Advances in QCD and Higgs phenomenology

Annual Theory Meeting

Durham 2014

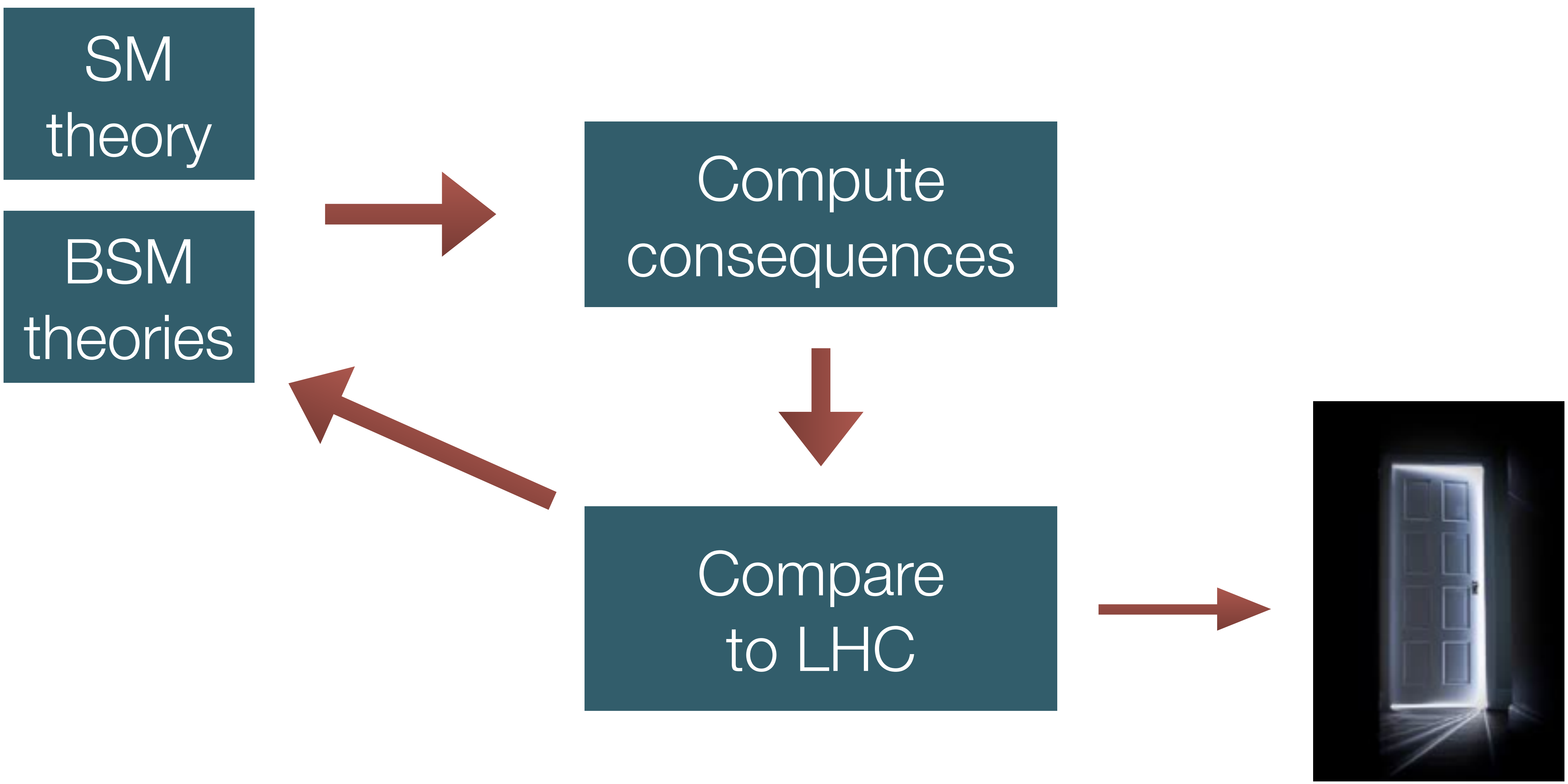
Babis Anastasiou
ETH Zurich

Breakthroughs in phenomenology

Perturbative Methods in QCD

HIGGS DISCOVERY

LHC precision physics

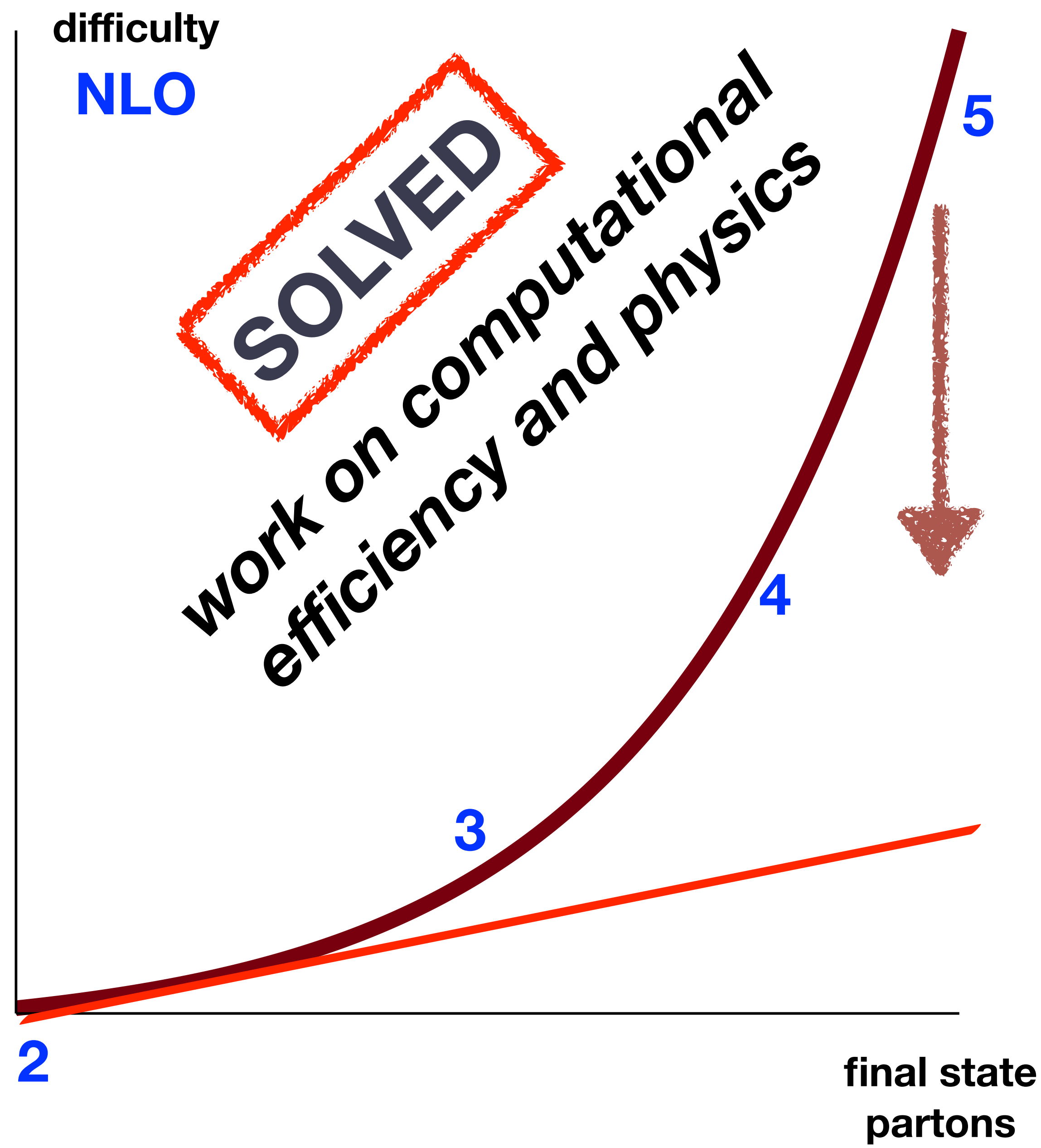


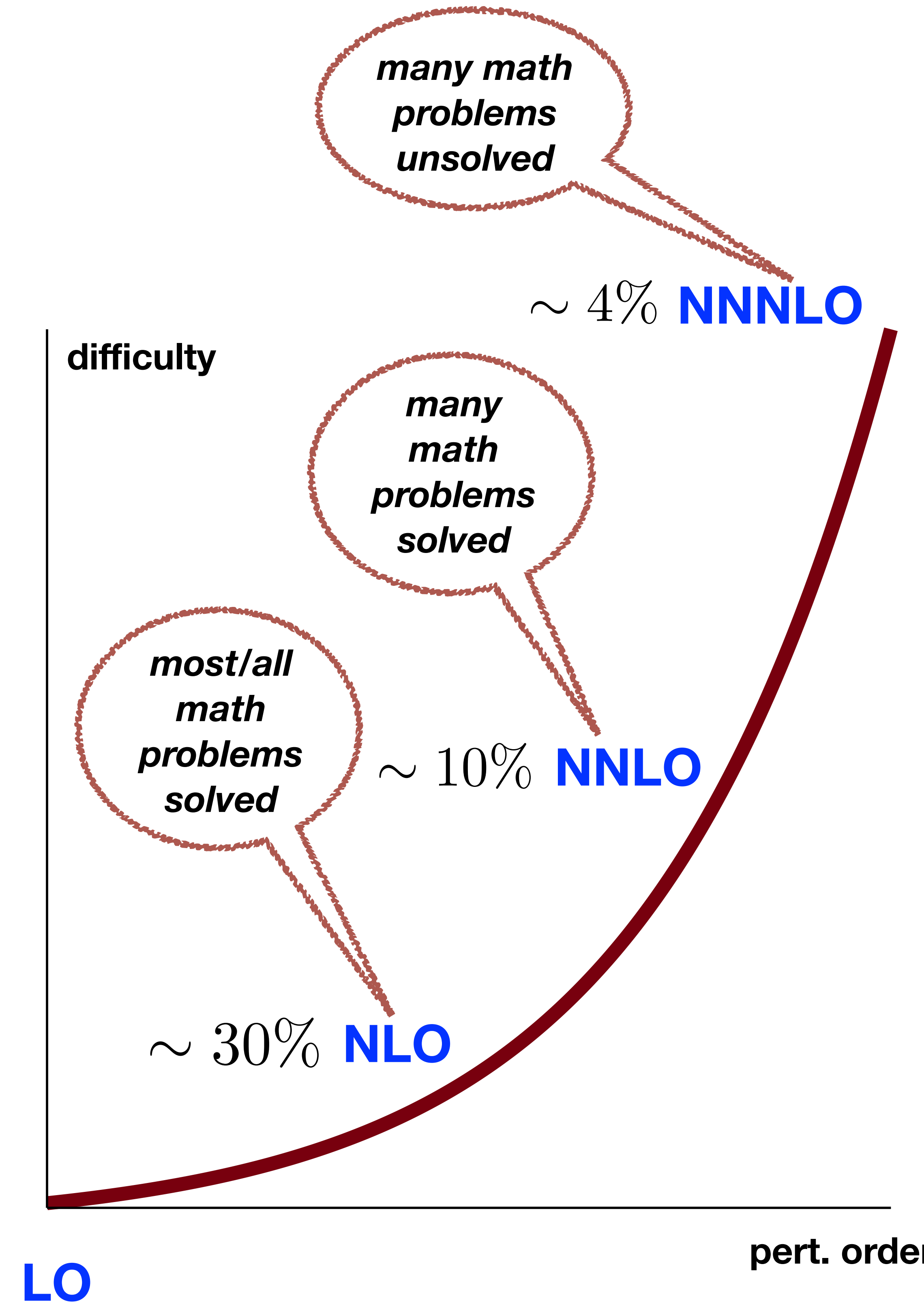
A “Feynman diagram” of finding *New Laws*

Not a review talk of pert. QCD.
Excellent review talk by Thomas Gehrmann in the 2013 meeting.

- Some highlights since then
- Focus on the youngest branch of pert. QCD computations: N3LO

Perturbative QCD

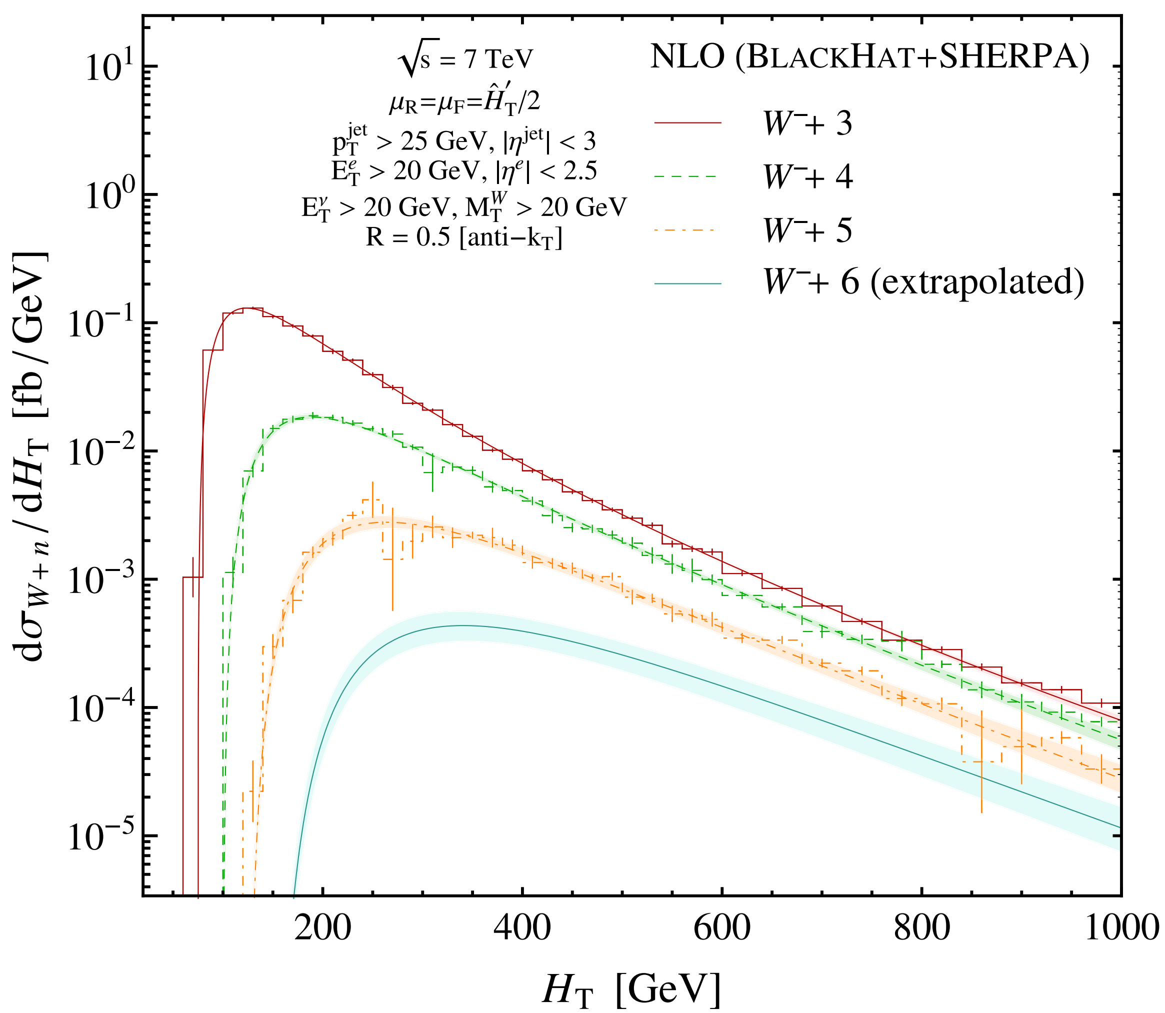


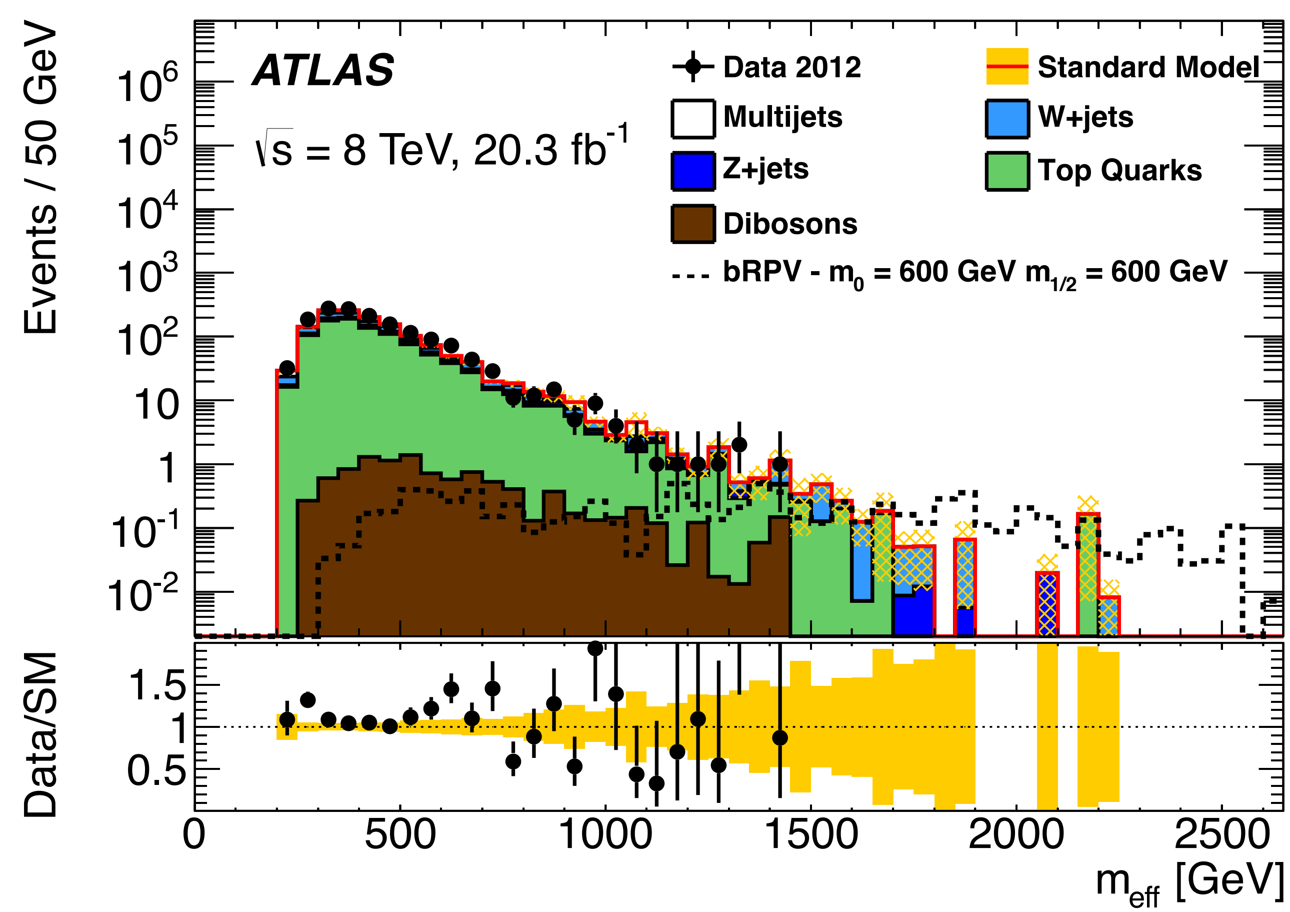


$$\sigma = \sigma_0 \alpha_s^n + \sigma_1 \alpha_s^{n+1} + \sigma_2 \alpha_s^{n+2} + \dots$$

NLO for very high multiplicities

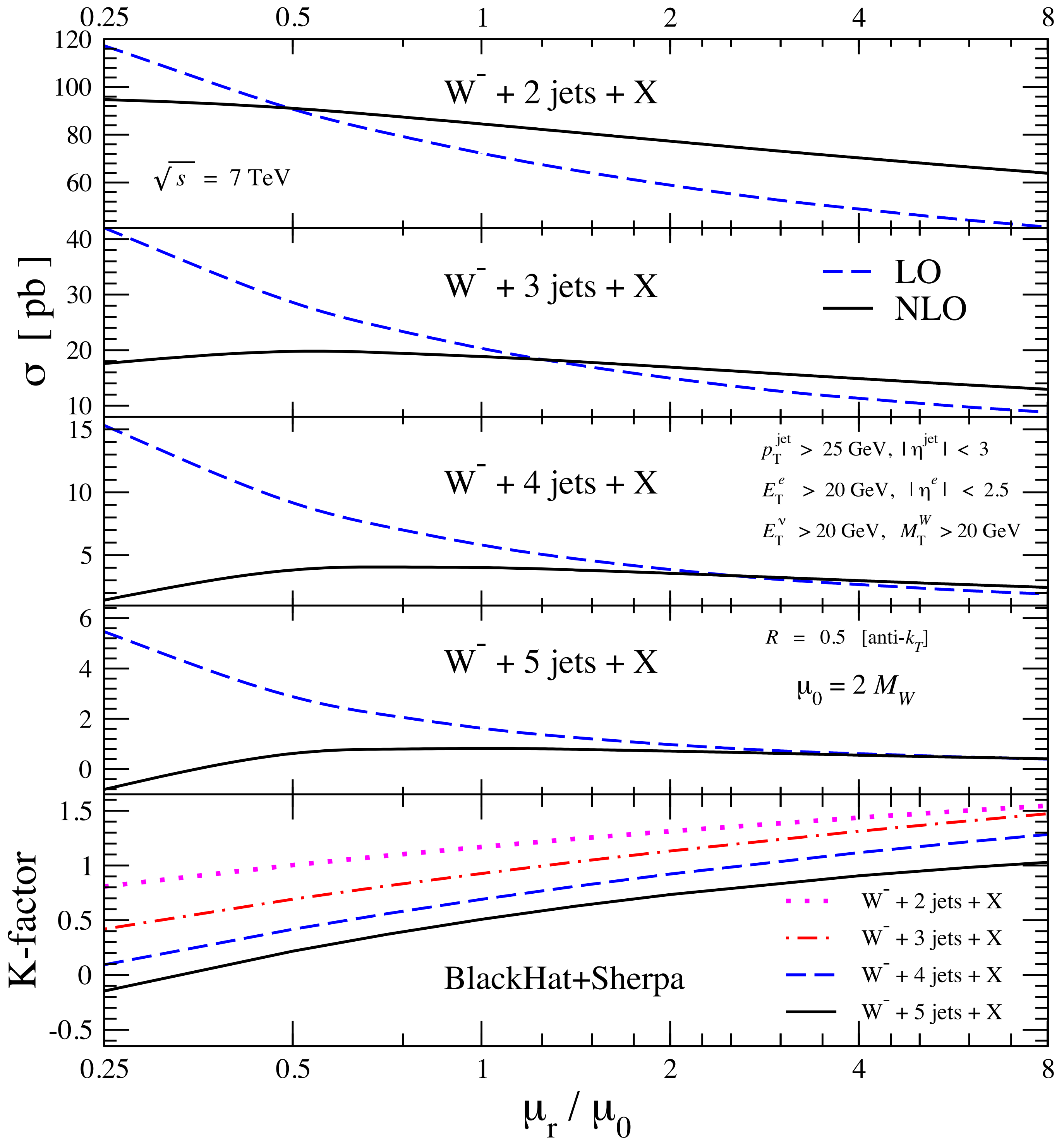
- Backgrounds to new physics
- Experiments can push their searches to small corners of phase-space





(a) $\tau + e$ bRPV SR, $N_{\mathrm{jet}} \geq 4$

NLO for very high multiplicities



- Stability in varying renormalisation and factorisation scales.
- Better modelling of jets, which at NLO acquire a structure with content, size and non-zero invariant mass.
- Quantitatively accurate estimate of the cross-section

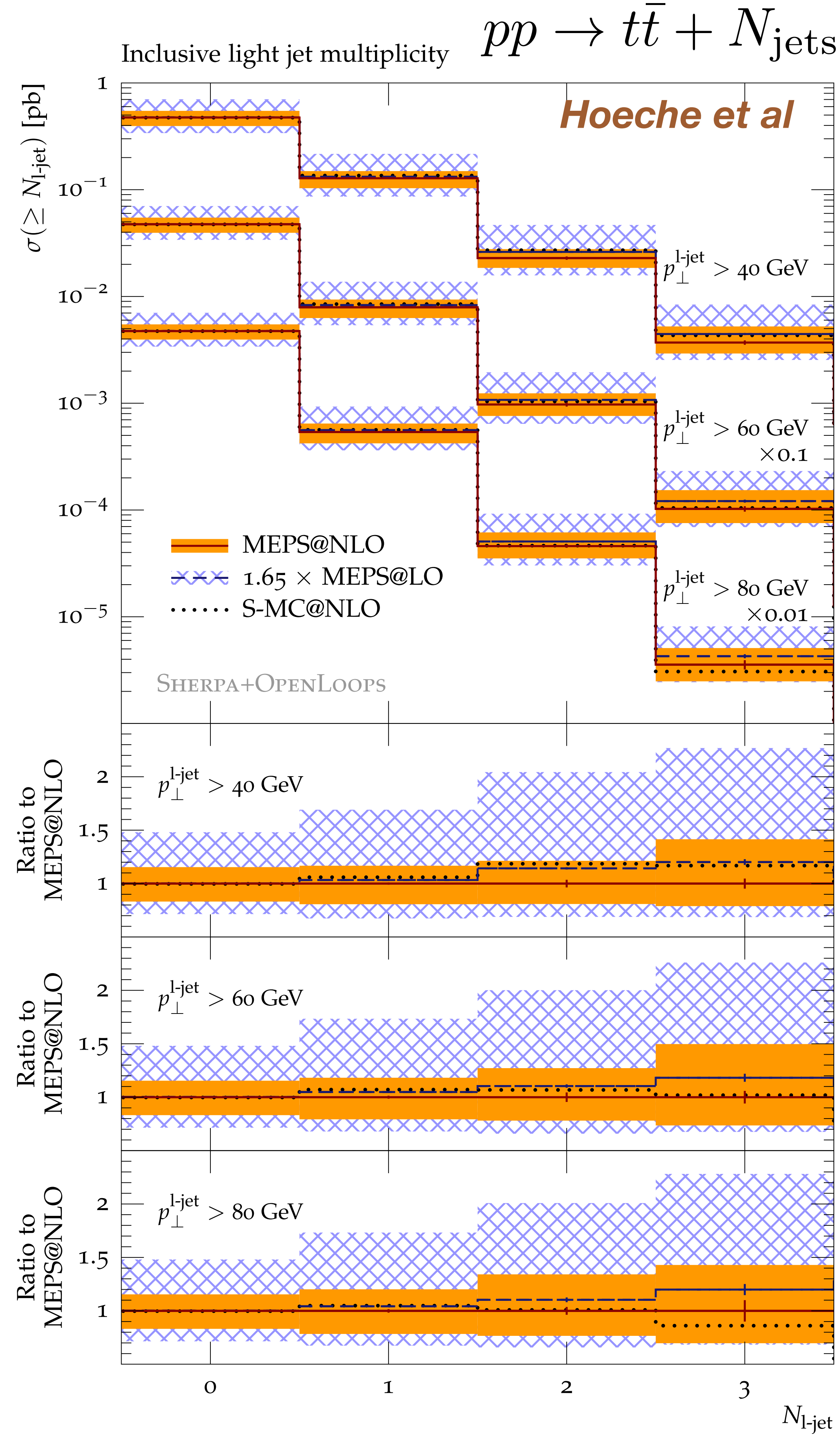
NLO merging with parton showers

- Methods for merging parton showers and NLO calculations for final states with varying jet multiplicity.
- Enormous progress in automation of NLO and NLO merging.

MADGRAPH5_aMC@NLO

	Process	Syntax	Cross section (pb) LO 13 TeV	Cross section (pb) NLO 13 TeV
	Heavy quarks+vector bosons			
e.1	$pp \to W^{\pm} b\bar{b}$ (4f)	`p p > wpm b b~`	$3.074 \pm 0.002 \cdot 10^{2}$ $^{+42.3\%}_{-29.2\%}$ $^{+2.0\%}_{-1.6\%}$	$8.162 \pm 0.034 \cdot 10^{2}$ $^{+29.8\%}_{-23.6\%}$ $^{+1.5\%}_{-1.2\%}$
e.2	$pp \to Z b\bar{b}$ (4f)	`p p > z b b~`	$6.993 \pm 0.003 \cdot 10^{2}$ $^{+33.5\%}_{-24.4\%}$ $^{+1.0\%}_{-1.4\%}$	$1.235 \pm 0.004 \cdot 10^{3}$ $^{+19.9\%}_{-17.4\%}$ $^{+1.0\%}_{-1.4\%}$
e.3	$pp \to \gamma b\bar{b}$ (4f)	`p p > a b b~`	$1.731 \pm 0.001 \cdot 10^{3}$ $^{+51.9\%}_{-34.8\%}$ $^{+1.6\%}_{-2.1\%}$	$4.171 \pm 0.015 \cdot 10^{3}$ $^{+33.7\%}_{-27.1\%}$ $^{+1.4\%}_{-1.9\%}$
e.4*	$pp \to W^{\pm} b\bar{b} j$ (4f)	`p p > wpm b b~ j`	$1.861 \pm 0.003 \cdot 10^{2}$ $^{+42.5\%}_{-27.7\%}$ $^{+0.7\%}_{-0.7\%}$	$3.957 \pm 0.013 \cdot 10^{2}$ $^{+27.0\%}_{-21.0\%}$ $^{+0.7\%}_{-0.6\%}$
e.5*	$pp \to Z b\bar{b} j$ (4f)	`p p > z b b~ j`	$1.604 \pm 0.001 \cdot 10^{2}$ $^{+42.4\%}_{-27.6\%}$ $^{+0.9\%}_{-1.1\%}$	$2.805 \pm 0.009 \cdot 10^{2}$ $^{+21.0\%}_{-17.6\%}$ $^{+0.8\%}_{-1.0\%}$
e.6*	$pp \to \gamma b\bar{b} j$ (4f)	`p p > a b b~ j`	$7.812 \pm 0.017 \cdot 10^{2}$ $^{+51.2\%}_{-32.0\%}$ $^{+1.0\%}_{-1.5\%}$	$1.233 \pm 0.004 \cdot 10^{3}$ $^{+18.9\%}_{-19.9\%}$ $^{+1.0\%}_{-1.5\%}$
e.7	$pp \to t\bar{t} W^{\pm}$	`p p > t t~ wpm`	$3.777 \pm 0.003 \cdot 10^{-1}$ $^{+23.9\%}_{-18.0\%}$ $^{+2.1\%}_{-1.6\%}$	$5.662 \pm 0.021 \cdot 10^{-1}$ $^{+11.2\%}_{-10.6\%}$ $^{+1.7\%}_{-1.3\%}$
e.8	$pp \to t\bar{t} Z$	`p p > t t~ z`	$5.273 \pm 0.004 \cdot 10^{-1}$ $^{+30.5\%}_{-21.8\%}$ $^{+1.8\%}_{-2.1\%}$	$7.598 \pm 0.026 \cdot 10^{-1}$ $^{+9.7\%}_{-11.1\%}$ $^{+1.9\%}_{-2.2\%}$
e.9	$pp \to t\bar{t} \gamma$	`p p > t t~ a`	$1.204 \pm 0.001 \cdot 10^{0}$ $^{+29.6\%}_{-21.3\%}$ $^{+1.6\%}_{-1.8\%}$	$1.744 \pm 0.005 \cdot 10^{0}$ $^{+9.8\%}_{-11.0\%}$ $^{+1.7\%}_{-2.0\%}$
e.10*	$pp \to t\bar{t} W^{\pm} j$	`p p > t t~ wpm j`	$2.352 \pm 0.002 \cdot 10^{-1}$ $^{+40.9\%}_{-27.1\%}$ $^{+1.3\%}_{-1.0\%}$	$3.404 \pm 0.011 \cdot 10^{-1}$ $^{+11.2\%}_{-14.0\%}$ $^{+1.2\%}_{-0.9\%}$
e.11*	$pp \to t\bar{t} Z j$	`p p > t t~ z j`	$3.953 \pm 0.004 \cdot 10^{-1}$ $^{+46.2\%}_{-29.5\%}$ $^{+2.7\%}_{-3.0\%}$	$5.074 \pm 0.016 \cdot 10^{-1}$ $^{+7.0\%}_{-12.3\%}$ $^{+2.5\%}_{-2.9\%}$
e.12*	$pp \to t\bar{t} \gamma j$	`p p > t t~ a j`	$8.726 \pm 0.010 \cdot 10^{[illegible]}$ $^{+45.4\%}_{-29.1\%}$ $^{+2.3\%}_{-2.6\%}$	$1.135 \pm 0.004 \cdot 10^{0}$ $^{+7.5\%}_{-12.2\%}$ $^{+2.2\%}_{-2.5\%}$
e.13*	$pp \to t\bar{t} W^{-} W^{+}$ (4f)	`p p > t t~ w+ w-`	$6.675 \pm 0.00[illegible] \cdot 10^{-3}$ $^{+30.9\%}_{-21.9\%}$ $^{+2.1\%}_{-2.0\%}$	$9.904 \pm 0.026 \cdot 10^{-3}$ $^{+10.9\%}_{-11.8\%}$ $^{+2.1\%}_{-2.1\%}$
e.14*	$pp \to t\bar{t} W^{\pm} Z$	`p p > t t~ wpm z`	$2.404 \pm 0.002 \cdot 10^{-3}$ $^{+26.6\%}_{-19.6\%}$ $^{+2.5\%}_{-1.8\%}$	$3.525 \pm 0.010 \cdot 10^{-3}$ $^{+10.6\%}_{-10.8\%}$ $^{+2.3\%}_{-1.6\%}$
e.15*	$pp \to t\bar{t} W^{\pm} \gamma$	`p p > t t~ wpm a`	$2.718 \pm 0.003 \cdot 10^{-3}$ $^{+25.4\%}_{-18.9\%}$ $^{+2.3\%}_{-1.8\%}$	$3.927 \pm 0.013 \cdot 10^{-3}$ $^{+10.3\%}_{-10.4\%}$ $^{+2.0\%}_{-1.5\%}$
e.16*	$pp \to t\bar{t} Z Z$	`p p > t t~ z z`	$1.34[illegible] \pm 0.014 \cdot 10^{-3}$ $^{+29.3\%}_{-21.1\%}$ $^{+1.7\%}_{-1.5\%}$	$1.840 \pm 0.007 \cdot 10^{-3}$ $^{+7.9\%}_{-9.9\%}$ $^{+1.7\%}_{-1.5\%}$
e.17*	$pp \to t\bar{t} Z \gamma$	`p p > t t~ z a`	$[illegible].548 \pm 0.003 \cdot 10^{-3}$ $^{+30.1\%}_{-21.5\%}$ $^{+1.7\%}_{-1.6\%}$	$3.656 \pm 0.012 \cdot 10^{-3}$ $^{+9.7\%}_{-11.0\%}$ $^{+1.8\%}_{-1.9\%}$
e.18*	$pp \to t\bar{t} \gamma\gamma$	`p p > t t~ a a`	$3.272 \pm 0.006 \cdot 10^{-3}$ $^{+28.4\%}_{-20.6\%}$ $^{+1.3\%}_{-1.1\%}$	$4.402 \pm 0.015 \cdot 10^{-3}$ $^{+7.8\%}_{-9.7\%}$ $^{+1.4\%}_{-1.4\%}$

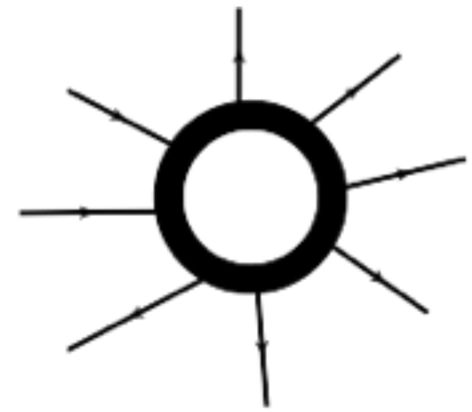
	Process	Syntax	Cross section (pb) LO 13 TeV	Cross section (pb) NLO 13 TeV
	Heavy quarks and jets			
d.1	$pp \to jj$	`p p > j j`	$1.162 \pm 0.001 \cdot 10^{6}$ $^{+24.9\%}_{-18.8\%}$ $^{+0.8\%}_{-0.9\%}$	$1.580 \pm 0.007 \cdot 10^{6}$ $^{+8.4\%}_{-9.0\%}$ $^{+0.7\%}_{-0.9\%}$
d.2	$pp \to jjj$	`p p > j j j`	$8.940 \pm 0.021 \cdot 10^{4}$ $^{+43.8\%}_{-28.4\%}$ $^{+1.2\%}_{-1.4\%}$	$7.791 \pm 0.037 \cdot 10^{4}$ $^{+2.1\%}_{-23.2\%}$ $^{+1.1\%}_{-1.3\%}$
d.3	$pp \to b\bar{b}$ (4f)	`p p > b b~`	$3.743 \pm 0.004 \cdot 10^{3}$ $^{+25.2\%}_{-18.9\%}$ $^{+1.5\%}_{-1.8\%}$	$6.438 \pm 0.028 \cdot 10^{3}$ $^{+15.9\%}_{-13.3\%}$ $^{+1.5\%}_{-1.7\%}$
d.4*	$pp \to b\bar{b}j$ (4f)	`p p > b b~ j`	$1.050 \pm 0.002 \cdot 10^{3}$ $^{+44.1\%}_{-28.5\%}$ $^{+1.6\%}_{-1.8\%}$	$1.327 \pm 0.007 \cdot 10^{3}$ $^{+6.8\%}_{-11.6\%}$ $^{+1.5\%}_{-1.8\%}$
d.5*	$pp \to b\bar{b}jj$ (4f)	`p p > b b~ j j`	$1.852 \pm 0.006 \cdot 10^{2}$ $^{+61.8\%}_{-35.6\%}$ $^{+2.1\%}_{-2.4\%}$	$2.471 \pm 0.012 \cdot 10^{2}$ $^{+8.2\%}_{-16.4\%}$ $^{+2.0\%}_{-2.3\%}$
d.6	$pp \to b\bar{b}b\bar{b}$ (4f)	`p p > b b~ b b~`	$5.050 \pm 0.007 \cdot 10^{-1}$ $^{+61.7\%}_{-35.6\%}$ $^{+2.9\%}_{-3.4\%}$	$8.736 \pm 0.034 \cdot 10^{-1}$ $^{+20.9\%}_{-22.0\%}$ $^{+2.9\%}_{-3.4\%}$
d.7	$pp \to t\bar{t}$	`p p > t t~`	$4.584 \pm 0.003 \cdot 10^{2}$ $^{+29.0\%}_{-21.1\%}$ $^{+1.8\%}_{-2.0\%}$	$6.741 \pm 0.023 \cdot 10^{2}$ $^{+9.8\%}_{-10.9\%}$ $^{+1.8\%}_{-2.1\%}$
d.8	$pp \to t\bar{t}j$	`p p > t t~ j`	$3.135 \pm 0.002 \cdot 10^{2}$ $^{+45.1\%}_{-29.0\%}$ $^{+2.2\%}_{-2.5\%}$	$4.106 \pm 0.015 \cdot 10^{2}$ $^{+8.1\%}_{-12.2\%}$ $^{+2.1\%}_{-2.5\%}$



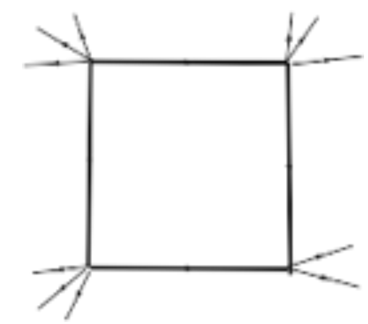
NLO: why is it solved?

- One-loop amplitudes in gauge theories = (Tree-amplitudes in gauge-theories) and (Integrals in scalar field theories)

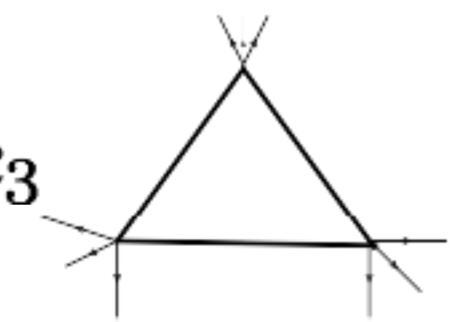
- Singularities of tree-amplitudes due to radiation of a single parton understood for arbitrary processes.



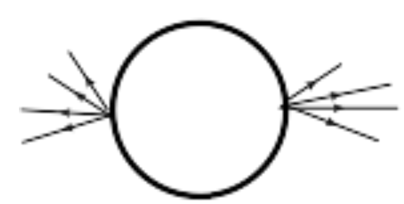
$=$ c_4



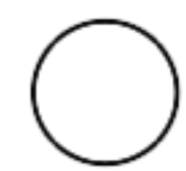
$+c_3$



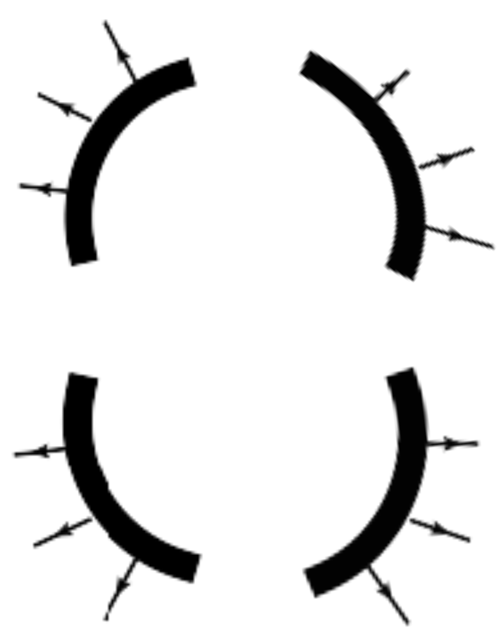
$+c_2$



$+c_1$



$c_4 =$



$= Tree \times Tree \times Tree \times Tree$

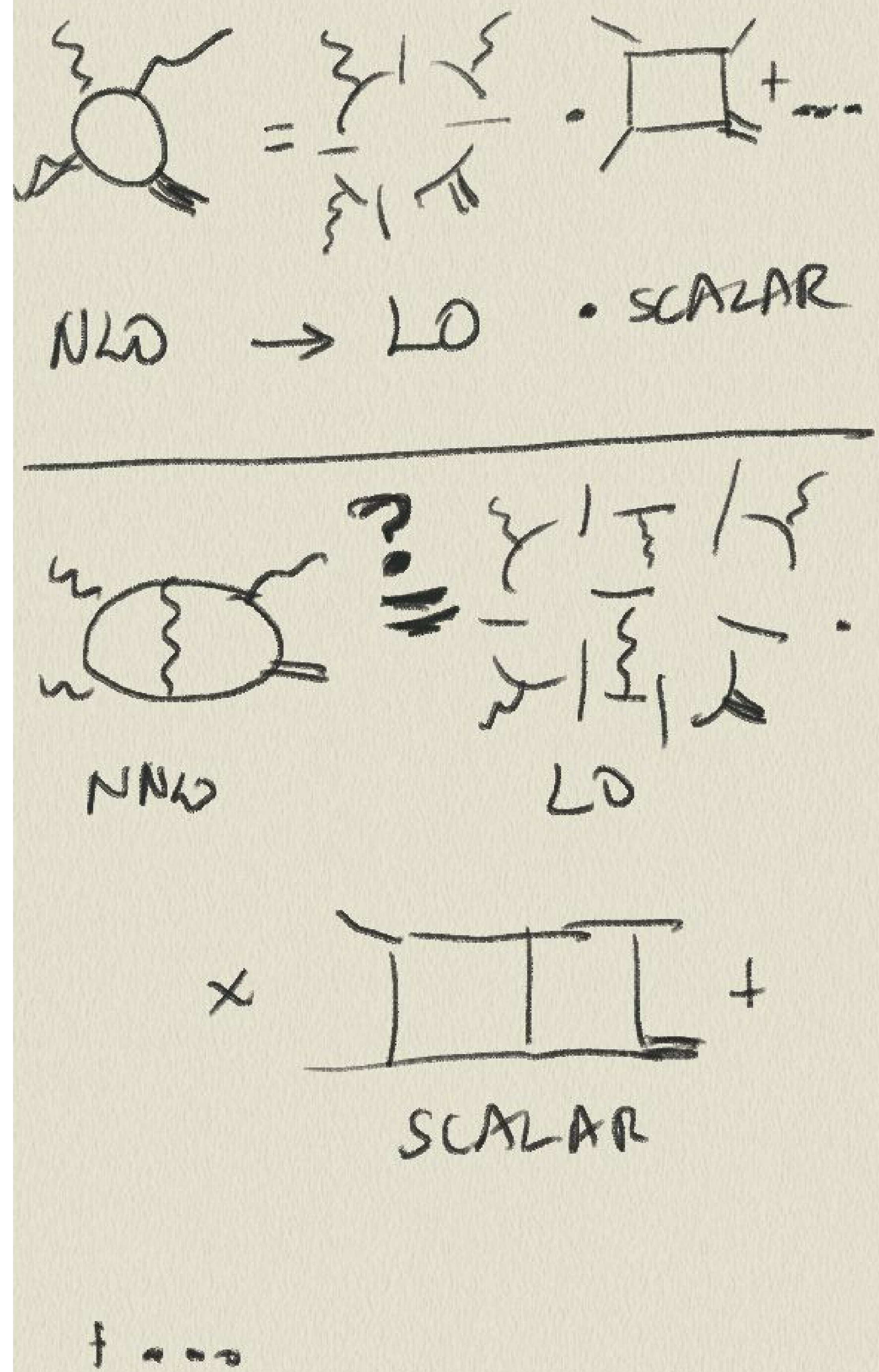
From NLO to NNLO

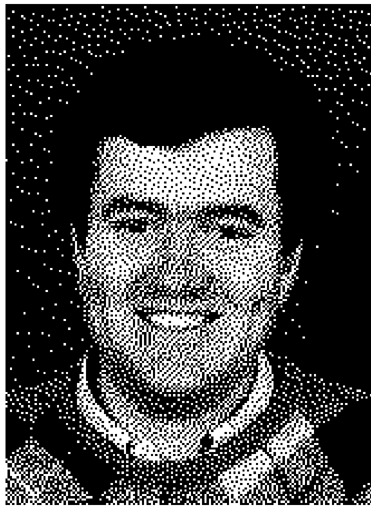
- A very beautiful structure of perturbation theory at NLO, where we can reduce the cross-section calculations to a few scalar integrals and LO calculations (infrared limits, master integral coefficients)

- It makes one dream that also higher orders NNLO, NNNLO, etc may be reduced to a few scalar integrals and LO calculations.

- Such a structure has not arisen yet.

- But progress is fast over the last decade with increasingly sophisticated methods.





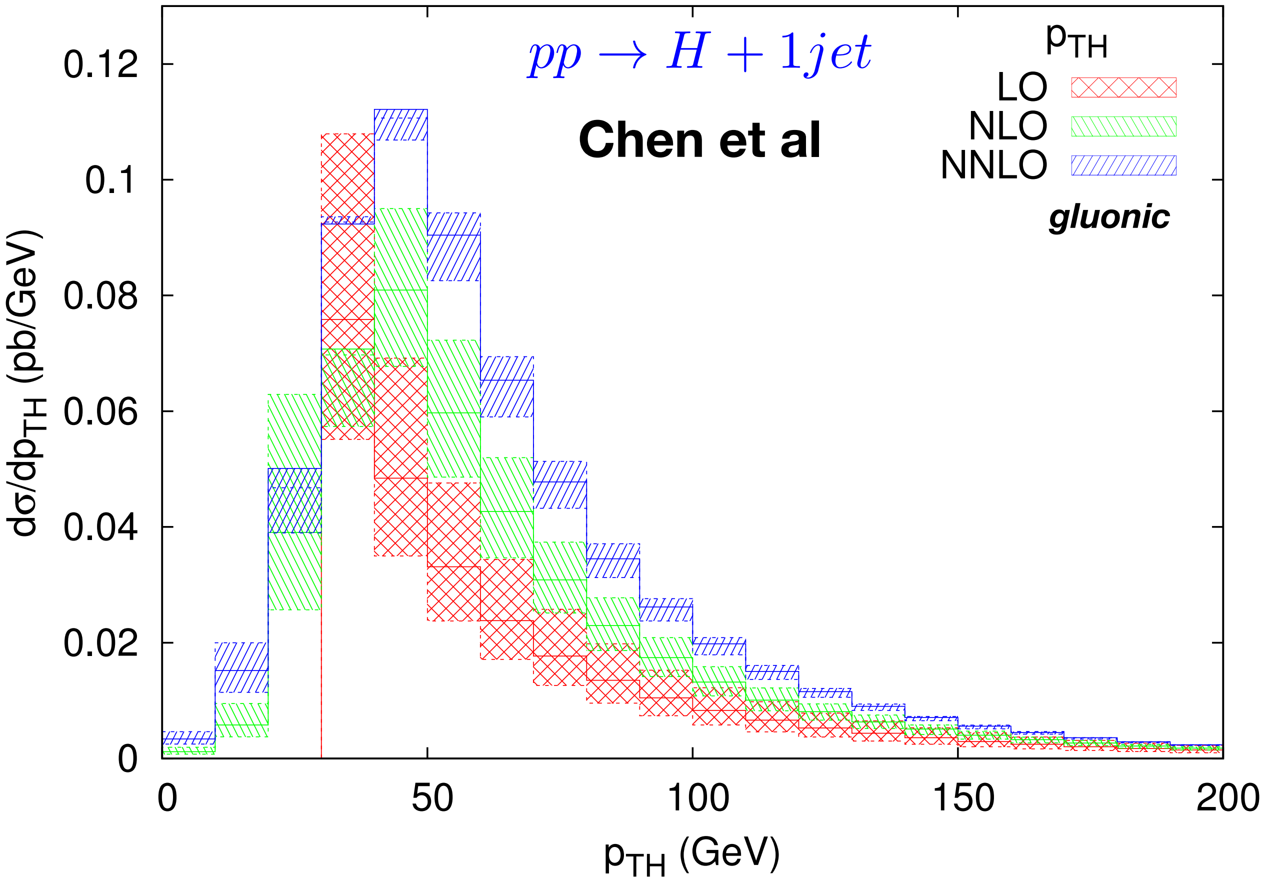
“However, while the NLO program has been extremely successful in reducing the theoretical uncertainty, the improvements in the data are even more impressive, to the extent that the theoretical error tends to dominate.”

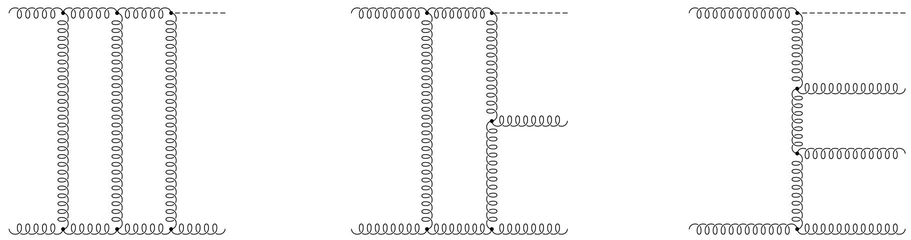
“One way to improve the theoretical predictions is to incorporate next-to-next-to-leading order (NNLO) effects.”

“Although a complete NNLO jet calculation is some way off, and for $2 \to 2$ processes the two-loop double box integrals are not even known yet, some encouraging steps have been taken in this direction.”

arXiv:hep-ph/9805481

NNLO computations in 2014 for LHC



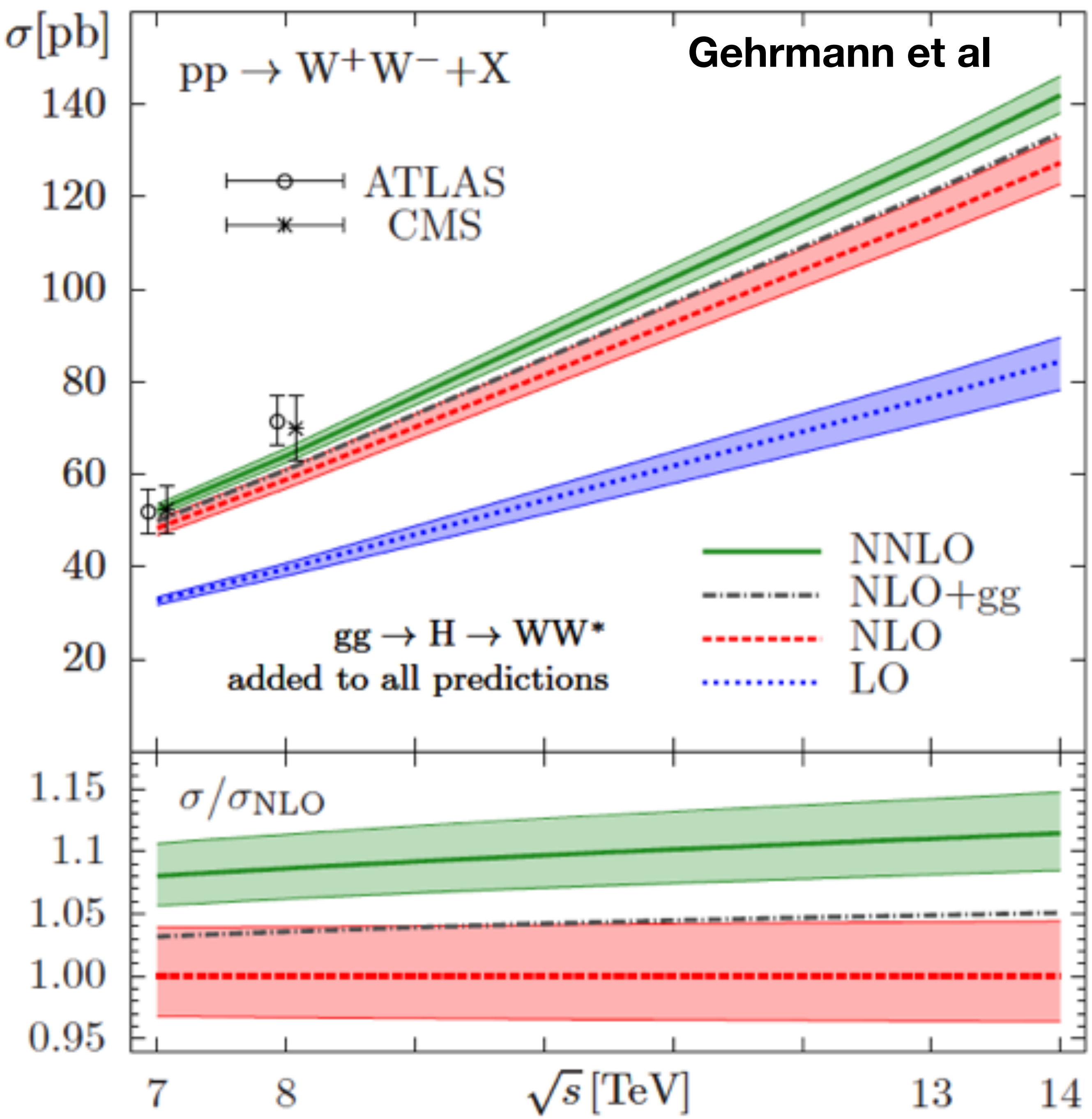


$$d\sigma_{gg,NNLO} = \int_{d\Phi_3} \left[d\sigma^{RR}_{gg,NNLO} - d\sigma^{S}_{gg,NNLO}\right] + \int_{d\Phi_2} \left[d\sigma^{RV}_{gg,NNLO} - d\sigma^{T}_{gg,NNLO}\right] + \int_{d\Phi_1} \left[d\sigma^{VV}_{gg,NNLO} - d\sigma^{U}_{gg,NNLO}\right]$$

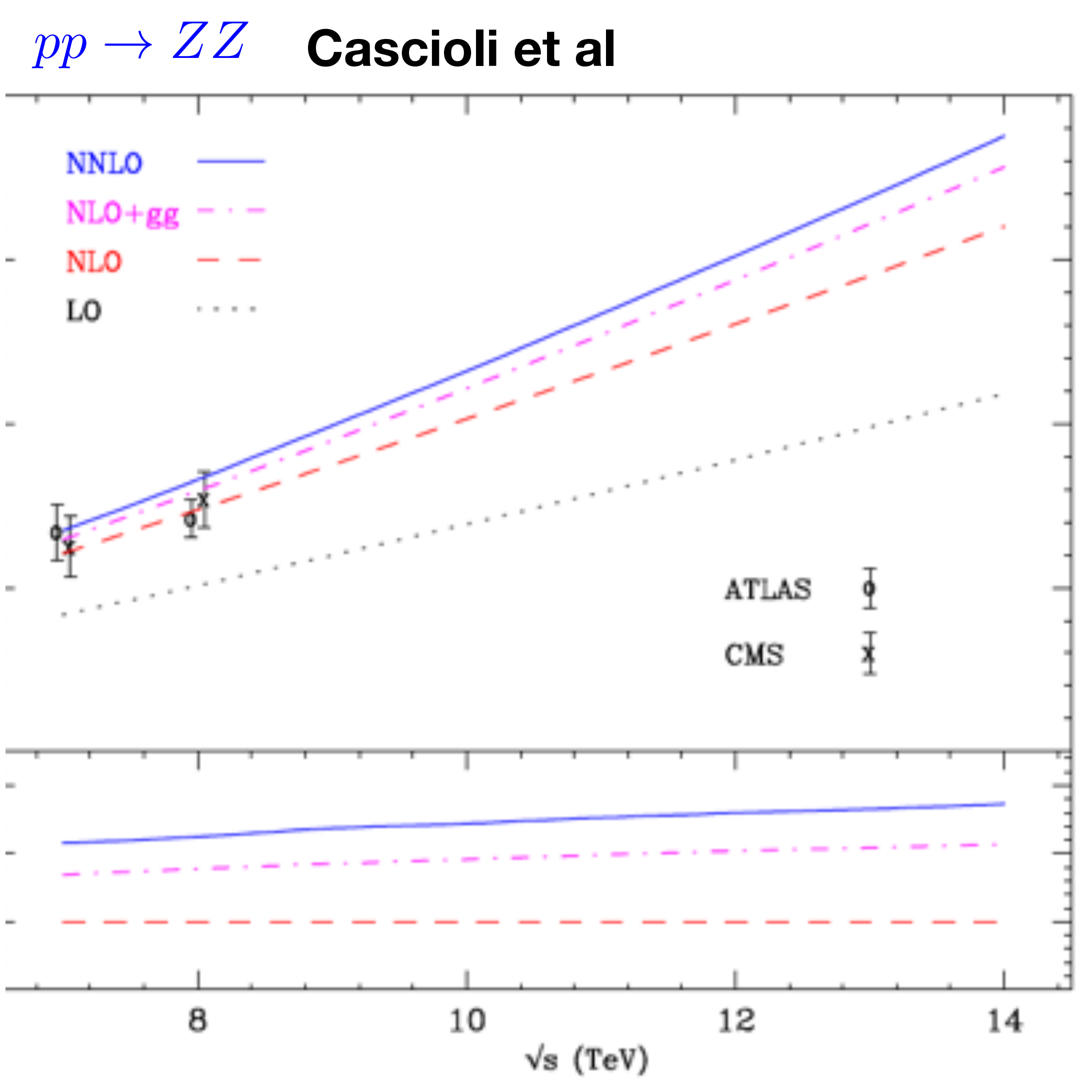
$$\sigma_{LO} = 2.72^{+1.22}_{-0.78} \ \text{pb}\,,$$
$$\sigma_{NLO} = 4.38^{+0.76}_{-0.74} \ \text{pb}\,,$$
$$\sigma_{NNLO} = 6.34^{+0.28}_{-0.49} \ \text{pb}\,,$$

Chen et al, also by Boughezal et al

- H+1jet with two different methods.
- Reaching maturity in understanding the universal infrared structure of QCD radiation at NNLO.
- NNLO effects are not cosmetic: large corrections
- More reliable modelling of "edges"



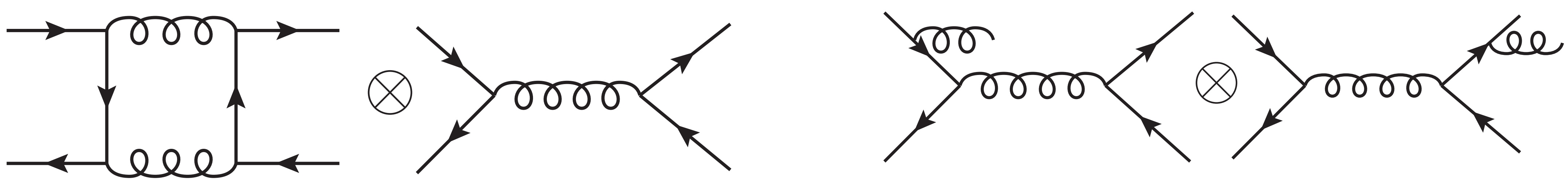
σ[pb]
pp → W⁺W⁻+X
Gehrmann et al
ATLAS
CMS
gg → H → WW*
added to all predictions
NNLO
NLO+gg
NLO
LO
σ/σNLO
√s [TeV]



pp → ZZ
Cascioli et al
NNLO
NLO+gg
NLO
LO
ATLAS
CMS
√s (TeV)

Top-charge asymmetry

A perturbative QCD effect



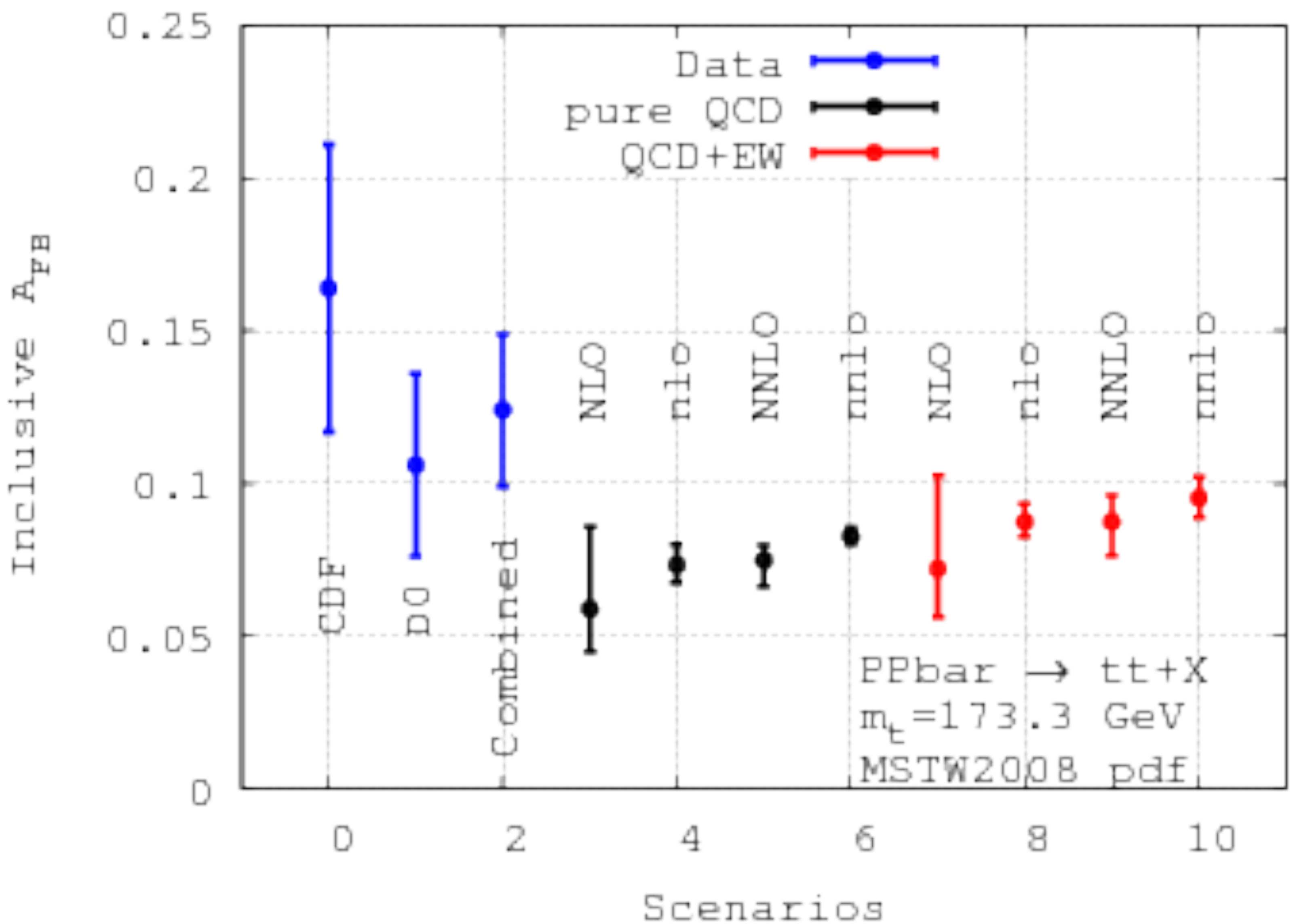
Sensitive to new physics

- *Gluon excitations*
- *Coloured scalars*
- *New gauge bosons*

Tevatron measurements left room for beyond QCD asymmetry

Top-charge asymmetry

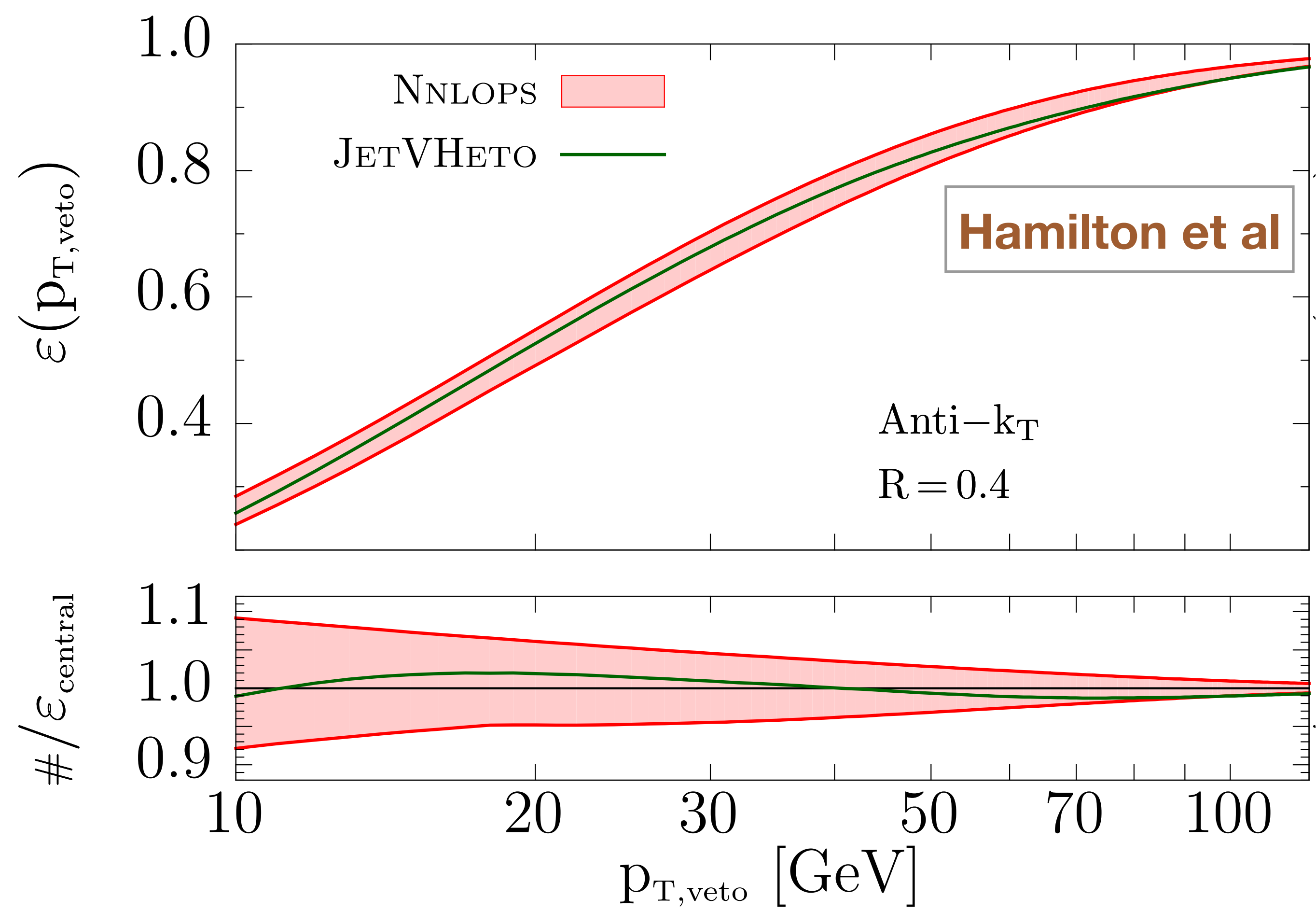
NNLO: Czakon, Fiedler, Mitov



- NNLO is the first non-trivial order.
- An important missing piece of the puzzle for a long time.
- Computed this Fall: brings Standard Model prediction in closer agreement with the final D0 and CDF measurements.

Higgs at NNLO with Parton Shower

- Advances in merging parton shower and varying jet multiplicity at NLO.
- Fully differential Higgs production at NNLO
- Merging of NNLO and parton-shower for Higgs and Drell-Yan production
  - Higgs+2jets @ LO + PS
  - Higgs +1jet @ NLO+PS
  - Higgs@NLO +PS
  - Higgs rapidity @ NNLO
- A very important breakthrough for a very accurate description of Higgs signal events.
- Before NNLOPS we were reweighing NLOPS to the NNLO cross-section. No need to do this anymore.



Commitment and breakthroughs

Process	2-Loop	Monte-Carlo
H (gluon fusion)	1999	2005
Drell-Yan	1987	2006
LEP 3-jets	2002	2008
diphoton	2001	2011
WH	1987	2011
H (bottom fusion)	1999	2012
top-pair	2013	2013
WW	2014	2014
ZZ	2014	2014
H+1jet	2012	2013 (gluons)
jet inclusive	2001	2013 (gluons)

Future of NNLO precision QCD

- Spectacular results have been achieved at NNLO.
- Understanding of infrared divergences and developing the subtraction formalisms for their cancelation is maturing.
- Ever more powerful methods for the computation of two-loop amplitudes are now emerging.
- All 2->2 LHC processes can be computed at NNLO in the next few years. First 2->3 processes by 2020?

“However, while the NNLO program has been extremely successful in reducing the theoretical uncertainty, the improvements in the data are even more impressive, to the extent that the theoretical error tends to dominate.”

“One way to improve the theoretical predictions is to incorporate next-to-next-to-next-to-leading order (N3LO) effects.”

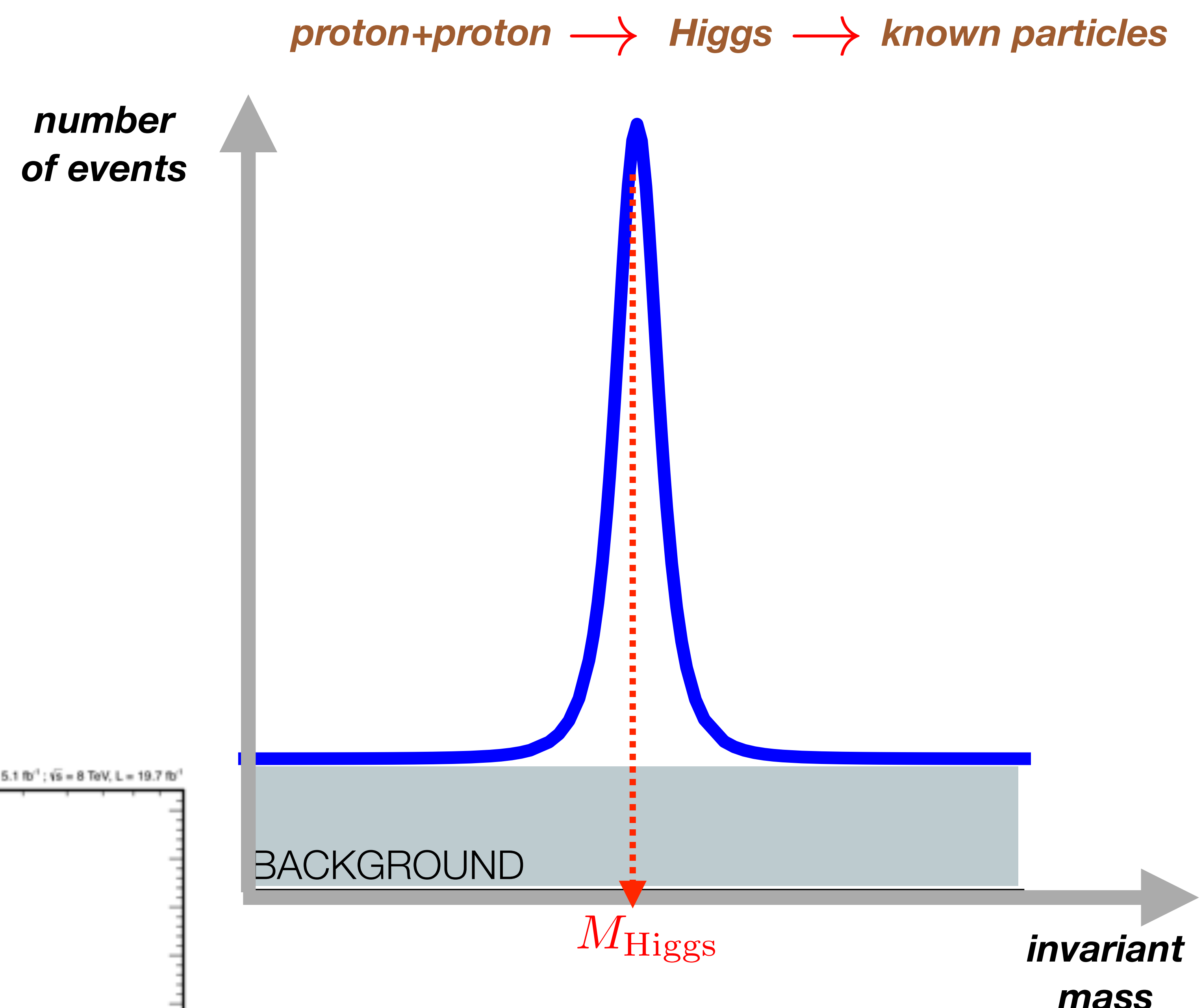
“Although NNLO calculations for many basic LHC processes are missing and our NNLO methods are under development, some encouraging steps have been taken towards N3LO.”

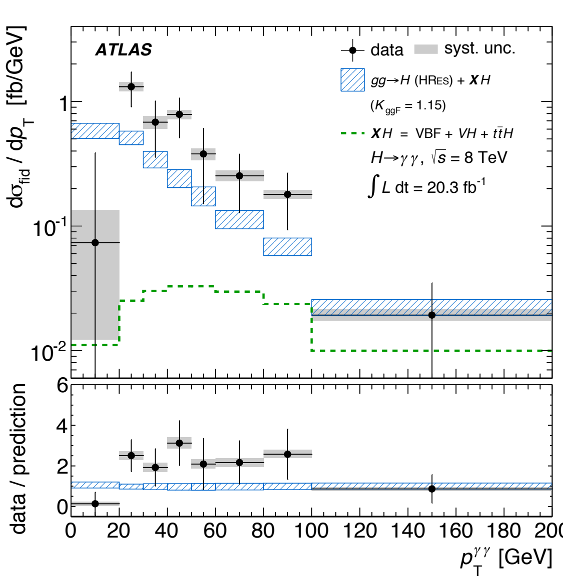
Higgs precision physics

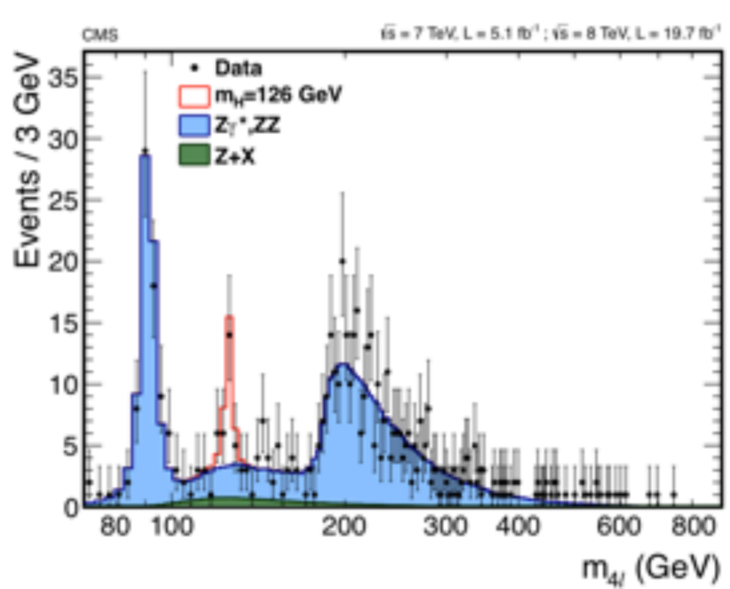
- Higgs couplings: are there other fundamental sources to the mass of elementary particles other than the Higgs field?
- Are there heavy particles which interact with the Higgs boson?
- Can the Higgs boson decay to dark matter?
- What gives mass to the Higgs boson? Why 125 GeV?
- How does the Higgs boson interact with itself?
- ….

Higgs signals

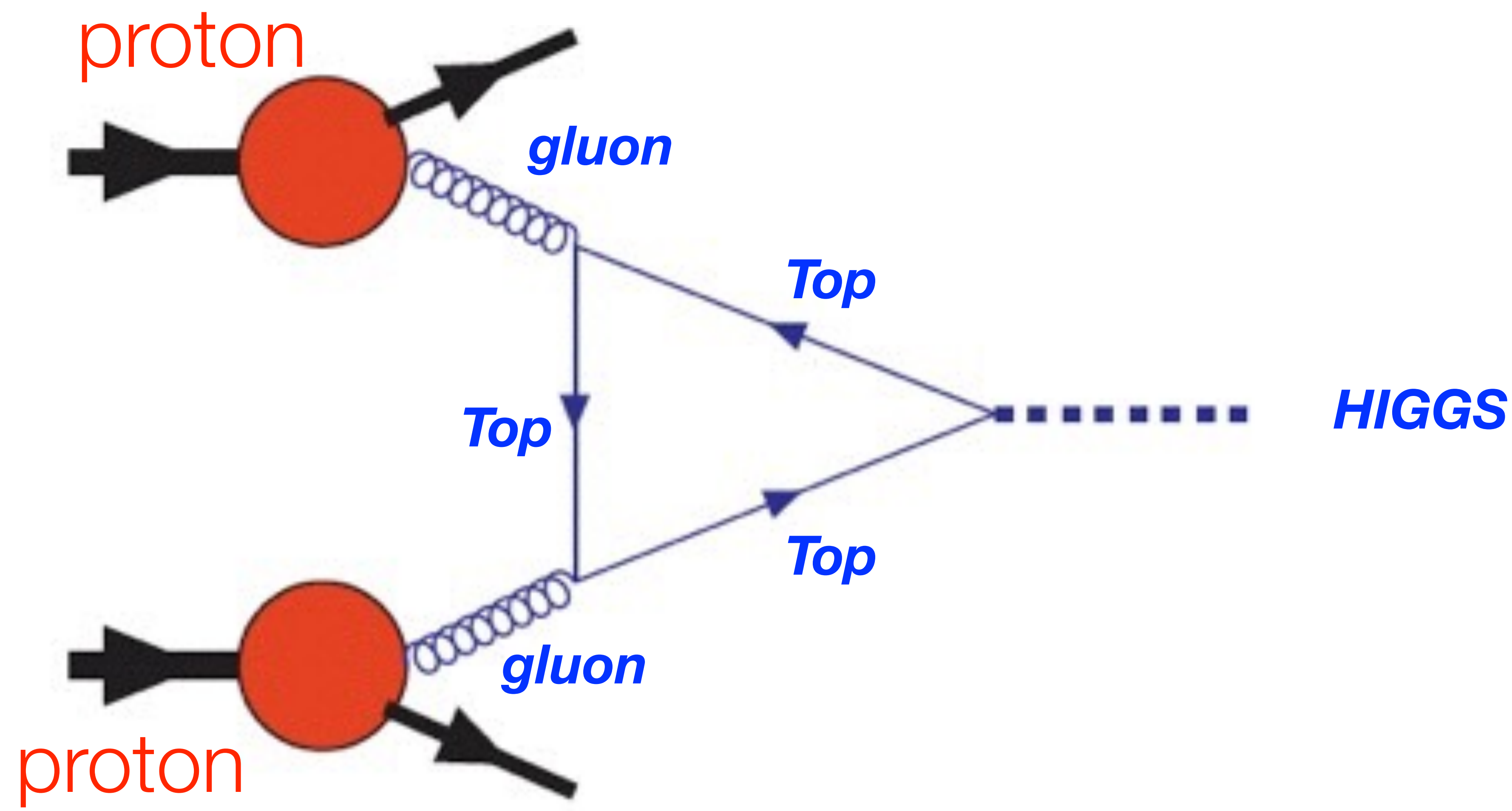
- ***Higgs mass***
- ***Signal strengths***
- ***Width and shape of resonance?***
- ***Differential distributions***





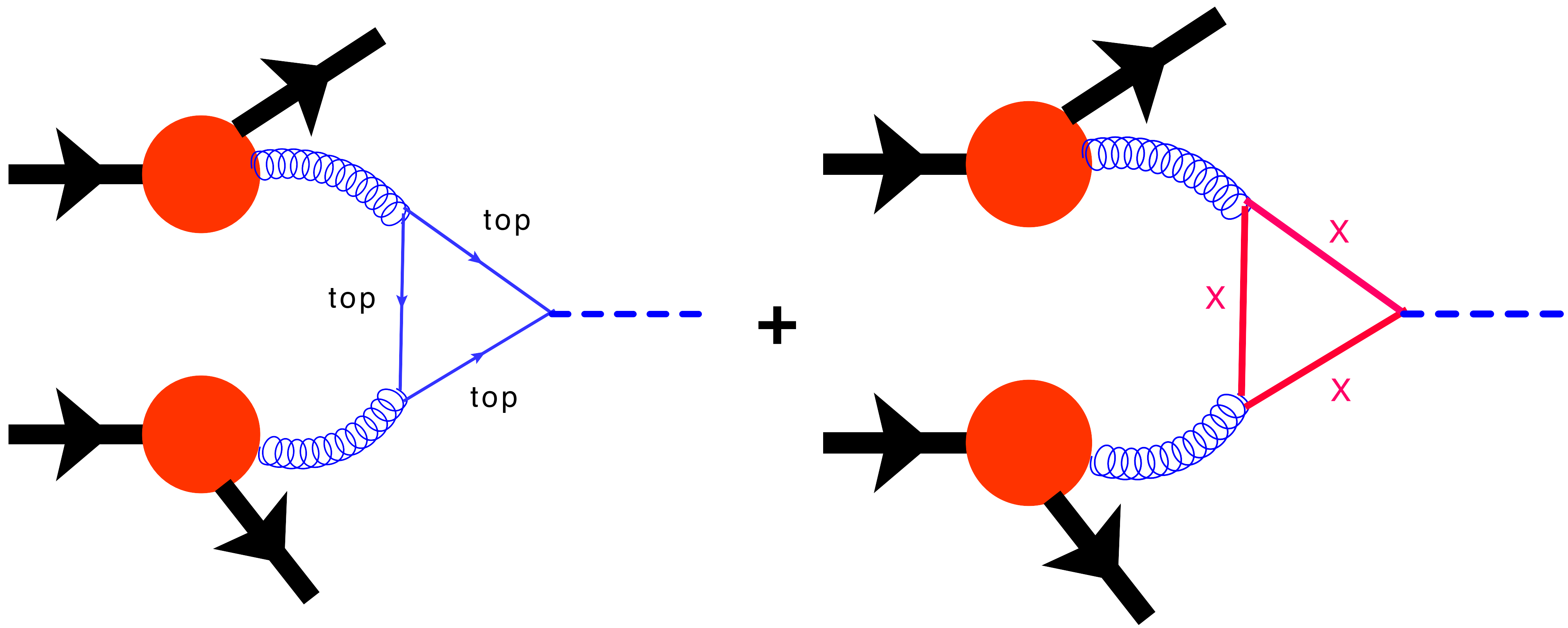


Other mass generation mechanisms?



$$\sigma(pp \to H) \sim \frac{(\text{Higgs-Top coupling})^2}{M_{\text{top}}^2} \sim \frac{(M_{\text{top}} \text{ from Higgs field})^2}{(M_{\text{top}} \text{ from all sources})^2}$$

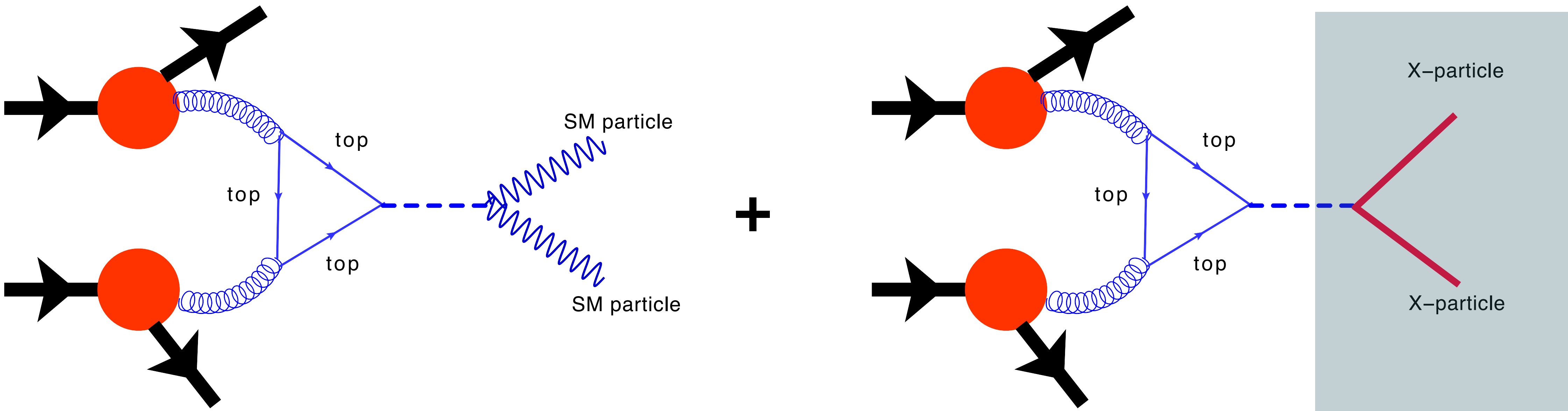
New particles?



New heavy particles may couple to the Higgs boson.
Hard to produce directly. But can alter the cross-section:

$$\sigma(pp \to H) \sim \left| \frac{(M_{\text{top}} \text{ from Higgs field})}{(M_{\text{top}} \text{ from all sources})} + \frac{(M_{\text{X}} \text{ from Higgs field})}{(M_{\text{X}} \text{ from all sources})} \right|^2$$

New particles?

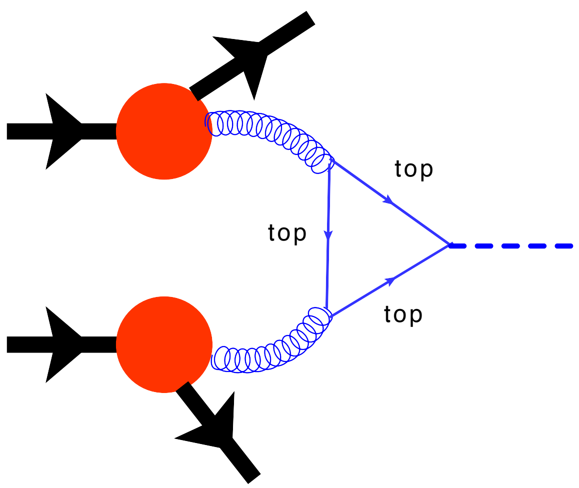


dark matter?

New light particles may couple to the Higgs boson.
Hard to detect (e.g. dark matter). Can alter the branching ratios:

$$\sum_{\text{detectable signals}} BR(H \to \text{known particles}) + \boxed{BR(H \to \text{dark matter})} = 1$$

Role of precise theory predictions



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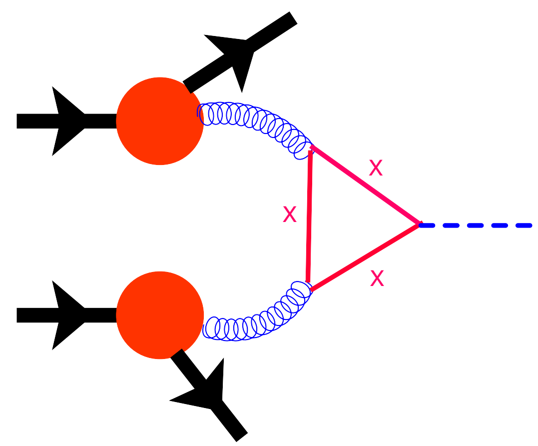
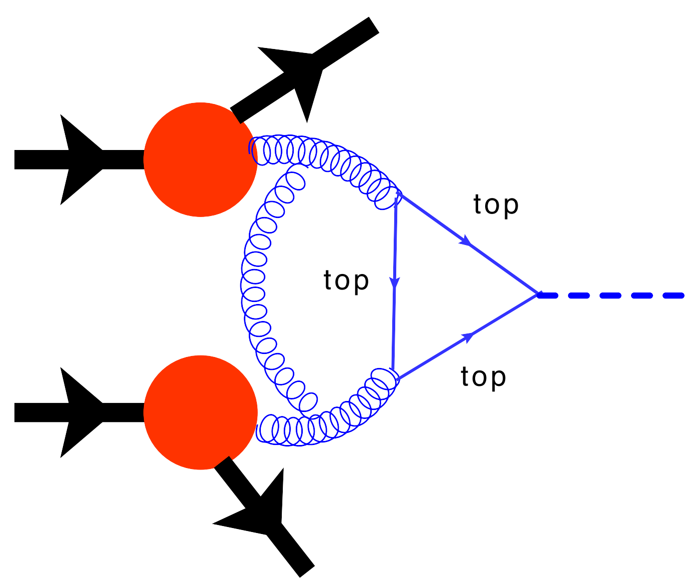


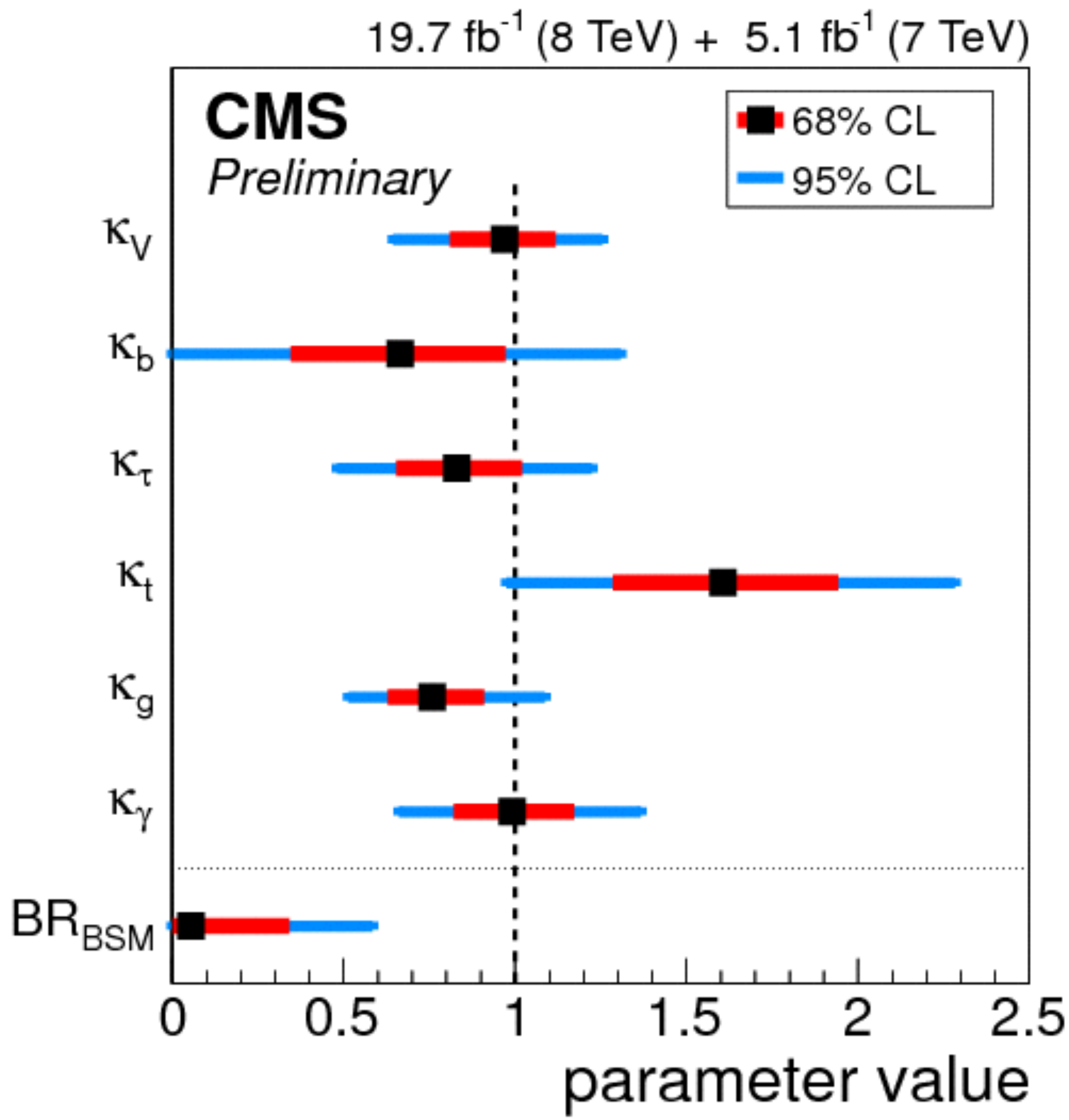
NEW PHYSICS

or



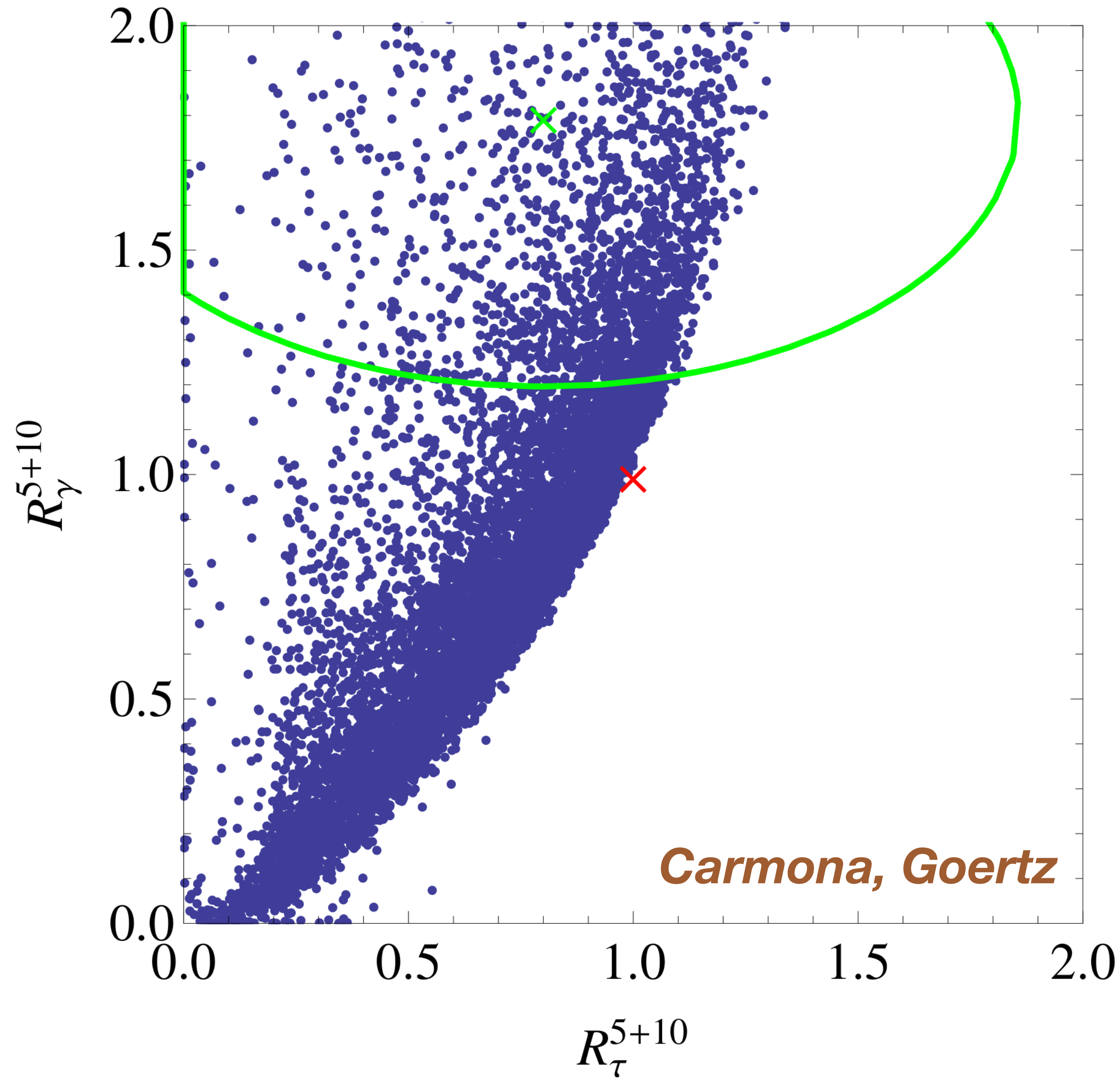
omitted Standard Model effects?

Higgs boson precision physics

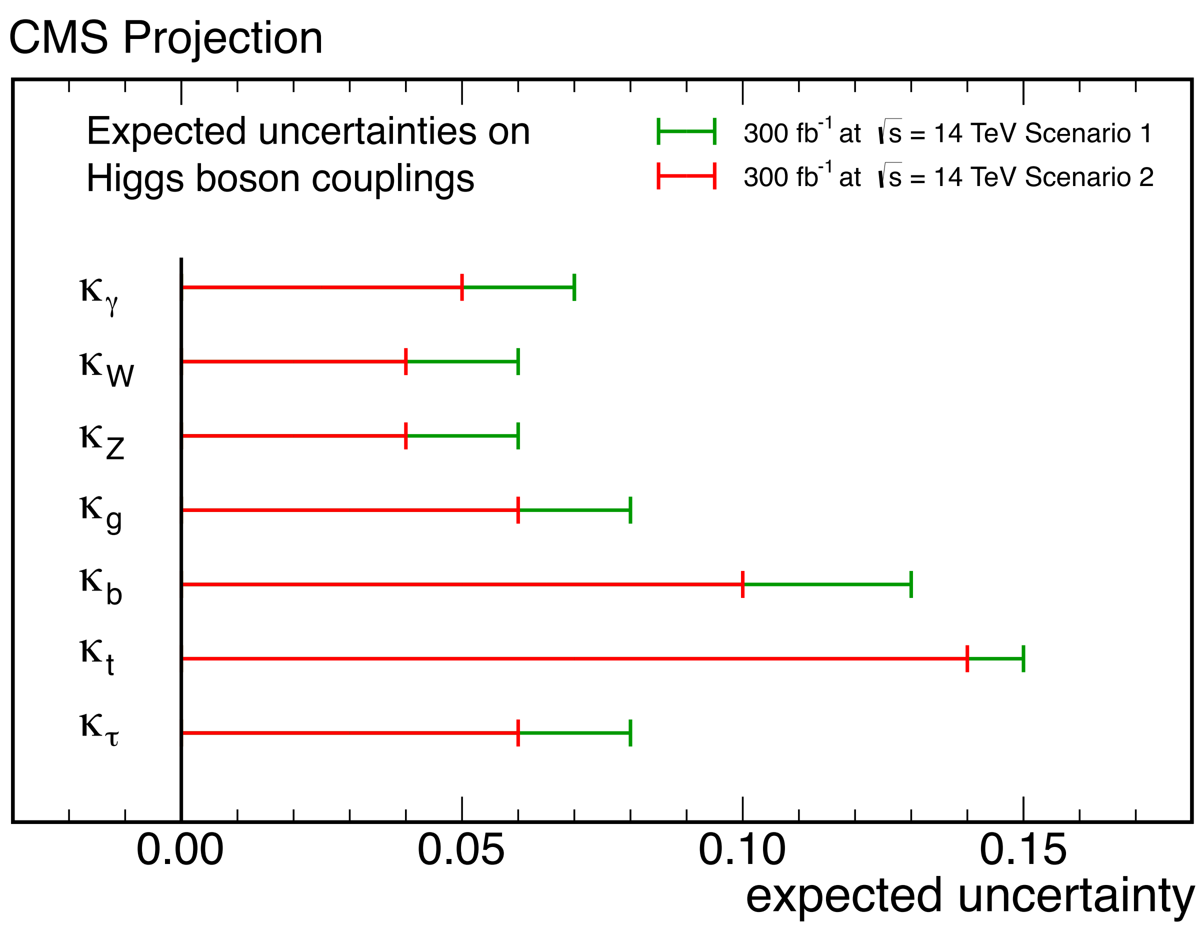


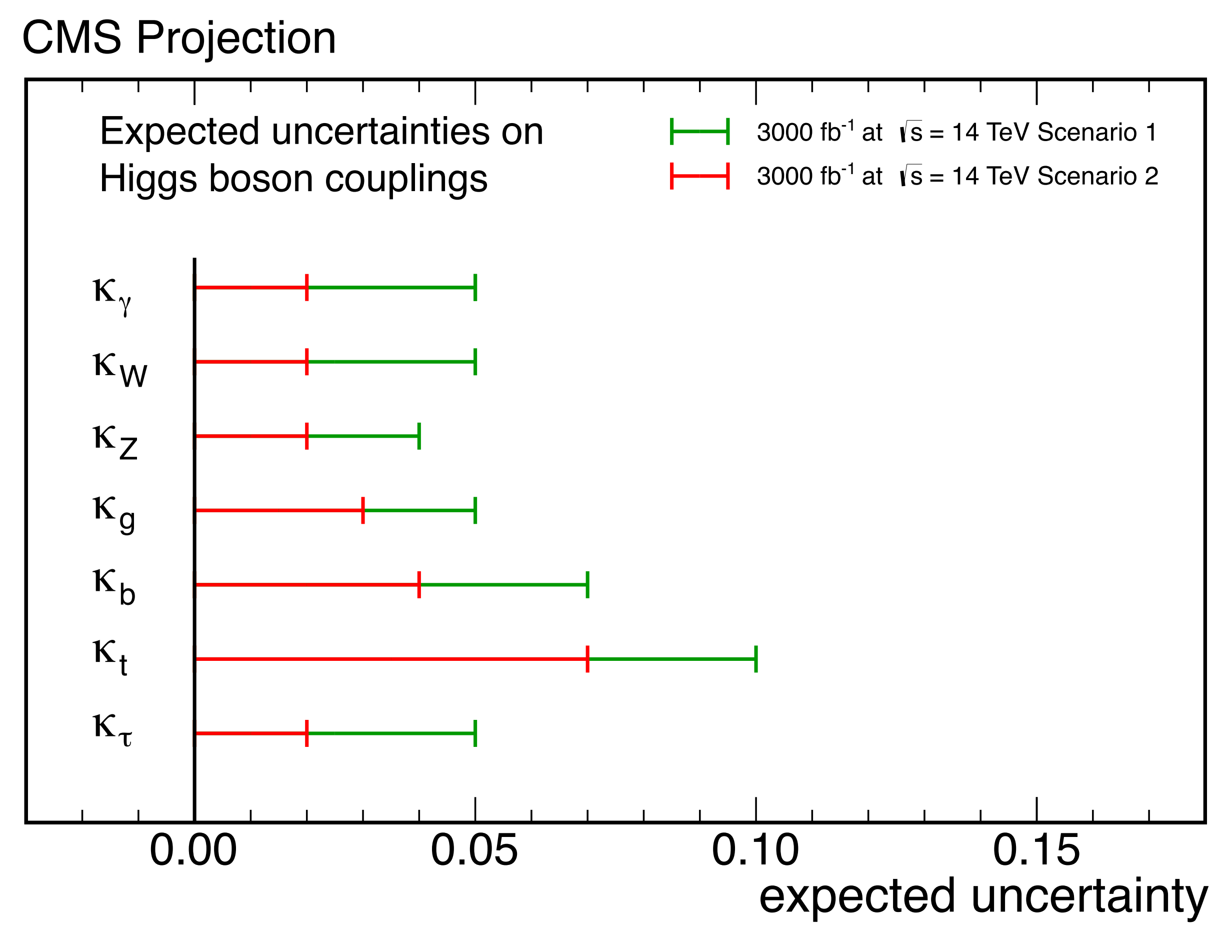
What precision do we need?

- As good as we can!
- Well motivated models beyond the Standard Model can give cross-sections which are very different than the Standard Model
- but also very close to the Standard Model…
- better precision = better discriminating power



Projection of precision in Higgs couplings





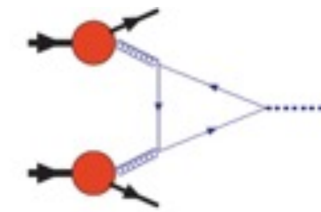
10 times more collisions
~2 times more energy

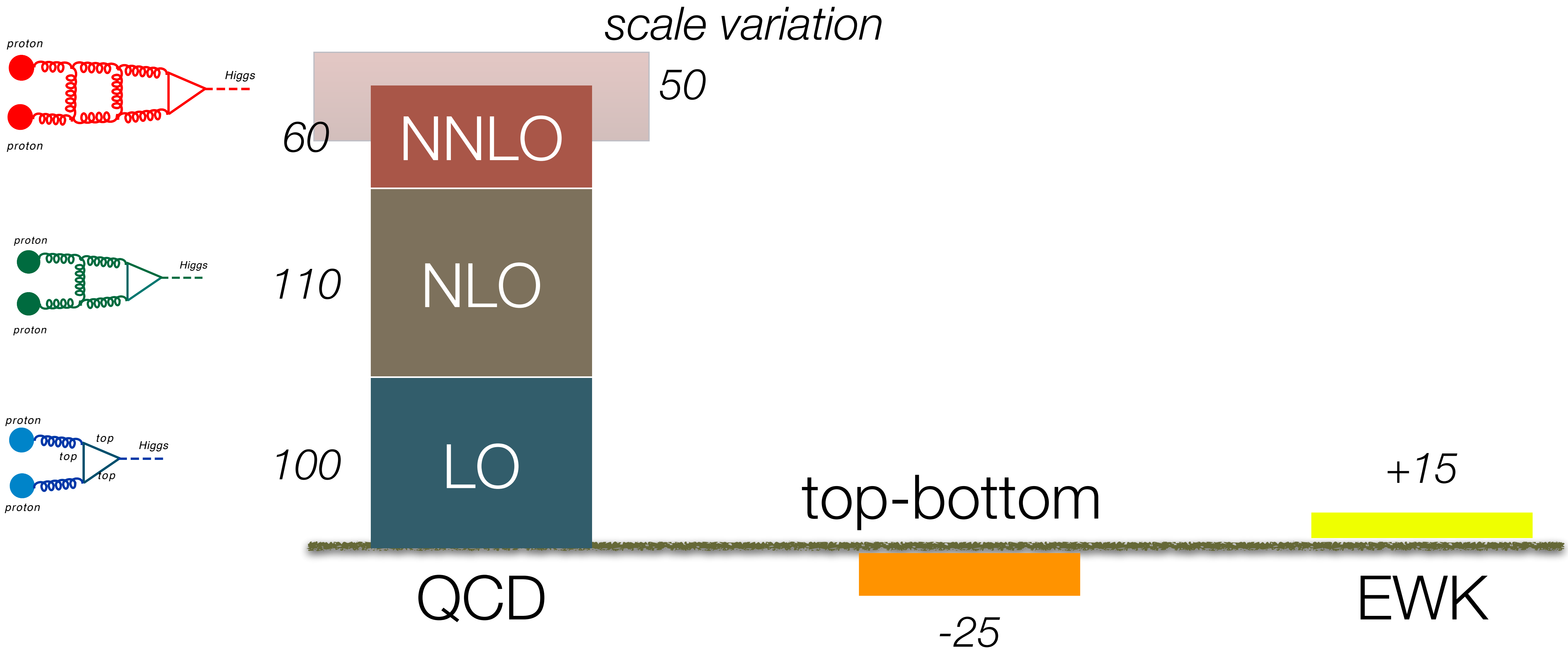
100 times more collisions
~2 times more energy

Green bars: *no improvement in theory precision*

Red bars: *improvement in theory precision by factor of 2*

Gluon fusion cross-section in fixed order perturbation theory





LHC 14TeV

$\mu = M_H = 125\,\mathrm{GeV}$

MSTWNNLO2008

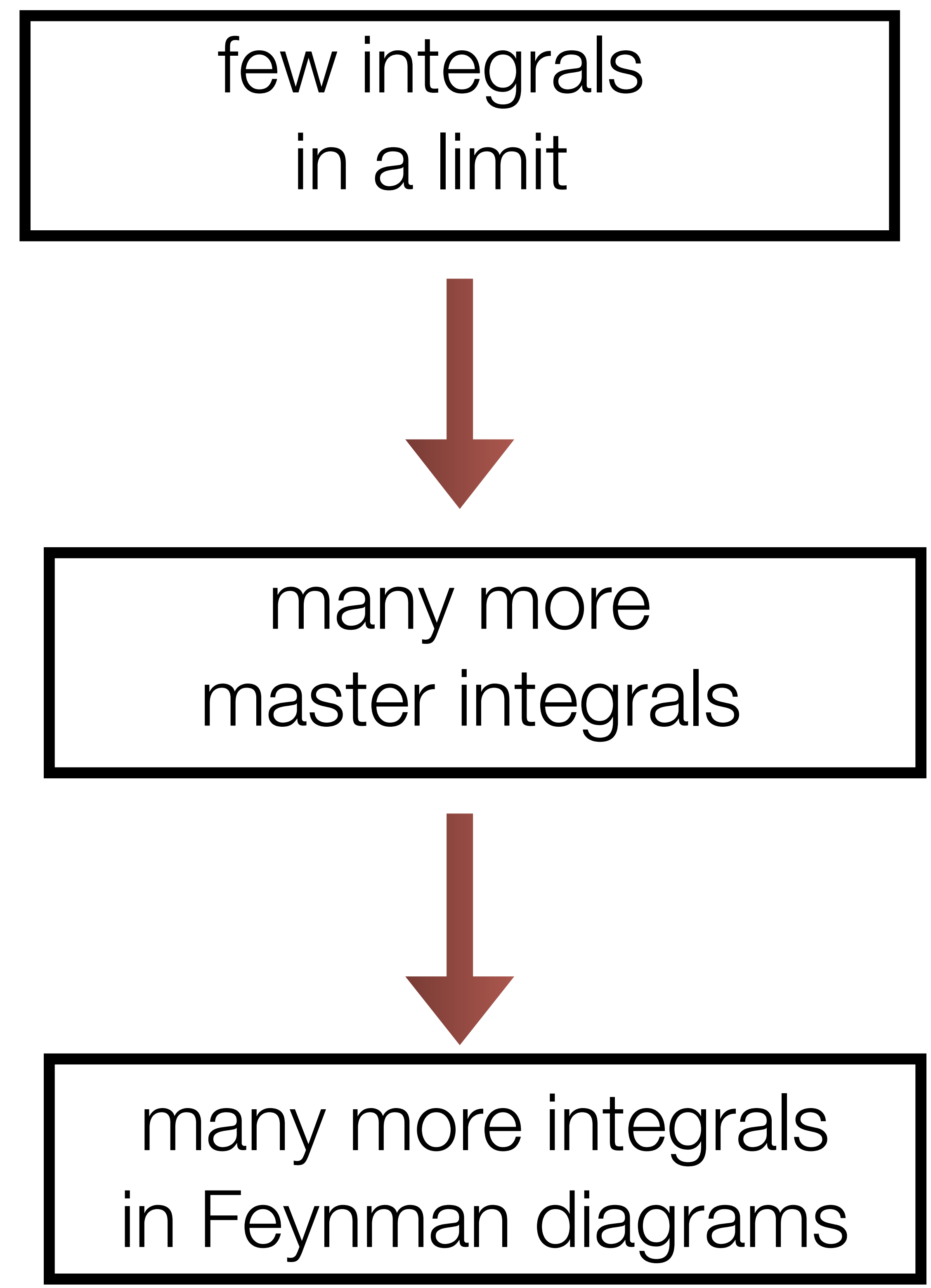
NNLO is not enough!

HOWTO compute the Higgs cross-section at N3LO

- Reduce the enormous amount of Feynman integrals to a more manageable number of master integrals

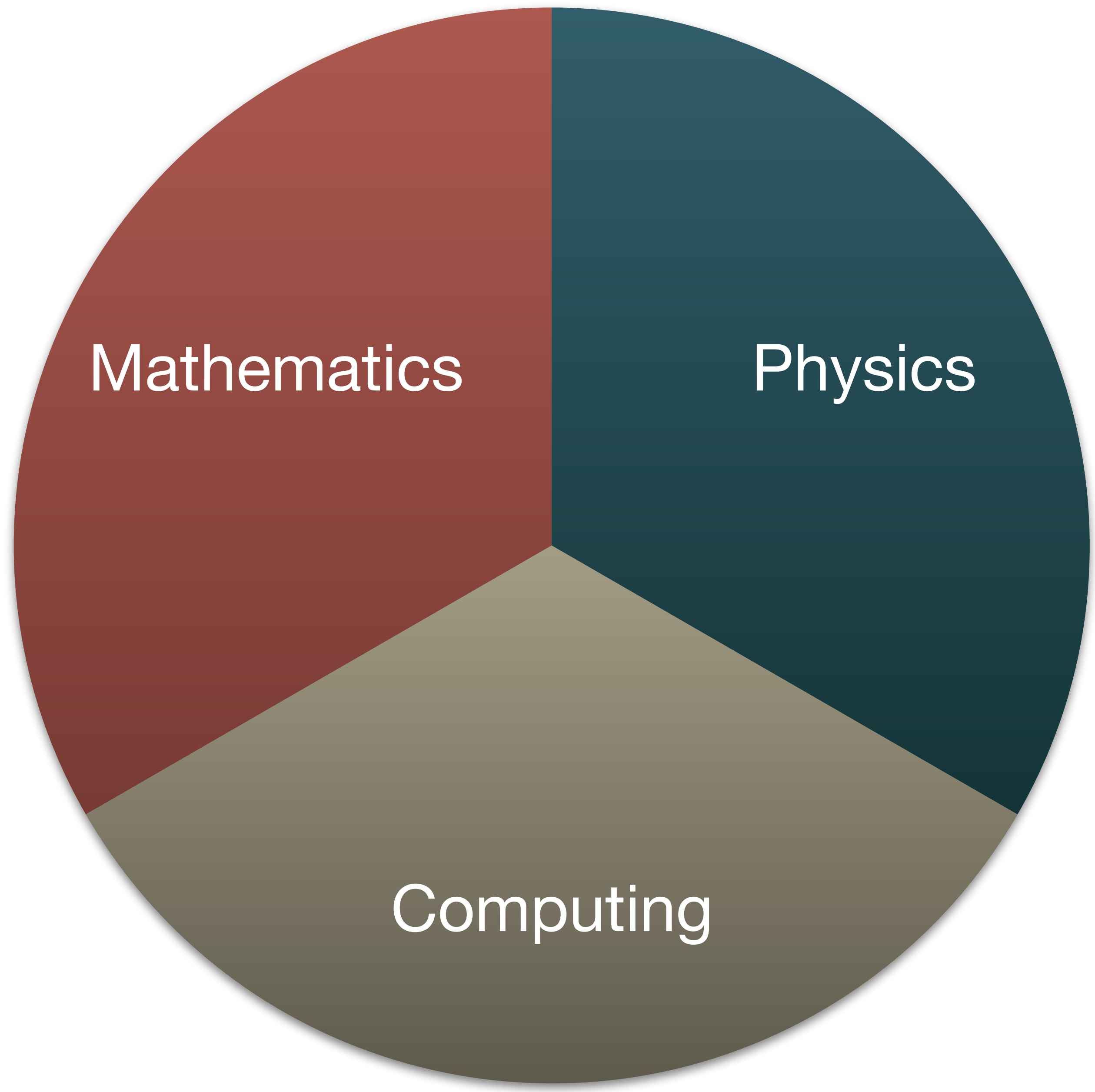
- Compute the master integrals in a kinematic limit (threshold).

- Use the kinematic limit as “boundary condition” and extend the calculation to arbitrary kinematics.



N3LO TECHNIQUES

- Unitarity (“reverse”)
- Intense Linear Algebra
- Solution of coupled systems of differential equations
- Algebraic properties of generalised polylogarithms
- Asymptotic Expansions
- Nested high-dimensionality integrations
- Numerical methods



Many methods originate from NNLO experience: can be applied at any perturbative computation.

Asymptotic expansions

An integral depending on a small parameter ($\delta \to 0^+$*):*

$$I = \int_{-\infty}^{\infty} dx \, \frac{1}{(x^2 + \delta^2)\left[(x-1)^2 + \delta^2\right]} = \frac{2\pi}{\delta(1 + 4\delta^2)}$$

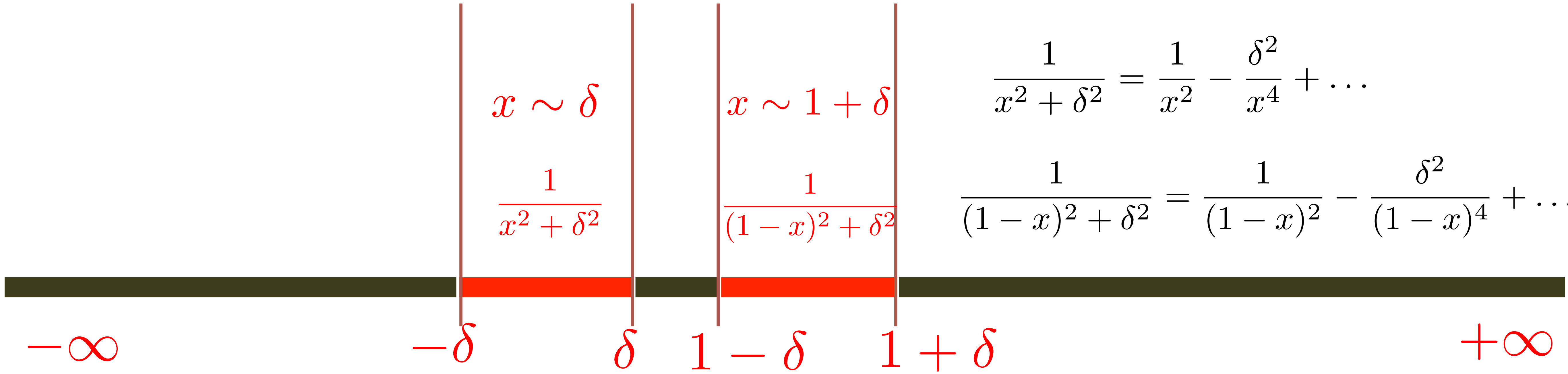
Knowing the analytic result it is easy to expand:

$$I = \frac{2\pi}{\delta}\left[1 - (4\delta^2) + (4\delta^2)^2 - (4\delta^2)^3 + \ldots\right]$$

If we did not know the answer, could we at least get a few terms of the expansion in the small parameter?

NOT EASY: As $\delta \to 0^+$ the integral diverges!

Need to expand around an infinite value!



$$\frac{1}{x^2+\delta^2} = \frac{1}{x^2} - \frac{\delta^2}{x^4} + \ldots$$

$$\frac{1}{(1-x)^2+\delta^2} = \frac{1}{(1-x)^2} - \frac{\delta^2}{(1-x)^4} + \ldots$$

$$I = \int_{-\infty}^{-\delta} + \int_{-\delta}^{+\delta} + \int_{+\delta}^{1-\delta} + \int_{1-\delta}^{1+\delta} + \int_{1+\delta}^{\infty}$$

Hard, Soft A, Hard, Soft B, Hard

- Slice the integration region: **Hard**+**SoftA**+**SoftB**
- **Hard**: no denominator becomes singular. **SoftA,B**: one denominator becomes singular.
- In each region, Taylor expansions are legitimate! **We can expand the integrand and integrate within the boundaries of the region.**

Strategy of regions

Let's introduce some regulators

$$I = \int_{-\infty}^{\infty} dx \frac{1}{\left[(x^2+\delta^2)\right]^{1+a}\left[(x-1)^2+\delta^2\right]^{1+b}} \qquad a, b \text{ small}$$

and expand the ***integrand*** *around the first singular point,*
without restricting the integration *within the SoftA region:*

$$x \sim \delta : \qquad I_{x\sim\delta} = \sum_{n=0}^{\infty} \frac{(1+b,n)}{n!} \int_{-\infty}^{\infty} dx \frac{\left(2x - x^2 - \delta^2\right)^n}{\left[x^2+\delta^2\right]^{1+a}}$$

Exchanging the summation and integration is illegitimate.
But let's go on! The a,b regulators allow to perform all integrations.
After we do so we find that we can set the regulators to zero. We obtain:

$$I_{x\sim\delta} = \frac{\pi}{\delta}\left[1 - (4\delta^2) + (4\delta^2)^2 - (4\delta^2)^3 + \ldots\right] = \frac{I}{2}$$

Now expand around the second singular point: $x \sim 1+\delta$

$$I_{x\sim 1+\delta} = \sum_{n=0}^{\infty} \frac{(1+a,n)}{n!} \int_{-\infty}^{\infty} dx \frac{\left(2(1-x)-(1-x)^2-\delta^2\right)^n}{\left[(1-x)^2+\delta^2\right]^{1+b}}$$

Performing, an unrestricted integration and setting the regulators to zero:

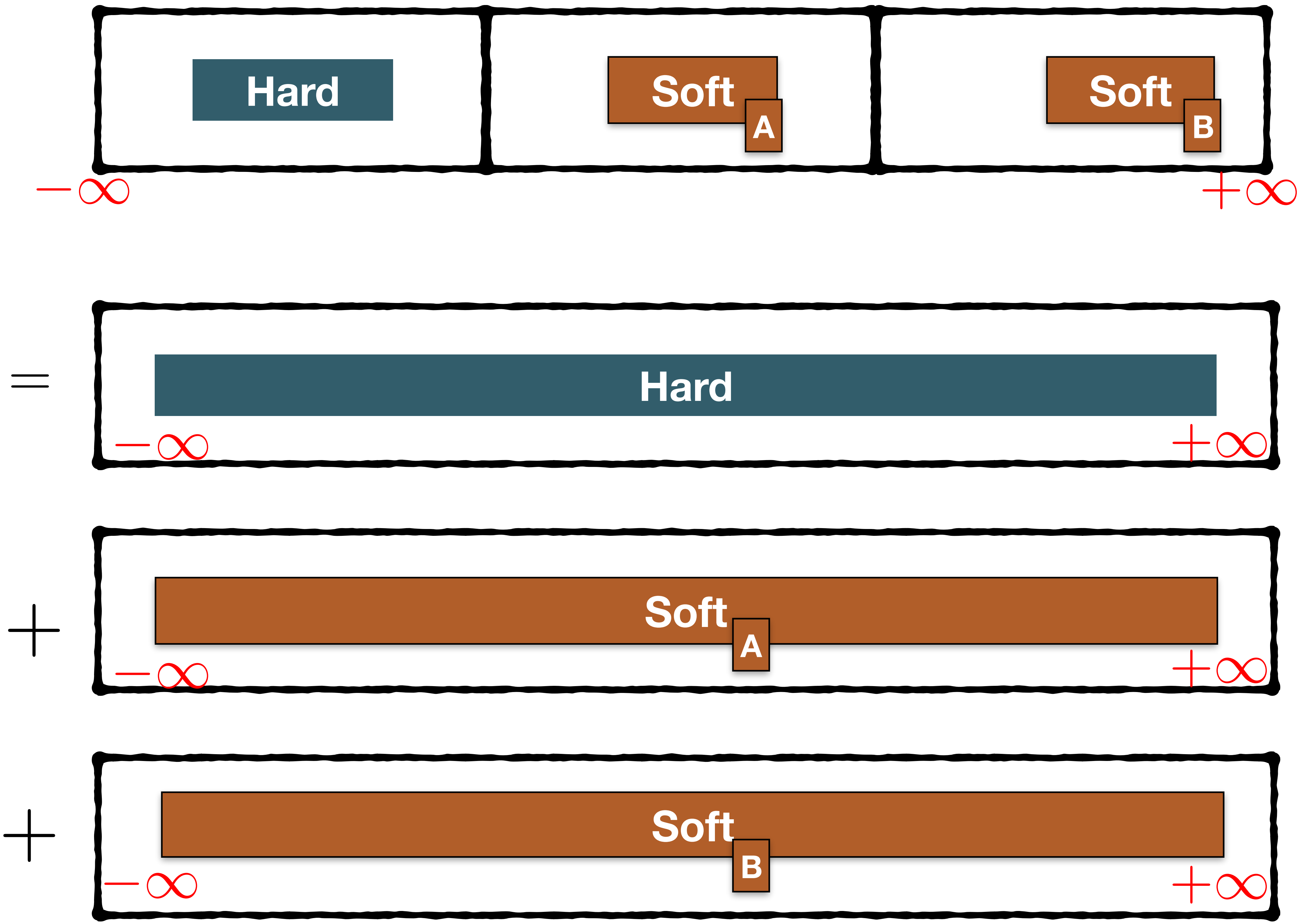
$$I_{x\sim 1+\delta} = \frac{\pi}{\delta}\left[1-(4\delta^2)+(4\delta^2)^2-(4\delta^2)^3+\ldots\right] = \frac{I}{2}$$

Now expand around a point away from the singularities:

$$I_{x\nsim 0,1} = \sum_{n,m=0}^{\infty} \frac{(1+a,n)}{n!}\frac{(1+a,m)}{m!} \int_{-\infty}^{\infty} dx \frac{(\delta^2)^{n+m}}{(x^2)^{1+a+n}\left[(1-x)^2\right]^{1+b+m}} = 0,$$

as $a,b \to 0.$

Strategy of regions



Strategy of regions (***Beneke,Smirnov; Jantzen***)

- The total integral is the sum of all of its regions, where each one of them is extended to cover the full integration domain.
- Counterintuitive: the integration domains are overlapping! Apparently, the overlaps vanish.
- Not always clear why. Regularisation seems to play a role.
- This observation appears to hold in general for all sorts of Feynman integrals.
- Basis for the formulation of effective theories such as SCET and the proof of factorisation theorems

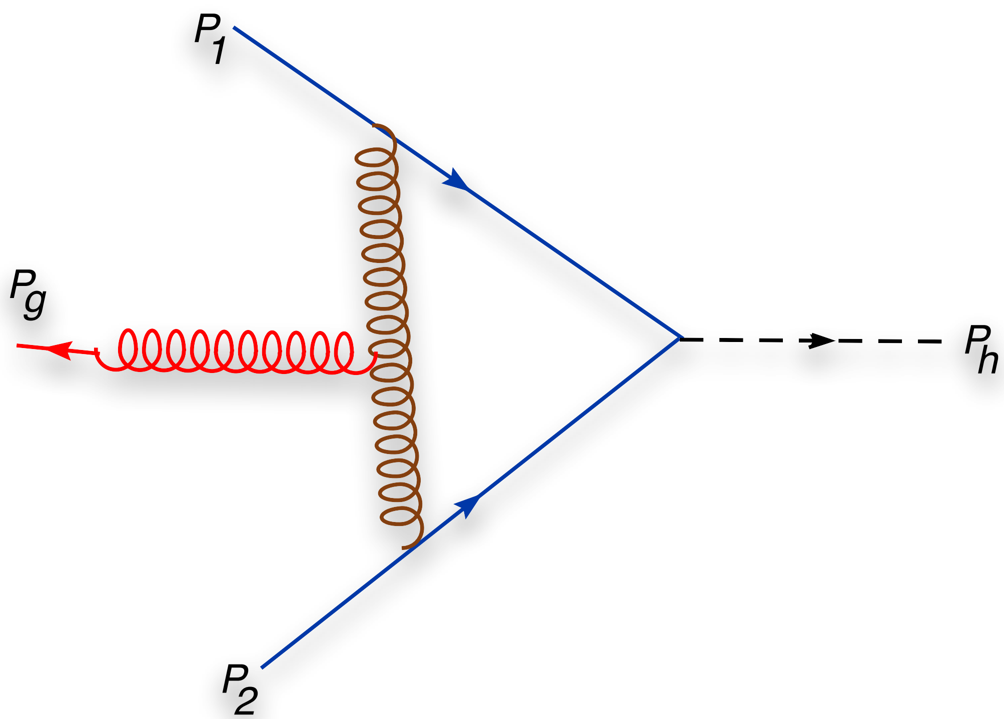
Application in Higgs production

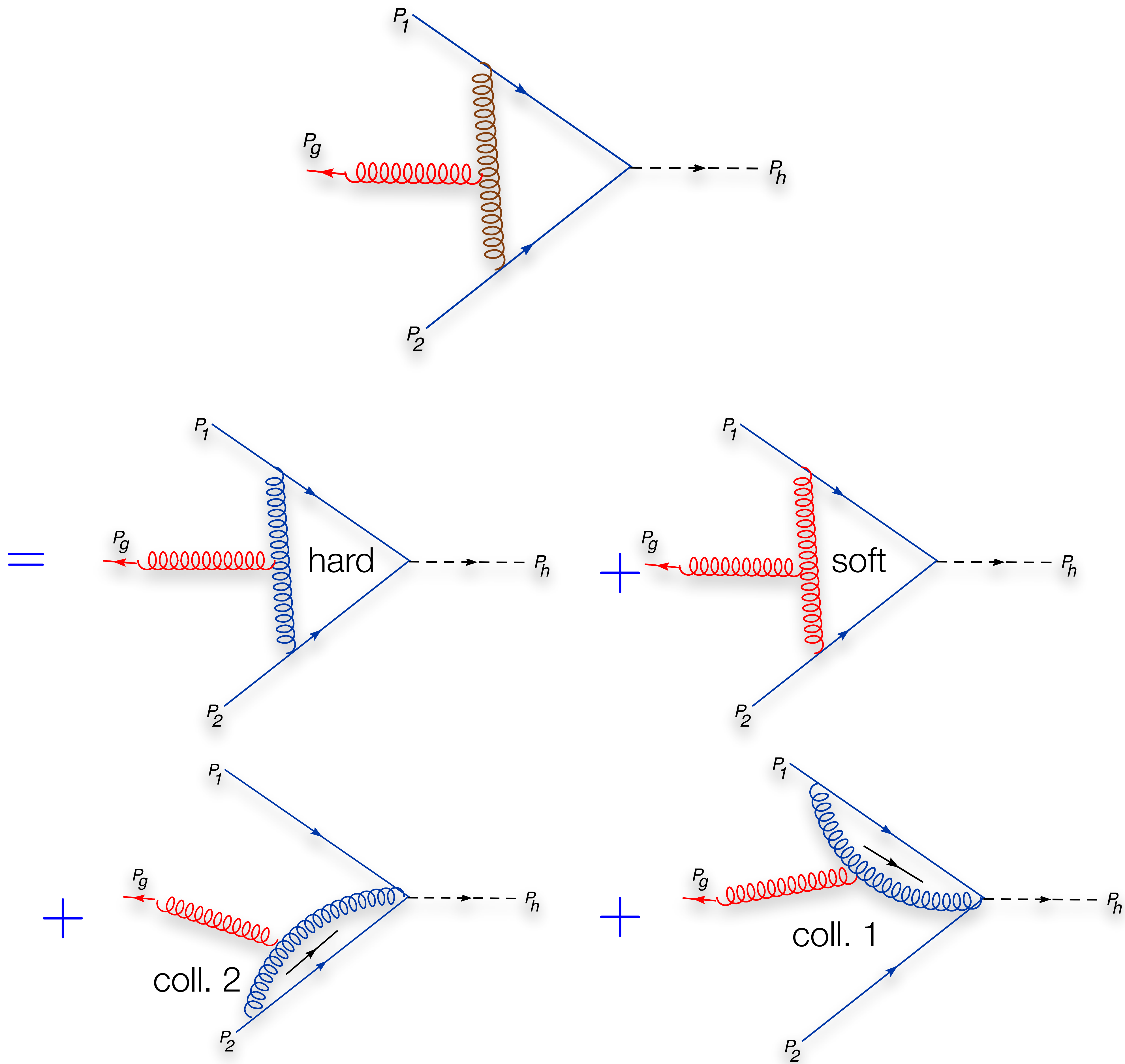
- The equality of the full integral with the sum of regions is a statement valid at all orders in the small-parameter expansion, not just the zero limit.
- Can be used to calculate as many sub-leading terms in the small parameter as we can technically calculate.
- Small parameter in Higgs boson production: Recoil energy of the Higgs boson:

$$\delta = 1 - \frac{(P_1 + P_2)^2}{M_h^2}$$

- Final state gluon radiation is suppressed by this factor:

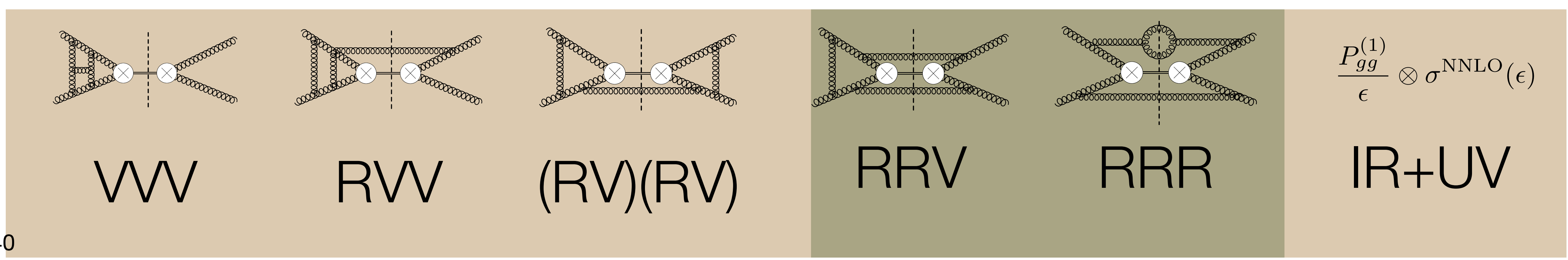
$$P_g \propto \delta$$




P1
Pg
Ph
P2
hard
soft
coll. 2
coll. 1

Progress in N3LO corrections

	1st term	2nd term	Full
VVV	-	-	yes
RVV	yes	yes	yes
(RV)(RV)	yes	yes	yes
RRV	yes	yes	no
RRR	yes	yes	no
IR+UV	yes	yes	yes



Progress in N3LO corrections

$$z = \frac{M_h^2}{s}$$

	$\frac{1}{(1-z)_+}$	$(1-z)^0$	ALL
$\delta(1-z)$	—	—	yes
$\log^5(1-z)$	yes	yes	yes
$\log^4(1-z)$	yes	yes	yes
$\log^3(1-z)$	yes	yes	yes
$\log^2(1-z)$	yes	yes	no
$\log^1(1-z)$	yes	yes	no
$\log^0(1-z)$	yes	yes	no

These results constitute the state-of-the-art beyond NNLO

Progress in N3LO corrections

	$\frac{1}{(1-z)_+}$	$(1-z)^0$	ALL
$\delta(1-z)$	$-$	$-$	5.1%
$\log^5(1-z)$	93.72%	115.33%	205.63%
$\log^4(1-z)$	20.01%	101.07%	113.88%
$\log^3(1-z)$	-39.30%	-32.15%	-78.50%
$\log^2(1-z)$	-52.45%	-89.41%	?
$\log^1(1-z)$	-22.88%	-55.50%	?
$\log^0(1-z)$	-5.85%	-14.31%	?

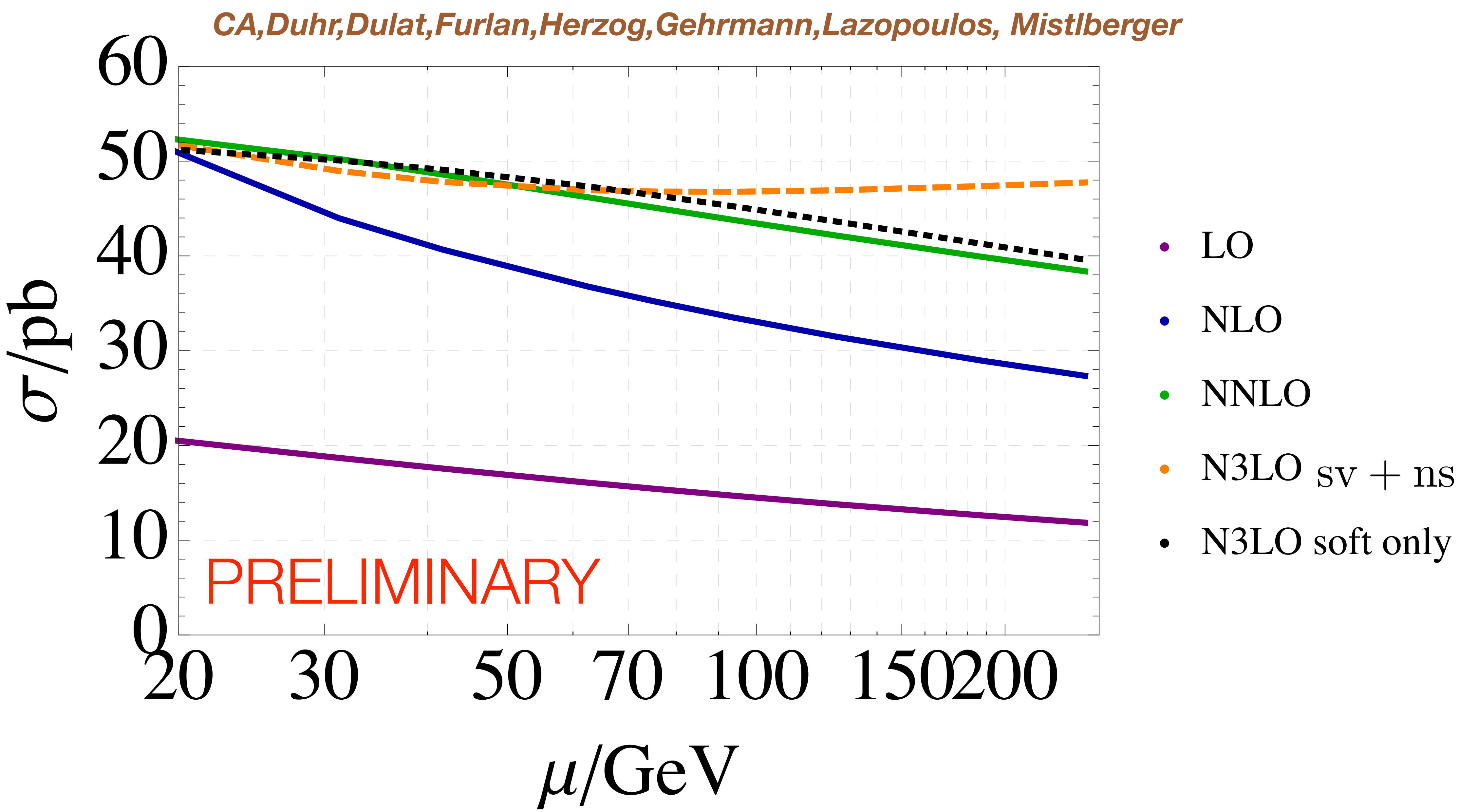
CA,Duhr,Dulat,Furlan,Herzog,Gehrmann,Mistlberger

LHC 14TeV

$\mu = M_H = 125\,\mathrm{GeV}$

MSTWNNLO2008

Numerical impact



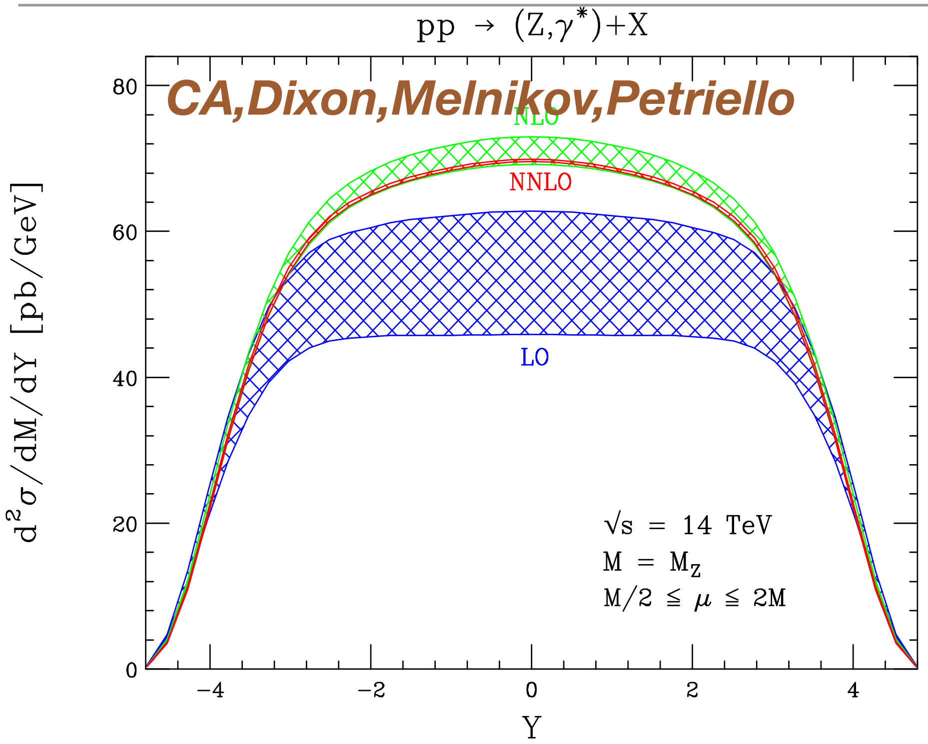
What “easy things” to expect at N3LO

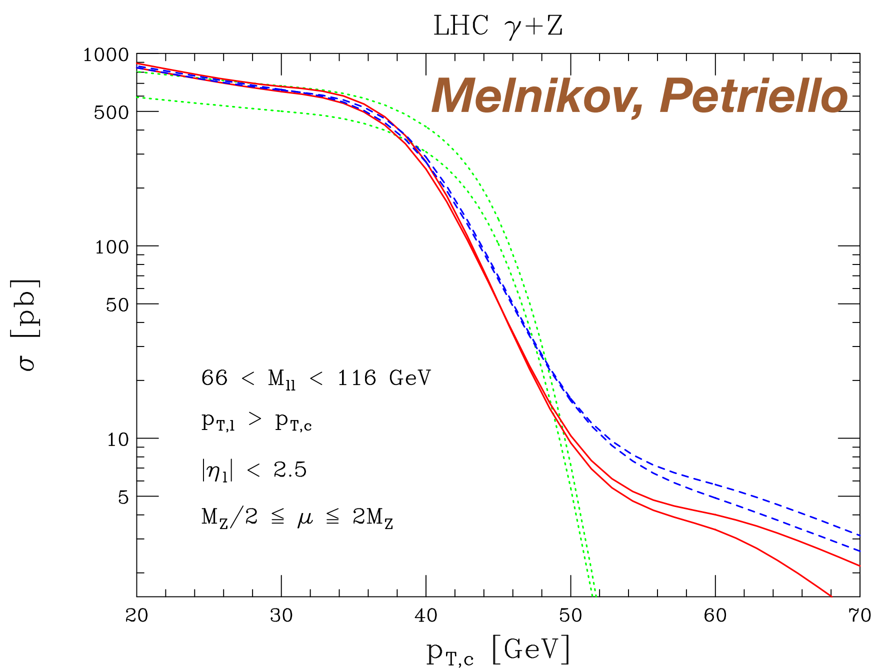
- Soon the full N3LO computation for the gluon fusion process.
- Once this done, it is straightforward to compute the N3LO inclusive cross-section for all 2 to 1 processes:

$$pp \to W$$
$$pp \to Z$$
$$b\bar{b} \to H$$
$$pp \to WH$$
$$pp \to ZH$$

- Already, these have been extracted in the threshold limit from our gluon-fusion computation. **Ravindran et al**

Differential distributions at N3LO for 2 to1 processes?

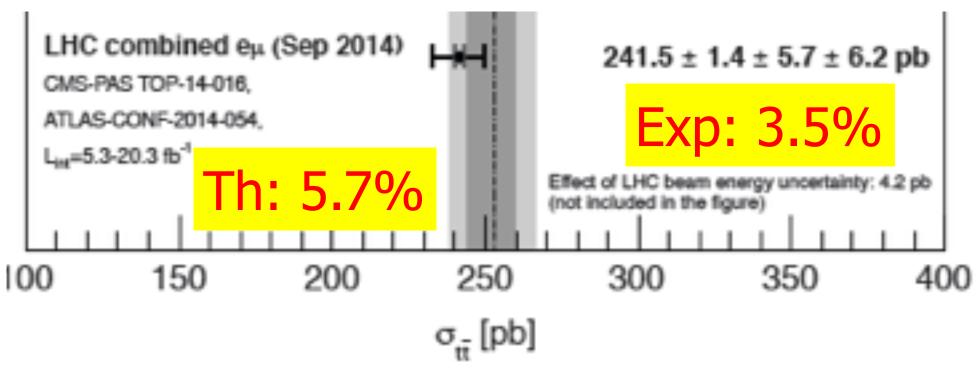




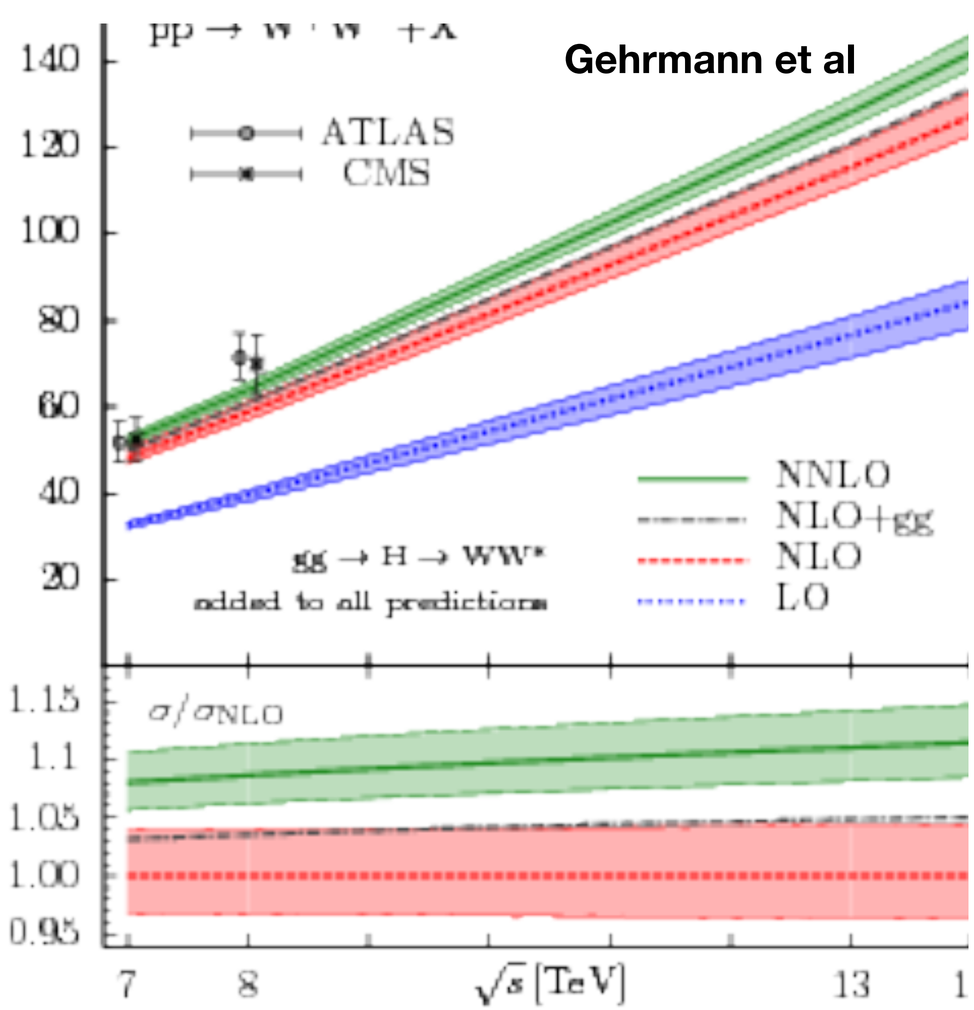
- Distributions give us more information, but they require further development of our mathematical and computational tools.
- The established Drell-Yan precision at NNLO is astonishing.
- N3LO computations will bring theory uncertainties to per mille level.
- Many applications: pdf's, electroweak precisions measurements.
- Necessary to maintain high precision for elevated lepton triggers.
- CAN IT BE DONE?

N3LO cross-sections for 2->2 processes?

from Albert's talk



Gehrmann et al



- Experimental precision has already surpassed NNLO theoretical precision for top-pair production.
- In quark initiated processes, (e.g. WW,ZZ, etc) the gluon-gluon channel only opens at NNLO. Enhanced by pdf's. N3LO is needed to stabilise this contribution.
- Another leap in computational methods is needed; barely achieved at NNLO.
- CAN IT BE DONE?

CONCLUSIONS

- LHC offers great opportunities for precision studies with the discovery of the Higgs boson.

- Answer important questions:
  - is the Higgs mechanism the main source of mass for elementary particles?
  - does the Higgs couple to more complicated physics, like light dark matter and heavy new particles?
  - what gives mass to the Higgs boson itself?
  - etc

- Enormous progress in perturbative QCD.
  - NLO solved
  - NNLO maturing
  - new chapter started at N3LO

- High precision theory + High precision experiment = Progress