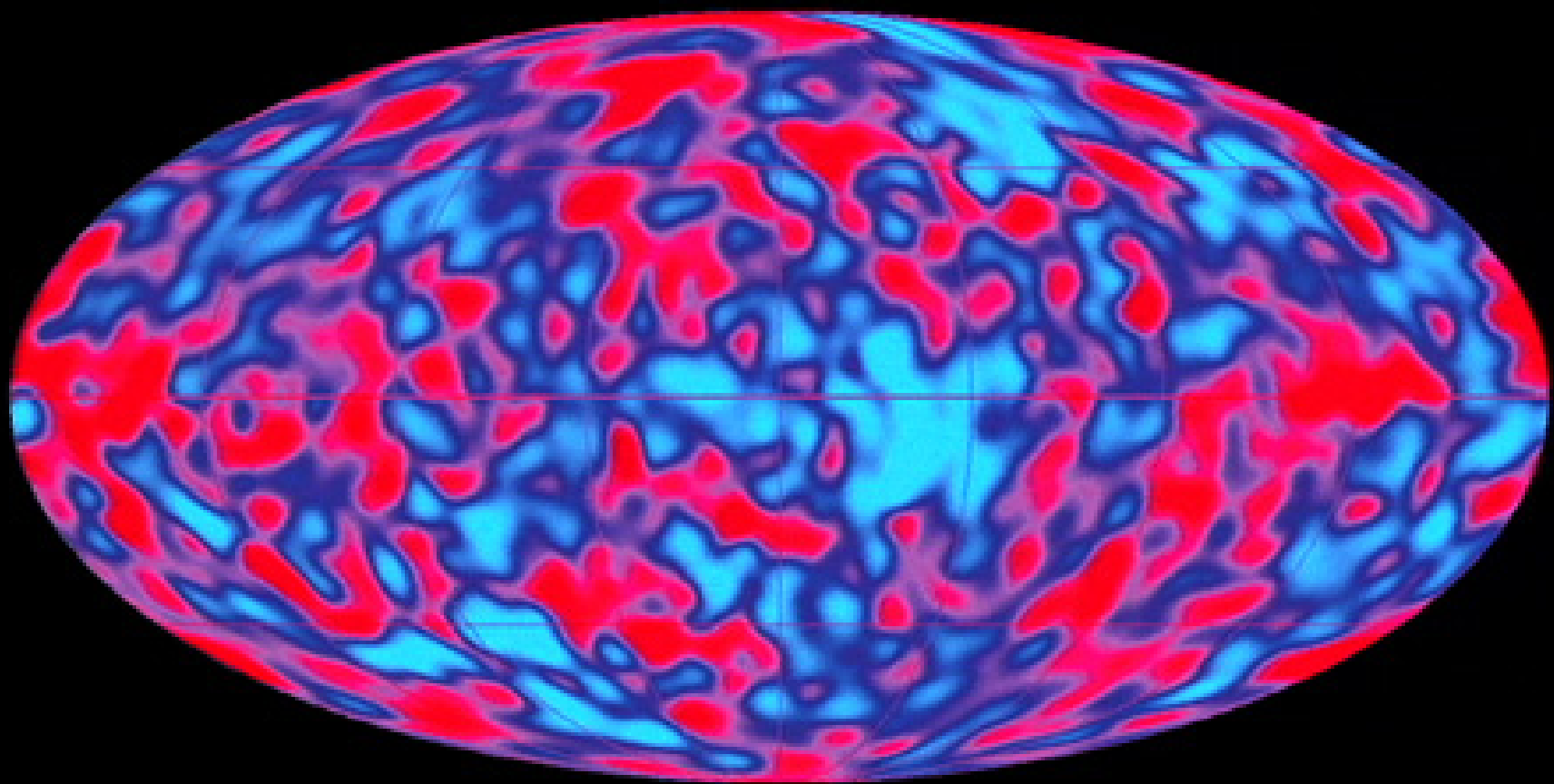
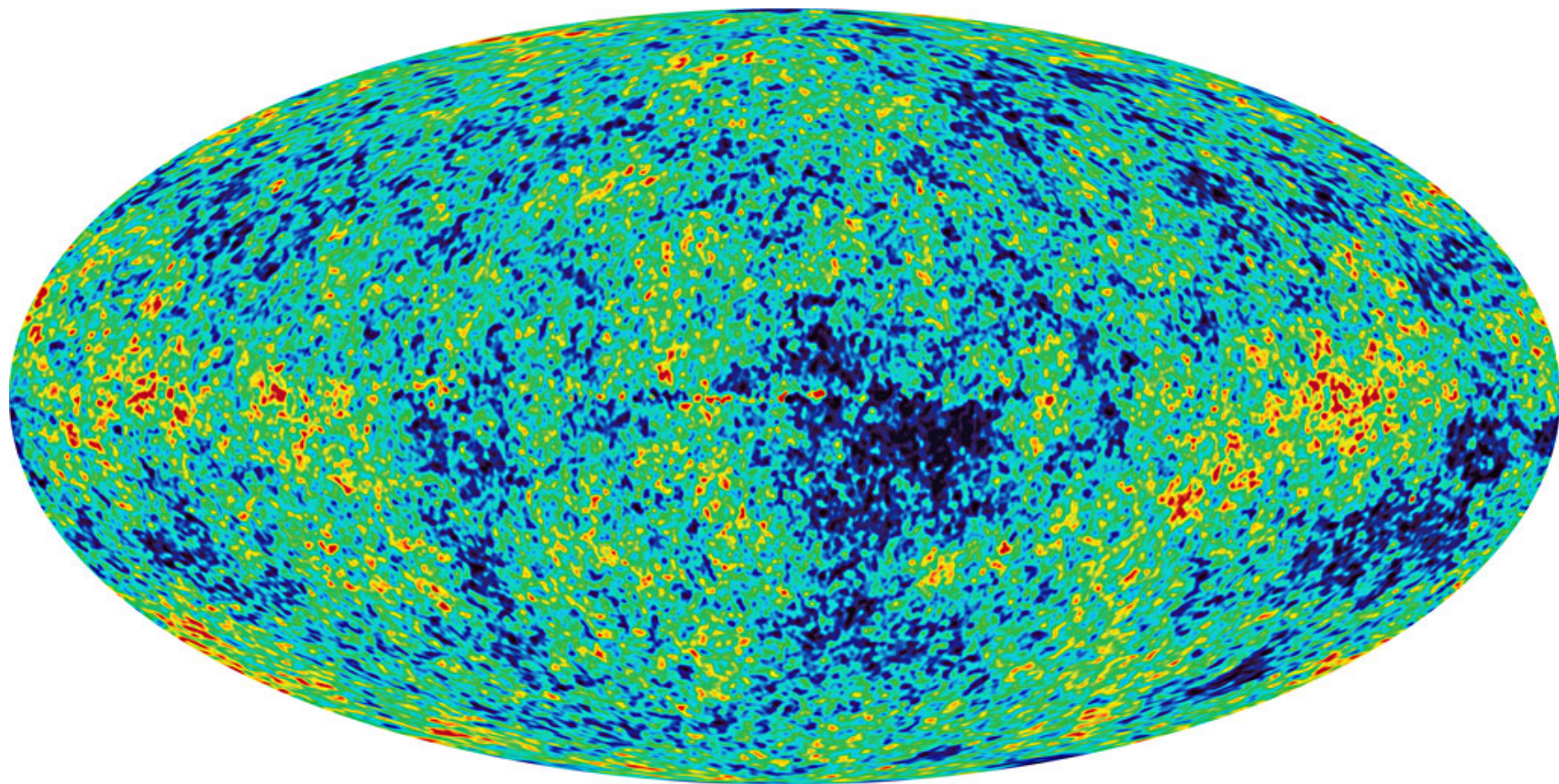


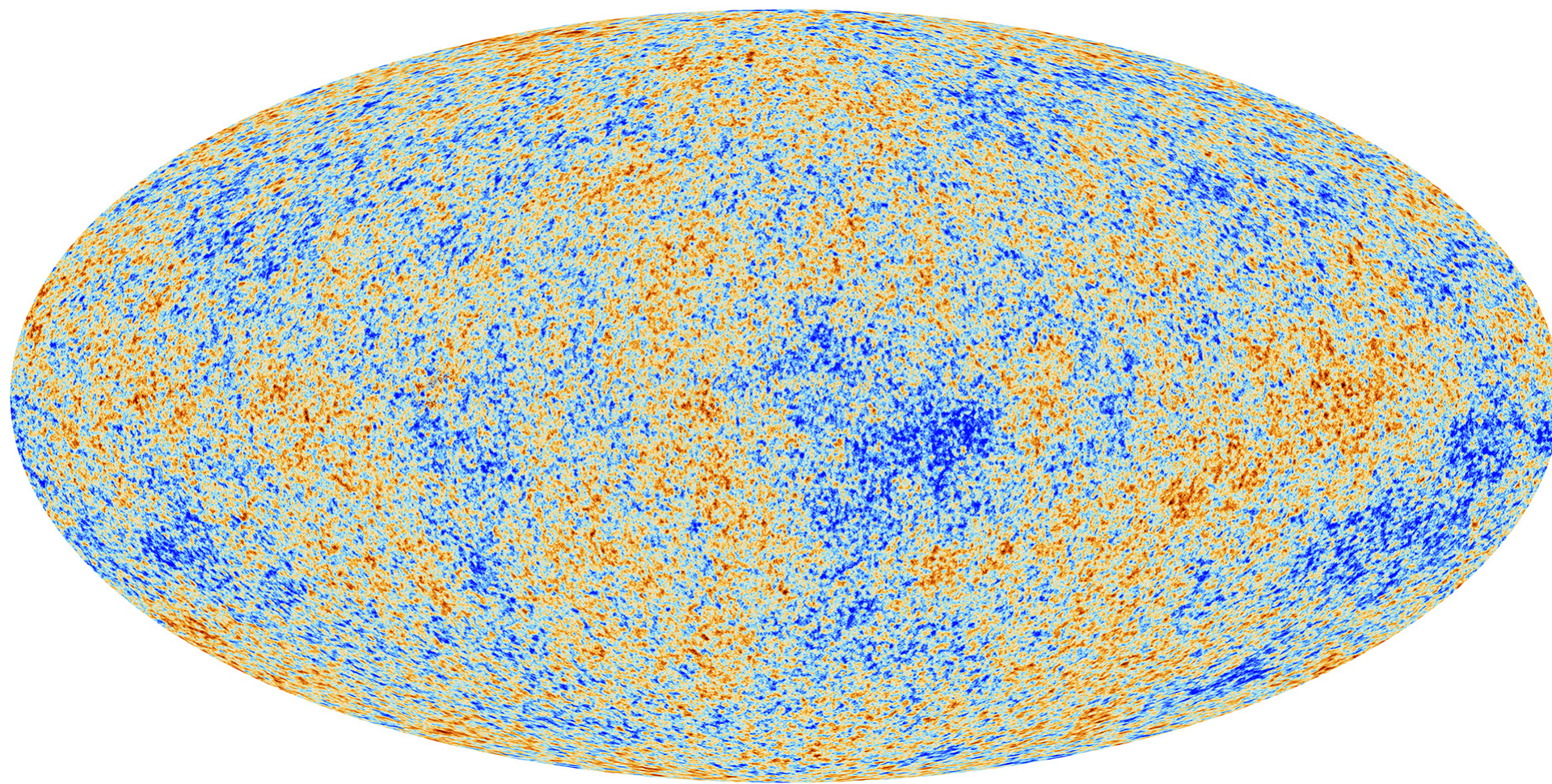
CMB:Theory

Peter Coles (Sussex)
(@telescoper)

with many thanks to Antony Lewis!







The Elephant in the Room



BICEP2



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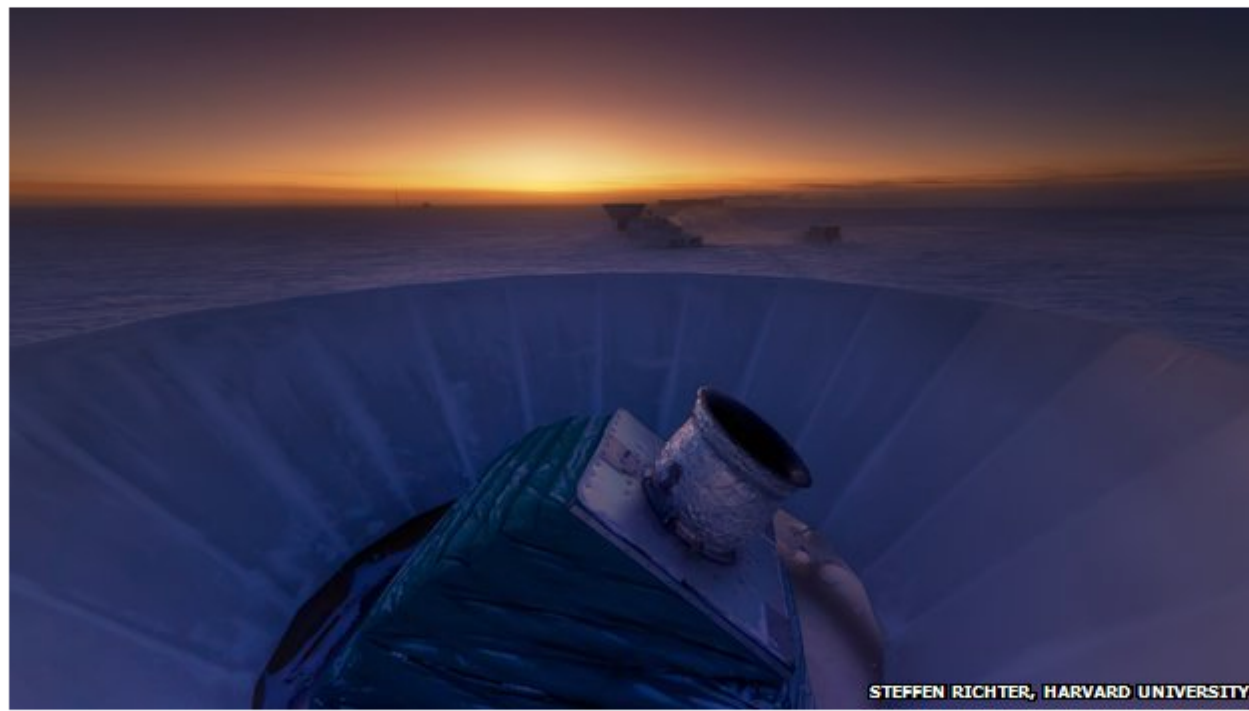
17 March 2014 Last updated at 14:46

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Cosmic inflation: 'Spectacular' discovery hailed

By Jonathan Amos

Science correspondent, BBC News



STEFFEN RICHTER, HARVARD UNIVERSITY

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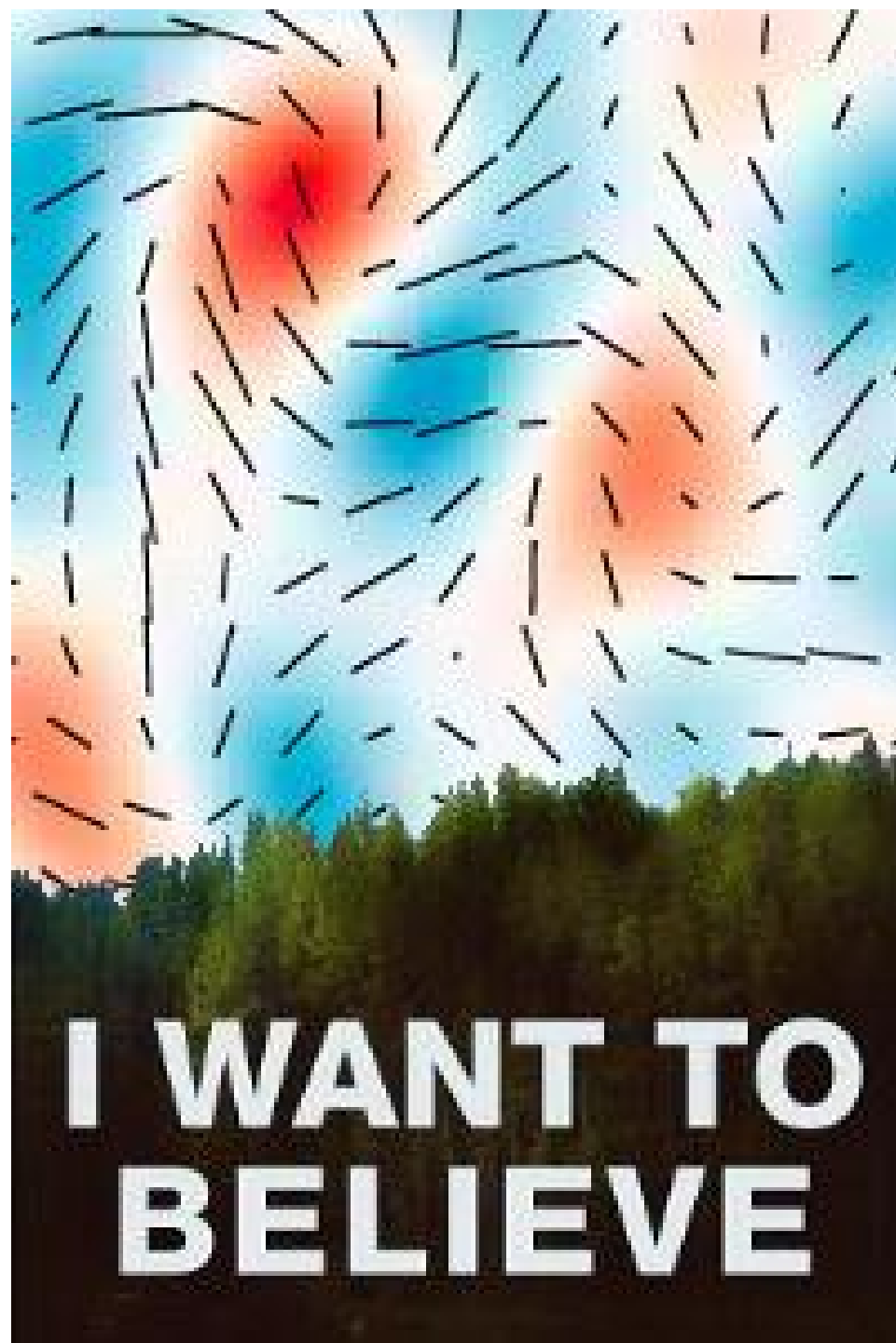
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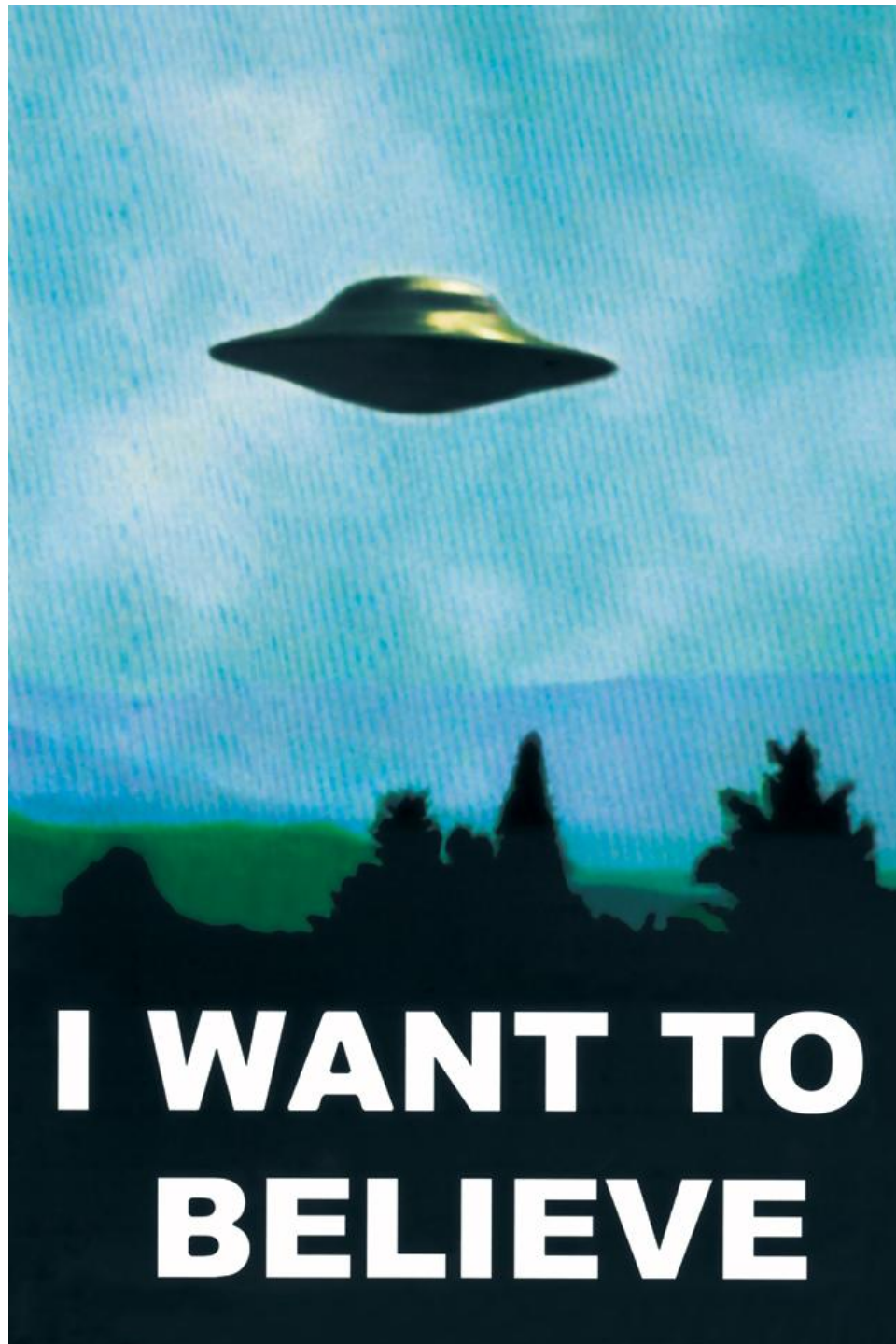


'I earned my stripes'

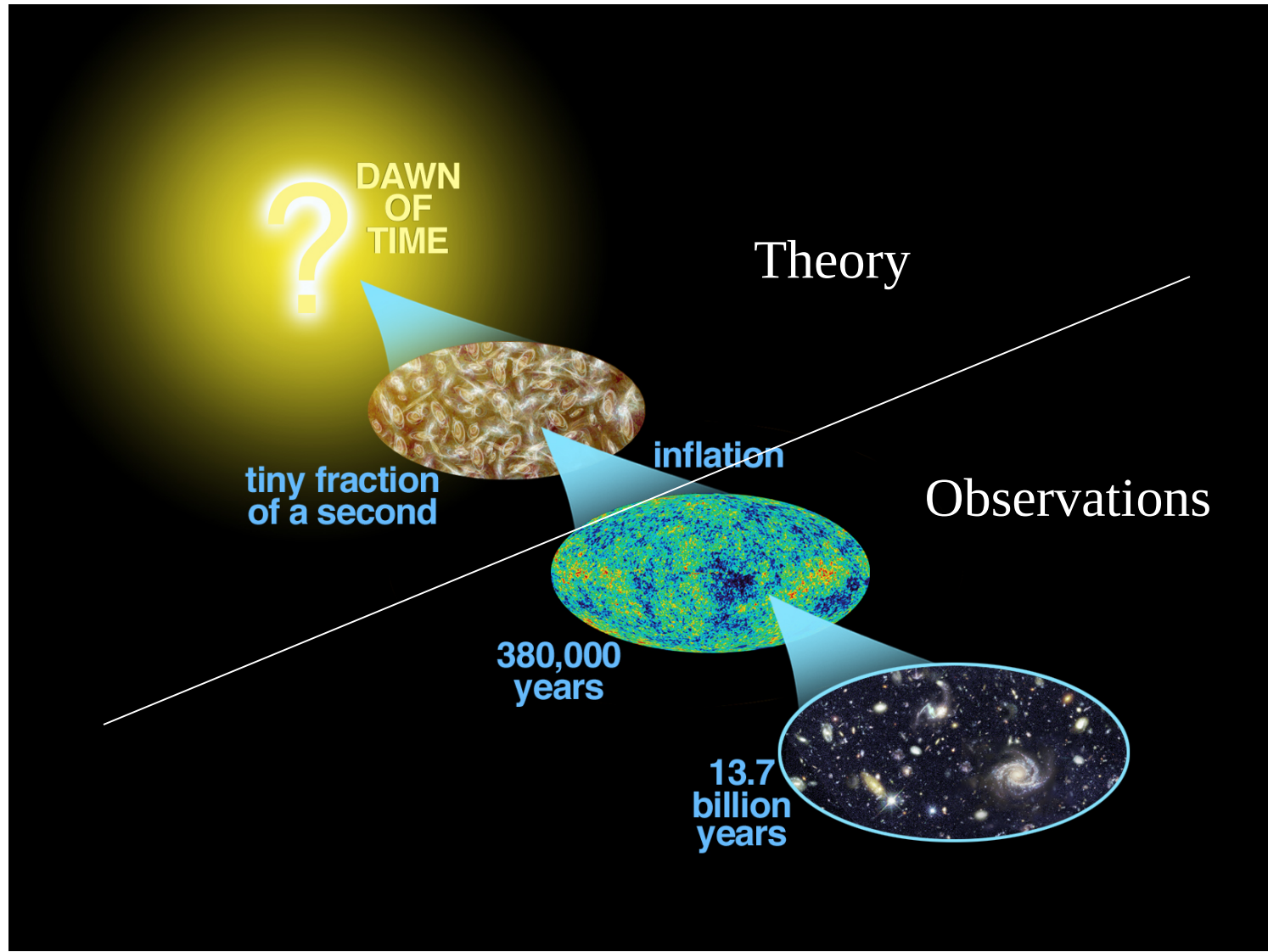
Kylie Minogue on her pop



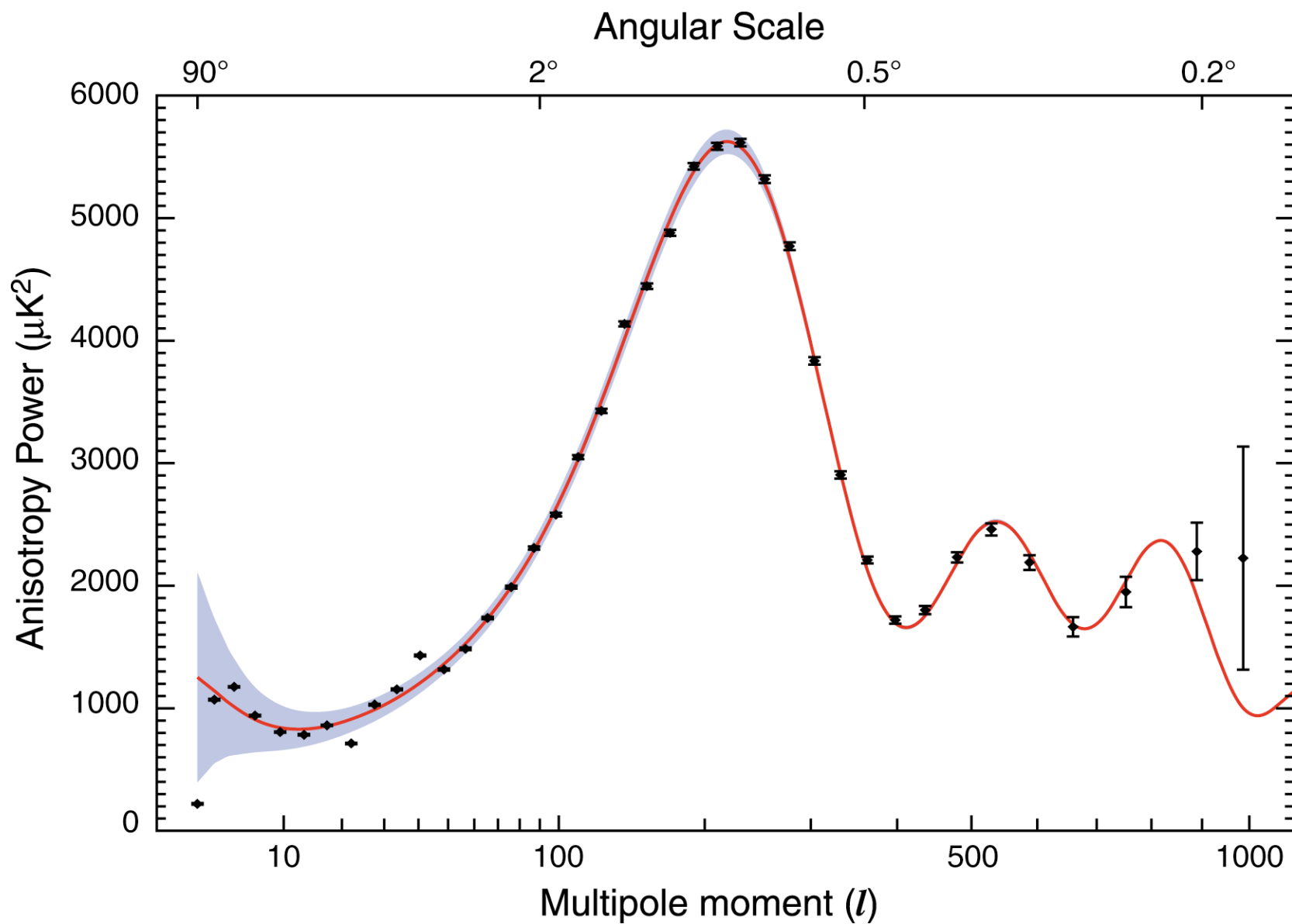
**I WANT TO
BELIEVE**

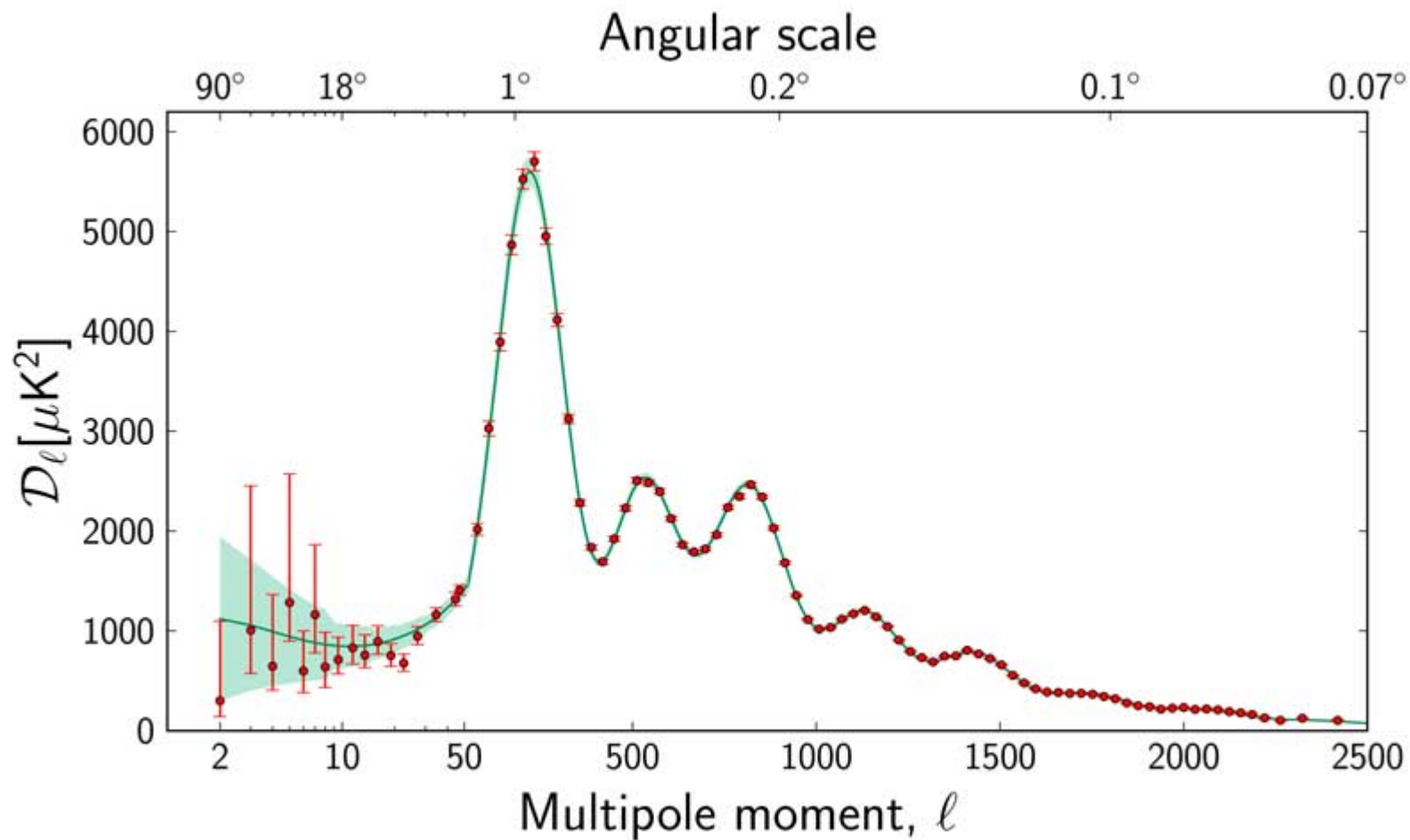


**I WANT TO
BELIEVE**



Source: NASA/WMAP Science Team





Planck results: cosmological parameters

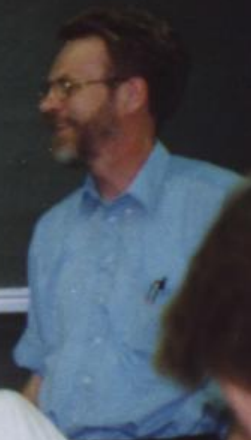
Main result: the standard 6-parameter Λ CDM model remains a good fit to CMB data.

Parameter	<i>Planck</i>		<i>Planck+lensing</i>		<i>Planck+WP</i>	
	Best fit	68% limits	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033	0.022242	0.02217 ± 0.00033	0.022032	0.02205 ± 0.00028
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031	0.11805	0.1186 ± 0.0031	0.12038	0.1199 ± 0.0027
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068	1.04150	1.04141 ± 0.00067	1.04119	1.04131 ± 0.00063
τ	0.0925	0.097 ± 0.038	0.0949	0.089 ± 0.032	0.0925	$0.089^{+0.012}_{-0.014}$
n_s	0.9624	0.9616 ± 0.0094	0.9675	0.9635 ± 0.0094	0.9619	0.9603 ± 0.0073
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072	3.098	3.085 ± 0.057	3.0980	$3.089^{+0.024}_{-0.027}$

Some interesting derived numbers:


H_0	67.11	67.4 ± 1.4	68.14	67.9 ± 1.5	67.04	67.3 ± 1.2
Ω_Λ	0.6825	0.686 ± 0.020	0.6964	0.693 ± 0.019	0.6817	$0.685^{+0.018}_{-0.016}$
Ω_m	0.3175	0.314 ± 0.020	0.3036	0.307 ± 0.019	0.3183	$0.315^{+0.016}_{-0.018}$

	BJ	AB	JP	PC	VI	RIM	PK
H_0	40	35	65	75	60	70	60
Ω_0	0.2	1	0.35	0.2	1	0.3	0.2
Λ_0	0	0	0	0	0	0.7	0
K	-1	0	-1	-1	0	0	-1
t_0	15	14	29	12	18	14	16



Precision Cosmology

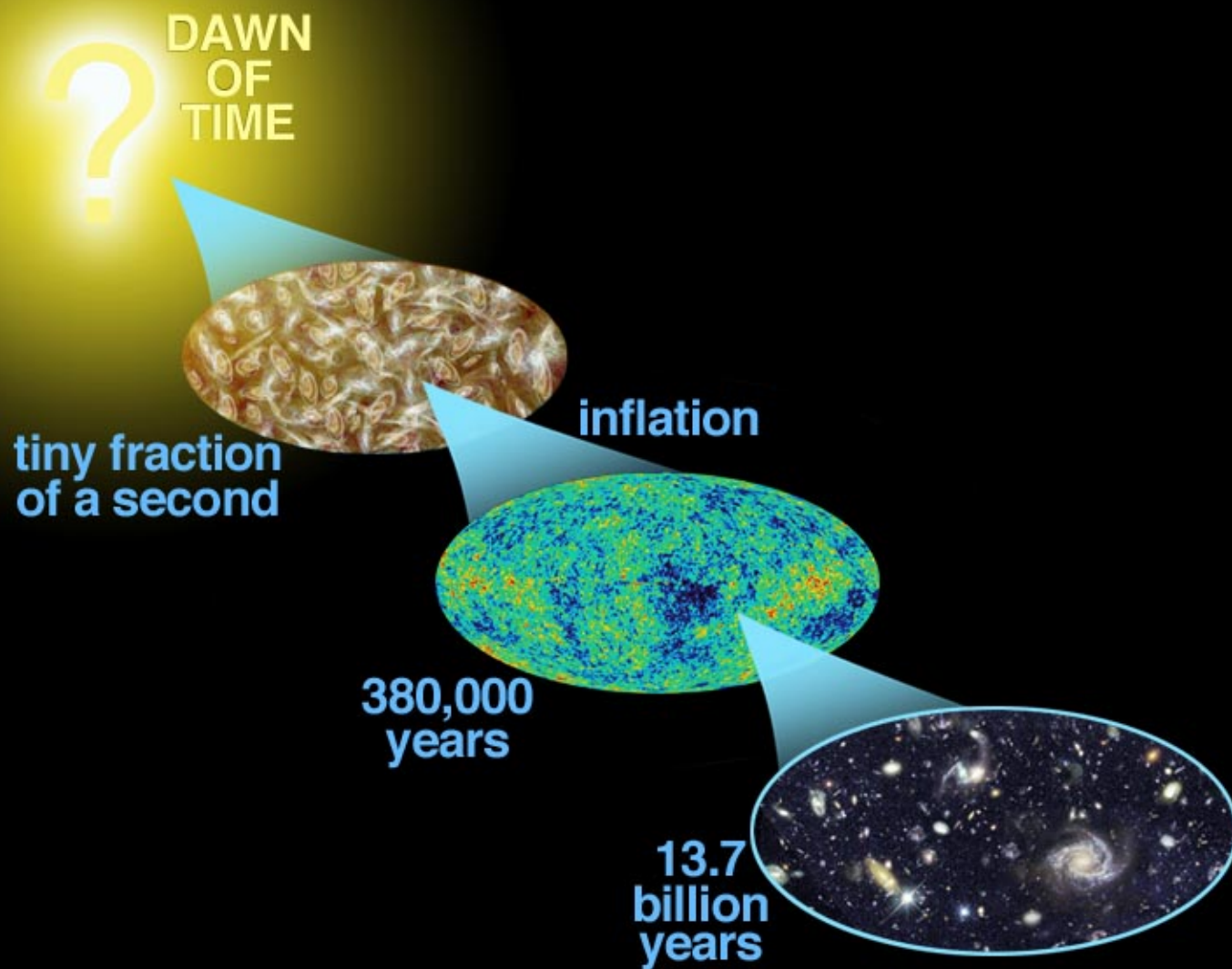
“...as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns -- the ones we don't know we don't know.”

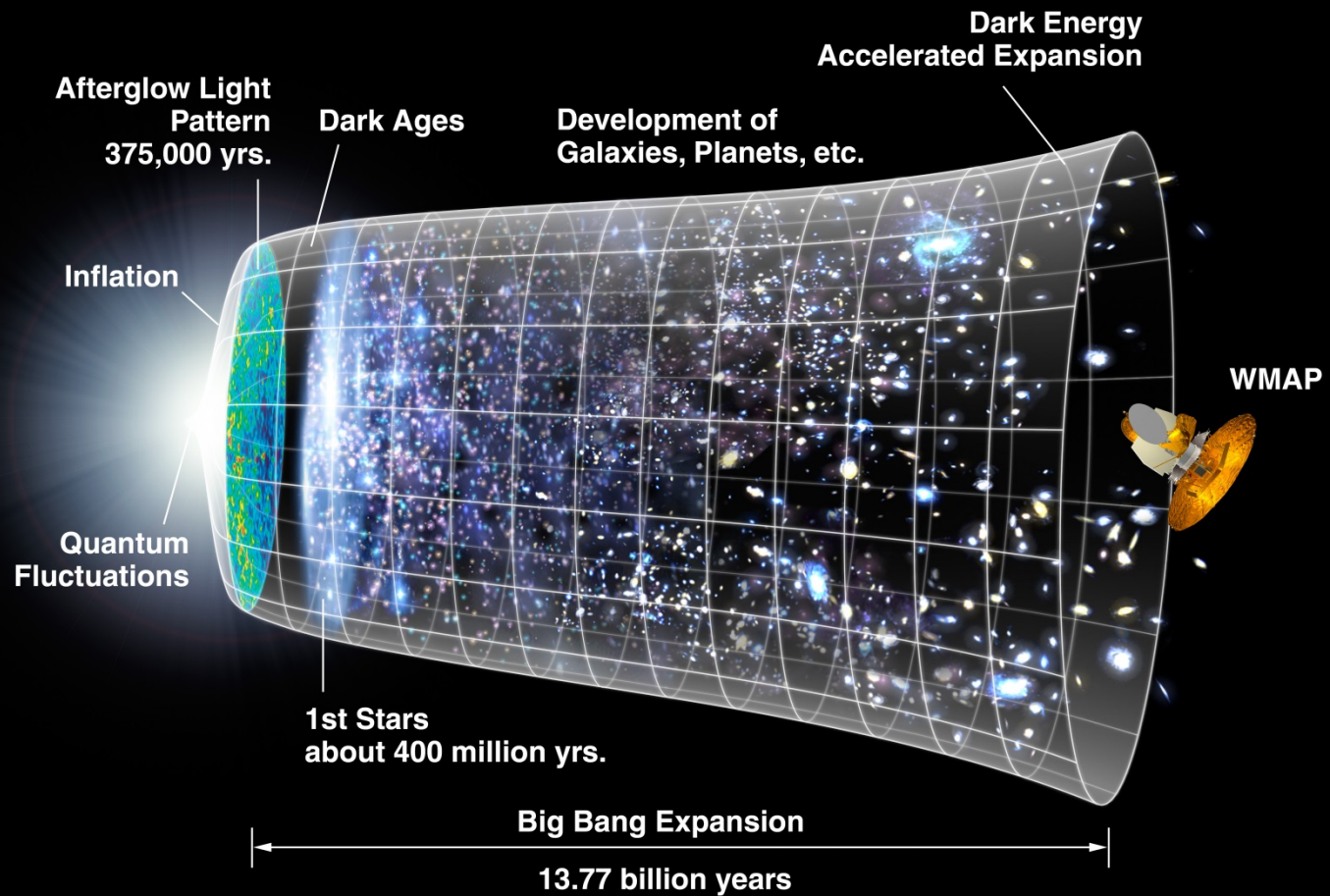
A man with dark, curly hair and a mustache, wearing a dark suit and a white shirt, is shown from the chest up. He is holding a handgun in his right hand, pointing it towards the right. The background is a plain, light-colored wall. The text "SAY 'PRECISION COSMOLOGY' ONE MORE TIME..." is overlaid in white, serif font on the right side of the image.

SAY "PRECISION
COSMOLOGY"
ONE MORE TIME...

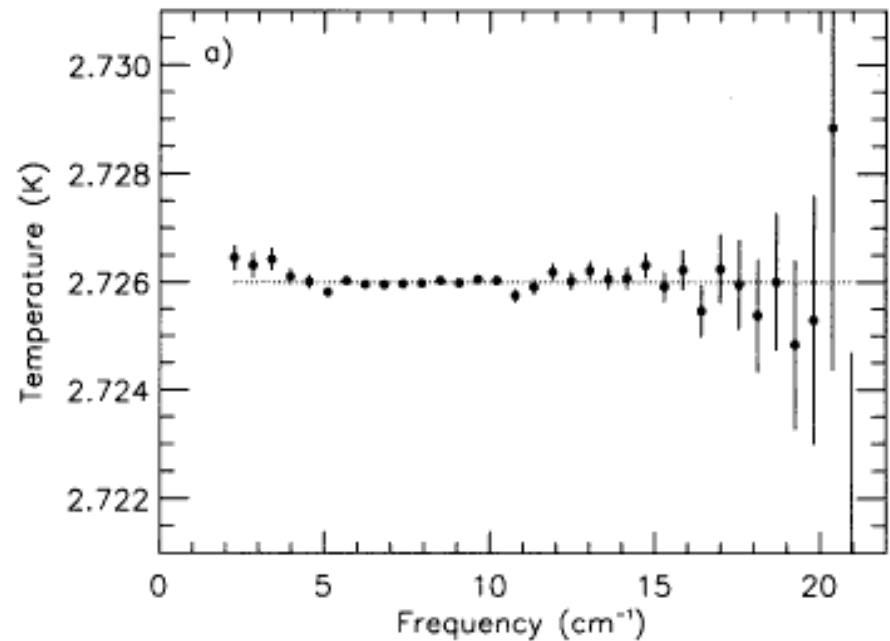
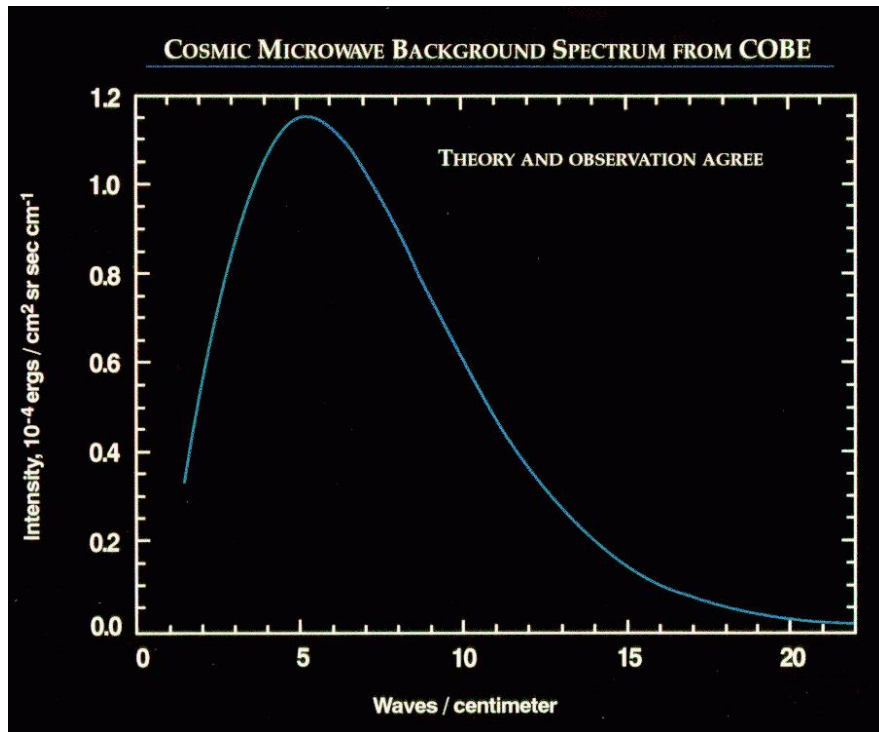
Outline

- Introduction and *basic* physics
- CMB temperature power spectrum and observables
 - Beyond the power spectrum?
 - Parameter estimation
 - Primordial perturbations
- CMB Polarization: E and B modes
 - CMB lensing (if time)





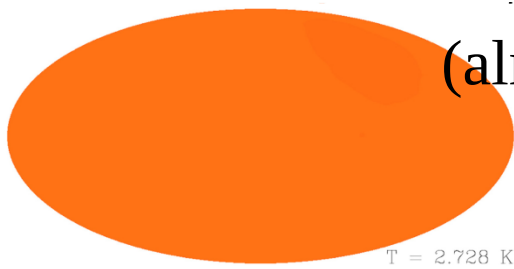
Black body spectrum observed by COBE



Residuals [Mather et al 1994](#)

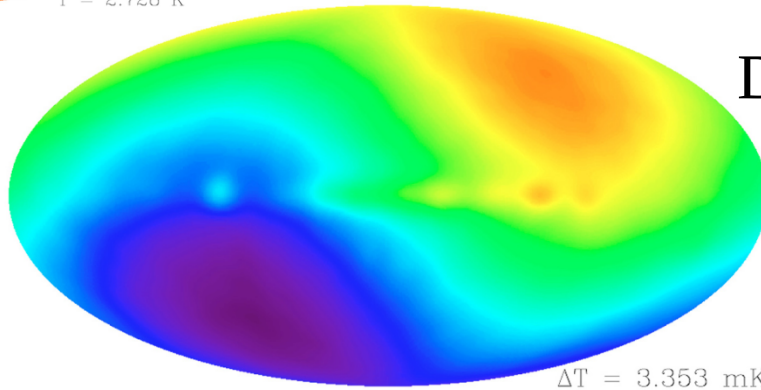
close to thermal equilibrium:
temperature today of 2.726K ($\sim 3000\text{K}$ at $z \sim 1000$ because $\nu \sim (1+z)$)

(almost) uniform 2.726K blackbody



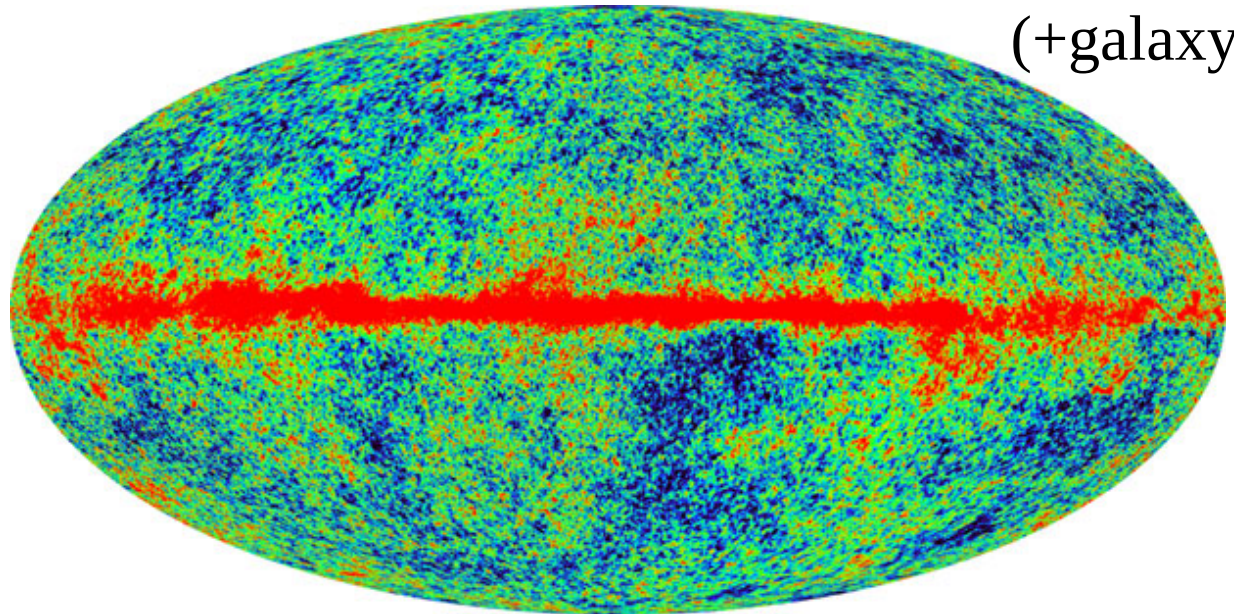
$T = 2.728 \text{ K}$

Dipole (local motion)



$\Delta T = 3.353 \text{ mK}$

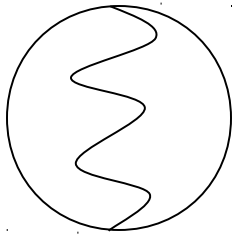
$O(10^{-5})$ perturbations
(+galaxy)



Observations:
the microwave
sky today

Can we predict the primordial perturbations?

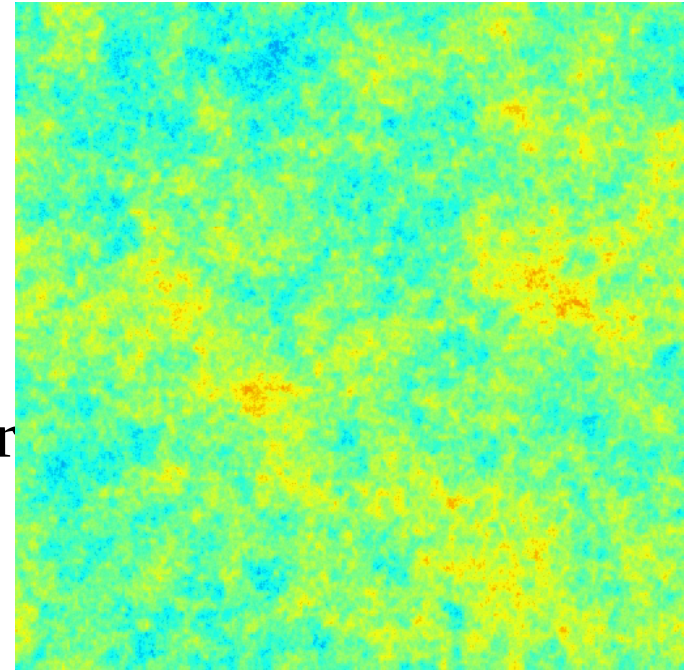
- Maybe..



Quantum Mechanics

“waves in a box” calculation
vacuum state, etc...

Inflation
make $>10^{30}$ times bigger



After inflation
Huge size, amplitude $\sim 10^{-5}$

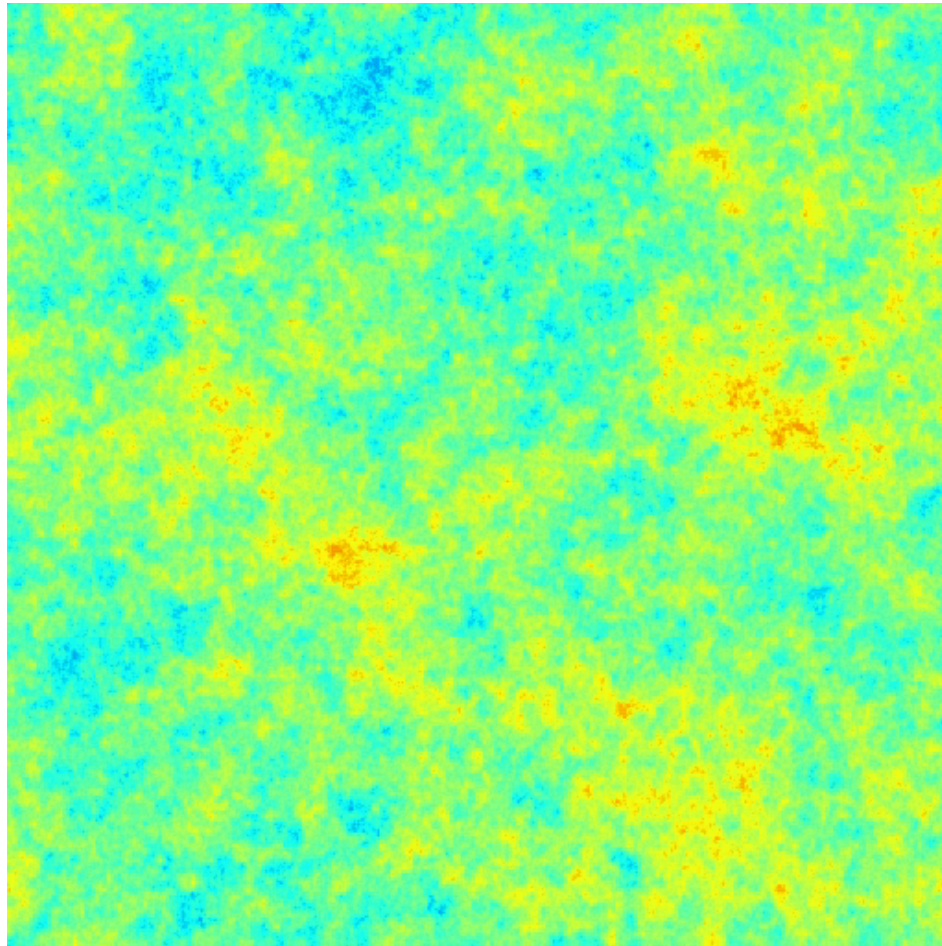


The Meaning of Inflation (OED)

1. The action of inflating or distending with air or gas
2. The condition of being inflated with air or gas, or being distended or swollen as if with air
3. The condition of being puffed up with vanity, pride or baseless notions
4. The quality of language or style when it is swollen with big or pompous words; turgidity, bombast

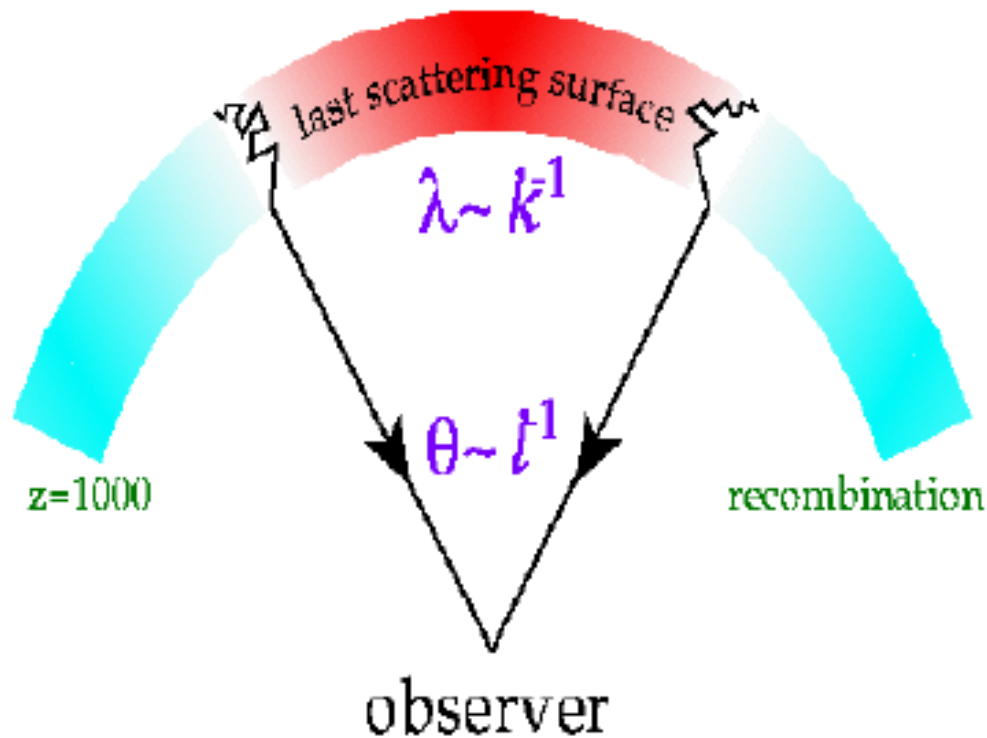
Perturbation evolution

photon/baryon plasma + dark matter, neutrinos



Characteristic scales: sound wave travel distance; diffusion damping length

Observed ΔT as function of angle on the sky



Theory of perturbation evolution

Physics Ingredients

- **Linear Physics:** Perturbations $\sim 10^{-5}$ so Fourier modes evolve independently
- **Thomson scattering** (non-relativistic electron-photon scattering)
 - tightly coupled before recombination: ‘tight-coupling’ approximation (baryons follow electrons because of very strong em coupling)
- **Background recombination physics** (full multi-level calculation)
- **Linearized General Relativity**
- **Boltzmann equation** (how angular distribution function evolves with scattering)

CMB power spectrum C_l


- Theory: Linear physics + Gaussian primordial fluctuations

$$a_{lm} = \int d\Omega \Delta T Y_{lm}^*$$

Theory prediction $C_l = \langle |a_{lm}|^2 \rangle$

- variance (average over all possible sky realizations)
- statistical isotropy implies independent of m

Initial conditions
+ cosmological parameters

linearized GR
+ Boltzmann equations 

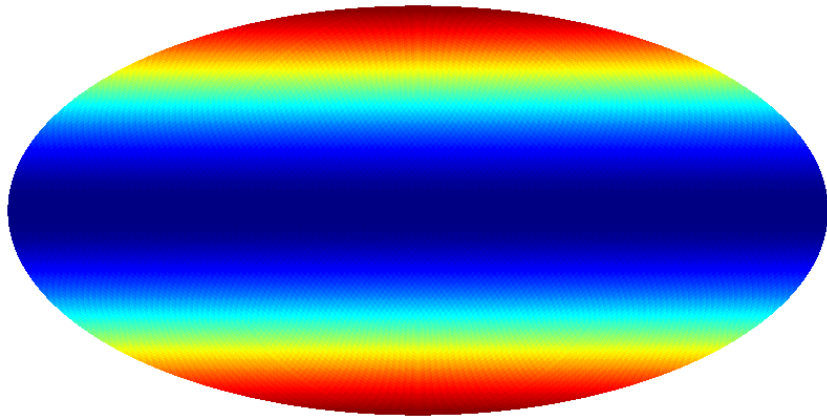
C_l

CMBFAST: cmbfast.org

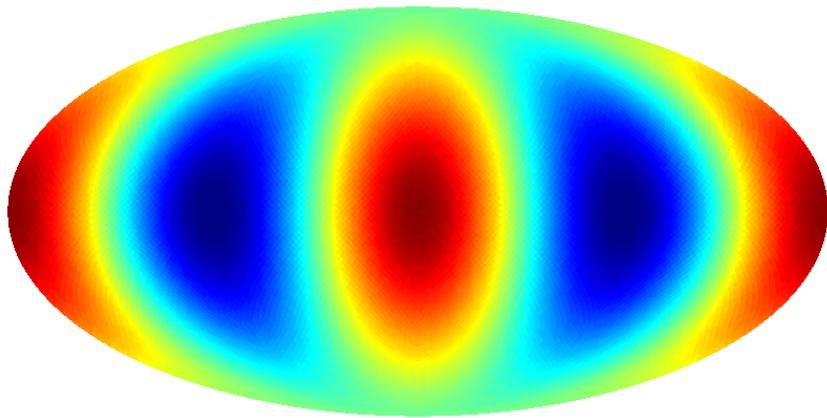
CAMB: camb.info

CMBEASY: cmbeasy.org

COSMICS, etc..

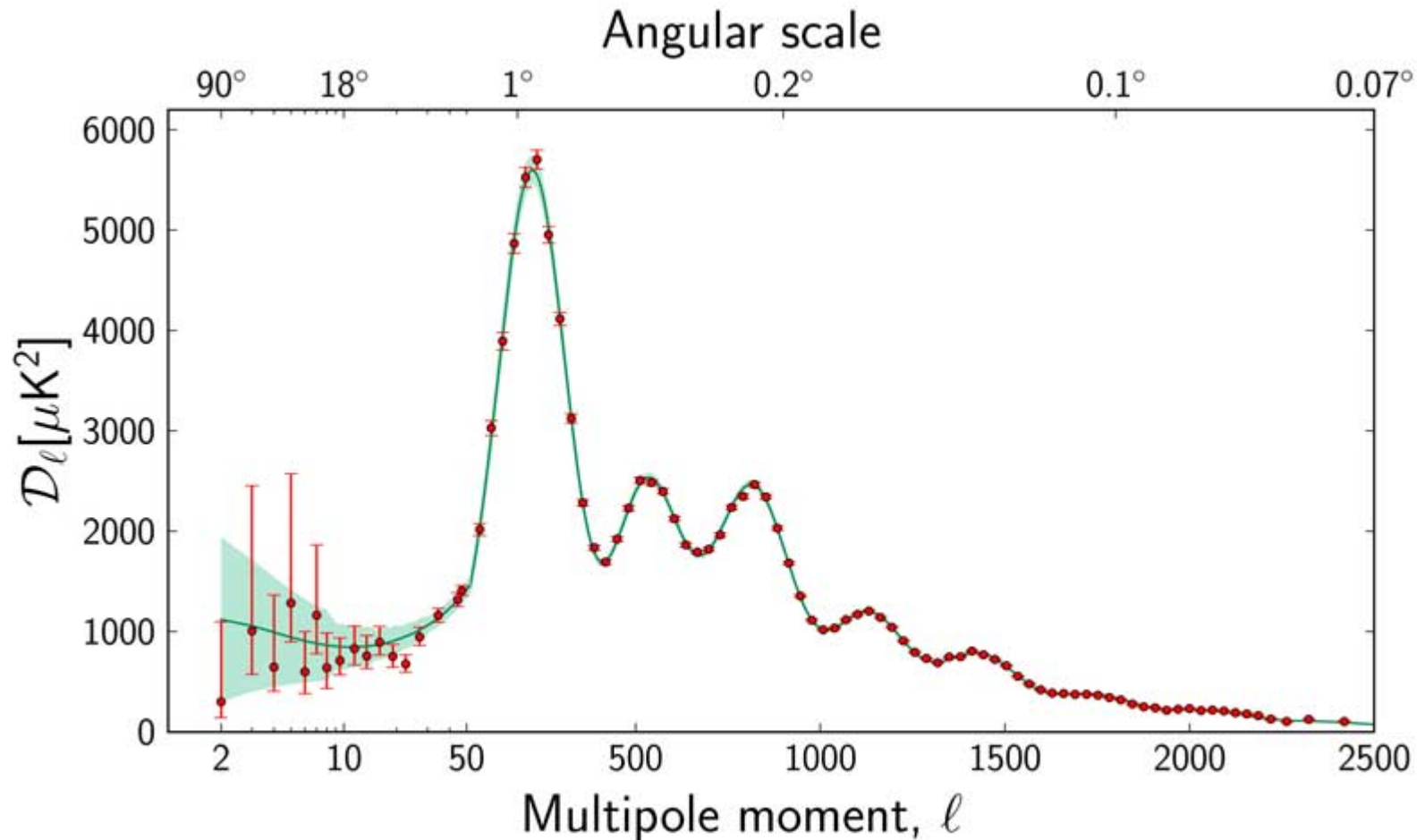


Zonal
($m=0$)

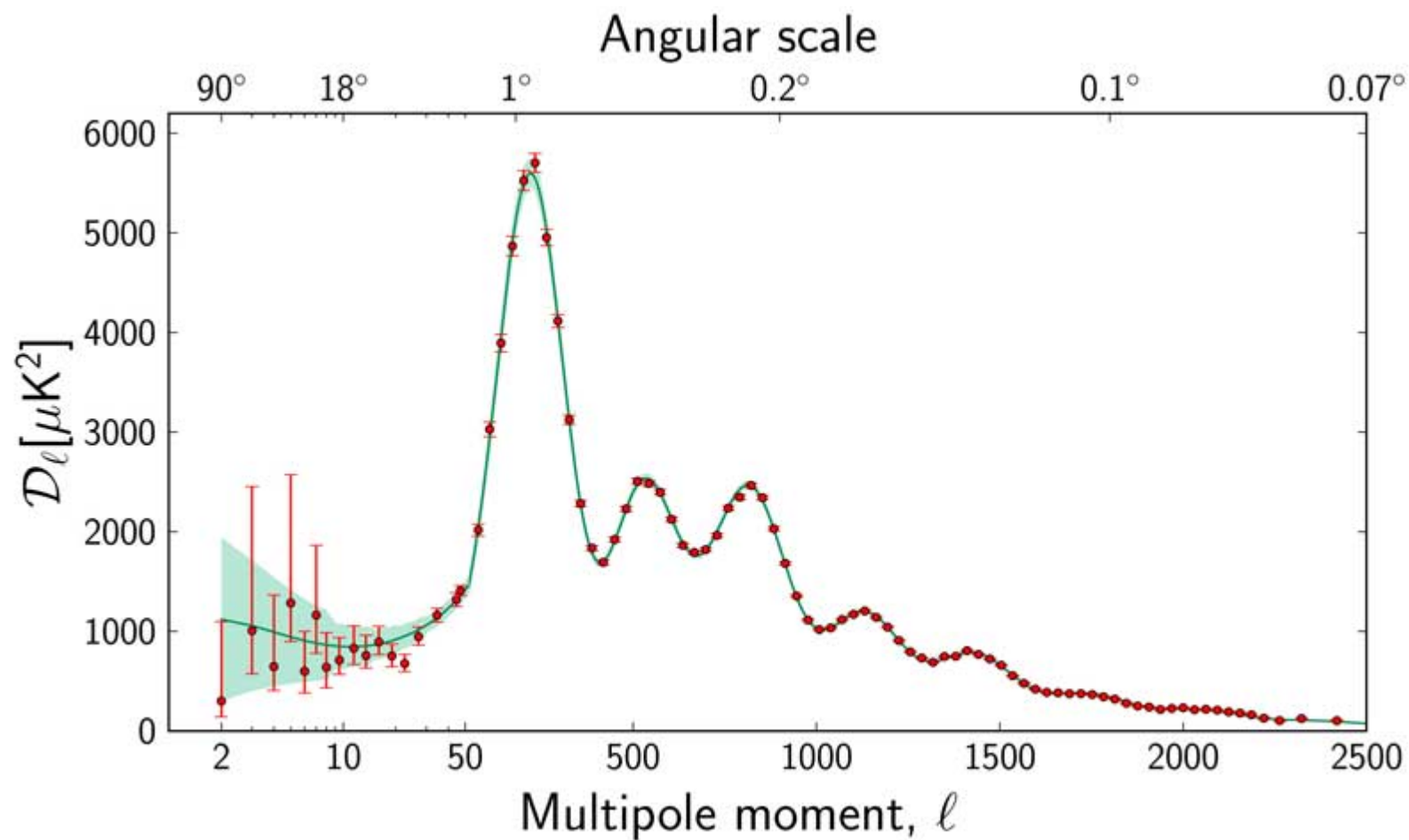


Sectoral
($m=1$)

Parity Violation?



Digression !



The diagram consists of two orange ovals. The top oval is labeled 'Theories' and the bottom oval is labeled 'Observations'. Between them are two orange arrows: one pointing up from 'Observations' to 'Theories' and one pointing down from 'Theories' to 'Observations'. To the left of the upward arrow is the word 'Bayesian' and to the right of the downward arrow is the word 'Frequentist'.

Theories

Bayesian

Frequentist

Observations

**Cosmology is a massive
exercise in data
compression...**

**....but it is worth looking at
the information that has
been thrown away to check
that it makes sense!**

Beyond the Power Spectrum

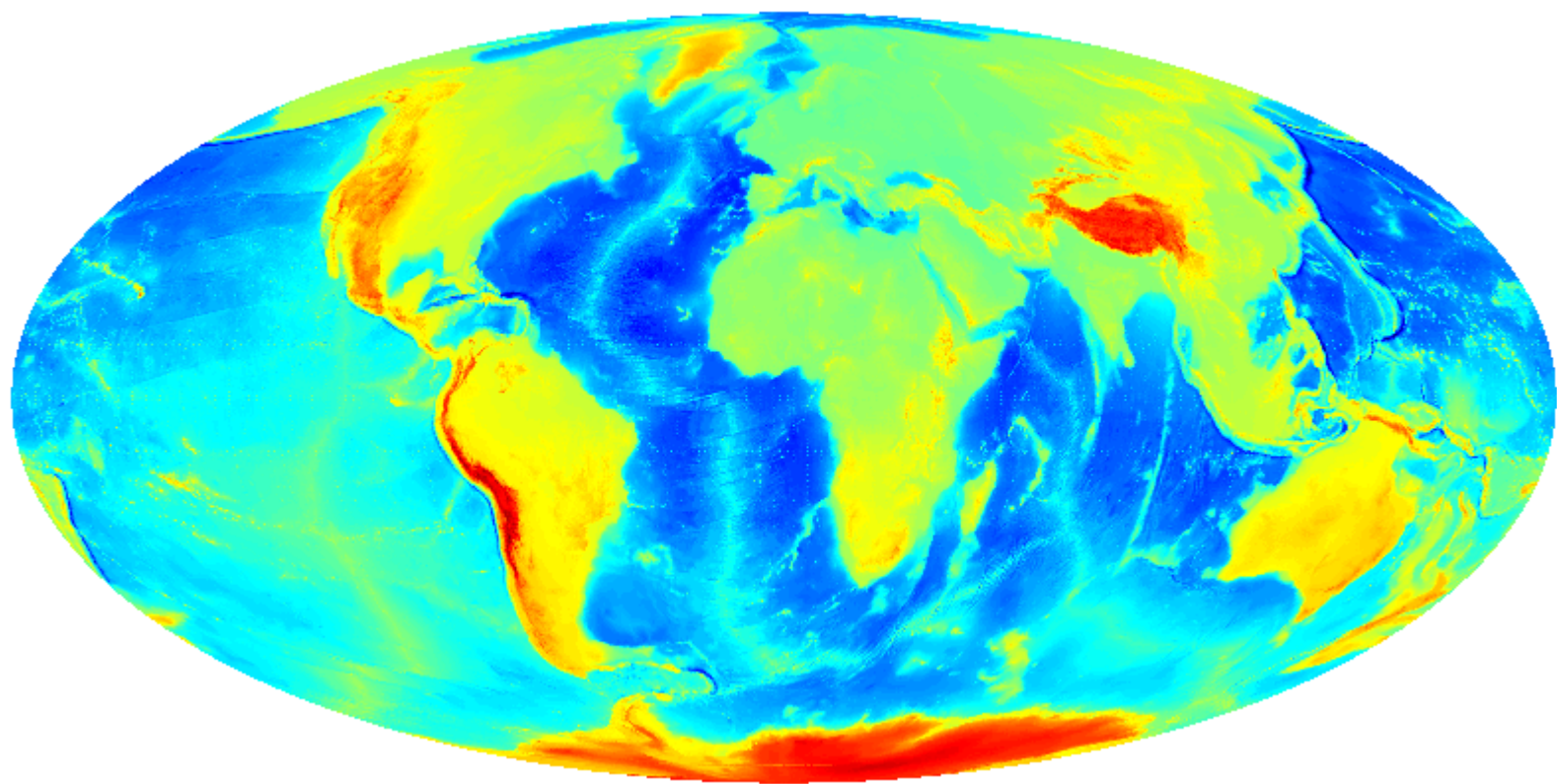
- So far what we have discovered is largely based on second-order statistics...
- This is fine as long as we don't throw away important clues...
- ..ie if the fluctuations are statistically homogeneous and isotropic, and Gaussian..

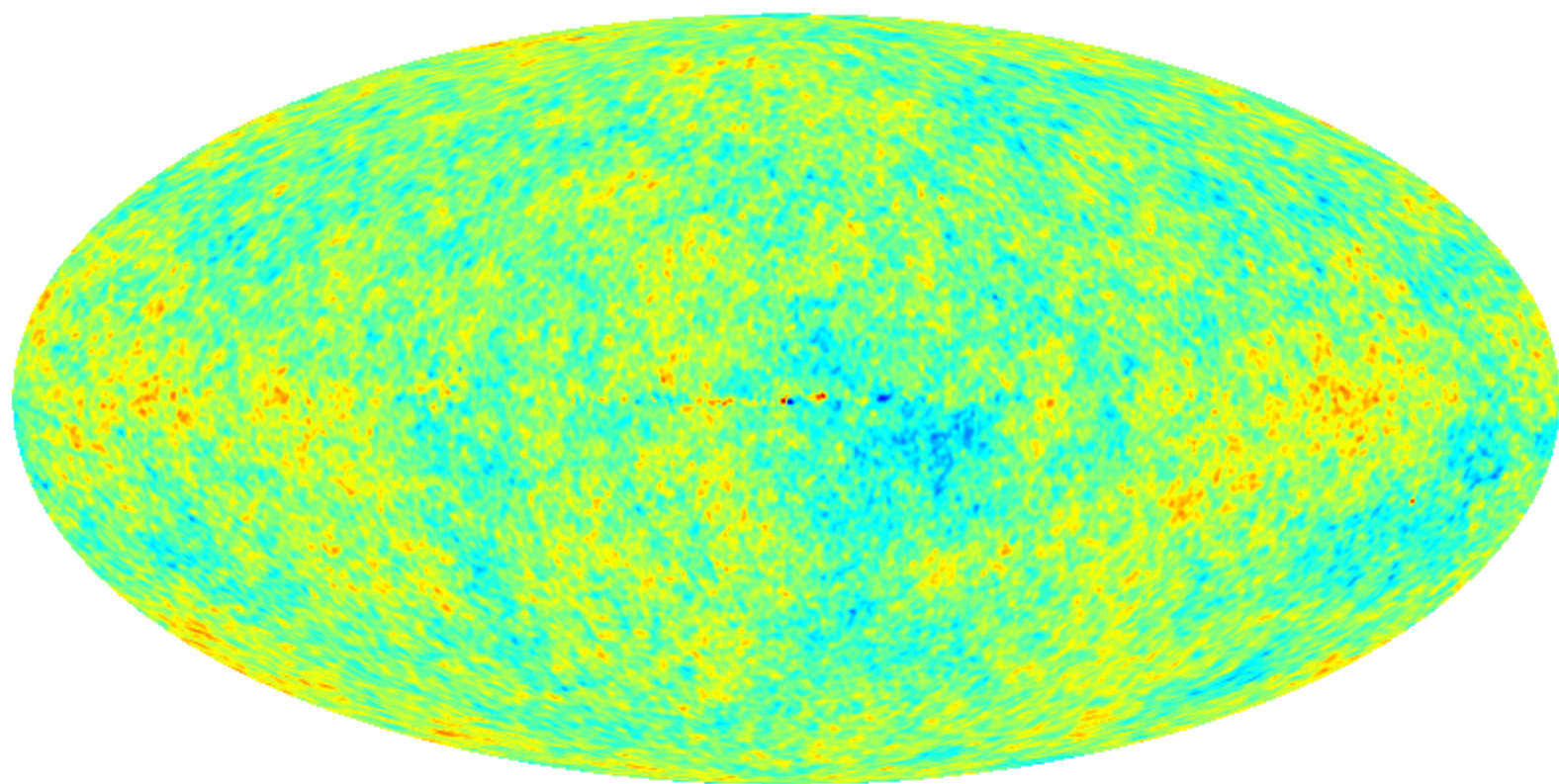
Weirdness in Phases

$$\frac{\Delta T(\theta, \varphi)}{T} = \sum \sum a_{l,m} Y_{lm}(\theta, \varphi)$$


$$a_{l,m} = |a_{l,m}| \exp[i\phi_{l,m}]$$

For a homogeneous and isotropic Gaussian random field (on the sphere) the **phases** are independent and uniformly distributed. Non-random phases therefore indicate weirdness..

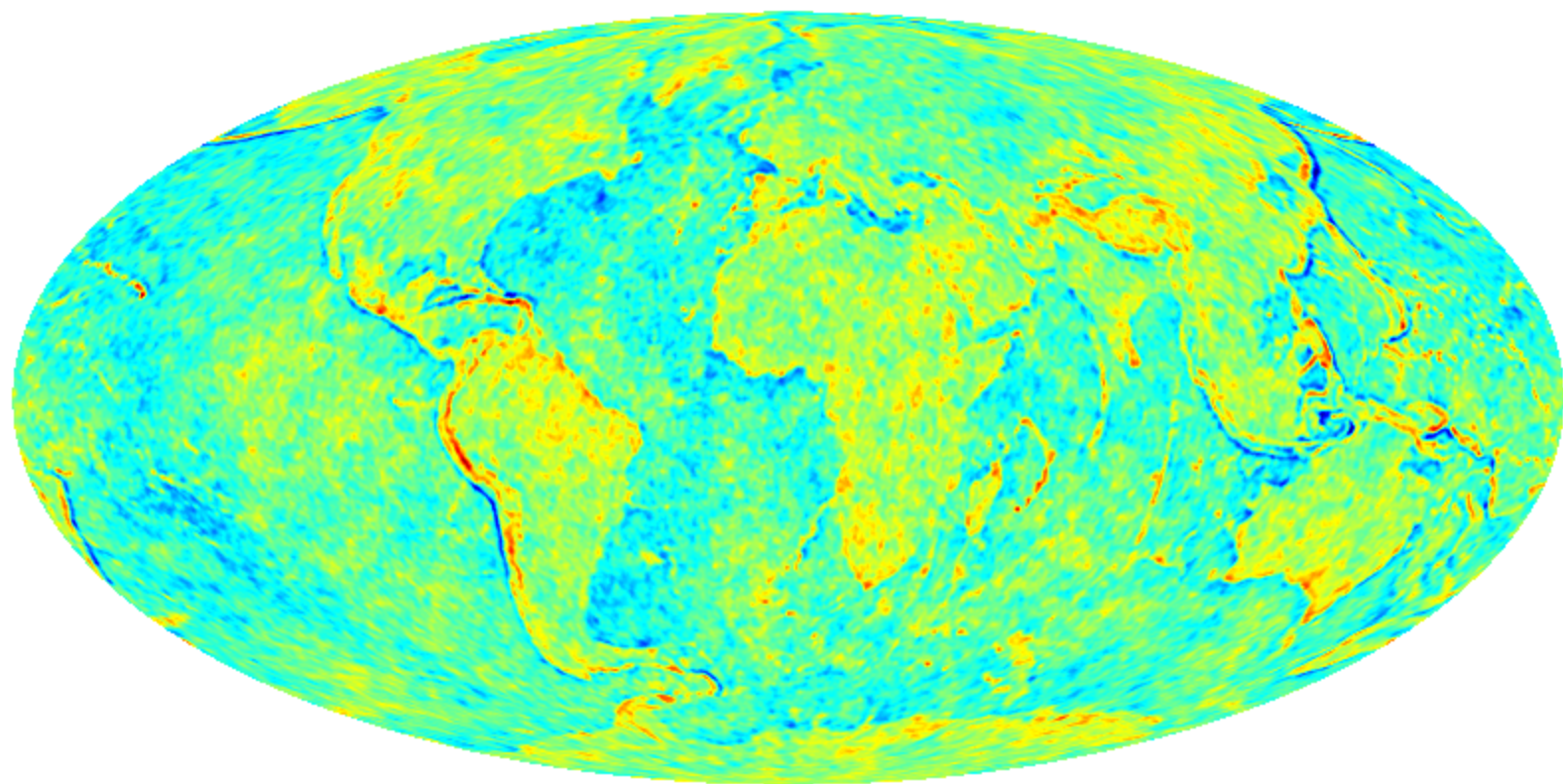




-0.593483



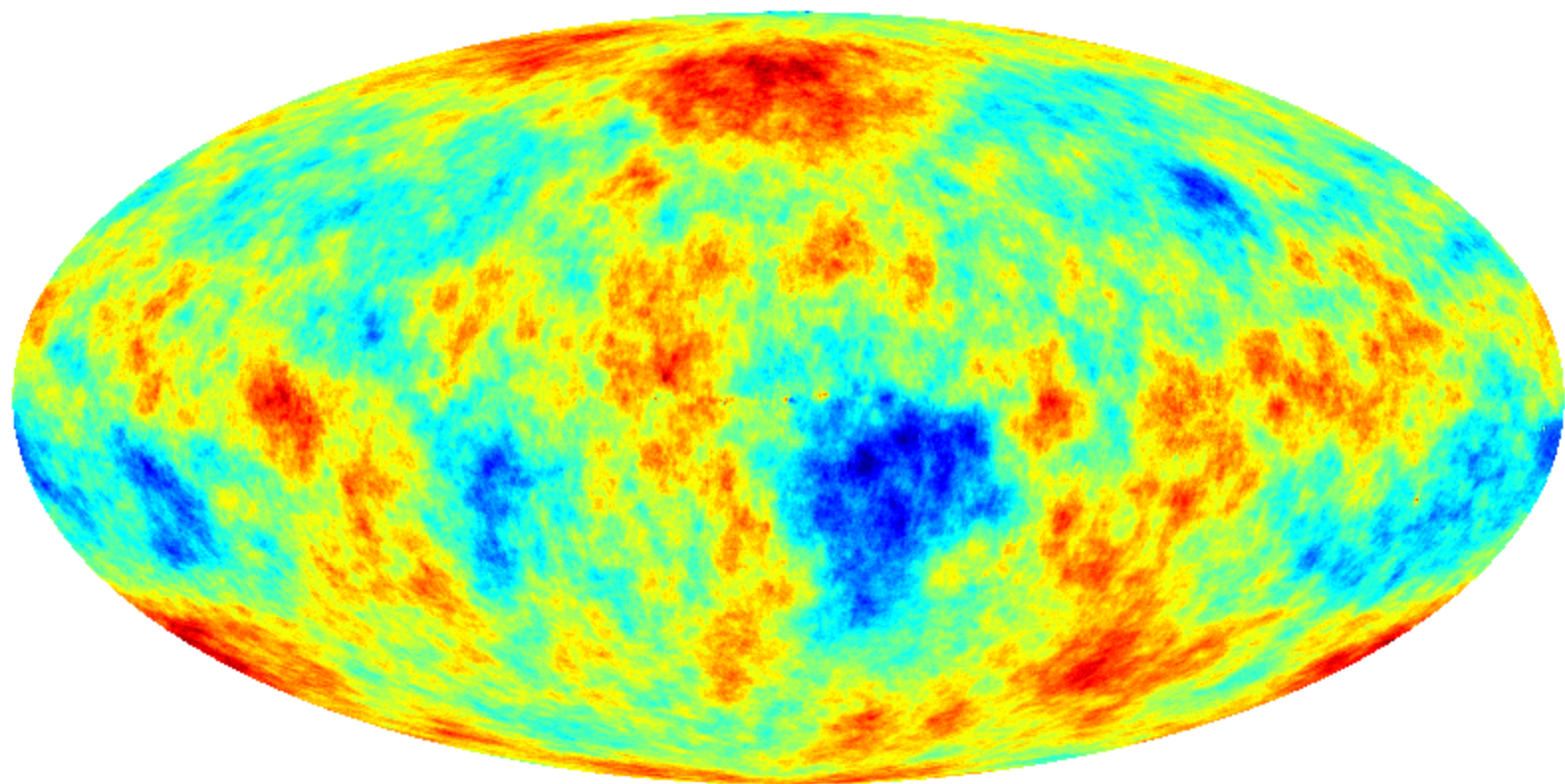
+0.530173



-0.446332



+0.450228

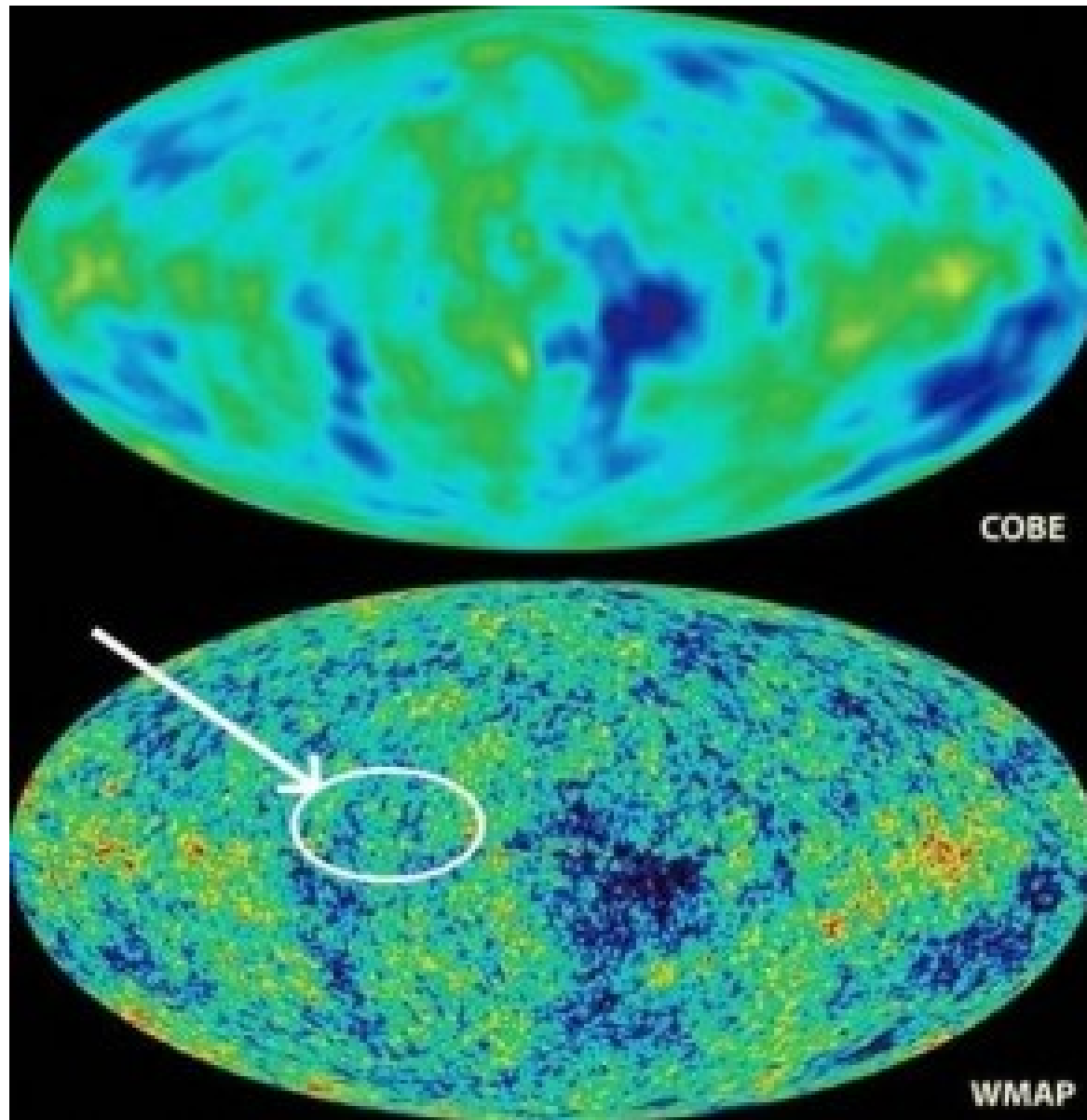


“If tortured sufficiently, data
will confess to almost
anything”

Fred Menger

Beware the Prosecutor's Fallacy!

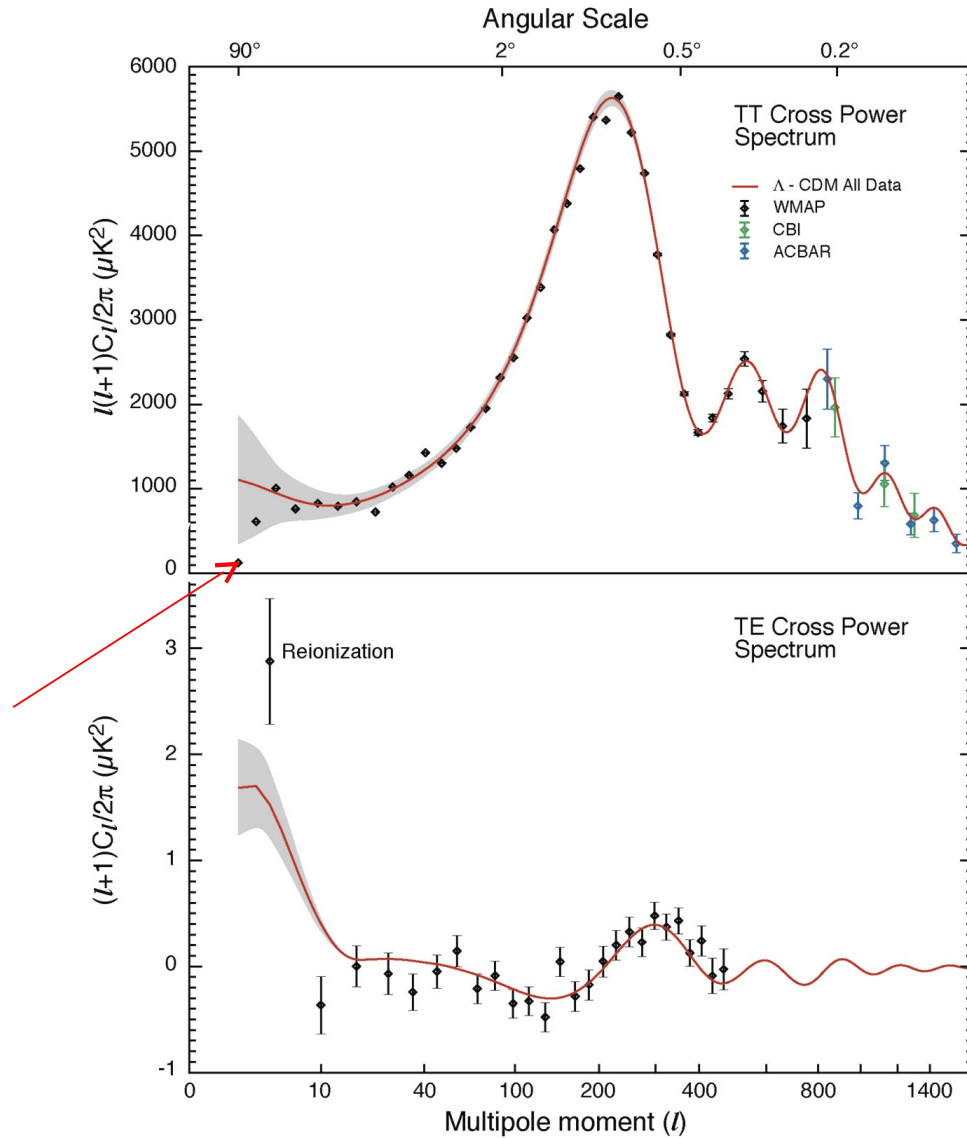
$$P(A|M) \neq P(M|A)$$



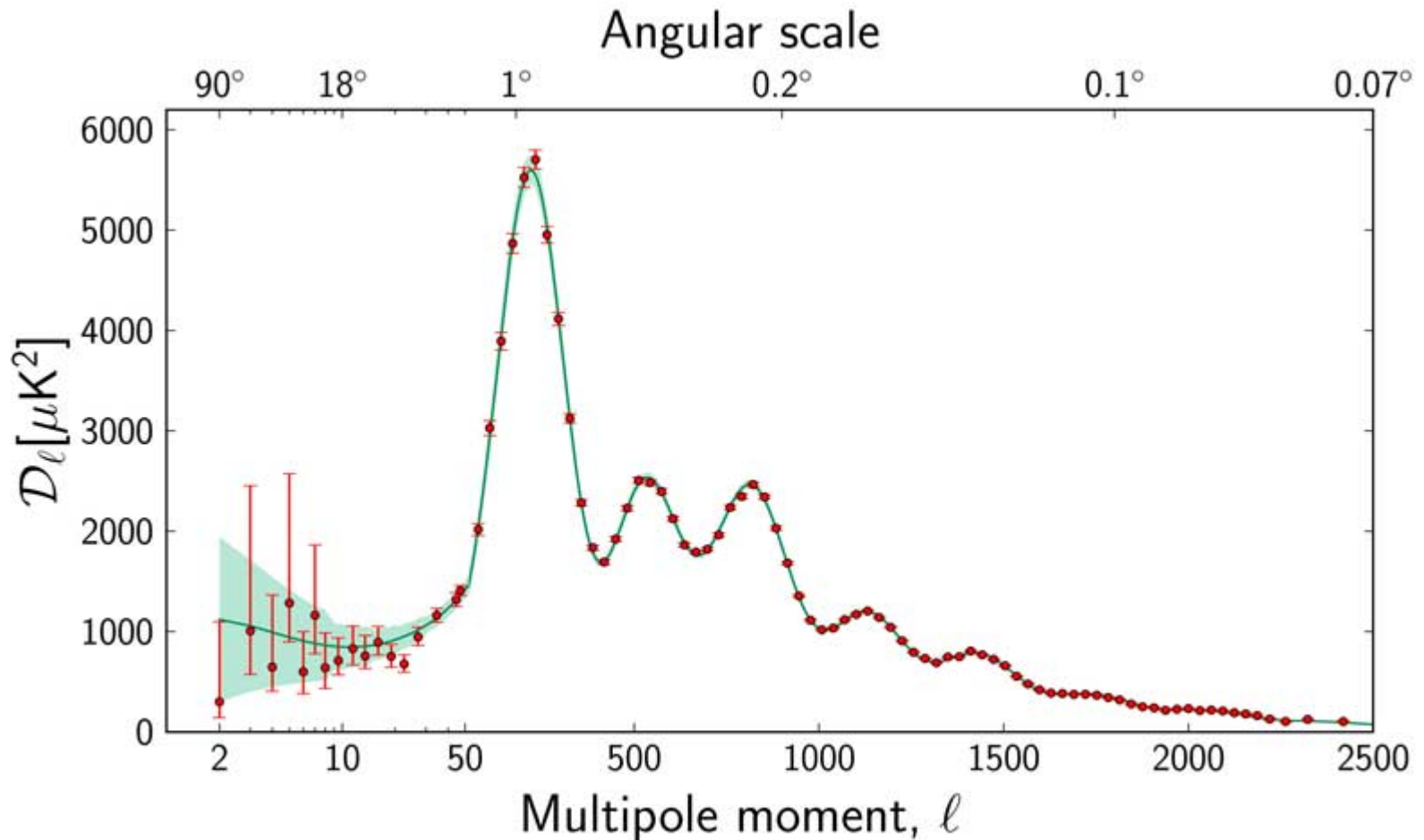
CMB Anomalies

- Type I – obvious problems with data (e.g. foregrounds)
- Type II – anisotropies and alignments (North-South, Axis of Evil..)
- Type III – localized features, e.g. “The Cold Spot”
- Type IV – Something else (even/odd multipoles, magnetic fields, ?)

Low Quadrupole?

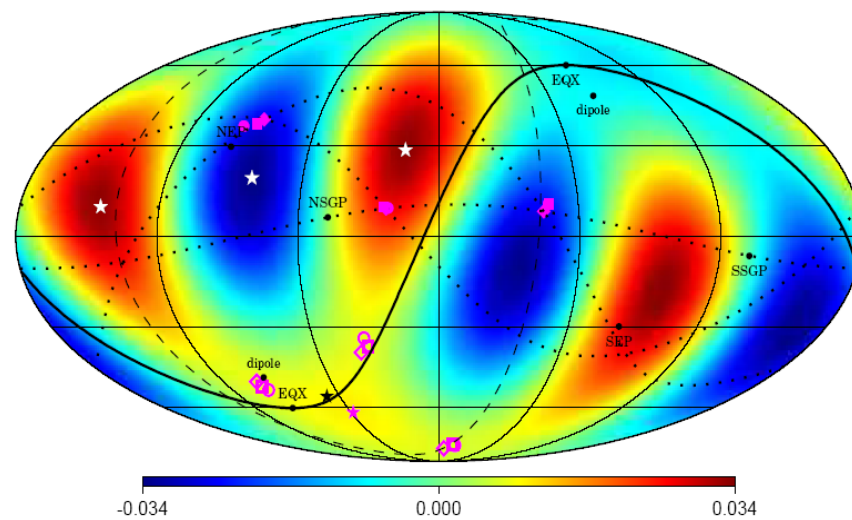
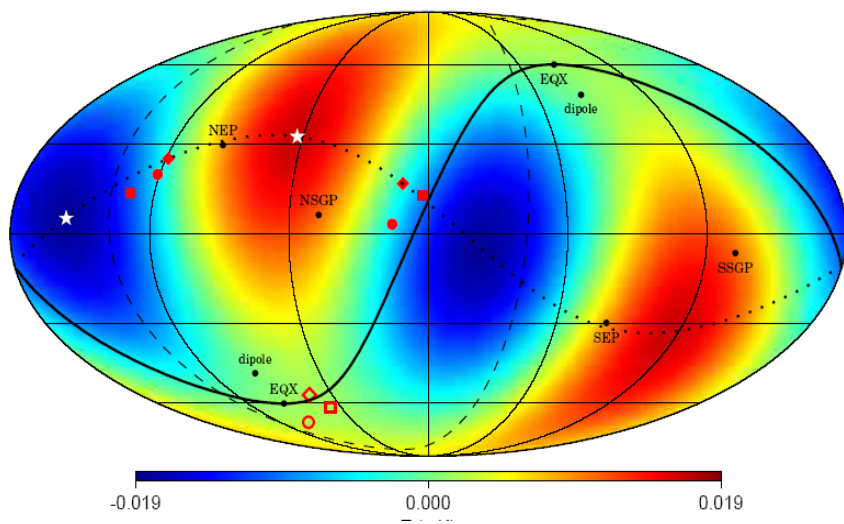


Parity Violation?



The *Axle* of Elvis

The *Axis* of Evil



(from Copi et al. 2005)

Maximum asymmetry positions

(from Hansen et al. 2004)

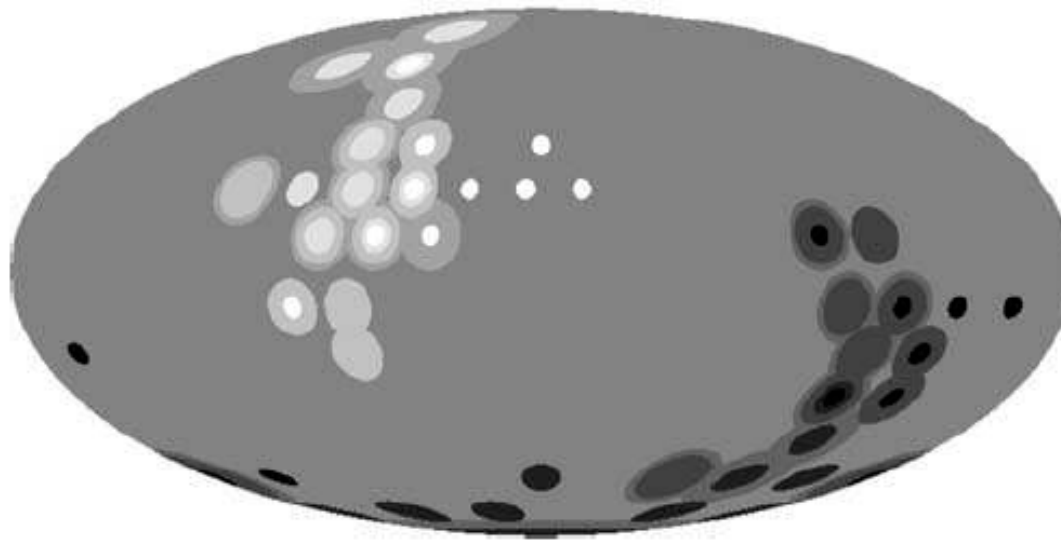
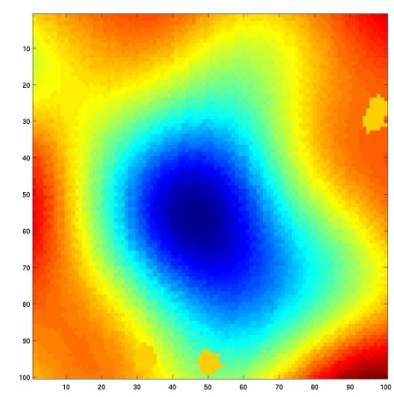
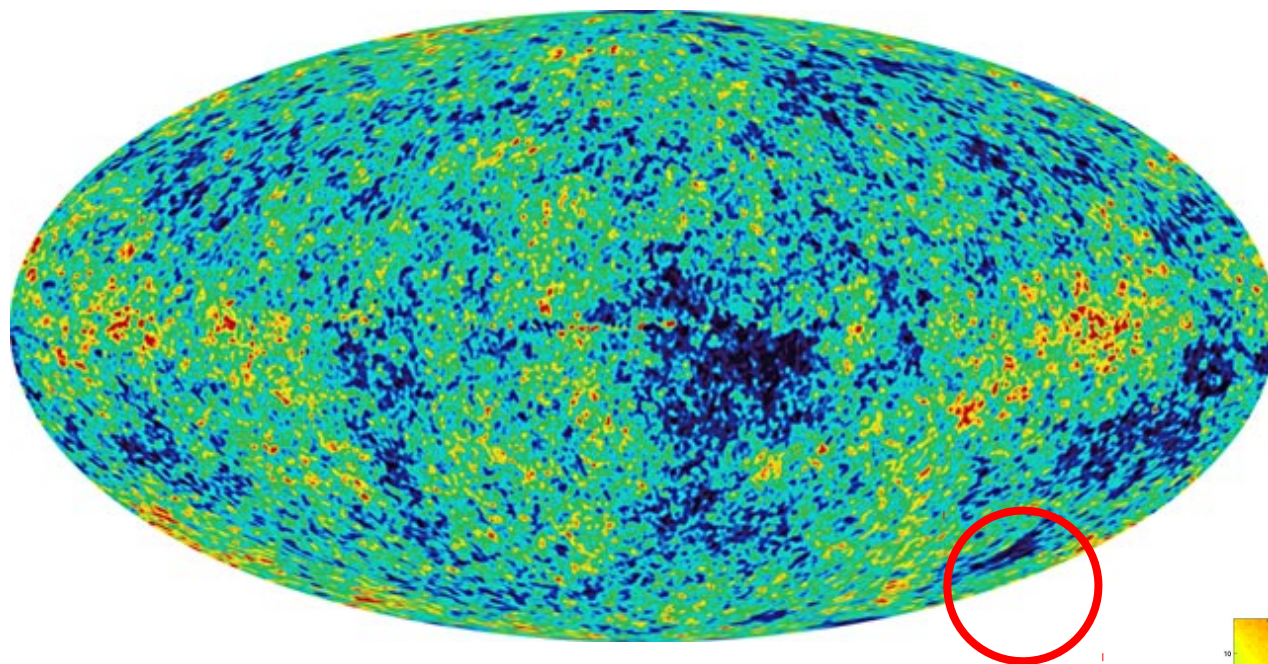
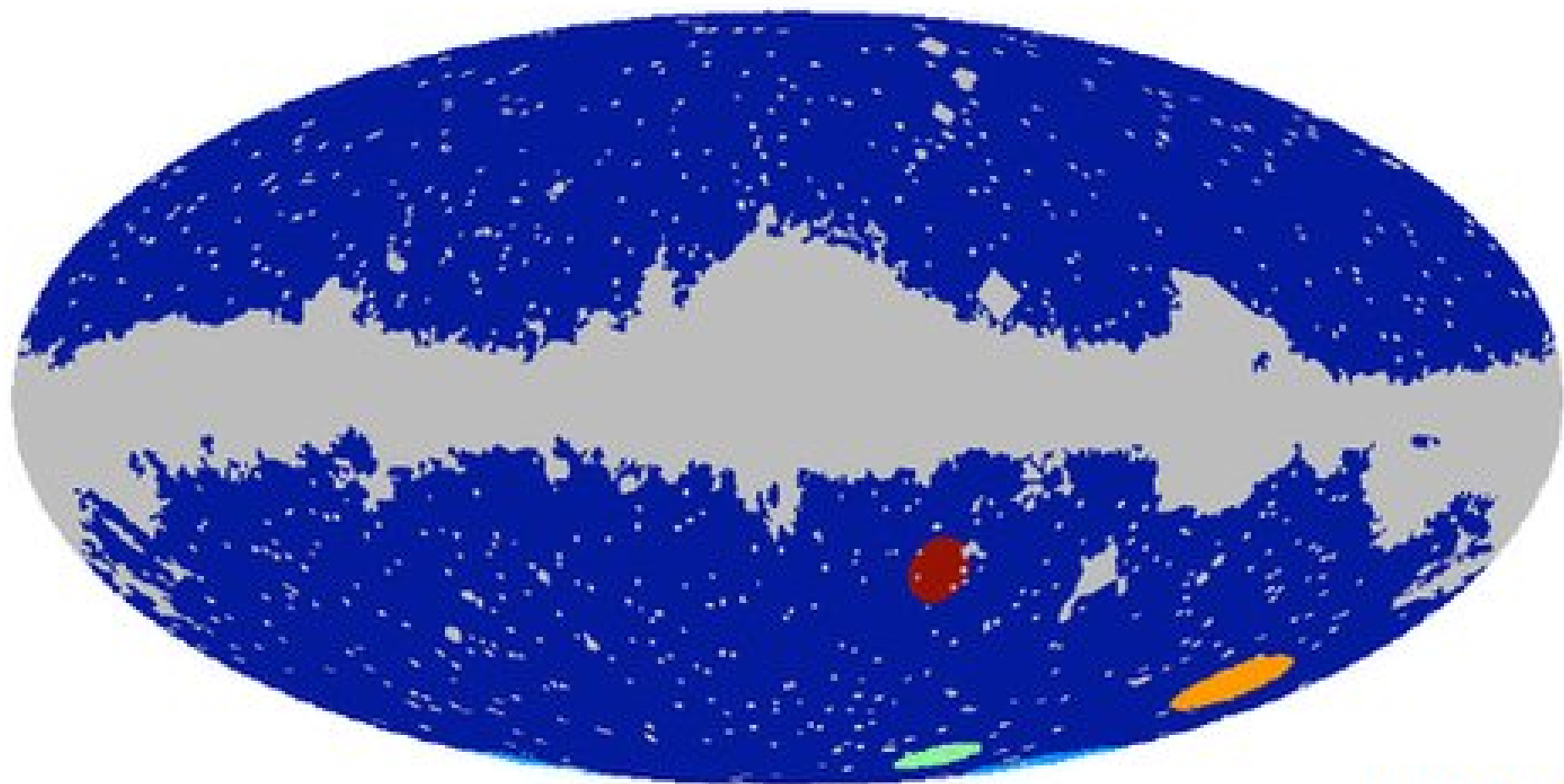


Figure 24. The discs show the positions of the hemispheres with the 10 highest (black discs) and 10 lowest (white discs) bin values. The power-spectrum bins considered were $\ell = 2-40$ (large discs), $\ell = 8-40$ (second-largest discs), $\ell = 5-16$ (second-smallest discs) and $\ell = 29-40$ (smallest discs).





FEENEY ET AL

The hottest hotspot on the microwave sky

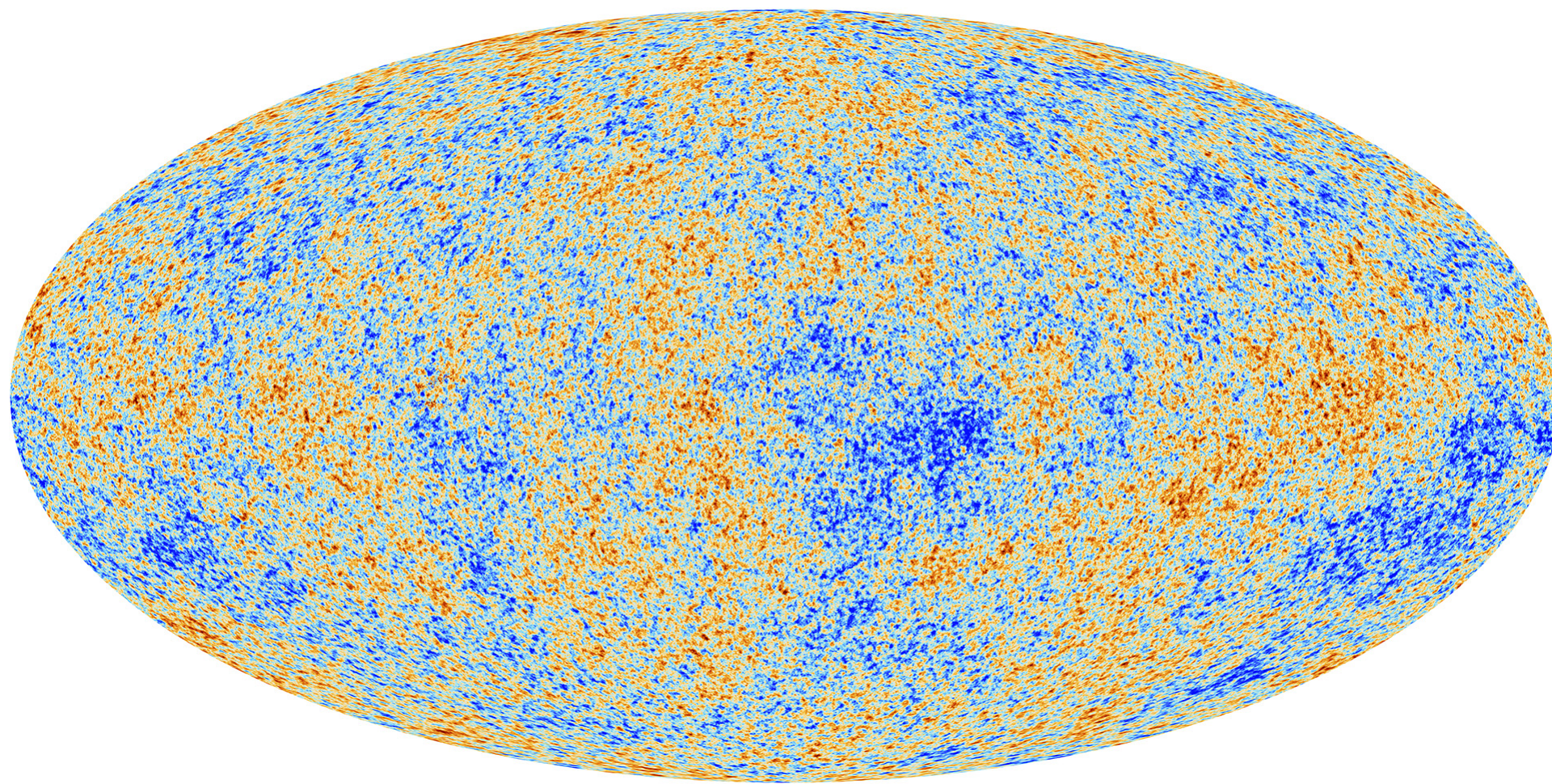
Peter Coles *Astronomy Centre, University of Sussex, Brighton BN1 9QH*

Accepted 1987 September 18. Received 1987 August 20; in original form 1987 July 13

Summary. Approximate confidence limits on the temperature of the hottest hotspot expected on the microwave sky are derived under the condition that the coherence angle of the temperature fluctuations is a very small fraction of the total sky. We apply the result to temperature anisotropies expected in galaxy formation models where the universe is dominated by cold dark matter and discuss its possible use for discriminating between high peaks in system noise and true sky fluctuations.

1 Introduction

Anisotropies of the cosmic microwave background provide one of the few direct tests of theories of galaxy formation. Theoreticians have predicted the rms amplitudes and covariance functions of the fluctuations produced by various models (Bond & Efstathiou 1984, Bond, Efstathiou & Silk 1981; Wilson & Silk 1980) and more recently, using techniques developed for the study of biased galaxy formation, have examined more detailed statistical properties of temperature fluctuations (Sazhin 1985; Zabotin & Nasel'skii 1985; Bond & Efstathiou 1987; Vittorio & Juszkiewicz 1987; Coles & Barrow 1987; Coles 1987, in preparation). For the most part, these analyses concentrate on *local* properties of the microwave sky such as the mean number density



A. There's no problem at all with Λ CDM...

B. There are interesting indications...

C. There's definitely evidence of new physics

Sources of CMB anisotropy

Sachs Wolfe: Potential wells at last scattering cause redshifting as photons climb out

Photon density perturbations: Overdensities of photons look hotter

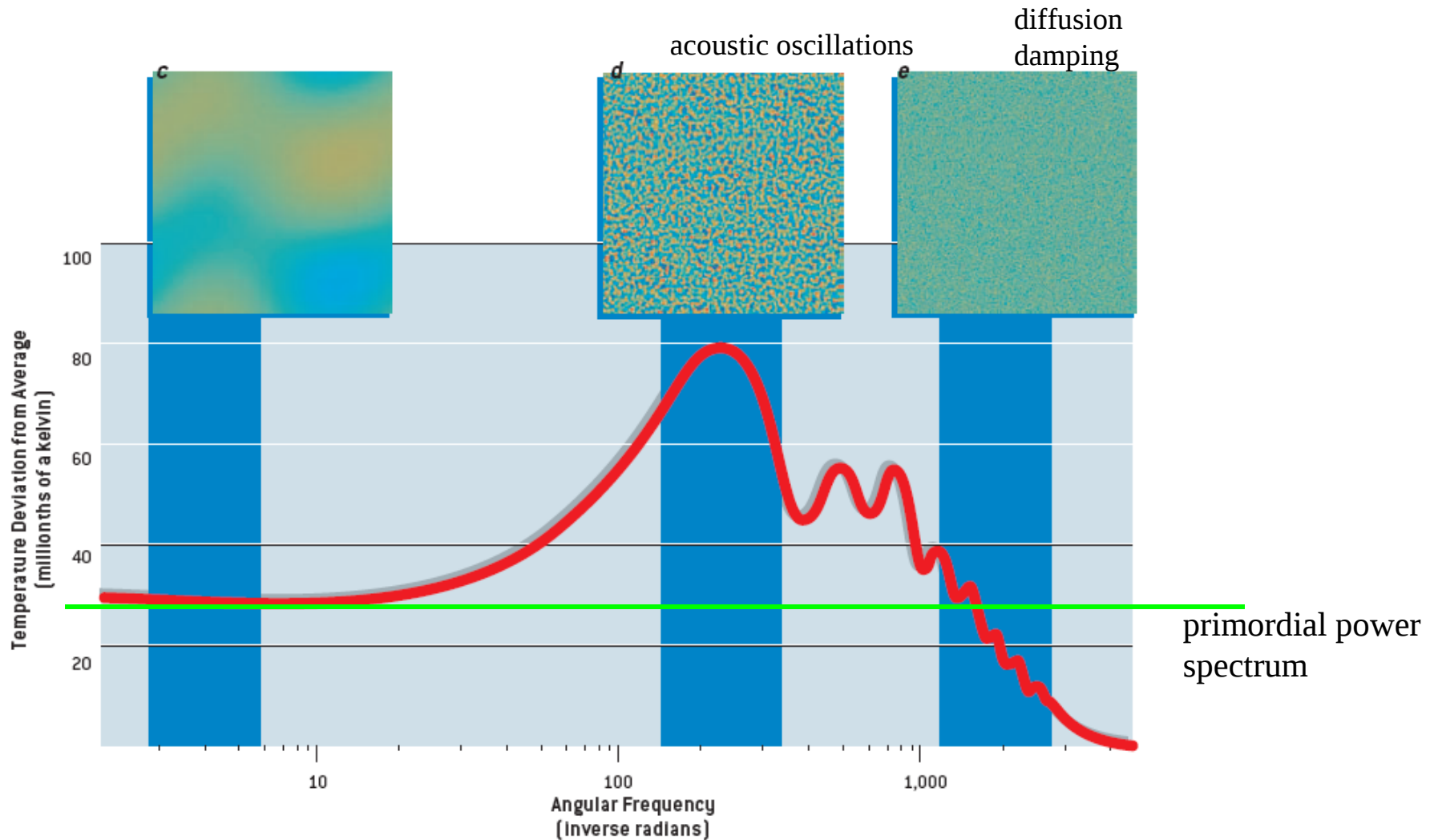
Doppler: Velocity of photon/baryons at last scattering gives Doppler shift

Integrated Sachs Wolfe: Evolution of potential along photon line of sight: net red- or blue-shift as photon climbs in and out of varying potential wells

Others: Photon quadrupole/polarization at last scattering, second-order effects, etc.

CMB temperature power spectrum

Primordial perturbations + later physics



finite thickness

Why The Wiggles?

Think in k-space: modes of different size

- Co-moving Poisson equation: $(k/a)^2 \Phi = \kappa \delta\rho / 2$
- potentials approx constant on super-horizon scales
- radiation domination $\rho \sim 1/a^4$

$$\rightarrow \delta\rho/\rho \sim k^2 a^2 \Phi$$

\rightarrow since $\Phi \sim \text{constant}$, super-horizon density perturbations grow $\sim a^2$



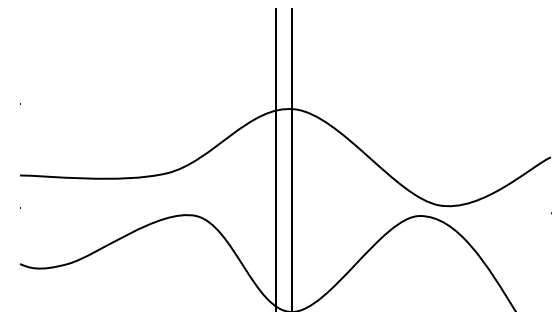
- After entering horizon pressure important: perturbation growth slows, then bounces back

\rightarrow series of acoustic oscillations (sound speed $\sim c/\sqrt{3}$)



- CMB anisotropy (mostly) from a surface at fixed redshift: phase of oscillation at time of last scattering depends on time since entering the horizon

\rightarrow k-dependent oscillation amplitude in the observed CMB



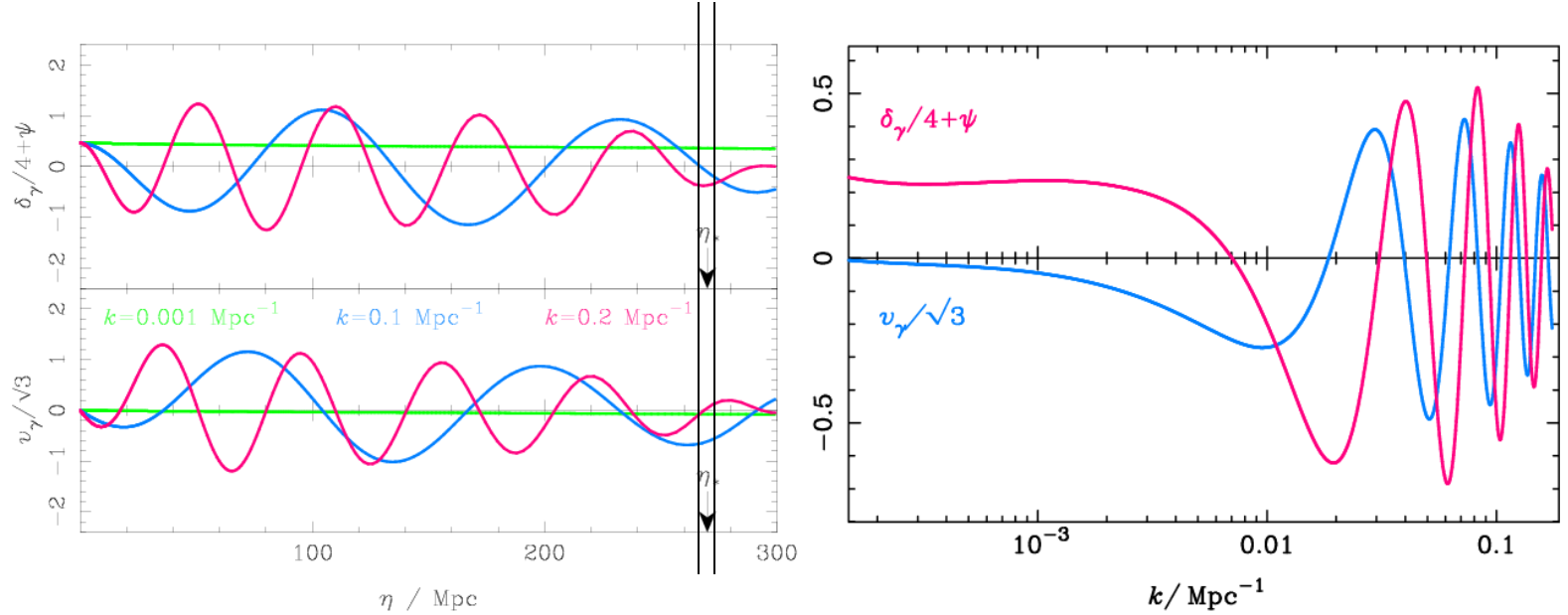
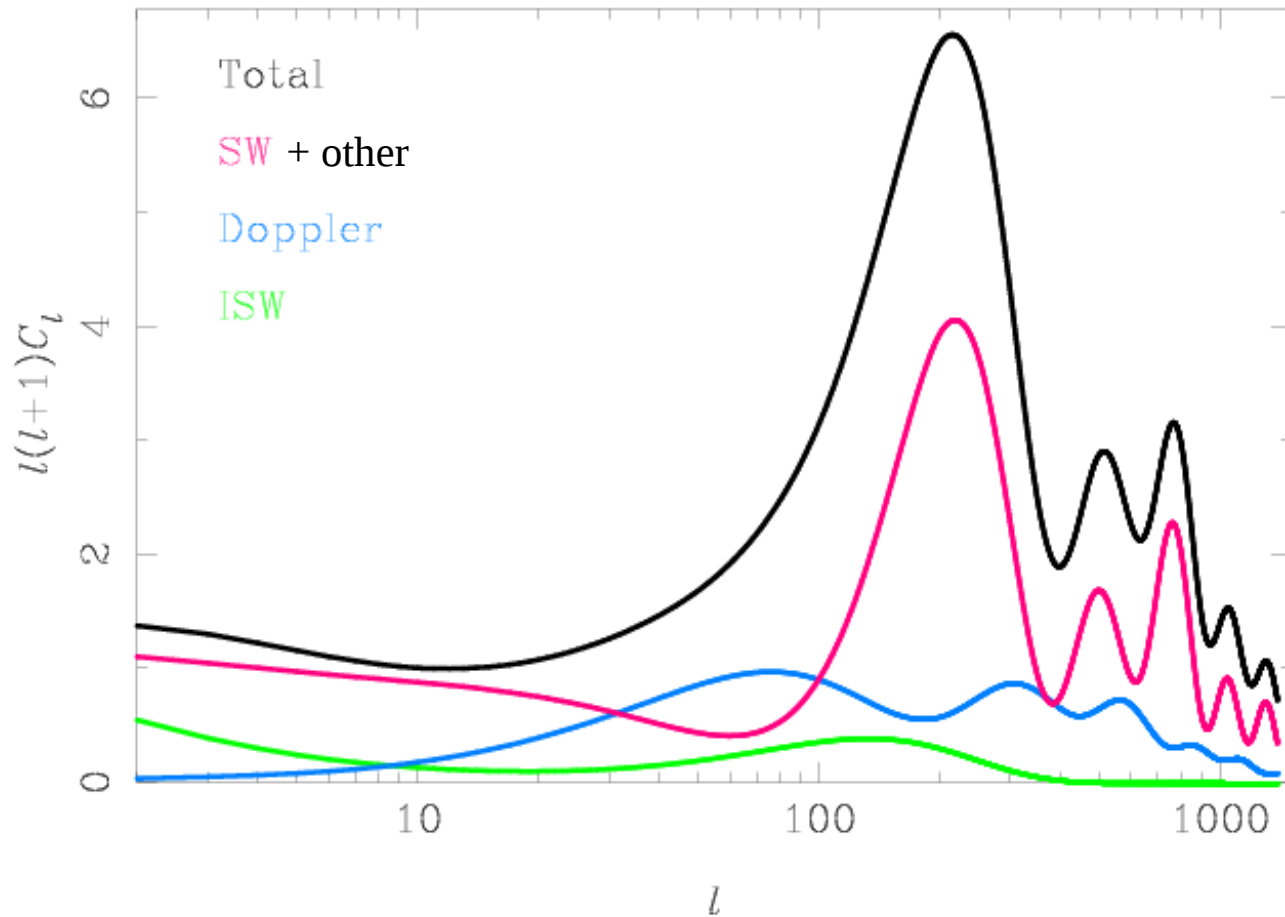


Fig. 3. Evolution of the combination $\delta_\gamma/4 + \psi$ (top left) and the photon velocity v_γ (bottom left) which determine the temperature anisotropies produced at last scattering (denoted by the arrow at η_*). Three modes are shown with wavenumbers $k = 0.001, 0.1$ and 0.2 Mpc^{-1} , and the initial conditions are adiabatic. The fluctuations at the time of last scattering are shown as a function of linear scale in the right-hand plot.

Contributions to temperature C_l



What can we learn from the CMB?

- Initial conditions

What types of perturbations, power spectra, distribution function (Gaussian?); => learn about inflation or alternatives.
(distribution of ΔT ; power as function of scale; polarization and correlation)

- What and how much stuff

Matter densities (Ω_b , Ω_{cdm}); neutrino mass
(details of peak shapes, amount of small scale damping)

- Geometry and topology

global curvature Ω_k of universe; topology
(angular size of perturbations; repeated patterns in the sky)

- Evolution

Expansion rate as function of time; reionization
- Hubble constant H_0 ; dark energy evolution w = pressure/density
(angular size of perturbations; $l < 50$ large scale power; polarization)

- Astrophysics

S-Z effect (clusters), foregrounds, etc.

Cosmic Variance

Use estimator for variance: $C_l^{obs} = \frac{1}{2l+1} \sum_m |a_{lm}|^2$

Assume a_{lm} gaussian:

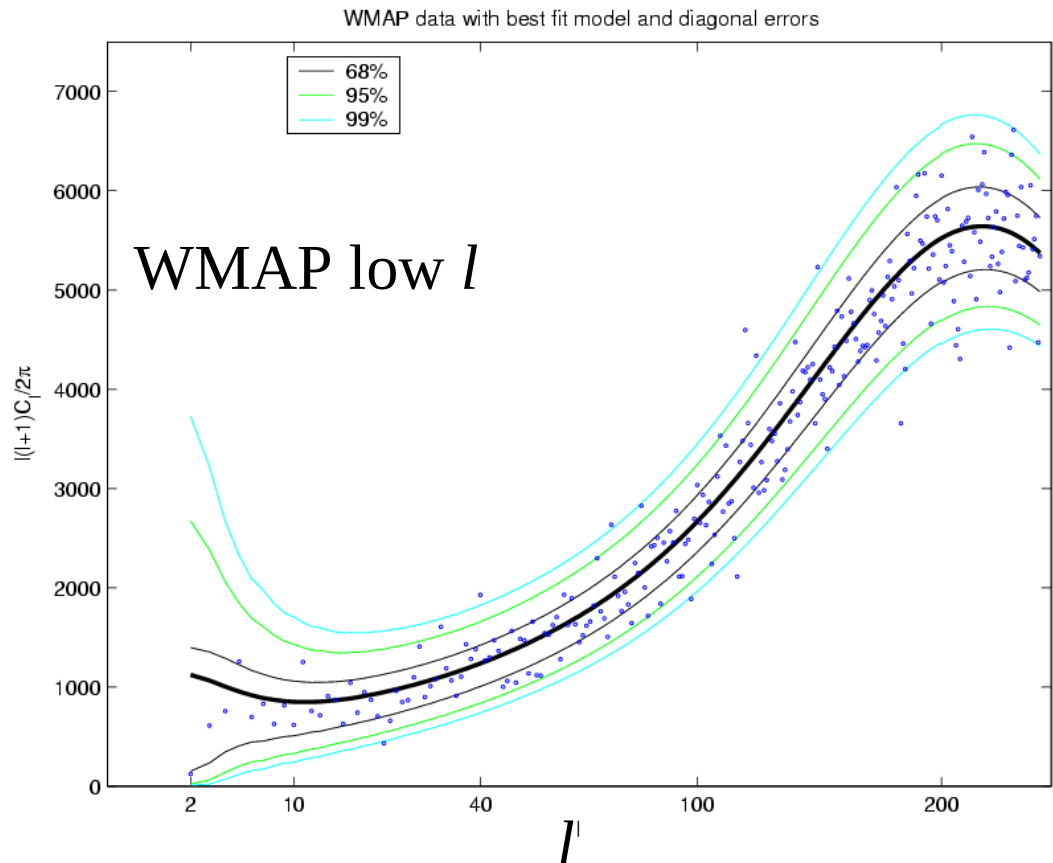
$$C_l^{obs} \sim \chi^2 \text{ with } 2l+1 \text{ d.o.f.}$$

“Cosmic Variance”

$$\langle |\Delta C_l^{obs}|^2 \rangle \approx \frac{2C_l^2}{2l+1}$$

$$P(C_l | C_l^{obs})$$

- inverse gamma distribution
(+ noise, sky cut, etc).



Parameter Estimation

- Can compute $P(\{\theta\} \mid \text{data}) = P(C_l(\{\theta\}) \mid c_l^{obs})$
- Often want marginalized constraints. e.g.

$$\langle \theta_1 \rangle = \int \theta_1 P(\theta_1 \theta_2 \theta_3 \dots \theta_n \mid \text{data}) d\theta_1 d\theta_2 \dots d\theta_n$$

- BUT: Large n integrals very hard to compute!
- If we instead *sample* from $P(\{\theta\} \mid \text{data})$ then it is easy:

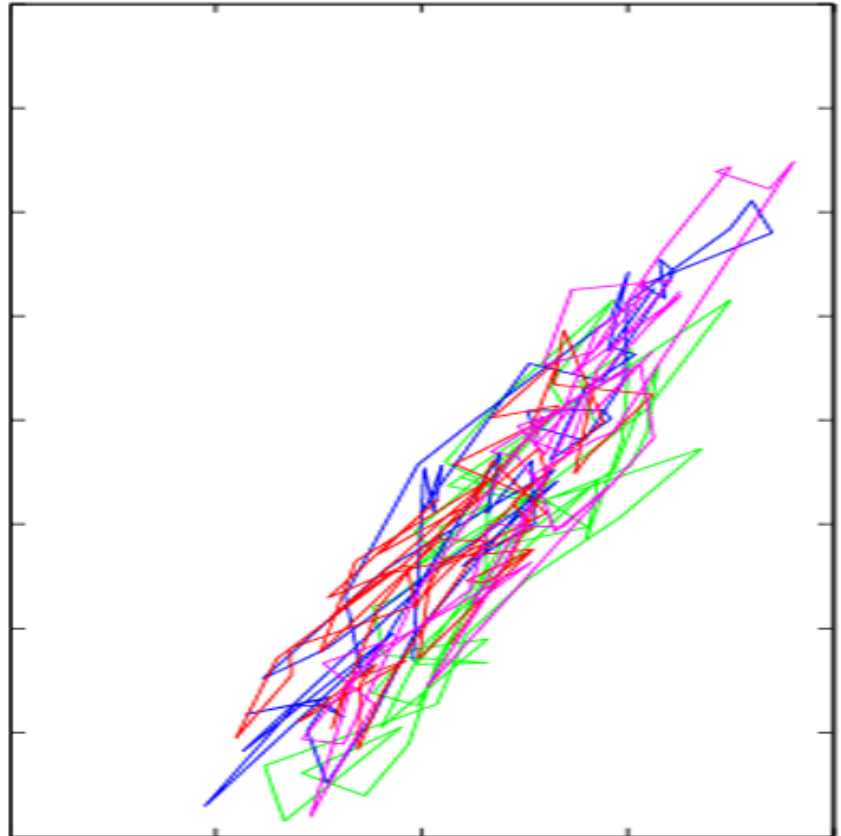
$$\langle \theta_1 \rangle \approx \frac{1}{N} \sum_i \theta_1^{(i)}$$



Can easily learn everything we need from set of samples

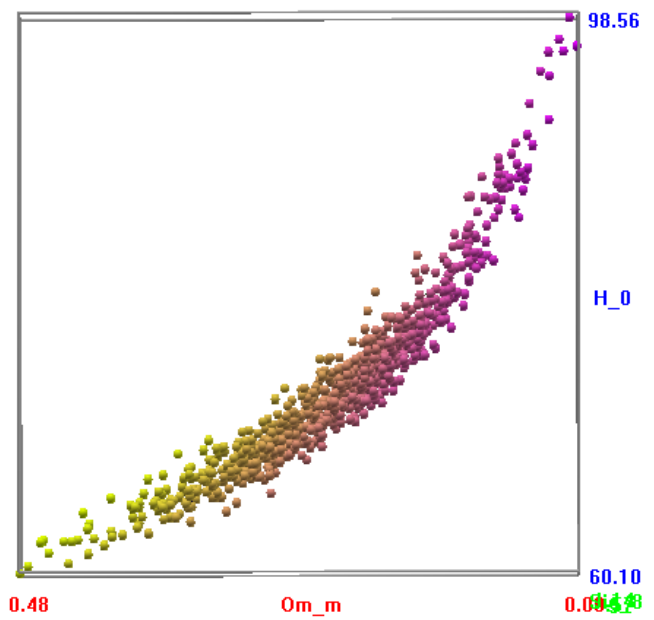
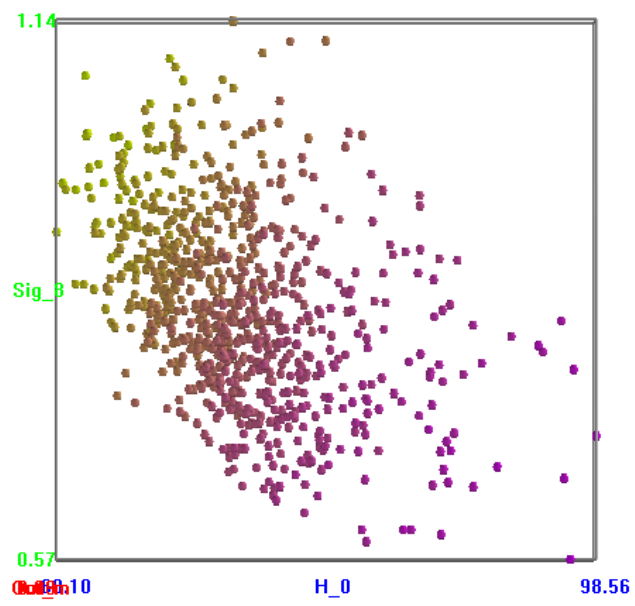
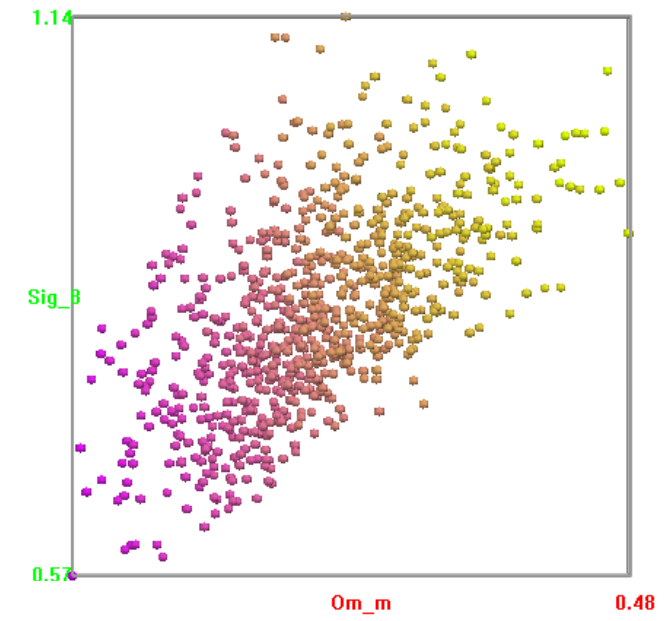
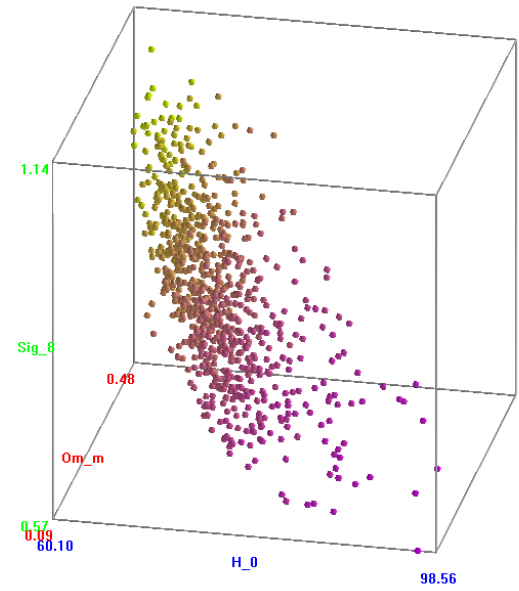
Markov Chain Monte Carlo sampling

- Metropolis-Hastings algorithm
- Number density of samples proportional to probability density
- At its best scales linearly with number of parameters (as opposed to exponentially for brute integration)



This is now standard method for parameter estimation. Public CosmoMC code available at <http://cosmologist.info/cosmomc> (Lewis, Bridle: astro-ph/0205436)

Samples in
6D parameter
space

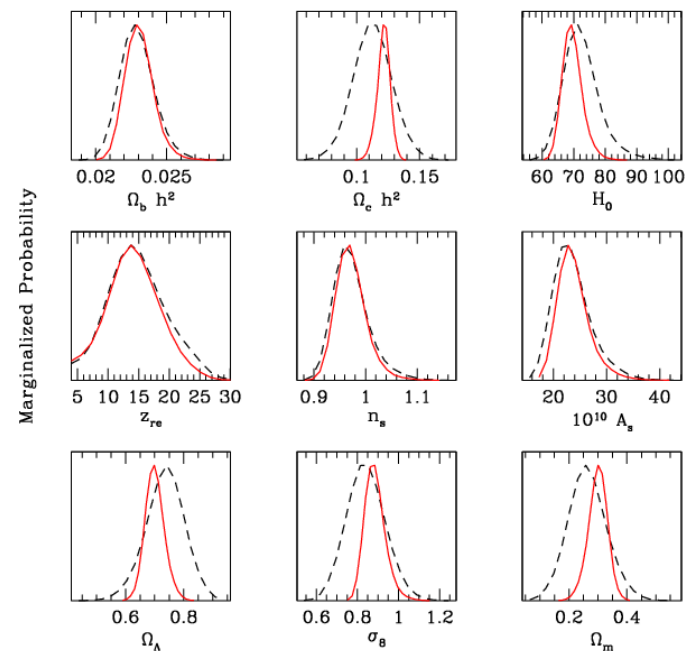
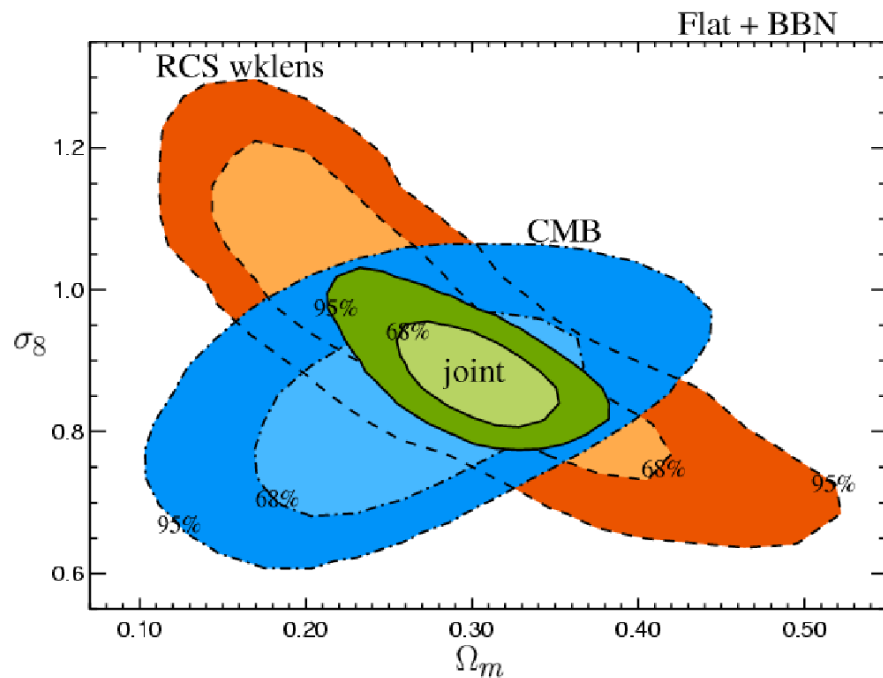


CMB data
alone
color = optical
depth

Plot number density of samples as function of parameters

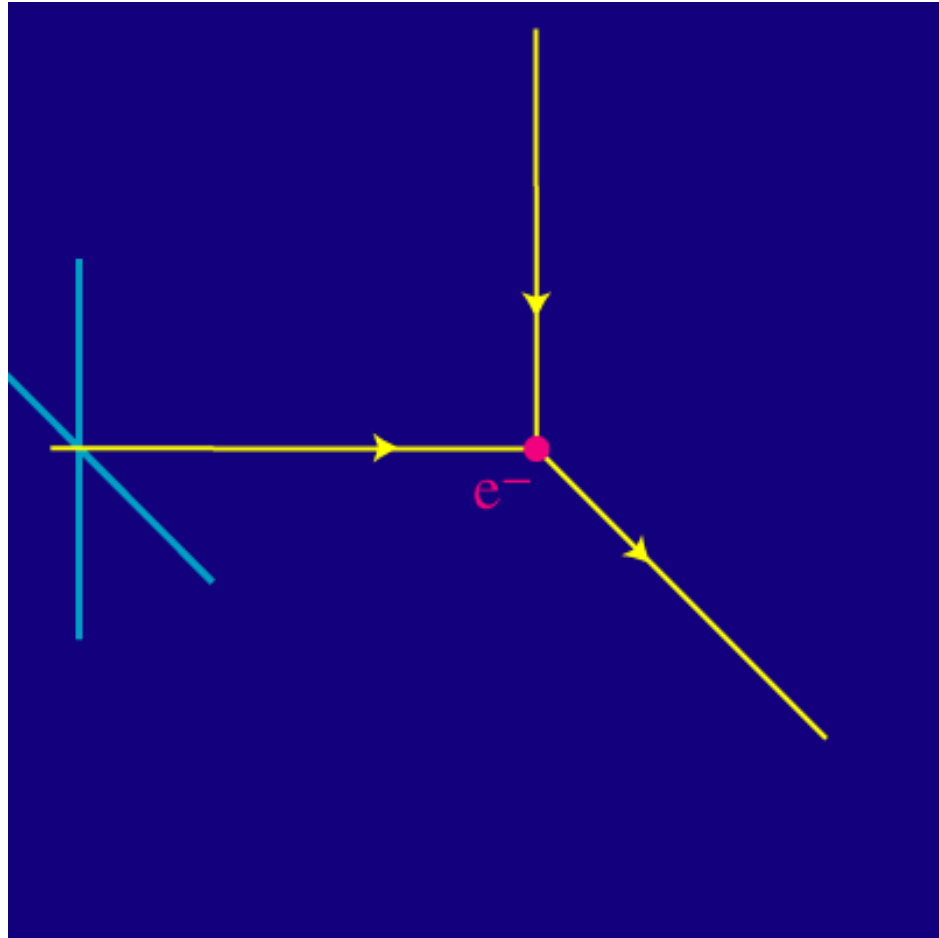
Often better constraint by combining with other data

e.g. CMB+galaxy lensing +BBN prior



Contaldi, Hoekstra, Lewis: astro-ph/0302435

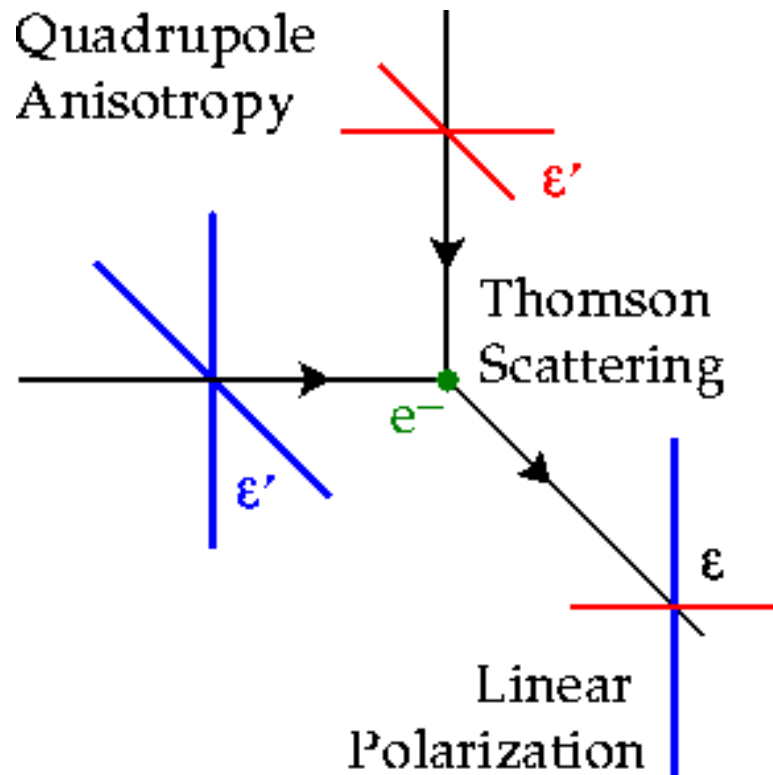
Thomson Scattering Polarization



W Hu

CMB Polarization

Generated during last scattering (and reionization) by Thomson scattering of anisotropic photon distribution



Stokes' Parameters



$Q \rightarrow -Q, U \rightarrow -U$ under 90 degree rotation

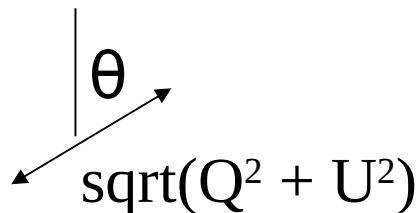
$Q \rightarrow U, U \rightarrow -Q$ under 45 degree rotation



Spin-2 field $Q + i U$

or Rank 2 trace free symmetric tens

$$P = \begin{pmatrix} Q & U \\ U & -Q \end{pmatrix}$$



$$\theta = \frac{1}{2} \tan^{-1} U/Q$$

E and B polarization

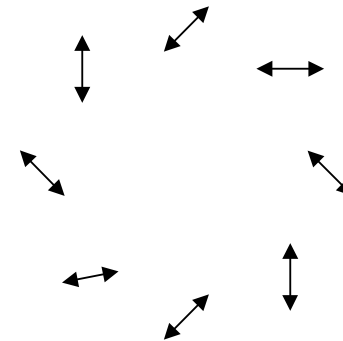
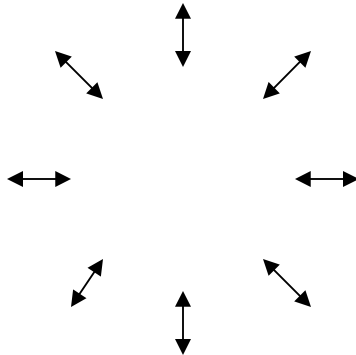
$$\mathcal{P}_{ab} = \nabla_{\langle a} \nabla_{b \rangle} P_E - \epsilon^c_{ (a} \nabla_{b \rangle} \nabla_c P_B$$



“gradient” modes
E polarization

“curl” modes
B polarization

e.g.



E and B harmonics

- Expand scalar P_E and P_B in spherical harmonics
- Expand P_{ab} in tensor spherical harmonics

$$\mathcal{P}_{ab} = \frac{1}{\sqrt{2}} \sum_{lm} \left(E_{lm} Y_{(lm)ab}^G + B_{lm} Y_{(lm)ab}^C \right)$$

$$E_{lm} = \sqrt{2} \int_{4\pi} dS Y_{(lm)}^{G\,ab*} \mathcal{P}_{ab} \qquad B_{lm} = \sqrt{2} \int_{4\pi} dS Y_{(lm)}^{C\,ab*} \mathcal{P}_{ab}$$

Harmonics are orthogonal over the full sky:

E/B decomposition is exact and lossless on the full sky

Zaldarriaga, Seljak: astro-ph/9609170

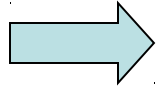
Kamionkowski, Kosowsky, Stebbins: astro-ph/9611125

Primordial Perturbations

fluid at redshift $< 10^9$

- Photons
- Nearly massless neutrinos
Free-streaming (no scattering) after neutrino decoupling at $z \sim 10^9$
- Baryons + electrons
tightly coupled to photons by Thomson scattering
- Dark Matter
Assume cold. Coupled only via gravity.
- Dark energy
probably negligible early on

Perturbations $O(10^{-5})$



- Linear evolution
- Fourier k mode evolves independently
- Scalar, vector, tensor modes evolve independently
- Various linearly independent solutions

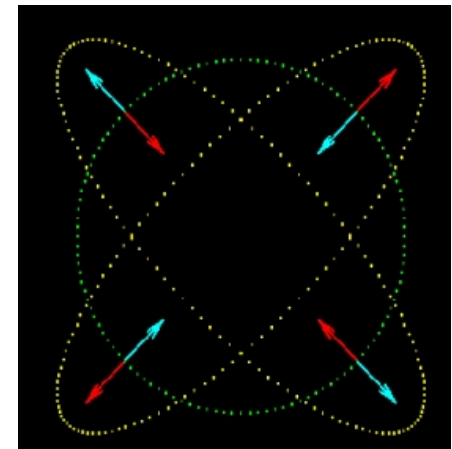
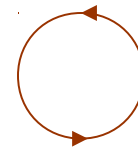
Scalar modes: Density perturbations, potential flows

$\delta\rho, \nabla\delta\rho, \text{etc}$

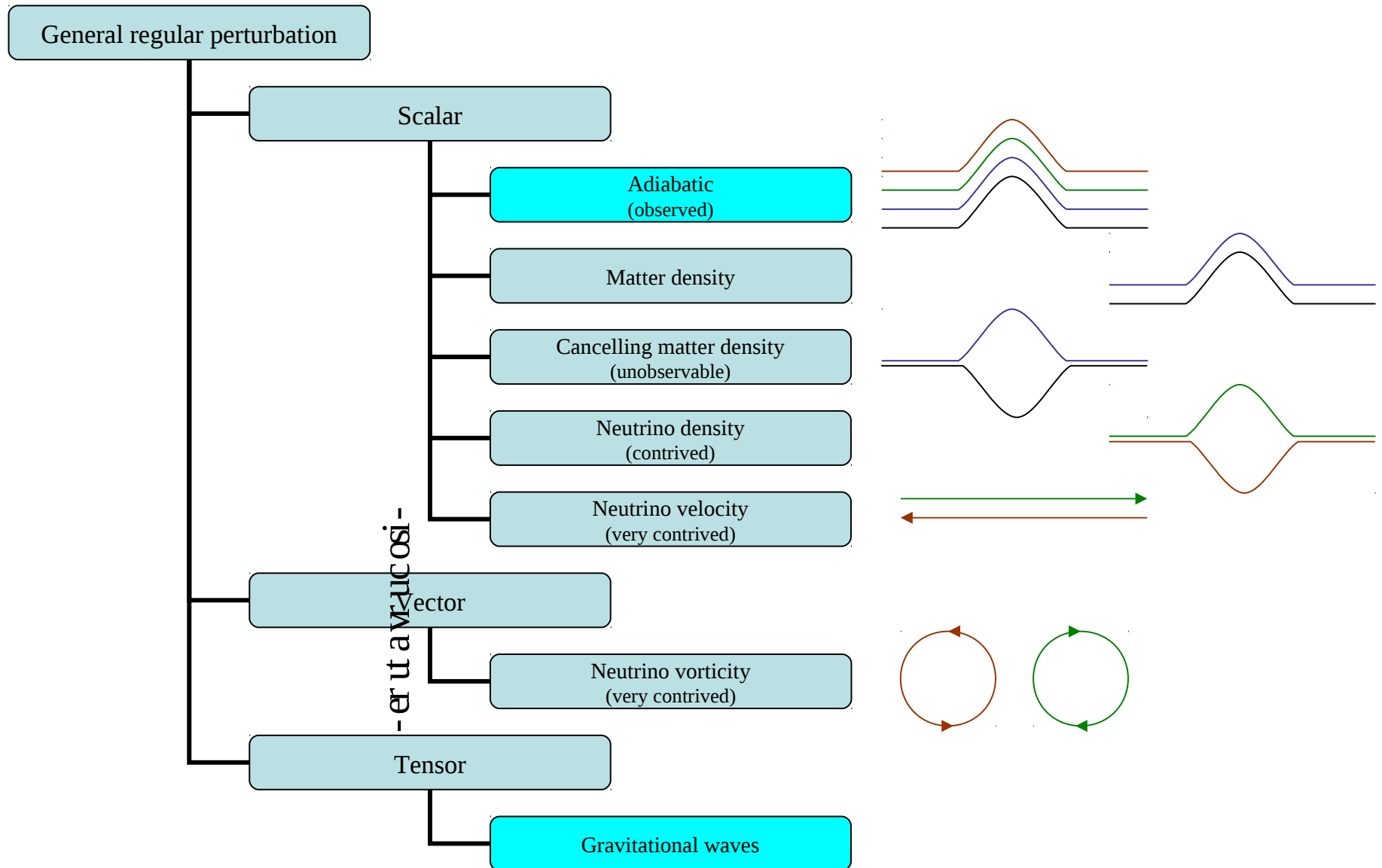
Vector modes: Vortical perturbations

velocities, v ($\nabla \cdot v = 0$)

Tensor modes: Anisotropic space distortions
– gravitational waves



General regular linear primordial perturbation



+ irregular modes, neutrino n-pole modes, n-Tensor modes [Rebhan and Schwarz: gr-qc/9403032](https://arxiv.org/abs/gr-qc/9403032)
 + other possible components, e.g. defects, magnetic fields, exotic stuff...

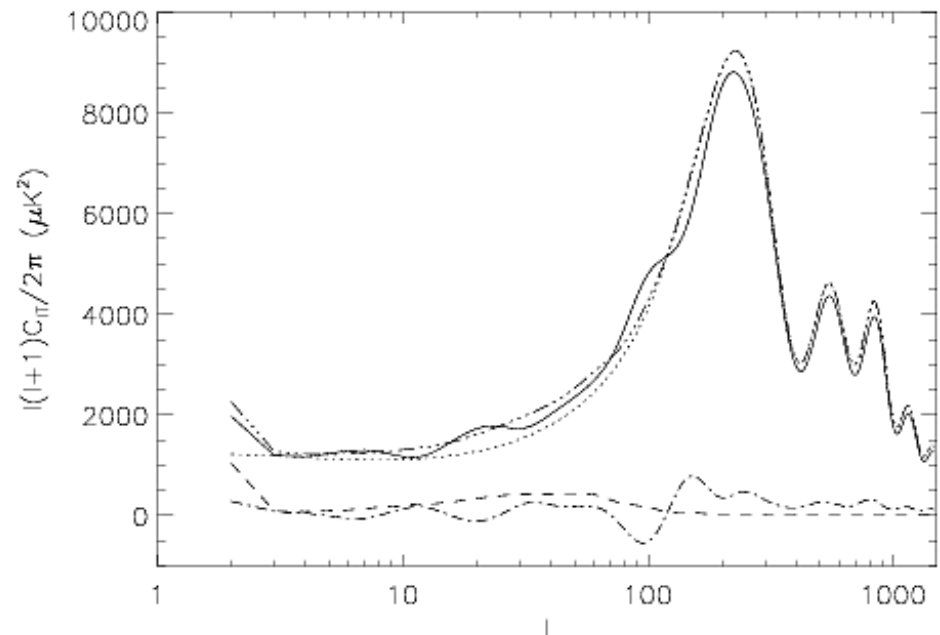
Decaying modes

- Generally $\sim a^{-1}$, a^{-2} or $a^{-1/2}$
- E.g. decaying vector modes unobservable at late times unless ridiculously large early on

Adiabatic decay $\sim a^{-1/2}$ after neutrino decoupling.

possibly observable if generated around or after neutrino decoupling

Otherwise have to be very large (non-linear?) at early times



CMB Polarization Signals

- E polarization from scalar, vector and tensor modes
- B polarization only from vector and tensor modes (curl grad = 0)
+ non-linear scalars

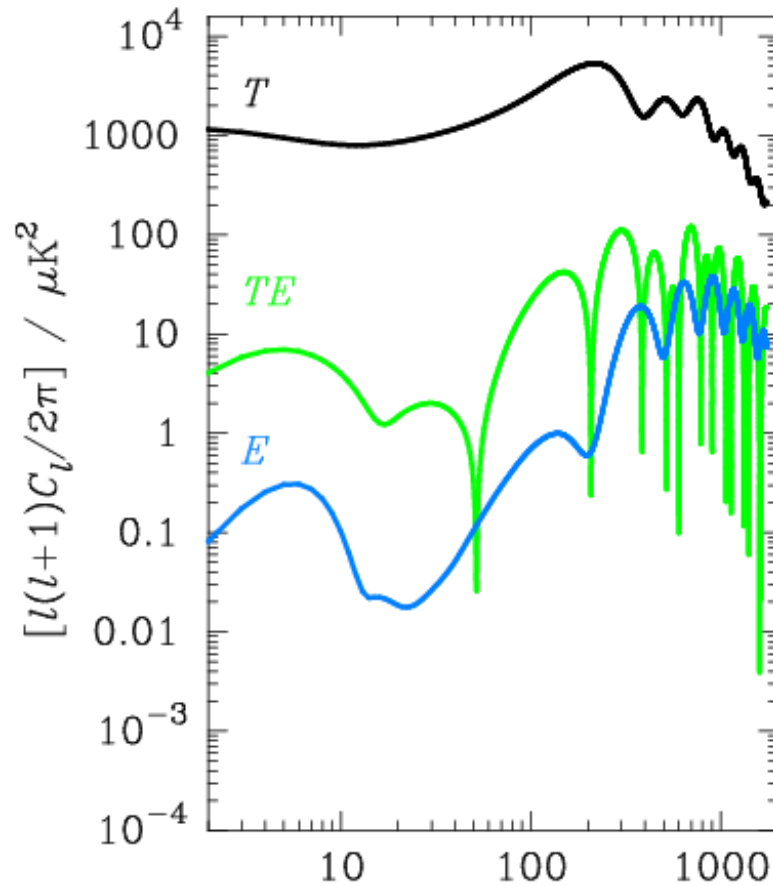
Average over possible realizations (statistically isotropic):

$$\langle E_{l'm'}^* E_{lm} \rangle = \delta_{l'l} \delta_{m'm} C_l^{EE} \quad \langle B_{l'm'}^* B_{lm} \rangle = \delta_{l'l} \delta_{m'm} C_l^{BB}$$

Parity symmetric ensemble: $\langle E_{l'm'}^* B_{lm} \rangle = 0$

Power spectra contain all the useful information if the field is Gaussian

Scalar adiabatic mode

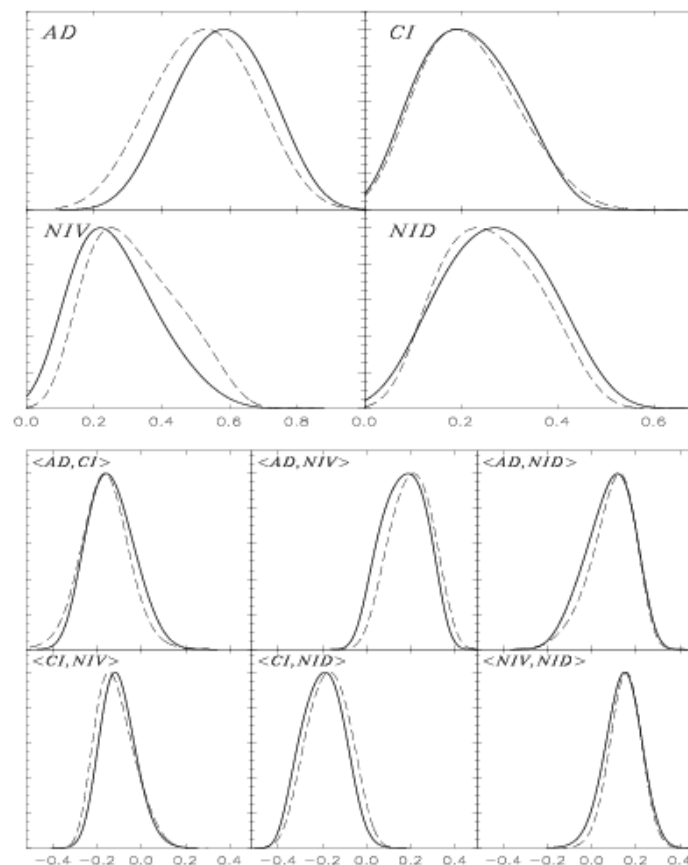
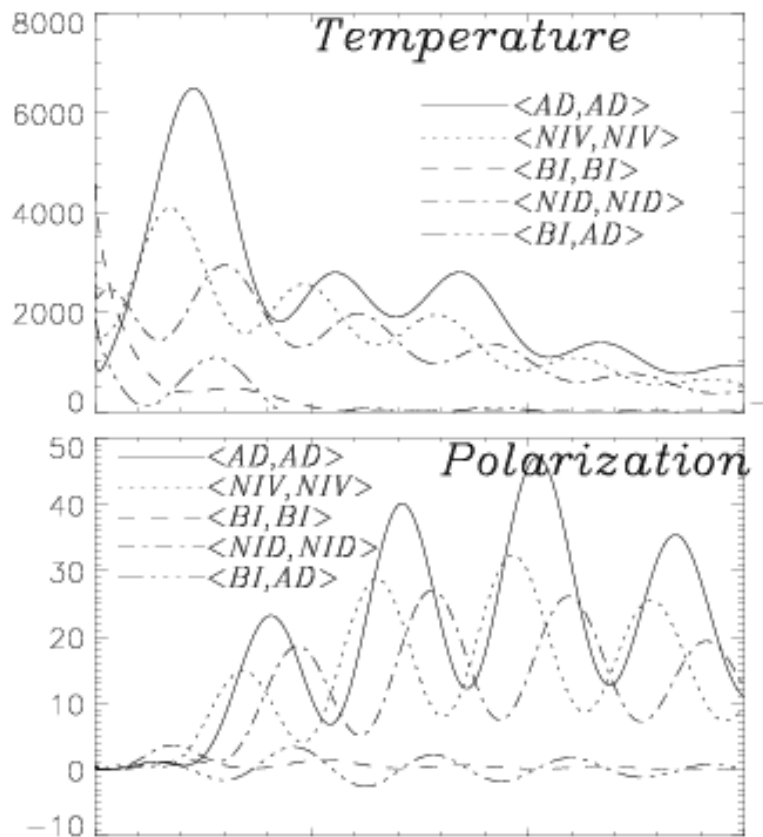


E polarization only

correlation to temperature T-E

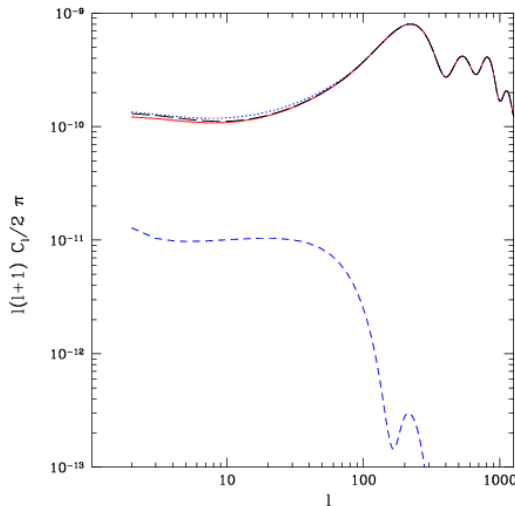
General isocurvature models

- General mixtures currently poorly constrained



Primordial Gravitational Waves (tensor modes)

- Well motivated by some inflationary models
 - Amplitude measures inflaton potential at horizon crossing
 - distinguish models of inflation
- Observation would rule out other models
 - ekpyrotic scenario predicts exponentially small amplitude
 - small also in many models of inflation, esp. two field e.g. curvaton
- Weakly constrained from CMB temperature anisotropy

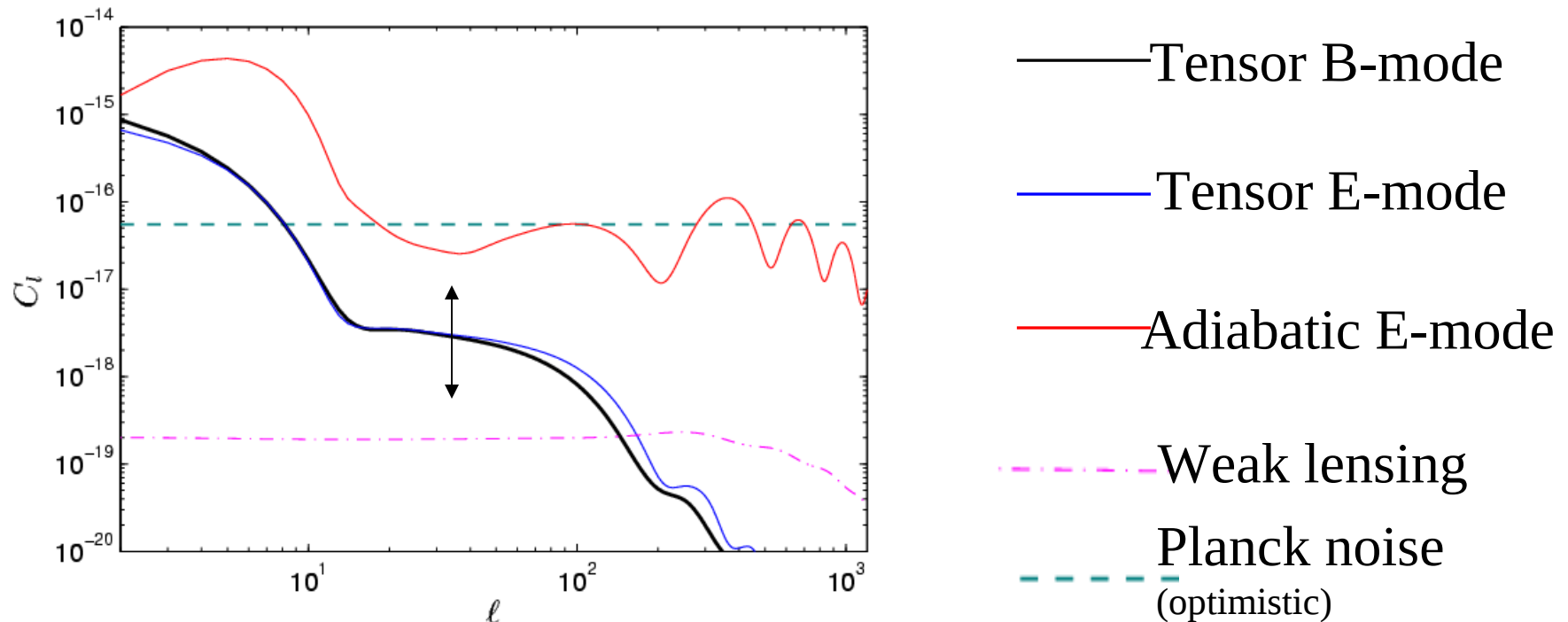


- cosmic variance limited to 10%
- degenerate with other parameters (tilt, reionization, etc)



Look at CMB polarization:
'B-mode' smoking gun

CMB polarization from primordial gravitational waves (tensors)



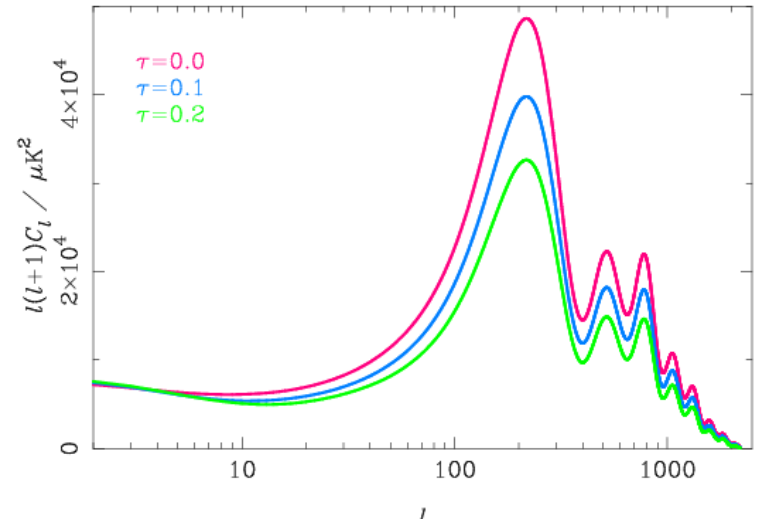
- Amplitude of tensors unknown
- Clear signal from B modes – there are none from scalar modes
- Tensor B is always small compared to adiabatic E

Seljak, Zaldarriaga: [astro-ph/9609169](https://arxiv.org/abs/astro-ph/9609169)

Reionization

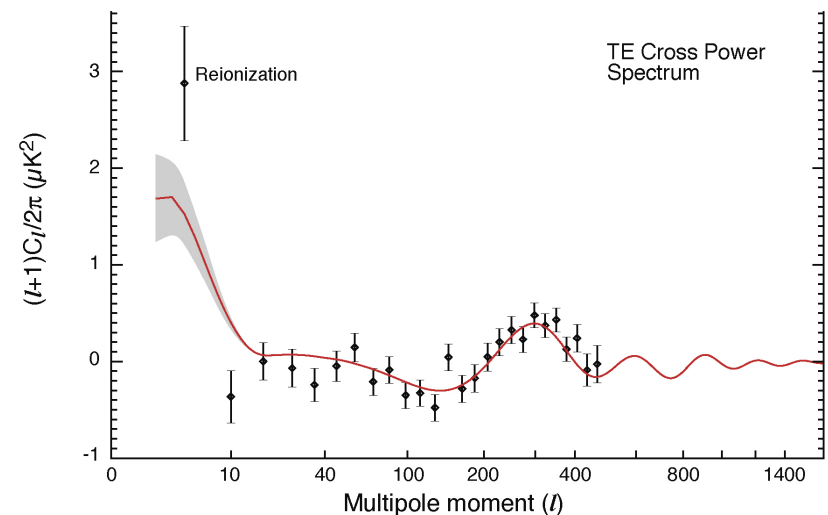
Ionization since $z \sim 6-20$ scatters CMB photons

Temperature signal similar to tensors



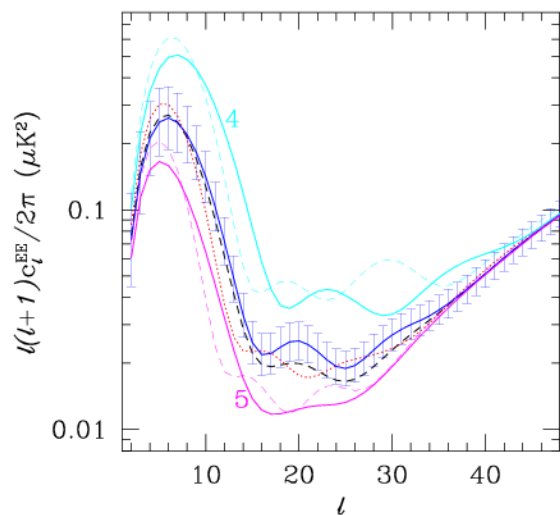
Quadrupole at reionization implies large scale polarization signal

Measure optical depth with T-E correlation

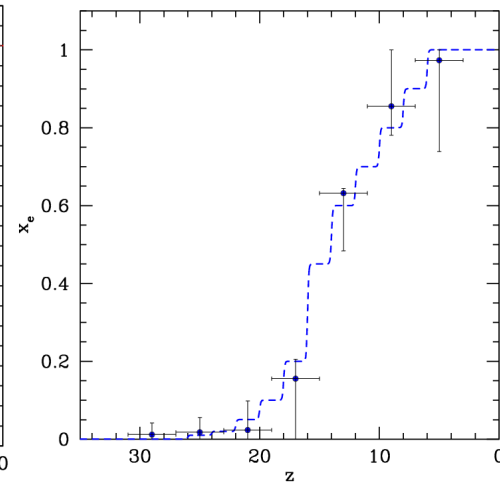
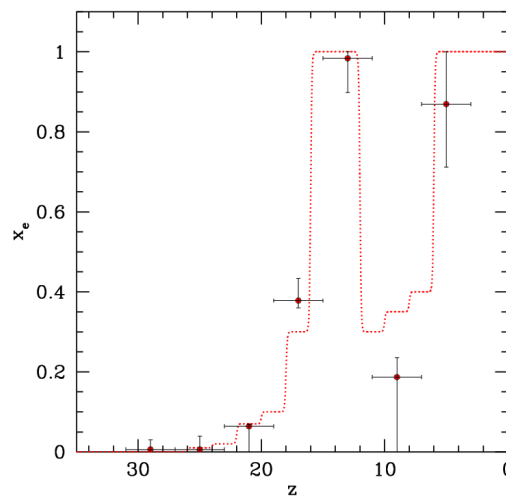


Cosmic variance limited data – resolve structure in EE power spectrum

(Weakly) constrain ionization history



Holder et al: astro-ph/0302404



Weller, Lewis, Battye (in prep)

Other B-modes?

- Topological defects

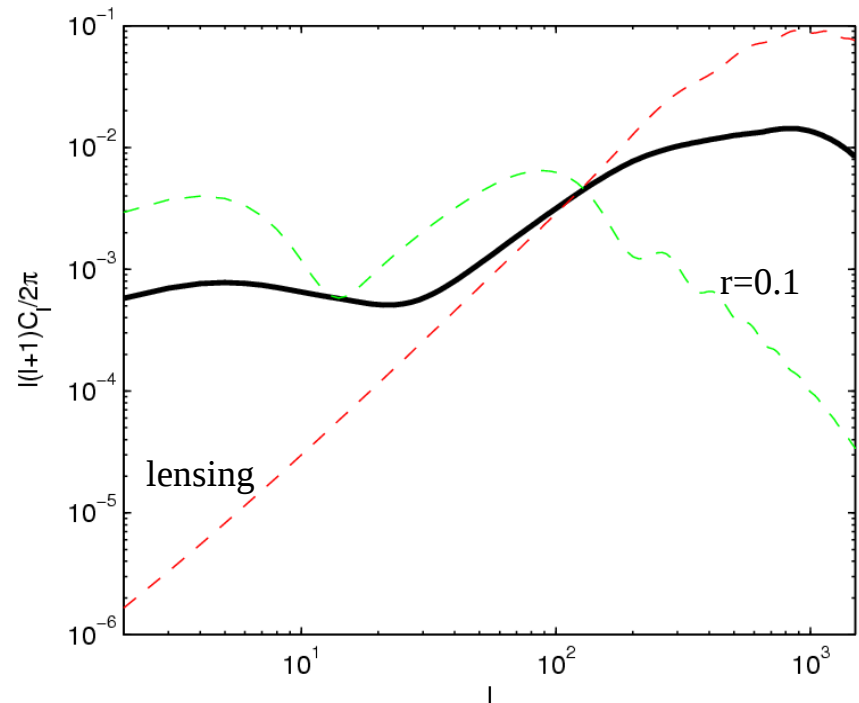
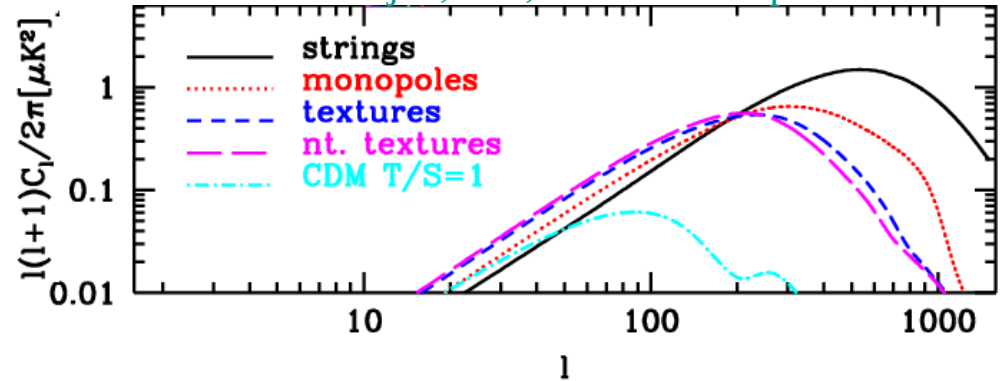
Non-Gaussian signals

global defects:

10% local strings from
brane inflation:

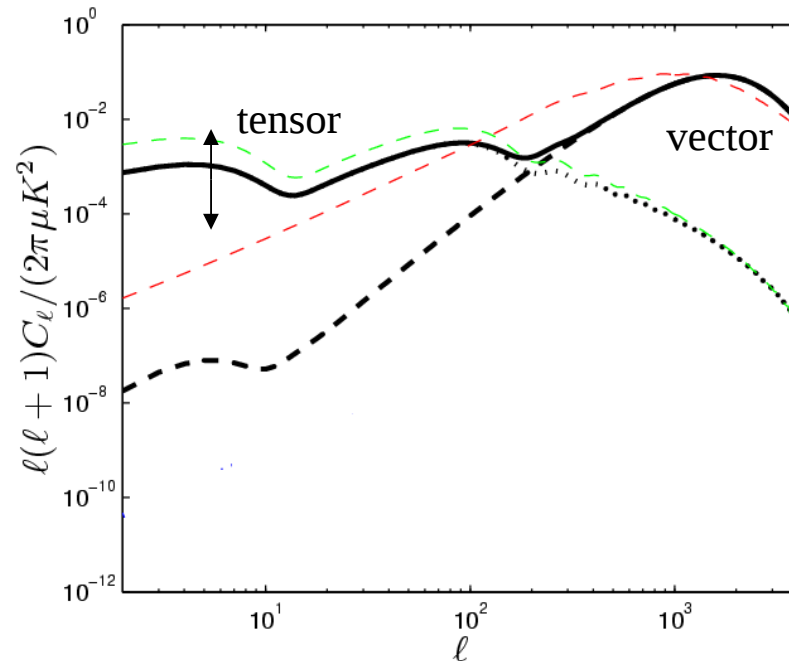
Pogosian, Tye, Wasserman, Wyman:
hep-th/0304188

Seljak, Pen, Turok: astro-ph/9704231



Primordial magnetic fields

- motivation?



Tensor amplitude uncertain.

Non-Gaussian signal.

Check on galaxy/cluster evolution models.

the initial properties of the magnetic field. (c) Concerning studies of generation of cosmic microwave background (CMBR) anisotropies due to primordial magnetic fields of $B \sim 10^{-9}$ Gauss on $\gtrsim 10$ Mpc scales, such fields are not only impossible to generate in early causal magnetogenesis scenarios but also seemingly ruled out by distortions of the CMBR spectrum due to magnetic field dissipation on smaller scales and the overproduction of cluster magnetic fields. (d) The most promising detection

Banerjee and Jedamzik: [astro-ph/0410032](#)

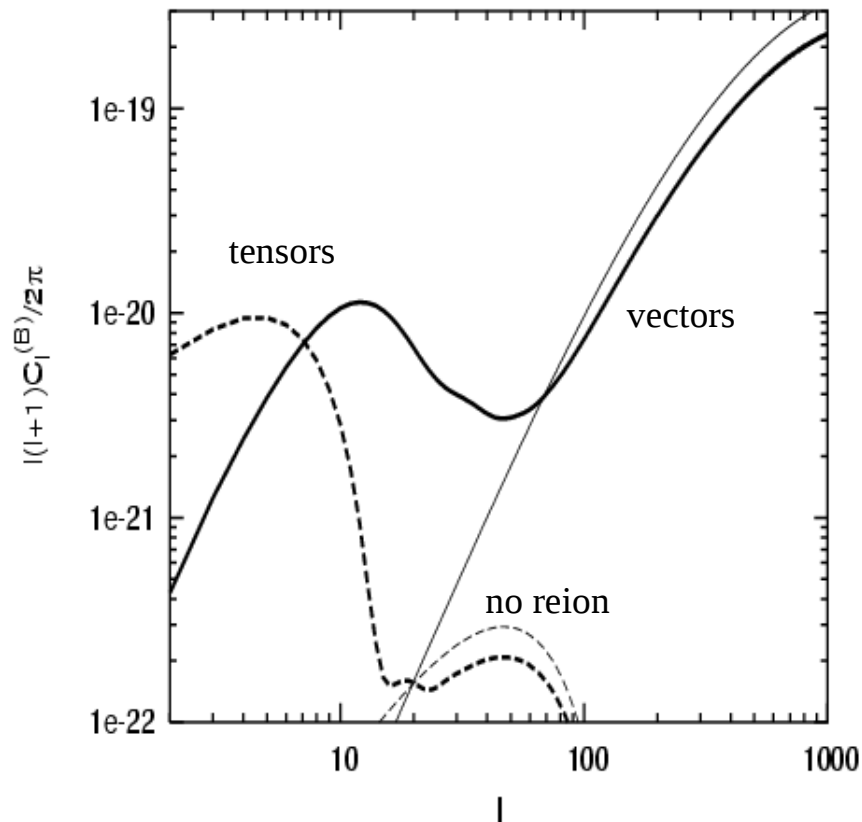
- Also Faraday rotation B-modes at low frequencies

Kosowsky, Loeb: [astro-ph/9601055](#), Scoccola, Harari, Mollerach: [astro-ph/0405396](#)

- Small second order effects, e.g.

Second order vectors and tensors:

Mollerach, Harari, Matarrese: [astro-ph/0310711](#)

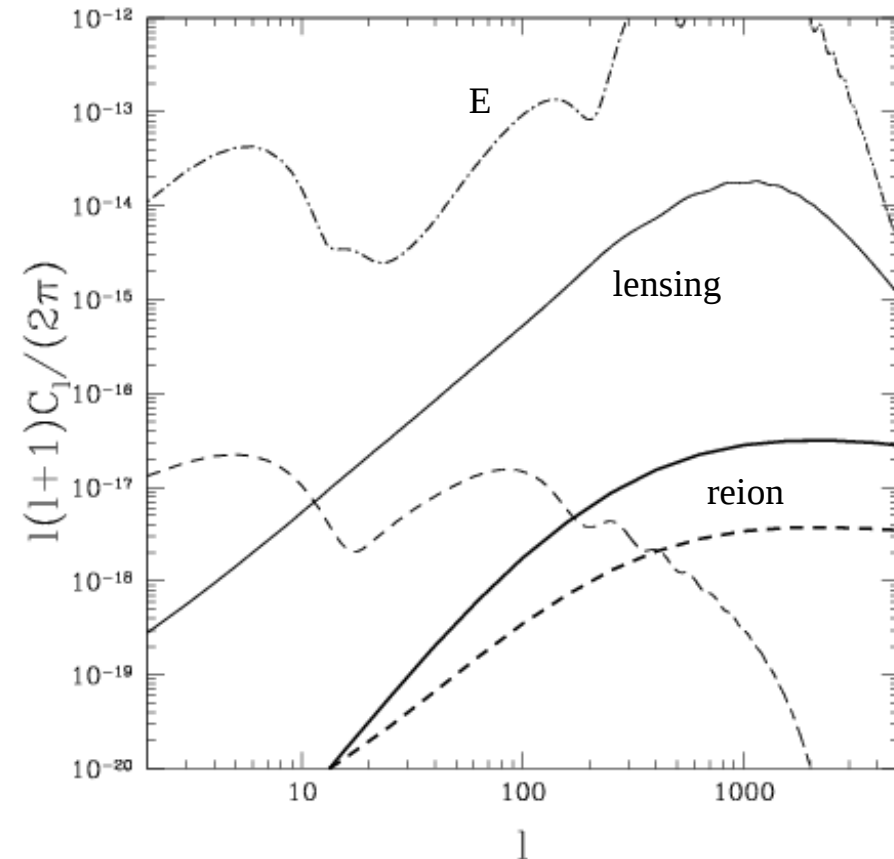


non-Gaussian

Inhomogeneous reionization

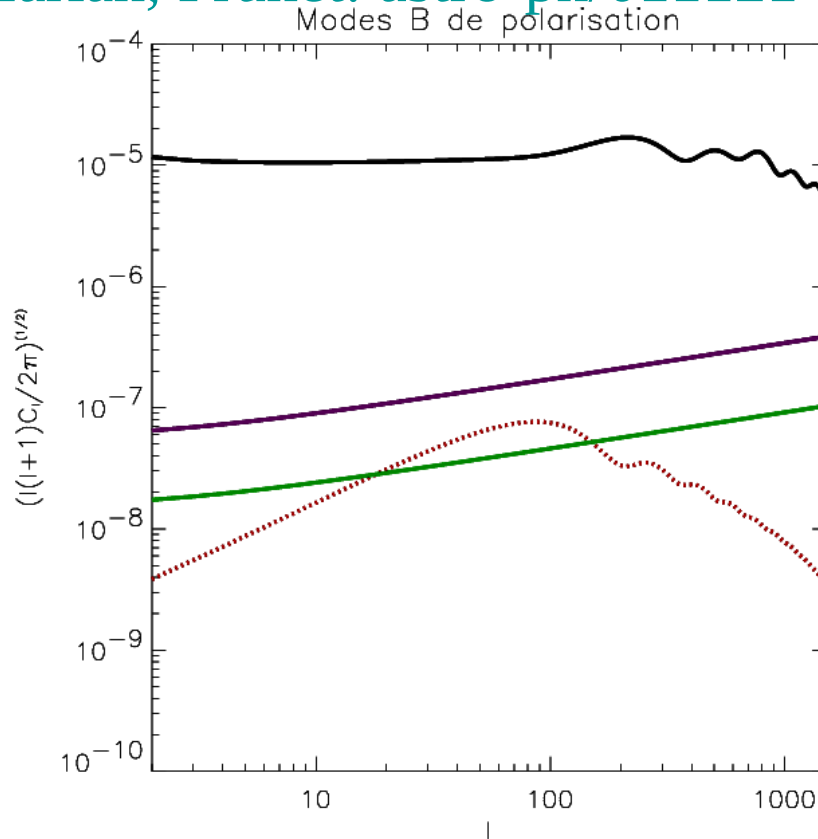
Santon, Cooray, Haiman, Knox, Ma:

[astro-ph/0305471](#) • [Hu](#) • [astro-ph/0007103](#)

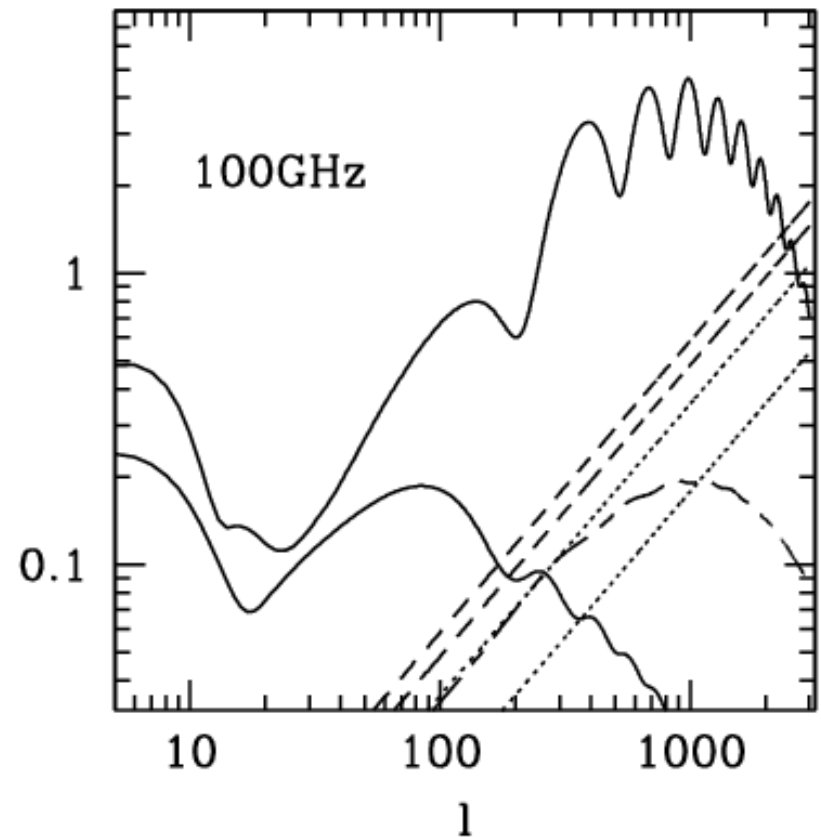


- Systematics and foregrounds, e.g.

Galactic dust (143 and 217 GHz):
Lazarian, Prunet: [astro-ph/0111214](#)



Extragalactic radio sources:
Tucci et al: [astro-ph/0307073](#)



B modes potentially a good diagnostic of foreground subtraction problems or systematics

Partial sky E/B separation problem

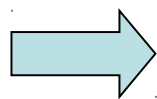
$$\mathcal{P}_{ab} = \nabla_{\langle a} \nabla_{b \rangle} P_E - \epsilon^c_{ (a} \nabla_{b \rangle} \nabla_c P_B$$

Pure E: $\nabla^a \nabla^b \mathcal{P}_{ab} = (\nabla^2 + 2) \nabla^2 P_E$

Pure B: $\epsilon^b_c \nabla^c \nabla^a \mathcal{P}_{ab} = (\nabla^2 + 2) \nabla^2 P_B$

Inversion non-trivial with boundaries

Likely important as reionization signal same scale as galactic cut



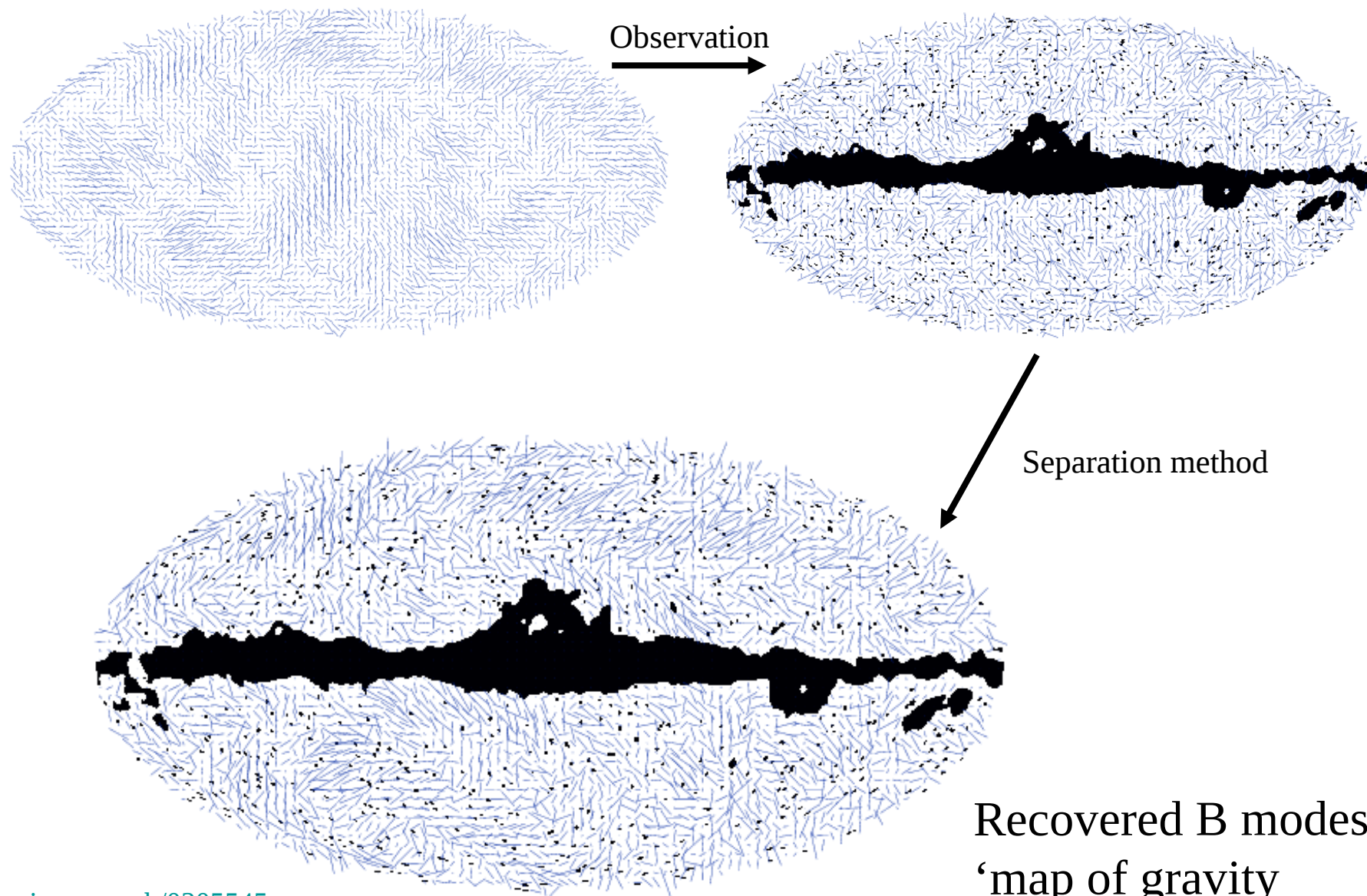
Use set of E/B/mixed harmonics that are orthogonal and complete

over the observed section of the sphere.

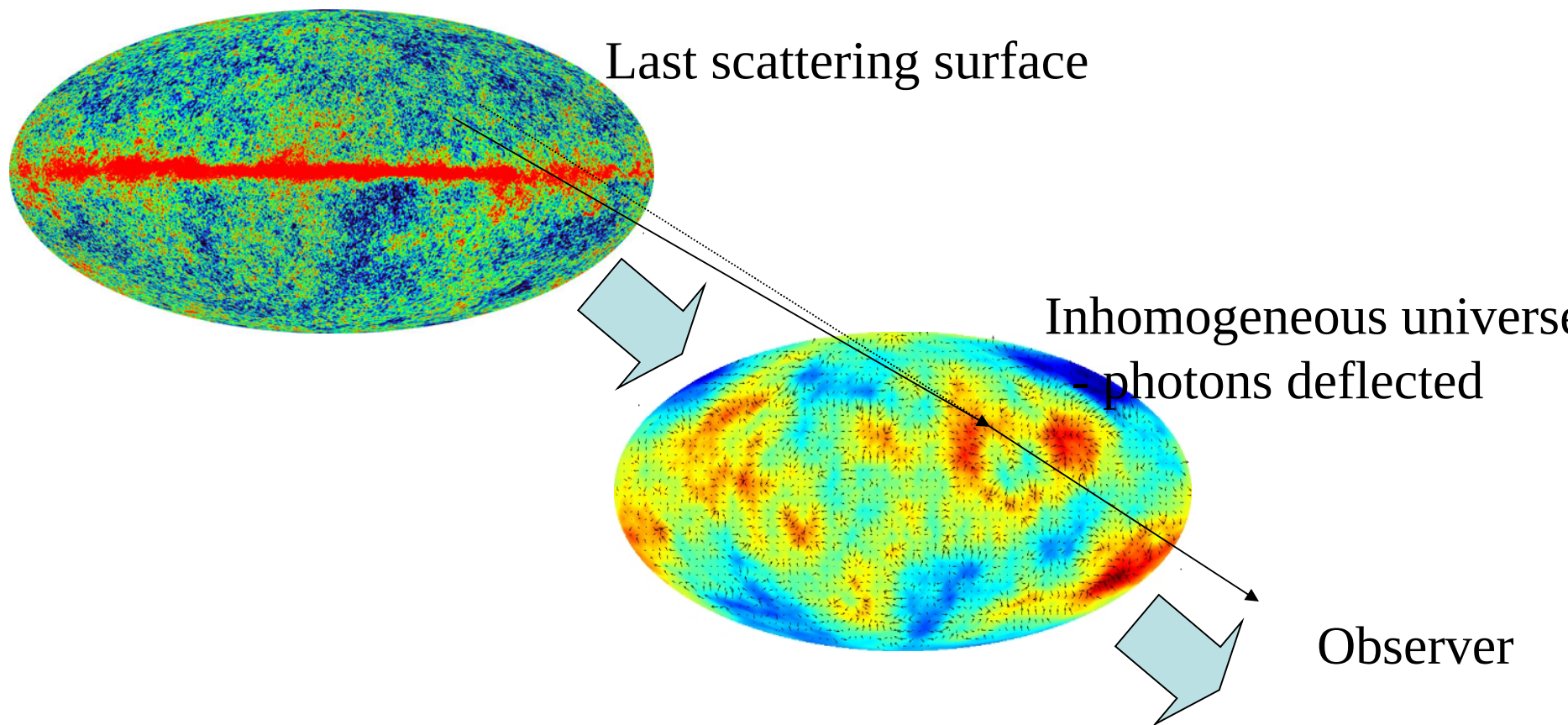
Project onto the 'pure' B modes to extract B.

Underlying B-modes

Part-sky mix with scalar E

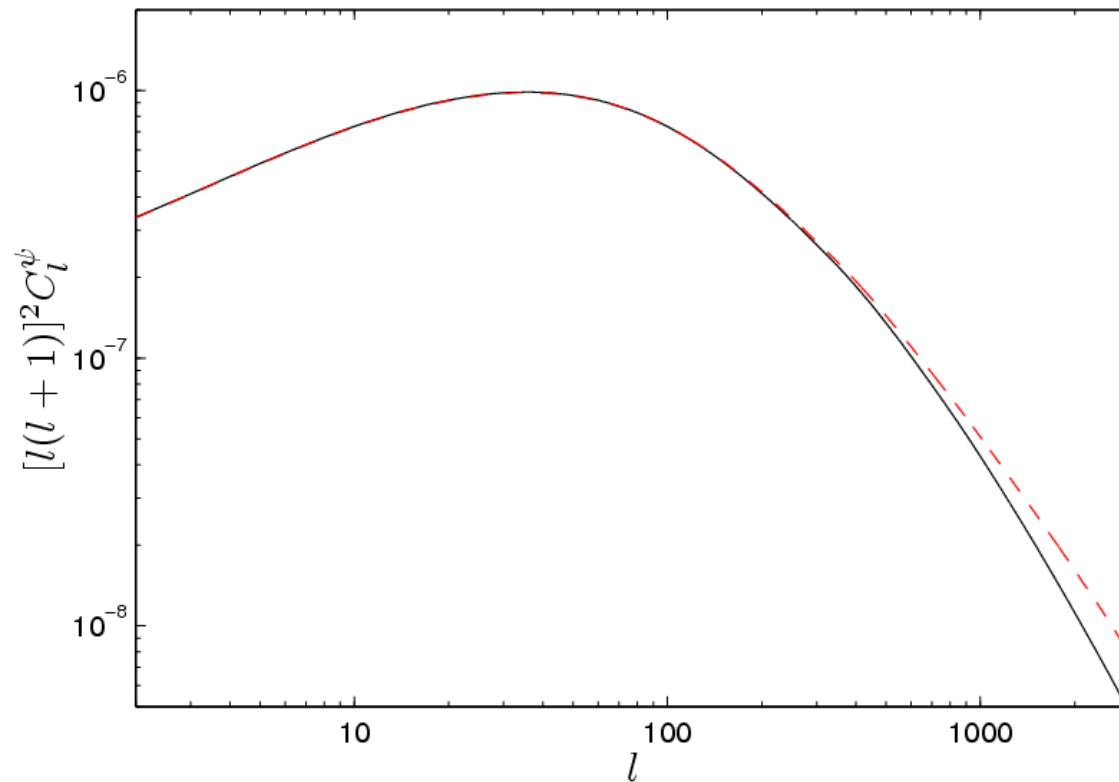


Weak lensing of the CMB



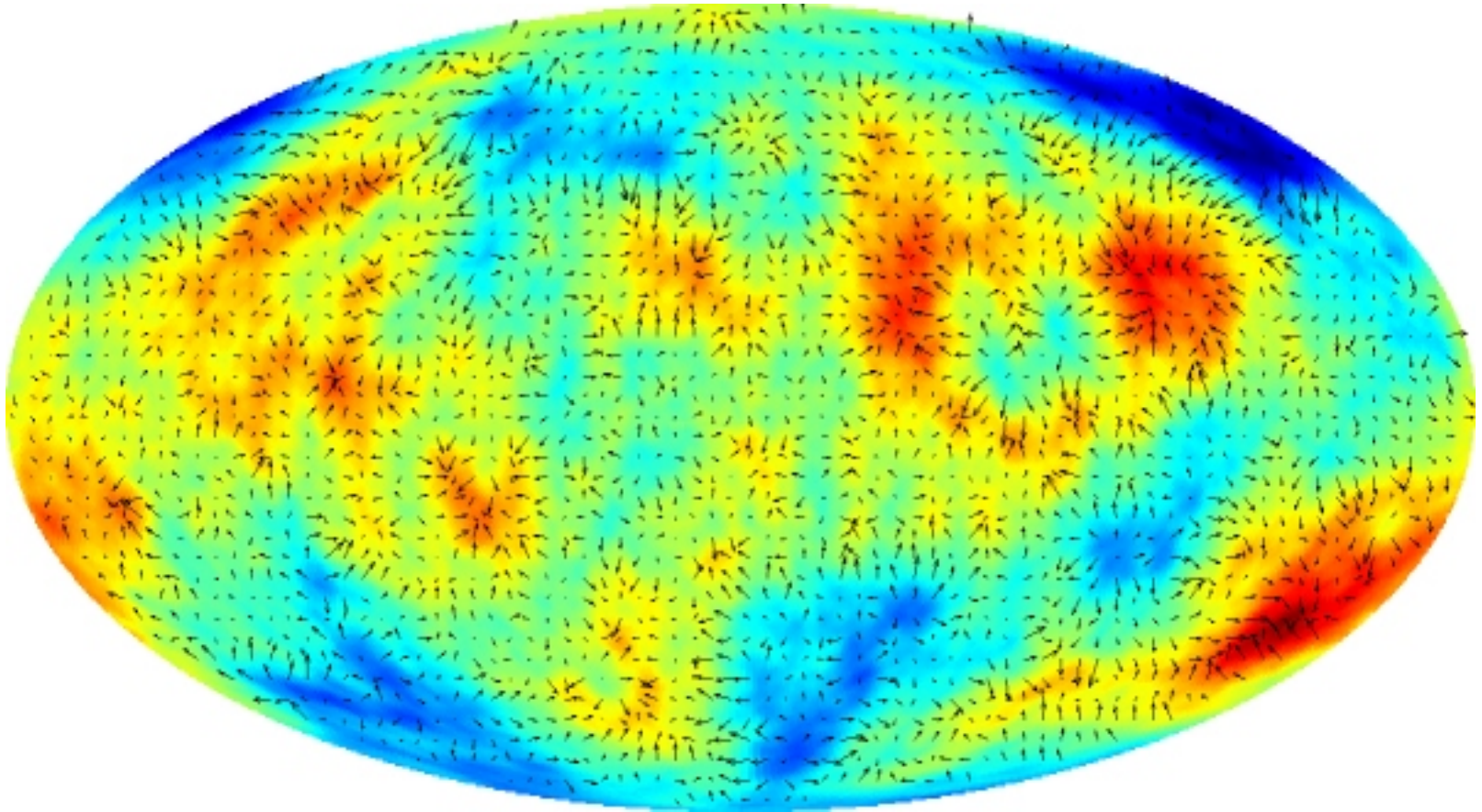
Lensing Potential

$$\bar{X}(\mathbf{n}) = X(\mathbf{n}') = X(\mathbf{n} + \nabla\psi(\mathbf{n}))$$



Deflections $O(10^{-3})$, but coherent on degree scales \rightarrow important!

Lensing potential and deflection angles



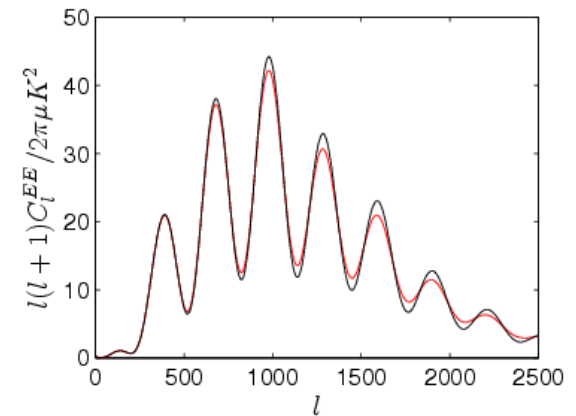
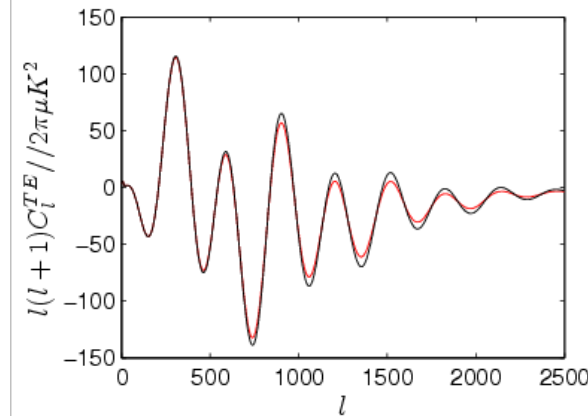
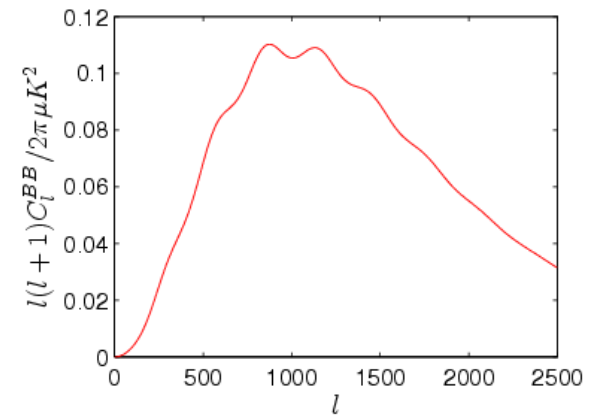
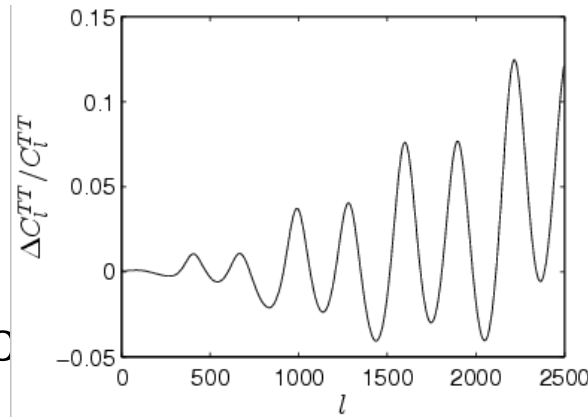
- Changes power spectra
- Makes distribution non-Gaussian

Lensed CMB power spectra

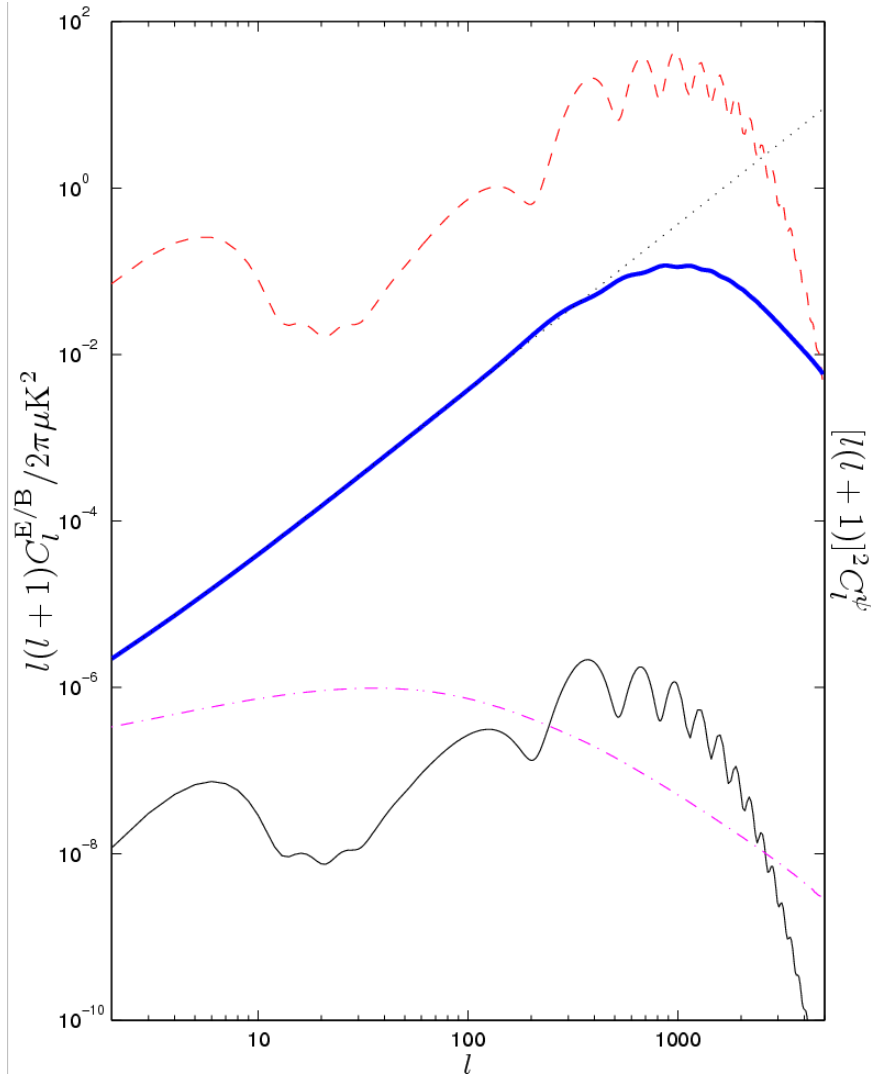
Few % on temperature

10% on TE/EE polarizatio

New lensed BB signal



Lensing of CMB polarization



Nearly white BB spectrum on large scales

Potential confusion with tensor modes

Lensing effect can be largely subtracted if only scalar modes + lensing present, but approximate and complicated (especially posterior statistics).

Hirata, Seljak : [astro-ph/0306354](https://arxiv.org/abs/astro-ph/0306354), Okamoto, Hu: [astro-ph/0301031](https://arxiv.org/abs/astro-ph/0301031)

Other non-linear effects

- **Thermal Sunyaev-Zeldovich**
Inverse Compton scattering from hot gas: frequency dependent signal
- **Kinetic Sunyaev-Zeldovich (kSZ)**
Doppler from bulk motion of clusters; patchy reionization;
(almost) frequency independent signal
- **Ostriker-Vishniac (OV)**
same as kSZ but for early linear bulk motion
- **Rees-Sciama**
Integrated Sachs-Wolfe from evolving non-linear potentials: frequency independent
- **General second order**
includes all of the above + more

Conclusions

- CMB contains a lot of useful information!
- “Precision” Cosmology – many parameters constrained
- E-mode and TE measure optical depth, constraining models of reionization
- B-mode tells us about energy scale of inflation
- Weak Lensing can generate B-modes (already observed)