QCD & Monte Carlo Event Generators

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PART I: INTRODUCTION

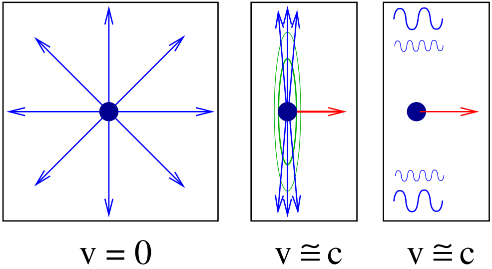
QCD BASICS

SCALES & KINEMATICS

Contents

An electromagnetic analogy

- consider a charge Z moving at constant velocity v



- at $v = 0$: radial E field only
- at $v = c$: B field emerges: $\vec{E} \perp \vec{B}$, $\vec{B} \perp \vec{v}$, $\vec{E} \perp \vec{v}$,

  energy flow $\sim$ Poynting vector $\vec{S} \sim \vec{E} \times \vec{B}$, $\parallel \vec{v}$
- approximate classical fields by "equivalent quanta": photons

- spectrum of photons:

(in dependence on energy ω and transverse distance $b_\perp$)

$$\mathrm{d}n_\gamma = \frac{Z^2\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}b_\perp^2}{b_\perp^2} \xrightarrow{\text{electron}(Z=1)} \frac{\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}b_\perp^2}{b_\perp^2}$$

- Fourier transform to transverse momenta $k_\perp$:

$$\mathrm{d}n_\gamma = \frac{\alpha}{\pi}\cdot\frac{\mathrm{d}\omega}{\omega}\cdot\frac{\mathrm{d}k_\perp^2}{k_\perp^2}$$

note: divergences for $k_\perp \to 0$ (collinear) and $\omega \to 0$ (soft)

- therefore: Fock state for lepton = superposition (coherent):

$$|e\rangle_{\rm phys} = |e\rangle + |e\gamma\rangle + |e\gamma\gamma\rangle + |e\gamma\gamma\gamma\rangle + \ldots$$

photon fluctuations will “recombine”

- lifetime of electron–photon fluctuations: $e(P) \to e(p) + \gamma(k)$
- first estimate: use uncertainty relation and Lorentz time dilation
  - $P^2 = (p+k)^2 = M^2_{\rm virt}$ the virtual mass of the incident electron
  - life time = life time in rest frame · time dilation

$$\tau \sim \frac{1}{M_{\rm virt}} \cdot \frac{E}{M_{\rm virt}} = \frac{E}{(p+k)^2} \sim \frac{E}{2Ek(1-\cos\theta)} \approx \frac{k}{k^2\sin^2\theta/2} \approx \frac{\omega}{k_\perp^2}$$

- second estimate: use uncertainty relation and assume only photon off-shell
  - energy balance of photon

$$P^2 = 2p\cdot k + k^2\,, \quad \text{therefore}\ \ k^2 \approx -k_\perp^2 \approx -2p\cdot k < 0\,.$$

  - assume photon momentum to be $k^\mu = (\omega, \vec{k}_\perp, k_\|)$,
    shift in energy for photon going on-shell: $\delta\omega \sim k_\perp^2/\omega$, therefore

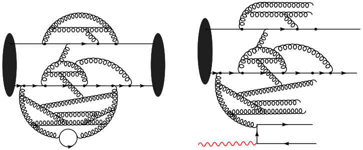
$$\tau_\gamma \sim \frac{1}{\delta\omega} \approx \frac{\omega}{k_\perp^2} \approx \frac{\omega}{\omega^2\sin^2\theta} \approx \frac{1}{\omega\theta^2}$$

- lifetime larger with smaller transverse momentum

(i.e. with larger transverse distance)

QED Initial and Final State Radiation

- physical interpretation:
  equivalent quanta = quantum manifestation of accompanying fields
- in absence of interaction: recombination enforced by coherence
- but: hard interaction possibly "kicks out" quantum
  $\longrightarrow$ coherence broken
  $\longrightarrow$ equivalent (virtual) quanta become real
  $\longrightarrow$ emission pattern unravels



- alternative idea:
  initial state radiation of photons off incident electron

- consider final state radiation in $\gamma^* \to \ell\bar{\ell}$
  (electron velocities/momenta labelled as v and v'/p and p')
- classical electromagnetic spectrum from radiation function:

(this is from Jackson or any other reasonable book on ED)

$$\frac{\mathrm{d}^2 I}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{e^2}{4\pi^2}\left|\vec{\epsilon}^{\,*}\cdot\left(\frac{\vec{v}}{1-\vec{v}\cdot\vec{n}}-\frac{\vec{v}'}{1-\vec{v}'\cdot\vec{n}}\right)\right|^2,$$

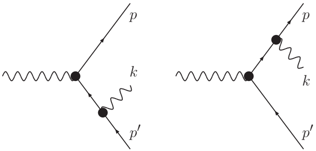
with ϵ the polarisation vector and $\vec{n}(\Omega)$ the direction of the radiation

- recast with four–momenta, equivalent photon spectrum:

$$\begin{aligned}\mathrm{d}N &= \frac{\mathrm{d}^3 k}{(2\pi)^3 2k_0}\,\frac{\alpha}{\pi}\left|\epsilon^*_\mu\left(\frac{p^\mu}{p\cdot k}-\frac{p'^\mu}{p'\cdot k}\right)\right|^2\\ &= \frac{\mathrm{d}^3 k}{(2\pi)^3 2k_0}\,\frac{\alpha}{\pi}\left|\mathrm{W}_{pp';k}\right|^2\end{aligned}$$

with the eikonal $\mathrm{W}_{pp';k}$

- repeat exercise in QFT, Feynman diagrams:



$$
\begin{aligned}
\mathcal{M}_{X\to e^+e^-\gamma} &= e\bar{u}(p)\left[\Gamma\,\frac{\not{p}'-\not{k}}{(p'-k)^2}\,\gamma^\mu - \gamma^\mu\,\frac{\not{p}+\not{k}}{(p+k)^2}\,\Gamma\right]u(p')\epsilon^*_\mu(k) \\
\stackrel{\text{soft}}{\longrightarrow}\quad e\epsilon^*_\mu(k)\left[\frac{p^\mu}{p\cdot k}-\frac{p'^\mu}{p'\cdot k}\right]\,\bar{u}(p')\Gamma u(p) &= e\mathcal{M}_{X\to e^+e^-\gamma}\cdot \mathrm{W}_{pp';k}
\end{aligned}
$$

- manifestation of Low's theorem:
  soft radiation independent of spin ($\to$ classical)

(radiation decomposes into soft, classical part with logs – i.e. dominant – and hard collinear part)

DGLAP equations for QED

(Dokshitser–Gribov–Lipatov–Altarelli–Parisi Equations)

- define probability to find electron or photon in electron:

$$\begin{aligned}\text{at LO in } \alpha(\text{noemission}):\ \ell(x,\, k_\perp^2) &= \delta(1-x)\\ \text{and}\quad \gamma(x,\, k_\perp^2) &= 0\end{aligned}$$

(introduced $x =$ energy fraction w.r.t. physical state)

- including emissions:
  - probabilities change
  - energy fraction ξ of lepton parton w.r.t. the physical lepton object reduced by some fraction $z = x/\xi$
  - reminder: differential of photon number w.r.t. $k_\perp^2$:

$$\mathrm{d}n_\gamma = \frac{\alpha}{\pi}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\frac{\mathrm{d}\omega}{\omega} \leftrightarrow \frac{\mathrm{d}n_\gamma}{\mathrm{d}\log k_\perp^2} = \frac{\alpha}{\pi}\frac{\mathrm{d}x}{x}$$

- evolution equations (trivialised)

$$\frac{\mathrm{d}\ell(x,\,k_\perp^2)}{\mathrm{d}\log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi}\int\limits_x^1 \frac{\mathrm{d}\xi}{\xi}\mathcal{P}_{\ell\ell}\left(\frac{x}{\xi},\,\alpha(k_\perp^2)\right)\ell(\xi,\,k_\perp^2)$$

$$\frac{\mathrm{d}\gamma(x,\,k_\perp^2)}{\mathrm{d}\log k_\perp^2} = \frac{\alpha(k_\perp^2)}{2\pi}\int\limits_x^1 \frac{\mathrm{d}\xi}{\xi}\mathcal{P}_{\gamma\ell}\left(\frac{x}{\xi},\,\alpha(k_\perp^2)\right)\ell(\xi,\,k_\perp^2)\,.$$

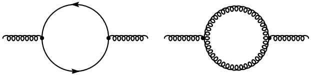
- $k_\perp^2$ plays the role of "resolution parameter"
- the $\mathcal{P}_{ab}(z)$ are the splitting functions, encoding quantum mechanics of the "splitting cross section", for example (at LO)

$$\mathcal{P}_{\ell\ell}(z) = \left(\frac{1+z^2}{1-z}\right)_+ + \frac{3}{2}\delta(1-z)$$

- if $\gamma\to\ell\bar{\ell}$ splittings included, have to add entries/splitting functions into evolution equations above

Running of α_s and bound states

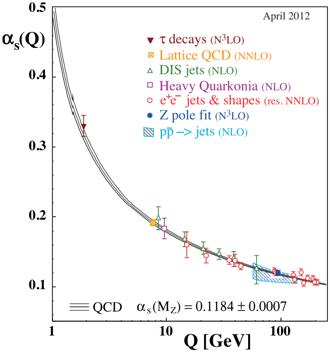
- quantum effect due to loops:
  couplings change with scale
- running driven by β–function



$$\beta(\alpha_s) = \mu_R^2 \frac{\partial \alpha_s(\mu_R^2)}{\partial \mu_R^2} = \frac{\beta_0}{4\pi}\alpha_s^2 + \frac{\beta_1}{(4\pi)^2}\alpha_s^3 + \ldots$$

with

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_R n_f$$
$$\beta_1 = \frac{34}{3} C_A^2 - \frac{20}{3} C_A T_R n_f - 4\, C_F T_R n_f$$



- Casimir operators in the fundamental and adjoint representation:

$$C_F = \frac{N_c^2-1}{2N_c} \quad \text{and} \quad C_A = N_c$$

  with $N_c = 3$ colours and $T_R = 1/2$.
- n_f = the number of (quark) flavours
- the Casimirs correspond to quark and gluon colour charges
- explicit expression for strong coupling

$$\alpha_s(\mu_R^2) \equiv \frac{g_s^2(\mu_R^2)}{4\pi} = \frac{1}{\frac{\beta_0}{4\pi}\log\frac{\mu_R^2}{\Lambda_{\rm QCD}{}^2}}$$

  with $\Lambda_{\rm QCD}$ the Landau pole of QCD, $\Lambda_{\rm QCD} \approx 250\,{\rm MeV}$.

Hadrons in initial state: DGLAP equations of QCD

- similar to QED case:
  define probabilities (at LO) to find a parton q – quark or gluon – in hadron h at energy fraction x and resolution parameter/scale Q:
  parton distribution function (PDF) $f_{q/h}(x,\,Q^2)$
- scale-evolution of PDFs: DGLAP equations

$$\frac{\partial}{\partial \log Q^2}\left(\begin{array}{c} f_{q/h}(x,\,Q^2) \\ f_{g/h}(x,\,Q^2) \end{array}\right)$$

$$= \frac{\alpha_s(Q^2)}{2\pi}\int\limits_x^1 \frac{\mathrm{d}z}{z}\left(\begin{array}{cc} \mathcal{P}_{qq}\left(\frac{x}{z}\right) & \mathcal{P}_{qg}\left(\frac{x}{z}\right) \\ \mathcal{P}_{gq}\left(\frac{x}{z}\right) & \mathcal{P}_{gg}\left(\frac{x}{z}\right) \end{array}\right)\left(\begin{array}{c} f_{q/h}(z,\,Q^2) \\ f_{g/h}(z,\,Q^2) \end{array}\right),$$

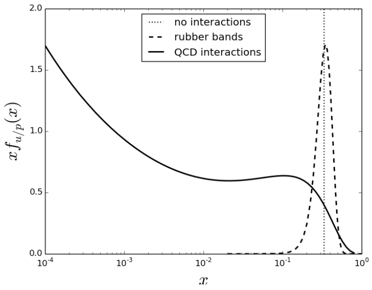
- QCD splitting functions:

$$\begin{aligned}
\mathcal{P}_{qq}^{(1)}(x) &= C_F\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\delta(1-x)\right] = \left[P_{qq}^{(1)}(x)\right]_+ + \gamma_q^{(1)}\delta(1-x)\\
\mathcal{P}_{qg}^{(1)}(x) &= T_R\left[x^2+(1-x)^2\right] = P_{qg}^{(1)}(x)\\
\mathcal{P}_{gq}^{(1)}(x) &= C_F\left[\frac{1+(1-x)^2}{x}\right] = P_{gq}^{(1)}(x)\\
\mathcal{P}_{gg}^{(1)}(x) &= 2C_A\left[\frac{x}{(1-x)_+}+\frac{1-x}{x}+x(1-x)\right]\\
&\quad +\frac{11C_A-4n_fT_R}{6}\delta(1-x) = \left[P_{gg}^{(1)}(x)\right]_+ + \gamma_g^{(1)}\delta(1-x)\,.
\end{aligned}$$

- remark: IR regularisation by $+$–prescription & terms $\sim\delta(1-x)$ from physical conditions on splitting functions

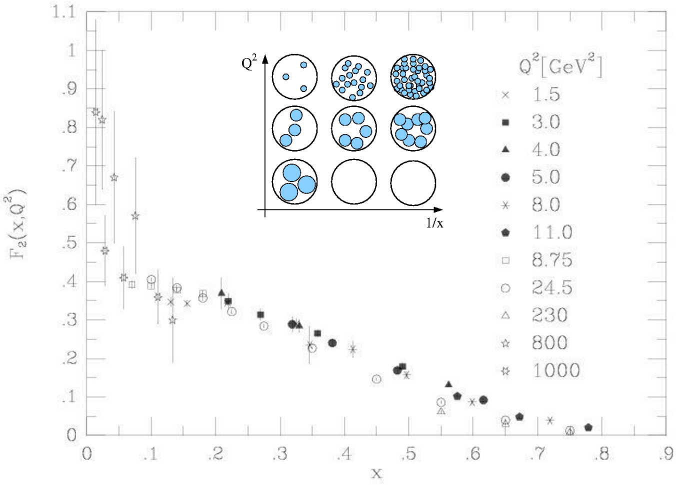
(flavour conservation for $q\to qg$ and momentum conservation for $g\to gg,\,q\bar{q}$)

- simple idea: proton $= |uud\rangle$ (valence quarks) only
- naively: no interactions $\longrightarrow f_{u,d/p} \sim \delta\left(x - \frac{1}{3}\right)$
- elastic interactions $\longrightarrow$ Gaussian smearing
- strong interactions: develop "sea" = soft partons will depend on resolution scale remember: $\mathrm{d}n \propto \log\omega \log k_\perp^2$



- in fact, due to $g \to gg$, sea increases much faster,

$$f_{\mathrm{sea}/p}(x,\, Q^2) \sim x^{-\lambda}\,, \quad \lambda \approx 1\,.$$


$F_2(x,Q^2)$
x
Q^2
1/x
Q^2[GeV^2]
1.5
3.0
4.0
5.0
8.0
11.0
8.75
24.5
230
800
1000
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
1.1

Hadron production: Scales

- consider QCD final state radiation
- pattern for $q \to qg$ similar to $\ell \to \ell\gamma$ in QED:

$$\mathrm{d}w^{q\to qg} = \frac{\alpha_s(k_\perp^2)}{2\pi}\, C_F\, \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \frac{\mathrm{d}\omega}{\omega} \left[1 + \left(1 - \frac{\omega}{E}\right)^2\right]$$

$$\stackrel{\omega=E(1-z)}{=} \frac{\alpha_s(k_\perp^2)}{2\pi}\, C_F\, \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \mathrm{d}z\, \frac{1+z^2}{1-z} = \frac{\alpha_s(k_\perp^2)}{2\pi}\, C_F\, \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\, \mathrm{d}z\, P_{qg}^{(1)}(z)\,.$$

- divergent structures for:

$z \to 1$ (soft divergence)	$\longleftrightarrow$	infrared/soft logarithms
$k_\perp^2 \to 0$ (collinear/mass divergence)	$\longleftrightarrow$	collinear logarithms

- cut regularise with cut-off $k_{\perp,\mathrm{min}} \sim 1\mathrm{GeV} > \Lambda_{\mathrm{QCD}}$

- find two perturbative regimes:
  - a regime of jet production, where $k_\perp \sim k_\parallel \sim \omega \gg k_{\perp,\rm min}$ and emission probabilities scale like $w \sim \alpha_s(k_\perp) \ll 1$; and
  - a regime of jet evolution, where $k_{\perp,\rm min} \le k_\perp \ll k_\parallel \le \omega$ and therefore emission probabilities scale like $w \sim \alpha_s(k_\perp)\log^2 k_\perp^2 \gtrsim 1$.
- in jet production:
  standard fixed–order perturbation theory
- in jet evolution regime,
  perturbative parameter not α_s any more
  but rather towers of $\exp\left[\alpha_s \log k_\perp^2 \log k_\parallel\right]$
- induces counting of leading logarithms (LL), $\alpha_s L^{2n}$,
  next-to leading logarithms (NLL), $\alpha_s L^{2n-1}$, etc.

- consider spatio-temporal structure in classical QED case
  - assume a charge comes into existence at $t = 0$ with $v = 0$: in its rest frame radial $\vec{E}$ spreads out in sphere $r' \leq t'$
  - assume charge moves with $v \to 1$ and Lorentz factor E/m: then in lab frame field at radial distance $r_\perp$ will arrive at $t = \gamma t' = Er_\perp/m$
- translate to classical QCD:
  - light quarks with constituent mass $m \approx \Lambda_{\rm QCD} \approx 1/R$ or $m = m_Q$ for heavy quarks

(assume here typical hadron radius R)

  - identify $r_\perp$ with typical hadronic size R
  - then: hadronization time

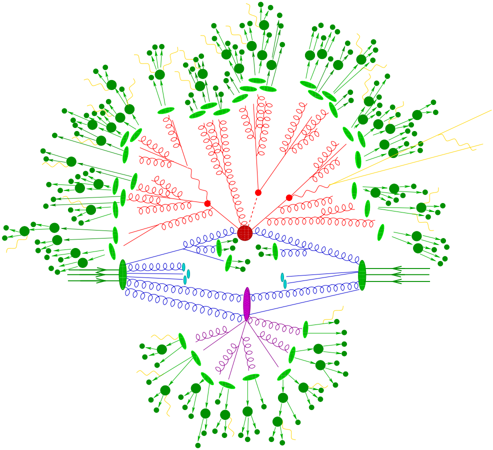
$$t^{(\rm had)} \approx \begin{cases} ER^2 & \text{for light quarks} \\ \dfrac{ER}{m_Q} & \text{for heavy quarks.} \end{cases}$$

- repeat exercise in quantum mechanics
- confining forces associated with gluons with $k \approx k_\perp \approx k_\parallel \approx 1/R \approx m$ in hadronic rest frame
- demand hadronization time $\geq$ formation time:

$$t^{(\mathrm{form})} \approx \frac{k_\parallel}{k_\perp^2} \leq k_\parallel R^2 \approx t^{(\mathrm{had})}$$

- therefore $k_\perp \geq 1/R = \mathcal{O}\left(\mathrm{few}\,\Lambda_{\mathrm{QCD}}\right)$
- therefore: breakdown of perturbative picture at scales/transverse momenta $\mathcal{O}\left(\mathrm{few}\,\Lambda_{\mathrm{QCD}}\right)$
- "gluers" replace gluons
- transition to bound states (phase transition)
- no first–principle understanding: $\Longrightarrow$ models

Summary



GOING MONTE CARLO

GENERAL IDEAS & TECHNIQUES

Contents

Prelude: Selecting from a distribution

- a typical Monte Carlo/simulation problem:
  - wanted: random numbers $x \in [x_{\rm min}, x_{\rm max}]$, distributed according to (probability) density $f(x)$, i.e.

$$\mathcal{P}(x \in [x', x' + {\rm d}x']) = f(x'){\rm d}x'$$

  - but: only "usual" random numbers $\#$ available: "flat" in $[0, 1]$
- exact solution:
  - must know integral F of density f and its inverse F^{-1}
  - x given by

$$\int\limits_{x_{\rm min}}^{x} {\rm d}x' f(x') = \# \int\limits_{x_{\rm min}}^{x_{\rm max}} {\rm d}x' f(x')$$

  and therefore

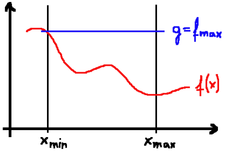
$$x = F^{-1}\left[F(x_{\rm min}) + \#\left(F(x_{\rm max}) - F(x_{\rm min})\right)\right]$$

- case above is very untypical – integral sometimes known, inverse practically never
- need a work-around solution: "Hit-or-miss"
  (solution, if exact case does not work.)
- construct good "over-estimator" $g(x)$ (G and G^{-1} known):

$$g(x) > f(x) \;\; \forall x \in [x_{\rm min},\, x_{\rm max}]$$

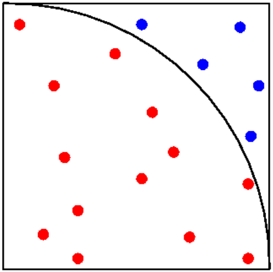
- exact algorithm to select a trial-x according to over-estimator g
- accept x with probability $f(x)/g(x)$
- obvious fall-back choice for $g(x)$:

$$g(x) = {\rm Max}_{[x_{\rm min},\, x_{\rm max}]}\{f(x)\}.$$



Monte Carlo integration

- underlying idea: determination of π with random number generator



$$\frac{\text{Hits}}{\text{Misses} + \text{Hits}} \longrightarrow \frac{\pi}{4}$$

Throw random points (x,y), with x, y in [0,1]

For hits: $(x^2 + y^2) < r^2 = 1$

- MC integration: estimate integral by N probes

$$I_f^{(a,b)} = \int\limits_a^b \mathrm{d}x f(x)$$

$$\longrightarrow \langle I_f^{(a,b)} \rangle = \frac{b-a}{N} \sum_{i=1}^{N} f(x_i) = \langle f \rangle_{a,b}$$

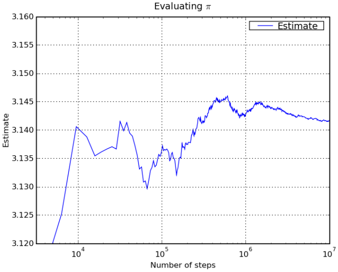
where x_i homogeneously distributed in $[a, b]$

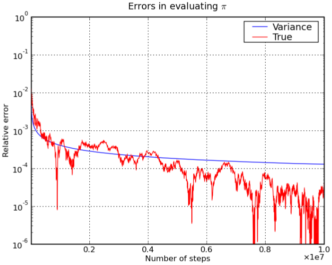
- error estimate from statistical sample $\Longrightarrow$ standard deviation

$$\langle E_f^{(a,b)}(N) \rangle = \sigma = \left[\frac{\langle f^2 \rangle_{a,b} - \langle f \rangle_{a,b}^2}{N} \right]^{1/2}$$

- independent of the number of integration dimensions!
  $\Longrightarrow$ method of choice for high-dimensional integrals.

Monte Carlo integration: basic idea



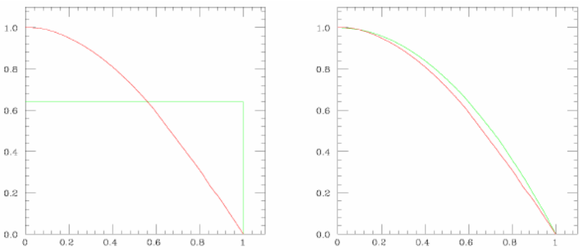


Monte Carlo integration: refinements

- want to minimise number of potentially expensive function calls
  $\Longrightarrow$ need to improve convergence of MC integration.
- first basic idea: sample in regions, where f largest

  ($\Longrightarrow$ corresponds to a Jacobean transformation of integral)
- alternative algorithm: minimise error by "smoothing" integrand ("importance sampling")
  - assume a function $g(x)$ similar to $f(x)$.
  - $f(x)/g(x)$ smooth $\Longrightarrow$ $\langle E(f/g)\rangle$ small
  - must sample according to $\mathrm{d}x\, g(x)$ rather than $\mathrm{d}x$:
    $g(x)$ plays role of probability distribution; we know already how to deal with this!
- works, if $f(x)$ is well-known, but hard to generalise.

- importance sampling
- consider $f(x) = \cos\frac{\pi x}{2}$ and $g(x) = 1 - x^2$:



$$I = \int_0^1 dx \cos\tfrac{\pi}{2}x = 0.637 \pm 0.308/\sqrt{N}$$

$$\begin{aligned} I &= \int_0^1 dx(1-x^2)\,\frac{\cos\frac{\pi}{2}x}{1-x^2} \\ &= \int d\rho\,\frac{\cos\frac{\pi}{2}x}{1-x^2}[x(\rho)] \\ &= 0.637 \pm 0.032/\sqrt{N} \end{aligned}$$

- yet another idea: decompose integral in M sub-integrals

$$\langle I(f)\rangle = \sum_{j=1}^{M}\langle I_j(f)\rangle$$

$$\langle E(f)\rangle^2 = \sum_{j=1}^{M}\langle E_j(f)\rangle^2$$

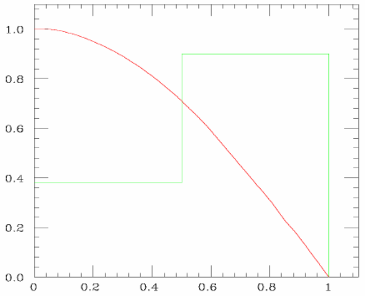
- overall variance smallest, if "equally distributed".

($\Longrightarrow$ sample, where the fluctuations are.)

("stratified sampling")

- algorithm:
  - divide interval in bins (variable bin-size or weight);
  - adjust such that variance identical in all bins.

- stratified sampling
- consider $f(x) = \cos\frac{\pi x}{2}$ and $g(x) = 1 - x^2$:



$$\langle I \rangle = 0.637 \pm 0.147/\sqrt{N}$$

- a hybrid of stratified and importance sampling: replace independent bins of stratified sampling with independent functions of importance sampling
- "bins" – with weight α_i – of "eigenfunctions" – $g_i(x)$:
  $\Longrightarrow \quad g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x})$.
- in particle physics, this is the method of choice for parton level event generation:
  - translate each Feynman diagram into one or more channels
  - optimise interplay of channels, cuts, etc. through weights α_i
  - optional: add VEGAS to "best" channels

Traditional MC simulation

- a classical example: two-dimensional Ising model:

(spins s_i fixed on 2-D lattice with nearest neighbour interactions.)

$$\mathcal{H} = -J \sum_{\langle ij \rangle} s_i s_j$$

- evaluation of observable $\mathcal{O}$ by summing over all micro states $\phi_{\{i\}}$, given as spin ensembles

(similar to path integral in QFT.)

$$\langle \mathcal{O} \rangle = \int \mathcal{D}\phi_{\{i\}} \,\mathrm{Tr}\, \left\{ \mathcal{O}(\phi_{\{i\}}) \exp\left[-\frac{\mathcal{H}(\phi_{\{i\}})}{k_B T} \right] \right\}$$

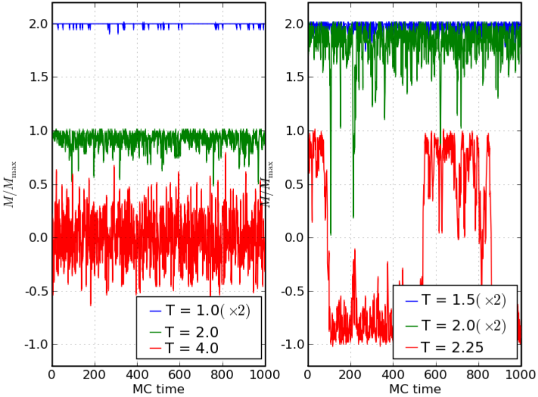
- typical problem in such calculations (integrations!):
  phase space too large $\Longrightarrow$ need to sample.

- Metropolis algorithm simulates the canonical ensemble, summing/integrating over micro-states with MC method.
- necessary ingredient: interactions among spins in probabilistic language (this will come back to us!)
- algorithm:
  - go over the spins,
  - check whether they flip:
    - compare $\mathcal{P}_{\rm flip}$ with random number
    - $\mathcal{P}_{\rm flip}$ from energies of the two micro-states (before and after flip) and Boltzmann factors
  - repeat to equilibrium.
  - evaluate observables directly during run &take thermal average (average over many steps).

- why does this work? detailed balance!
  - consider one spin flip, connecting micro-states 1 and 2.
  - rate of transitions given by the transition probabilities $\mathcal{W}$
  - if $E_1 > E_2$ then $\mathcal{W}_{1\to 2} = 1$ and $\mathcal{W}_{2\to 1} = \exp\left(-\frac{E_1 - E_2}{k_B T}\right)$
  - in thermal equilibrium, both transitions equally often:
    $$\mathcal{P}_2 \mathcal{W}_{2\to 1} = \mathcal{P}_1 \mathcal{W}_{1\to 2}$$
    takes into account that the respective states are occupied according to their Boltzmann factors.

    $(\mathcal{P}_i \sim \exp(-E_i/k_B T))$
- in principle, all systems in thermal equilibrium can be studied with Metropolis - just need to write transition probabilities in accordance with detailed balance, as above $\Longrightarrow$ general simulation strategy in thermodynamics.

- example results on a 10×10 lattice



PART II: MONTE CARLO

FOR PERTURBATIVE QCD

MONTE CARLO FOR

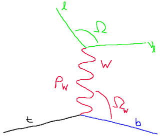
PARTON LEVEL

Contents

Simulating hard processes (signals & backgrounds)

- Simple example: $t \to bW^+ \to b\bar{l}\nu_l$:

$$|\mathcal{M}|^2 = \frac{1}{2}\left(\frac{8\pi\alpha}{\sin^2\theta_W}\right)^2 \frac{p_t\cdot p_\nu \; p_b\cdot p_l}{(p_W^2-M_W^2)^2+\Gamma_W^2 M_W^2}$$



- Phase space integration (5-dim):

$$\Gamma = \frac{1}{2m_t}\frac{1}{128\pi^3}\int \mathrm{d}p_W^2 \frac{\mathrm{d}^2\Omega_W}{4\pi}\frac{\mathrm{d}^2\Omega}{4\pi}\left(1-\frac{p_W^2}{m_t^2}\right)|\mathcal{M}|^2$$

- 5 random numbers $\Longrightarrow$ four-momenta $\Longrightarrow$ "events".
- Apply smearing and/or arbitrary cuts.
- Simply histogram any quantity of interest - no new calculation for each observable

Calculating matrix elements efficiently

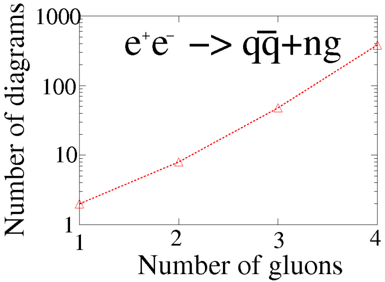
- stating the problem(s):
  - multi-particle final states for signals & backgrounds.
  - need to evaluate $\mathrm{d}\sigma_N$:

$$\int\limits_{\text{cuts}} \left[\prod_{i=1}^{N} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i}\right] \delta^4\left(p_1 + p_2 - \sum_i q_i\right) \left|\mathcal{M}_{p_1 p_2 \to N}\right|^2 .$$

  - problem 1: factorial growth of number of amplitudes.
  - problem 2: complicated phase-space structure.
  - solutions: numerical methods.

- example for factorial growth: $e^+e^- \to q\bar{q} + ng$

n	$\#_{\rm diags}$
0	1
1	2
2	8
3	48
4	384

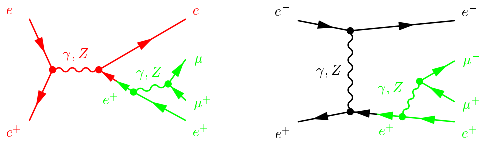


- obvious: traditional textbook methods (squaring, completeness relations, traces) fail
  $\Longrightarrow$ result in proliferation of terms $(\mathcal{M}_i\mathcal{M}_j^*)$
- better ideas of efficient ME calculation:
  $\Longrightarrow$ realise: amplitudes just are complex numbers,
  $\Longrightarrow$ add them before squaring!
- remember: spinors, gamma matrices have explicit form
  could be evaluated numerically (brute force)
  but: Rough method, lack of elegance, CPU-expensive
- can do better with smart basis for spinors (see next slide)
- this is still on the base of traditional Feynman diagrams!

- helicity method:
  - introduce basic helicity spinors (needs to “gauge”-vectors)
  - write everything as spinor products, e.g.
    $$\bar{u}(p_1,\,h_1)u(p_2,\,h_2) = \text{complex numbers.}$$
  - have completeness relations such as
    $$(\not{p}+m) \Longrightarrow \frac{1}{2}\sum_h \left[\left(1+\frac{m^2}{p^2}\right)\bar{u}(p,\,h)u(p,\,h) + \left(1-\frac{m^2}{p^2}\right)\bar{v}(p,\,h)v(p,\,h)\right]$$
  - there are other genuine expressions . . .
  - translate Feynman diagrams into “helicity amplitudes”: complex-valued functions of momenta & helicities.
  - spin-correlations etc. nearly come for free.

- taming the factorial growth in the helicity method
  - by reusing pieces: calculate only once!
  - factoring out: reduce number of multiplications!

  can be implemented as a-posteriori manipulations of amplitudes.



- better method: recursion relations (recycling built in).
  best candidate so far: off-shell recursions

(Dyson-Schwinger, Berends-Giele etc.)

- improvement: off-shell recursion relations
- general idea: recursively construct generalised currents $\mathcal{J}_\alpha(\pi)$ for a set π of external particles on their mass shell plus one internal one

$$\mathcal{J}_\alpha(\pi) = P_\alpha(\pi) \left\{ \sum_{\mathcal{P}_2(\pi)} \sum_{\mathcal{V}_\alpha^{\alpha_1\alpha_2}} \left[S(\pi_1,\, \pi_2)\, \mathcal{V}_\alpha^{\alpha_1\alpha_2} \mathcal{J}_{\alpha_1}(\pi_1)\mathcal{J}_{\alpha_2}(\pi_2)\right] \right.$$
$$\left. + \sum_{\mathcal{P}_3(\pi)} \sum_{\mathcal{V}_\alpha^{\alpha_1\alpha_2\alpha_3}} \left[S(\pi_1,\, \pi_2,\, \pi_3)\, \mathcal{V}_\alpha^{\alpha_1\alpha_2\alpha_3} \mathcal{J}_{\alpha_1}(\pi_1)\mathcal{J}_{\alpha_2}(\pi_2)\mathcal{J}_{\alpha_3}(\pi_3)\right] \right\}$$

  - $P_\alpha(\pi)$ denotes the propagator denominator
  - $S(\pi_1,\, \pi_2)$ and $S(\pi_1,\, \pi_2,\, \pi_3)$ for symmetry factors
  - $\mathcal{V}_\alpha$ for three– and four–particle vertices
  - go over all permutations of external particles π

- recursion relations particularly powerful due to massive recycling as integral part of structure $\longrightarrow$ bookkeeping problem only
- there are sub–classes of particularly simple amplitudes:
  maximally helicity violating (MHV) amplitudes
- all-gluon amplitudes with helicities given

(Parke–Taylor/Berends–Giele amplitudes)

$$\begin{aligned}
\mathcal{A}(1^+, 2^+, \dots, i^-, \dots, j^-, \dots\dots n^+) &= ig_s^{n-2} \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle \dots \langle (n-1)n\rangle\langle n1\rangle} \\
\mathcal{A}(1^-, 2^-, \dots, i^+, \dots, j^+, \dots\dots n^-) &= ig_s^{n-2} \frac{[ij]^4}{[12][23] \dots [(n-1)n][n1]} \,.
\end{aligned}$$

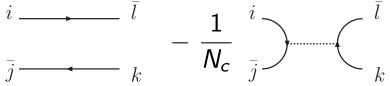
- terms $[ij]$ etc. are products of two–component left– and right–handed Weyl spinors (particularly simple)
- note: all-sign identical amplitudes vanish due to the conservation of angular momentum

- in principle also factorial growth with number of colours
- sampling over colours improves situation.

(but still, e.g. naively $\simeq (n-1)!$ permutations/colour-ordering for n external gluons).

- improved scheme: colour dressing

$$T^a_{i\bar{j}}T^a_{k\bar{l}} = \delta_{i\bar{l}}\delta_{k\bar{j}} - \frac{1}{N_c}\delta_{i\bar{j}}\delta_{k\bar{l}} \longleftrightarrow$$



- works very well with Berends-Giele recursions

Final State	BG		BCF		CSW	
	CO	CD	CO	CD	CO	CD
$2g$	0.24	0.28	0.28	0.33	0.31	0.26
$3g$	0.45	0.48	0.42	0.51	0.57	0.55
$4g$	1.20	1.04	0.84	1.32	1.63	1.75
$5g$	3.78	2.69	2.59	7.26	5.95	5.96
$6g$	14.2	7.19	11.9	59.1	27.8	30.6
$7g$	58.5	23.7	73.6	646	146	195
$8g$	276	82.1	597	8690	919	1890
$9g$	1450	270	5900	127000	6310	29700
$10g$	7960	864	64000	-	48900	-

Time [s] for the evaluation of 10^4 phase space points, sampled over helicities & colour.

Phase spacing for professionals

("Amateurs study strategy, professionals study logistics")

- democratic, process-blind integration methods:
  - Rambo/Mambo: Flat & isotropic

    R.Kleiss, W.J.Stirling & S.D.Ellis, Comput. Phys. Commun. **40** (1986) 359;
  - HAAG/Sarge: Follows QCD antenna pattern

    A.van Hameren & C.G.Papadopoulos, Eur. Phys. J. C **25** (2002) 563.
- multi-channelling: each Feynman diagram related to a phase space mapping (= "channel"), optimise their relative weights

  R.Kleiss & R.Pittau, Comput. Phys. Commun. **83** (1994) 141.
- main problem: practical only up to $\mathcal{O}(10\mathrm{k})$ channels.
- some improvement by building phase space mappings recursively: more channels feasible, efficiency drops a bit.

basic idea of multichannel sampling (again):
use a sum of functions $g_i(\vec{x})$ as Jacobean $g(\vec{x})$.

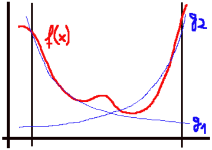
$\Longrightarrow \quad g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x})$;

$\Longrightarrow$ condition on weights like stratified sampling;

(“combination” of importance & stratified sampling).

algorithm for one iteration:

- select g_i with probability $\alpha_i \to \vec{x}_j$.
- calculate total weight $g(\vec{x}_j)$ and partial weights $g_i(\vec{x}_j)$
- add $f(\vec{x}_j)/g(\vec{x}_j)$ to total result and $f(\vec{x}_j)/g_i(\vec{x}_j)$ to partial (channel-) results.
- after N sampling steps, update a-priori weights.

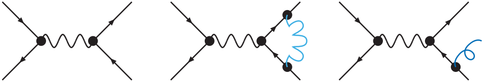


this is the method of choice for parton level event generation!

- quality measure for integration performance: unweighting efficiency
- want to generate events "as in nature".
- basic idea: use hit-or-miss method;
    - generate $\vec{x}$ with integration method,
    - compare actual $f(\vec{x})$ with maximal value during sampling $\Longrightarrow$ "Unweighted events".
- comments:
    - unweighting efficiency, $w_{\rm eff} = \langle f(\vec{x}_j)/f_{\rm max}\rangle$ = number of trials for each event.
    - expect $\log_{10} w_{\rm eff} \approx 3 - 5$ for good integration of multi-particle final states at tree-level.
    - maybe acceptable to use $f_{\rm max,eff} = Kf_{\rm max}$ with $K < 1$.
      problem: what to do with events where $f(\vec{x}_j)/f_{\rm max,eff} > 1$?
      answer: Add $\mathrm{int}[f(\vec{x}_j)/f_{\rm max,eff}] = k$ events and perform hit-or-miss on $f(\vec{x}_j)/f_{\rm max,eff} - k$.

Including higher order corrections

- obtained from adding diagrams with additional:
  loops (virtual corrections) or
  legs (real corrections)



- effect: reducing the dependence on μ_R & μ_F
  NLO allows for meaningful estimate of uncertainties
- additional difficulties when going NLO:
  ultraviolet divergences in virtual correction
  infrared divergences in real and virtual correction

  enforce
  UV regularisation & renormalisation
  IR regularisation & cancellation

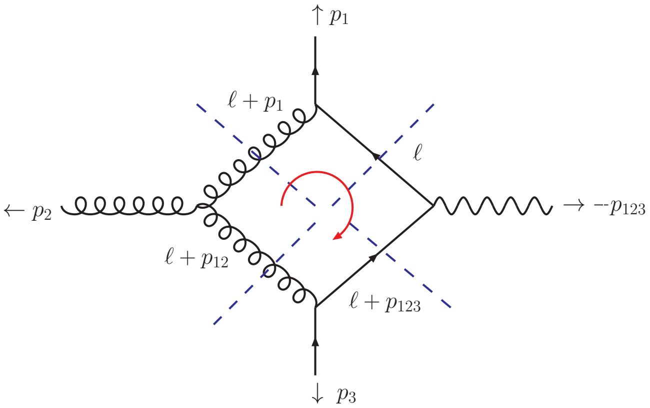
(Kinoshita–Lee–Nauenberg–Theorem)

- traditional bottleneck of higher–order calculations: virtual parts
- algorithm before about 2005:
  - Passarino–Veltman reduction of tensors in numerator
    (replace $2p \cdot k = (p+k)^2 - p^2 - k^2$)
  - reduce to scalar master integrals of the form

$$\int \frac{\mathrm{d}^D k}{[(p_1+k)^2(p_2+k)^2 \ldots]}$$

  - further reduce to integrals with up to four propagators only
    (but careful: introduces instabilities through "Gram determinants")

- about 2005: begin of "NLO revolution"
- basic idea: reduce to master integrals numerically by cutting



- coefficients of master integrals emerge as solutions of linear equations

Cancellation of infrared divergences

- need a mechanism to cancel IR divergences for higher multiplicities in final states
- toy model in one dimension:

$$|\mathcal{M}^R_{m+1}|^2 = \frac{1}{x}\,R(x) \quad \text{and} \quad |\mathcal{M}^V_m|^2 = \frac{1}{\epsilon}\,V\,,$$

where x = gluon energy & regularised in $d = 4 - 2\epsilon$ dimensions.
Cross section in d dimensions with jet measure F^J:

$$\sigma = \int\limits_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}}\,R(x)F^J_1(x) + \frac{1}{\epsilon}\,VF^J_0$$

- infrared safety of jet measure: $F^J_1(0) = F^J_0$
  $\Longrightarrow$ "a soft/collinear parton has no effect."
  (tricky issue - without it, no reliable NLO calculation!)
- KLN theorem: $R(0) = V$.

- rewrite toy-model cross section as

$$\begin{aligned}\sigma &= \int\limits_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} R(x) F_1^J(x) - \int\limits_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} V F_0^J + \int\limits_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} V F_0^J + \frac{1}{\epsilon} V F_0^J \\ &= \int\limits_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} \left(R(x) F_1^J(x) - V F_0^J\right) + \mathcal{O}(1) V F_0^J\,.\end{aligned}$$

- two separately finite integrals, with no large numbers to be added/subtracted.
- subtraction terms are universal (analytic bit can be calculated once and for all).
- this has been automated in two schemes: Catani-Seymour and Frixione-Kunszt-Signer

- general structure of NLO calculation for N-body production

$$\begin{aligned}
\mathrm{d}\sigma &= \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{B}}\mathcal{V}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{R}}\mathcal{R}_N(\Phi_{\mathcal{R}}) \\
&= \mathrm{d}\Phi_{\mathcal{B}}\left(\mathcal{B}_N + \mathcal{V}_N + \mathcal{I}_N^{(\mathcal{S})}\right) + \mathrm{d}\Phi_{\mathcal{R}}\left(\mathcal{R}_N - \mathcal{S}_N\right)
\end{aligned}$$

- phase space factorisation assumed here $(\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}} \otimes \Phi_1)$

$$\int \mathrm{d}\Phi_1 \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) = \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}})$$

- process independent subtraction kernels

$$\begin{aligned}
\mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1) \\
\mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}})
\end{aligned}$$

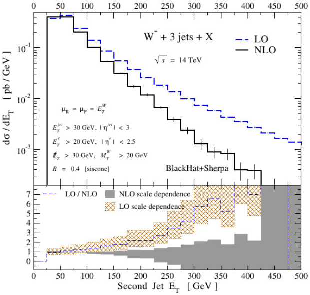
with universal $\mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1)$ and $\mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}})$

- in Catani–Seymour invertible phase space mapping

$$\Phi_{\mathcal{R}} \longleftrightarrow \Phi_{\mathcal{B}} \otimes \Phi_1$$

Aside: choices . . .

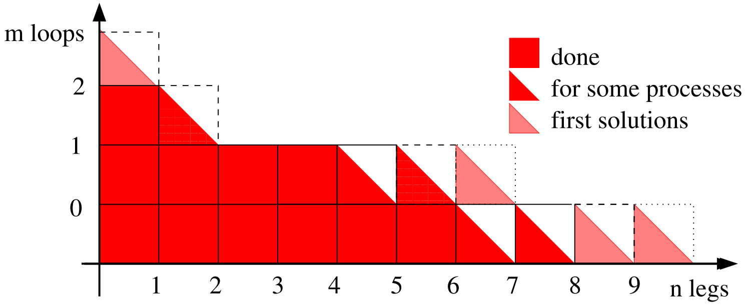
- common lore: NLO calculations reduce scale uncertainties
- this is, in general, true. however:
  unphysical scale choices will yield unphysical results



- so maybe we have to be a bit smarter than just running NLO code

Availability of exact calculations (hadron colliders)

- fixed order matrix elements ("parton level") are exact to a given perturbative order. (and often quite a pain!)
- important to understand limitations:
  only tree-level and one-loop level fully automated, beyond: prototyping



Survey of existing parton-level tools @ tree–level

	Models	$2 \to n$	Ampl.	Integ.	public?	lang.
ALPGEN	SM	$n = 8$	rec.	Multi	yes	Fortran
AMEGIC++	SM, UFO	$n = 6$	hel.	Multi	yes	C++
COMIX	SM, UFO	$n = 8$	rec.	Multi	yes	C++
COMPHEP	SM, LANHEP	$n = 4$	trace	1Channel	yes	C
HELAC	SM	$n = 8$	rec.	Multi	yes	Fortran
MADEVENT	SM, UFO	$n = 6$	hel.	Multi	yes	Python/Fortran
WHIZARD	SM, UFO	$n = 8$	rec.	Multi	yes	O'Caml

Survey of existing parton-level tools @ NLO

	type	technology dependencies on other codes
LoopTools	integrals	
OneLoop	integrals	
QCDLoop	integrals	
Collier	reduction	
CutTools	reduction	OPP
FormCalc	reduction	PV
Ninja	reduction	Laurent expansion
Samurai	reduction	
BlackHat	library (amplitudes)	OPP (unitarity)
McFm	library (full calculation)	PV & OPP
MJet	library (amplitudes)	OPP
GoSam	generator (amplitudes)	OPP Samurai +Ninja +
MadLoop	generator (full calculation)	OL+OPP CutTools +
OpenLoops	generator (amplitudes)	OL+OPP Collier +CutTools +
Recola	generator (amplitudes)	TR Collier +CutTools +
Helac-NLO	generator (full calculation)	OPP CutTools +

GOING MONTE CARLO

PARTON SHOWERS – THE BASICS

Contents

An analogy: Radioactive decays

- consider radioactive decay of an unstable isotope with half-life τ.

(and ignore factors of ln 2.)

- "survival" probability after time t is given by

$$\mathcal{S}(t) = \mathcal{P}_{\rm nodec}(t) = \exp[-t/\tau]$$

(note "unitarity relation": $\mathcal{P}_{\rm dec}(t) = 1 - \mathcal{P}_{\rm nodec}(t)$.)

- probability for an isotope to decay at time t:

$$\frac{{\rm d}\mathcal{P}_{\rm dec}(t)}{{\rm d}t} = -\frac{{\rm d}\mathcal{P}_{\rm nodec}(t)}{{\rm d}t} = \frac{1}{\tau}\exp(-t/\tau)$$

- now: connect half-life with width $\Gamma = 1/\tau$.
- probability for the isotope to decay at any fixed time t determined by Γ.

- spice things up now: add time–dependence, $\Gamma = \Gamma(t')$
- rewrite

$$\Gamma t \longrightarrow \int\limits_0^t \mathrm{d}t'\Gamma$$

- decay-probability at a given time t is given by

$$\frac{\mathrm{d}\mathcal{P}_{\mathrm{dec}}(t)}{\mathrm{d}t} = \Gamma(t)\,\exp\left[-\int\limits_0^t \mathrm{d}t'\Gamma(t')\right] = \Gamma(t)\,\mathcal{P}_{\mathrm{nodec}}(t)$$

(unitarity strikes again: $\mathrm{d}\mathcal{P}_{\mathrm{dec}}(t)/\mathrm{d}t = -\mathrm{d}\mathcal{P}_{\mathrm{nodec}}(t)/\mathrm{d}t$.)

- interpretation of l.h.s.:
  - first term is for the actual decay to happen.
  - second term is to ensure that no decay before t
    $\Longrightarrow$ conservation of probabilities.
    the exponential is - of course - called the Sudakov form factor.

The pattern of QCD radiation

- a detour: Altarelli-Parisi equation, once more
- AP describes the scaling behaviour of the parton distribution function

(which depends on Bjorken-parameter and scale Q^2)

$$\frac{\mathrm{d}q(x,\, Q^2)}{\mathrm{d}\ln Q^2} = \int\limits_x^1 \frac{\mathrm{d}y}{y}\left[\alpha_s(Q^2)P_q(x/y)\right] q(y, Q^2)$$

- term in square brackets determines the probability that the parton emits another parton at scale Q^2 and Bjorken-parameter y

(after the splitting, $x \to yx + (1-y)x$.)

- driving term: Splitting function $P_q(x)$
  important property: universal, process independent

- differential cross section for gluon emission in $e^+e^- \to$ jets

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

singular for $x_{1,2} \to 1$.

- rewrite with opening angle θ_{qg} and gluon energy fraction $x_3 = 2E_g/E_{\mathrm{c.m.}}$:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}\cos\theta_{qg}\mathrm{d}x_3} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\left[\frac{2}{\sin^2\theta_{qg}}\frac{1+(1-x_3)^2}{x_3} - x_3\right]$$

singular for $x_3 \to 0$ ("soft"), $\sin\theta_{qg} \to 0$ ("collinear").

- re-express collinear singularities

$$\begin{aligned}\frac{2\mathrm{d}\cos\theta_{qg}}{\sin^2\theta_{qg}} &= \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{qg}}{1+\cos\theta_{qg}} \\ &= \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{\bar{q}g}}{1-\cos\theta_{\bar{q}g}} \approx \frac{\mathrm{d}\theta^2_{qg}}{\theta^2_{qg}} + \frac{\mathrm{d}\theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}}\end{aligned}$$

- independent evolution of two jets (q and $\bar{q}$)

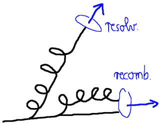
$$\mathrm{d}\sigma_{ee\to 3j} \approx \sigma_{ee\to 2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2_{jg}}{\theta^2_{jg}} P(z)\,,$$

- note: same form for any $t \propto \theta^2$:
- transverse momentum $k_\perp^2 \approx z^2(1-z)^2E^2\theta^2$
- invariant mass $q^2 \approx z(1-z)E^2\theta^2$

$$\frac{\mathrm{d}\theta^2}{\theta^2} \approx \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \approx \frac{\mathrm{d}q^2}{q^2}$$

- parametrisation-independent observation:
  (logarithmically) divergent expression for $t \to 0$.
- practical solution: cut-off Q_0^2.
  $\Longrightarrow$ divergence will manifest itself as $\log Q_0^2$.
- similar for $P(z)$: divergence for $z \to 0$ cured by cut-off.

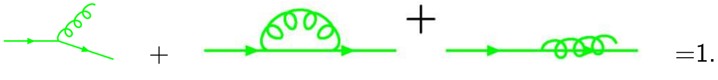
- what is a parton?
  collinear pair/soft parton recombine!
- introduce resolution criterion $k_\perp > Q_0$.



- combine virtual contributions with unresolvable emissions:
  cancels infrared divergences $\Longrightarrow$ finite at $\mathcal{O}(\alpha_s)$

  (Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

- unitarity: probabilities add up to one
  $\mathcal{P}(\text{resolved}) + \mathcal{P}(\text{unresolved}) = 1$.



- the Sudakov form factor, once more
- differential probability for emission between q^2 and $q^2 + \mathrm{d}q^2$:

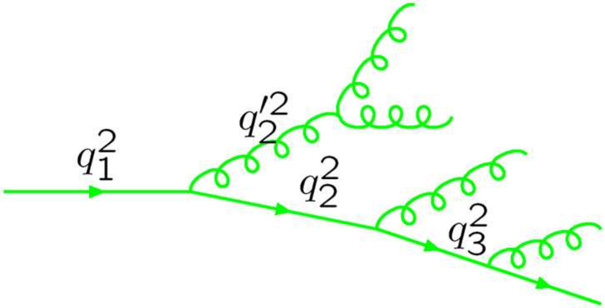
$$\mathrm{d}\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2}\int\limits_{z_{\min}}^{z_{\max}} \mathrm{d}z P(z) =: \mathrm{d}q^2\,\Gamma(q^2)$$

- from radioactive example: evolution equation for Δ

$$-\frac{\mathrm{d}\Delta(Q^2,\,q^2)}{\mathrm{d}q^2} = \Delta(Q^2,\,q^2)\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}q^2} = \Delta(Q^2,\,q^2)\Gamma(q^2)$$

$$\Longrightarrow\ \Delta(Q^2,\,q^2) = \exp\left[-\int\limits_{q^2}^{Q^2}\mathrm{d}k^2\Gamma(k^2)\right]$$

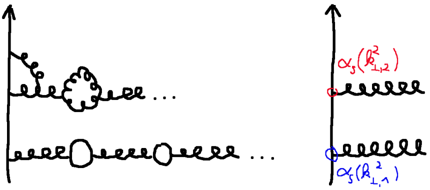
- maximal logs if emissions ordered
- impacts on radiation pattern: in each emission t becomes smaller



$$q_1^2 > q_2^2 > q_3^2,\ q_1^2 > {q_2'}^2$$

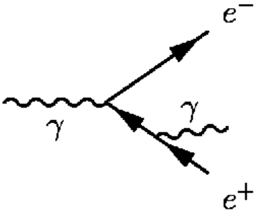
Quantum improvements

- improvement: inclusion of various quantum effects
- trivial: effect of summing up higher orders (loops) $\alpha_s \to \alpha_s(k_\perp^2)$



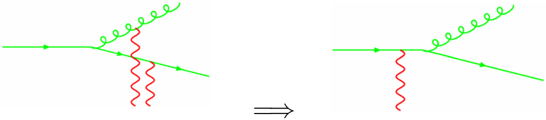
- much faster parton proliferation, especially for small $k_\perp^2$.
- avoid Landau pole: $k_\perp^2 > Q_0^2 \gg \Lambda^2_{\mathrm{QCD}} \Longrightarrow Q_0^2 =$ physical parameter.

- soft limit for single emission also universal
- problem: soft gluons come from all over (not collinear!)
  quantum interference? still independent evolution?
- answer: not quite independent.
- consider case in QED



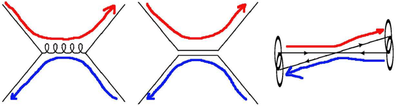
- assume photon into e^+e^- at θ_{ee} and photon off electron at θ
  photon momentum denoted as k
- energy imbalance at vertex: $k_\perp^\gamma \sim k_{||}\theta$, hence $\Delta E \sim k_\perp^2/k_{||} \sim k_{||}\theta^2$.
- formation time for photon emission: $\Delta t \sim 1/\Delta E \sim k_{||}/k_\perp^2 \sim 1/(k_{||}\theta^2)$.
- ee-separation: $\Delta b \sim \theta_{ee}\Delta t$
- must be larger than transverse wavelength of photon:
  $\theta_{ee}/(k_{||}\theta^2) > 1/k_\perp = 1/(k_{||}\theta)$
- thus: $\theta_{ee} > \theta$ must be satisfied for photon to form
- angular ordering as manifestation of quantum coherence

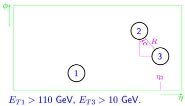
- pictorially:

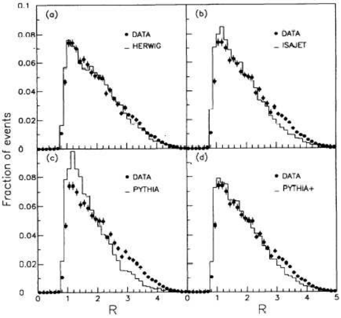


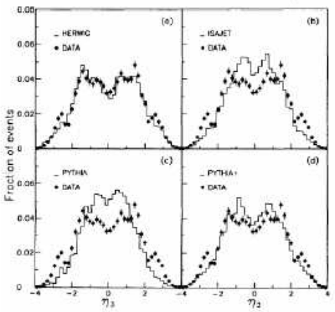
gluons at large angle from combined colour charge!

- experimental manifestation:
  ΔR of $2^{\rm nd}$ & $3^{\rm rd}$ *jetinmulti − jeteventsinpp − collisions*









Parton showers, compact notation

- Sudakov form factor (**no-decay** probability)

$$\Delta^{(\mathcal{K})}_{ij,k}(t, t_0) = \exp\left[-\int\limits_{t_0}^{t} \frac{\mathrm{d}t}{t}\,\frac{\alpha_s}{2\pi}\int \mathrm{d}z\,\frac{\mathrm{d}\phi}{2\pi}\quad \underbrace{\mathcal{K}_{ij,k}(t,\, z,\, \phi)}_{\substack{\text{splitting kernel for}\\ (ij)\to ij\ \text{(spectator } k)}} \right]$$

- evolution parameter t defined by kinematics

generalised angle (Herwig ++) or transverse momentum (Pythia, Sherpa)

- will replace $\frac{\mathrm{d}t}{t}\mathrm{d}z\frac{\mathrm{d}\phi}{2\pi} \longrightarrow \mathrm{d}\Phi_1$
- scale choice for strong coupling: $\alpha_s(k_\perp^2)$

resums classes of higher logarithms

- regularisation through cut-off t_0

- "compound" splitting kernels $\mathcal{K}_n$ and Sudakov form factors $\Delta_n^{(\mathcal{K})}$ for emission off n-particle final state:

$$\mathcal{K}_n(\Phi_1) = \frac{\alpha_s}{2\pi}\sum_{\text{all }\{ij,k\}}\mathcal{K}_{ij,k}(\Phi_{ij,k})\,,\quad \Delta_n^{(\mathcal{K})}(t,t_0) = \exp\left[-\int\limits_{t_0}^{t}\mathrm{d}\Phi_1\,\mathcal{K}_n(\Phi_1)\right]$$

- consider first emission only off Born configuration

$$\mathrm{d}\sigma_B = \mathrm{d}\Phi_N\,\mathcal{B}_N(\Phi_N)$$
$$\cdot\underbrace{\left\{\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0) + \int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\left[\mathcal{K}_N(\Phi_1)\Delta_N^{(\mathcal{K})}(\mu_N^2,t(\Phi_1))\right]\right\}}_{\text{integrates to unity}\ \longrightarrow\ \text{"unitarity" of parton shower}}$$

- further emissions by recursion with $Q^2 = t$ of previous emission

- analyse connection to Q_T resummation formalism
- consider standard Collins-Soper-Sterman formalism (CSS):

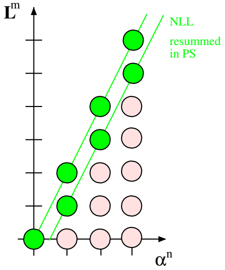
$$\frac{\mathrm{d}\sigma_{AB\to X}}{\mathrm{d}y\mathrm{d}Q_\perp^2} = \mathrm{d}\Phi_X\,\mathcal{B}_{ij}(\Phi_X)\cdot\underbrace{\int\frac{\mathrm{d}^2 b_\perp}{(2\pi)^2}\exp(i\vec{b}_\perp\cdot\vec{Q}_\perp)}_{\text{guarantee 4-mom conservation}}\underbrace{\tilde{W}_{ij}(b;\,\Phi_X)}_{\text{higher orders}}$$

with

$$\tilde{W}_{ij}(b;\,\Phi_X) = \overbrace{C_i(b;\,\Phi_X,\,\alpha_s)C_j(b;\,\Phi_X,\,\alpha_s)}^{\text{collinear bits}}\overbrace{H_{ij}(\alpha_s)}^{\text{loops}}$$
$$\underbrace{\exp\left[-\int\limits_{1/b_\perp^2}^{Q_X^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left(A(\alpha_s(k_\perp^2))\log\frac{Q_X^2}{k_\perp^2}+B(\alpha_s(k_\perp^2))\right)\right]}_{\text{Sudakov form factor, } A,\ B \text{ expanded in powers of } \alpha_s}$$

- analyse structure of emissions above
- logarithmic accuracy in $\log\frac{\mu_N}{k_\perp}$ (a la CSS)
  possibly up to next-to leading log,
  - if evolution parameter $\sim$ transverse momentum,
  - if argument in α_s is $\propto\ k_\perp$ of splitting,
  - if $K_{ij,k}\to$ terms $A_{1,2}$ and B_1 upon integration

  (OK, if soft gluon correction is included, and if $K_{ij,k}\ \to$ AP splitting kernels)



- in CSS $k_\perp$ typically is the transverse momentum of produced system, in parton shower of course related to the cumulative effect of explicit multiple emissions
- resummation scale $\mu_N\approx\mu_F$ given by (Born) kinematics –
  simple for cases like $q\bar{q}'\to V$, $gg\to H$, …
  tricky for more complicated cases

ROUND III: PRECISION MONTE CARLO

FIRST IMPROVEMENTS:

ME CORRECTIONS

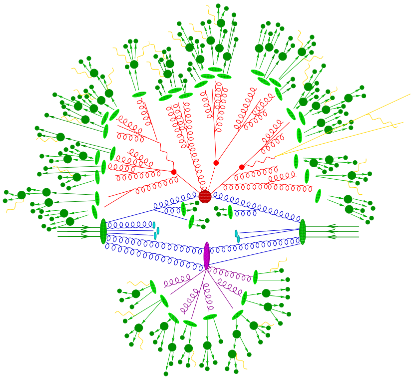
Contents

Improving event generators

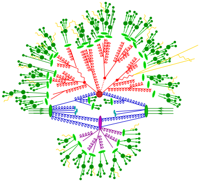
The inner working of event generators

... simulation: divide et impera

- hard process:
  fixed order perturbation theory
  traditionally: Born-approximation
- bremsstrahlung:
  resummed perturbation theory
- hadronisation:
  phenomenological models
- hadron decays:
  effective theories, data
- "underlying event":
  phenomenological models



. . . and possible improvements
possible strategies:



- improving the phenomenological models:
  - "tuning" (fitting parameters to data)
  - replacing by better models, based on more physics

    (my hot candidate: "minimum bias" and "underlying event" simulation)
- improving the perturbative description:
  - inclusion of higher order exact matrix elements and correct connection to resummation in the parton shower:

    "NLO-Matching" & "Multijet-Merging"
  - systematic improvement of the parton shower: next-to leading (or higher) logs & colours

- remember structure of NLO calculation for N-body production

$$\begin{aligned}\mathrm{d}\sigma &= \mathrm{d}\Phi_{\mathcal{B}}\mathcal{B}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{B}}\mathcal{V}_N(\Phi_{\mathcal{B}}) + \mathrm{d}\Phi_{\mathcal{R}}\mathcal{R}_N(\Phi_{\mathcal{R}}) \\ &= \mathrm{d}\Phi_{\mathcal{B}}\left(\mathcal{B}_N + \mathcal{V}_N + \mathcal{I}_N^{(\mathcal{S})}\right) + \mathrm{d}\Phi_{\mathcal{R}}\left(\mathcal{R}_N - \mathcal{S}_N\right)\end{aligned}$$

- phase space factorisation assumed here ($\Phi_{\mathcal{R}} = \Phi_{\mathcal{B}} \otimes \Phi_1$)

$$\int \mathrm{d}\Phi_1 \mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) = \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}})$$

- process independent subtraction kernels

$$\begin{aligned}\mathcal{S}_N(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1) \\ \mathcal{I}_N^{(\mathcal{S})}(\Phi_{\mathcal{B}} \otimes \Phi_1) &= \mathcal{B}_N(\Phi_{\mathcal{B}}) \otimes \mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}})\end{aligned}$$

with universal $\mathcal{S}_1(\Phi_{\mathcal{B}} \otimes \Phi_1)$ and $\mathcal{I}_1^{(\mathcal{S})}(\Phi_{\mathcal{B}})$

- in Catani–Seymour invertible phase space mapping

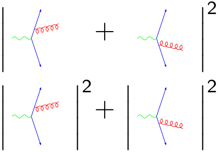
$$\Phi_{\mathcal{R}} \longleftrightarrow \Phi_{\mathcal{B}} \otimes \Phi_1$$

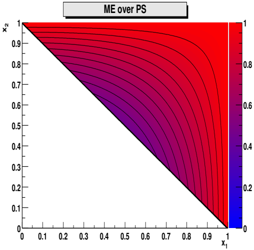
Matrix element corrections

- parton shower ignores interferences typically present in matrix elements
- pictorially

ME :

PS :





- form many processes $\mathcal{R}_N < \mathcal{B}_N \times \mathcal{K}_N$
- typical processes: $q\bar{q}' \to V$, $e^-e^+ \to q\bar{q}$, $t \to bW$
- practical implementation: shower with usual algorithm, but reject first/hardest emissions with probability $\mathcal{P} = \mathcal{R}_N/(\mathcal{B}_N \times \mathcal{K}_N)$

- analyse **first** emission, given by

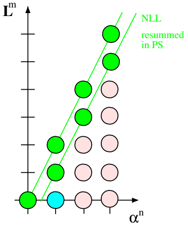
$$\mathrm{d}\sigma_B = \mathrm{d}\Phi_N\, \mathcal{B}_N(\Phi_N)$$

$$\cdot \underbrace{\left\{ \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t_0) + \int_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1 \left[\frac{\mathcal{R}_N(\Phi_N \times \Phi_1)}{\mathcal{B}_N(\Phi_N)} \Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2, t(\Phi_1)) \right] \right\}}_{\text{once more: integrates to unity} \longrightarrow \text{“unitarity” of parton shower}}$$

- radiation given by $\mathcal{R}_N$ (correct at $\mathcal{O}(\alpha_s)$)

  (but modified by logs of higher order in α_s from $\Delta_N^{(\mathcal{R}/\mathcal{B})}$)

- emission phase space constrained by μ_N
- also known as "soft ME correction"
  hard ME correction fills missing phase space
- used for "power shower":
  $\mu_N \to E_{pp}$ and apply ME correction



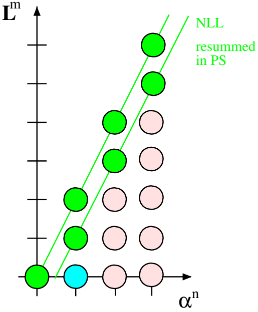
PRECISION MONTE CARLO

NLO MATCHING

Contents

NLO matching: Basic idea

- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- adjust ("match") terms:
  - cross section at NLO accuracy &
    correct hardest emission in PS to exactly reproduce ME at order α_s
    ($\mathcal{R}$-part of the NLO calculation)
    (this is relatively trivial)
  - maintain (N)LL-accuracy of parton shower
    (this is not so simple to see)



PowHeg

- reminder: $\mathcal{K}_{ij,k}$ reproduces process-independent behaviour of $\mathcal{R}_N/\mathcal{B}_N$ in soft/collinear regions of phase space

$$\mathrm{d}\Phi_1\,\frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)}\;\xrightarrow{\text{IR}}\;\mathrm{d}\Phi_1\,\frac{\alpha_s}{2\pi}\,\mathcal{K}_{ij,k}(\Phi_1)$$

- define modified Sudakov form factor (as in ME correction)

$$\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,t_0)=\exp\left[-\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\frac{\mathcal{R}_N(\Phi_{N+1})}{\mathcal{B}_N(\Phi_N)}\right],$$

- assumes factorisation of phase space: $\Phi_{N+1}=\Phi_N\otimes\Phi_1$
- typically will adjust scale of α_s to parton shower scale

- define local K-factors
- start from Born configuration Φ_N with NLO weight:

("local K-factor")

$$\begin{aligned}\mathrm{d}\sigma_N^{(\mathrm{NLO})} &= \mathrm{d}\Phi_N\,\bar{\mathcal{B}}(\Phi_N)\\ &= \mathrm{d}\Phi_N\left\{\mathcal{B}_N(\Phi_N)+\underbrace{\mathcal{V}_N(\Phi_N)+\mathcal{B}_N(\Phi_N)\otimes\mathcal{S}}_{\tilde{\mathcal{V}}_N(\Phi_N)}\right.\\ &\qquad\left.+\int\mathrm{d}\Phi_1\left[\mathcal{R}_N(\Phi_N\otimes\Phi_1)-\mathcal{B}_N(\Phi_N)\otimes\mathrm{d}\mathcal{S}(\Phi_1)\right]\right\}\end{aligned}$$

- by construction: exactly reproduce cross section at NLO accuracy
- note: second term vanishes if $\mathcal{R}_N\equiv\mathcal{B}_N\otimes\mathrm{d}\mathcal{S}$

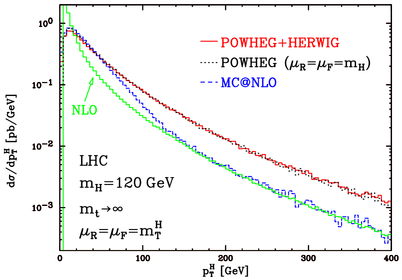
(relevant for MC@NLO)

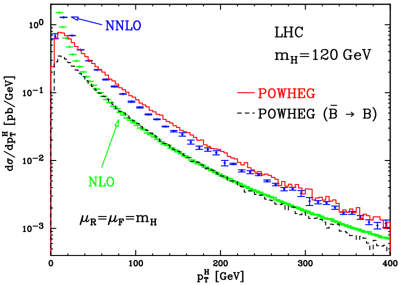
- analyse accuracy of radiation pattern
- generate emissions with $\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0)$:

$$\mathrm{d}\sigma_N^{(\mathrm{NLO})} = \mathrm{d}\Phi_N\,\bar{\mathcal{B}}(\Phi_N)$$
$$\times \underbrace{\left\{\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1\,\frac{\mathcal{R}_N(\Phi_N \otimes \Phi_1)}{\mathcal{B}_N(\Phi_N)}\,\Delta_N^{(\mathcal{R}/\mathcal{B})}(\mu_N^2,\, k_\perp^2(\Phi_1))\right\}}_{\text{integrating to yield 1 - ``unitarity of parton shower''}}$$

- radiation pattern like in ME correction
- pitfall, again: choice of upper scale μ_N^2 (this is vanilla POWHEG!)
- apart from logs: which configurations enhanced by local K-factor

(K-factor for inclusive production of X adequate for $X+$ jet at large $p_\perp$?)



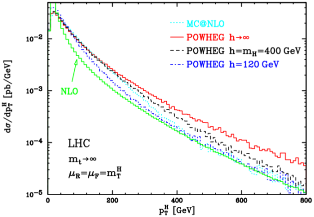


- large enhancement at high $p_{T,h}$
- can be traced back to large NLO correction
- fortunately, NNLO correction is also large $\to\sim$ agreement

- improving POWHEG
- split real-emission ME as

$$\mathcal{R} = \mathcal{R}\left(\underbrace{\frac{h^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(S)}} + \underbrace{\frac{p_\perp^2}{p_\perp^2 + h^2}}_{\mathcal{R}^{(F)}}\right)$$

- can "tune" h to mimick NNLO - or other (resummation) result



- differential event rate up to first emission

$$\mathrm{d}\sigma = \mathrm{d}\Phi_B \bar{\mathcal{B}}^{(\mathrm{R}^{(\mathrm{S})})}\left[\Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, t_0) + \int\limits_{t_0}^{s} \mathrm{d}\Phi_1 \frac{\mathcal{R}^{(S)}}{\mathcal{B}} \Delta^{(\mathcal{R}^{(S)}/\mathcal{B})}(s,\, k_\perp^2)\right]$$

$$+\mathrm{d}\Phi_R\, \mathcal{R}^{(F)}(\Phi_R)$$

MC@NLO

- MC@NLO paradigm: divide $\mathcal{R}_N$ in soft ("S") and hard ("H") part:

$$\mathcal{R}_N = \mathcal{R}_N^{(S)} + \mathcal{R}_N^{(H)} = \mathcal{B}_N \otimes \mathrm{d}\mathcal{S}_1 + \mathcal{H}_N$$

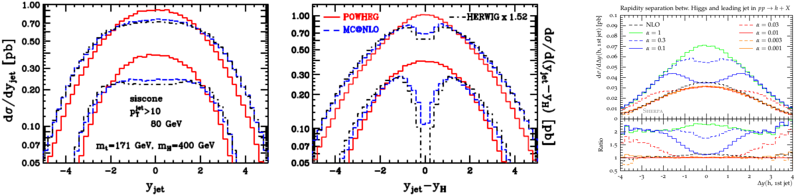
- identify subtraction terms and shower kernels $\mathrm{d}\mathcal{S}_1 \equiv \sum\limits_{\{ij,k\}} \mathcal{K}_{ij,k}$

(modify $\mathcal{K}$ in 1st emission to account for colour)

$$\begin{aligned}\mathrm{d}\sigma_N \quad = \quad & \mathrm{d}\Phi_N \underbrace{\tilde{\mathcal{B}}_N(\Phi_N)}_{\mathcal{B}+\tilde{\mathcal{V}}} \left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\, t_0) + \int\limits_{t_0}^{\mu_N^2} \mathrm{d}\Phi_1\, \mathcal{K}_{ij,k}(\Phi_1)\, \Delta_N^{(\mathcal{K})}(\mu_N^2,\, k_\perp^2)\right] \\ & + \mathrm{d}\Phi_{N+1}\, \mathcal{H}_N\end{aligned}$$

- effect: only resummed parts modified with local K-factor

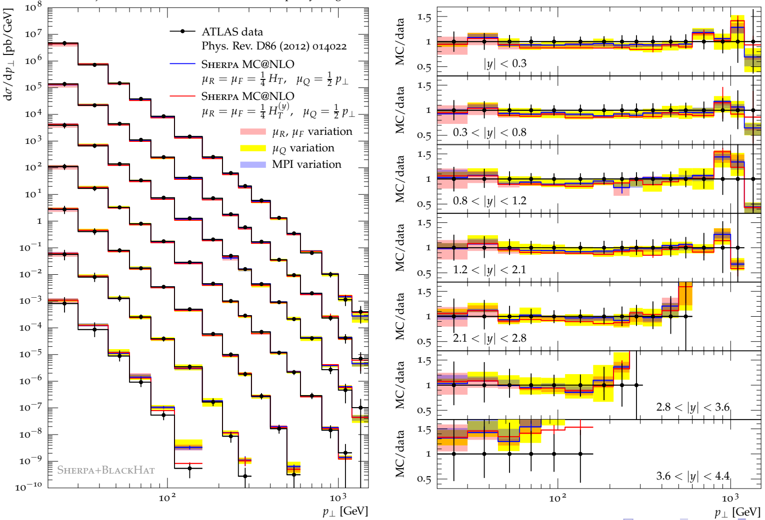
- phase space effects: shower vs. fixed order



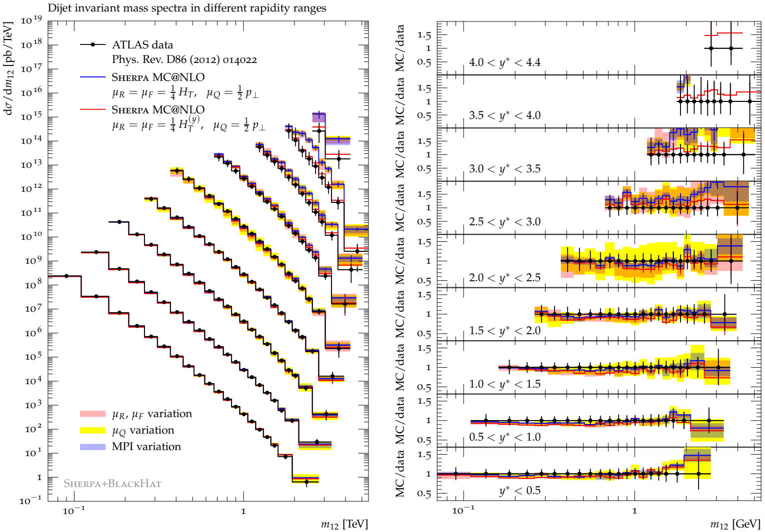
- problem: impact of subtraction terms on local K-factor (filling of phase space by parton shower)
- studied in case of $gg \to H$ above
- proper filling of available phase space by parton shower paramount

MC@NLO for light jets: jet-$p_\perp$

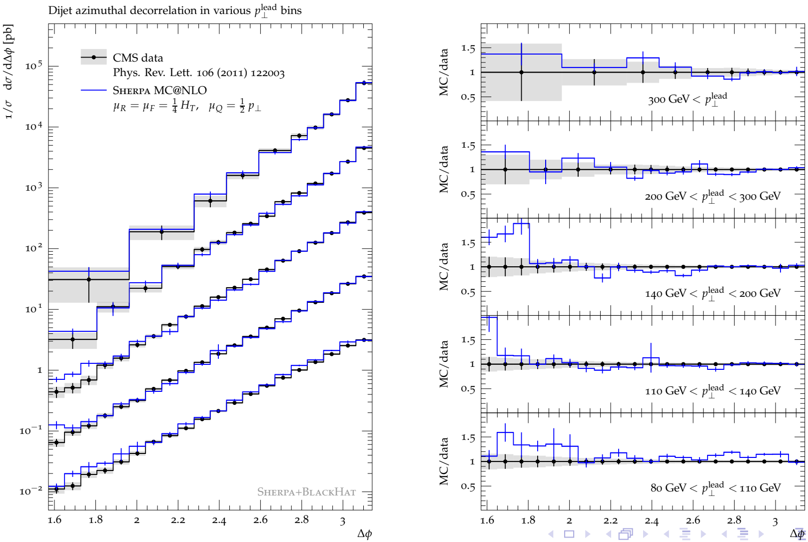
Inclusive jet transverse momenta in different rapidity ranges



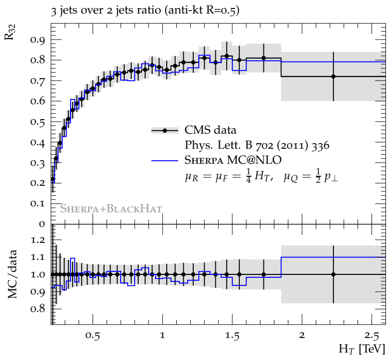
MC@NLO for light jets: dijet mass

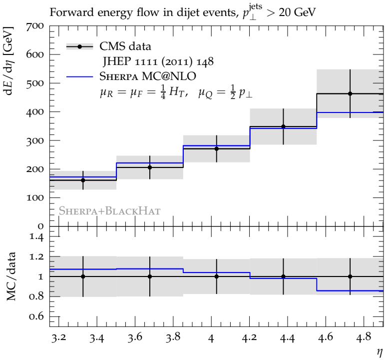


MC@NLO for light jets: azimuthal decorrelations

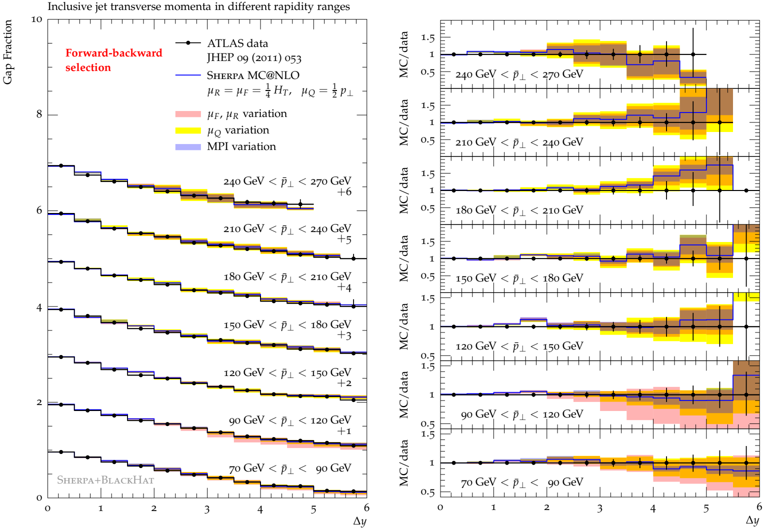


MC@NLO for light jets: R_{32} & forward energy flow





MC@NLO for light jets: jet vetoes



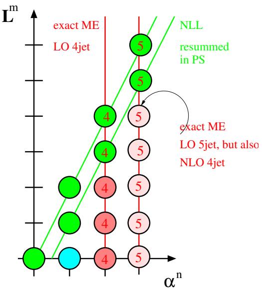
PRECISION MONTE CARLO

MULTIJET MERGING

Contents

Multijet merging: basic idea

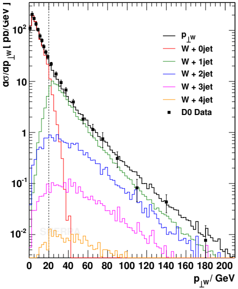
- parton shower resums logarithms
  fair description of collinear/soft emissions
  jet evolution (where the logs are large)
- matrix elements exact at given order
  fair description of hard/large-angle emissions
  jet production (where the logs are small)
- combine ("merge") both:
  result: "towers" of MEs with increasing number of jets evolved with PS
  - multijet cross sections at Born accuracy
  - maintain (N)LL accuracy of parton shower



- separate regions of jet production and jet evolution with jet measure Q_J

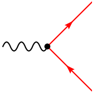
("truncated showering" if not identical with evolution parameter)

- matrix elements populate hard regime
- parton showers populate soft domain

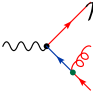


Why it works: jet rates with the parton shower

- consider jet production in $e^+e^- \to$ hadrons
  Durham jet definition: relative transverse momentum $k_\perp > Q_J$
- fixed order: one factor α_S and up to $\log^2 \frac{E_{\mathrm{c.m.}}}{Q_J}$ per jet
- use Sudakov form factor for resummation &
  replace approximate fixed order by exact expression:



$$\mathcal{R}_2(Q_J) \;=\; \left[\Delta_q(E^2_{\mathrm{c.m.}},\, Q_J^2)\right]^2$$



$$\mathcal{R}_3(Q_J) \;=\; 2\Delta_q(E^2_{\mathrm{c.m.}},\, Q_J^2)\int\limits_{Q_J^2}^{E^2_{\mathrm{c.m.}}} \frac{\mathrm{d}k_\perp^2}{k_\perp^2}\left[\frac{\alpha_s(k_\perp^2)}{2\pi}\mathrm{d}z\mathcal{K}_q(k_\perp^2, z)\right.$$
$$\left.\times\Delta_q(E^2_{\mathrm{c.m.}},\, k_\perp^2)\Delta_q(k_\perp^2,\, Q_J^2)\Delta_g(k_\perp^2,\, Q_J^2)\right]$$

Multijet merging at LO

- expression for first emission

$$\mathrm{d}\sigma = \mathrm{d}\Phi_N\,\mathcal{B}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_0) + \int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,\,t_{N+1})\Theta(Q_J - Q_{N+1})\right]$$
$$+\mathrm{d}\Phi_{N+1}\,\mathcal{B}_{N+1}\,\Delta_N^{(\mathcal{K})}(\mu_{N+1}^2,\,t_{N+1})\Theta(Q_{N+1} - Q_J)$$

- note: $N+1$-contribution includes also $N+2,\ N+3,\ \ldots$

(no Sudakov suppression below t_{n+1}, see further slides for iterated expression)

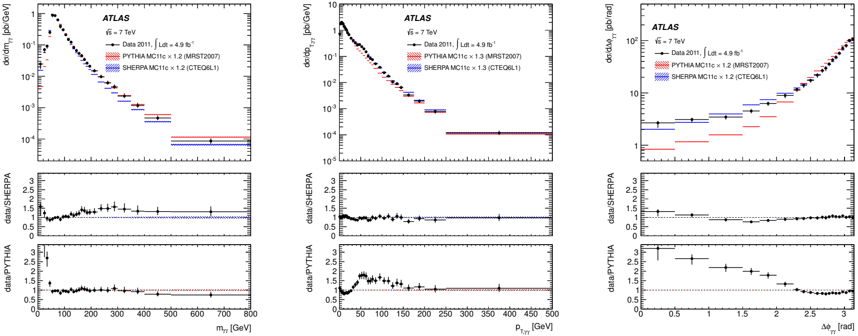
- potential occurence of different shower start scales: $\mu_{N,N+1,\ldots}$
- "unitarity violation" in square bracket: $\mathcal{B}_N\mathcal{K}_N \longrightarrow \mathcal{B}_{N+1}$

(cured with UMEPS formalism, L. Lonnblad & S. Prestel, JHEP 1302 (2013) 094 & S. Platzer, arXiv:1211.5467 [hep-ph] & arXiv:1307.0774 [hep-ph])

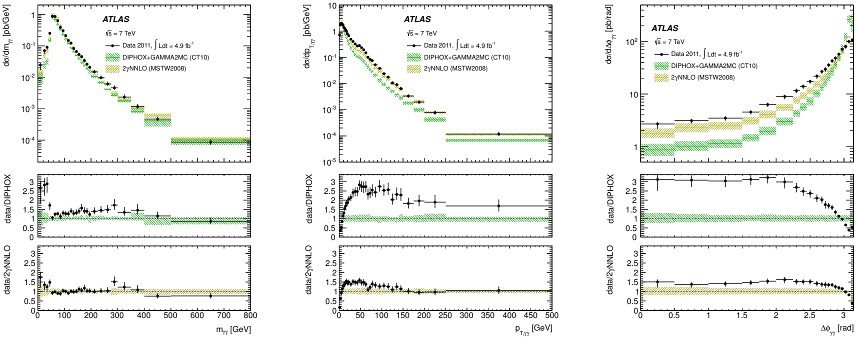
$$
\begin{aligned}
d\sigma \;=\; & \sum_{n=N}^{n_{\max}-1} \left\{ d\Phi_n\,\mathcal{B}_n\, \overbrace{\left[\prod_{j=N}^{n-1}\Theta(Q_{j+1}-Q_J)\right]}^{(n-N)\text{ extra jets}} \overbrace{\left[\prod_{j=N}^{n-1}\Delta_j^{(\mathcal{K})}(t_j,\,t_{j+1})\right]}^{\text{no emissions off internal lines}} \right. \\
& \left. \times \left[\underbrace{\Delta_n^{(\mathcal{K})}(t_n,t_0)}_{\text{no emission}} + \underbrace{\int_{t_0}^{t_n} d\Phi_1\,\mathcal{K}_n\Delta_n^{(\mathcal{K})}(t_n,t_{n+1})\Theta(Q_J-Q_{n+1})}_{\text{next emission no jet \& below last ME emission}} \right] \right. \\
& + d\Phi_{n_{\max}}\,\mathcal{B}_{n_{\max}} \left[\prod_{j=N}^{n_{\max}-1}\Theta(Q_{j+1}-Q_J)\right] \left[\prod_{j=N}^{n_{\max}-1}\Delta_j^{(\mathcal{K})}(t_j,\,t_{j+1})\right] \\
& \times \left[\Delta_{n_{\max}}^{(\mathcal{K})}(t_{n_{\max}},t_0) + \int_{t_0}^{t_{n_{\max}}} d\Phi_1\,\mathcal{K}_{n_{\max}}\Delta_{n_{\max}}^{(\mathcal{K})}(t_{n_{\max}},t_{n_{\max}+1}) \right]
\end{aligned}
$$

Di-photons @ ATLAS: $m_{\gamma\gamma}$, $p_{\perp,\gamma\gamma}$, and $\Delta\phi_{\gamma\gamma}$ in showers

(arXiv:1211.1913 [hep-ex])



Aside: Comparison with higher order calculations



A step towards multijet-merging at NLO: MENLOPS

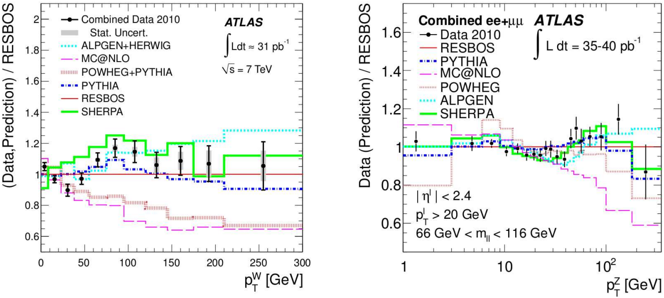
- combine matching for lowest multiplicity with multijet merging
- interpolating local K-factor for reweighting hard emissions

$$k_N(\Phi_{N+1}) = \frac{\tilde{\mathcal{B}}_N}{\mathcal{B}_N}\left(1-\frac{\mathcal{H}_N}{\mathcal{B}_{N+1}}\right)+\frac{\mathcal{H}_N}{\mathcal{B}_{N+1}} \longrightarrow \begin{cases} \tilde{\mathcal{B}}_N/\mathcal{B}_N & \text{for soft emission} \\ 1 & \text{for hard emission} \end{cases}$$

$$\begin{aligned} \mathrm{d}\sigma \;=\; & \mathrm{d}\Phi_N\,\tilde{\mathcal{B}}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0)+\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\right] \\ & +\mathrm{d}\Phi_{N+1}\,\mathcal{H}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1}) \\ & +\mathrm{d}\Phi_{N+1}\,k_N\,\mathcal{B}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_{N+1}-Q_J) \end{aligned}$$

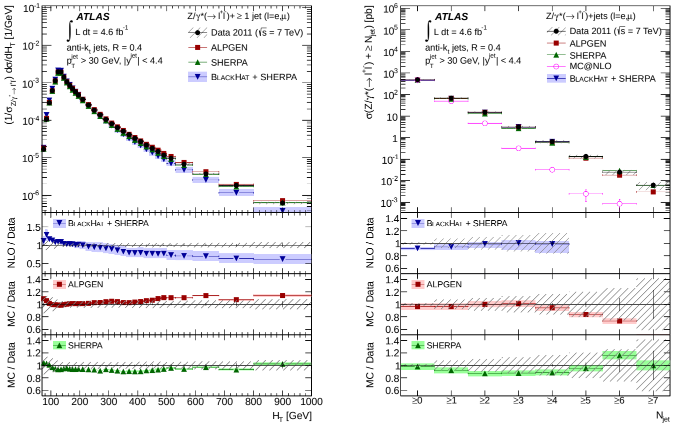
Transverse momentum of W & Z boson

ATLAS, arXiv:1108.6308, arXiv:1107.2381



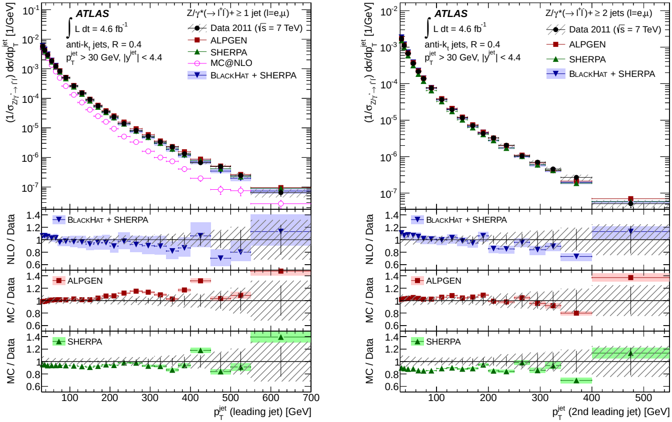
Z+jets: inclusive quantities

ATLAS, arXiv:1111.2690



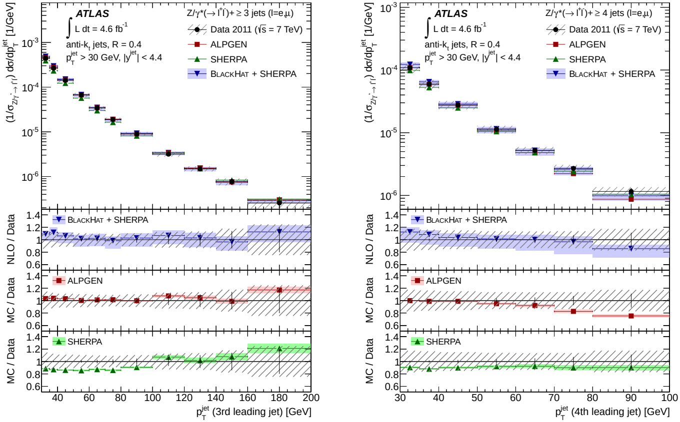
Z+jets: jet transverse momenta

ATLAS, arXiv:1111.2690



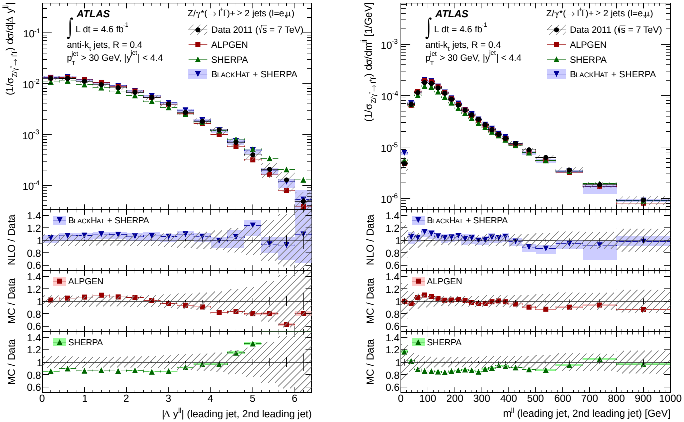
Z+jets: jet transverse momenta

ATLAS, arXiv:1111.2690



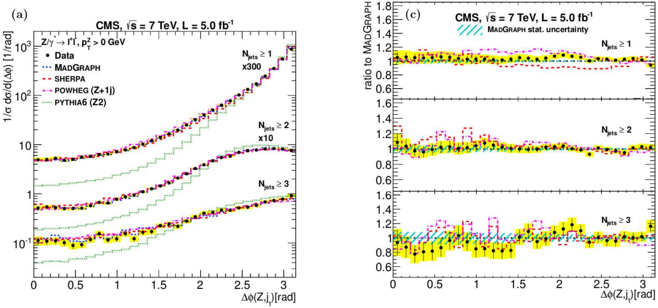
Z+jets: correlation of leading jets

ATLAS, arXiv:1111.2690



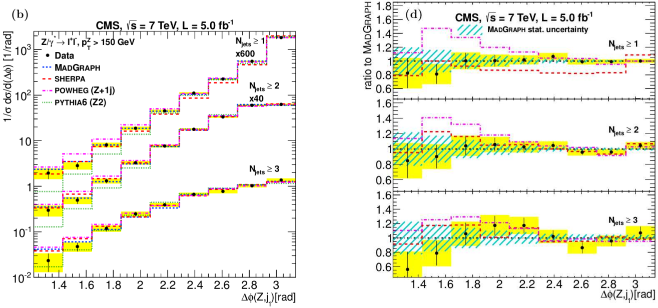
Z+jets: $\Delta\phi_{Zj}$ in unboosted sample

CMS, arXiv:1301.1646



Z+jets: $\Delta\phi_{Zj}$ in boosted sample

CMS, arXiv:1301.1646



Multijet-merging at NLO: MEPS@NLO

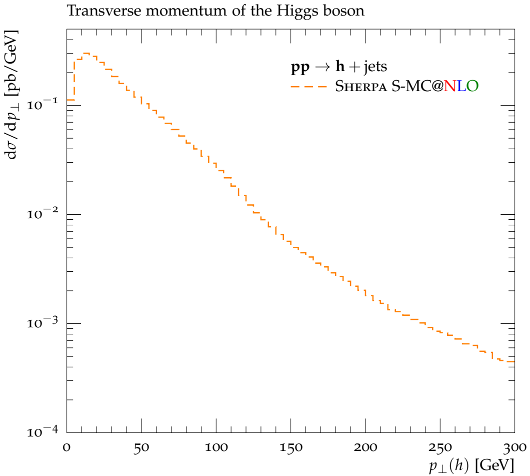
- basic idea like at LO: towers of MEs with increasing jet multi (but this time at NLO)
- combine them into one sample, remove overlap/double-counting

  maintain NLO and (N)LL accuracy of ME and PS
- this effectively translates into a merging of MC@NLO simulations and can be further supplemented with LO simulations for even higher final state multiplicities

First emission(s), once more

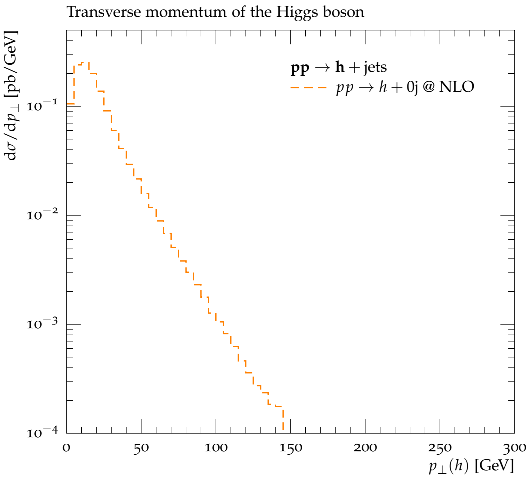
$$
\begin{aligned}
\mathrm{d}\sigma \;=\;& \mathrm{d}\Phi_N\,\tilde{\mathcal{B}}_N\left[\Delta_N^{(\mathcal{K})}(\mu_N^2,t_0)+\int\limits_{t_0}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\right]\\
&+\mathrm{d}\Phi_{N+1}\,\mathcal{H}_N\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Theta(Q_J-Q_{N+1})\\
&+\mathrm{d}\Phi_{N+1}\,\tilde{\mathcal{B}}_{N+1}\left(1+\frac{\mathcal{B}_{N+1}}{\tilde{\mathcal{B}}_{N+1}}\int\limits_{t_{N+1}}^{\mu_N^2}\mathrm{d}\Phi_1\,\mathcal{K}_N\right)\Theta(Q_{N+1}-Q_J)\\
&\qquad\cdot\left[\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_0)+\int\limits_{t_0}^{t_{N+1}}\mathrm{d}\Phi_1\,\mathcal{K}_{N+1}\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\right]\\
&+\mathrm{d}\Phi_{N+2}\,\mathcal{H}_{N+1}\Delta_N^{(\mathcal{K})}(\mu_N^2,t_{N+1})\Delta_{N+1}^{(\mathcal{K})}(t_{N+1},t_{N+2})\Theta(Q_{N+1}-Q_J)+\dots
\end{aligned}
$$

$p_\perp^H$ in MEPS@NLO



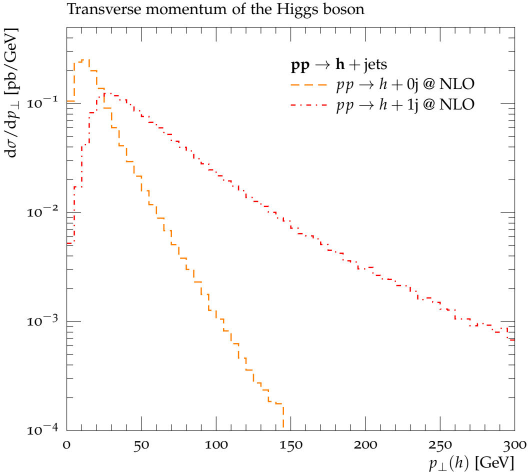
- first emission by MC@NLO

$p_\perp^H$ in MEPS@NLO



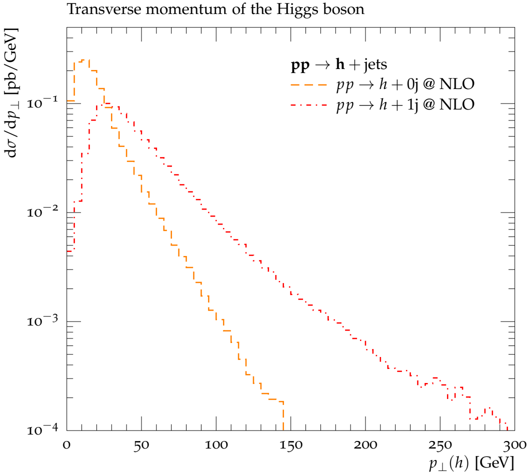
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



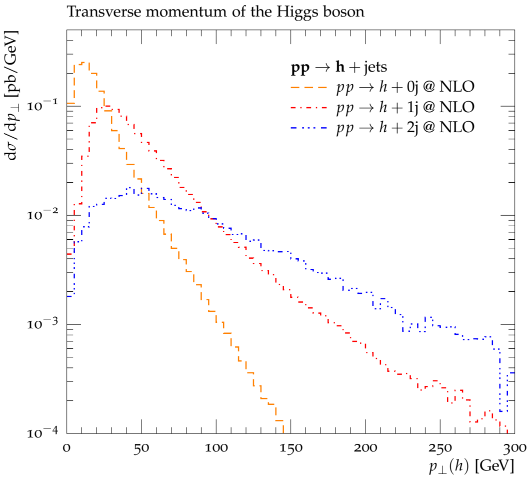
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h+$jet for $Q_{n+1} > Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



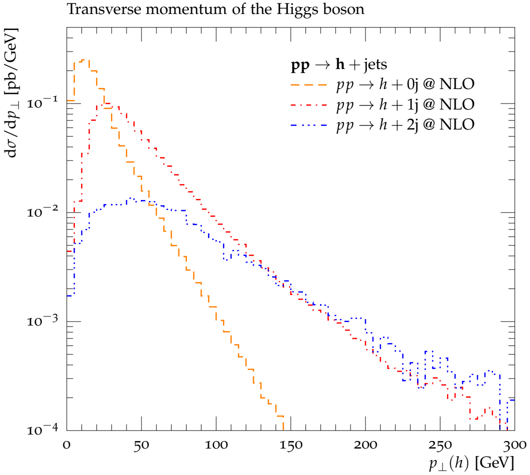
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\rm cut}$

$p_\perp^H$ in MEPs@NLO



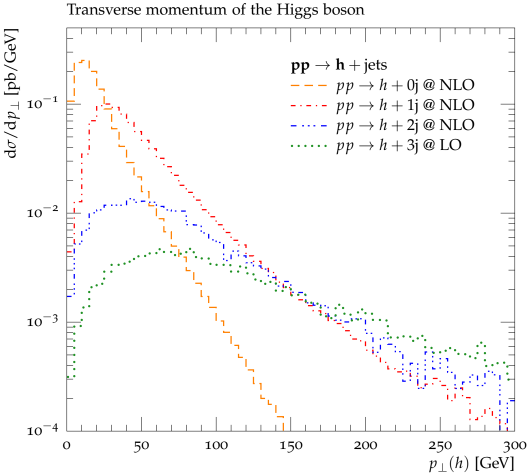
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h+\text{jet}$ for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h+\text{jet}$ to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h+2\text{jets}$ for $Q_{n+2} > Q_{\rm cut}$

$p_\perp^H$ in MEPS@NLO



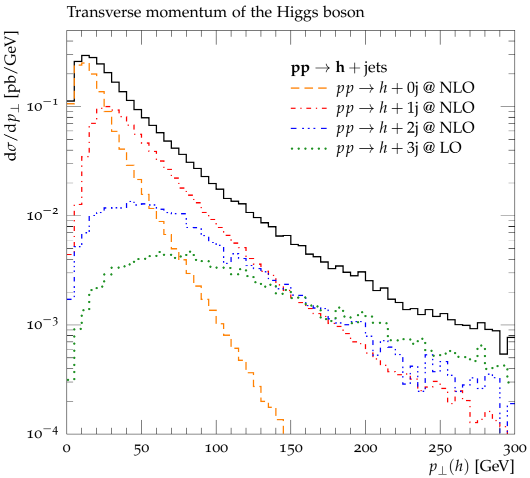
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h+\text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h+\text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \to h+2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate

$p_\perp^H$ in MEPS@NLO



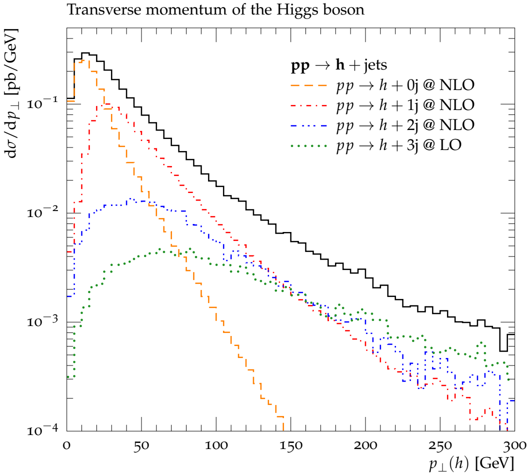
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h +$ jet for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h +$ jet to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \to h +$ 2jets for $Q_{n+2} > Q_{\text{cut}}$
- iterate

$p_\perp^H$ in MEPS@NLO



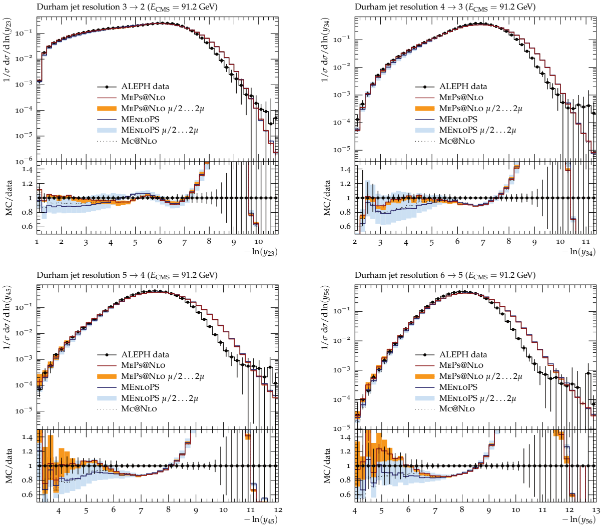
- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\text{cut}}$
- MC@NLO $pp \to h + \text{jet}$ for $Q_{n+1} > Q_{\text{cut}}$
- restrict emission off $pp \to h + \text{jet}$ to $Q_{n+2} < Q_{\text{cut}}$
- MC@NLO $pp \to h + 2\text{jets}$ for $Q_{n+2} > Q_{\text{cut}}$
- iterate
- sum all contributions

$p_\perp^H$ in MEPS@NLO

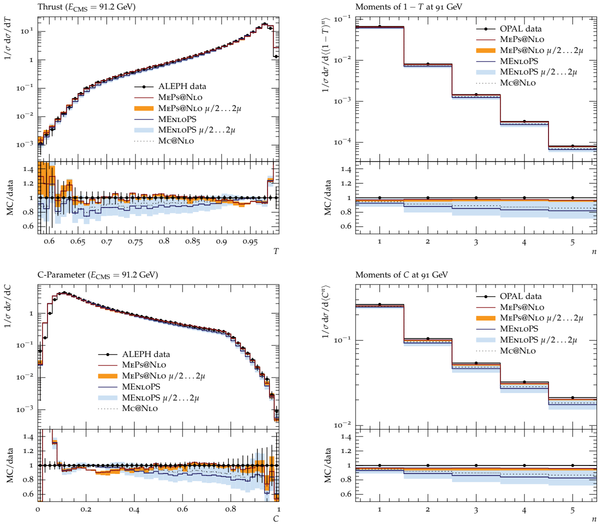


- first emission by MC@NLO , restrict to $Q_{n+1} < Q_{\rm cut}$
- MC@NLO $pp \to h + $jet for $Q_{n+1} > Q_{\rm cut}$
- restrict emission off $pp \to h + $jet to $Q_{n+2} < Q_{\rm cut}$
- MC@NLO $pp \to h + $2jets for $Q_{n+2} > Q_{\rm cut}$
- iterate
- sum all contributions
- eg. $p_\perp(h)$>200 GeV has contributions fr. multiple topologies

MEPS@NLO: example results for $e^-e^+ \to$ hadrons

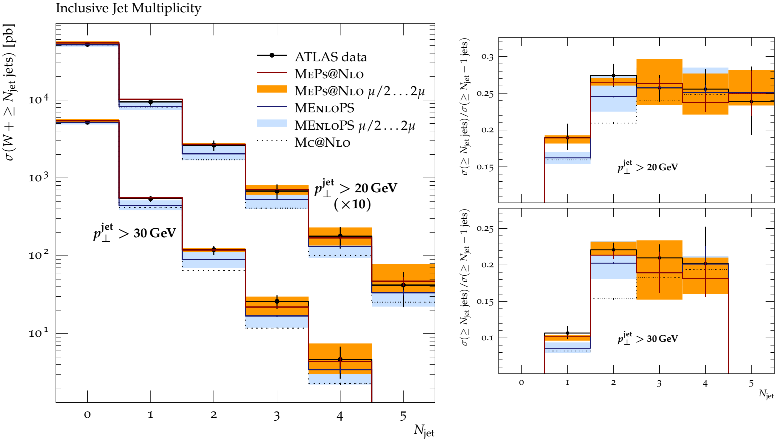


MEPS@NLO: example results for $e^-e^+ \to$ hadrons

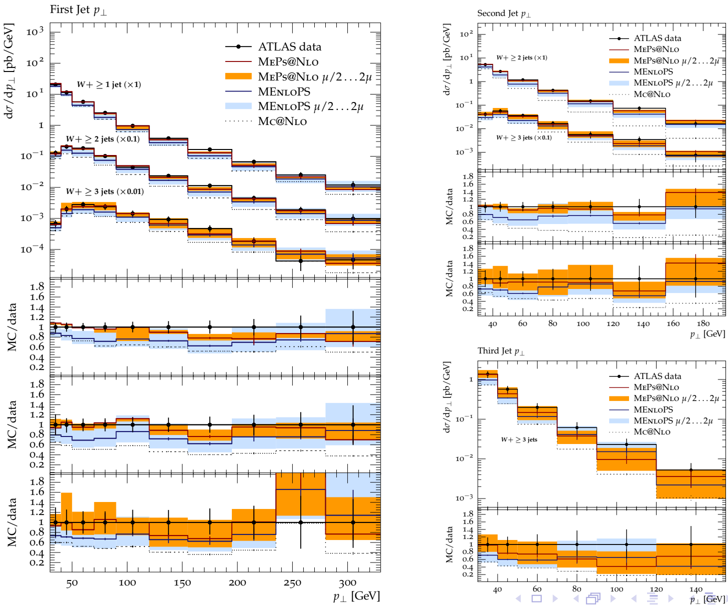


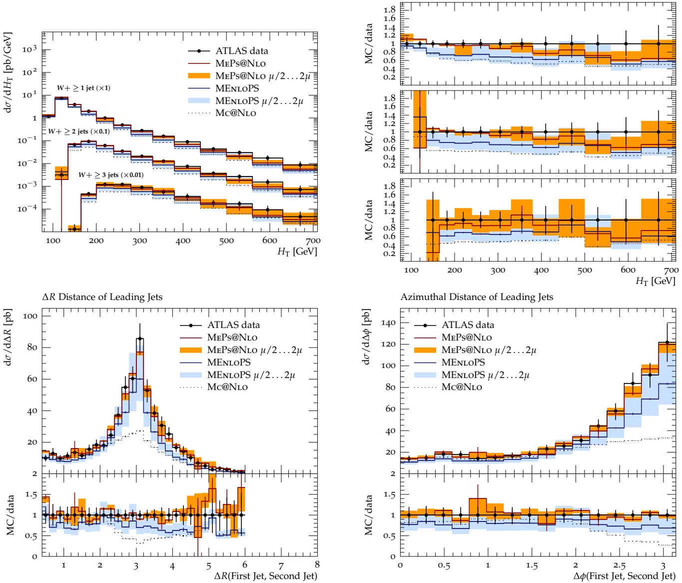
Example: MEPS@NLO for W+jets

(up to two jets @ NLO, from BlackHat, see arXiv: 1207.5031 [hep-ex])



Multijet merging at NLO

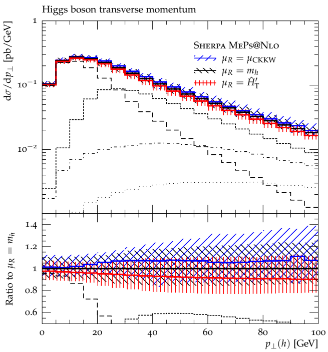


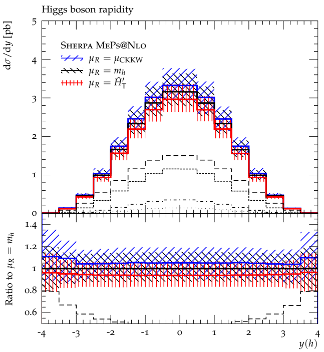

ATLAS data
MEPS@NLO
MEPS@NLO $\mu/2\dots2\mu$
MENLOPS
MENLOPS $\mu/2\dots2\mu$
MC@NLO
$W+\geq 1$ jet ($\times 1$)
$W+\geq 2$ jets ($\times 0.1$)
$W+\geq 3$ jets ($\times 0.01$)
$d\sigma/dH_T$ [pb/GeV]
H_T [GeV]
MC/data
ΔR Distance of Leading Jets
$d\sigma/d\Delta R$ [pb]
ΔR(First Jet, Second Jet)
Azimuthal Distance of Leading Jets
$d\sigma/d\Delta\phi$ [pb]
$\Delta\phi$(First Jet, Second Jet)

Results for Higgs boson production through gluon fusion

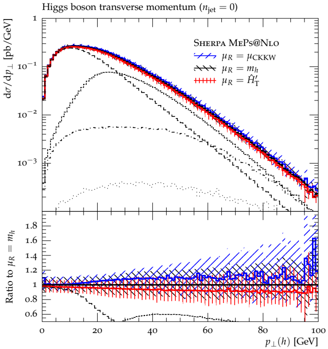
- parton-shower level, Higgs boson does not decay
- setup & cuts:
  jets: anti-kt, $p_\perp \geq 20$ GeV, $R = 0.4$, $|\eta| \leq 4.5$
  dijet cuts: at least 2 jets with $p_\perp \geq 25$ GeV
  WBF cuts: $m_{jj} \geq 400$ GeV, $\Delta y_{jj} \geq 2.8$
- jet multiplicity plots:
  0-jet excl.: no jet with $p_\perp \geq \{20, 25, 30\}$ GeV
  2-jet incl.: at least two jets with $p_\perp \geq \{20, 25, 30\}$ GeV
- SHERPA with $H + \{0,1,2\}^{(NLO)} + \{3\}^{(LO)}$ jets, $Q_{\rm cut} = 20 GeV$

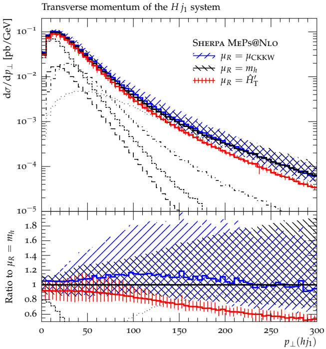
Inclusive observables for $gg \to H$



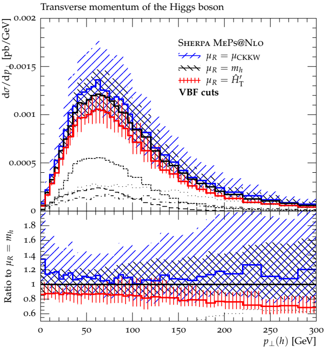


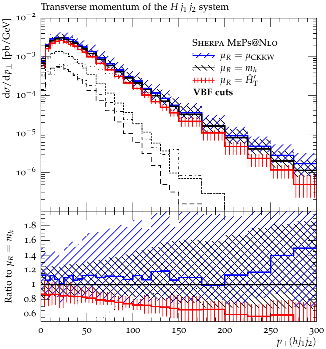
Exclusive observables for $gg \to H$



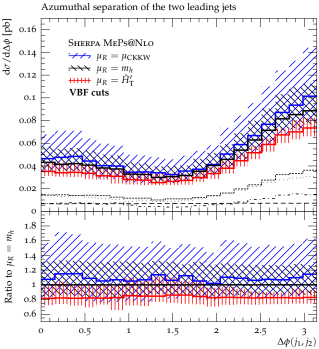


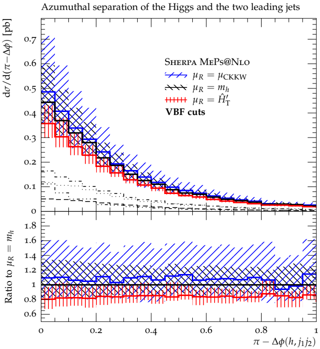
$gg \to H$ after WBF cuts





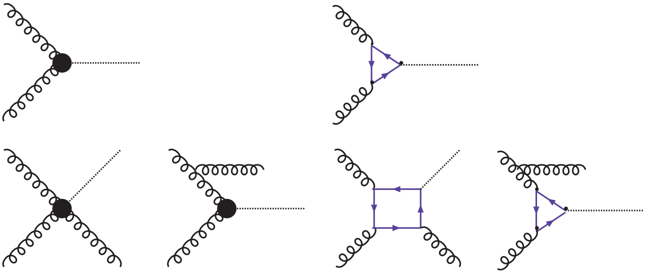
$gg \to H$ after WBF cuts





Quark mass effects

- include effects of quark masses

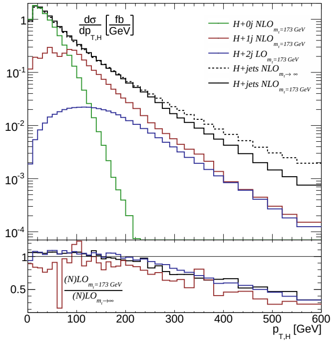


- reweight NLO HEFT with LO ratio:

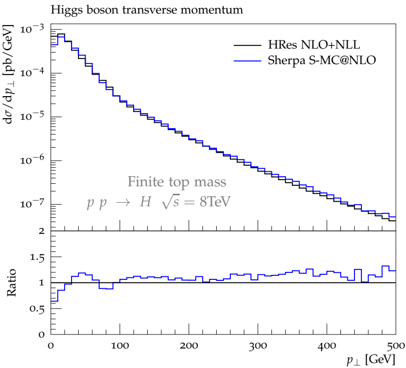
$$\mathrm{d}\sigma^{(\mathrm{NLO})}_{\mathrm{mass}} \approx \mathrm{d}\sigma^{(\mathrm{NLO})}_{\mathrm{HEFT}} \times \frac{\mathrm{d}\sigma^{(\mathrm{LO})}_{\mathrm{mass}}}{\mathrm{d}\sigma^{(\mathrm{LO})}_{\mathrm{HEFT}}}$$

Quark mass effects – results

- top mass effect in MEPS@NLO (on Higgs–$p_\perp$)



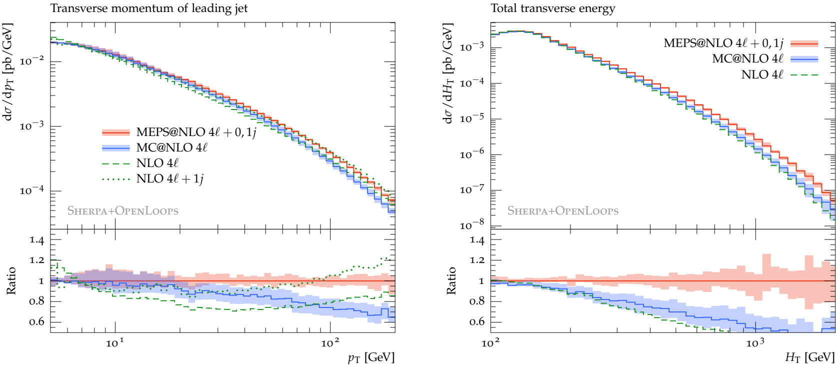
- comparison S-MC@NLO– HRES (top–loop only)



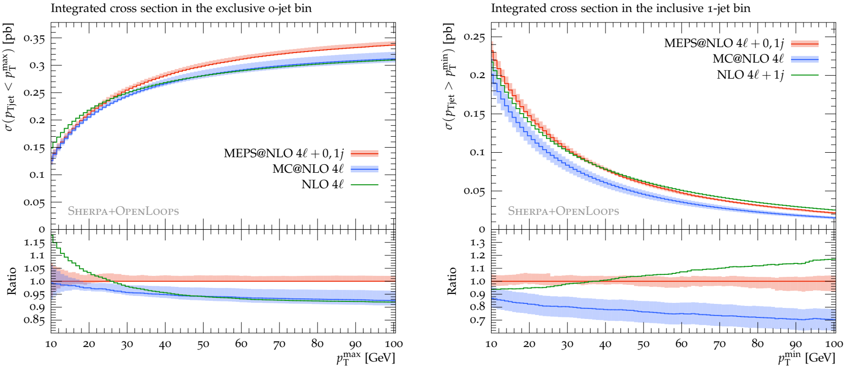
b–mass effects

- b–mass effects more tricky
- relevant only for (negative) interference of top– and bottom–loops (bottom^2 double Yukawa - supressed)
- but: cannot start shower at m_H
  radiation "sees" bottom at all scales above m_b
  $\Longrightarrow$ must use full theory there
- p_T spectrum naively "squeezed" – funny shapes
- LO multijet merging improves situation

Higgs backgrounds: inclusive observables in W^+W^-+jets

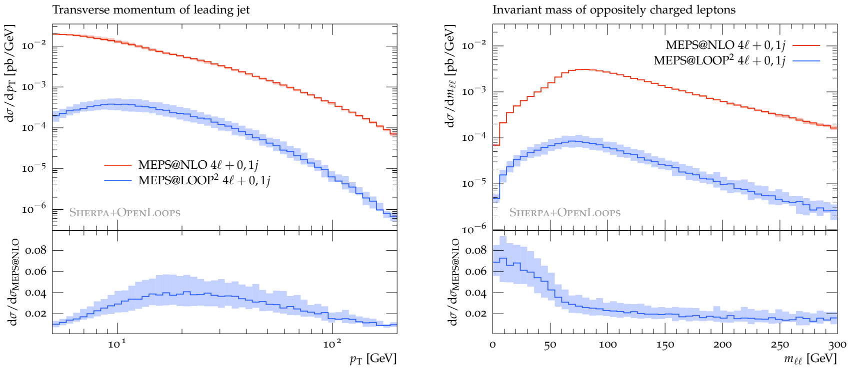


Higgs backgrounds: jet vetoes in W^+W^-+jets

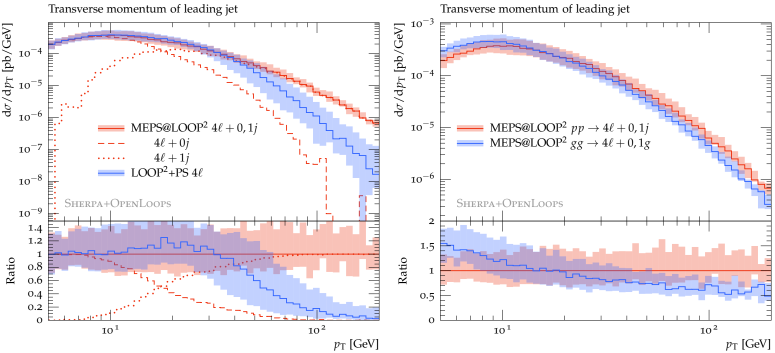


Higgs backgrounds: gluon-induced processes W^+W^-+jets

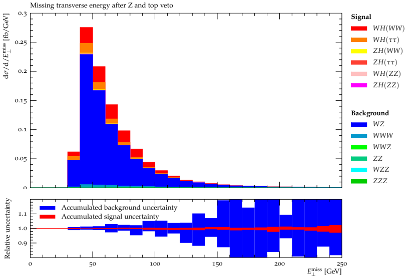
- include (LO-) merged loop^2 contributions of $gg \to VV$ (+1 jet)

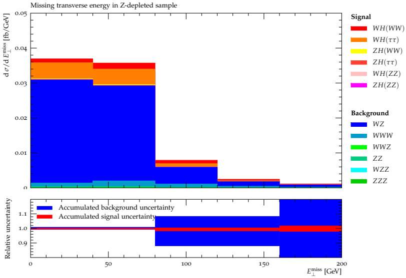


Higgs backgrounds: jet vetoes in W^+W^-+jets

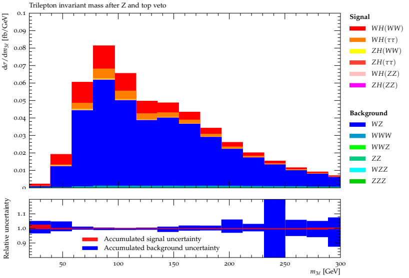


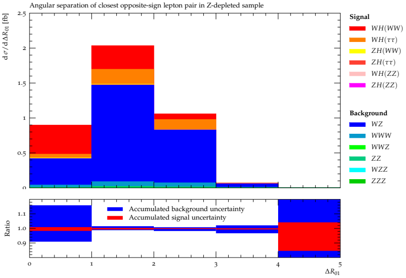
Relevant observables for $VH \to 3\ell$: $\not{E}_T$





Relevant observables for $VH \to 3\ell$: m_{123} & ΔR_{01}





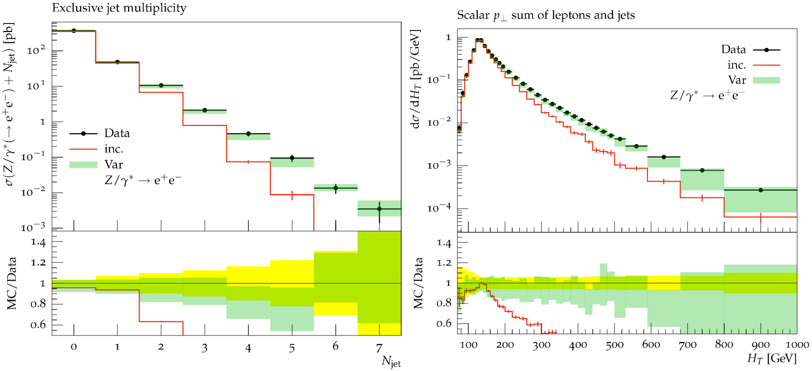
Differences between MEPS@NLO, UNLOPS & FxFx

	FxFx	MEPS@NLO	UNLOPS
ME	all internal	$\mathcal{V}$ external COMIX or AMEGIC++ $\mathcal{V}$ from OPENLOOPS, , MJET, . . .	all external
shower	external HERWIG or PYTHIA	intrinsic	intrinsic PYTHIA
Δ_N $\Theta(Q_J)$	analytical a-posteriori	from PS per emission	from PS per emission
Q_J-range	relatively high (but changed)	$>$ Sudakov regime $\approx$ 10%	$\approx$ Sudakov regime $\approx$ 10%

FxFx: validation in Z+jets

(Data from ATLAS, 1304.7098, with HERWIG++)

(green: 0, 1, 2 jets + uncertainty band from scale and PDF variations, red: MC@NLO)



FxFx: validation in Z+jets

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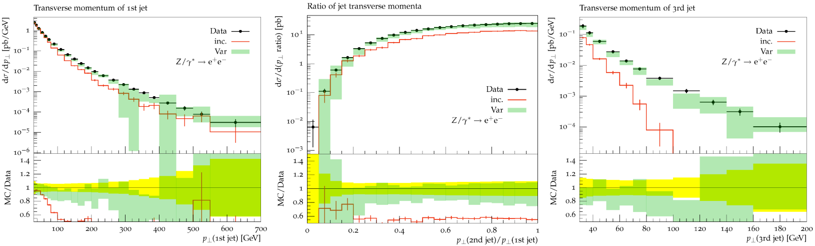
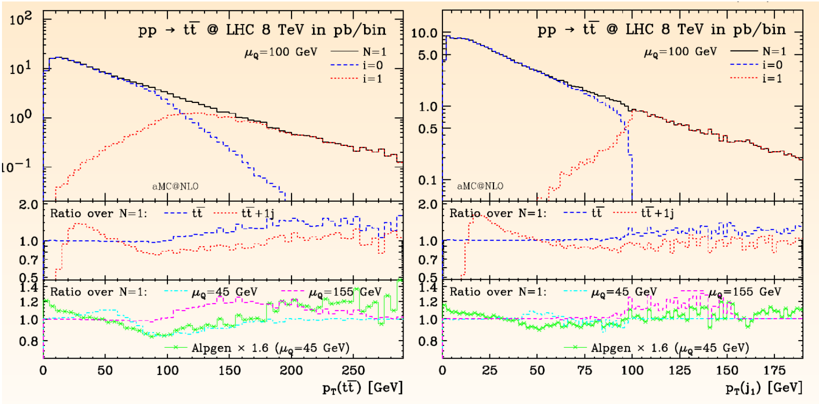


FxFx: Q_J dependence in $t\bar{t}$

(R.Frederix & S.Frixione, JHEP 1212 (2012) 061)



Aside: merging without Q_J - the MINLO approach

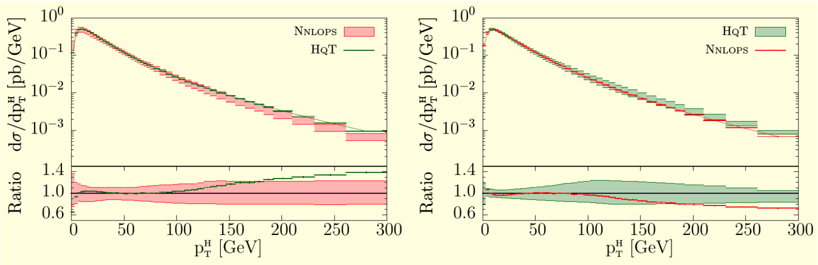
(K.Hamilton, P.Nason, C.Oleari & G.Zanderighi, JHEP 1305 (2013) 082)

- based on POWHEG + shower from PYTHIA or HERWIG
- up to today only for singlet S production, gives NNLO + PS
- basic idea:
  - use S+jet in POWHEG
  - push jet cut to parton shower IR cutoff
  - apply analytical NNLL Sudakov rejection weight for intrinsic line in Born configuration

    (kills divergent behaviour at order α_S)
  - don't forget double-counted terms
  - reweight to NNLO fixed order

for H production

(K.Hamilton, P.Nason, E.Re & G.Zanderighi, JHEP 1310 (2013) 222)



ROUND IV: SIMULATING SOFT QCD

SIMULATING SOFT QCD

HADRONISATION

Contents

QCD radiation, once more

- remember QCD emission pattern

$$\mathrm{d}w^{q\to qg} = \frac{\alpha_s(k_\perp^2)}{2\pi}\,C_F\,\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\,\frac{\mathrm{d}\omega}{\omega}\,\left[1+\left(1-\frac{\omega}{E}\right)\right]\,.$$

- spectrum cut-off at small transverse momenta and energies by onset of hadronization, at scales $R \approx 1\,\mathrm{fm}/\Lambda_{\mathrm{QCD}}$
- two (extreme) classes of emissions: gluons and gluers
  determined by relation of formation and hadronization times

- gluers formed at times R, with momenta $k_\parallel \sim k_\perp \sim \omega \gtrsim 1/R$
- assuming that hadrons follow partons,

$$\mathrm{d}N_{\text{(hadrons)}} \sim \int\limits_{k_\perp > 1/R}^{Q} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \frac{C_F\,\alpha_s(k_\perp^2)}{2\pi} \left[1 + \left(1 - \frac{\omega}{E}\right)\right] \frac{\mathrm{d}\omega}{\omega}$$

$$\sim \frac{C_F \alpha_s(1/R^2)}{\pi} \log(Q^2 R^2) \frac{\mathrm{d}\omega}{\omega}$$

or - relating their energyn with that of the gluers -

$$\mathrm{d}N_{\text{(hadrons)}}/\mathrm{d}\log\epsilon = \text{const.}\,,$$

a plateau in log of energy (or in rapidity)

- impact of additional radiation
- new partons must separate before they can hadronize independently
- therefore, one more time

$$\begin{aligned} t^{\rm form} &\sim \frac{k_\parallel}{k_\perp^2} & & \\ t^{\rm sep} &\sim R\theta &\sim& \; t^{\rm form}\,(Rk_\perp) \\ t^{\rm had} &\sim k_\parallel R^2 &\sim& \; t^{\rm form}\,(Rk_\perp)^2\,. \end{aligned}$$

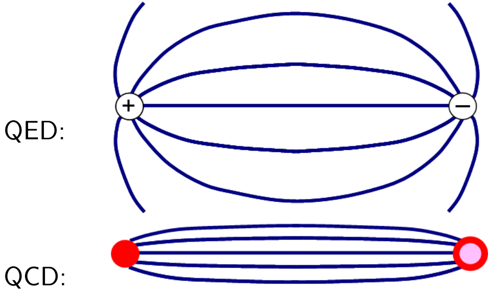
- for gluers $Rk_\perp \approx 1$: all times the same
- naively; new & more hadrons following new partons
- but: colour coherence
  primary and secondary partons not separated enough in

$$1/R \lesssim \omega_{\rm (hadron)} \lesssim 1/(R\theta)$$

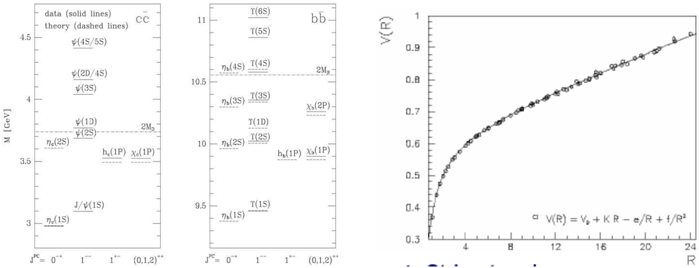
  and therefore no independent radiation

Hadronisation: General thoughts

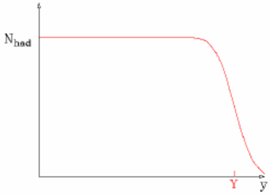
- confinement the striking feature of low–scale sotrng interactions
- transition from partons to their bound states, the hadrons
- the Meissner effect in QCD

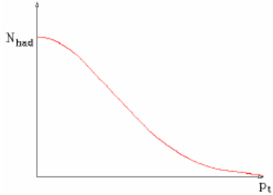


- linear QCD potential in Quarkonia – like a string



- combine some experimental facts into a naive parameterisation
- in $e^+e^- \to$ hadrons: exponentially decreasing $p_\perp$, flat plateau in y for hadrons





- try "smearing": $\rho(p_\perp^2) \sim \exp(-p_\perp^2/\sigma^2)$

- use parameterisation to "guesstimate" hadronisation effects:

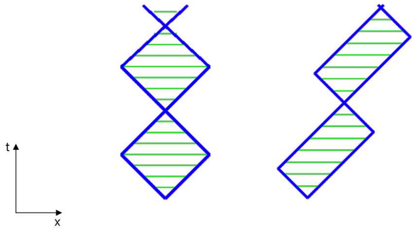
$$
\begin{aligned}
E &= \int_0^Y \mathrm{d}y\mathrm{d}p_\perp^2\,\rho(p_\perp^2)p_\perp \cosh y = \lambda \sinh Y \\
P &= \int_0^Y \mathrm{d}y\mathrm{d}p_\perp^2\,\rho(p_\perp^2)p_\perp \sinh y = \lambda(\cosh Y - 1) \approx E - \lambda \\
\lambda &= \int \mathrm{d}p_\perp^2\,\rho(p_\perp^2)p_\perp = \langle p_\perp \rangle \,.
\end{aligned}
$$

- estimate $\lambda \sim 1/R_{\rm had} \approx m_{\rm had}$, with $m_{\rm had}$ 0.1-1 GeV.
- effect: jet acquire non-perturbative mass $\sim 2\lambda E$ ($\mathcal{O}(10\mathrm{GeV})$ for jets with energy $\mathcal{O}(100\mathrm{GeV})$).

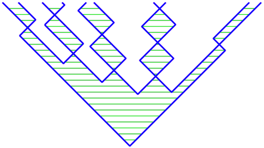
- similar parametrization underlying Feynman-Field model for independent fragmentation
- recursively fragment $q \to q' +$ had, where
  - transverse momentum from (fitted) Gaussian;
  - longitudinal momentum arbitrary (hence from measurements);
  - flavour from symmetry arguments + measurements.
- problems: frame dependent, “last quark”, infrared safety, no direct link to perturbation theory, ...

The string model

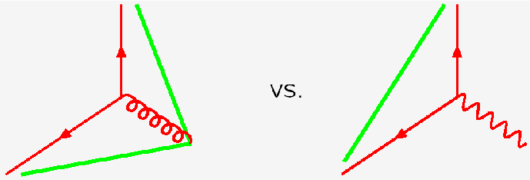
- a simple model of mesons: yoyo strings
  - light quarks ($m_q = 0$) connected by string, form a meson
  - area law: $m^2_{\rm had} \propto$ area of string motion
  - L=0 mesons only have 'yo-yo' modes:



- turn this into hadronisation model $e^+e^- \to q\bar{q}$ as test case
- ignore gluon radiation: $q\bar{q}$ move away from each other, act as point-like source of string
- intense chromomagnetic field within string:
  more $q\bar{q}$ pairs created by tunnelling and string break-up
- analogy with QED (Schwinger mechanism):
  $\mathrm{d}\mathcal{P} \sim \mathrm{d}x\mathrm{d}t \exp\left(-\pi m_q^2/\kappa\right)$, $\kappa =$ "string tension".



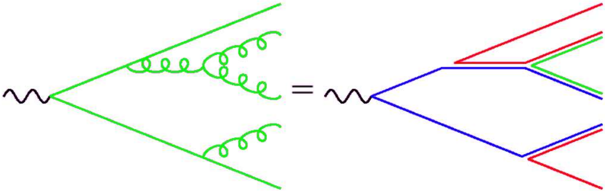
- string model = well motivated model, constraints on fragmentation (Lorentz-invariance, left-right symmetry, ...)
- how to deal with gluons?
- interpret them as kinks on the string $\Longrightarrow$ the string effect



- infrared-safe, advantage: smooth matching with PS.

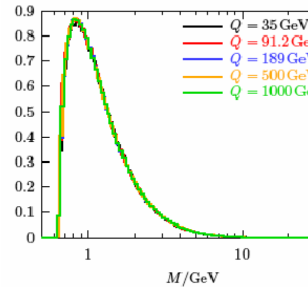
The cluster model

- underlying idea: preconfinement/LPHD
  - typically, neighbouring colours will end in same hadron
  - hadron flows follow parton flows $\longrightarrow$ don't produce any hadrons at places where you don't have partons
  - works well in large–N_c limit with planar graphs
- follow evolution of colour in parton showers



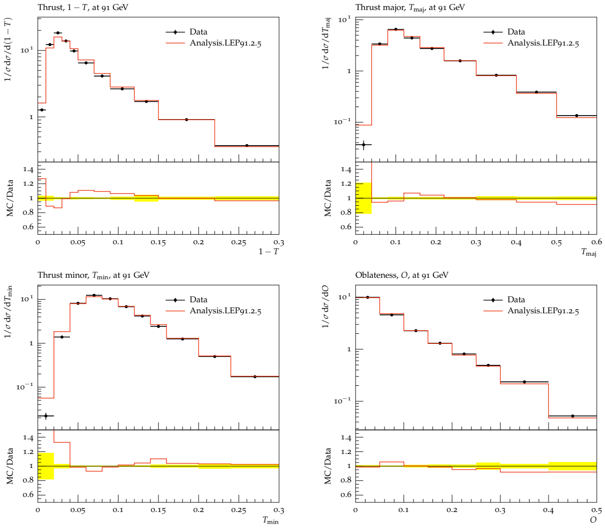
- paradigm of cluster model: clusters as continuum of hadron resonances
- trace colour through shower in $N_c \to \infty$ limit
- force decay of gluons into $q\bar{q}$ or $\bar{d}d$ pairs, form colour singlets from neighbouring colours, usually close in phase space
- mass of singlets: peaked at low scales $\approx Q_0^2$
- decay heavy clusters into lighter ones or into hadrons (here, many improvements to ensure leading hadron spectrum hard enough, overall effect: cluster model becomes more string-like)
- if light enough, clusters will decay into hadrons
- naively: spin information washed out, decay determined through phase space only $\to$ heavy hadrons suppressed (baryon/strangeness suppression)

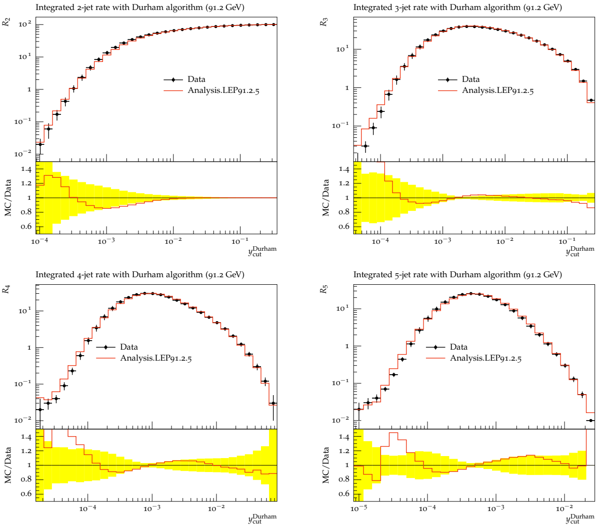
- self–similarity of parton shower will end with roughly the same **local** distribution of partons, with roughly the same invairant mass for colour singlets
- adjacent pairs colour connected, form colourless (white) clusters.
- clusters ("≈ excited hadrons) decay into hadrons

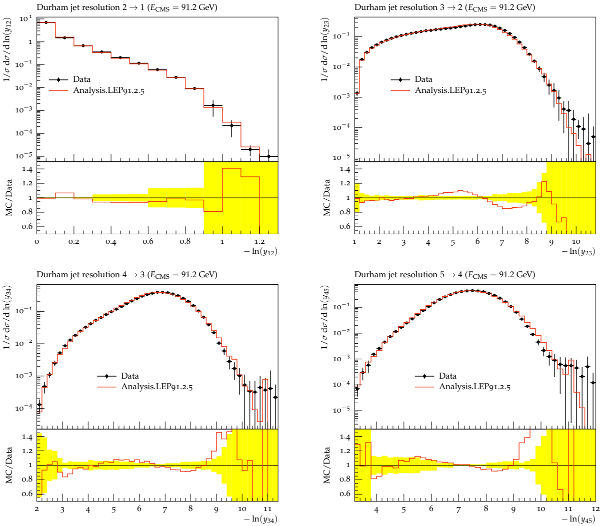


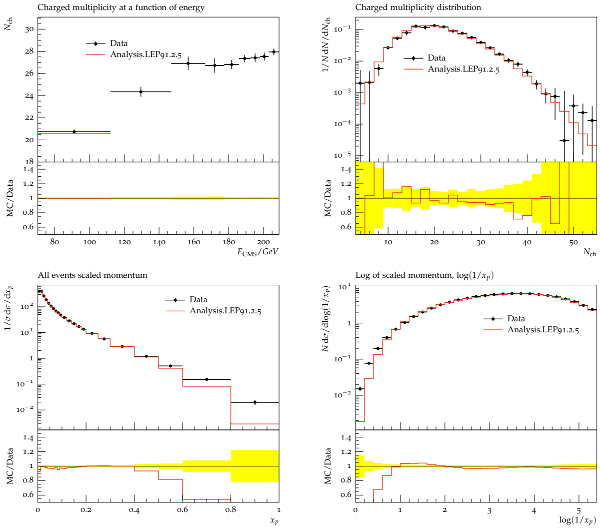
Observables

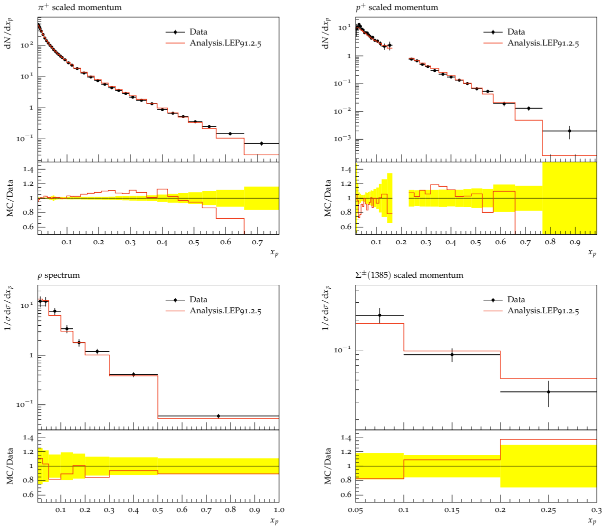
- in the following a selection of data from the LEP collaboration relevant for the tuning of hadronisation models
- all compared with an actual tune of SHERPA
- typically, PYTHIA does as good (or sometimes even slightly better)
- so, this is the level we talk about these days, agreement of 5% or better over large ranges of observables and scales

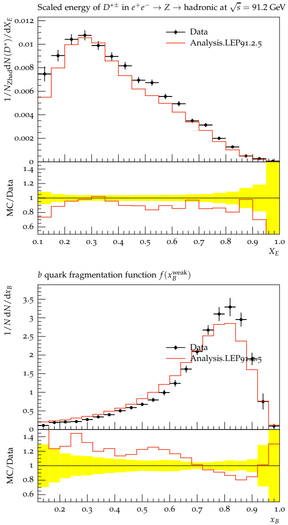

Thrust, $1-T$, at 91 GeV
$1/\sigma\, d\sigma/d(1-T)$
Data
Analysis.LEP91.2.5
MC/Data
$1-T$
Thrust major, $T_{\rm maj}$, at 91 GeV
$1/\sigma\, d\sigma/dT_{\rm maj}$
Data
Analysis.LEP91.2.5
MC/Data
$T_{\rm maj}$
Thrust minor, $T_{\rm min}$, at 91 GeV
$1/\sigma\, d\sigma/dT_{\rm min}$
Data
Analysis.LEP91.2.5
MC/Data
$T_{\rm min}$
Oblateness, O, at 91 GeV
$1/\sigma\, d\sigma/dO$
Data
Analysis.LEP91.2.5
MC/Data
O


Integrated 2-jet rate with Durham algorithm (91.2 GeV)
Integrated 3-jet rate with Durham algorithm (91.2 GeV)
Integrated 4-jet rate with Durham algorithm (91.2 GeV)
Integrated 5-jet rate with Durham algorithm (91.2 GeV)
Data
Analysis.LEP91.2.5
R_2
R_3
R_4
R_5
MC/Data
$y_{\mathrm{cut}}^{\mathrm{Durham}}$


Durham jet resolution 2 → 1 (E_{CMS} = 91.2 GeV)
Durham jet resolution 3 → 2 (E_{CMS} = 91.2 GeV)
Durham jet resolution 4 → 3 (E_{CMS} = 91.2 GeV)
Durham jet resolution 5 → 4 (E_{CMS} = 91.2 GeV)
Data
Analysis.LEP91.2.5
MC/Data
$1/\sigma\, d\sigma/d\ln(y_{12})$
$1/\sigma\, d\sigma/d\ln(y_{23})$
$1/\sigma\, d\sigma/d\ln(y_{34})$
$1/\sigma\, d\sigma/d\ln(y_{45})$
$-\ln(y_{12})$
$-\ln(y_{23})$
$-\ln(y_{34})$
$-\ln(y_{45})$


Charged multiplicity at a function of energy
N_{ch}
Data
Analysis.LEP91.2.5
MC/Data
E_{CMS}/GeV
Charged multiplicity distribution
$1/N\,dN/dN_{ch}$
Data
Analysis.LEP91.2.5
MC/Data
N_{ch}
All events scaled momentum
$1/\sigma\,d\sigma/dx_p$
Data
Analysis.LEP91.2.5
MC/Data
x_p
Log of scaled momentum, $\log(1/x_p)$
$N\,d\sigma/d\log(1/x_p)$
Data
Analysis.LEP91.2.5
MC/Data
$\log(1/x_p)$


π^+ scaled momentum
$\mathrm{d}N/\mathrm{d}x_p$
Data
Analysis.LEP91.2.5
MC/Data
x_p
p^+ scaled momentum
$\mathrm{d}N/\mathrm{d}x_p$
Data
Analysis.LEP91.2.5
MC/Data
x_p
ρ spectrum
$1/\sigma\,\mathrm{d}\sigma/\mathrm{d}x_p$
Data
Analysis.LEP91.2.5
MC/Data
x_p
$\Sigma^\pm(1385)$ scaled momentum
$1/\sigma\,\mathrm{d}\sigma/\mathrm{d}x_p$
Data
Analysis.LEP91.2.5
MC/Data
x_p


Scaled energy of $D^{*\pm}$ in $e^+e^- \to Z \to$ hadronic at $\sqrt{s} = 91.2$ GeV
Data
Analysis.LEP91.2.5
$1/N_{Z\mathrm{had}} dN(D^*)/dX_E$
MC/Data
X_E
J/ψ scaled momentum
Data
Analysis.LEP91.2.5
$1/\sigma\, d\sigma/dx_p/0.1$
MC/Data
x_p
b quark fragmentation function $f(x_B^{\mathrm{weak}})$
Data
Analysis.LEP91.2.5
$1/N\, dN/dx_B$
MC/Data
x_B
b quark fragmentation function $f(x_B^{\mathrm{weak}})$
Data
Analysis.LEP91.2.5
$1/N\, dN/dx_B$
MC/Data
x_B

- practicalities of hadronisation models: parameters
  - kinematics of string or cluster decay: 2-5 parameters
  - must "pop" quark or diquark flavours in string or cluster decay — cannot be completely democratic or driven by masses alone
    $\longrightarrow$ suppression factors for strangeness, diquarks 2-10 parameters
  - transition to hadrons, cannot be democratic over multiplets
    $\longrightarrow$ adjustment factors for vectors/tensors etc. 2-6 parameters
- tuned to LEP data, overall agreement satisfying
- validity for hadron data not quite clear

  (beam remnant fragmentation not in LEP.)
- there are some issues with inclusive strangeness/baryon production

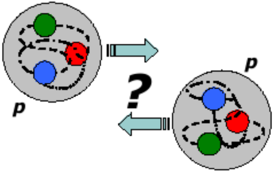
SIMULATING SOFT QCD

UNDERLYING EVENT

Contents

Multiple parton scattering

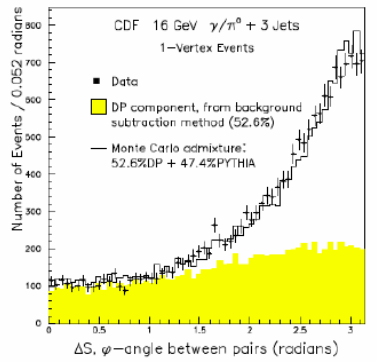
- hadrons = extended objects!
- no guarantee for one scattering only.
- running of α_S
  $\Longrightarrow$ preference for soft scattering.



- first experimental evidence for double–parton scattering: events with $\gamma + 3$ jets:
  - cone jets, $R = 0.7$, $E_T > 5$ GeV; $|\eta_j| < 1.3$;
  - "clean sample": two softest jets with $E_T < 7$ GeV;
- cross section for DPS

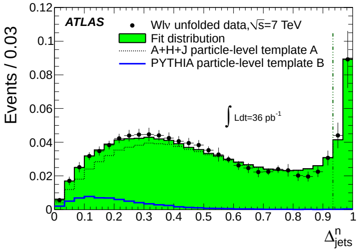
$$\sigma_{\rm DPS} = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{\rm eff}}$$

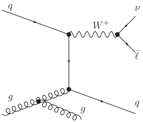
$\sigma_{\rm eff} \approx 14 \pm 4$ mb.

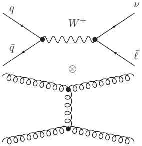


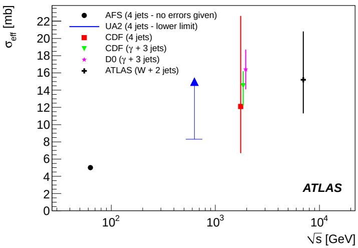
Multiple parton scattering

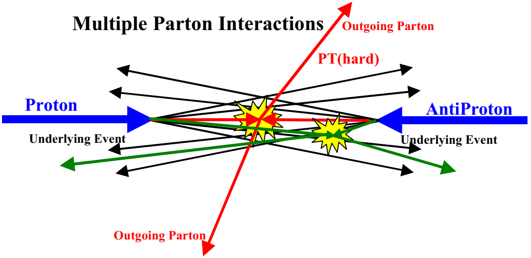
- more measurements, also at LHC
- ATLAS results from $W + 2$ jets











but: how to define the underlying event?

1. everything apart from the hard interaction, but including IS showers, FS showers, remnant hadronisation.
2. remnant-remnant interactions, soft and/or hard.
3. lesson: hard to define

- origin of MPS: parton–parton scattering cross section exceeds hadron–hadron total cross section

$$\sigma_{\rm hard}(p_{\perp,\rm min}) \;=\; \int\limits_{p^2_{\perp,\rm min}}^{s/4} {\rm d}p_\perp^2\,\frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} > \sigma_{pp,\rm total}$$

  for low $p_{\perp,\rm min}$

- remember

$$\frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} \;=\; \int\limits_0^1 {\rm d}x_1{\rm d}x_2\, f(x_1,\,q^2)f(x_2,\,q^2)\,\frac{{\rm d}\hat\sigma_{2\to 2}}{{\rm d}p_\perp^2}$$

- $\langle\sigma_{\rm hard}(p_{\perp,\rm min})/\sigma_{pp,\rm total}\rangle \geq 1$
- depends strongly on cut-off $p_{\perp,\rm min}$ (energy-dependent)!

Modelling the underlying event

- take the old PYTHIA model as example:
  - start with hard interaction, at scale $Q^2_{\rm hard}$.
  - select a new scale $p_\perp^2$ from

$$\exp\left[-\frac{1}{\sigma_{\rm norm}}\int\limits_{p_\perp^2}^{Q^2_{\rm hard}}{\rm d}{p'_\perp}^2\,\frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}{p'_\perp}^2}\right]$$

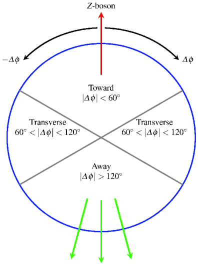
    with constraint $p_\perp^2 > p^2_{\perp,\rm min}$
  - rescale proton momentum ("proton-parton = proton with reduced energy").
  - repeat until no more allowed $2\to 2$ scatter

Modelling the underlying event

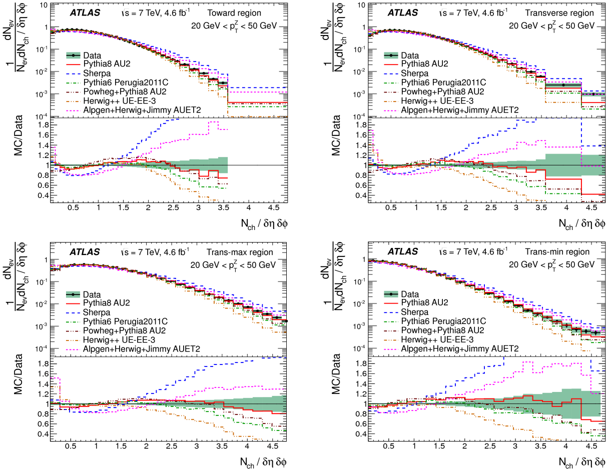
- possible refinements:
  - may add impact-parameter dependence $\longrightarrow$ more fluctuations
  - add parton showers to UE
  - "regularisation" to dampen sharp dependence on $p_{\perp,\mathrm{min}}$: replace $1/\hat{t}$ in MEs by $1/(t+t_0)$, also in α_s.
  - treat intrinsic $k_\perp$ of partons ($\rightarrow$ parameter)
  - model proton remnants ($\rightarrow$ parameter)

Some results for MPS in Z production

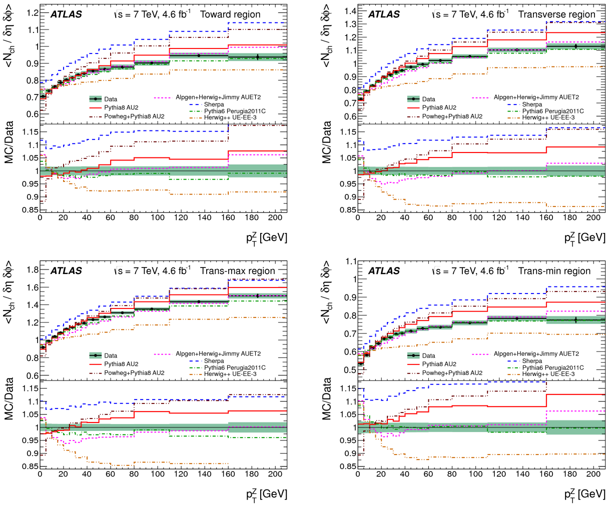
- observables sensitive to MPS
- classical analysis: transverse regions in QCD/jet events
- idea: find the hardest system, orient event into regions:
  - toward region along system
  - away region back-to-back
  - transverse regions
- typically each in 120°



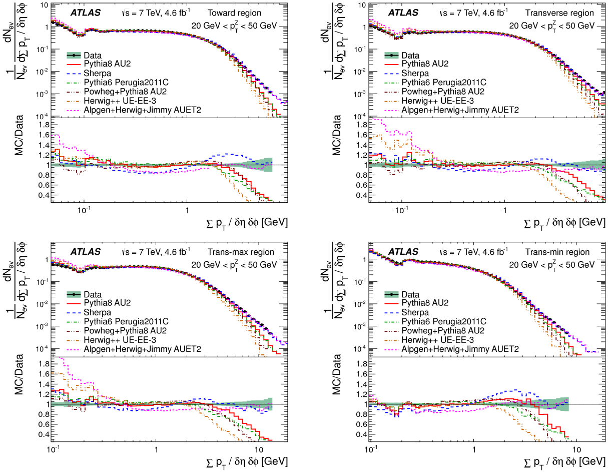
Some results in Z production



Some results in Z production



Some results in Z production



- see some data comparison in Minimum Bias
- practicalities of underlying event models: parameters
  - profile in impact parameter space — 2-3 parameters
  - IR cut-off at reference energy, its energy evolution, dampening paramter and normalisation cross section — 4 parameters
  - treating colour connections to rest of event — 2-5 parameters
- tuned to LHC data, overall agreement satisfying
- energy extrapolation not exactly perfect, plus other process categories such as diffraction etc..

SUMMARY

Summary

- Systematic improvement of event generators by including higher orders has been at the core of QCD theory and developments in the past decade:
  - multijet merging (“CKKW”, “MLM”)
  - NLO matching (“MC@NLO”, “POWHEG”)
  - MENLOPS NLO matching & merging
  - MEPS@NLO (“SHERPA”, “UNLOPS”, “MINLO”, “FxFx”)

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- multijet merging an important tool for many relevant signals and backgrounds - pioneering phase at LO & NLO over
- complete automation of NLO calculations done
  $\longrightarrow$ must benefit from it!

(it’s the precision and trustworthy & systematic uncertainty estimates!)

Famous last screams

- in Run-II we'll be in for a ride:

  more statistics
  more energy
  more channels
  more precision
  more fun

- . . . and all with QCD . . .

oh, and btw.: the first NNLO+PS are out!