H+Dijets (or more) with High Energy Jets (HEJ)

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Overview of Talk

Multi-Jet Predictions

A new approach to resummation for multiple, hard, wide-angle emissions in the **hard scattering**:

High Energy Jets

Predictions for dijets, W+jets, H+jets,...

Theory vs. Data

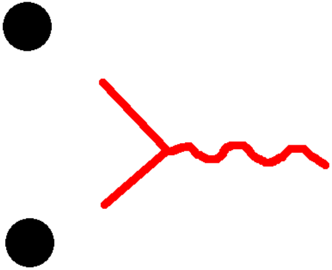
Data vs. description of hard, higher order effects
Thoughts on what not to do

Drivers of Perturbative Corrections

1. **Collinear** (jet profile)
2. **Soft** (p_t-hierarchies)
3. **Opening of phase space** (semi-hard emissions - not related to a divergence of $|M|^2$).
   Think (e.g.) multiple jets of fixed p_t, with increasing rapidity span (span=max difference in rapidity of two hard jets=Δy).
   All calculations will agree that number of additional jets increases - but the amount of radiation will differ (wildly) - e.g. due to **limitations** on the **number** (NLO) or **hardness** (shower) of additional radiation **allowed by theoretical assumptions**.
4. **Virtual Corrections: Disparate scales**
   In $\hat{s}/p_t^2 \sim \Delta y$ ("High Energy": $\hat{s} \to \infty, p_t$ fixed)

Corrections from 3) and 4) are **systematically summed** with **High Energy Jets** (including contributions also from soft emissions).

"High Energy" can mean slightly different things



Consider first the **production of *W*-boson** in a hadronic collision.
One-scale partonic process: m_W.

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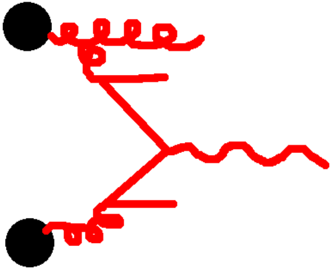

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If $\sqrt{s} \gg m_W$ (i.e. high energy hadronic cms) the pdfs at $x = m_W/\sqrt{s}$ will be completely dominated by the gluon component.

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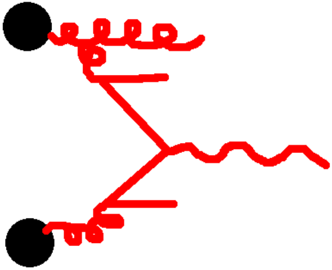

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This is **not** the "High Energy Limit" we will be discussing. Even at 14TeV, Wjj receives only a small perturbative contribution from gg-states.

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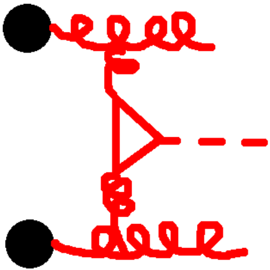

Will instead be discussing the limit of large **partonic** centre of mass energy: $s > \hat{s}(\gg p_t^2)$. Relevant for e.g. *hjj* (where cuts on large m_{jj} is often imposed). But what really is the difference of the two "High Energy Limits?" The diagrams look the same!

For "$\sqrt{s} \gg m_H$": Emissions of gluons considered process-independent. Fundamental process: off-shell gluon fusion

For the limit $\hat{s} \to \infty$, p_t fixed: Standard DGLAP pdfs. Describe the on-shell scattering matrix element at large invariant mass. (hjj dominated by qg-initial states!)

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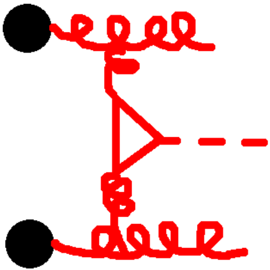

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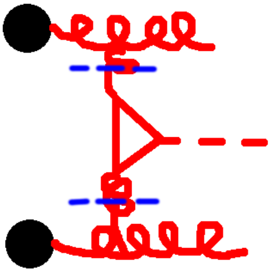

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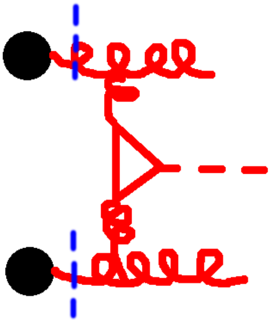

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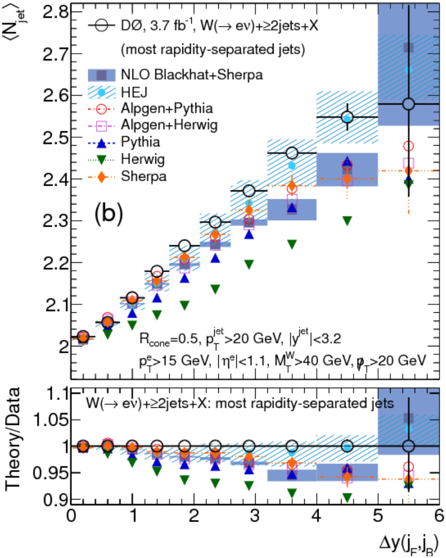
Increasing Rapidity Span → Increasing Number of Jets

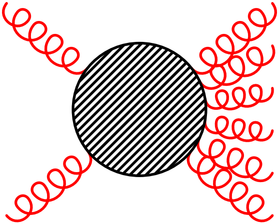


W+dijets@Tevatron (D0)

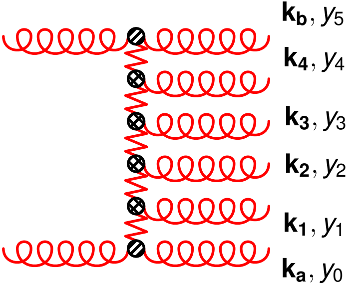
$\Delta y(j_f, j_b)$: Rapidity difference between most forward and backward hard jet

All models show a clear increase in the number of hard jets as the rapidity span $\Delta y(j_f, j_b)$ increases.

Radiation patter important to understand for applying **central jet veto**, extracting *CP*-**properties** of the Higgs boson coupling. Similar behaviour in pure dijet, W/Z+dijet, H+dijet



High Energy Limit
$$\xrightarrow{}$$
$|\hat{t}|$ fixed, $\hat{s} \to \infty$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2} \left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)} \frac{V^{J_i}(q_i, q_{i+1})}{q_i^2 q_{i+1}^2} \right) e^{\omega(q_{n+1})(y_n - y_{n+1})} \frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1} \mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|} \right)$$

to all orders in α_s.

At LL only gluon production; at NLL also quark–anti-quark pairs produced. Approximation of any-jet rate possible.

Comparison of 3-jet scattering amplitudes

Universal behaviour of scattering amplitudes in the **High Energy Limit**:

$$\forall i \in \{2, \ldots, n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i, j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}^{MRK}_{gg\to g\cdots g}\right|^2 = \frac{4\ s^2}{N_C^2 - 1}\ \frac{g^2\ C_A}{|p_{1\perp}|^2}\ \left(\prod_{i=2}^{n-1} \frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\ \frac{g^2\ C_A}{|p_{n\perp}|^2}.$$

$$\left|\overline{\mathcal{M}}^{MRK}_{qg\to qg\cdots g}\right|^2 = \frac{4\ s^2}{N_C^2 - 1}\ \frac{g^2\ C_F}{|p_{1\perp}|^2}\ \left(\prod_{i=2}^{n-1} \frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\ \frac{g^2\ C_A}{|p_{n\perp}|^2},$$

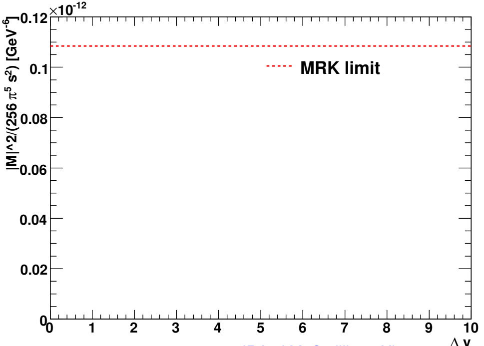
$$\left|\overline{\mathcal{M}}^{MRK}_{qQ\to qg\cdots Q}\right|^2 = \frac{4\ s^2}{N_C^2 - 1}\ \frac{g^2\ C_F}{|p_{1\perp}|^2}\ \left(\prod_{i=2}^{n-1} \frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\ \frac{g^2\ C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (**BFKL equation**).
However, **how well** does this actually **approximate the amplitude?**

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.
Limit ensures logarithmic accuracy



JRA, J.M. Smillie, arXiv:0908.2786

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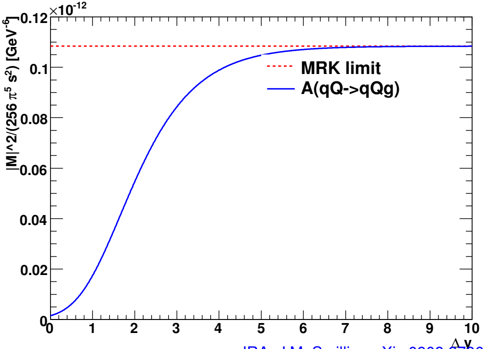

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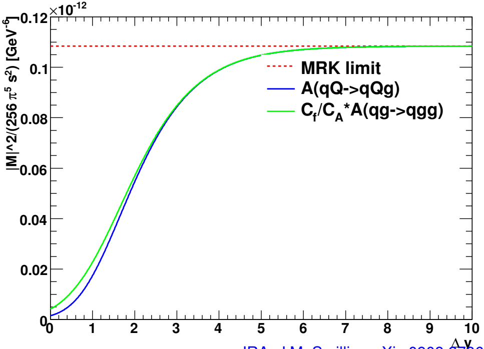

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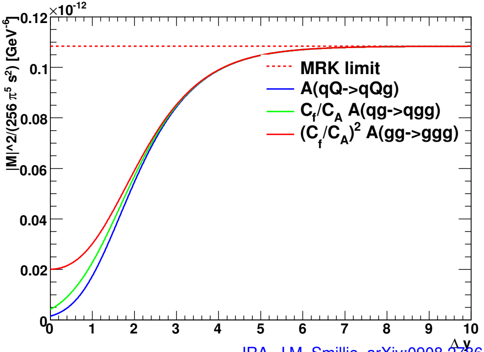

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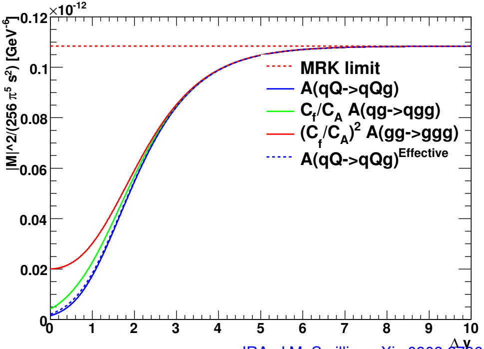

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High Energy Jets (HEJ):

1) Inspiration from Fadin&Lipatov: dominance by *t*-channel
2) No kinematic approximations in invariants (denominator)
3) Accurate definition of currents (coupling through *t*-channel exchange)
4) Gauge invariance. Not just asymptotically.

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$:

$$M_{q^- Q^- \to q^- Q^-} = \langle 1|\mu|a\rangle \frac{g^{\mu\nu}}{t} \langle 2|\nu|b\rangle$$

***t*-channel factorised**: Contraction of (local) currents across *t*-channel pole

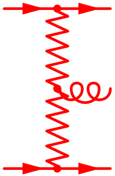
$$\left|\overline{\mathcal{M}}^t_{qQ\to qQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\ C_F\ \frac{1}{t_1}\right) \cdot \left(g^2\ C_F\ \frac{1}{t_2}\right).$$

Extend to $2 \to n \ldots$

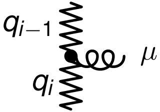
J.M.Smillie and JRA: arXiv:0908.2786

Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

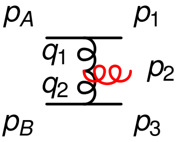


$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

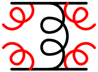


$$V^\mu(q_{i-1}, q_i)$$

$$j^\nu = \overline{\psi}\gamma^\nu\psi$$



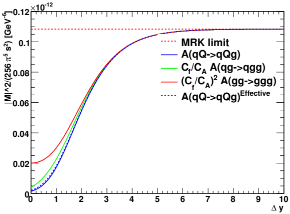
$$\begin{aligned}V^\rho(q_1, q_2) = &-(q_1+q_2)^\rho \\ &+\frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_A}+\frac{p_2\cdot p_B}{p_A\cdot p_B}+\frac{p_2\cdot p_n}{p_A\cdot p_n}\right)+p_A\leftrightarrow p_1 \\ &-\frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_B}+\frac{p_2\cdot p_A}{p_B\cdot p_A}+\frac{p_2\cdot p_1}{p_A\cdot p_1}\right)-p_B\leftrightarrow p_3.\end{aligned}$$



Building Blocks for an Amplitude

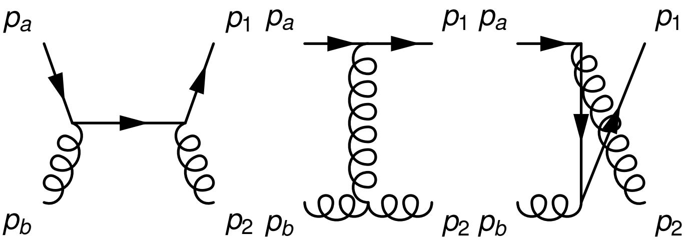
$p_g \cdot V = 0$ can easily be checked (**exact** gauge **invariance**).
The lowest order approximation for $qQ \to qgQ$ is given by

$$\left|\overline{\mathcal{M}}^t_{qQ\to qgQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\,\left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\,C_F\,\frac{1}{t_1}\right)\cdot\left(g^2\,C_F\,\frac{1}{t_2}\right) \cdot \left(\frac{-g^2 C_A}{t_1 t_2}\,V^\mu(q_1,q_2)V_\mu(q_1,q_2)\right)$$



Quark-Gluon Scattering

"What happens in $2 \to 2$-processes with gluons? Surely the t-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \to q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p_b^-}{p_2^-}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p_2^-}{p_b^-}} \right) \langle b|\sigma|2\rangle \; \times \langle 1|\sigma|a\rangle.$$

Complete t-channel factorisation!

J.M.Smillie and JRA

Quark-Gluon Scattering

The *t*-channel current generated by a gluon in qg scattering is that generated by a quark, but with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

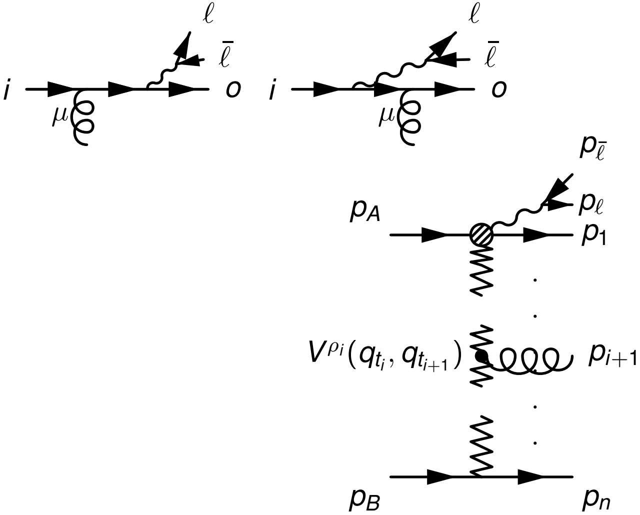
instead of C_F. Tends to C_A in MRK limit.

Similar results for e.g. $g^+g^- \to g^+g^-$. **Exact, complete *t*-channel factorisation**.

By using the formalism of **current-current scattering**, we get a better description of the ***t*-channel poles of the amplitude** than by using just the BFKL kinematic limit.

W+Jets

Two currents to calculate for $W + jets$:



Performing the Explicit Resummation

Soft divergence from real radiation:

$$\left|\mathcal{M}_t^{p_a p_b \to p_0 p_1 p_2 p_3}\right|^2 \stackrel{\mathbf{p}_1^2 \to 0}{\longrightarrow} \left(\frac{4 g_s^2 C_A}{\mathbf{p}_1^2}\right) \left|\mathcal{M}_t^{p_a p_b \to p_0 p_2 p_3}\right|^2$$

Integrate over the soft part $\mathbf{p}_1^2 < \lambda^2$ of phase space in $D = 4 + 2\varepsilon$ dimensions

$$\int_0^\lambda \frac{\mathrm{d}^{2+2\varepsilon}\mathbf{p}\ \mathrm{d}y_1}{(2\pi)^{2+2\varepsilon}\ 4\pi} \left(\frac{4 g_s^2 C_A}{\mathbf{p}^2}\right) \mu^{-2\varepsilon}$$
$$= \frac{4 g_s^2 C_A}{(2\pi)^{2+2\varepsilon} 4\pi}\ \Delta y_{02}\ \frac{\pi^{1+\varepsilon}}{\Gamma(1+\varepsilon)} \frac{1}{\varepsilon} (\lambda^2/\mu^2)^\varepsilon$$

Pole in ε cancels with that from the virtual corrections

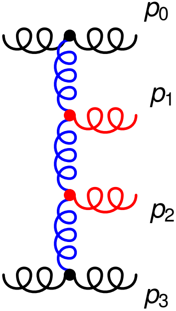
$$\frac{1}{t_1} \to \frac{1}{t_1} \exp\left(\hat{\alpha}(t) \Delta y_{02}\right) \qquad \hat{\alpha}(t) = -\frac{g_s^2 C_A \Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} \left(\mathbf{q}^2/\mu^2\right)^\varepsilon.$$

Expression for the Regularised Amplitude

$$\overline{\left|\mathcal{M}_{\mathrm{HEJ}}^{\mathrm{reg}}(\{p_i\})\right|}^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{f_1 f_2 \to f_1 f_2}\right\|^2 \cdot \left(g^2\ K_{f_1}\ \frac{1}{t_1}\right) \cdot \left(g^2\ K_{f_2}\ \frac{1}{t_{n-1}}\right)$$

$$\cdot \prod_{i=1}^{n-2} \left(g^2 C_A \left(\frac{-1}{t_i t_{i+1}} V^\mu(q_i, q_{i+1}) V_\mu(q_i, q_{i+1}) - \frac{4}{\mathbf{p}_i^2}\ \theta\left(\mathbf{p}_i^2 < \lambda^2\right) \right)\right)$$

$$\cdot \prod_{j=1}^{n-1} \exp\left[\omega^0(q_j,\lambda)(y_{j-1}-y_j)\right], \qquad \omega^0(q_j,\lambda) = -\frac{\alpha_s N_C}{\pi} \log \frac{\mathbf{q}_j^2}{\lambda^2}.$$



Resummed (and Matched) Cross Section

The cross section is calculated as phase space integrals over explicit n-body phase space

$$\sigma_{2j}^{\text{resum,match}} = \sum_{f_1,f_2} \sum_{n=2}^{\infty} \prod_{i=1}^{n} \left(\int \frac{\mathrm{d}^2 \mathbf{p}_{i\perp}}{(2\pi)^3} \int \frac{\mathrm{d}y_i}{2} \right) \frac{\overline{\left|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})\right|}^2}{\hat{s}^2}$$

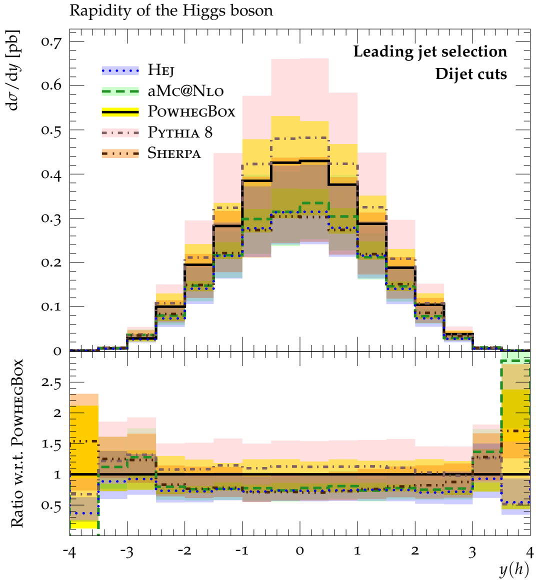
$$\times \quad \mathcal{O}_{2j}(\{p_i\}) \times \sum_m \mathcal{O}^e_{mj}(\{p_i\})\ w_{m-\text{jet}}$$

$$\times \quad x_a f_{A,f_1}(x_a, Q_a)\ x_2 f_{B,f_2}(x_b, Q_b)\ (2\pi)^4\ \delta^2\!\left(\sum_{i=1}^{n} \mathbf{p}_{i\perp}\right).$$

Matching to fixed order (tree-level so far) is obtained by clustering the n-parton phase space point into m-jet momenta and multiply by ratio of full to approximate matrix element:

$$w_{m-\text{jet}} \equiv \frac{\overline{\left|\mathcal{M}^{f_1 f_2 \to f_1 g \cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}{\overline{\left|\mathcal{M}^{t, f_1 f_2 \to f_1 g \cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}.$$

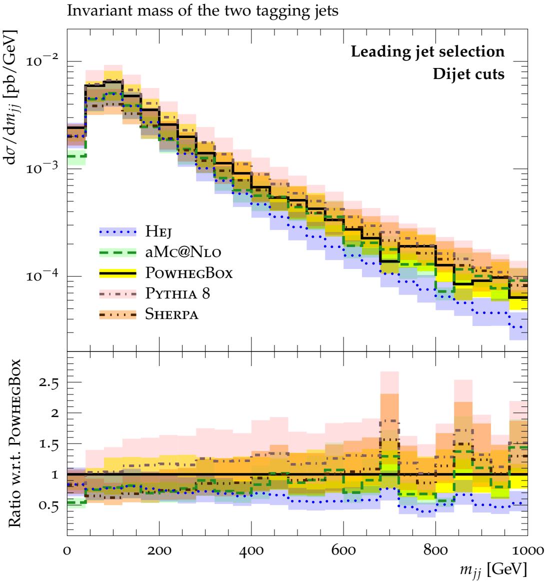
Les Houches Comparison of HJJ Predictions



Good agreement of inclusive *Hjj*-cross section and differential distributions (e.g. rapidity of the Higgs boson).

Variations within the uncertainty quoted for each calculation.

Les Houches Comparison of HJJ Predictions

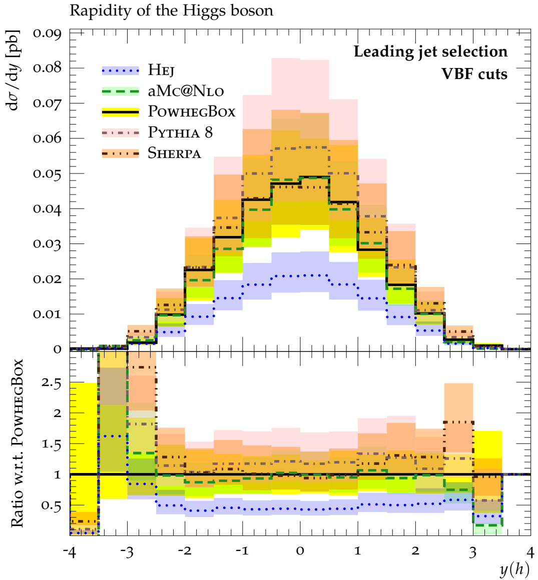


Differences arising at large invariant mass between the hard jets.
(as expected)

Vector-Boson-Fusion cuts select region of large m_{jj}.

(We will revisit large m_{jj} in W+Dijets).

Les Houches Comparison of HJJ Predictions



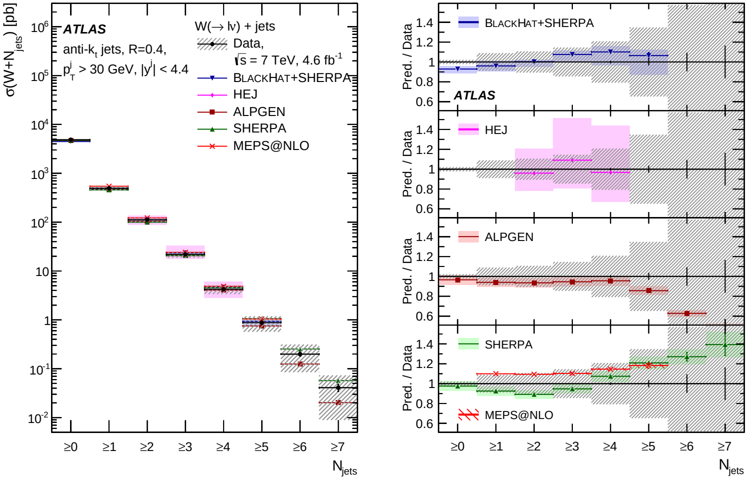
The difference in the distribution of m_{jj} (and Δy_{12}) induce a difference in the cross section after VBF-cuts.

The difference in behaviour between shower-approaches and HEJ appear at large rapidities and large m_{jj} - where HEJ resums virtual corrections that are not treated systematically in any of the other approaches.

Theory vs. Data

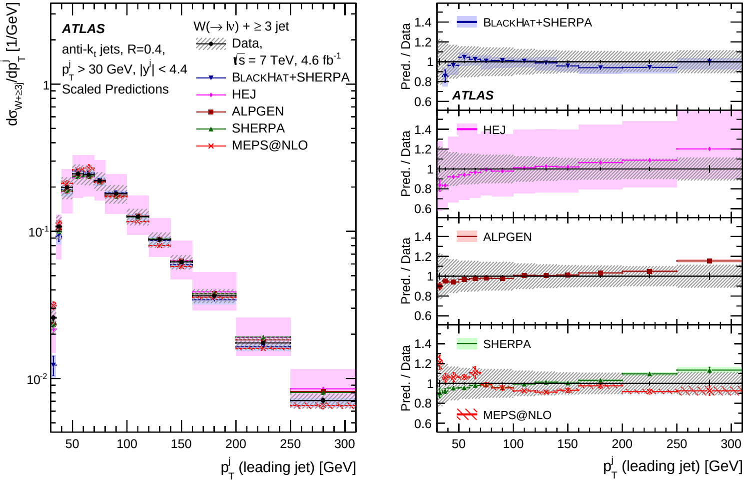
Example: ATLAS W+jets.

W+DiJets



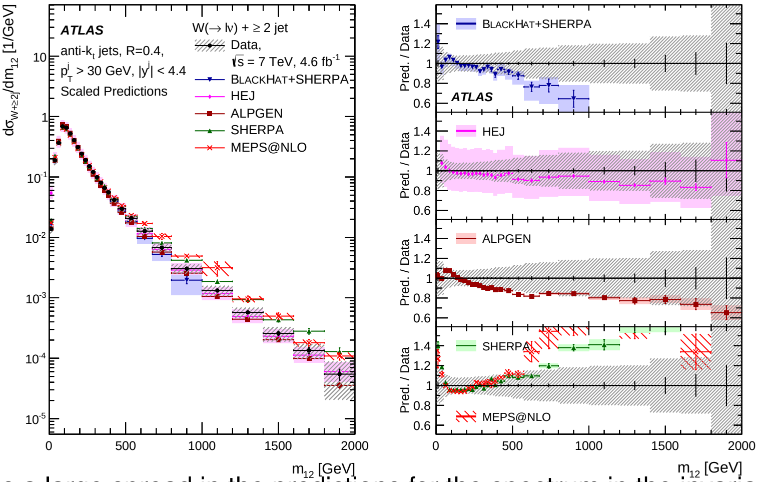
Good agreement between all predictions and data - on inclusive quantities.

W+DiJets



For standard p_t-based observables, all predictions give a reasonable description (NLO is very good!).

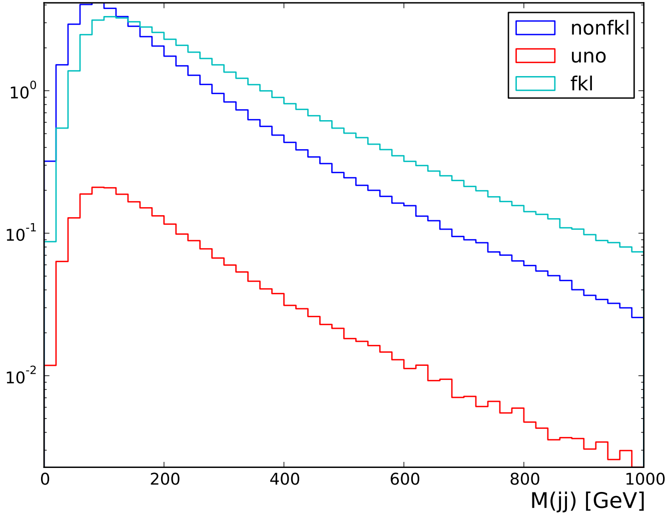
W+DiJets



There is a large spread in the predictions for the spectrum in the invariant mass between the two hardest jets. Here, the terms systematically dealt with in HEJ are important, and HEJ gives a good description.

Note: hjj interesting for $m_{jj} > 400 - 600$GeV.

W+DiJets



Sub-leading corrections small at large mjj - matching corrections small.

First set of sub-leading corrections included in the all-order treatment.

Summary

- Hadron colliders probes hard (=jets) perturbative corrections beyond pure NLO . . . already at 2, 7TeV!
- ***High Energy Jets**** provides a new approach to the perturbative description of proton collider physics
  . . . and compares favourably to data in several analyses
  . . . several ongoing improvements in the formal accuracy of the perturbative approximations

* http://cern.ch/hej