Effective Field Theories and Higgs Physics

Lecture 1

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Outline

- Introduction
- Dimensional Analysis and Power Counting
- Examples
- Loops
- Spontaneously broken theories and Sigma Models
- Standard Model as an EFT

EFT lectures: hep-ph/9606222,

Field Redefinition example: hep-ph/9701294

An EFT is just a field theory, and you calculate the same way as in QED or QCD.

The only other condition is a power counting argument which tells you what contributions are relevant, analgous to the counting of α in QED.

Basic Idea

Effective field theory ideas are “obvious,” but non-trivial to actually use them correctly in quantum field theory.

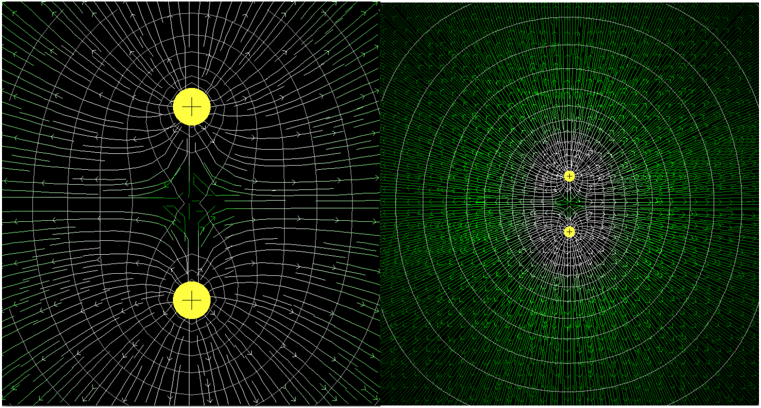
- You can make quantitative predictions of observable phenomena without knowing everything.
- The computations have some small (non-zero) error.
- Can improve on the accuracy by adding a finite number of additional parameters, in a systematic way.
- Key concept is locality — as a result one can factorize quantities into some short distance parameters (coefficients in the Lagrangian), and long distance operator matrix elements.

Example: H atom

- Chemistry and atomic physics depend on the interactions of atoms.
- The interaction Hamiltonian contains non-relativistic electrons and nuclei interacting via a Coulomb potential, plus electromagnetic radiation.
- The only property of the nucleus we need is the electric charge Z.
- The quark structure of the proton, weak interactions, GUTs, etc. are irrelevant.

- A more accurate calculation includes recoil corrections and needs the proton mass m_p.
- The fine structure needs m_e, m_p
- The Hyperfine interaction needs the proton magnetic moment μ_p
- Anomalous magnetic moment of the electron (calculable in QED)
- Charge radius, ...
- Weak interactions, ...
- If one is interested in atomic parity violation, weak interactions are the leading contribution, and cannot be treated as a small correction.

Multipole Expansion



The field far away looks just like a point charge.

System of size a.

$$V(r) = \sum c_{lm} Y_{lm}(\Omega) \frac{1}{r} \left(\frac{a}{r}\right)^l$$

At the classical level, expand in a/r.

c_{lm} expected to be of order unity, once a^l has been factored out.

Need more multipoles for a better description of the field.

The multipoles c_{lm} are localized at the origin. A local description in terms of a finite number of parameters, to a given order in a/r.

Effective theory is a local quantum field theory with a finite number of low energy parameters.

There is a systematic expansion in a small parameter like a/r for the multipole expansion. [called power counting]

Keep as many terms as you need to reach the desired accuracy.

It is a quantum theory — one can compute radiative corrections (loops), renormalize the theory, etc. just as for QED or QCD.

All the non-trivial effects are due to quantum corrections. Otherwise, just do a series expansion.

EFT is the low-energy limit of a “full theory”

It is not a Lagrangian with form-factors $e \to e\, F(q^2/M^2)$
These are non-local, contain an infinite amount of information, and lead to a violation of power counting.

It is not just a series expansion of amplitudes in the full theory

$$F(q^2/M^2) \to F(0) + F'(0)\frac{q^2}{M^2} + \ldots$$

though it looks like this at tree-level.

The EFT is an interacting quantum theory in its own right.

One can compute using it without ever referring to the full theory from which it came.

If you are given a full theory, can compute the EFT Lagrangian — matching.

The EFT has a different divergence structure from the full theory. The renormalization procedure is part of the definition of a field theory, not some irrelevant detail.

Reasons for using EFT

- Every theory is an effective theory: Can compute in the standard model, even if there are new interactions at (not much) higher energies.
- Greatly simplifies the calculation by only including the relevant interactions: Gives an explicit power counting estimate for the interactions.
- Deal with only one scale at a time: For example the B meson decay rate depends on M_W, m_b and Λ_{QCD}, and one can get horribly complicated functions of the ratios of these scales. In an EFT, deal with only one scale at a time, so there are no functions, only constants.

- Makes symmetries manifest: QCD has spontaneously broken chiral symmetry, which is manifest in the chiral Lagrangian, and heavy quark spin-flavor symmetry which is manifest in HQET. These symmetries are only true for certain limits of QCD, and so are hidden in the QCD Lagrangian.

$$b\uparrow,\ b\downarrow, c\uparrow, c\downarrow$$

- Sum logs: Use renormalization group improved perturbation theory. The running of constants is not small, e.g.

$$\alpha_s(M_Z) \sim 0.118, \qquad \alpha_s(m_b) \sim 0.22.$$

Fixed order perturbation theory breaks down. Sum logs of the ratios of scales (such as M_Z/m_b).

- Efficient way to characterize new physics: Can include the effects of new physics in terms of higher dimension operators. All the information about the dynamics is encoded in the coefficients. [This also shows it is difficult to discover new physics using low-energy measurements.]
- Include non-perturbative effects: Can include $\Lambda_{\rm QCD}/m$ corrections in a systematic way through matrix elements of higher dimension operators. The perturbative corrections and power corrections are tied together. [Renormalons]

Dimensional Analysis

Effective Lagrangian (neglect topological terms)

$$L = \sum c_i O_i = \sum L_D$$

is a sum of local, gauge and Lorentz invariant operators.

The functional integral is

$$\int \mathcal{D}\phi \; e^{iS}$$

so S is dimensionless.

Kinetic terms:

$$S = \int \mathrm{d}^d x \, \bar{\psi} \, i\not{D} \, \psi, \qquad S = \int \mathrm{d}^d x \, \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi$$

so

$$0 = -d + 2\,[\psi] + 1, \qquad 0 = -d + 2\,[\phi] + 2$$

Dimensions given by

$$[\phi] = (d-2)/2, \qquad [\psi] = (d-1)/2, \qquad [D] = 1, \qquad [gA_\mu] = 1$$

Field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \ldots$ so A_μ has the same dimension as a scalar field.

$$[g] = 1 - (d-2)/2 = (4-d)/2$$

In $d = 4$,

$$[\phi] = 1, \qquad [\psi] = 3/2, \qquad [A_\mu] = 1, \qquad [D] = 1, \qquad [g] = 0$$

Only Lorentz invariant renormalizable interactions (with $D \leq 4$) are

$$\begin{aligned}
D = 0: &\quad 1 \\
D = 1: &\quad \phi \\
D = 2: &\quad \phi^2 \\
D = 3: &\quad \phi^3, \bar{\psi}\psi \\
D = 4: &\quad \phi\bar{\psi}\psi, \phi^4
\end{aligned}$$

and kinetic terms which include gauge interactions.

Renormalizable interactions have coefficients with mass dim ≥ 0.

In $d = 2$,

$$[\phi] = 0, \qquad [\psi] = 1/2, \qquad [A_\mu] = 0, \qquad [D] = 1, \qquad [g] = 1$$

so an arbitrary potential $V(\phi)$ is renormalizable. Also $(\bar{\psi}\psi)^2$ is renormalizable.

In $d = 6$,

$$[\phi] = 2, \qquad [\psi] = 5/2, \qquad [A_\mu] = 2, \qquad [D] = 1, \qquad [g] = -1$$

Only allowed interaction is ϕ^3.

What Fields to use for EFT?

Not always obvious: Low energy QCD described in terms of meson fields.

NRQCD/NRQED and SCET: Naive guess does not work. Need multiple gluon fields.

The Sine-Gordon model is the massive Thirring model.
Two theories in 1+1 dimensions

$$L = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{\alpha}{\beta^2}\cos\beta\phi, \qquad L = \bar\psi\left(i\partial\!\!\!/ - m\right)\psi - \frac{1}{2}g\left(\bar\psi\gamma^\mu\psi\right)^2,$$

$$\frac{\beta^2}{4\pi} = \frac{1}{1+g/\pi}.$$

Effective Lagrangian:

$$L_D = \sum_D \frac{O_D}{M^{D-d}}$$

so in $d = 4$,

$$L_{eft} = L_{D\leq 4} + \frac{O_5}{M} + \frac{O_6}{M^2} + \ldots$$

An infinite number of terms (and parameters)

Power Counting

If one works at some typical momentum scale p, and neglects terms of dimension D and higher, then the error in the amplitudes is of order

$$\left(\frac{p}{M}\right)^{D-4}$$

A non-renormalizable theory is just as good as a renormalizable theory for computations, provided one is satisfied with a finite accuracy.

Usual renormalizable case given by taking $M \to \infty$.

Photon-Photon Scattering

(a) (b)

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha^2}{m_e^4} \left[c_1 \left(F_{\mu\nu} F^{\mu\nu} \right)^2 + c_2 \left(F_{\mu\nu} \tilde{F}^{\mu\nu} \right)^2 \right].$$

(Terms with only three field strengths are forbidden by charge conjugation symmetry.)

e^4 from vertices, and $1/16\pi^2$ from the loop.

An explicit computation gives

$$c_1 = \frac{1}{90}, \qquad c_2 = \frac{7}{90}.$$

Scattering amplitude

$$A \sim \frac{\alpha^2 \omega^4}{m_e^4}$$

and

$$\sigma \sim \left(\frac{\alpha^2 \omega^4}{m_e^4}\right)^2 \frac{1}{\omega^2} \frac{1}{16\pi} \sim \frac{\alpha^4 \omega^6}{16\pi m_e^8} \times \frac{15568}{22275}$$

$$A \propto \frac{1}{m_e^4} \Rightarrow \sigma \propto \omega^6$$

determined by the operator dimension.

Rayleigh Scattering

Scattering of light from atoms

$$L = \psi^\dagger \left(i\partial_t - \frac{p^2}{2M} \right) \psi + a_0^3 \, \psi^\dagger \psi \left(c_1 E^2 + c_2 B^2 \right)$$

$$A \sim c_i \, a_0^3 \, \omega^2$$

$$\sigma \propto a_0^6 \, \omega^4 .$$

Scattering goes as the fourth power of the frequency, so blue light is scattered about 16 times mores strongly than red.

a_0^3 dimensional analysis.

Landau-Yang Theorem

Assume the Higgs is a spin-1 particle, h_μ. Then $h \to \gamma\gamma$ is given by a Lagrangian

$$L = \partial^\mu h_\mu F_{\alpha\beta} F^{\alpha\beta}, \qquad h^\mu F_{\mu\alpha} \partial_\nu F^{\nu\alpha}, \qquad \text{etc}$$

But for a physical spin-one particle with momentum p and polarization ϵ

$$p \cdot \epsilon = 0$$

and for a massless particle $p^2 = 0$, so

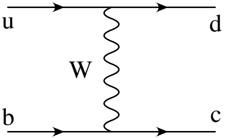
$$h \not\to \gamma\,\gamma \qquad \text{for spin-one}$$

with all particles on-shell.

Low energy weak interactions

W boson interacts with a current:

$$-\frac{ig}{\sqrt{2}}\; V_{ij}\; \bar{q}_i\, \gamma^\mu\, P_L\, q_j,$$



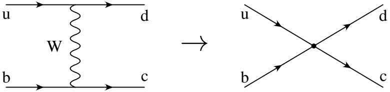
The tree-level amplitude is

$$A = \left(\frac{ig}{\sqrt{2}}\right)^2 V_{cb} V_{ud}^*\; (\bar{c}\, \gamma^\mu\, P_L\, b)\, (\bar{d}\, \gamma^\nu\, P_L\, u) \left(\frac{-i g_{\mu\nu}}{p^2 - M_W^2}\right),$$

For low momentum transfers, $p \ll M_W$:

$$\frac{1}{p^2 - M_W^2} = -\frac{1}{M_W^2}\left(1 + \frac{p^2}{M_W^2} + \frac{p^4}{M_W^4} + \ldots\right),$$

and retaining only a finite number of terms.



integrate out W

$$A = \frac{i}{M_W^2}\left(\frac{ig}{\sqrt{2}}\right)^2 V_{cb} V_{ud}^* \; (\bar{c}\,\gamma^\mu\, P_L\, b)\,(\bar{d}\,\gamma_\mu\, P_L\, u) + \mathcal{O}\left(\frac{1}{M_W^4}\right).$$

$$L = -\frac{4G_F}{\sqrt{2}} V_{cb} V_{ud}^* \; (\bar{c}\,\gamma^\mu\, P_L\, b)\,(\bar{d}\,\gamma_\mu\, P_L\, u) + \mathcal{O}\left(\frac{1}{M_W^4}\right),$$

$$\frac{G_F}{\sqrt{2}} \equiv \frac{g^2}{8M_W^2}.$$

Effective Lagrangian for μ decay

$$L = -\frac{4G_F}{\sqrt{2}}\; (\bar{e}\,\gamma^\mu\, P_L\,\nu_e)\,(\bar{\nu}_\mu\,\gamma^\mu\, P_L\,\mu) + \mathcal{O}\left(\frac{1}{M_W^4}\right),$$

Gives the standard result for the muon lifetime at lowest order,

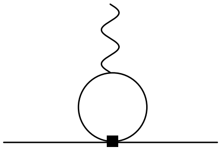
$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\; f\left(\frac{m_e^2}{m_\mu^2}\right)$$

EFT gives the full dependence on low energy parameters.

$$f(\rho) = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho, \qquad \rho = \frac{m_e^2}{m_\mu^2}$$

The advantages of EFT show up in higher order calculations

Loops



The dimension-six operator gives a contribution

$$\frac{1}{M_W^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m^2} \sim \frac{\Lambda^2}{M_W^2} \sim \mathcal{O}(1)$$

Similarly, a dimension eight operator has vertex k^2/M_W^4, and gives a contribution

$$\frac{1}{M_W^4}\int \frac{d^4k}{(2\pi)^4}\frac{k^2}{k^2-m^2} \sim \frac{\Lambda^4}{M_W^4} \sim \mathcal{O}(1)$$

Would need to know the entire effective Lagrangian, since all terms are equally important. The reason for this breakdown is using a cutoff procedure with a dimensionful parameter Λ.

More generally, need to make sure that dimensionful parameters at the high scale do not occur in the numerator after evaluating Feynman diagrams.

In doing weak interactions, one should not have M_G or M_P appear in the numerator.

Need a renormalization scheme which maintains the power counting.

Dimensional Regularization

$d = 4 - 2\epsilon$:

$$\mu^{2\epsilon} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\left(k^2\right)^a}{\left(k^2 - M^2\right)^b}$$

$$= \frac{\mu^{2\epsilon}}{(4\pi)^{d/2}} \frac{(-1)^{a-b}\Gamma(d/2+a)\Gamma(b-a-d/2)}{\Gamma(d/2)\Gamma(b)} \left(M^2\right)^{d/2+a+b}$$

Integral defined by analytic continuation.

Convert all integrals to this form using Feynman parameters for the denominator.

$\overline{\text{MS}}$

Need to use a mass independent subtraction scheme such as $\overline{\text{MS}}$:

μ can only occur in logarithms, so

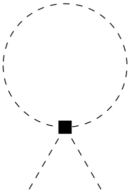
$$\frac{1}{M_W^2}\mu^{2\epsilon}\int d^d k\, \frac{1}{k^2-m^2} \sim \frac{m^2}{M_W^2}\log\frac{\mu^2}{m^2},$$

$$\frac{1}{M_W^4}\mu^{2\epsilon}\int d^d k\, \frac{k^2}{k^2-m^2} \sim \frac{m^4}{M_W^4}\log\frac{\mu^2}{m^2},$$

Expanding $1/(k^2-M_W^2)$ in a power series ensures that there is no pole for $k \sim M_W$, and so M_W cannot appear in the numerator.

Dimensional regularization is like doing integrals using residues. Relevant scales given by poles of the denominator.

No Quadratic Divergences



The standard argument for the hierarchy problem is that there are quadratic divergences in corrections to the Higgs mass.

But the actual integral gives

$$\mu^{2\epsilon}\int d^d k\,\frac{1}{k^2-m_H^2}\sim m_H^2\log\frac{\mu^2}{m_H^2}$$

not

$$\mu^{2\epsilon}\int d^d k\,\frac{1}{k^2-m_H^2}\sim \Lambda^2$$

Power Counting Formula

Manifest power counting in p/M.

Loop graphs consistent with the power counting, since one can never get any M's in the numerator.

If the vertices have $1/M^a$, $1/M^b$, etc. then any amplitude (including loops) will have

$$\frac{1}{M^a}\frac{1}{M^b}\ldots = \frac{1}{M^{a+b+\ldots}}$$

Correct dimensions due to factors of the low scale in the numerator, represented generically by p. (Could be a mass)

Loop graph with insertions of operators O_k of dimension d_k:

$$d - 4 = \sum_k (d_k - 4)$$

Power Counting Formula

Only a finite number of terms to any given order in $1/M$.

Order $1/M$: L_5 at tree level

Order $1/M^2$: L_6 at tree level,
or loop graphs with two insertions of L_5.

General power counting result:

- you can count the powers of M.
- you can count powers of p

Power counting formula for χPT:

$$A \sim p^r, \qquad r = 2L + 2 + \sum_k n_k (k-2)$$

where n_k is the number of vertices of order p^k.

Naive Dimensional Analysis

A.M. and H. Georgi, NPB 234 (1984) 189

A power counting scheme keeping track of 4π factors: An effective Lagrangian term has the form

$$L = f^2\Lambda^2 \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^a \left(\frac{\phi}{f}\right)^b \left(\frac{D}{\Lambda}\right)^c \left(\frac{gF_{\mu\nu}}{\Lambda^2}\right)^d \qquad \Lambda \sim 4\pi f$$

Λ is the scale of variation of form factors, and is what we called M on the previous slide.

This makes a big difference in estimating the coefficients of higher dimension terms. E.g. a 4-quark operator has coefficient

$$L = \frac{1}{f^2}\left(\overline{\psi}\gamma^\mu\psi\right)^2 \qquad \text{not} \qquad \frac{1}{\Lambda^2}\left(\overline{\psi}\gamma^\mu\psi\right)^2$$

The difference is ~ 150.

EFT vs κ

Experimental analysis using κ factors, so $h \to \gamma\gamma$, or *ZWW* vertex gets multiplied by κ. Fine as a way of presenting data.

Theoretically, this does not include the powerful constraints from gauge invariance and locality.

A local operator

$$\epsilon_{abc} W^a_{\mu\nu} W^b_{\nu\alpha} W^c_{\alpha\mu}$$

gives a specific momentum and polarization depedence. Also relates *WWW* and *WWWW* vertices. Similarly 4-fermion operators in weak decays such as neutron decay change the rate, angular and spin correlations, etc. Correlated changes in different observables.

Power counting based on the dimension of the operator. $\gamma\gamma \to \gamma\gamma$ had the lowest dimension operator of dimension 8. SMEFT *S* and *T* are dim 6, and *U* is dim 8.