar fields with non-canonical kinetic terms. Includes models with derivative self-couplings which become important in vicinity of massive sources. The strong coupling boosts the kinetic terms so after canonical normalisation the coupling of fluctuations to matter is weakened -- screening via Vainshtein mechanism

Similar fine tuning to Quintessence -- vital in brane-world modifications of gravity, massive gravity, degravitation models, DBI model, Gallileons, ...

3. Symmetron fields [Hinterbichler and Khoury 2010 ...]

vev of scalar field depends on local mass density: vev large in low density regions and small in high density regions. Also coupling of scalar to matter is prop to vev, so couples with grav strength in low density regions but decoupled and screened in high density regions.

4. Interacting Dark Energy [Kodama & Sasaki (1985), Wetterich (1995), Amendola (2000) + many others…]

Idea: why not directly couple dark energy and dark matter?

$$\begin{aligned} \text{Ein eqn} &: \quad G_{\mu\nu} = 8\pi G T_{\mu\nu} \\ \text{General covariance} &: \quad \nabla_\mu G^\mu_\nu = 0 \rightarrow \nabla_\mu T^\mu_\nu = 0 \end{aligned}$$

$$T_{\mu\nu} = \sum_i T^{(i)}_{\mu\nu} \rightarrow \nabla_\mu T^{\mu\,(i)}_\nu = -\nabla_\mu T^{\mu\,(j)}_\nu \ \text{ is ok}$$

Couple dark energy and dark matter fluid in form:

$$\begin{aligned} \nabla_\mu T^{\mu\,(\phi)}_\nu &= \sqrt{\frac{2}{3}}\kappa\beta(\phi)T^{\alpha\,(m)}_\alpha \nabla_\nu\phi \\ \nabla_\mu T^{\mu\,(m)}_\nu &= -\sqrt{\frac{2}{3}}\kappa\beta(\phi)T^{\alpha\,(m)}_\alpha \nabla_\nu\phi \end{aligned}$$

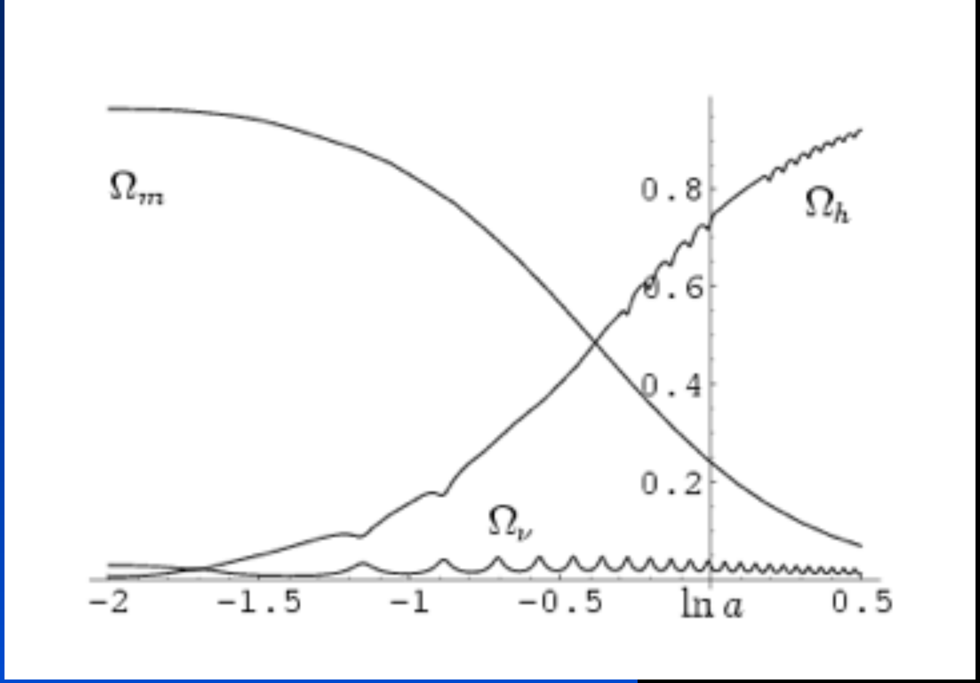
Including neutrinos -- 2 distinct DM families -- resolve coincidence problem [Amendola et al (2007)]

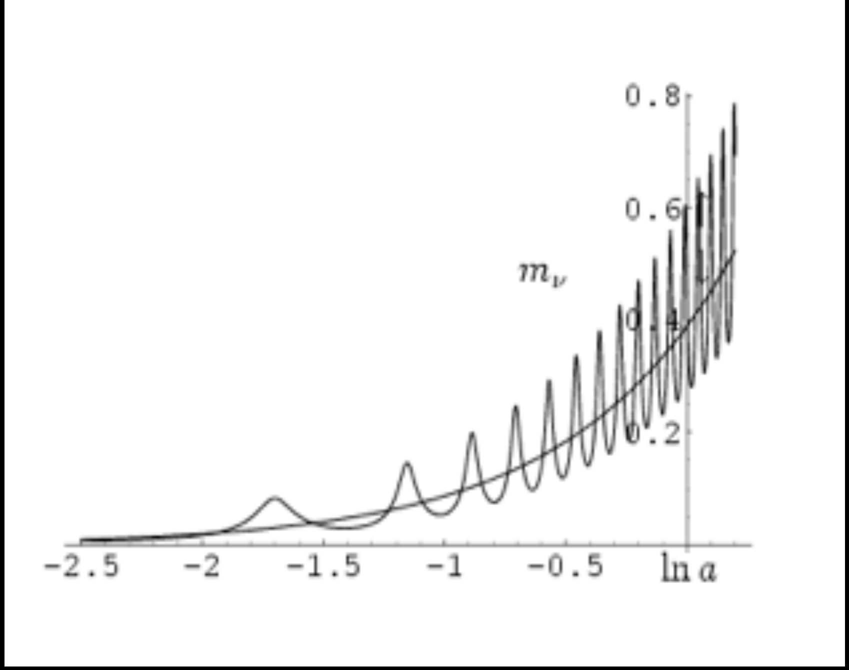
Depending on the coupling, find that the neutrino mass grows at late times and this triggers a transition to almost static dark energy.

Trigger scale set by when neutrinos become non-rel

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV$$

$$w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$





m_ν

Perturbations in Interacting Dark Energy Models [Baldi et al (2008), Tarrant et al (2010), Baugh et al (2010)]

Perturb everything linearly : Matter fluid example

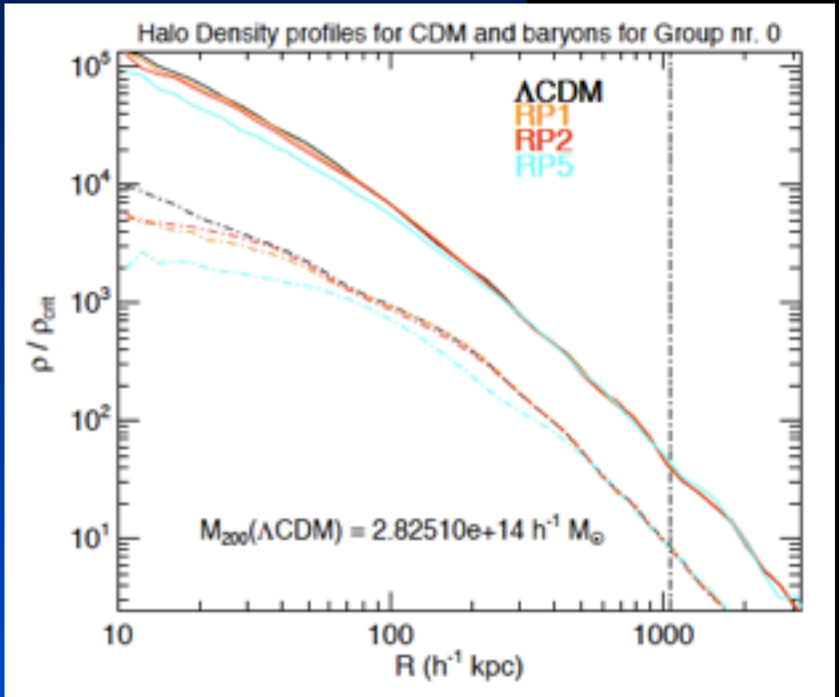
$$\ddot{\delta}_c + \left(2H - 2\beta\frac{\dot{\phi}}{M}\right)\dot{\delta}_c - \frac{3}{2}H^2[(1+2\beta^2)\Omega_c\delta_c + \Omega_b\delta_b] = 0$$

extra friction

modified grav interaction

vary DM particle mass

Include in simulations of structure formation : GADGET [Springel (2005)]



Density decreases as coupling β increases

Halo mass function modified.

Halos remain well fit by NFW profile.

Density decreases compared to ΛCDM as coupling β increases.

Scale dep bias develops from fifth force acting between CDM particles. enhanced as go from linear to smaller non-linear scales.

Still early days -- but this is where there should be a great deal of development.

Dark Energy Effects

Interactions with standard model particles inevitable even if indirect.

Light scalar fields that interact with std model fields mediate fifth forces

but we dont see any long range fifth forces on earth or in the solar system.

Screening !

Dark energy changes the way photons propagate through B fields. The polarised photon can fluctuate into a DE scalar particle leading to a modification of apparent polarisation and luminosity of the sources.

Two tests   Burrage, Davis Shaw, 2008,2009

Look for evidence of DE through changes in the scatter of luminosities of high energy sources.

Look for evidence of correlation between poln and freq of starlight .

Dark Energy Direct Detection Experiment [Burrage, EC, Hinds]

Atom Interferometry

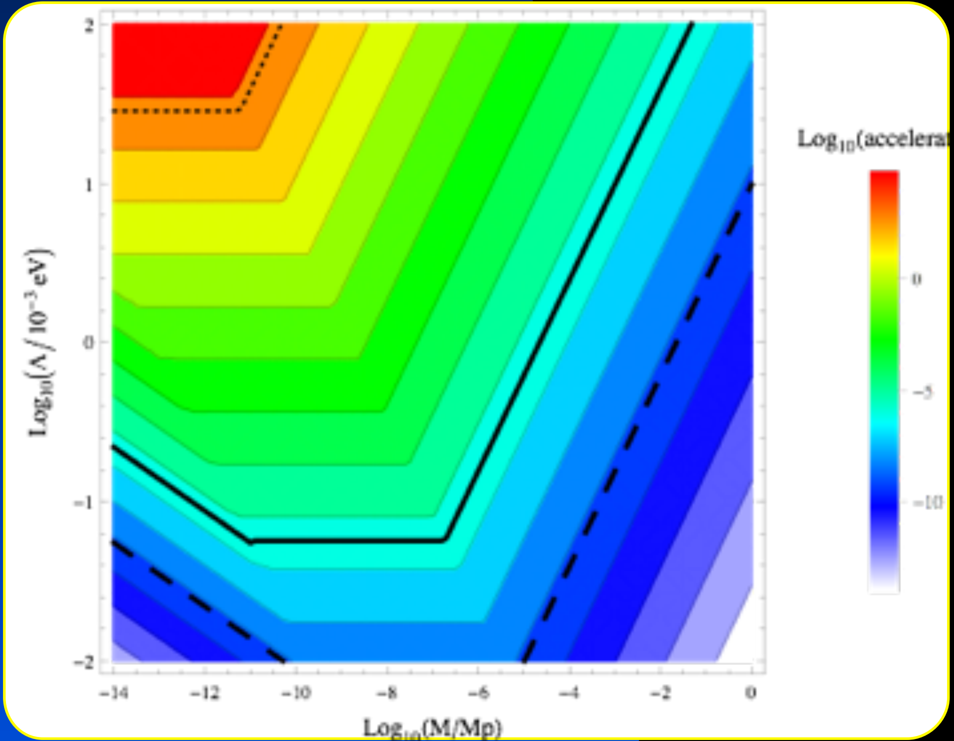
Idea: Individual atoms in a high vacuum chamber are too small to screen the chameleon field and so are very sensitive to it - can detect it with high sensitivity. Can use atom interferometry to measure the chameleon force - or more likely constrain the parameters !

$$\nabla^2\phi = -\frac{\Lambda^2}{\phi^2} + \frac{\rho}{M}$$

$$F_r = \frac{GM_AM_B}{r^2}\left[1 + 2\lambda_A\lambda_B\left(\frac{M_P}{M}\right)^2\right]$$

$$\lambda_i = 1 \text{ for } \rho_i R_i^2 < 3M\phi_{bg}$$

$$\lambda_i = \frac{3M\phi_{bg}}{\rho_i R_i^2} \text{ for } \rho_i R_i^2 > 3M\phi_{bg}$$



Sph source A and test object B near middle of chamber experience force between them - usually λ<<1 in cosmology but for atom λ=1 - reduced suppression

Modifying Gravity rather than looking for Dark Energy - non trivial

Any theory deviating from GR must do so at late times yet remain consistent with Solar System tests. Potential examples include:

f(R), f(G) gravity -- coupled to higher curv terms, changes the dynamical eqns for the spacetime metric.

Modified source gravity -- gravity depends on nonlinear function of the energy.

Gravity based on the existence of extra dimensions -- DGP gravity

We live on a brane in an infinite extra dimension. Gravity is stronger in the bulk, and therefore wants to stick close to the brane -- looks locally four-dimensional. Tightly constrained -- both from theory [ghosts] and observations

Scalar-tensor theories including higher order scalar-tensor lagrangians -- recent examples being Galileon models

Massive gravity - single massive graviton bounds m>O(1meV) from demand perturbative down to O(1)mm - too large to conform with GR at large distances

[Burrage et al 2013]

More general f (R) models [Loads of people]

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R + f(R)}{2\kappa^2} + \mathcal{L}_\mathrm{m} \right]$$

No Λ

Usually f (R) struggles to satisfy both solar system bounds on deviations from GR and late time acceleration. It brings in extra light degree of freedom --> fifth force constraints.

Ans: Make scalar dof massive in high density solar vicinity and hidden from solar system tests by chameleon mechanism.

Requires form for f (R) where mass of scalar is large and positive at high curvature.

Issue over high freq oscillations in R and singularity in finite past.

In fact has to look like a standard cosmological constant [Song et al, Amendola et al]

What should we do to help determine the nature of DE ?

1. We need to define properly theoretically predicted observables, or determine optimum ways to parameterise consistency tests (i.e. how should we parameterise w(z)?)

2. Need to start including dynamical dark energy, interacting dark matter-dark energy and modified gravity models in large scale simulations -[Wyman et al 2013, Li et al 2013 Puchwein et al 2013, Jennings et al 2012, Barreira et al 2012, Brax et al 2013].

3. Include the gastrophysics + star formation especially when considering baryonic effects in the non-linear regimes - `mud wrestling'.

4. On the theoretical side, develop models that go beyond illustrative toy models. Extend Quintessential Axion models. Are there examples of actual Landscape predictions? De Sitter vaccua in string theory is non trivial -[see Burgess et al].

5. Recently massive gravity and galileon models have been developed which have been shown to be free of ghosts. What are their self-acceleration and consistency properties?

6. Will we be able to reconstruct the underlying Quintessence potential from observation?

7. Will we ever be able to determine whether $w \neq -1$?

8. Look for alternatives, perhaps we can shield the CC from affecting the dynamics through self tuning-- The Fab Four, Sequestering

9. Given the complexity (baroque nature ?) of some of the models compared to that of say Λ, should we be using Bayesian model selection criterion to help determine the relevance of any one model.

Things are getting very exciting with DES beginning to take data and future Euclid missions, LSST, as well as proposed giant telescopes, GMT, ELT, SKA - traveling in new directions !

Testing General Relativity on Cosmological Scales

$$G_{ab} = 8\pi G T^{(\text{known})}_{ab} + U_{ab}.$$

[Skordis (2009)]

U_{ab} -- encapsulates unknown fields/modifications.

Assume no more than second order field equations places constraints on number of derivatives of the extra fields in U_{ab}.

Bianchi Identity: $\nabla_a U^a{}_b = 0$

Obtains most general diffeomorphism invariant modification to Einstein's eqns for which bgd cosmology is ΛCDM, no extra fields present and no higher deriv than 2 in field equations. Does this by adding gauge invariant terms to Einstein eqns.

Active field of research currently !

Finally we return to the beginning -- Inflation

A period of accelerated expansion in the early Universe

Small smooth and coherent patch of Universe size less than (1/H) grows to size greater than the comoving volume that becomes entire observable Universe today.

Explains the homogeneity and spatial flatness of the Universe

and also explains why no massive relic particles predicted in say GUT theories

Leading way to explain observed inhomogeneities in the Universe

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G(\rho + 3p) --- Accn$$

$$If \;\; \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

What is Inflation?

Any epoch of the Universe's evolution during which the comoving Hubble length is decreasing. It corresponds to any epoch during which the Universe has accelerated expansion.

$$\frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0 \leftrightarrow \ddot{a} > 0$$

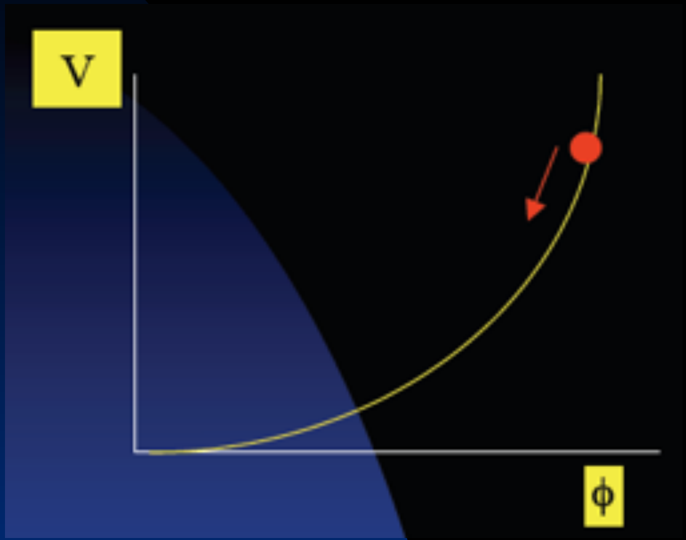
$$\boxed{\frac{\ddot{a}}{a} = -\frac{8\pi}{3}G\left(\rho + 3p\right) --- Accn}$$

$$\boxed{If \;\; \rho + 3p < 0 \Rightarrow \ddot{a} > 0}$$

For inflation require material with negative pressure. Not many examples. One is a scalar field!

Intro fundamental scalar field -- like Higgs

If Universe is dominated by the potential of the field, it will accelerate!

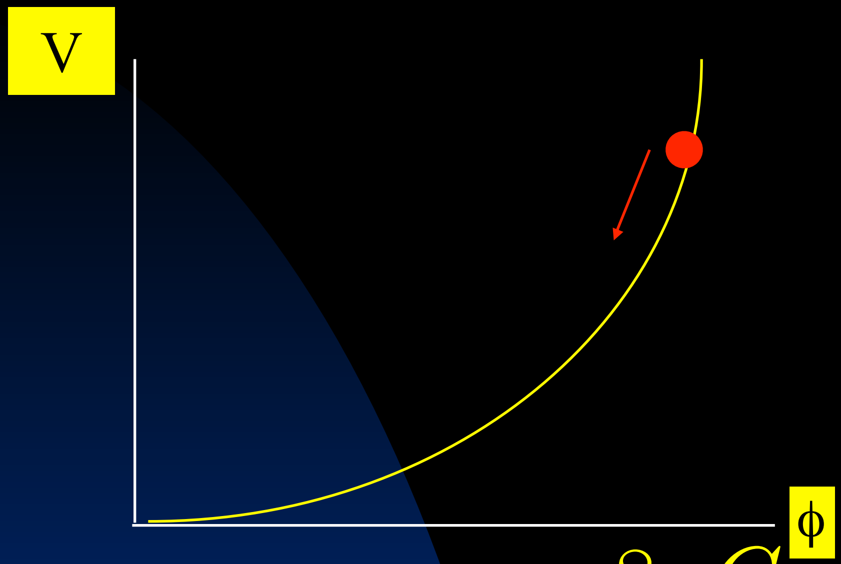


$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

We aim to constrain potential from observations.

During inflation as field slowly rolls down its potential, it undergoes quantum fluctuations which are imprinted in the Universe. Also leads to gravitational wave production.

Examples of inflation



Simplest case – homogeneous single scalar field

$$\rho_\phi = V(\phi) + \frac{\dot{\phi}^2}{2} \quad ; \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

EoM $$H^2 = \frac{8\pi G}{3}\rho_\phi \quad ; \quad \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Inflation $$\ddot{a} > 0 \leftrightarrow (\rho + 3p) < 0 \leftrightarrow \dot{\phi}^2 \ll V(\phi)$$ Slow roll approx

$$\Rightarrow \quad H^2 = \frac{8\pi G}{3}V(\phi) \quad ; \quad 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Also: $$\dot{H} = -4\pi G\dot{\phi}^2,$$

So, define a quantity which specifies how fast H changes during inflation

Prediction -- potential determines important quantities

Slow roll parameters [Liddle & Lyth 1992]

$$\epsilon = \frac{1}{16\pi G}\left[\frac{V'(\phi)}{V(\phi)}\right]^2$$

$$\eta = \frac{1}{8\pi G}\left[\frac{V''(\phi)}{V(\phi)}\right]$$

Inflation occurs when both of these slow roll conditions are << 1

End of inflation corresponds to ε=1

How much does the universe expand? Given by number of e-folds

$$N \equiv \ln\left(\frac{a(t_{\rm end})}{a(t_i)}\right) = \int_{t_i}^{t_e} H\,dt \simeq -\int_{\phi_i}^{\phi_e} \frac{V}{V'} d\phi$$

Last expression is true in the slow roll limit (for single field inflation).

Number of e-folds required

Solve say the Flatness problem:

Assume inflation until tend = 10^{-34} sec

Assume immediate radn dom until today, $t_0 = 10^{17}$ sec

Assume $\left| \Omega \ (t_0) - 1 \right| \leq 0.01$

Now $\left| \Omega - 1 \right| = \dfrac{k}{a^2 H^2}$; $\quad \mathrm{RD}: \left| \Omega - 1 \right| \propto t$

$$\left| \Omega \left(10^{-34}\ \mathrm{s}\right) - 1 \right| \leq 0.01 * 10^{-34} * 10^{-17} \leq 10^{-54}$$

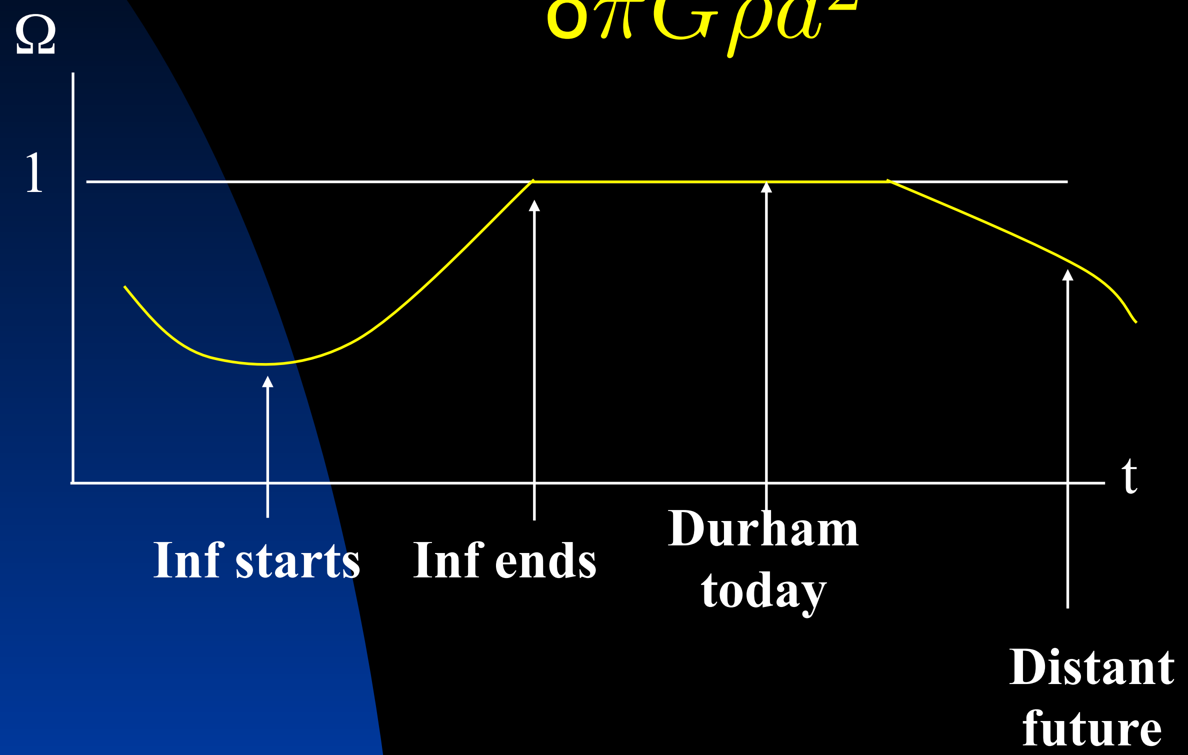
Inf $\left| \Omega - 1 \right| \propto 1 / a^2$ $\Rightarrow$ $\dfrac{\left| \Omega_{end} - 1 \right|}{\left| \Omega_{ini} - 1 \right|} = a_i^2 / a_e^2 = 10^{-54}$

$\Rightarrow$ $N = \ln \left[\dfrac{a_{tend}}{a_{tini}} \right] \approx 62$

Solving the big bang problems

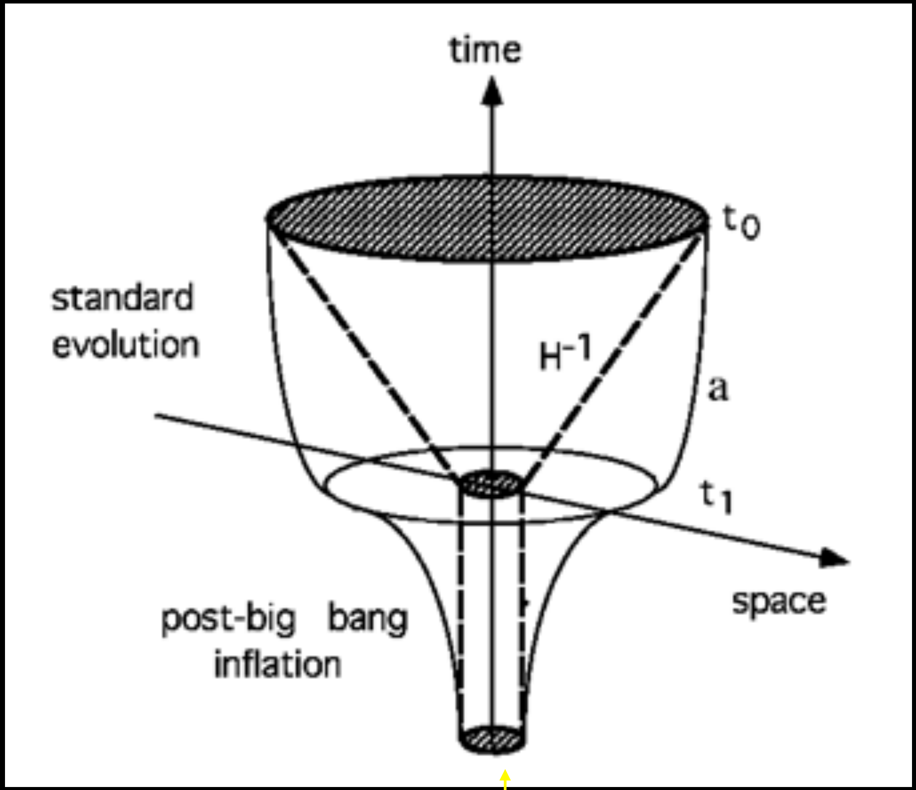
1. Flatness

$$\Omega^{-1} - 1 = -\frac{3k}{8\pi G\rho a^2} \propto a^{-2} \longrightarrow \exp(-2Ht)$$



2. Horizon problem:

Physical: H^{-1} const during inflation. Small initial patch can inflate. How likely is that? Question of initial conditions.



Initial causally connected region

3. Monopole problem: $\rho_{\text{mon}} \propto a^{-3} \rightarrow 0$ rapidly during inflation

Everything infact diluted away except for the inflaton field itself.

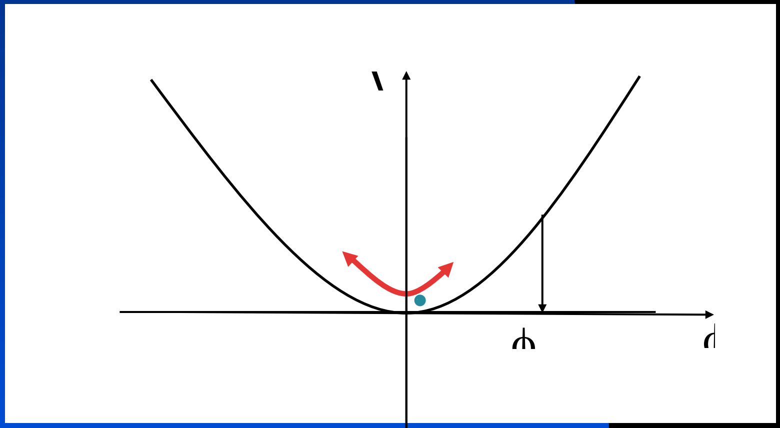
$T \propto a^{-1} \rightarrow 0$ rapidly during inflation

Hence need to reheat the universe at end of inflation

End of inflation

- Eventually SRA breaks down, as inflaton rolls to minima of its potential.

Experimental test of slow roll approximation – Aspen 2002



- Leaves a cold empty Universe apart from inflaton.
- Inflation has to end and the energy density of the inflaton field decays into particles. This is reheating and happens as the field oscillates around the minimum of the potential

End of inflation.

- Inflaton is coupled to other matter fields and as it rolls down to the minima it produces particles –perturbatively or through parametric resonance where the field produces many particles in a few oscillations.

- Dramatic consequences. Universe reheats, can restore previously broken symmetries, create defects again, lead to Higgs windings and sphaleron effects, generation of baryon asymmetry at ewk scale at end of a period of inflation.

- Important constraints: e.g.: gravitino production means : $T_{rh} < 10^9$ GeV -- often a problem!

The origins of perturbations -- the most important aspect of inflation

Idea: Inflaton field is subject to perturbations (quantum and thermal fluctuations). Those are stretched to superhorizon scales, where they become classical. They induce metric perturbations which in turn become later the first perturbations to seed the structures in the universe.

Also predict a cosmological gravitational wave background.

During inf $\boxed{\phi(\underline{x},t) = \phi_0(t) + \delta\phi(\underline{x},t)}$ ← Quantum fluc

Fourier modes: $\boxed{\delta\phi(\underline{x},t) = \sum_k \delta\phi_k(t)\, e^{ikx}}$ → Generates fluc in matter and metric

$\boxed{\delta_H^2(k)}$ Scalar pertn – spectra of gaussian adiabatic density pertns generated by flucns in scalar field and spacetime metric. Responsible for structure formation.

$\boxed{h_{\mu\nu}}$ $\boxed{A_G(k)}$ Tensor pertn in metric– gravitational waves.

Key features

During inflation comoving Hubble length (1/aH) decreases.

So, a given comoving scale can start inside (1/aH), be affected by causal physics, then later leave (1/aH) with the pertns generated being imprinted.

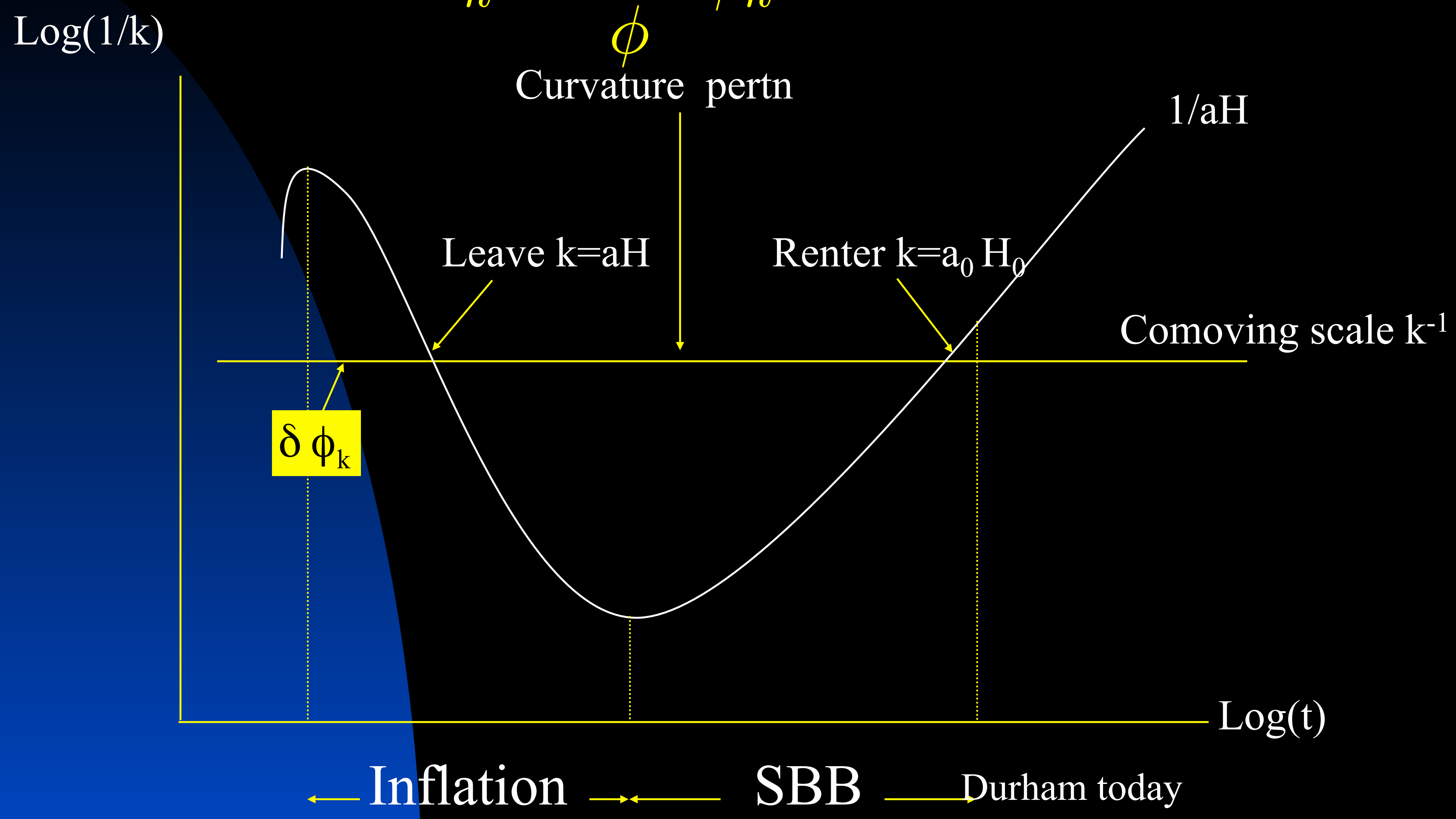
Quantum flucns in inflaton arise from uncertainty principle.

Pertns are created on wide range of scales and generated causally.

Size of irregularities depend on energy scale at which inflation occurs.

Pertn created causally, stretched by expansion.

$$\mathcal{R}_k = \frac{H}{\dot{\phi}}\delta\phi_k \simeq \text{const}$$



The power spectra

Focus on statistical measures of clustering.

Inflation predicts the amp of waves of a given k which obey gaussian statistics, the amplitude of each wave is chosen independently and randomly from its gaussian distribution. It predicts how the amplitude varies with scale — the power spectrum

Good approx -- power spectra as being power-laws with scale.

Density pertn

$$\delta_H^2(k) = \delta_H^2(k_0)\left[\frac{k}{k_0}\right]^{n-1}$$

Grav waves

$$A_G^2(k) = A_G^2(k_0)\left[\frac{k}{k_0}\right]^{n_G}$$

Four parameters

Some formulae

Power spectra $$P_\phi(k) = \frac{k^3}{2\pi^2}\left\langle \left|\delta\,\phi_k\right|^2 \right\rangle$$

Vacuum soln $$\left\langle \left|\delta\,\phi_k\right|^2 \right\rangle = \frac{H^2}{2\,k^3} \Rightarrow P_\phi(k) = \left.\left|\frac{H}{2\pi}\right|^2\right|_{k=aH(\mathrm{Exit})}$$

Amp of density pertn $$\delta_H^2(k) = \frac{4}{25}\left(\frac{H}{\dot\phi}\right)^2\left(\frac{H}{2\pi}\right)^2_{k=aH}$$

SRA $$\delta_H(k) \propto \kappa^{3/2}\left.\frac{V^{3/2}}{|V'|}\right]_{k=aH}$$

WMAP: 60 efolds before tend $\Rightarrow$ $\delta_H(k) \approx 1.91 * 10^{-5}$

$\Rightarrow$ $V^{1/4}/\varepsilon \le 10^{16}$ GeV -- Lyth

In other words the properties of the inflationary potential are constrained by the CMB

Tensor pertns : amp of grav waves. $\rightarrow$ $$A_G(k) \propto \kappa^2 V^{1/2} \Big]_{k=aH}$$

Note: Amp of perts depends on form of potential. Tensor pertns gives info directly on potential but difficult to detect.

Observational consequences.

Precision CMBR expts like WMAP and Planck → probing spectra.

Standard approx – power law.

$$\delta_H^2(k) \propto k^{n-1} \; ; A_G^2(k) \propto k^{n_G}$$

$$n - 1 = \frac{d \ln \delta_H^2}{d \ln k} \; ; n_G = \frac{d \ln A_G^2}{d \ln k}$$

Power law ok, only a limited range of scales are observable.

For range 1Mpc → 10^4 Mpc : $\Delta \ln k \approx 9$

Crucial eqn

$$\frac{d \ln k}{d \phi} = \kappa \frac{V}{V'}$$

$$n = 1 - 6\varepsilon + 2\eta \; ; n_G = -2\varepsilon$$

$n=1$; $n_G=0$ – Harrison Zeldovich

CMBR → Measure relative importance of density pertns and grav waves.

$$R = \frac{C_2^{GW}}{C_2^{S}} \approx 4\pi\varepsilon$$

$$\text{where } \frac{\Delta T}{T} = \sum a_{lm} Y_m^l(\theta, \varphi),\ C_l = \left\langle |a_{lm}|^2 \right\rangle$$

C_l -- radiation angular power spectrum.

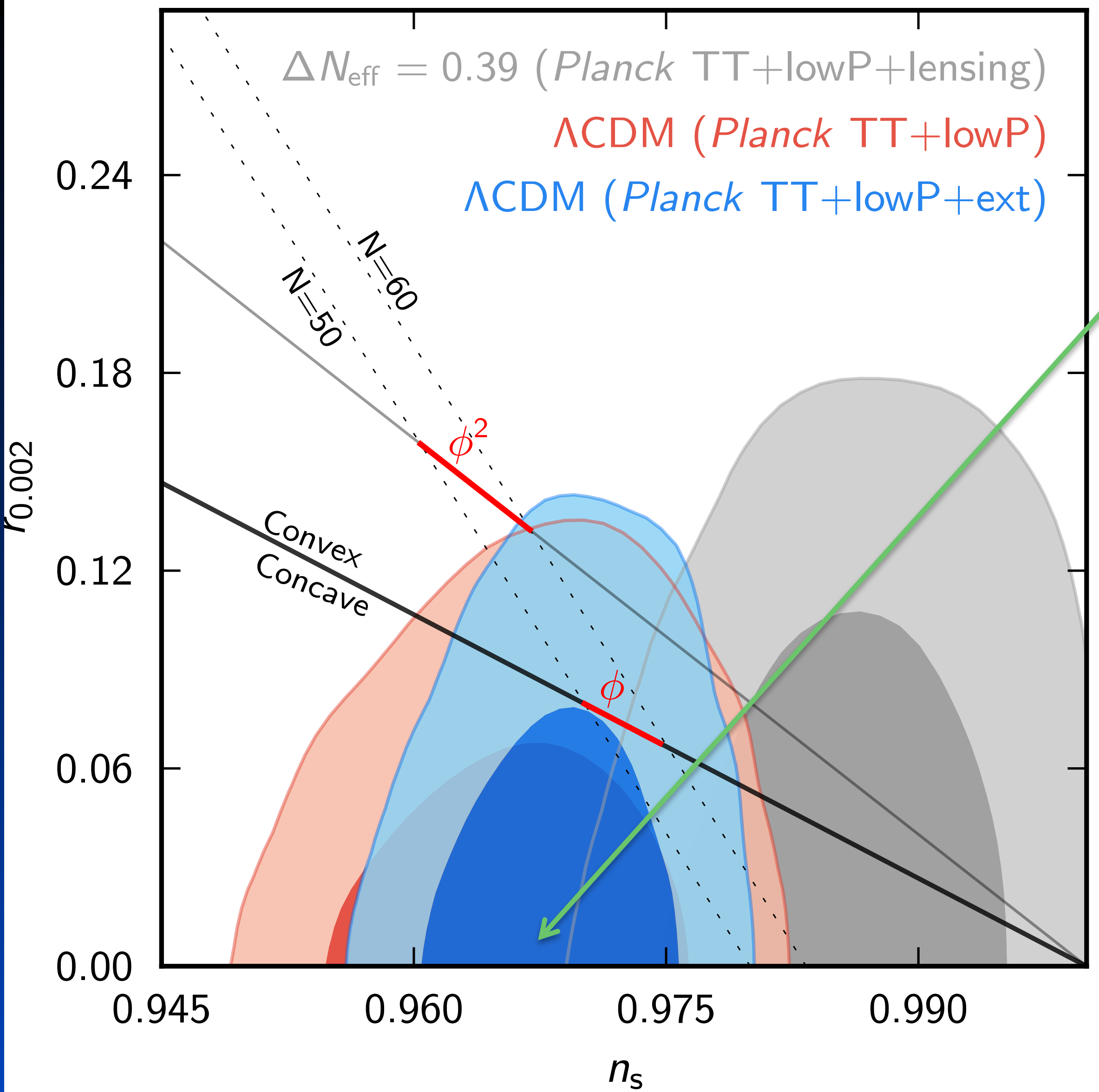
A unique test of inflation

$$R = -2\pi n_G$$

Indep of choice of inf model, relies on slow roll and power law approx. Unfortunately n_G too small for detection, but maybe Planck !

This is where the Bicep2 excitement was !

Planck collaboration 2014 - preliminary



Starobinsky (R^2) inflation

$n_s \approx 1 - 2/N \approx 0.967$
$r \approx 12/N^2 \approx 0.0033$
$dn_s/d\ln k \approx -2/N^2 \approx -0.0006$

...... but, there is plenty of room at the top

(and to the side!)

preliminary

Inflation model building today -- big industry

Multi-field inflation

Inflation in string theory and braneworlds

Inflation in extensions of the standard model

Cosmic strings formed at the end of inflation

The idea is clear though:

Use a combination of data (CMB, LSS, SN, BAO ...) to try and constrain models of the early universe through to models explaining the nature of dark energy today.

Things not explored - no time

1. Gravitational waves from pre-heating
2. Non-Gaussianity from multi-field inflation
3. Nature of perturbations (adiabatic v non-adiabatic)
4. Thermal inflation and warm inflation
5. Going beyond slow roll
6. Inflation model building -- how easy in string theory.
7. Where is the inflaton in particle physics ? How fine tuned is it?
8. Low energy inflation (i.e. TeV scale).
9. Singularity -- eternal inflation !
10. Impact of multiverse on inflation.
11. Alternatives: pre-big bang, cyclic/ekpyrotic, string cosmology, varying speed of light, quantum gravity ....

And so where are we today?

- Exciting time in cosmology -- Big Bang success.
- String - theory suggests we can consistently include gravity into particle physics.
- What started the big bang ?
- How did inflation emerge – if at all ?
- How did the spacetime dimensions split up?
- Where did the particle masses come from?
- Why are there just three families of particles?
- Why is the Universe accelerating today?
- What is the dark matter
- Where is all the anti-matter?

Thank you for listening and enjoy the rest of the YETI meeting.

Extra stuff for anyone interested

Perturbative Reheating:

1. Instantaneous reheating where vac energy is converted immediately to radiation with $T_{RH.}$

2. Reheat by slow decay of ϕ with the zero modes comoving energy density decaying into particles which scatter and thermalise. Assume decay width for this is same as for free ϕ.

Expect small decay width, as flatness of potential requires weak coupling of ϕ to other fields. Also in SUGR if coupling not weak, overproduce gravitinos during reheating.

Boltzmann eqn:

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0$$

$$\dot{\rho}_{\mathrm{rad}} + 4H\rho_{\mathrm{rad}} - \Gamma_\phi \rho_\phi = 0$$

T_{RH} – inflaton executes coherent oscillations about V_{min} after inflation.

$< \rho_\phi >_{osc} \propto a^{-3}$ Averaged over many coherent oscillations

$\rho_{\phi I}, a_I$ Values when coherent oscillations start.

Hubble expansion rate: $$H(a) = \sqrt{\frac{8\pi G}{3}\rho_{\phi I}\left(\frac{a_I}{a}\right)^3}$$

Equating: $H(a) = \Gamma_\phi$ gives $$\left(\frac{a_I}{a}\right) = \left(\frac{3G\Gamma_\phi^2}{8\pi\rho_{\phi I}}\right)^{1/3}$$

Assume at this moment all coherent energy density immediately transferred into radiation.

$$\rho_\phi = \rho_R \text{ where } \rho_\phi = \rho_{\phi I}\left(\frac{a_I}{a}\right)^3 \text{ and } \rho_R = \left(\frac{\pi^2}{30}\right) g_* T_{RH}^4$$

Hence: $$T_{RH} = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4}\sqrt{\Gamma_\phi M_P} = 0.2\left(\frac{200}{g_*}\right)^{1/4}\sqrt{\Gamma_\phi M_P}$$

Bound from Gravitino overproduction : $T_{RH} \le 10^9 - 10^{10}\,\mathrm{GeV}$

Preheating: Traschen & Brandenberger; Kofman, Linde & Starobinsky

Non-perturbative resonant transfer of energy to particles induced by the coherent oscillations of ϕ -- can be very efficient!

Assume ϕ oscillating about min of potential.

$$V(\phi) = \frac{m^2\phi^2}{2}; \text{ Write } \phi(t) = \Phi(t)\sin mt$$

In expanding universe Φ decreases due to redshift of momentum.

Assume scalar field X coupled to ϕ $$L_{int} = \frac{g^2 X^2 \phi^2}{2}$$

Mode eqn: $\chi_k = X_k a^{3/2}$: $\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\Phi^2(t)\sin^2(mt)\right)\chi_k = 0$

Minkowski space: Φ const

$$\chi_k'' + \left[A_k - 2q\cos(2z)\right]\chi_k = 0;$$

$$z = mt, \chi_k' \equiv d\chi_k/dz; \quad q = \frac{g^2\Phi^2}{4m^2}; \quad A_k = 2q + \frac{k^2}{m^2}$$

Mathieu equation

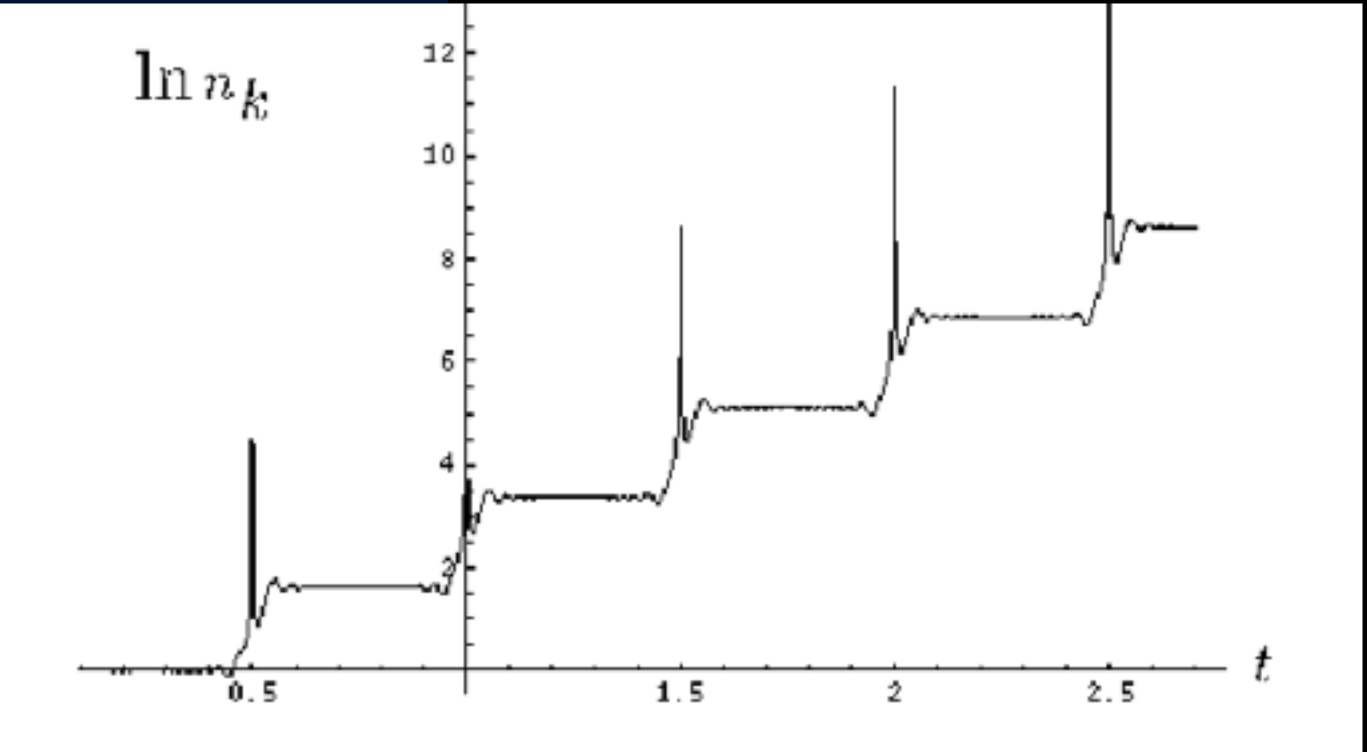
Exponential instability regions:

$$\chi_k \propto \exp(\mu_k z) \text{ where } \mu_k = \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{2k}{m} - 1\right)^2}$$

Max growth at 2k = m

Growth of modes leads to growth of occupation numbers of created particles

Number density = Energy of that mode/Energy of each particle (ω_k)

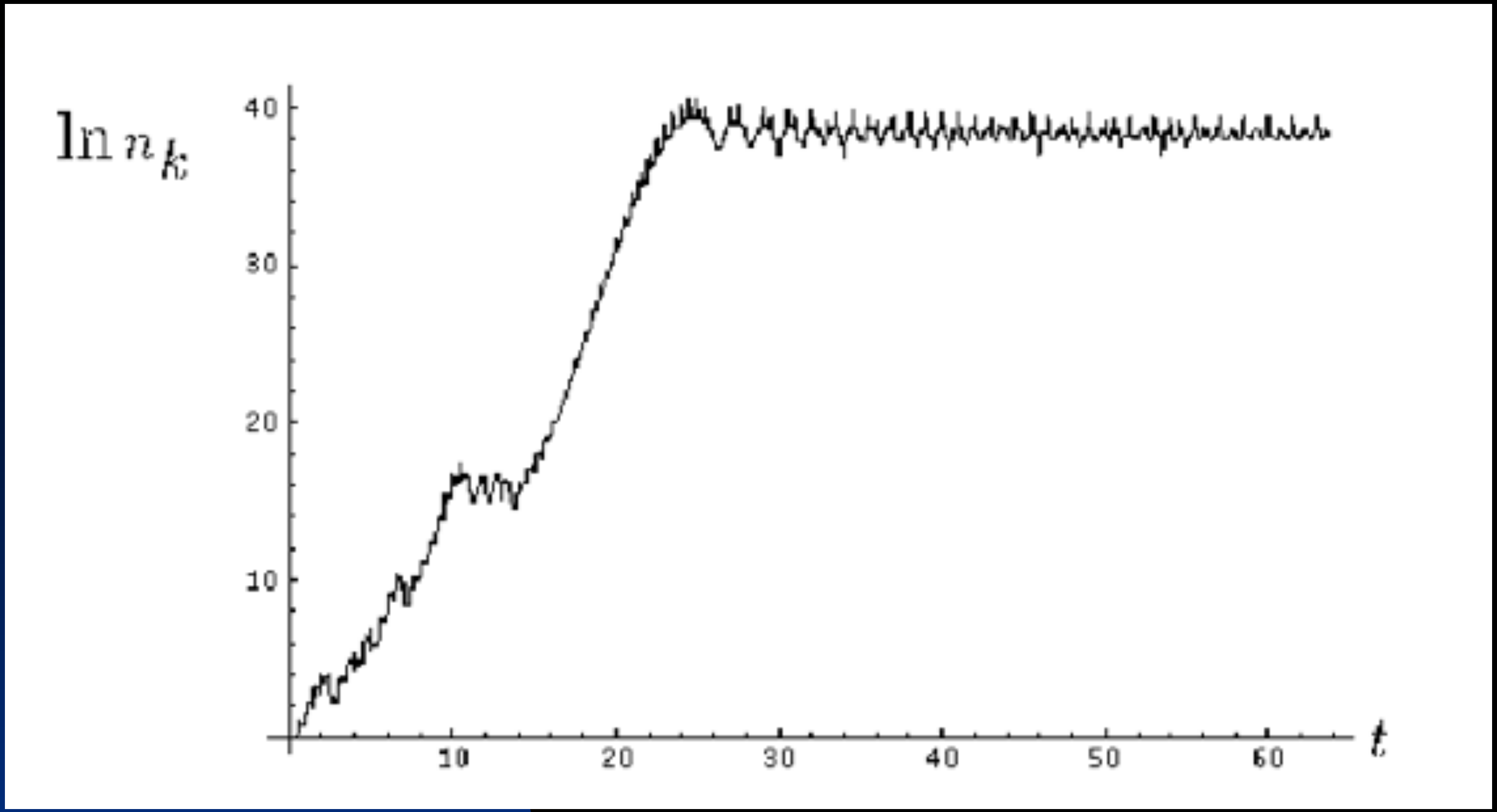


Kofman, Linde and Starobinsky (97)

Period of enhanced rate of energy transfer – preheating, because particles produced not in thermal eqm. Explosive growth every time $\phi(t)=0$.

$$n_k \propto \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2} \propto e^{2\mu_k z}$$

Still occurs when A,q not constant:



Kofman, Linde and Starobinsky (97)

Longer time evolution

This efficient quick transfer of energy means that can have large reheat temperatures, phase transitions, defect production and baryogenesis through production of particles with mass bigger than inflaton mass. Can also generate potentially obervable primordial gravitational waves from pre-heating.

Some examples – Chaotic Inflation

$$V(\phi)=\frac{m^2\phi^2}{2}$$ with $$\varepsilon=\frac{1}{2\kappa^2}\left[\frac{V'}{V}\right]^2 \quad ; \quad \eta=\frac{1}{\kappa^2}\left[\frac{V''}{V}\right]$$

Find: $$\varepsilon=\frac{2}{\kappa^2\phi^2}=\eta$$

SRA: $$H^2=\frac{8\pi G}{3}V(\phi) \quad ; \quad 3H\dot{\phi}+\frac{dV}{d\phi}=0$$

Inf soln: $$\phi(t)=\phi_i-\frac{\sqrt{2}mt}{\sqrt{3}\kappa} \quad ;$$

$$a(t)=a_i\exp\left[\frac{\kappa m}{\sqrt{6}}\left(\phi_i t-\frac{mt^2}{\sqrt{6}\kappa}\right)\right]$$

End of inflation:

$$\varepsilon = 1 \Rightarrow \phi_e = \frac{\sqrt{2}}{\kappa}$$

Num of e-folds:

$$N(\phi) = -\kappa^2 \int_{\phi}^{\phi_e} \frac{V}{V'} d\phi = \frac{\kappa^2 \phi^2}{4} - \frac{1}{2}$$

N=60:

$$\phi_{60} \approx \frac{16}{\kappa} > \phi_e$$

Scale just entering Hubble radius today, COBE scale

Amp of den pertn:

$$\delta_H(k) = \frac{\kappa^3}{\sqrt{75\pi}} \frac{V^{3/2}}{|V'|}\Bigg]_{k=aH}$$

Take to be 60 efolds before end of inflation.

Find:

$$\delta_H(k) = 12m\sqrt{G} \quad \text{where} \quad \kappa^2 \equiv 8\pi G$$

Amp of grav waves:

$$A_G(k) = \sqrt{\frac{32}{75}} \, GV^{1/2} \Big]_{k=aH}$$

60 efolds before end of inflation.

Find:

$$A_G(k) \approx 1.4 m \sqrt{G}$$

Normalise to COBE:

$$\delta_H\,(k) \approx 1.91 * 10^{-5}$$

Find:

$$m = 2 * 10^{13}\ \mathrm{GeV}$$

Constraint on inflaton mass!

Spectral indices

$$n = 1 - 6\varepsilon + 2\eta\,;\; n_G = -2\varepsilon$$

Slow roll

Use values 60 e-folds before end of inflation.

$$n = 0.97;\; n_G = -0.016$$

Close to scale inv

2. Models of Inflation—variety is the spice of life.

(where is the inflaton in particle physics?)

(Lyth and Riotto, Phys. Rep. 314, 1, (1998), Lyth and Liddle (2009)

Field theory

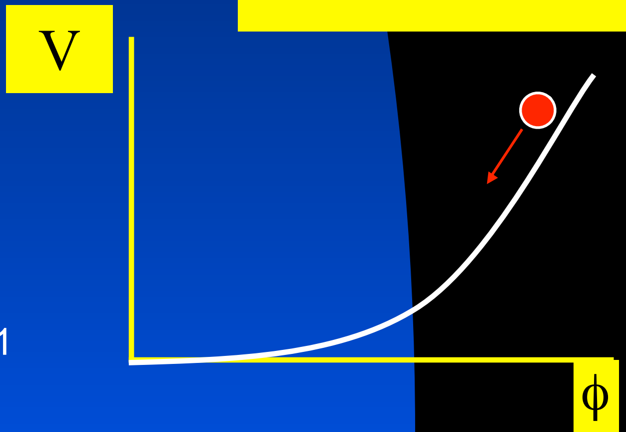
$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + M\phi^3 + \lambda\phi^4 + \sum_{d=5}^{\infty}\lambda_d M_P^{4-d}\phi^d$$

Quantum corrections give coefficients proportional to $\ln(\phi)$ and an additional term proportional to $\ln(\phi)$

1. Chaotic inflation .

$$V(\phi) \propto \phi^p; \;\; \phi >> M_P; \;\; n-1 = -(2+p)/2N;$$

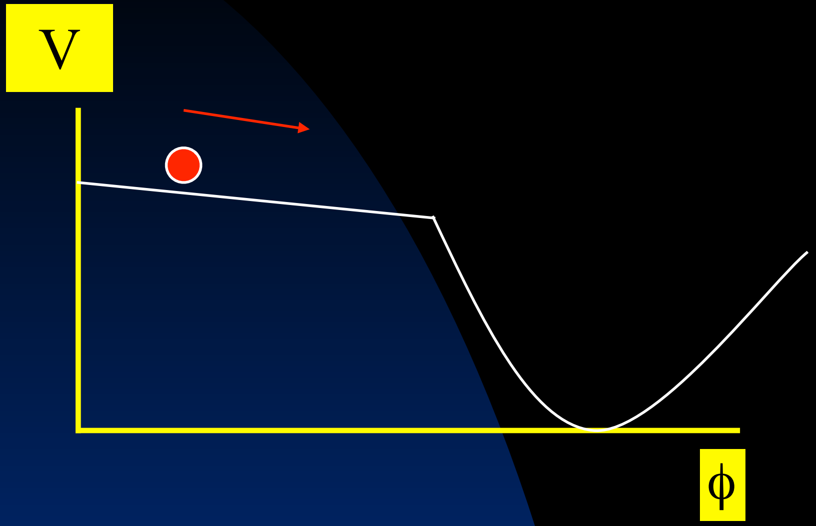
$$R = -2\pi n_G = {3.1p}/{N} \Rightarrow \text{sig grav waves.}$$



Inflates only for $\phi >> M_P$. Problem. Why only one term? All other models inflate at $\phi < M_P$ and give negligible grav. waves.

2. New inflation

$$V(\phi) = V_0 - c\phi^p + \ldots;\ \ p \ge 3;\ \ n - 1 = -\frac{2(p-1)}{(p-2)N}$$



$$V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \ldots; \Rightarrow n - 1 = -\frac{2M_P^2 m^2}{V_0}$$

$p = 2:$ modular, natural, quadratic inflation

3. Power-law inflation

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{p}}\,\frac{\phi}{m_P}\right);\ \ p > 1;\ \ n - 1 = -\frac{2}{p}$$

1. Very useful because have exact solutions without recourse to slow roll. Similarly perturbation eqns can be solved exactly.

2. No natural end to inflation

4. Natural inflation

$$V(\phi) = V_0\left(1 + \cos\frac{\phi}{f}\right);$$

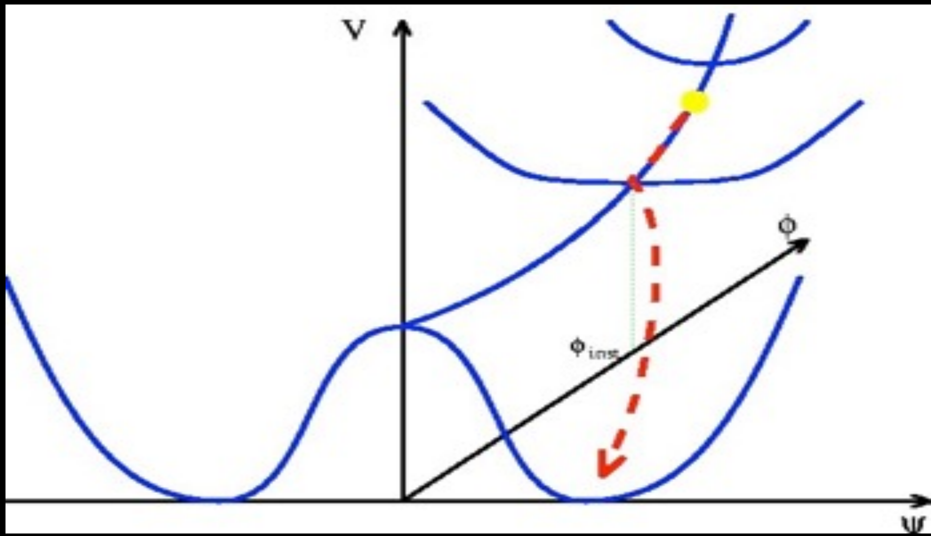
$$n - 1 < 0; \quad R - \text{negligible} - -\text{like New Inflation}$$

5. Hybrid inflation

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2;$$

$$n - 1 = \frac{2M_P^2 m^2}{V_0}$$

2 fields, inf ends when V_0 destabilised by 2nd non-inflaton field ψ

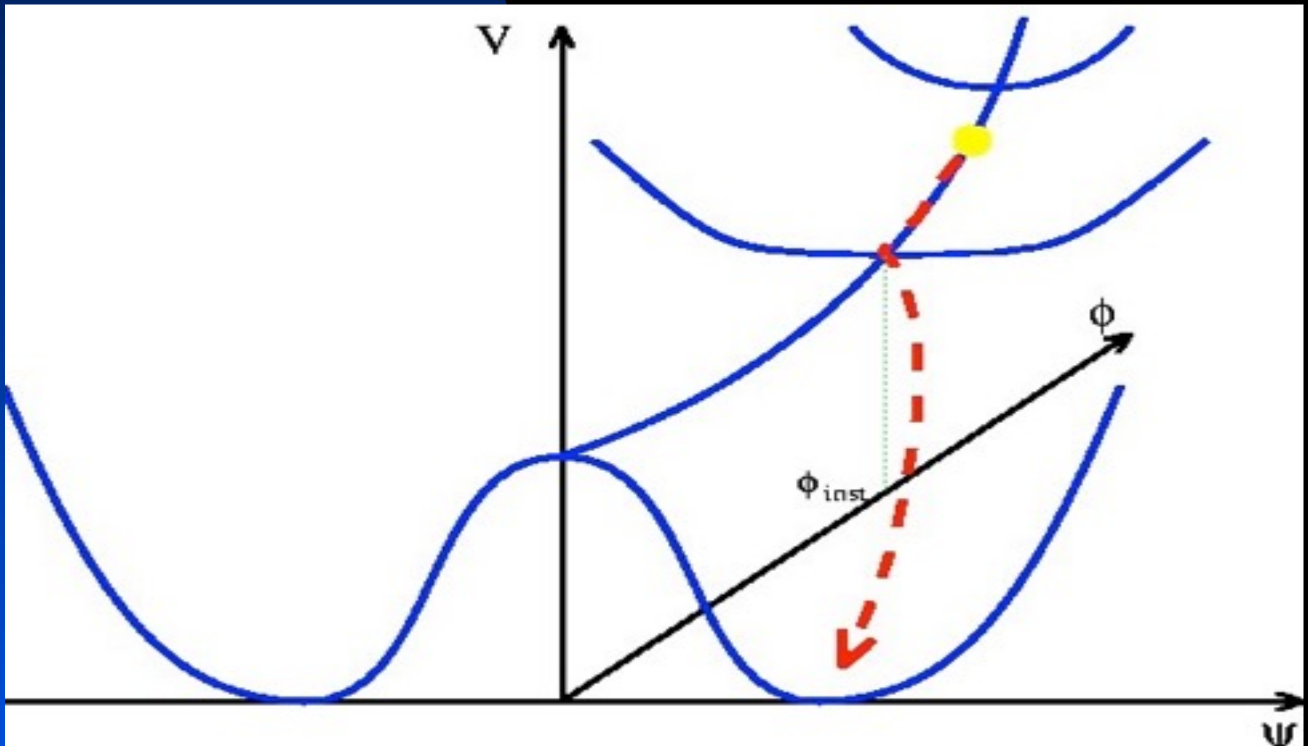


Two field inflation – more general

$$V(\phi,\psi) = \frac{1}{2}m_\phi{}^2\phi^2 + \frac{1}{2}g^2\phi^2|\chi|^2 + \frac{1}{4}\lambda\left(|\chi|^2 - \frac{m^2}{\lambda}\right)^2$$

Found in SUSY models.

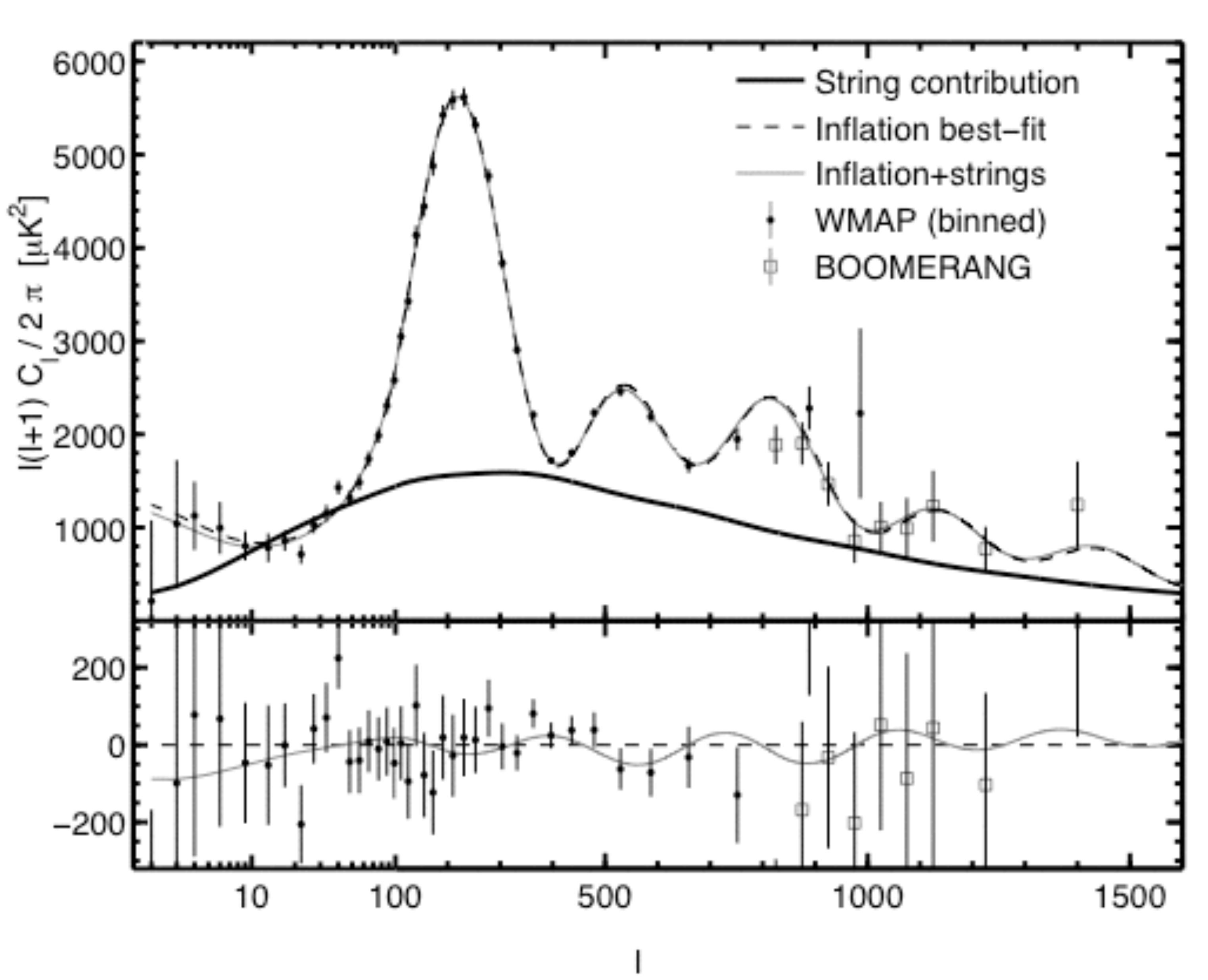
Better chance of success, plus lots of additional features, inc defect formation, ewk baryogenesis.



Inflation ends by triggering phase transition in second field.

Example of Brane inflation

Cosmic strings - may not do the full job but they can still contribute



Hybrid Inflation type models

String contribution < 11% implies $G\mu < 0.7 * 10^{-6}$.

Bevis et al 2007,2010.

Inflation in string theory -- non trivial

The η problem in Supergravity -- N=1 SUGR Lagrangian:

$$\mathcal{L} = -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} + V_F,$$

with

$$V_F = e^{K/M_p^2}\left[K^{\varphi\bar{\varphi}}D_\varphi W\overline{D_\varphi W} - \frac{3}{M_p^2}|W|^2\right]$$

and

$$D_\varphi = \partial_\varphi W + \frac{1}{M_p^2}\partial_\varphi K$$

$$K(\varphi,\bar{\varphi}) = K_0 + K_{\varphi\bar{\varphi}}\varphi\bar{\varphi} + \dots$$

Expand K about φ=0

$$\begin{aligned}\mathcal{L} &\approx -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} - V_0\left(1 + K_{\varphi\bar{\varphi}}|_{\varphi=0}\frac{\varphi\bar{\varphi}}{M_p^2} + \dots\right) \\ &= -\partial\phi\partial\bar{\phi} - V_0\left(1 + \frac{\phi\bar{\phi}}{M_p^2} + \dots\right),\end{aligned}$$

Canonically norm fields ϕ

Have model indep terms which lead to contribution to slow roll parameter η of order unity

$$\Delta\eta = M_p^2\frac{\Delta V''}{V_0} = 1.$$

So, need to cancel this generic term possibly through additional model dependent terms.

Ex 1: Warped D3-brane D3-antibrane inflation where model dependent corrections to V can cancel model indep contributions

[Kachru et al (03) -- KLMMT].

Find:

$$V(\phi) = V_0(\phi) + \beta H^2 \phi^2$$

β relates to the coupling of warped throat to compact CY space. Can be fine tuned to avoid η problem

Ex 2: DBI inflation -- simple -- it isn't slow roll as the two branes approach each other so no η problem

Ex 3: Kahler Moduli Inflation [Conlon & Quevedo 05]

Inflaton is one of Kahler moduli in Type IIB flux compactification. Inflation proceeds by reducing the F-term energy. No η problem because of presence of a symmetry, an almost no-scale property of the Kahler potential.

$$V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n \tau_n}}{\mathcal{V}^2},$$

Inflaton moduli: τ_n

$$V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n \tau_n}}{\mathcal{V}^2},$$

Find:

$$\begin{array}{rcl} 0.960 & < n < & 0.967, \\ -0.0006 & < \frac{dn}{d\ln k} < & -0.0008, \\ 0 & < |r| < & 10^{-10}, \end{array}$$

with large volume modulus

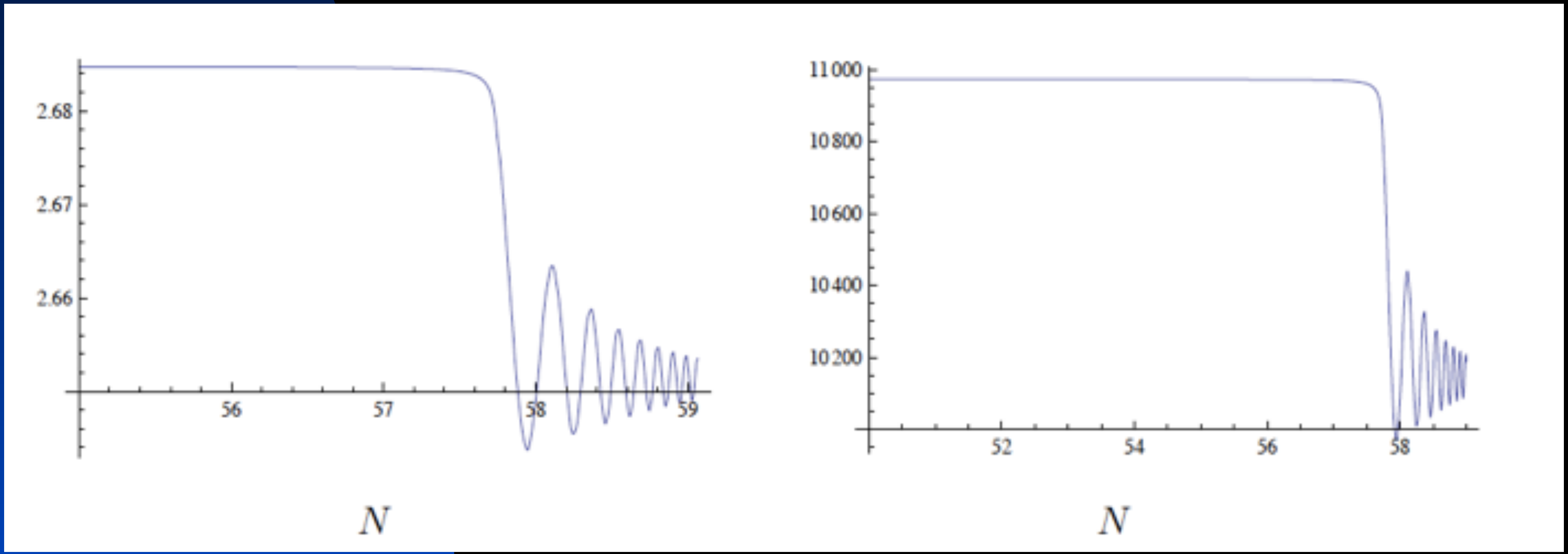
$$10^5 l_s^6 \le \mathcal{V} \le 10^7 l_s^6,$$

and

$$\begin{array}{rcl} \eta & \approx & -\frac{1}{N_e}, \\ \epsilon & < & 10^{-12}, \end{array}$$

for $N_e \approx$50-60 efolds with low energy scale

$$V_{inf} \sim 10^{13}\mathrm{GeV}.$$



Inflaton **[Blanco-Pillado et al 09]** **Volume modulus**

Can include curvaton as second evolving moduli -- Burgess et al 2010

Key inflationary parameters:

n: Perhaps Planck will finally determine whether it is unity or not.

r: Tensor-to-scalar ratio : considered as a smoking gun for inflation but also produced by defects and some inflation models produce very little.

dn/dln k : Running of the spectral index, usually very small -- probably too small for detection.

f_{NL}: Measure of cosmic non-gaussianity. Still consistent with zero, but tentative evidence of a non-zero signal in WMAP data which would provide an important piece of extra information to constrain models. For example, it could rule out single field models -- lots of current interest.

$G\mu$: string tension in Hybrid models where defects produced at end of period of inflation.

Also new perturbation generation mechanisms (e.g. Curvaton)

Perturbations not from inflaton but from extra field and then couple through to curvature perturbation