Conformal Bootstrap - a new way to compute in Quantum Field Theory

Slava Rychkov

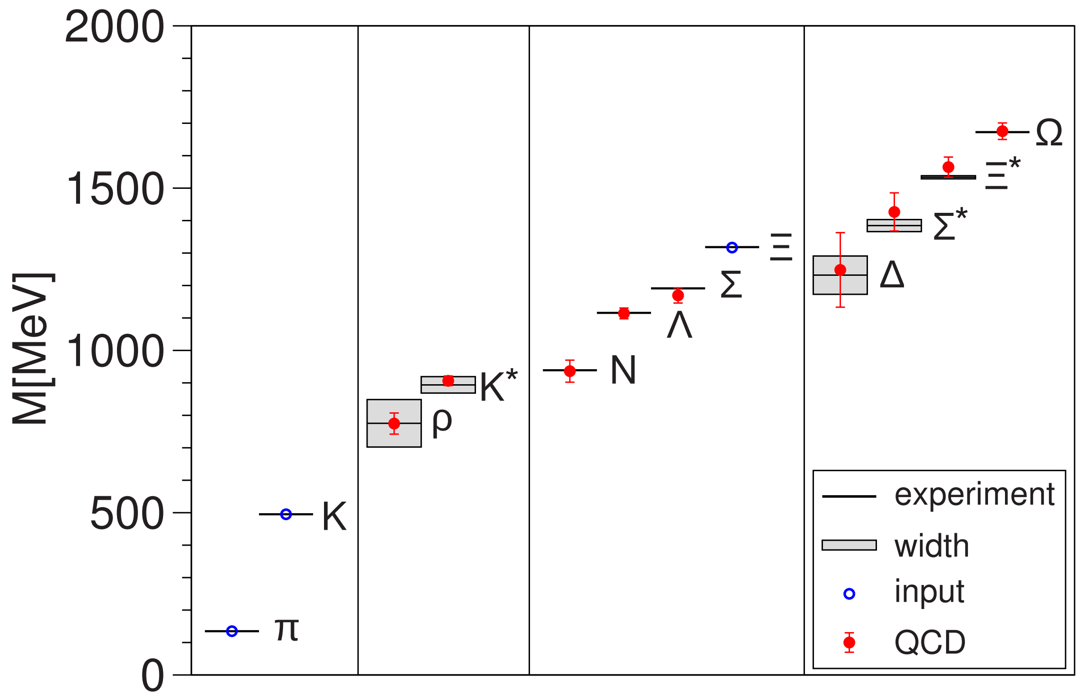
Annual Theory Meeting, Durham, 16/12/2014

Problem

Find an efficient algorithm to solve strongly coupled QFTs

Lattice field theory?

- Benchmark problem: Light hadron spectrum at % level



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Durr et al, Science 322(2008) 1224

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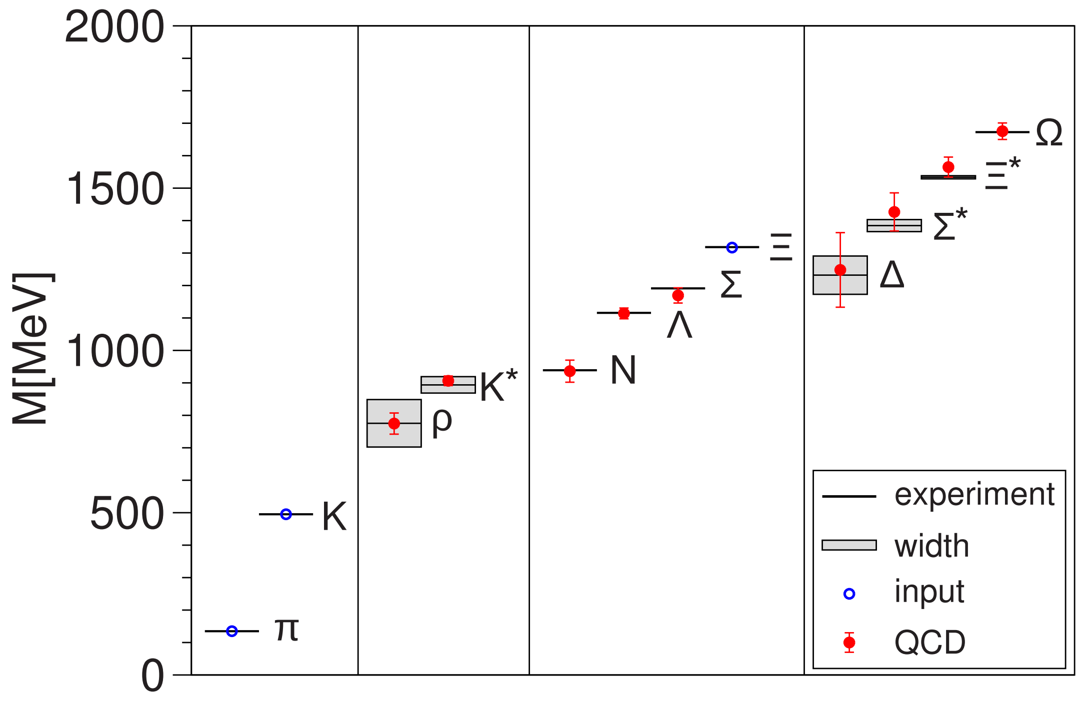

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- Cost: $\approx$ 1 supercomputer-year $\approx$ 100,000 single core-years

Today: Conformal theories

aka fixed points of RG flow

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Studying these fixed points is proving more difficult on the lattice than the ordinary QCD

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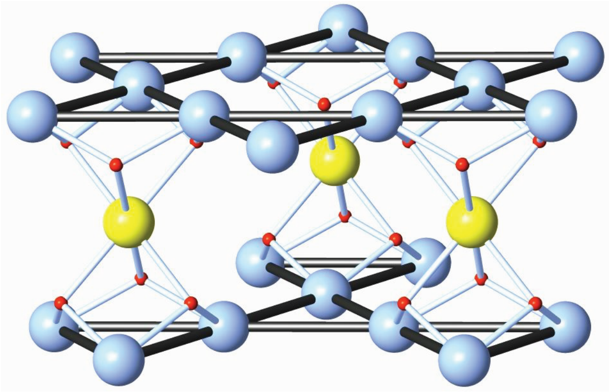
E.g. $N_f = 4$ for *Herbertsmithite* $ZnCu_3(OH)_6Cl_2$



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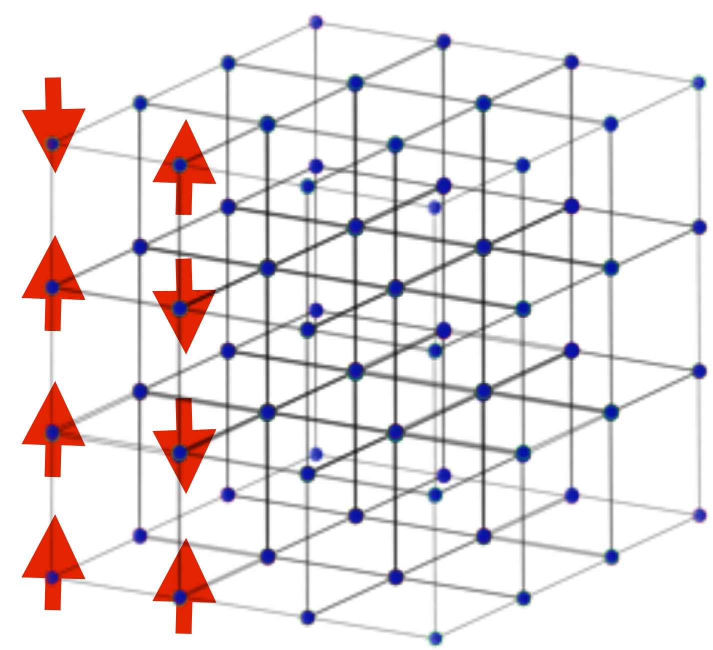
$$E = -J \sum_{\langle ij \rangle} s_i s_j$$

$$s_i = \pm 1$$



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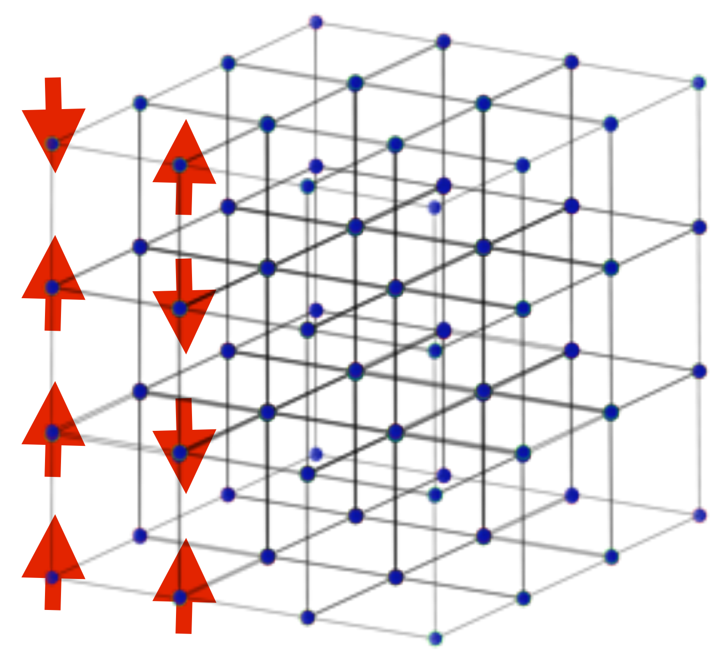

describes - Curie point of uniaxial magnets in 3d
- liquid-vapor phase transitions

Plan

1. CFT generalities

2. Conformal Bootstrap

CFT kinematics

i.e. everything you get for free

- Observables in CFT are correlation functions (no S-matrix)

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Example: each CFT contains a stress tensor operator $T_{\mu\nu}$, conserved and traceless, of dimension Δ=d

Operators transform covariantly under

Conformal transformations $x \to x' = f(x)$

E.g. for scalars:

$$\boxed{A(x') = \lambda(x)^{-\Delta} A(x)}$$

$\lambda = |\partial x' / \partial x|^{1/d}$ local scale factor

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Poincaré + dilatations + special conformal transformations

$$\delta_a x_\mu = 2(a.x)x^\mu - x^2 a^\mu$$

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$$\langle A_1(x_1)A_2(x_2)A_3(x_3)\rangle = f_{123}|x_{12}|^{\Delta_3-\Delta_1-\Delta_2}|x_{13}|^{\cdots}|x_{23}|^{\cdots}$$

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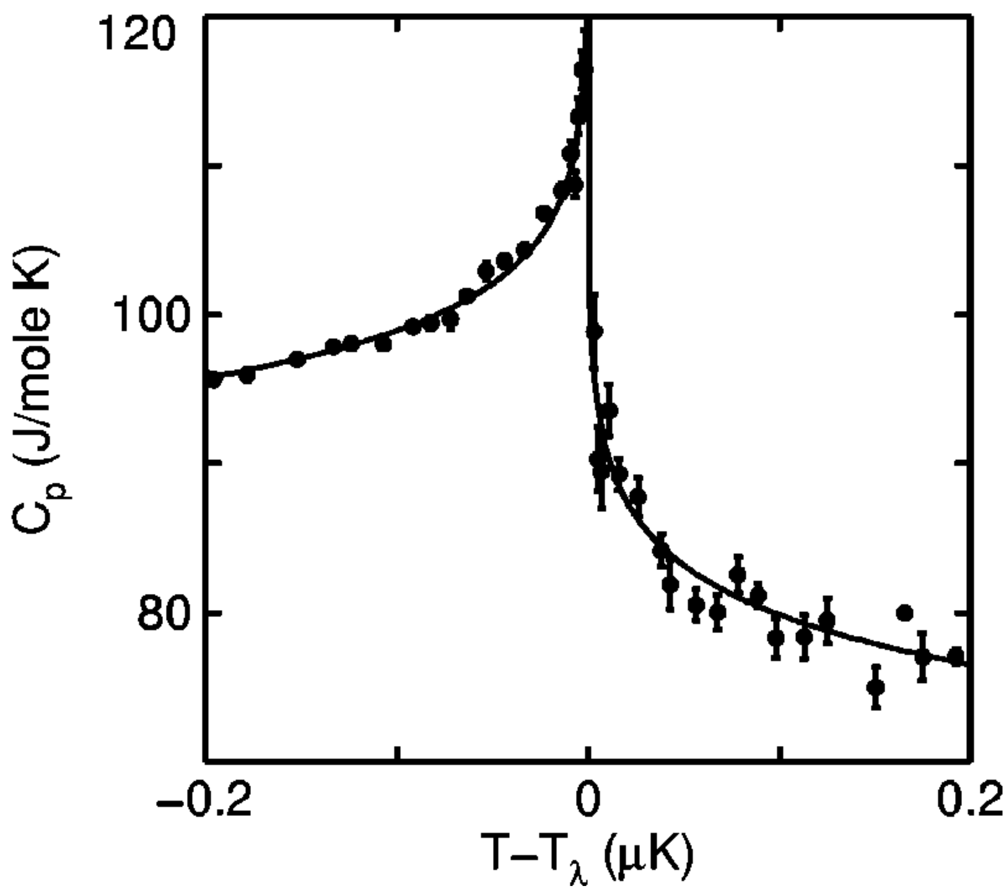
CFT data

to find these = to solve the CFT

Example: CFT data from experiment



Lipa et al, Phys.Rev.Lett. 76,944(1996)

λ-point of He-4

space shuttle measurement

Reached $T\text{-}T_c \sim$ nK $\Rightarrow$ $\xi/a \sim 10^6$

Specific heat exponent $\alpha = -\,0.0127(3)$

$$\Delta(\vec{\varphi}^2) = 1.5094(2) \quad \text{in the 3d } O(2) \text{ model}$$

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Common disadvantages:

- conformal symmetry left unused

- 3pt function coefficients f_{ijk} much harder to get than Δ_i (never done in practice)

Conformal Bootstrap

Original idea
Ferrara, Gatto, Grillo 1973; Polyakov 1974

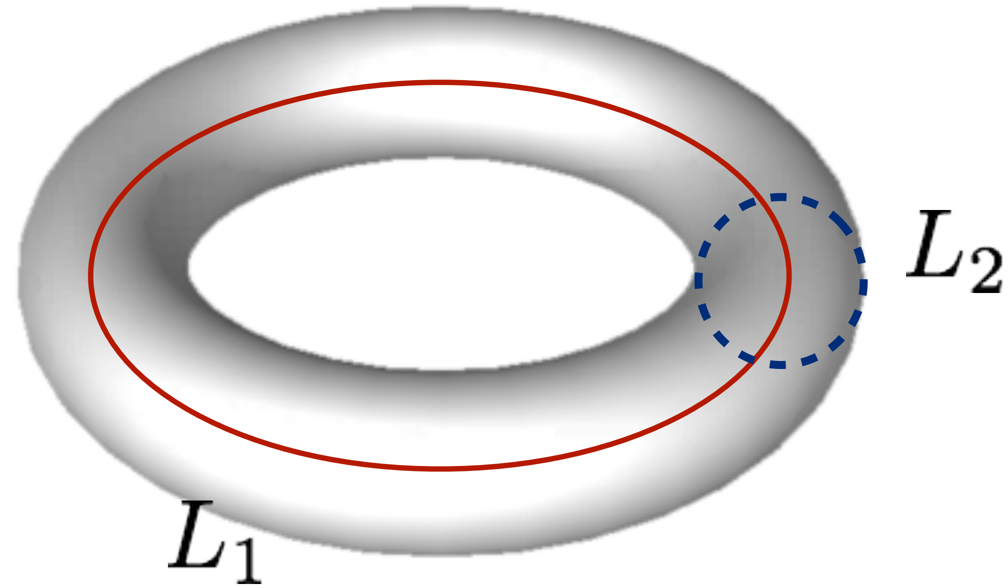
Concrete realization in 2d
Belavin, Polyakov, Zamolodchikov 1983

First results in 4d (motivated by BSM)
Rattazzi, S.R., Tonni, Vichi 2008
(Using 4d conformal blocks by Dolan,Osborn 2000)

First results in 3d (applications to Ising-3)
El-Showk, Paulos, Poland, S.R.,Simmons-Duffin,Vichi 2012

Analogy: Modular invariance

Put a Lorentz-inv theory on $S^1 \times S^1$



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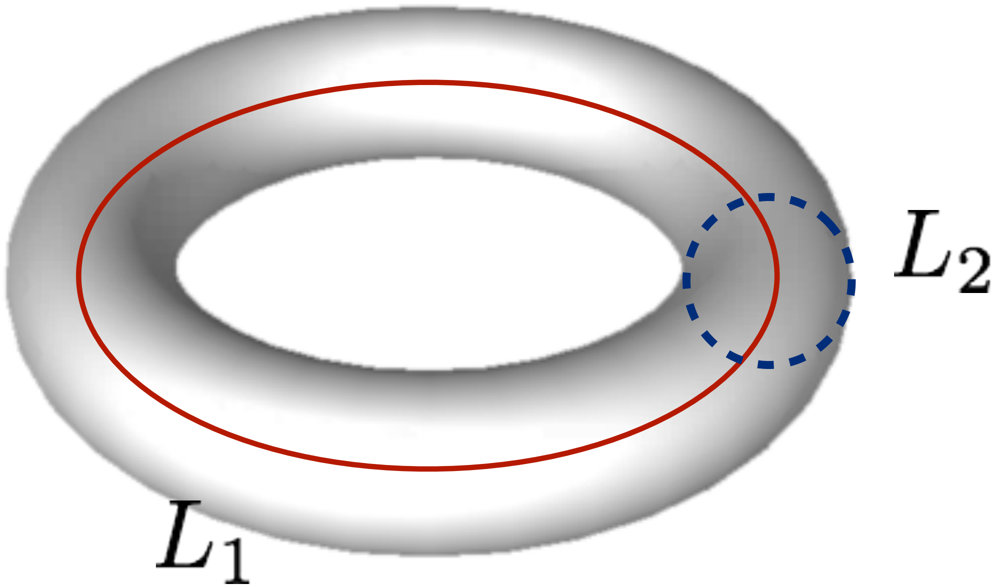

View L_1 as space and L_2 as β:

$$Z = \mathrm{Tr}\, e^{-L_2 \mathcal{H}(L_1)} = \sum_n e^{-L_2 E_n(L_1)}$$

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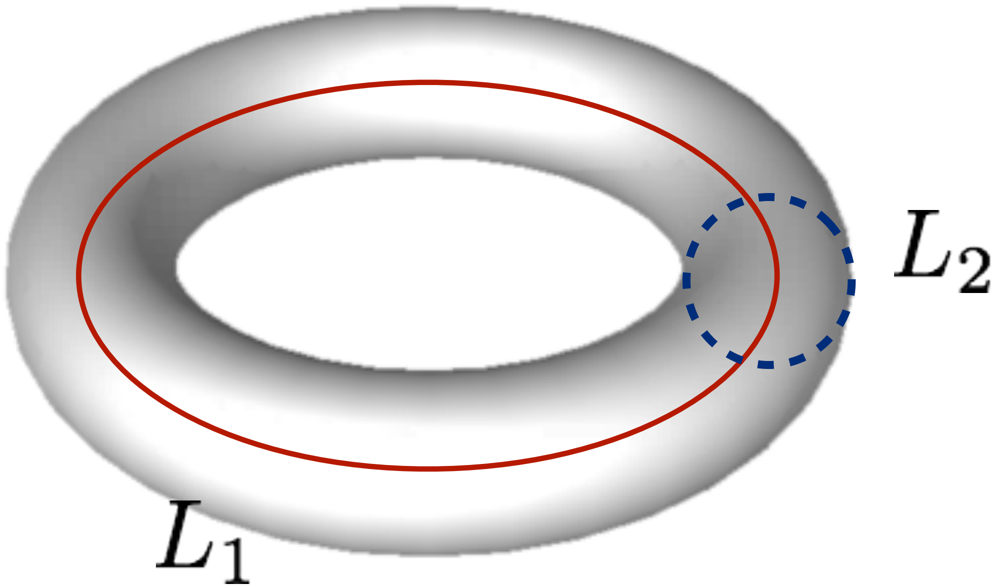

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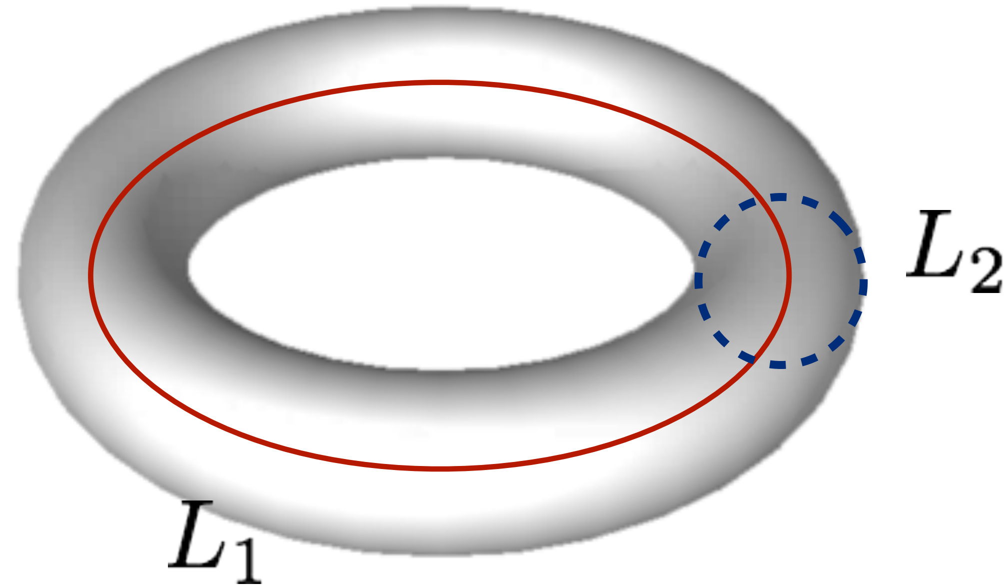

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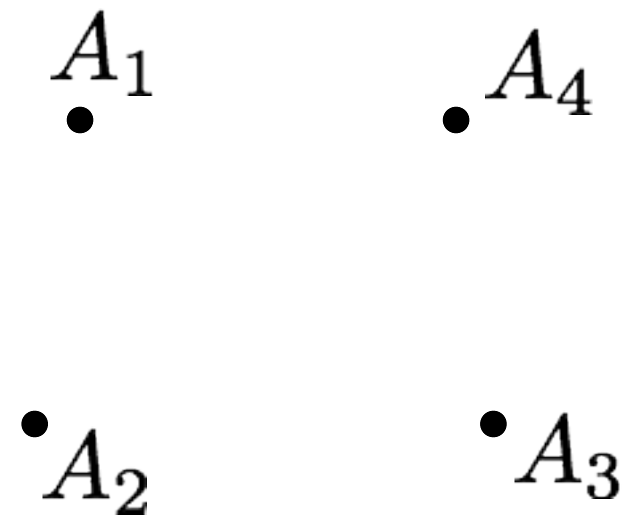
2d CFT: $E_n(L) = \frac{2\pi}{L}(\Delta_n - c/12)$ $\Rightarrow$ constraint on Δ's

Cardy 1986

Now real bootstrap

Consider 4-pt function:



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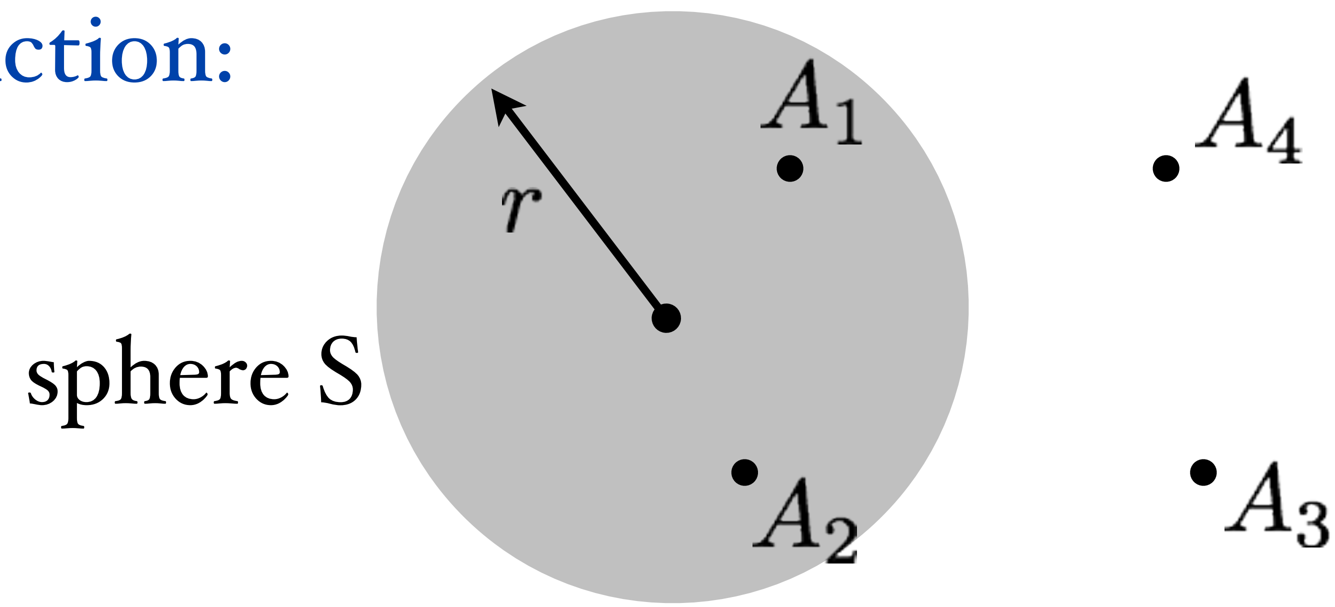

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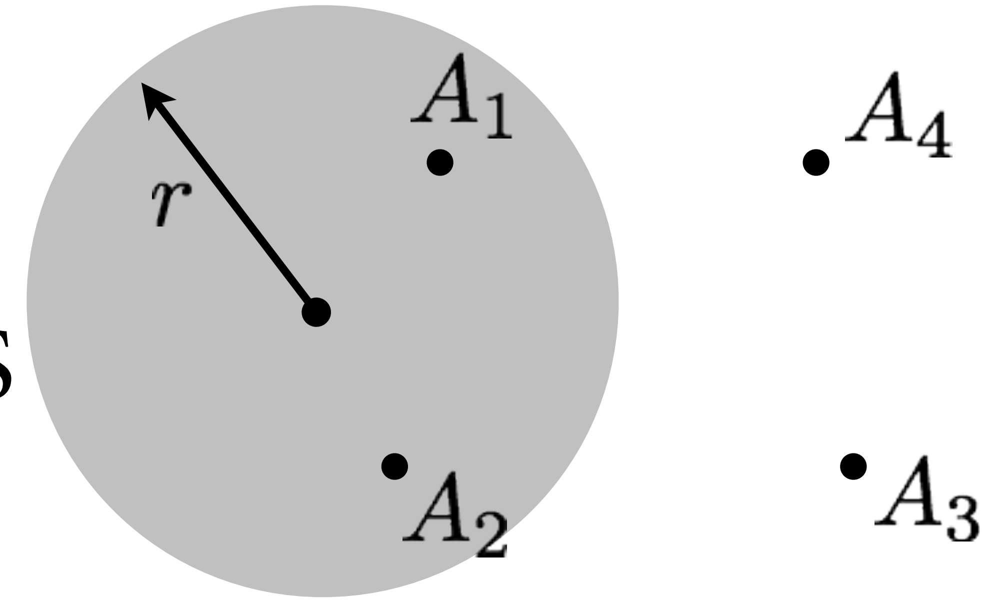

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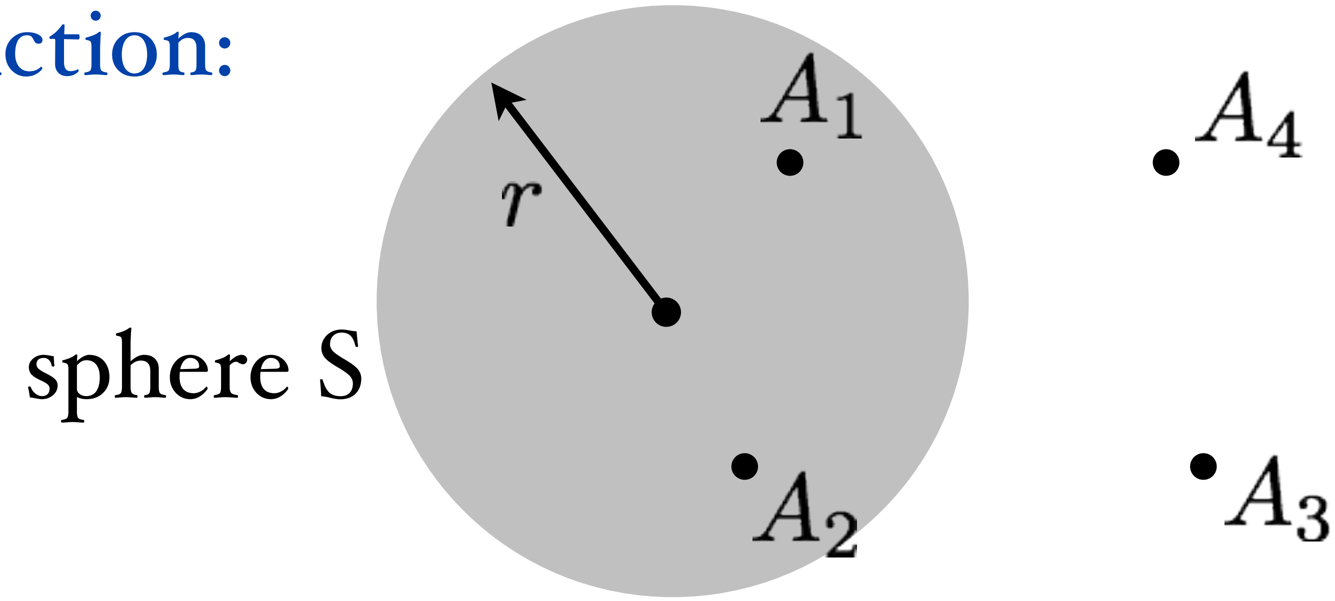

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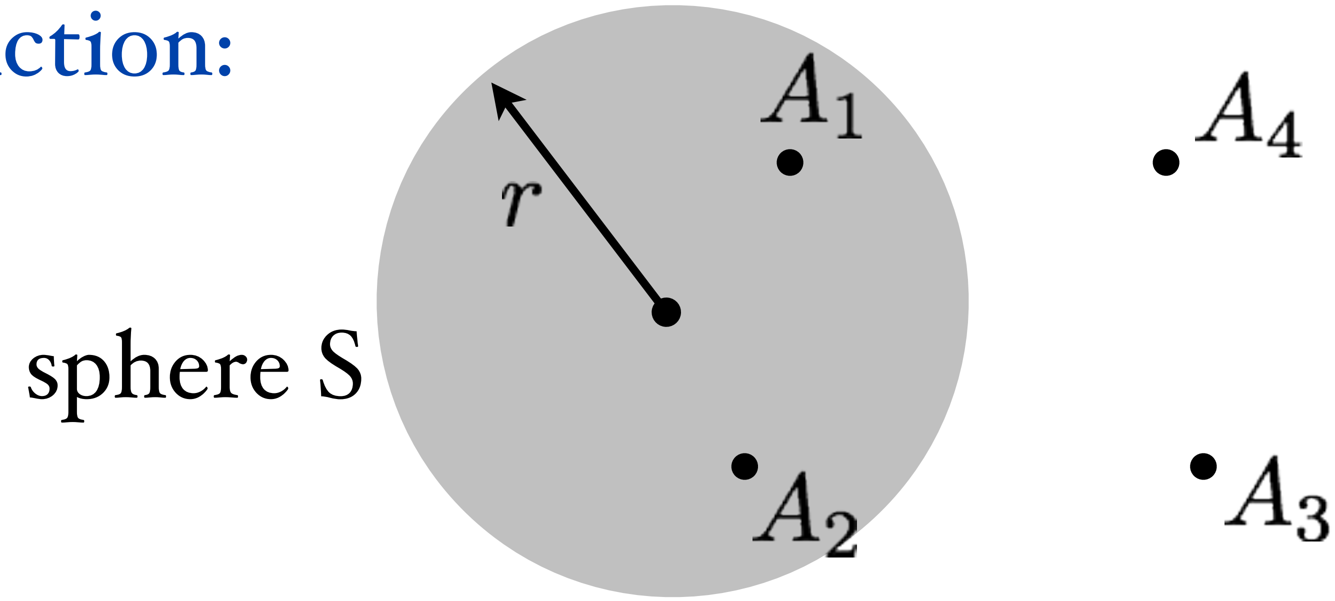

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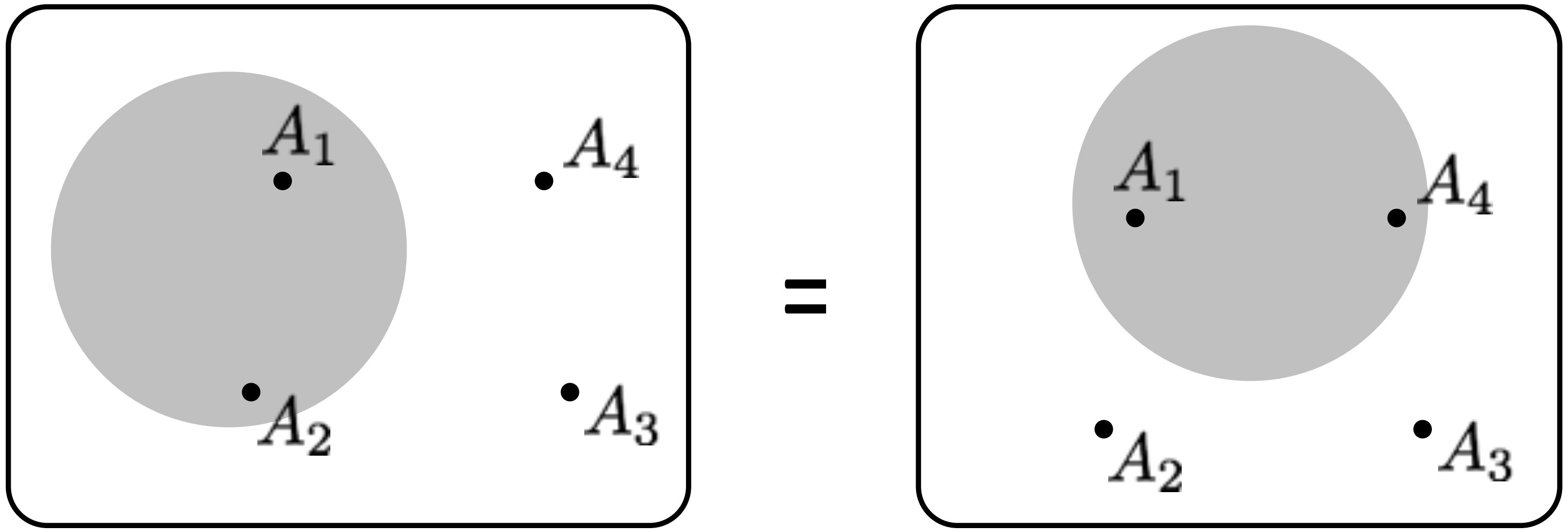
- Matrix elements:

$$\langle A_1 A_2 | n \rangle = f_{12n} \times (\text{kinematical factor})$$

conformal partial waves

Get

$$\langle A_1 A_2 A_3 A_4 \rangle = \sum_n f_{12n} f_{34n} G_n(x_1, x_2, x_3, x_4)$$



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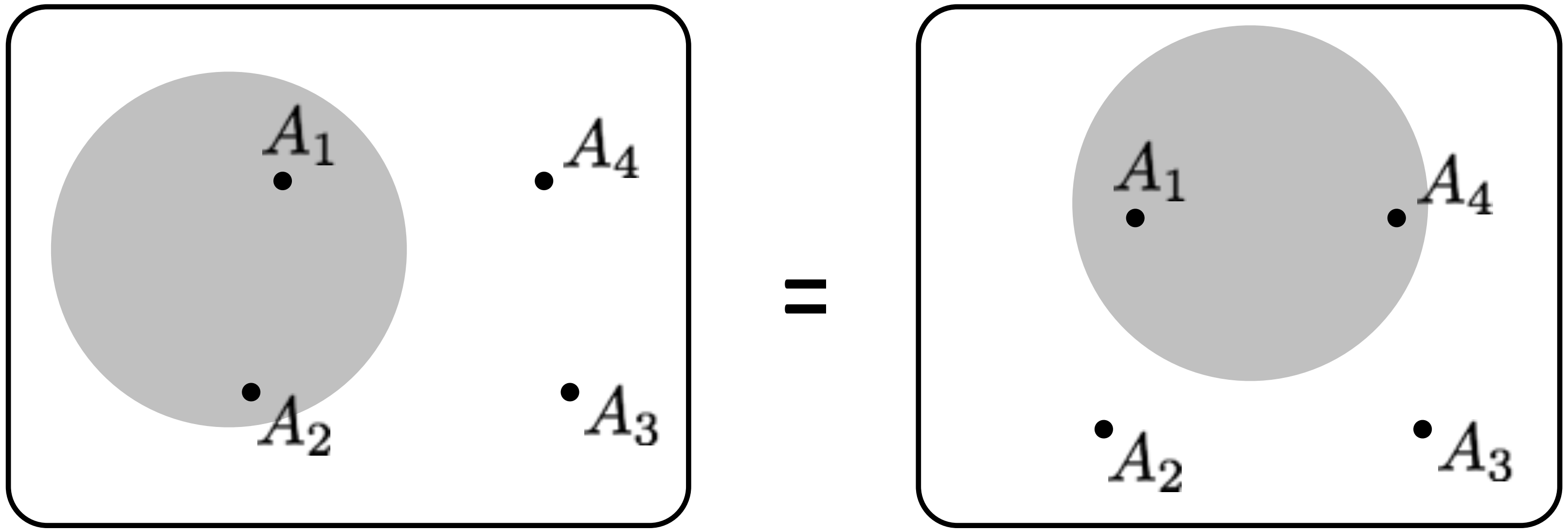

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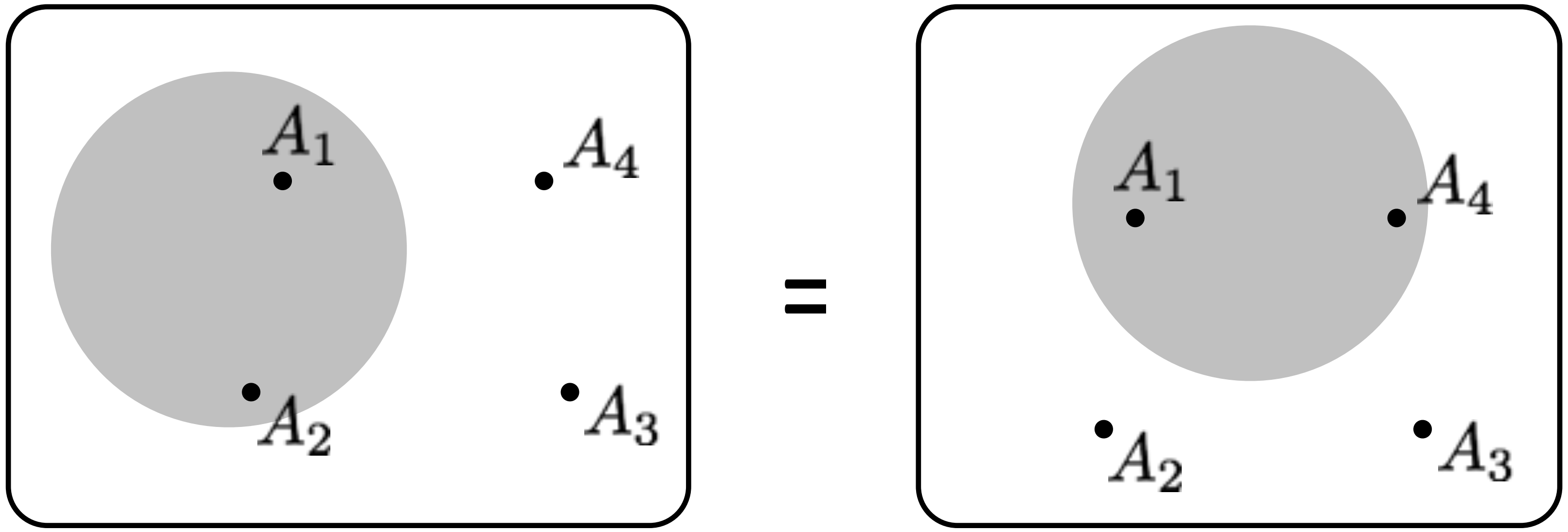

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conformal bootstrap eqn

1. *functional* constraint on CFT data - must be satisfied for every $x_1,\ldots,x_4$

2. one equation for every 4-point function

Concrete application: 3d Ising model CFT

Many other applications exist:

- Wilson-Fisher fixed point in 2<d<4
- O(N) models in d=3
- SUSY conformal theories in d=2,3,4,6 dimensions

...

Local operators

3d Ising model CFT can be def'd as IR fixed point of

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E.g. $[\phi^2]=1$ in the UV (free theory) becomes
$[\phi^2] \approx 1.41$ in the IR (CFT)

In bootstrap, ϕ-content of operators is irrelevant (only their dimension, spin, and Z_2 count)

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Very important experimental fact:
3d Ising CFT contains 2 and only 2 relevant (i.e. $\Delta<3$) scalar operators:

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Coupling $\langle\sigma\ \sigma\ \varepsilon\rangle$ allowed by Z_2 symmetry $\Rightarrow f_{\sigma\sigma\varepsilon} \neq 0$

More generally, consider OPE σ x σ

It contains all operators A such that $f_{\sigma\sigma A} \neq 0$
(infinitely many per each spin)

$$
\begin{aligned}
\sigma \times \sigma = \mathbf{1} + \varepsilon + \varepsilon' + \ldots \quad & (\ell = 0) \\
+ T_{\mu\nu} + T'_{\mu\nu} + \ldots \quad & (\ell = 2) \\
+ (\ell = 4, 6, \ldots) &
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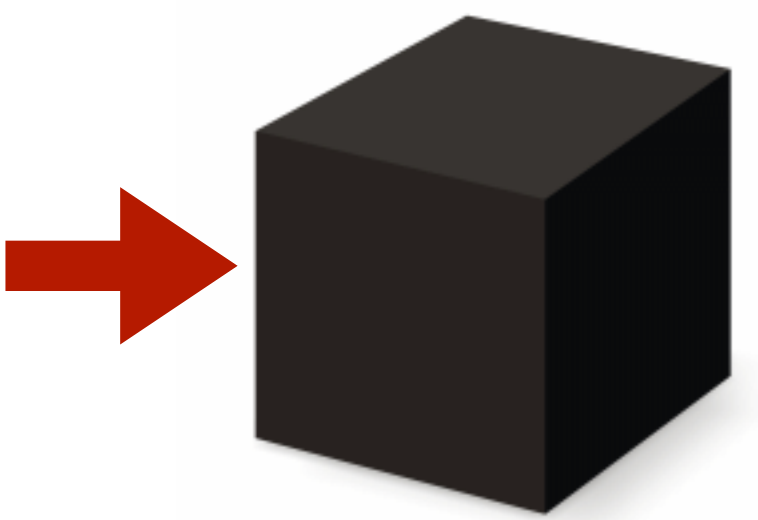
To solve the theory we must find dimensions and f's for all these operators

Bootstrap Oracle

The algorithm explores the space of CFT data
and rigorously rules out the impossible sets

subset of CFT data
e.g. $\Delta\sigma$, $\Delta\varepsilon$



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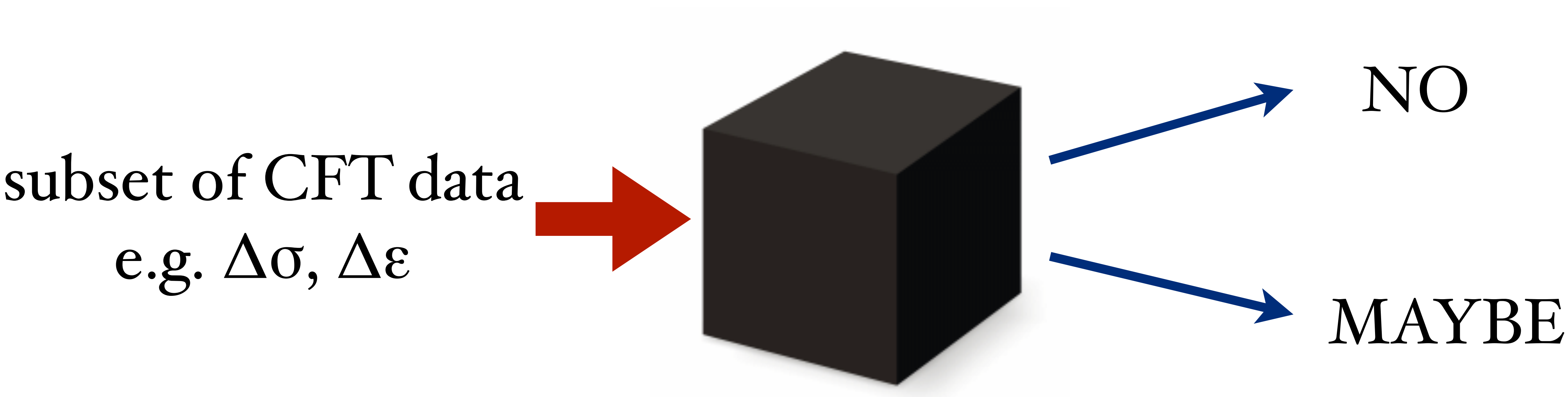
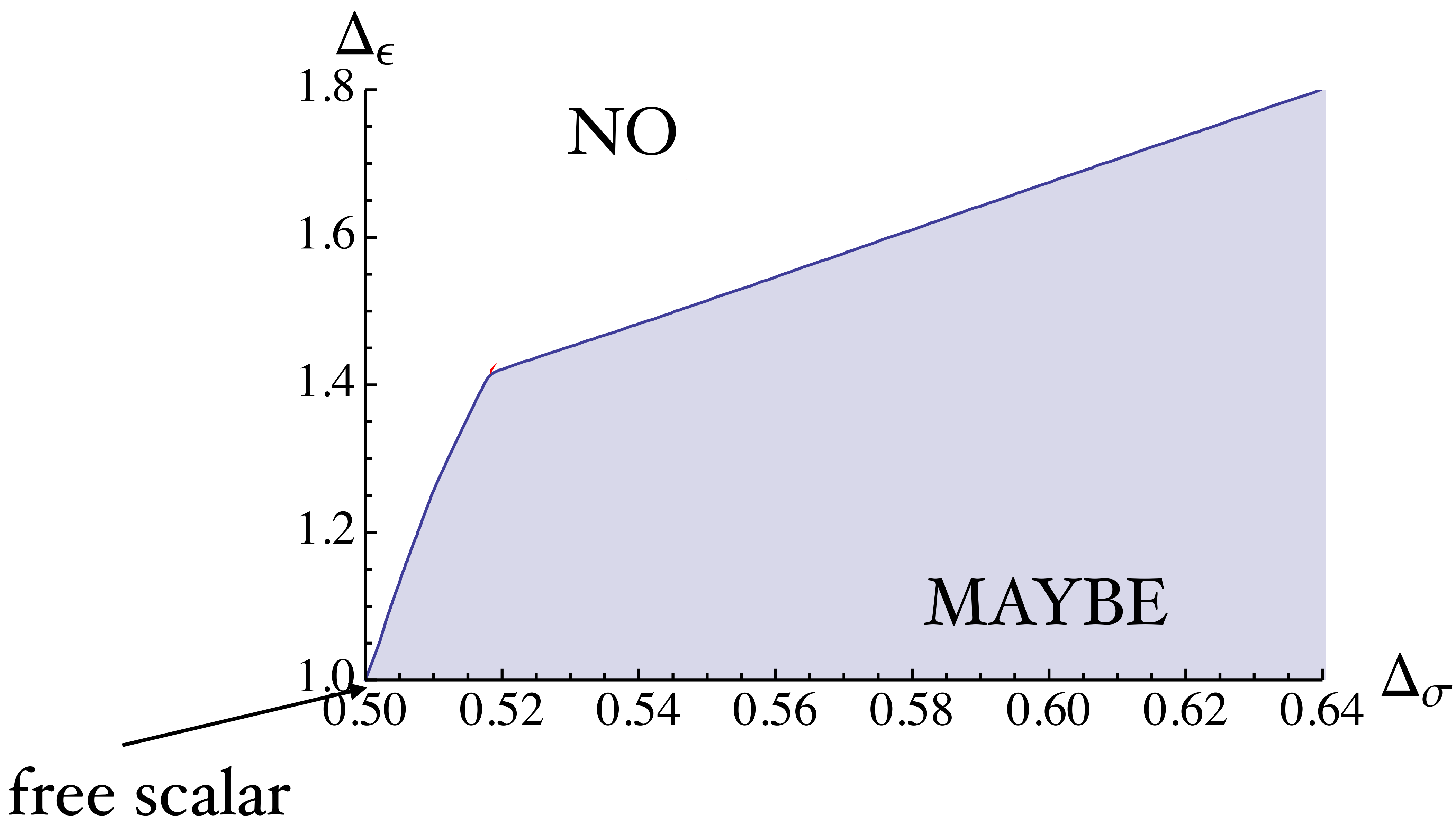


($\Delta\sigma$, $\Delta\varepsilon$)-oracle using <σσσσ>

El-Showk, Paulos, Poland, S.R., Simmons-Duffin, Vichi 2012



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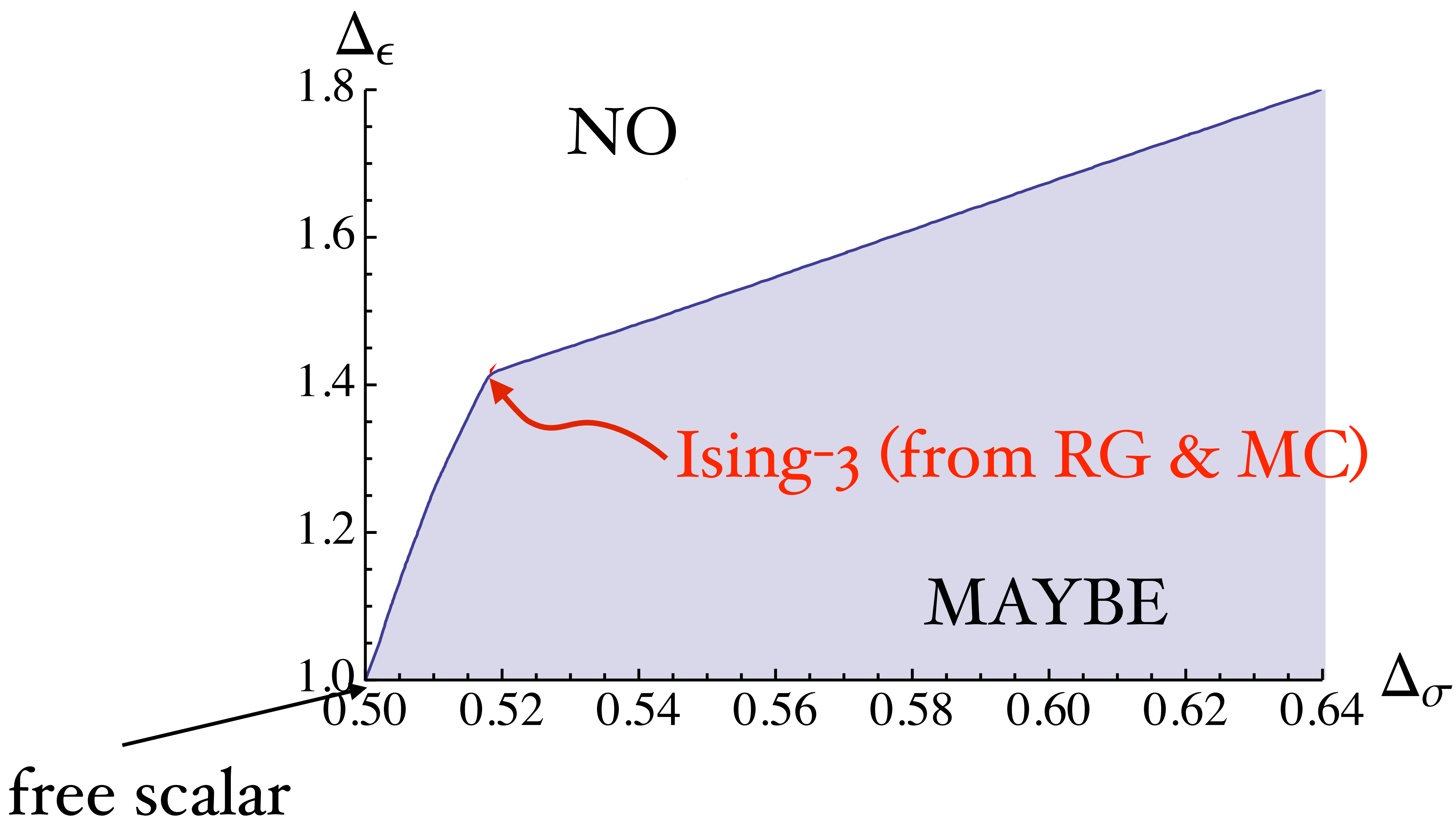

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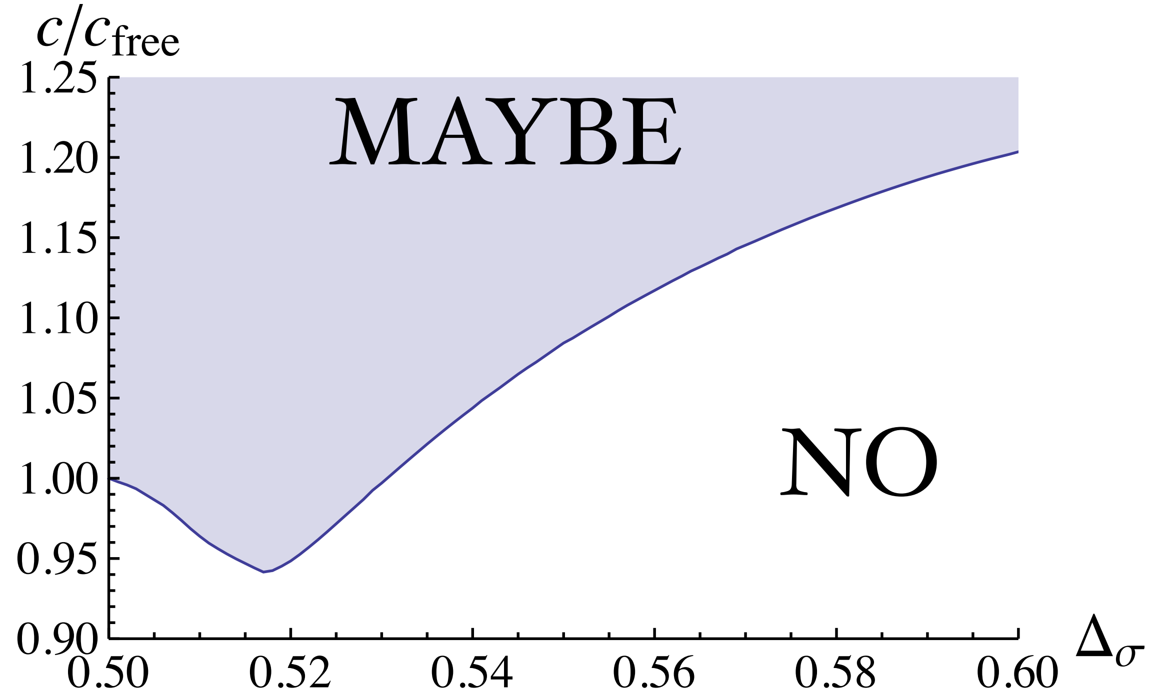

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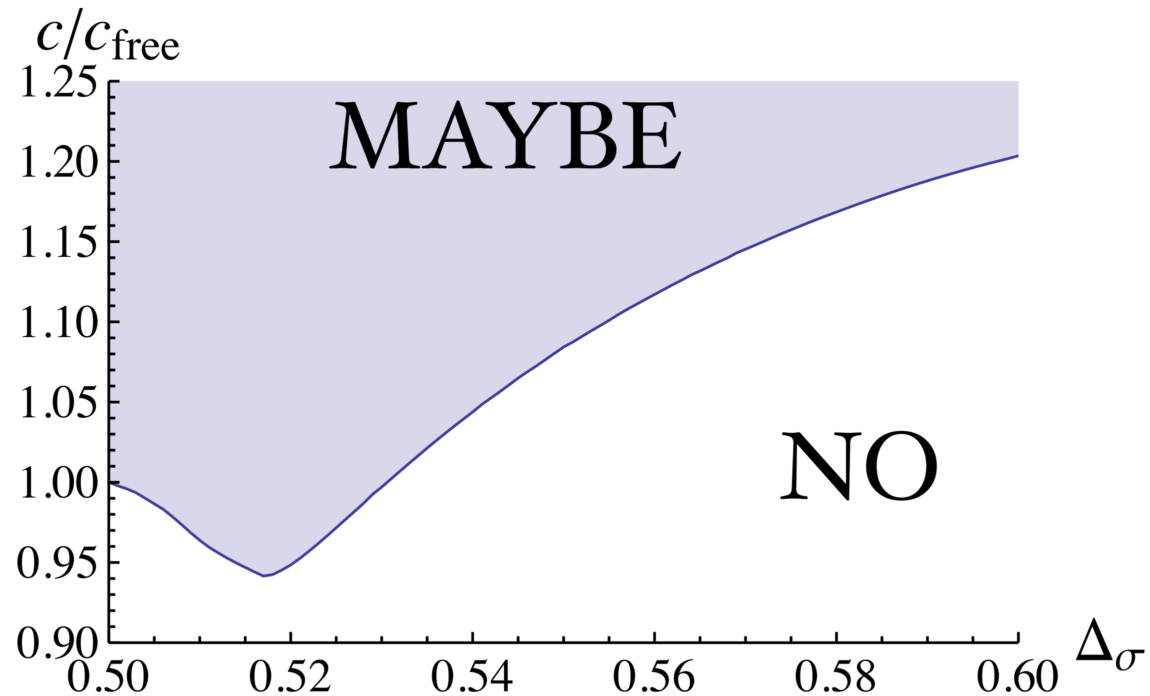
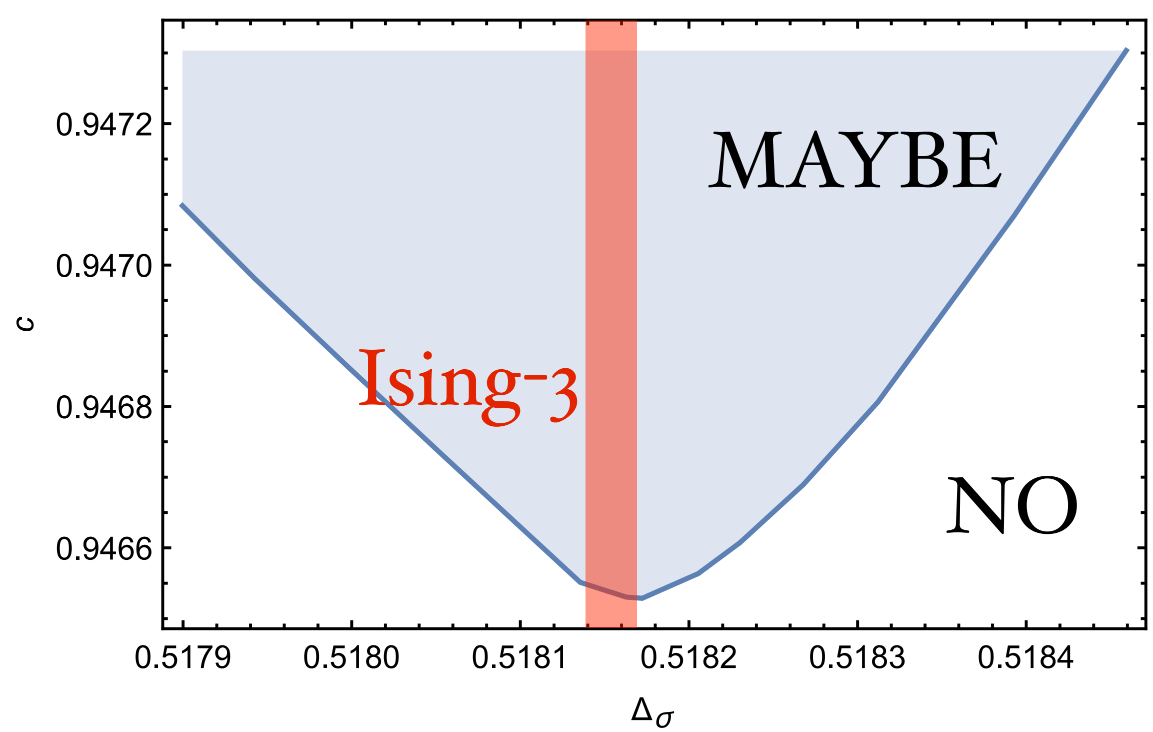


Zoom:



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known fact in d=2

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These are **world's most precise** determinations of 3d Ising model CFT data

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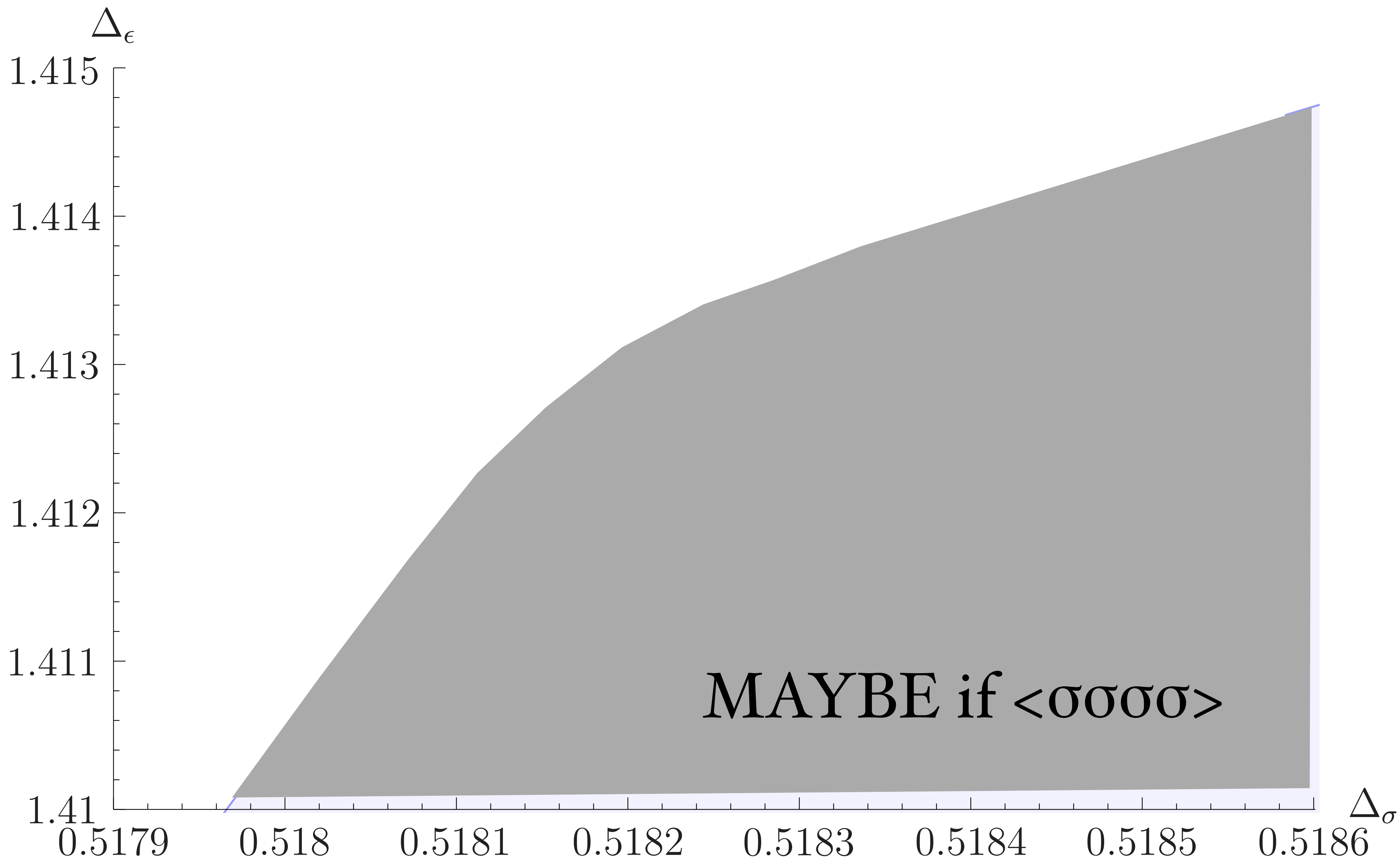
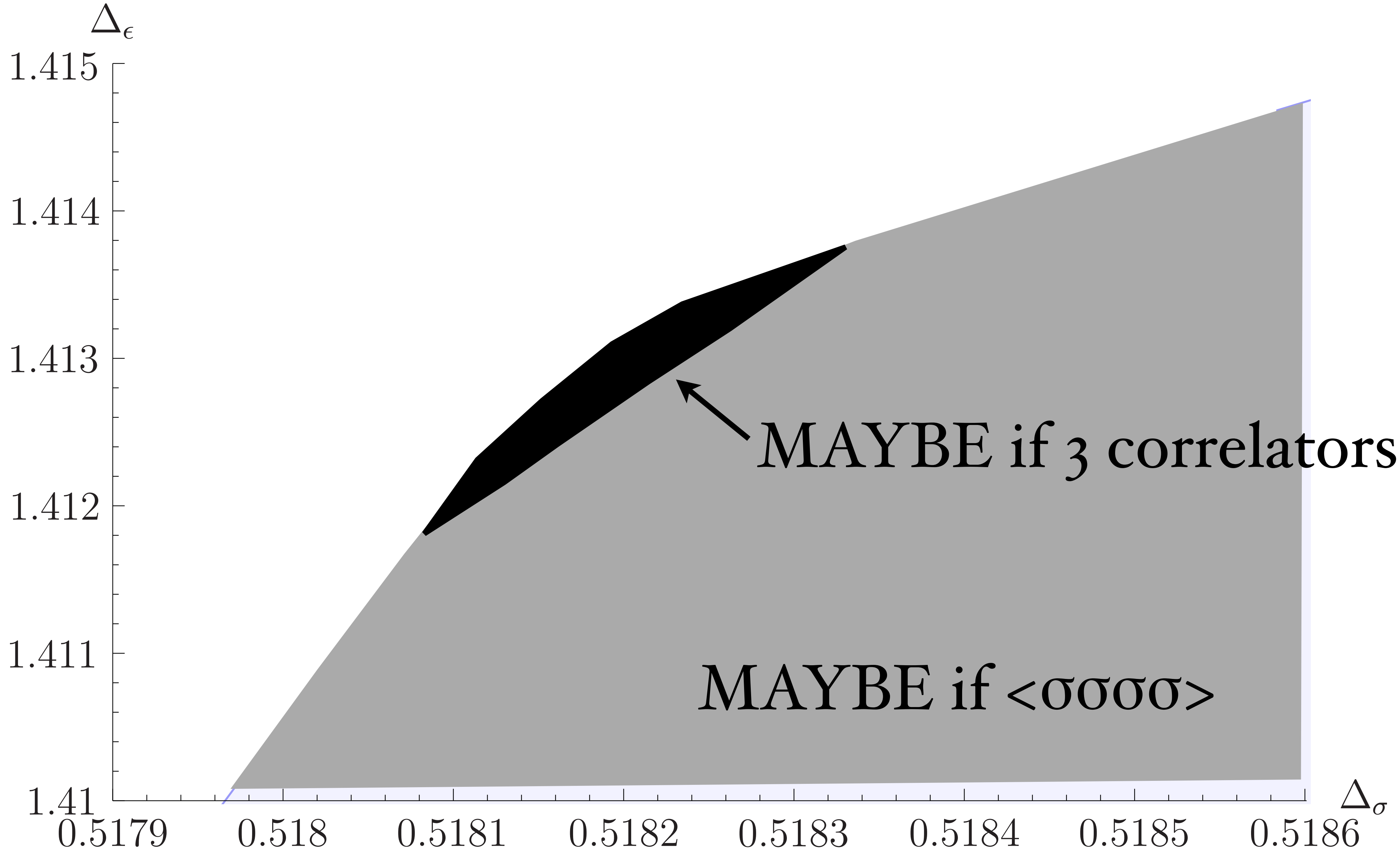
Kos,Poland,Simmons-Duffin 2014



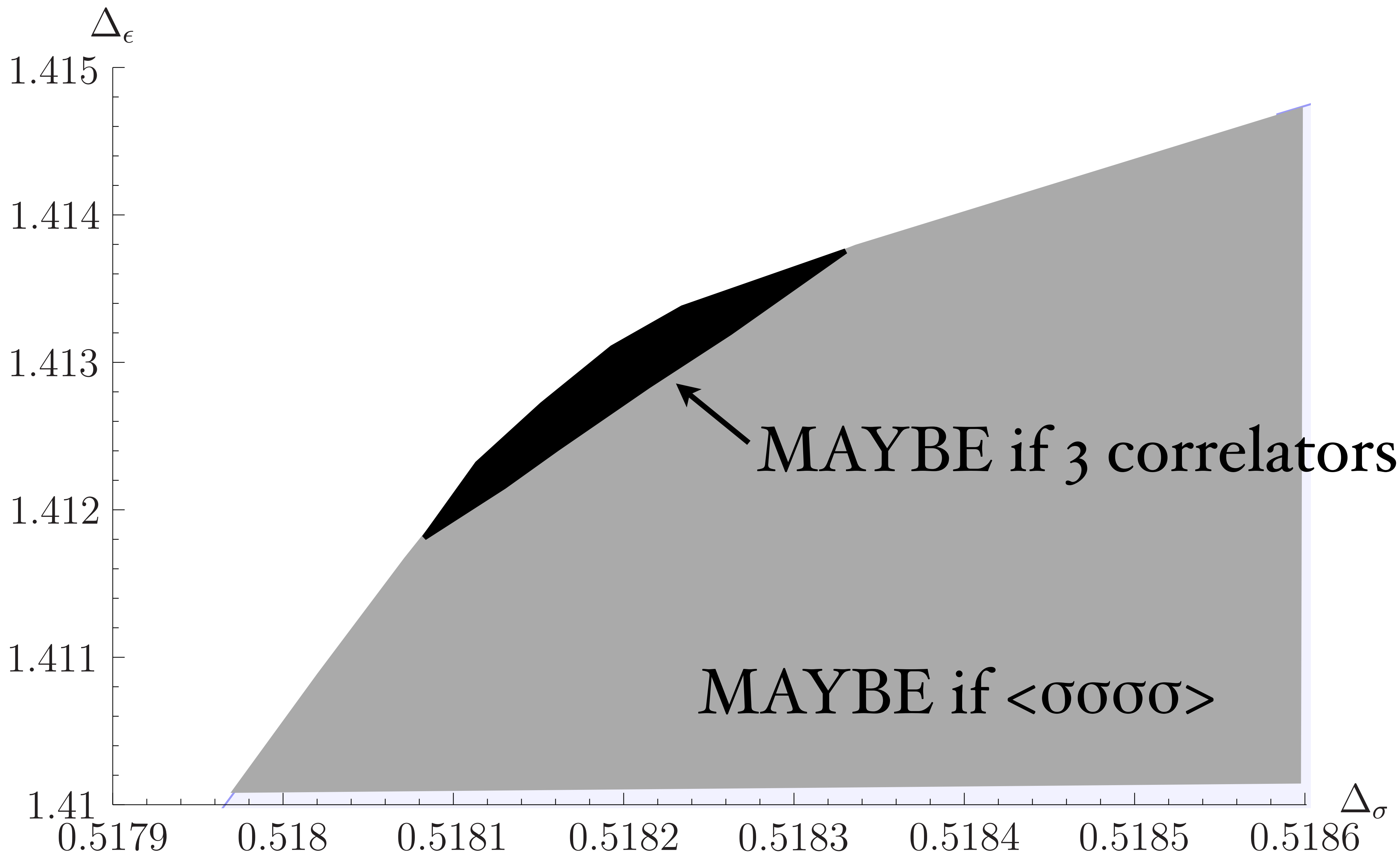
(Δσ, Δε)-oracle using <σσσσ>, <εεεε>, <σσεε>

Kos,Poland,Simmons-Duffin 2014



$(\Delta\sigma, \Delta\varepsilon)$-oracle using <σσσσ>, <εεεε>, <σσεε>

Kos,Poland,Simmons-Duffin 2014



Strongly coupled CFT computation with rigorous error bars

Oracle inner workings

Recall bootstrap: $$\sum_n f_{12n} f_{34n} \times (\ldots) = 2 \leftrightarrow 4$$

<u>Quadratic</u> equation for f's

But if identical operators : $$\sum_n f^2_{\sigma\sigma n} \times (\ldots) = 2 \leftrightarrow 4$$

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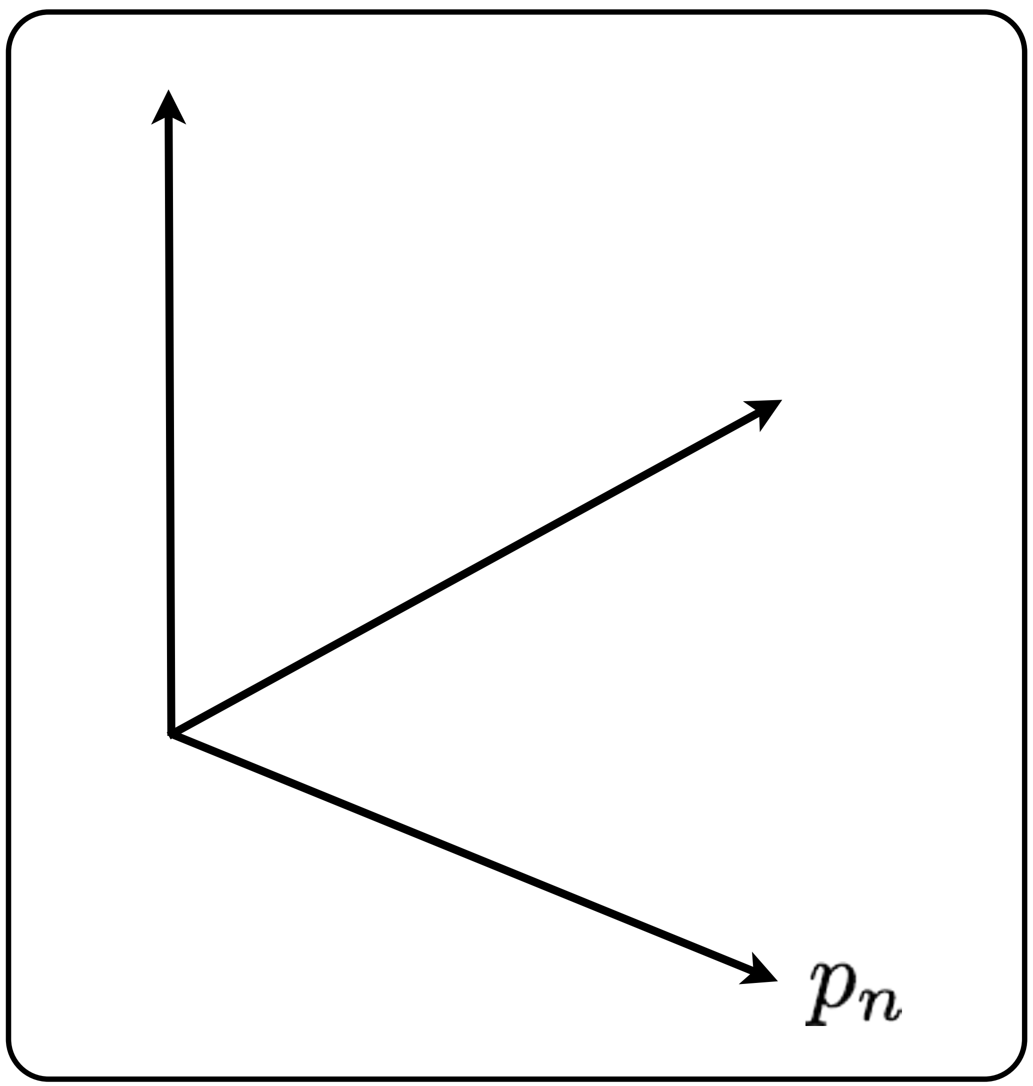

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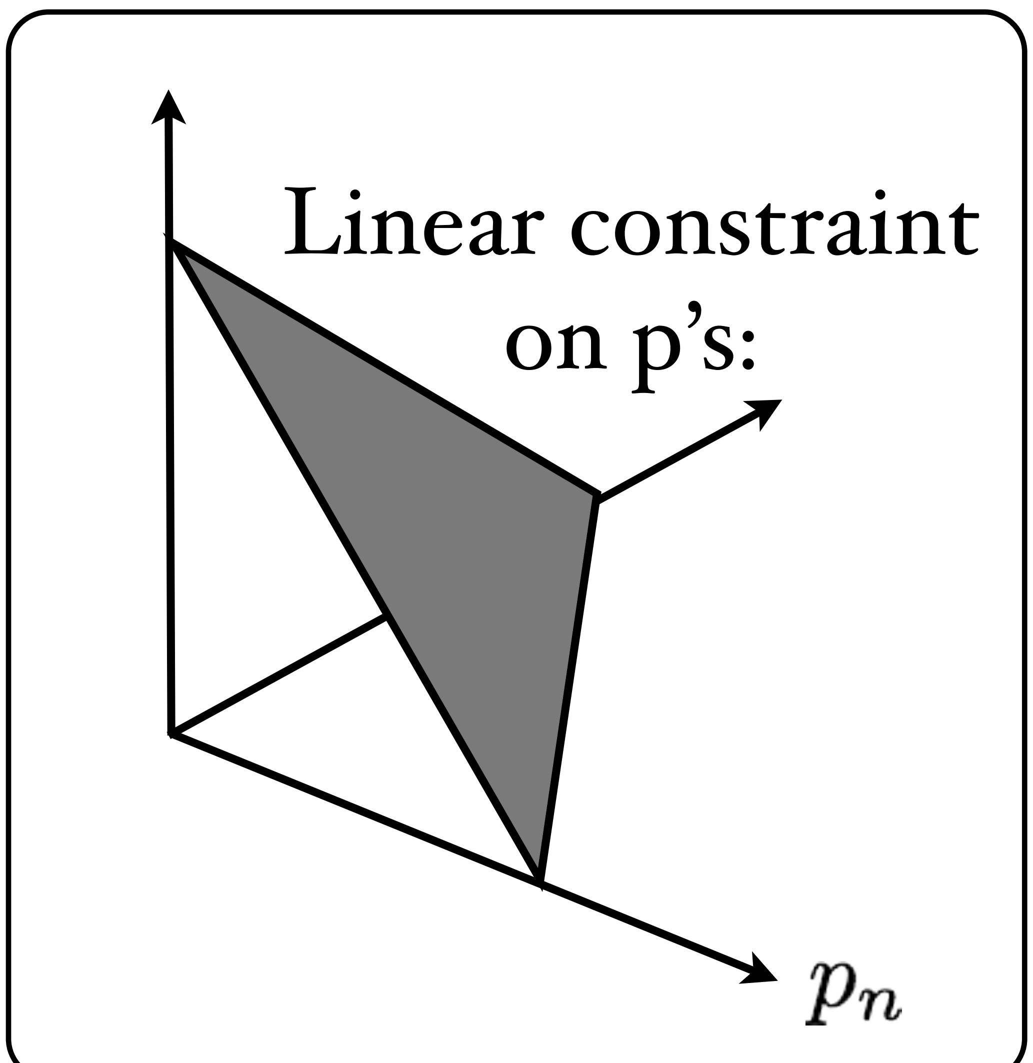

Is intersection nonempty?

Linear programming Dantzig 1947

Crucial: $p_n \geq 0$ is a convex condition

Several correlators <σσσσ>, <εεεε>, <σσεε>

$$\sum_n f_{\sigma\sigma n}^2 \times (\ldots) = 2 \leftrightarrow 4$$

$$\sum_n f_{\epsilon\epsilon n}^2 \times (\ldots) = 2 \leftrightarrow 4$$

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$$\sum f_{\sigma\sigma n} f_{\epsilon\epsilon n} \times (\ldots) = \sum f_{\sigma\epsilon n}^2 \times (\ldots)$$

non sign-definite

Several correlators <σσσσ>, <εεεε>, <σσεε>

$$\sum_n f^2_{\sigma\sigma n} \times (\ldots) = 2 \leftrightarrow 4 \qquad \sum_n f^2_{\epsilon\epsilon n} \times (\ldots) = 2 \leftrightarrow 4$$

$$\sum f_{\sigma\sigma n} f_{\epsilon\epsilon n} \times (\ldots) = \sum f^2_{\sigma\epsilon n} \times (\ldots)$$

non sign-definite

$$\mathbb{P}_n = \begin{pmatrix} f^2_{\sigma\sigma n} & f_{\sigma\sigma n} f_{\epsilon\epsilon n} \\ f_{\sigma\sigma n} f_{\epsilon\epsilon n} & f^2_{\epsilon\epsilon n} \end{pmatrix} \succeq 0$$

positive semidefinite $(\lambda_1 = 0, \lambda_2 = f^2_{\sigma\sigma n} + f^2_{\epsilon\epsilon n})$

Several correlators <σσσσ>, <εεεε>, <σσεε>

$$\sum_n f_{\sigma\sigma n}^2 \times (\ldots) = 2 \leftrightarrow 4 \qquad \sum_n f_{\epsilon\epsilon n}^2 \times (\ldots) = 2 \leftrightarrow 4$$

$$\sum f_{\sigma\sigma n} f_{\epsilon\epsilon n} \times (\ldots) = \sum f_{\sigma\epsilon n}^2 \times (\ldots)$$

non sign-definite

$$\mathbb{P}_n = \begin{pmatrix} f_{\sigma\sigma n}^2 & f_{\sigma\sigma n} f_{\epsilon\epsilon n} \\ f_{\sigma\sigma n} f_{\epsilon\epsilon n} & f_{\epsilon\epsilon n}^2 \end{pmatrix} \succeq 0$$

positive semidefinite $(\lambda_1 = 0, \lambda_2 = f_{\sigma\sigma n}^2 + f_{\epsilon\epsilon n}^2)$

Also a convex condition!

Linear programming → Semidefinite programming

Conclusions and future

- conformal bootstrap works in any d

- for some models (Ising-3, O(N)) even better than expected
  - lots of mileage out of a few constraints

Short-term: go through the list of known CFTs

Long-term: classify CFTs with a small number of low dimension operators

Backup

RG vs Monte Carlo and experiment

Ising-3

	Monte Carlo Hasenbusch 2010 (20 CPU-years)	RG Guida, Zinn-Justin 1998 (5-7 loops + Borel)
$\Delta(\varphi)$	0.51814(5)	0.51675(125)
$\Delta(\varphi^2)$	1.41275(25)	1.41370(330)

agrees, with O(10) larger errors

$O(2)$ d=3

	Experiment Lipa et al 1996	MC+HT Campostrini et al 2006	RG Guida, Zinn-Justin 1998
$\Delta(\varphi^2)$	1.5094(2)	1.5112(2)	1.5081(33)

8σ discrepancy

RG inconclusive

QCD conformal window

SU(3) massless gauge theory in d=3+1 with Nf fermions

Flows to a conformal IR fixed point for

$$N_F^* \leq N_F \leq 16$$

precise value (10-12) debated

Del Debbio, 1102.4046