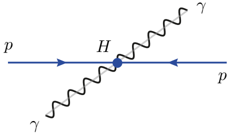
Hypothesis tests with unfolded differential distributions



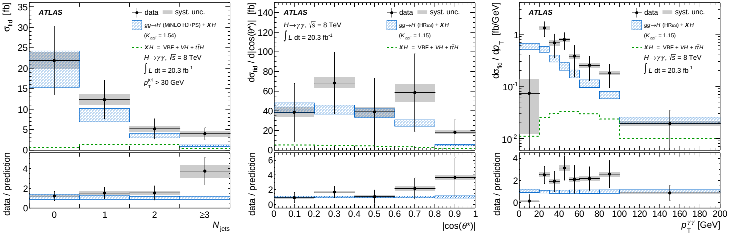
Florian Bernlochner

University of Bonn, Germany

December 8, 2014

Higgs + Jets Workshop
Durham 2014

New interesting results from the *ATLAS* experiment about the Higgs available:



* Differential Cross sections in $H \to 4\ell$ and $H \to \gamma\gamma$

  [JHEP09(2014)112] [Physics Letters B 738 (2014) 234-253]

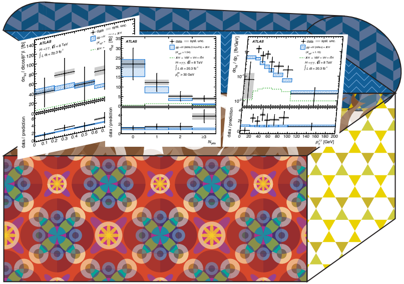
$\to$ Measured values published in **HepData**; unfolded to **particle level**

1. Minimal *underlying* physics dependence

   unfolding into truth fiducial region closely related to measured fiducial selection

2. Full set of *systematic* bin-by-bin cross correlations

Treasure chest for *theorists and phenomenologists*:



Do you have a new physics model and it impacts kinematics in the Higgs sector? Why not test it!

Like you know what a certain **dim-6 operator** makes the Higgs more boosted? Allowed **Spin** 2^+ coefficients?

→ Both measurements provide `Rivet` routines for **particle level fiducial regions**

→ Non-perturbative correction factors included to map parton to particle level predictions

Say, you have two hypotheses: SM and alternative theory

Neyman-Pearson Lemma: Likelihood ratio of both Hypotheses

$$\mathcal{L}_{\mathrm{alt}}/\mathcal{L}_{\mathrm{zero}}$$

most powerful discriminator *(called a test statistic)* you can build.

Applied to binned data: $-2\ln\left(\mathcal{L}_{\mathrm{alt}}/\mathcal{L}_{\mathrm{zero}}\right) = \chi^2_{\mathrm{alt}} - \chi^2_{\mathrm{zero}} = \Delta\chi^2$ where

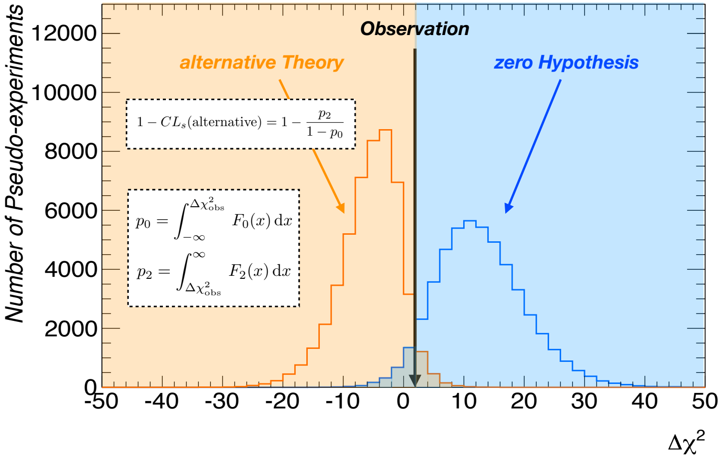
$$\chi^2_{hypo} = \left(\vec{x}_{\mathrm{data}} - \vec{x}_{\mathrm{hypo}}\right) C^{-1}_{\mathrm{hypo}} \left(\vec{x}_{\mathrm{data}} - \vec{x}_{\mathrm{hypo}}\right) .$$

To interpret an observed value of $\Delta\chi^2$ in data:

* Need to know how test statistic is distributed given either zero or alternative theory is the true underlying theory.
* Can be done using Monte Carlo Method with **pseudo-experiments**

Used Test Statistic and CL_s(alternative)

Example test statistic distribution for zero and alternative hypothesis:



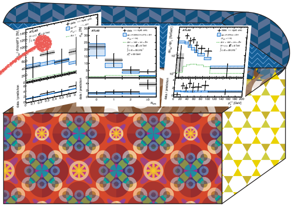
Typically determine: CL_s(alternative) and reject model at (arbitrary threshold), e.g. 95% or 90%

Confidence level of the alternative Hypothesis, penalized by the probability of the zero Hypothesis.

Getting the booty... yarr!

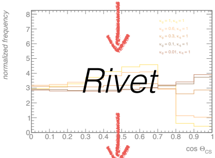
Flow of a typical differential analysis analysis:

Measurements



Predictions

(MG5_)[a]MC@NLO+ PS
or something else

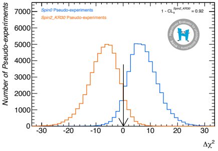


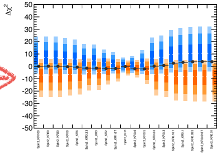

```
# Spin0 hypothesis predictiona and uncertainty covariance
Spin0.XSec:        4.28232 4.25835 ….
Spin0.XSecUncert:   0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
Spin0.FullyUncorrelatedUncertainties: 1

# Spin2 hypothesis predictiona and uncertainty covariance
Spin2.XSec:     3.04068 3.12015 ….
Spin2.XSecUncert:   0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
Spin2.FullyUncorrelatedUncertainties: 1
```

HypE -- Hypothesis Evaluator for unfolded distributions

or
something else





Hype – (Hyp)othesis (e)valuator for unfolded distributions

Software package that aims to provide simple path to hypotheses tests

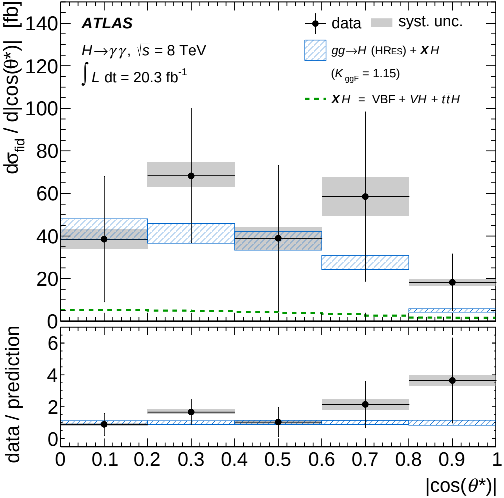
The Hype Team:
FB, Dag Gillberg, Robert Kowalewski, Michaela Queitsch-Maitland, Andy Pilkington

Features:

i. **Easily** test between zero and alternative hypotheses
ii. Additional **Plug-ins**: B *vs* S+B,..
iii. Can directly import **HepData** measurements
iv. Can **interface custom code**
v. Very performant: 1M toys in about 3s with fast toy option

Example Analysis: **Spin analysis with $H \to \gamma\gamma$ ATLAS Run 1 data**

Measured distribution sensitive to Spin of Higgs: $|\cos(\theta^*)|$



Actually measured in 10 bins, here merged to 5 to make the plot more aesthetically pleasing.

Example Analysis: **Spin analysis with $H \to \gamma\gamma$ ATLAS Run 1 data**

Zero Hypothesis: **SM** from MiNLO HJ + Py8 (ggF) + Powheg + Py8(VBF) + Py8(VH & $t\bar{t}H$)

Alternative Hypothesis: Spin 2^+

Effective Lagrangian of alternative hypothesis: *arXiv:1306.6464v3*

$$\mathcal{L} = -\frac{\kappa}{\Lambda} \sum_{f=q,\ell} \kappa_f \, T^f_{\mu\nu} X_2^{\mu\nu} - \frac{\kappa}{\Lambda} \sum_{V=Z,W,\gamma,g} \kappa_V \, T^V_{\mu\nu} X_2^{\mu\nu}$$

$H \to \gamma\gamma$ sensitive to variations in κ_q & κ_g

* Explore models with *free* parameter to change overall normalization: κ/Λ

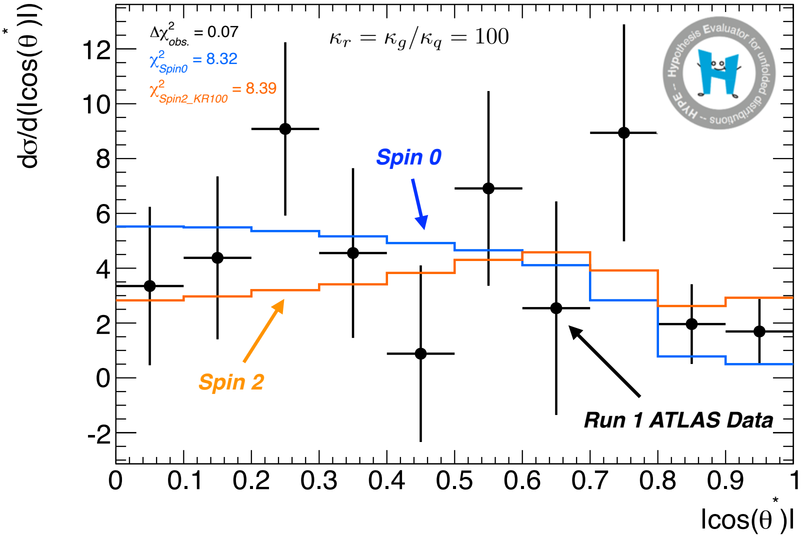
→ only relevant degree of freedom between various models: $\kappa_r = \kappa_g/\kappa_q$

* Perform a scan over 19 working points in κ_r ranging from **0.01 to 100**

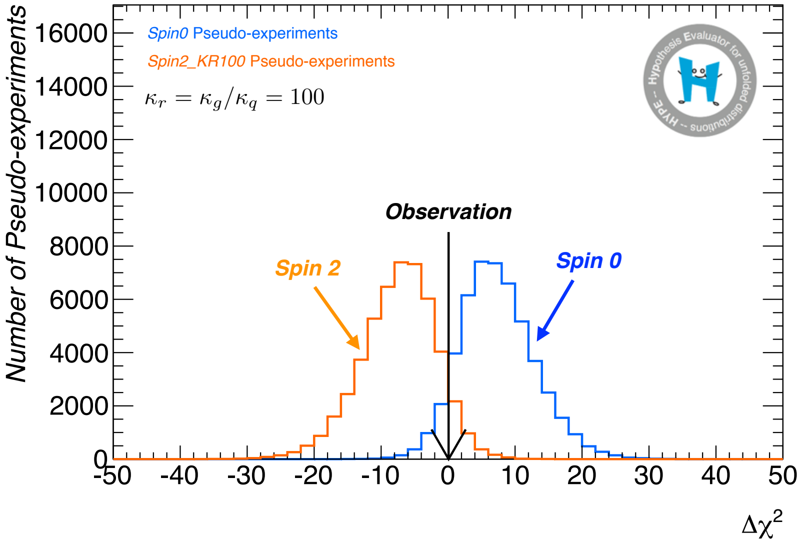
Predictions generated with aMC@NLO + Herwig++

Caveats: No theory uncertainties, no interference with background taken into account, private MC production!

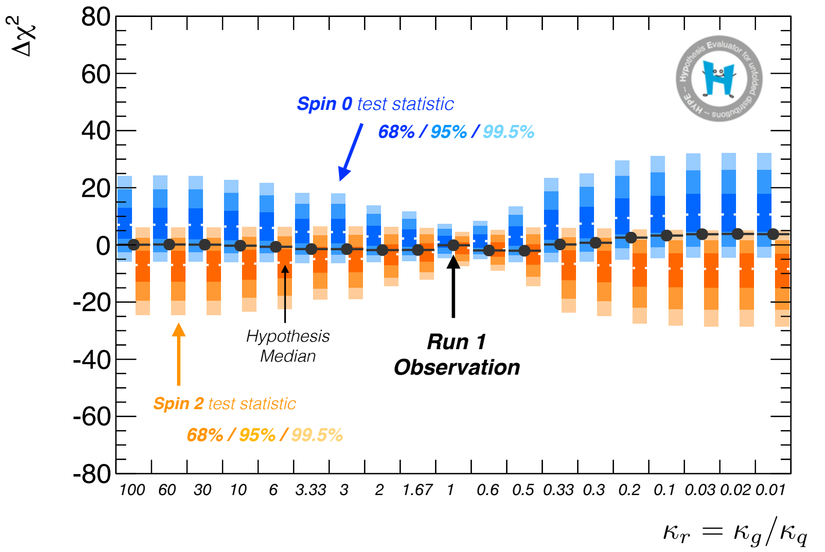
Fit result for $\kappa_r = 100$:



Test statistic for $\kappa_r = 100$:

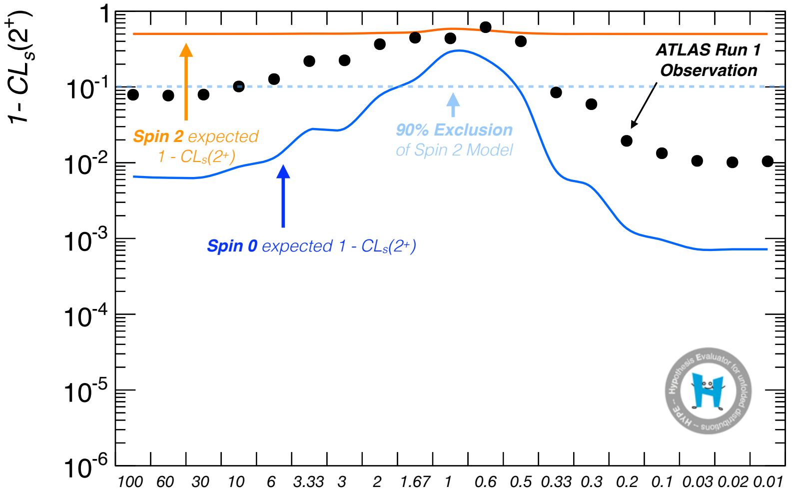


Scan for statistic for $\kappa_r = [100, 0.01]$:



Ratio scan from 100 to 0.01

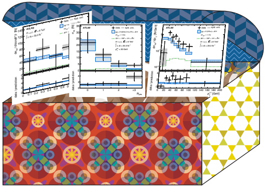
$1 - CL_s(2^+)$ for $\kappa_r = [100, 0.01]$:



$\kappa_r = \kappa_g / \kappa_q$

Can convert this into a limit of $\kappa_r \in [0.33, 10]$ 90% CL_s

Differential cross sections are a **treasure chest** full of booty to probe the SM



- * Measurements are unfolded into a **particle-level** fiducial volume, close to the measured one.
- * Full set of bin-by-bin systematics are on HepData.
- * Results empower people outside experiments to do neat stuff.

* We think this is pretty cool!

$\rightarrow$ We got even Hyped to do our own Spin 2 analysis: $\kappa_r \in [0.33, 10]$ 90% CL_s.

Backup

The Hype Approach to Pseudo-Experiments

Besides this normal implementation, Hype has a fast toy option:

$$\chi^2_{hypo} = (\vec{x}_{\rm data} - \vec{x}_{\rm hypo})\, C^{-1}\, (\vec{x}_{\rm data} - \vec{x}_{\rm hypo}) \ .$$

This option makes use of the asymptotic behaviour of $\Delta\chi^2$

* Reduces the problem of generating pseudo-experiments with N bins to the *two* or *more* relevant degrees of freedom

→ Cross terms cancelation in $\Delta\chi^2$; given fixed normalization test statistic normal distributed.

* Breaks down when floating normalization: $\vec{x}_{\rm hypo} \rightarrow \mu_{\rm hypo} \cdot \vec{x}_{\rm hypo}$

→ Problem now non-linear, normalization depends on pseudo-experiment.

→ Can be diagonalized in a new set of variables and solved for each pseudo-experiment; leaves only 2 effective degrees of freedom