QCD at the LHC focusing on Higgs production

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HiggsTools Annual School
Pré Saint Didier, 28/06 - 04/07 2015

QCD at the LHC

- Aim of the lecture:
  - ➡ Introduction to the basics of QCD calculations for the LHC.
  - ➡ Disclaimer: Had to make a (not unbiased!) choice of topics.
  - ➡ Will not cover partons showers, etc. (See F. Krauss's lecture)

- Part I:
  - ➡ QCD @ LHC: Factorisation, LO, NLO and beyond.

- Part II:
  - ➡ QCD for Higgs physics: large-mt limit, inclusive and differential ggF cross section, VBF.

QCD at the LHC

- The LHC is a proton collider.
- Protons are bound-states of quarks and gluons.
  - ➡ Need Quantum chromodynamics to describe LHC physics!
- QCD Lagrangian:

$$\mathcal{L} = -\frac{1}{4}\,\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \bar{q}_f i \not{D} q_f - m_f \bar{q}_f q_f + \frac{1}{2\xi}\left(\partial^\mu G^a_\mu\right)^2 + \bar{c}^a \partial^\mu D_\mu c^a$$

Gluons / pure Yang-Mills

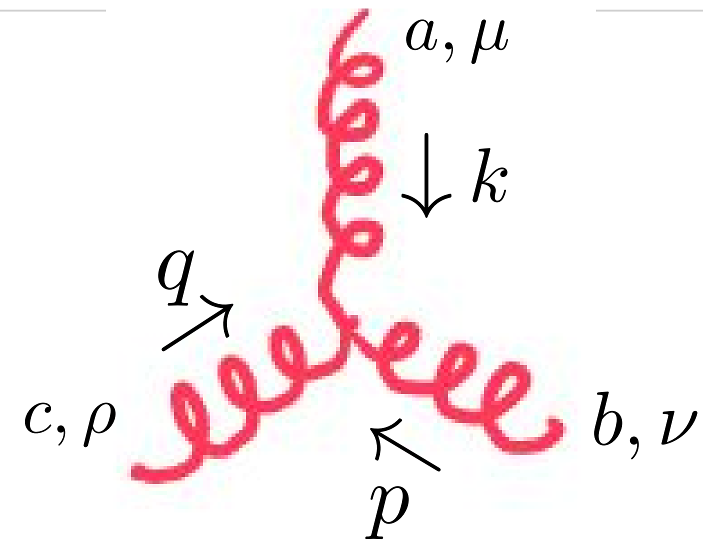
Quarks (different flavors)

Gauge fixing & ghosts

$$F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right] = \left(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g\sqrt{2} f^{abc} G^b_\mu G^c_\nu\right) T^a$$

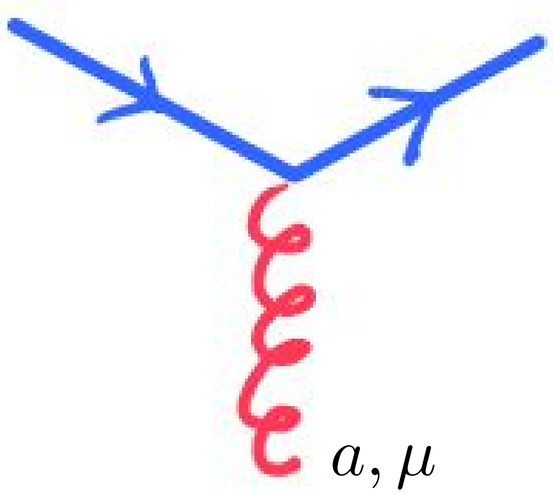
$$D_\mu = \partial_\mu - i g T^a G^a_\mu$$

QCD Feynman rules

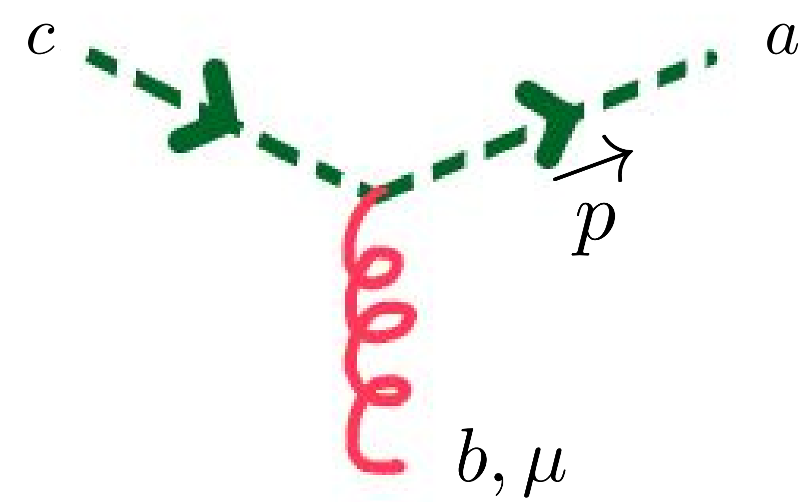


$$g_s\, f^{abc}[g^{\mu\nu}(k-p)^\rho + g^{\nu\rho}(p-q)^\mu + g^{\rho\mu}(q-k)^\nu]$$

$$-ig_s^2\,[f^{abe}f^{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f^{ade}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]$$



$$ig_s\,\gamma^\mu\, T^a$$



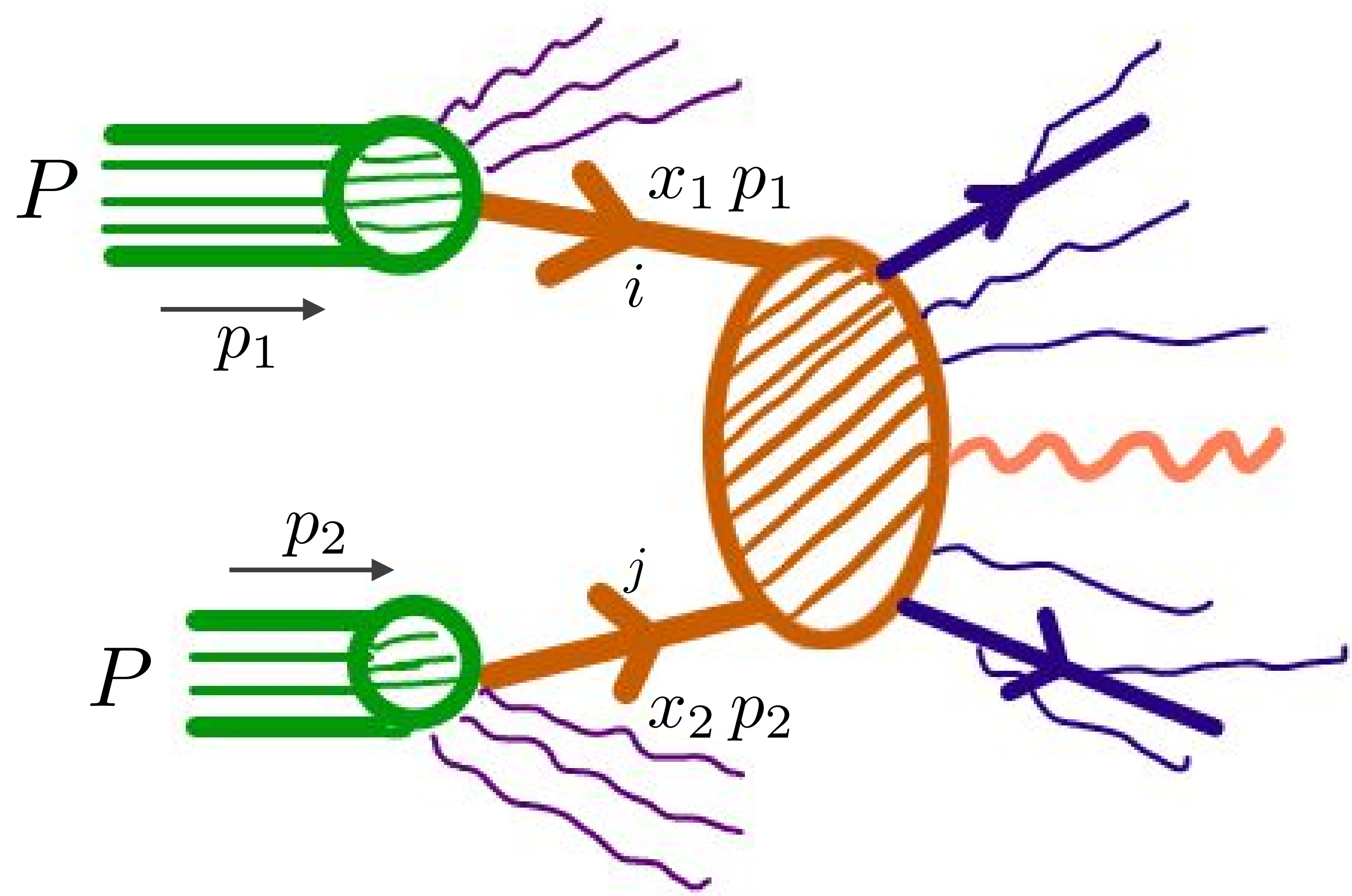
$$-g_s\, f^{abc}\, p^\mu$$

Factorisation

QCD factorisation

- We need a way to connect protons to quarks and gluons.

$$d\sigma = \sum_{i,j} \int_0^1 dx_1\, dx_2\, f_i(x_1, \mu_F^2)\, f_j(x_2, \mu_F^2)\, d\hat{\sigma}_{ij}(\mu_F^2)$$



QCD factorisation

$$d\sigma = \sum_{i,j} \int_0^1 dx_1\, dx_2\, f_i(x_1, \mu_F^2)\, f_j(x_2, \mu_F^2)\, d\hat{\sigma}_{ij}(\mu_F^2)$$

- $d\hat{\sigma}_{ij}$ is the partonic cross section.
  - ➡ calculable in perturbative QCD.
  - ➡ process-dependent.
- f_i are the parton distribution functions (PDFs).
  - ➡ non-perturbative, need to be extracted from measurements.
  - ➡ universal, process-independent.

- QCD factorisation is expected to hold up to terms $\mathcal{O}(\Lambda_{QCD}/\sqrt{S})$.

PDFs & DGLAP evolution

- The rhs of the factorisation formula depends on the factorisation scale μ_F.
  - ➡ Introduces a source of uncertainty order-by-order in perturbation theory.
  - ➡ μ_F typically of the order of the hard scale.
- μ_F - dependence governed by DGLAP equation:

$$\frac{d}{d\log\mu_F} f_i(x,\mu_F^2) = a_s(\mu_F^2)\sum_j P_{ij}(x, a_s(\mu_F^2)) \otimes f_j(x,\mu_F^2)$$

$$a_s(\mu_F^2) = \frac{\alpha_s(\mu_F^2)}{\pi} \qquad [f\otimes g](x) = \int_x^1 dt\, f(t)\, g\left(\frac{x}{t}\right)$$

➡ Coupled system of integro-differential equations.

PDFs & DGLAP evolution

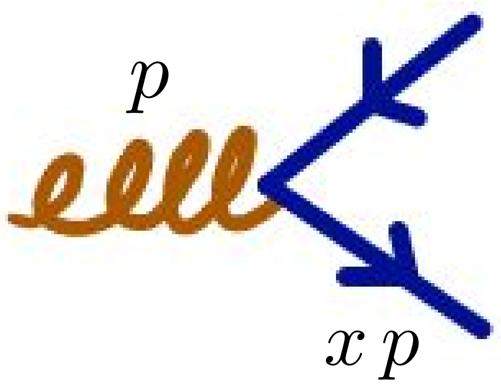
- P_{ij} are the Altarelli-Parisi splitting functions:

$$P_{ij}(x, a_s) = P_{ij}^{(0)}(x) + a_s\, P_{ij}^{(1)}(x) + a_s^2\, P_{ij}^{(2)}(x) + \dots$$

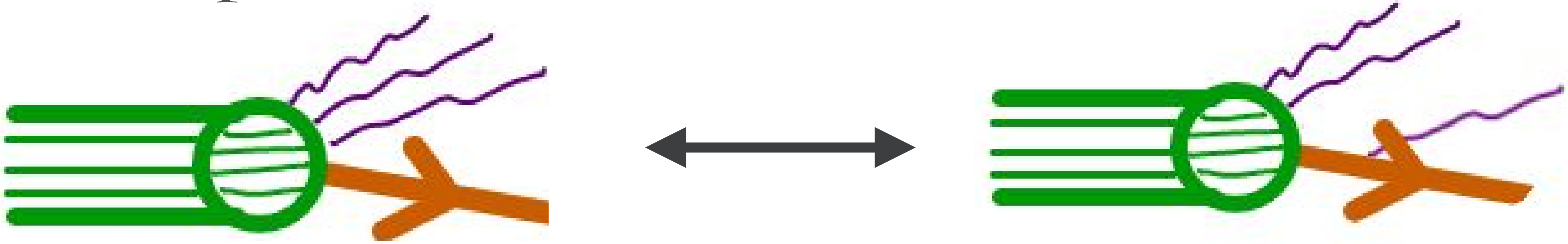
➡ Known up to a_s^2

- They describe the collinear splitting of a parton j into a parton i carrying a fraction x of the original momentum. E.g.:

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$



- Intuitive picture:

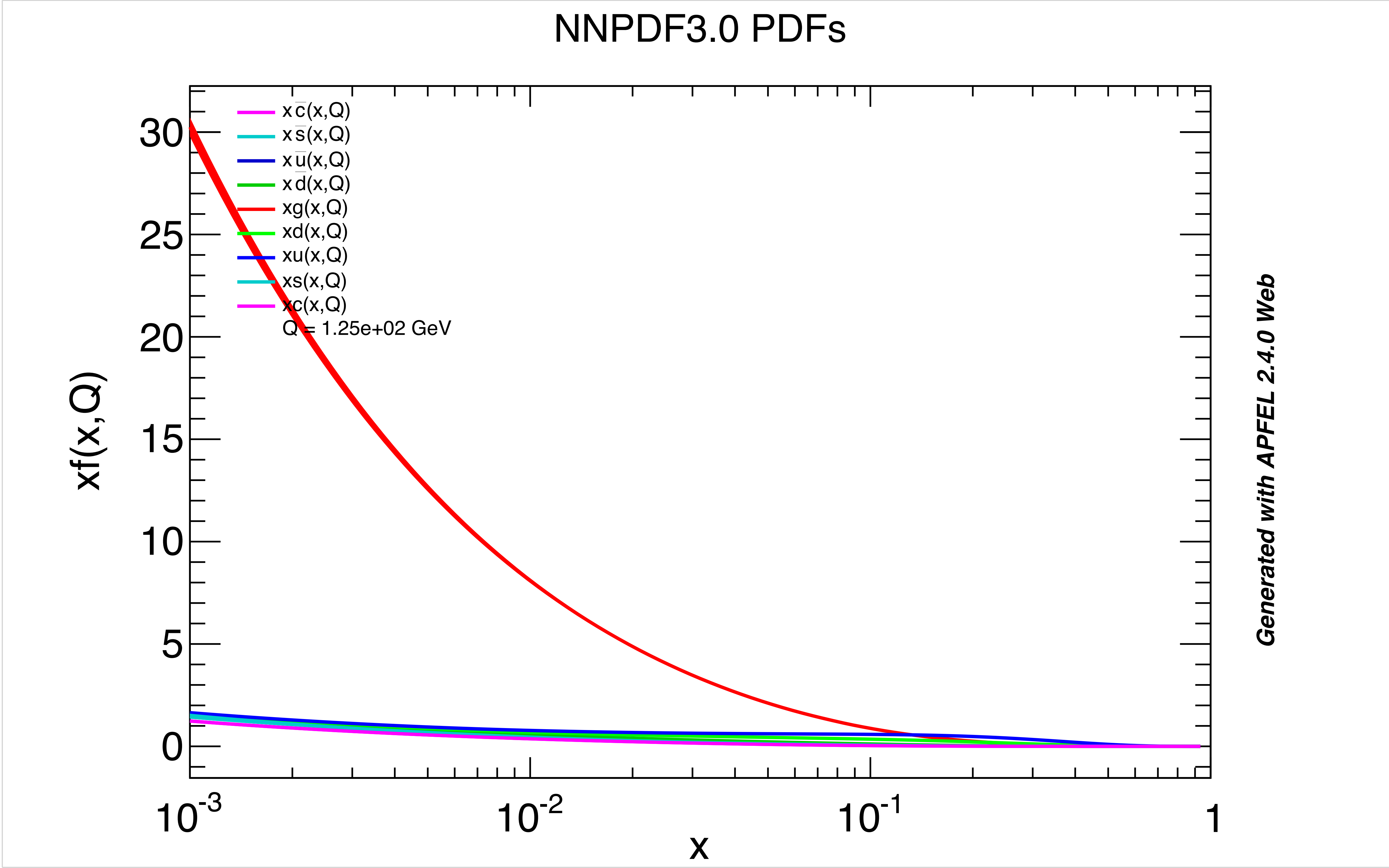


PDFs & DGLAP evolution

- If you need PDFs at a certain scale… where to get them from?
- There are several collaborations that are specialised in
  - ➡ fitting PDFs to data.
  - ➡ providing the evolution to arbitrary scales (up to a certain accuracy in a_s^2).
- Most common PDF set: MMHT, NNPDF, CTEQ, ABM,…
- Two public web sites that allow one to plot PDFs for different choices of x and the scale:

  http://apfel.mi.infn.it/
  http://hepdata.cedar.ac.uk/pdf/pdf3.html

PDFs


NNPDF3.0 PDFs
x c̄(x,Q)
x s̄(x,Q)
x ū(x,Q)
x d̄(x,Q)
xg(x,Q)
xd(x,Q)
xu(x,Q)
xs(x,Q)
xc(x,Q)
Q = 1.25e+02 GeV
30
25
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xf(x,Q)
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x
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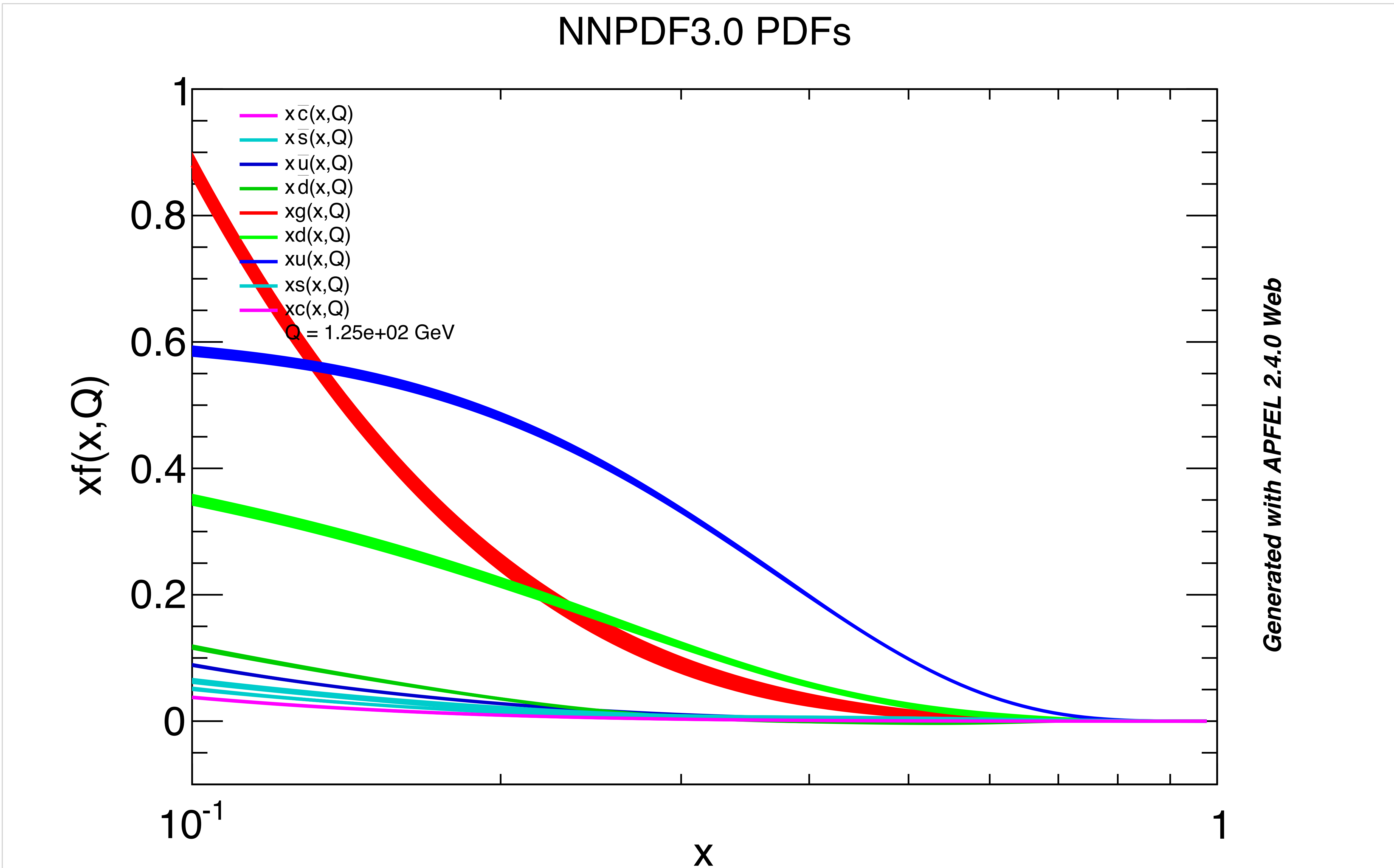
PDFs



NNPDF3.0 PDFs
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$x\bar{u}(x,Q)$
$x\bar{d}(x,Q)$
xg(x,Q)
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xs(x,Q)
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Q = 1.25e+02 GeV
1
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0.6
0.4
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xf(x,Q)
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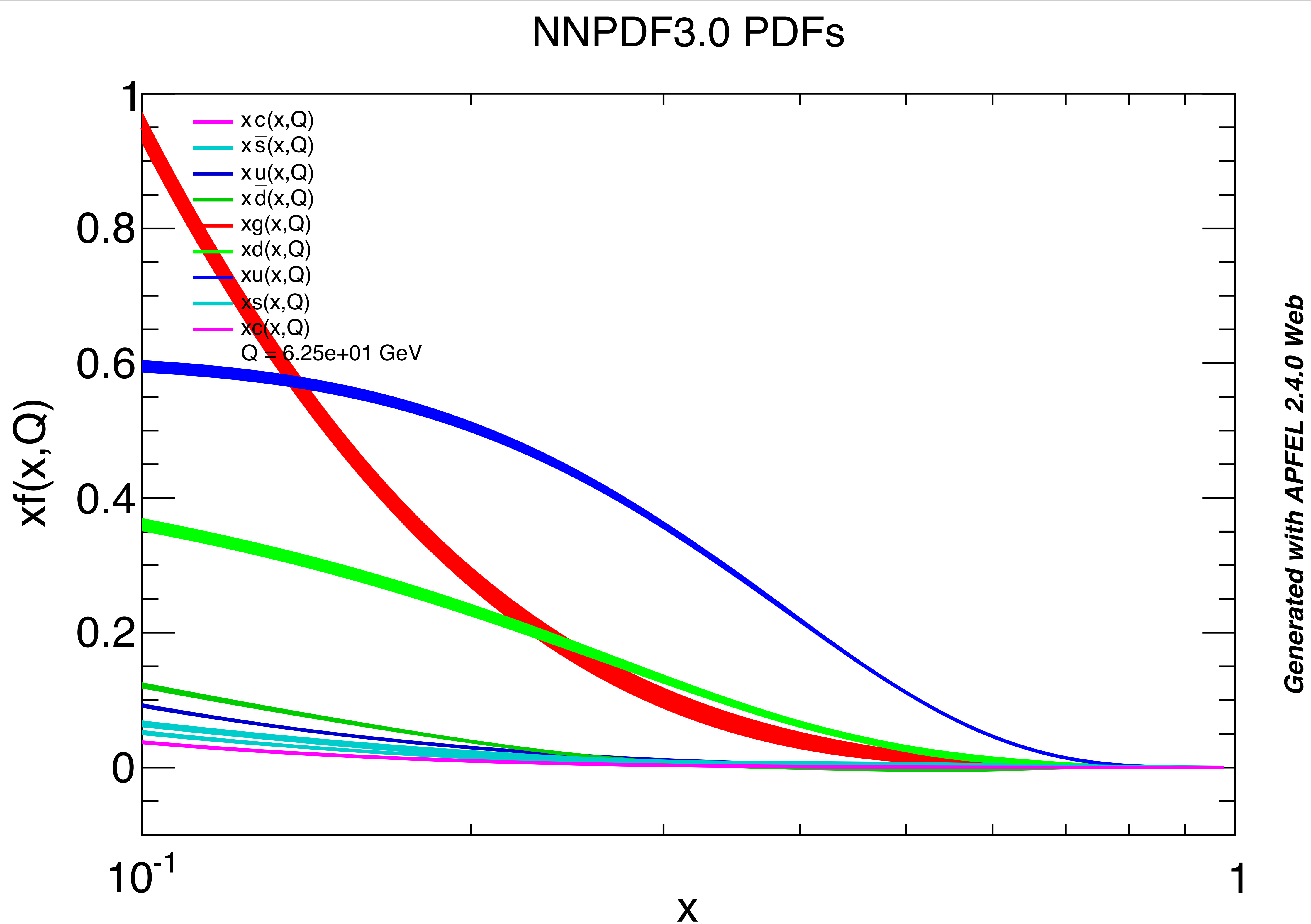
PDFs


NNPDF3.0 PDFs
$x\bar{c}(x,Q)$
$x\bar{s}(x,Q)$
$x\bar{u}(x,Q)$
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xg(x,Q)
xd(x,Q)
xu(x,Q)
xs(x,Q)
xc(x,Q)
Q = 6.25e+01 GeV
xf(x,Q)
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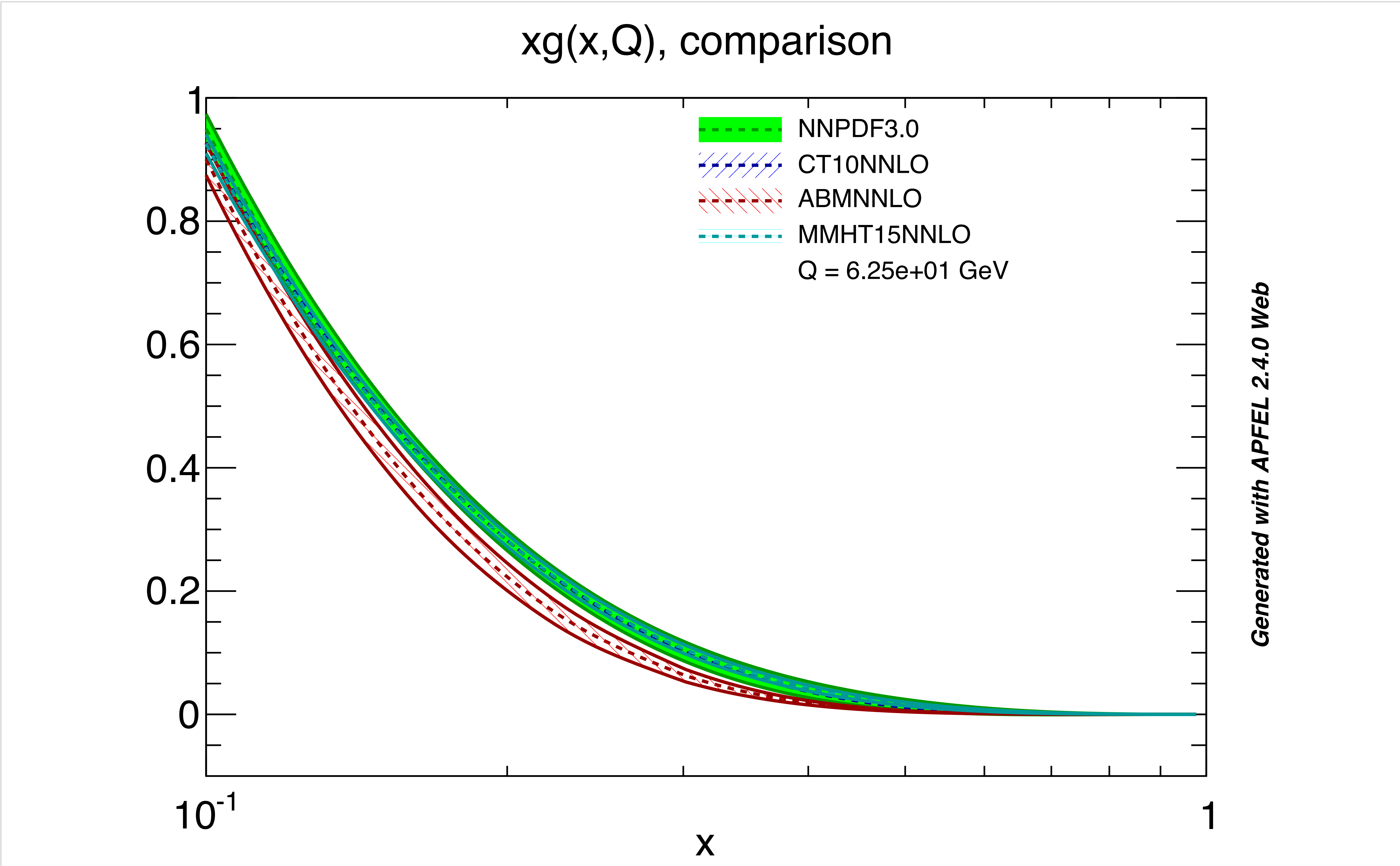
PDFs


xg(x,Q), comparison
NNPDF3.0
CT10NNLO
ABMNNLO
MMHT15NNLO
Q = 6.25e+01 GeV
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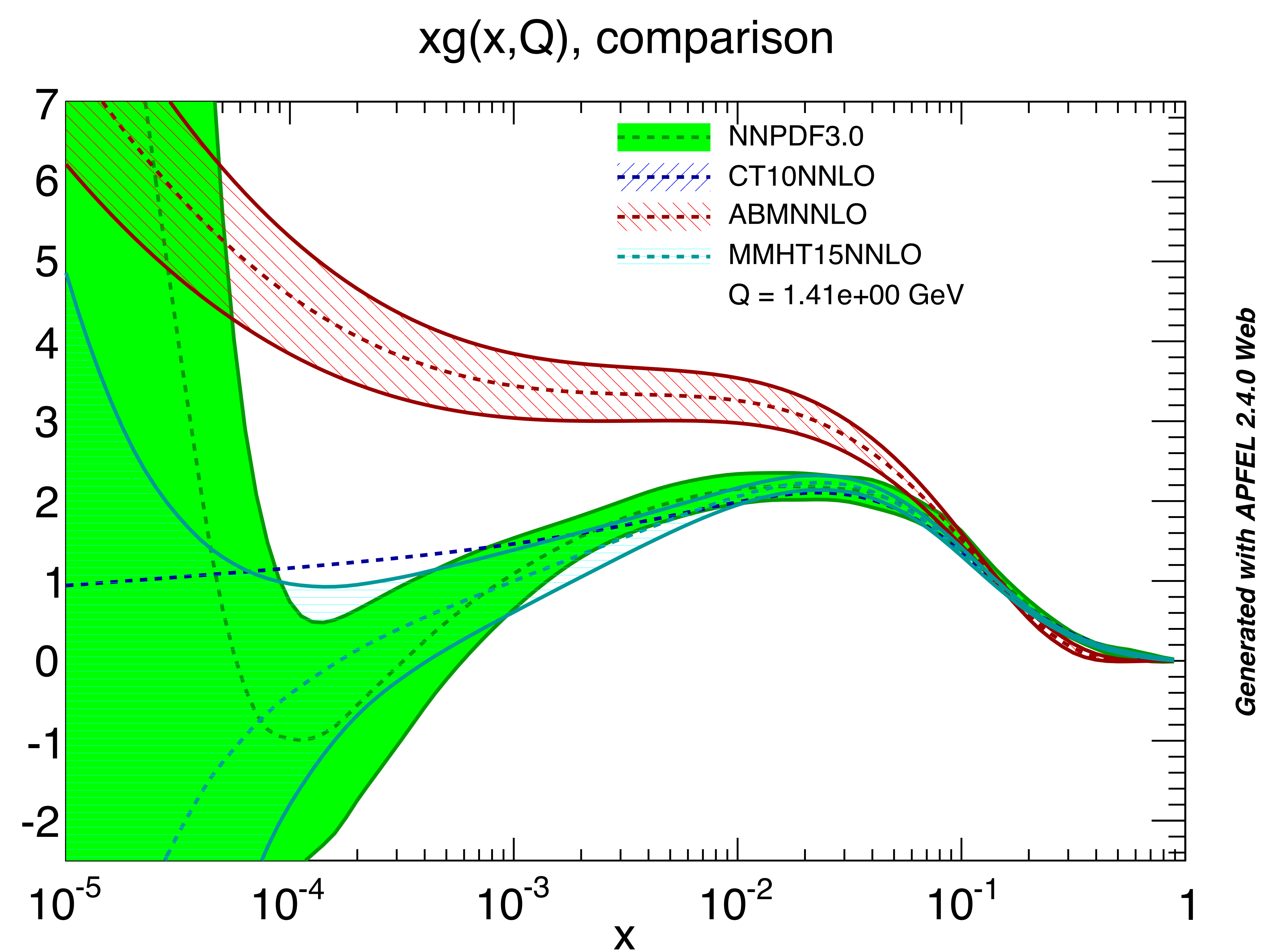
PDFs


xg(x,Q), comparison
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ABMNNLO
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Q = 1.41e+00 GeV
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-1
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10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

QCD @ LO

QCD @ leading order

- Consider total cross section for e+e- production via an off-shell photon at LO in QCD:
  - ➡ Concentrate on QCD part, i.e., production of an off-shell photon with virtuality Q2.

$$\hat{\sigma}_{q\bar{q}} = \frac{1}{2S} \int d\Phi_1 \, |\mathcal{M}_{q\bar{q}\to\gamma^*}|^2$$

$$d\Phi_1 = (2\pi)^4 \, \delta^{(4)}(p_\gamma - p_1 - p_2) \, \frac{d^4 p_\gamma}{(2\pi)^3} \, \delta_+(p_\gamma^2 - Q^2)$$

$$\delta_+(p^2 - m^2) \equiv \delta(p^2 - m^2) \, \theta(p^0)$$

$$|\mathcal{M}_{q\bar{q}\to\gamma^*}|^2 = \frac{4\pi\alpha e_q^2}{N_c} Q^2 \qquad \hat{\sigma}_{q\bar{q}} = \frac{8\pi^2\alpha e_q^2}{N_c \, Q^2} \, \delta(1-z) \qquad z = Q^2/\hat{s}$$

QCD @ leading order

- We need to fold the partonic cross section with the PDFs.
- Useful formula:

$$\sigma(S,Q^2) = \sum_{i,j} \int_0^1 dx_1\, dx_2\, f_1(x_1,\mu_F^2)\, f_2(x_2,\mu_F^2)\, \hat{\sigma}_{ij}(Q^2,\hat{s},\mu_F^2)$$

$$= \tau \sum_{i,j} \left[\mathcal{L}_{ij}(\tau/z,\mu_F^2) \otimes \frac{\hat{\sigma}_{ij}(Q^2,Q^2/z,\mu_F^2)}{z} \right](\tau)$$ ➡ Prove this!

$$\mathcal{L}_{ij}(z,\mu_F^2) = [f_i(x,\mu_F^2) \otimes f_j(x,\mu_F^2)](z) \qquad \tau = Q^2/S$$

- Final result for hadronic cross section:

$$\sigma^{LO}(S,Q^2) = \frac{16\pi^2\alpha}{N_c\, S} \sum_f e_f^2 \mathcal{L}_{q_f\bar{q}_f}(\tau,\mu_F^2)$$

QCD @ leading order

- We cannot fix the value of the factorisation scale at this point!
  - ➡ More on this later!
- We can also compute differential distributions of an observable.

  An observable $\mathcal{O}$ is given by a function $O(p_1,\ldots,p_n)$ of all momenta in the event. Its distribution is

$$\frac{d\hat{\sigma}_{ij}}{d\mathcal{O}} = \frac{1}{2\hat{s}}\int d\Phi_{n-2}\,\delta(\mathcal{O}-O(p_1,\ldots,p_n))\,|\mathcal{M}|^2$$

- Nowadays, there is no need to compute LO cross sections and distributions by hand!
  - ➡ Automated tools! MadGraph, Sherpa, CalcHep, …
  - ➡ + PS Monte Carlo generators: Pythia, Sherpa, Herwig.
  - ➡ See Frank's lecture.

Adding a jet

- Let us now add a jet to our process:

- Let's compute the total cross section for this process:

$$\hat{\sigma}_{q\bar{q}} = \frac{1}{2\hat{s}} \int d\Phi_2 \, |\mathcal{M}_{q\bar{q}\to\gamma^* g}|^2 \qquad |\mathcal{M}_{q\bar{q}\to\gamma^* g}|^2 = \sigma_0 \frac{2\alpha_s}{3N_c} \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2Q^2 \hat{s}}{\hat{u}\hat{t}} \right]$$

$$d\Phi_2 = (2\pi)^4 \delta^{(4)}(p_\gamma + k - p_1 - p_2) \frac{d^4 p_\gamma}{(2\pi)^3} \delta_+(p_\gamma^2 - Q^2) \frac{d^4 k}{(2\pi)^3} \delta_+(k^2)$$

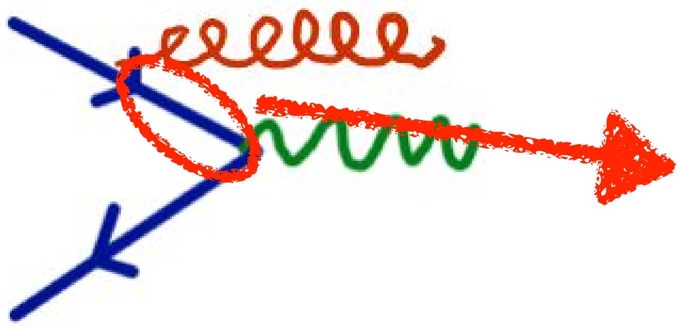
$$= \frac{\pi}{2} d\hat{t} \, d\hat{u} \, \delta(\hat{s} + \hat{t} + \hat{u} - Q^2)$$

$$\hat{t} = -\hat{s} \frac{1-z}{2} (1 - \cos\theta) \Rightarrow -\hat{s}(1-z) \leq \hat{t} \leq 0$$

The PS integral diverges!

IR divergences

- What is happening here?



$$\frac{1}{\hat{t}} = \frac{1}{E_1\, E_g\, (1 - \cos\theta)}$$

Singular if

$E_g \to 0$ or $\theta \to 0$

- This is a general feature: QCD amplitudes are singular whenever a gluon becomes soft or two (or more) massless partons becomes collinear.

- The divergence is universal, and does not depend on the details of the rest of the scattering process.

IR divergences

- How to avoid this divergence?
- Solution 1: Simply do not go into the singular region!

Practically, apply phase-space cuts to restrict the integration to the non-singular region:

$$\hat{\sigma}_{q\bar{q}}(p_{T,min}) = \frac{1}{2\hat{s}} \int d\Phi_2 \, |\mathcal{M}|^2 \, \theta(p_{T,g} > p_{T,min})$$

➡ The cross section depends on the cuts!

➡ This type of phase-space integrals is best handled using numerical methods.

IR divergences

- **Solution 2:** Additional jets belong to higher orders in perturbation theory.

$$\hat{\sigma}(q\bar{q} \to \gamma^*) = \mathcal{O}(\alpha) \qquad \text{LO QCD}$$

$$\hat{\sigma}(q\bar{q} \to \gamma^* g) = \mathcal{O}(\alpha\,\alpha_s) \qquad \text{NLO QCD}$$

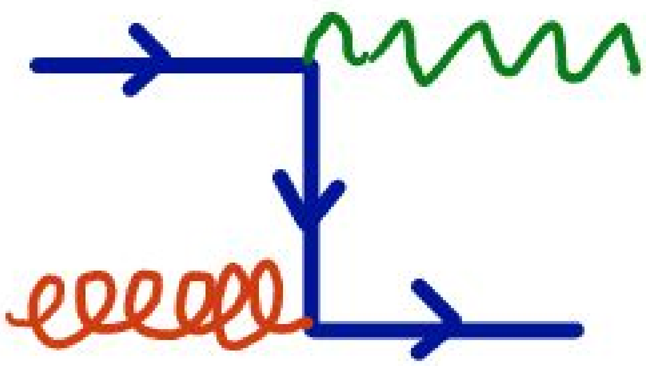
➡ If the gluon is soft or collinear, then the external state coincides with the external state of the Born configuration.

➡ We need to add together the loop corrections to the Born and the emissions of soft and collinear partons.

➡ This is a particular instance of the Kinoshita-Lee-Nauenberg (KLN) theorem: IR singularities cancel in inclusive-enough observables.

IR divergences

- Comment 1: There is a new channel opening!



➡ Need to include this channel.

➡ QCD factorisation requires to sum over initial states!

- Comment 2: PS cuts can give rise to large logarithms!

$$\hat{\sigma}_{q\bar{q}}(p_{T,min}) = \frac{1}{2\hat{s}} \int d\Phi_2 \, |\mathcal{M}|^2 \, \theta(p_{T,g} > p_{T,min}) \simeq \log \frac{p_{T,min}^2}{Q^2}$$

➡ If $p_{T,min}^2 \ll Q^2$, this logarithm can be large, at every order in perturbation theory.

➡ Need to resum these logarithms.

➡ cf. Frank's lectures.

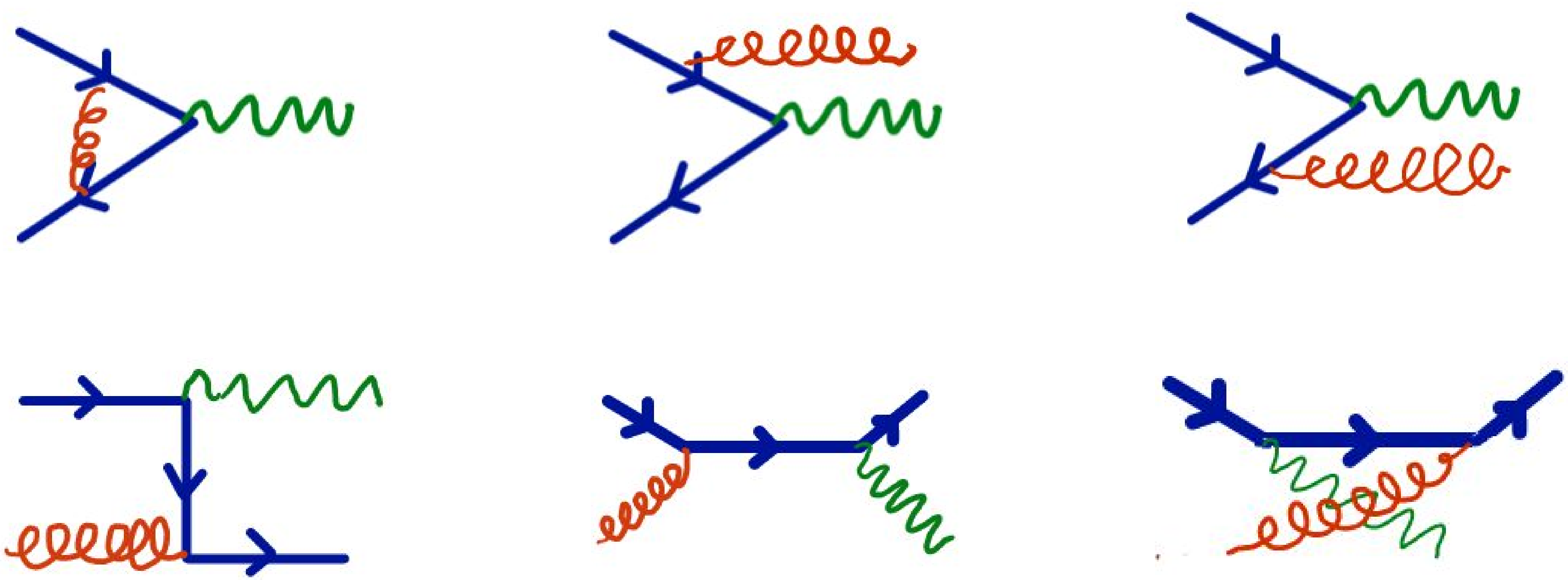
QCD @ NLO

QCD @ NLO

- Ingredients for NLO: $\hat{\sigma}_{ij}^{NLO} = \hat{\sigma}_{ij}^{V} + \hat{\sigma}_{ij}^{R}$

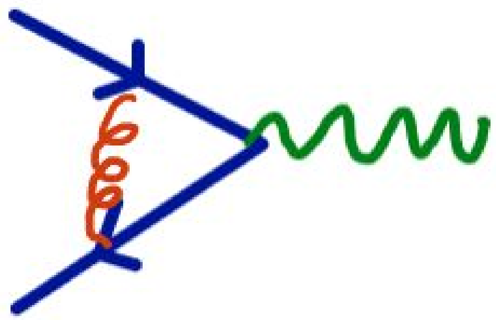
$$\hat{\sigma}_{ij}^{V} = \frac{1}{2\hat{s}} \int d\Phi_n \, 2\mathrm{Re}\mathcal{M}_n^{(0)} \mathcal{M}_n^{(1)*} \qquad \hat{\sigma}_{ij}^{R} = \frac{1}{2\hat{s}} \int d\Phi_{n+1} \left| \mathcal{M}_{n+1}^{(0)} \right|^2$$

- For our example:



Virtual corrections

- The phase-space is trivial, and the computation of the loop is not too difficult:



$$\hat{\sigma}^V_{q\bar{q}} = \sigma_0\, \delta(1-z)\, a_s\, (4\pi)^\epsilon\, e^{-\gamma_E \epsilon} \left\{ -\frac{4}{3\epsilon^2} + \frac{1}{\epsilon}\left[\frac{4}{3}\log\left(\frac{Q^2}{\mu^2}\right) - 2\right] - \frac{2}{3}\log^2\left(\frac{Q^2}{\mu^2}\right) + 2\log\left(\frac{Q^2}{\mu^2}\right) + \frac{7\pi^2}{9} - \frac{16}{3} + \mathcal{O}(\epsilon) \right\}$$

- We used DimReg to regulate the divergences in the loop.
  - ➡ In principle UV and IR poles.
  - ➡ Here only IR, because LO in a_s
  - ➡ Result depends on arbitrary scale μ introduced by DimReg.

- How can we combine this with the real corrections…?
  - ➡ The tree-level matrix element was independent of ϵ .

Dimensional regularisation

- We have to do everything in D-dimensions!
  - ➡ Conventional dimensional regularisation (CDR).
- D-dimensional phase space:

$$d\Phi_2 = \mu^{4\epsilon}\,(2\pi)^D\,\delta^{(D)}(p_\gamma + k - p_1 - p_2)\,\frac{d^D p_\gamma}{(2\pi)^{D-1}}\,\delta_+(p_\gamma^2 - Q^2)\,\frac{d^D k}{(2\pi)^{D-1}}\,\delta_+(k^2)$$

$$= \frac{\pi^{1-\epsilon}}{2\Gamma(1-\epsilon)}\,d\hat{t}\,d\hat{u}\,\left(\frac{\hat{u}\hat{t}}{\mu^4}\right)^{-\epsilon}\,\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)$$

vs. $\frac{\pi}{2}\,d\hat{t}\,d\hat{u}\,\delta(\hat{s}+\hat{t}+\hat{u}-Q^2)$ in D = 4.

➡ Phase space divergences regulated because $\int_0^1 \frac{d\hat{t}}{\hat{t}^{1+\epsilon}} = -\frac{1}{\epsilon}$.

- N.B.: We need everything in D dimensions, also the Born!
  (D-dimensional metric, Dirac matrices, etc.)

Real corrections

- Real corrections in D dimensions:

$$\hat{\sigma}^{R}_{q\bar{q}} = -\sigma_0\, a_s\, (4\pi)^{\epsilon} \left(\frac{Q^2}{\mu^2}\right)^{-\epsilon} \frac{4\Gamma(1-\epsilon)}{3\epsilon\,\Gamma(1-2\epsilon)} \left[z^{\epsilon}\,(1-z)^{1-2\epsilon} + 2z^{1+\epsilon}\,(1-z)^{-1-2\epsilon}\right]$$

  ➡ Finite in D-dimensions!

- This is supposed to cancel the poles of the virtuals:

$$\hat{\sigma}^{V}_{q\bar{q}} = \sigma_0\,\delta(1-z)\,a_s\,(4\pi)^{\epsilon}\,e^{-\gamma_E\epsilon}\left\{-\frac{4}{3\epsilon^2} + \frac{1}{\epsilon}\left[\frac{4}{3}\log\left(\frac{Q^2}{\mu^2}\right) - 2\right] - \frac{2}{3}\log^2\left(\frac{Q^2}{\mu^2}\right) + 2\log\left(\frac{Q^2}{\mu^2}\right) + \frac{7\pi^2}{9} - \frac{16}{3} + \mathcal{O}(\epsilon)\right\}$$

  ➡ No double pole in ϵ.

  ➡ No distribution $\delta(1-z)$.

Real corrections

- The real corrections are not integrable for $\epsilon = 0$ because there is a pole at $z = 1$

$$(1-z)^{-1-2\epsilon} = -\frac{1}{2\epsilon}\,\delta(1-z) + \sum_{k=0}^{\infty}\frac{(-2\epsilon)^k}{k!}\left[\frac{\log^k(1-z)}{1-z}\right]_+$$

➡ Additional pole is a soft singularity, because it happens for $\hat{s} = Q^2$

➡ + distribution: $\int_0^1 dz\left[\frac{\log^k(1-z)}{1-z}\right]_+ f(z) = -\int_0^1 dz\,\frac{f(z)-f(1)}{z-1}\,\log^k(1-z)$

- The partonic cross section is a distribution that needs to be convoluted with the parton luminosity!

$$\hat{\sigma}^R_{q\bar{q}} = \sigma_0\, a_s\,(4\pi)^\epsilon\, e^{\gamma_E \epsilon}\left\{\frac{4}{3\epsilon^2}\,\delta(1-z) + \frac{4}{3\epsilon}\left[1 + z - 2\left[\frac{1}{1-z}\right]_+ - \delta(1-z)\log\left(\frac{Q^2}{\mu^2}\right)\right] + \mathcal{O}(\epsilon^0)\right\}$$

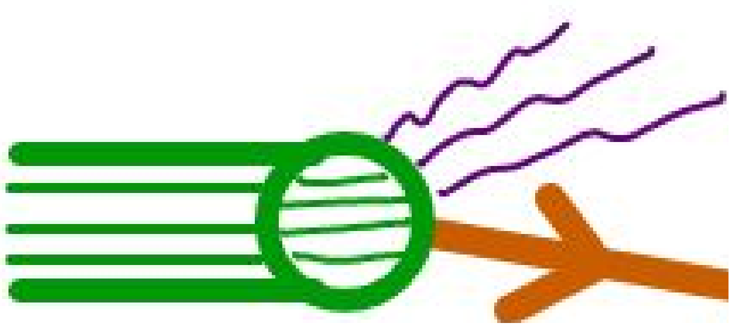
QCD @ NLO

$$\hat{\sigma}^V_{q\bar{q}} = \sigma_0\, \delta(1-z)\, a_s\, (4\pi)^\epsilon\, e^{-\gamma_E \epsilon} \left\{ \cancel{-\frac{4}{3\epsilon^2}} + \frac{1}{\epsilon}\left[\frac{4}{3}\log\left(\frac{Q^2}{\mu^2}\right) - 2\right] + \mathcal{O}(\epsilon^0) \right\}$$

$$\hat{\sigma}^R_{q\bar{q}} = \sigma_0\, a_s\, (4\pi)^\epsilon\, e^{\gamma_E \epsilon} \left\{ \cancel{\frac{4}{3\epsilon^2}}\delta(1-z) + \frac{4}{3\epsilon}\left[1 + z - 2\left[\frac{1}{1-z}\right]_+ - \delta(1-z)\log\left(\frac{Q^2}{\mu^2}\right)\right] + \mathcal{O}(\epsilon^0) \right\}$$

$$\hat{\sigma}^V_{q\bar{q}} + \hat{\sigma}^R_{q\bar{q}} = -2\sigma_0\, a_s\, (4\pi)^\epsilon\, e^{\gamma_E \epsilon}\, \frac{1}{\epsilon}\left[\frac{4}{3}\left[\frac{1}{1-z}\right]_+ + \delta(1-z) - \frac{4}{3}(1+z)\right] + \mathcal{O}(\epsilon^0)$$

$P^{(0)}_{qq}(z)$



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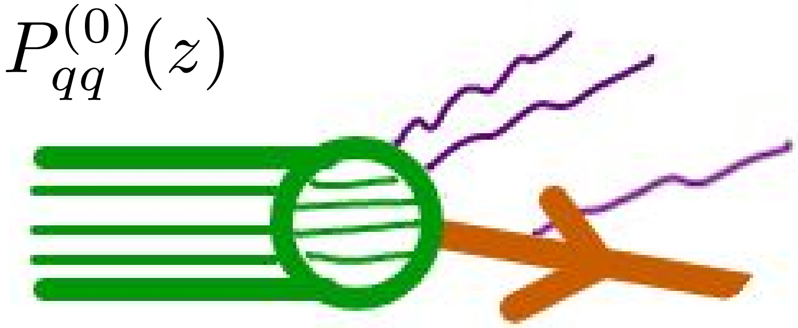


Mass factorisation

- The remaining divergence is proportional to a splitting function, and therefore process independent.
- We can 'renormalise' the PDFs such as to absorb this divergence

$$f_i^B(x) = (\Delta_{ij} \otimes f_j^R)(x) \qquad \Delta_{ij}(x, a_s) = \delta_{ij} + \frac{a_s}{\epsilon} P_{ij}^{(0)}(x) + \ldots$$

- After this procedure, the cross section is finite!
- The singularities have canceled, but there are still $\log \frac{Q^2}{\mu^2}$ the finite part.
    - ➡ After mass factorisation the cross section depends on the factorisation scale.
    - ➡ Want to choose $\mu^2 \sim Q^2$, because otherwise these logarithms will be large!

Subtraction

- If we had to work always with DimReg for the phase space, this would not be practical.
- In practise, one exploits the fact that the IR limit of tree amplitudes is universal to build universal counterterms:

$$\sigma^{NLO} = \int_n d\sigma_n^V + \int_{n+1} d\sigma_{n+1}^R$$

- If the subtraction is local in phase space, we can also do distributions.
  - ➡ Two popular schemes: Frixione-Kunszt-Signer and Catani-Seymour dipoles.

$$\frac{d\sigma^{NLO}}{d\mathcal{O}} = \int_n \left(d\sigma_n^V O_n + d\sigma_n^B O_{n+1} \int_1 \mathcal{C} \right) + \int_{n+1} \left(d\sigma_{n+1}^R - \mathcal{C}\, d\sigma_n^B \right) O_{n+1}$$

IR and collinear safety

$$\frac{d\sigma^{NLO}}{d\mathcal{O}} = \int_n \left(d\sigma_n^V \, O_n + d\sigma_n^B \, O_{n+1} \int_1 \mathcal{C} \right) + \int_{n+1} \left(d\sigma_{n+1}^R - \mathcal{C} \, d\sigma_n^B \right) \, O_{n+1}$$

- There is a mismatch in the integrand in the virtual!
  - ➡ Will in general not get finite answers for arbitrary observables!
- An observable is IR and collinear safe if

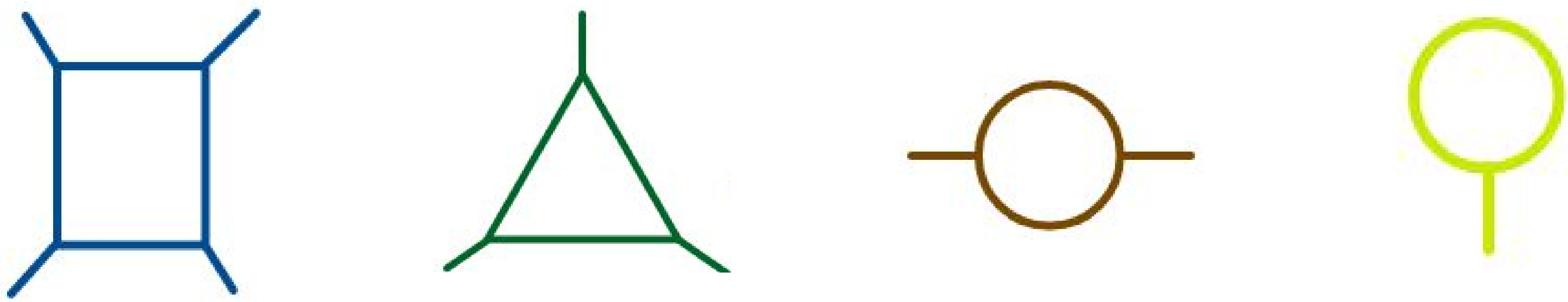
$$\lim_{i||j} O_n(p_1 \ldots p_i, p_j \ldots p_n) = O_{n-1}(p_1 \ldots p_i + p_j \ldots p_n)$$

$$\lim_{p_i \to 0} O_n(p_1 \ldots p_i \ldots p_n) = O_{n-1}(p_1 \ldots p_{i-1}, p_{i+1} \ldots p_n)$$

- If an observable is IR and collinear safe, then the previous construction gives a finite distribution.

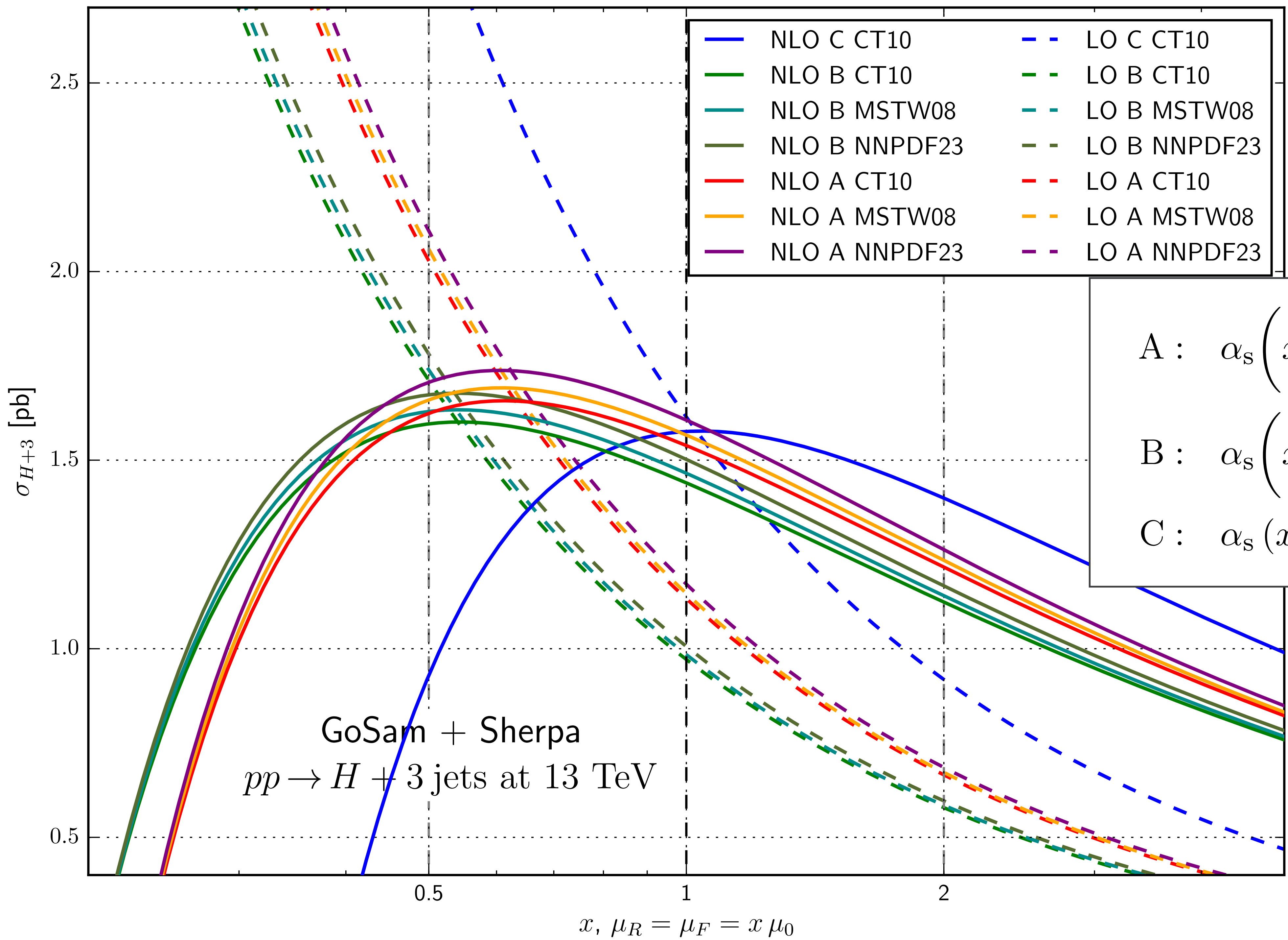
The ‘NLO revolution’

- At one-loop, we know the basis of integrals:



- A few years ago, several computer codes appeared that can compute the values of the coefficient numerically in an automated way!
  - ➡ Blackhat, Rocket, MadLoops, NJet, OpenLoops, GoSam,…
- Combined with automation for LO and FKS/CS, one can automate the whole NLO business.

H + 3j @ NLO



$$\text{A}: \quad \alpha_s\left(x \cdot \frac{\hat{H}'_T}{2}\right)^3 \alpha_s\left(x \cdot m_H\right)^2$$

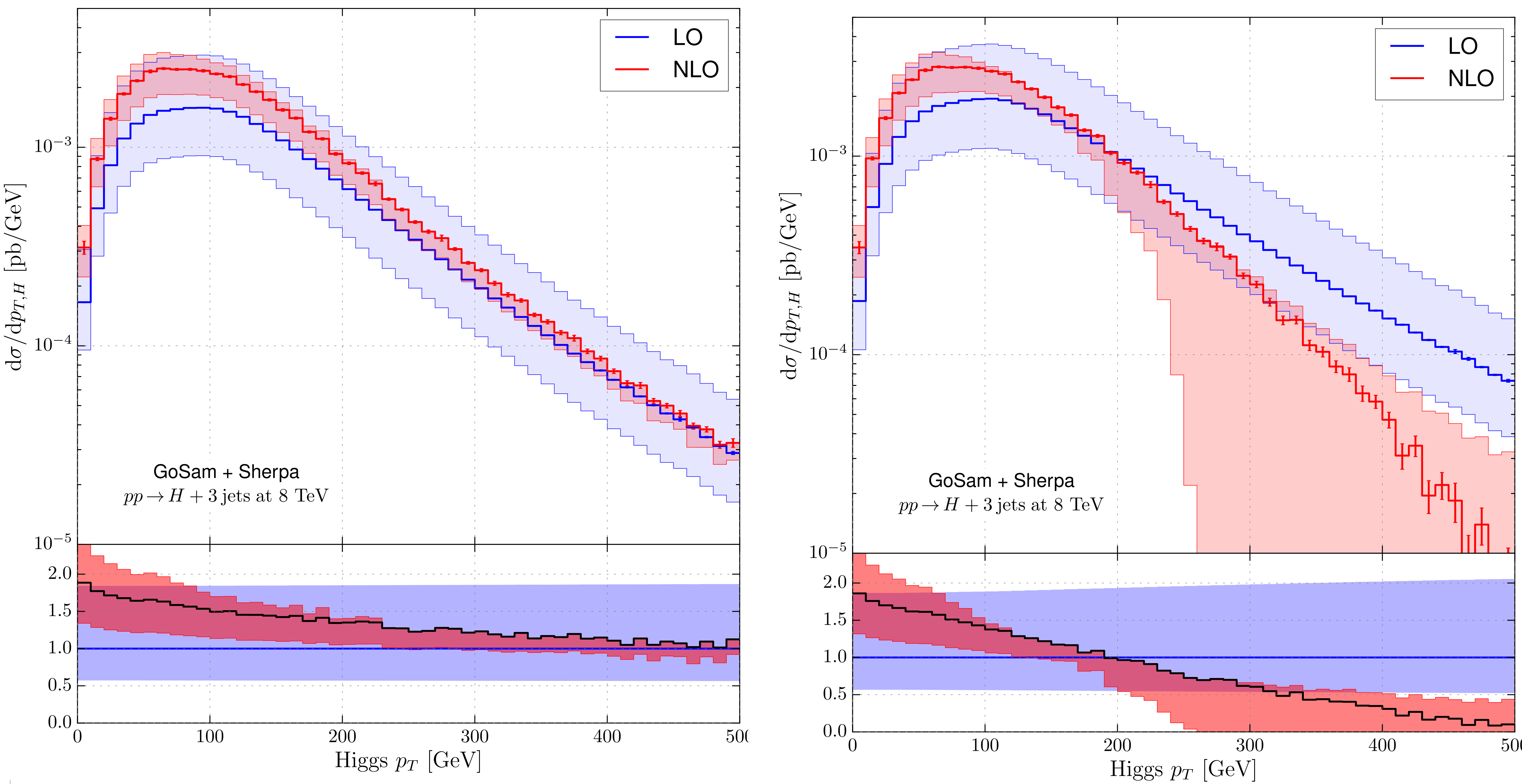
$$\text{B}: \quad \alpha_s\left(x \cdot \frac{\hat{H}'_T}{2}\right)^5$$

$$\text{C}: \quad \alpha_s\left(x \cdot m_H\right)^5 .$$

[Greiner, Hösche, Luisoni, Schönherr, Winter, Yundin]

H + 3j @ NLO

[Greiner, Hösche, Luisoni, Schönherr, Winter, Yundin]



Scale choice A

Scale choice C

NNLO and beyond

QCD @ NNLO

$$\hat{\sigma}_{ij}^{VV} = \frac{1}{2\hat{s}} \int d\Phi_n \left(2\mathrm{Re}\mathcal{M}_n^{(0)}\mathcal{M}_n^{(2)*} + |\mathcal{M}_n^{(1)}|^2\right)$$

$$\hat{\sigma}_{ij}^{RV} = \frac{1}{2\hat{s}} \int d\Phi_{n+1}\, 2\mathrm{Re}\mathcal{M}_{n+1}^{(0)}\mathcal{M}_{n+1}^{(1)*}$$

$$\hat{\sigma}_{ij}^{RR} = \frac{1}{2\hat{s}} \int d\Phi_{n+2}\, |\mathcal{M}_{n+2}^{(0)}|^2$$

- In principle, the whole story generalises in a straightforward manner.
- In practise, there is a huge jump in complexity when going from NLO to NNLO.

Virtual corrections

- At one loop, we know a complete basis of integrals in 4 dimensions.
- At two loops, we only know very few and specific integrals.
    - ➡ 2-to-2 massless, 2 scales (e.g. dijets): ~1999
    - ➡ 2-to-2 one leg off shell, 3 scales (e.g. Z+j): ~2000-01
    - ➡ 2-to-2 two legs off shell, 4 scales (e.g. ZZ): ~2014
    - ➡ We do not know any two-loop 2-to-3 integrals…
- Apart from these integrals, we only know a few very specific two-loop integrals:
    - ➡ ttbar (num.)
    - ➡ some integrals for electroweak corrections

Virtual corrections

- At one loop, all integrals can be expressed in terms of logarithms and dilogarithms:

$$\text{Li}_2(z) = -\int_0^z \frac{dt}{t} \log(1-t)$$

- Beyond one loop, more general functions appear:
  - ➡ multiple polylogarithms:

$$G(a_1,\ldots,a_n;z) = \int_0^z \frac{dt}{t-a_1} G(a_2,\ldots,a_n;t)$$

$$G(a;z) = \log\left(1-\frac{z}{a}\right) \qquad G(0,1;z) = -\text{Li}_2(z)$$

  - ➡ elliptic polylogarithms:

$$\mathcal{L}_n(z,q) = \sum_{k=-\infty}^{\infty} \text{Li}_n(z\,q^k) \qquad q = e^{2\pi i\tau}$$

Virtual corrections

- Beyond one loop, we do not know the basis of integrals.
  - ➡ We would still like to find a minimal set of integrals that we need to compute!
- Consider Feynman integrals with arbitrary powers of the propagators:

$$I(n_1,\ldots,n_k) = \int \frac{d^Dk_1\ldots d^Dk_L}{D_1^{n_1}\ldots D_k^{n_k}} \qquad n_i \in \mathbb{Z}$$

  - ➡ defines a function on a lattice $\mathbb{Z}^k$.
- Goal:
  - ➡ Find recursion relations on this lattice.
  - ➡ Solve these recursions to express all integrals in terms of a small set of 'master integrals'.

IBP identities

- We can shift exponents by differentiation!

$$\frac{\partial}{\partial k_i^\mu}\frac{1}{[(k_i+p)^2]^n} = -2n\,\frac{k_{i\mu}+p_\mu}{[(k_i+p)^2]^{n+1}}$$

- What is the rhs of the recursion...?

$$0 = \int d^D k_i \frac{\partial}{\partial k_i^\mu}\Big(\dots\Big)$$

- Theorem:

  In DimReg, integrals of total derivatives always vanish.

- N.B.:

  For recursions to close, we must be able to express all scalar products in terms of denominators (=topology).

IBP identities

- Example:

$$\mathrm{Bub}(n_1, n_2) = \int \frac{d^D k}{[k^2]^{n_1} [(k+p)^2]^{n_2}}$$

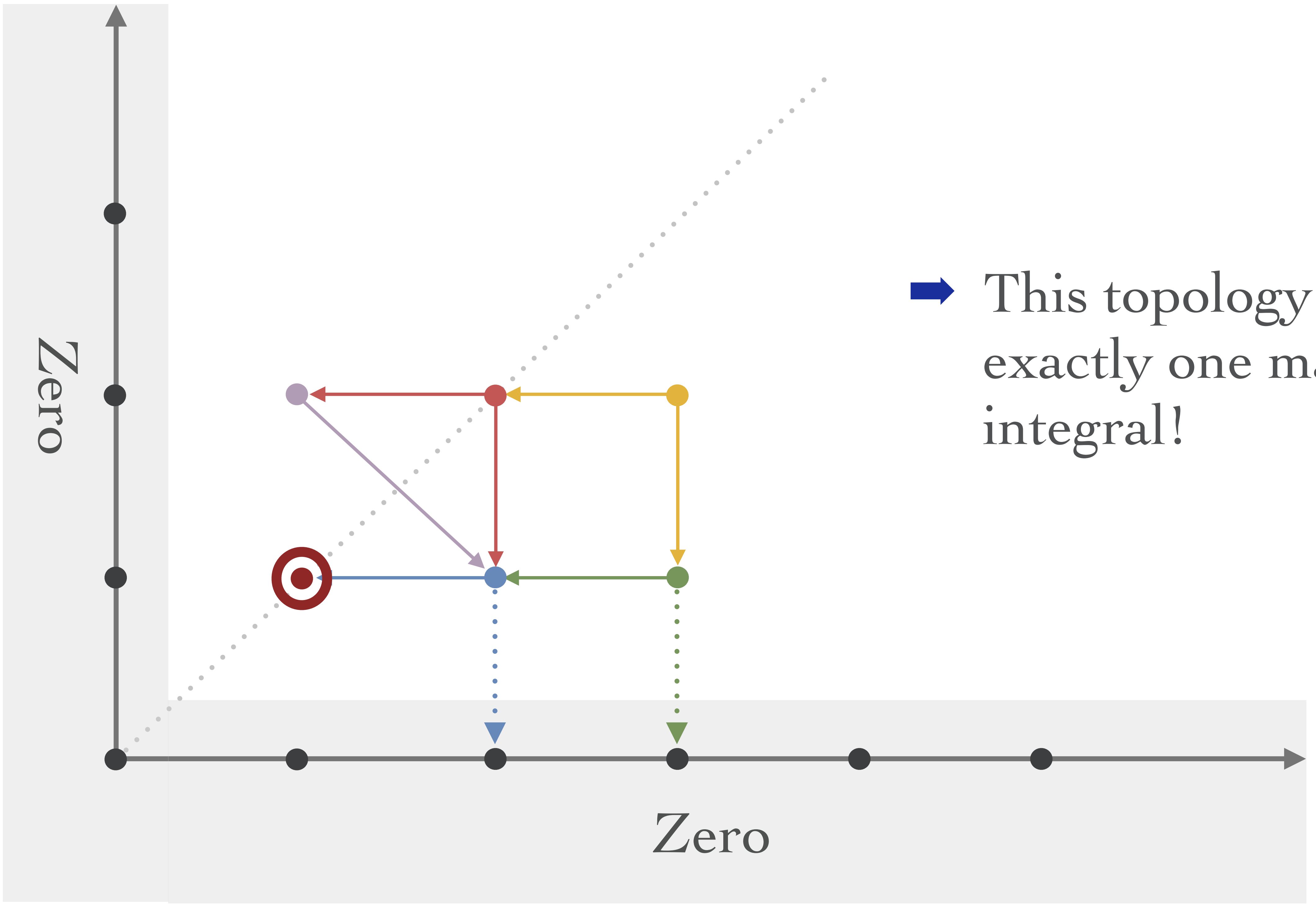
➡ IBP relations:

$$0 = \int d^D k \frac{\partial}{\partial k^\mu} k^\mu \left(\dots \right) \qquad\qquad 0 = \int d^D k \frac{\partial}{\partial k^\mu} p^\mu \left(\dots \right)$$

$$\begin{aligned} \mathrm{Bub}(n_1, n_2) &= \frac{n_1 + n_2 - 1 - D}{p^2 \left(n_2 - 1 \right)} \mathrm{Bub}(n_1, n_2 - 1) + \frac{1}{p^2} \mathrm{Bub}(n_1 - 1, n_2) \\ &= \frac{1}{p^2} \mathrm{Bub}(n_1, n_2 - 1) + \frac{n_1 + n_2 - 1 - D}{p^2 \left(n_1 - 1 \right)} \mathrm{Bub}(n_1 - 1, n_2) \end{aligned}$$

➡ N.B.: The integral vanishes unless $n_1, n_2 > 0$.

IBP identities



➡ This topology has exactly one master integral!

IBP identities

- Solving the recursions in the general case with many propagators is rather cumbersome.

- Other approach: Use the recursion relations to generate linear relations among integrals, and truncate the tower of relations.
  - ➡ Turns recursion relations into a finite-sized linear system.
  - ➡ Laporta's algorithm.

- There are several public (and private) computer codes that allow one to solve IBP relations.
  - ➡ FIRE, Reduze, LiteRed,…

Differential equations

- We can also differentiate a master integral w.r.t an external scale, e.g.

$$\frac{\partial}{\partial m_i^2} \frac{1}{[q_i^2 - m_i^2]^{n_i}} = \frac{n_i}{[q_i^2 - m_i^2]^{n_i+1}}$$

- We can IBP-reduce the lhs to master integrals.

- Conclusion:

  Master integrals satisfy systems of 1st order DEs among themselves!

- This gives an effective way to compute the master integrals.

IR divergences @ NNLO

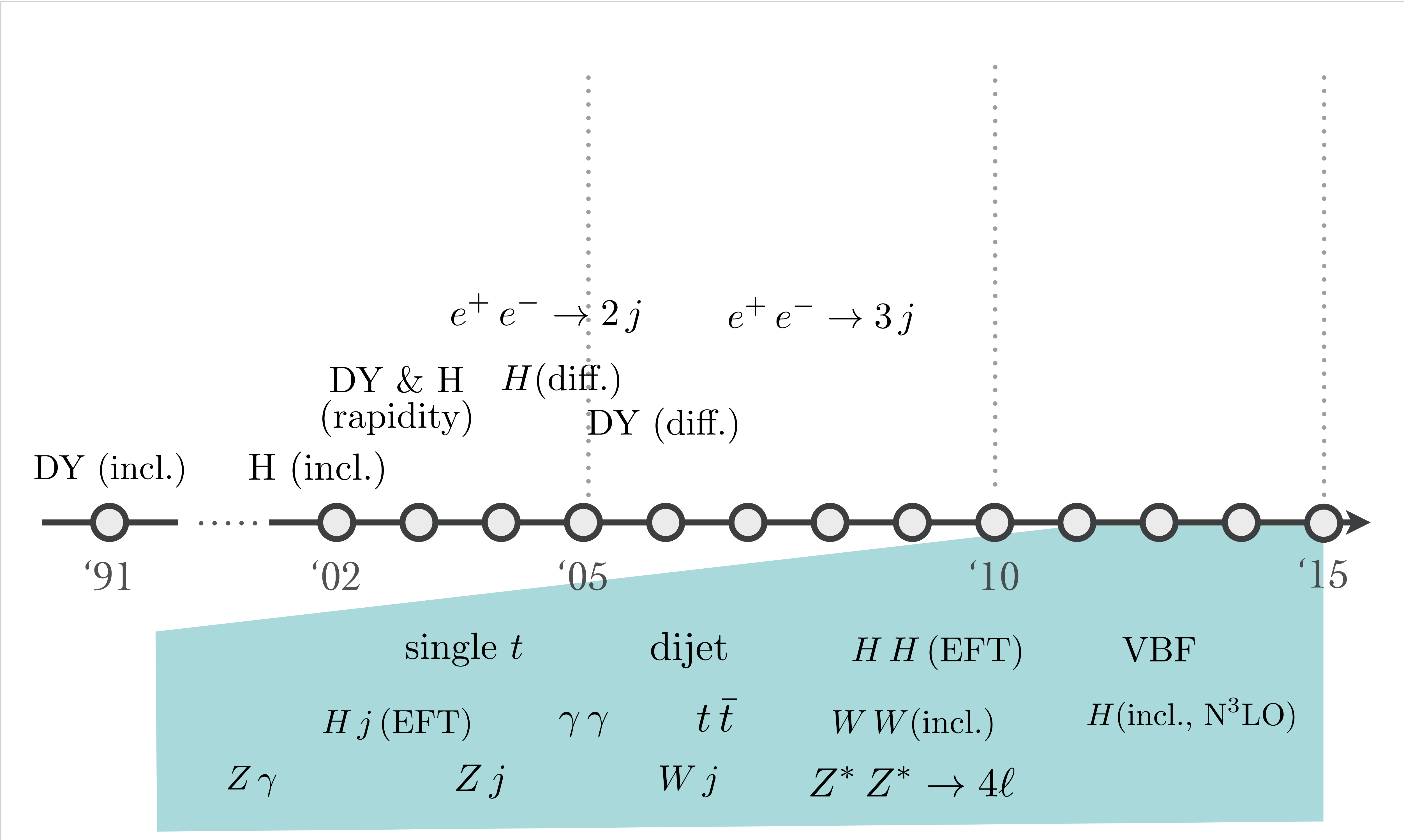
- Even if we can compute the virtual amplitudes, we still need to combine them with the real radiation contributions.

- We do not have a fully general subtraction scheme as we have at NLO, but a lot of progress in the last years:
  - ➡ Antenna subtraction. [Gehrmann, Gehrmann-de Ridder, Glover]
  - ➡ qT subtraction. [Catani, Grazzini]
  - ➡ Colourful NNLO [Somogyi, Tróscányi]
  - ➡ Stripper. [Czakon]
  - ➡ N-jettiness subtraction. [Boughezal, Focke, Liu, Petriello; Gaunt, Stahlhofen, Tackmann, Walsh]

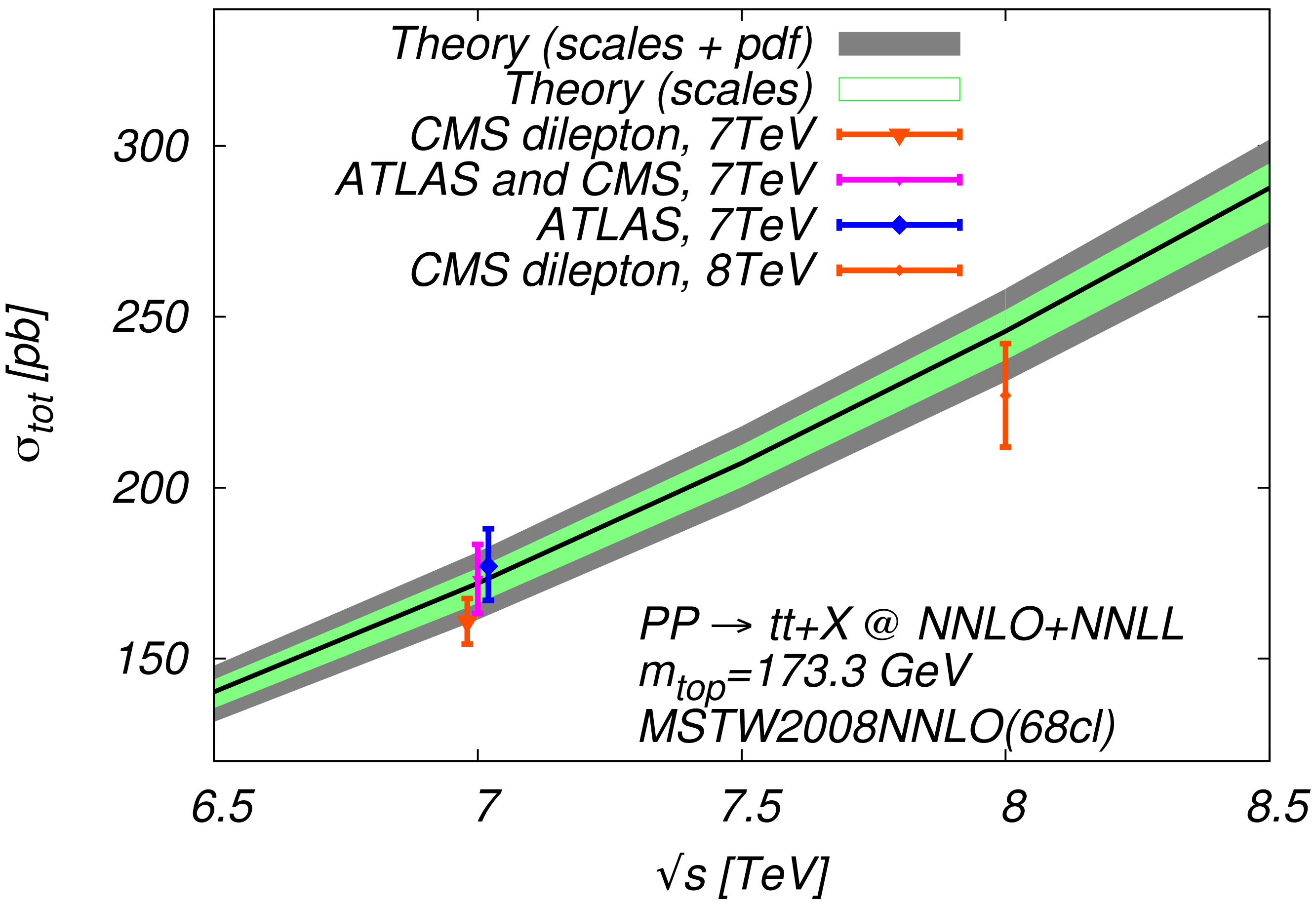
IR divergences @ NNLO

	Analytic	FS Colour	IS Colour	Local
Antenna	✓	✓	✓	✗
qT	✓	✗	✓	✓
Colourful	✓	✓	✗	✓
Stripper	✗	✓	✓	✓
N-jettiness	✓	✓	✓	✓

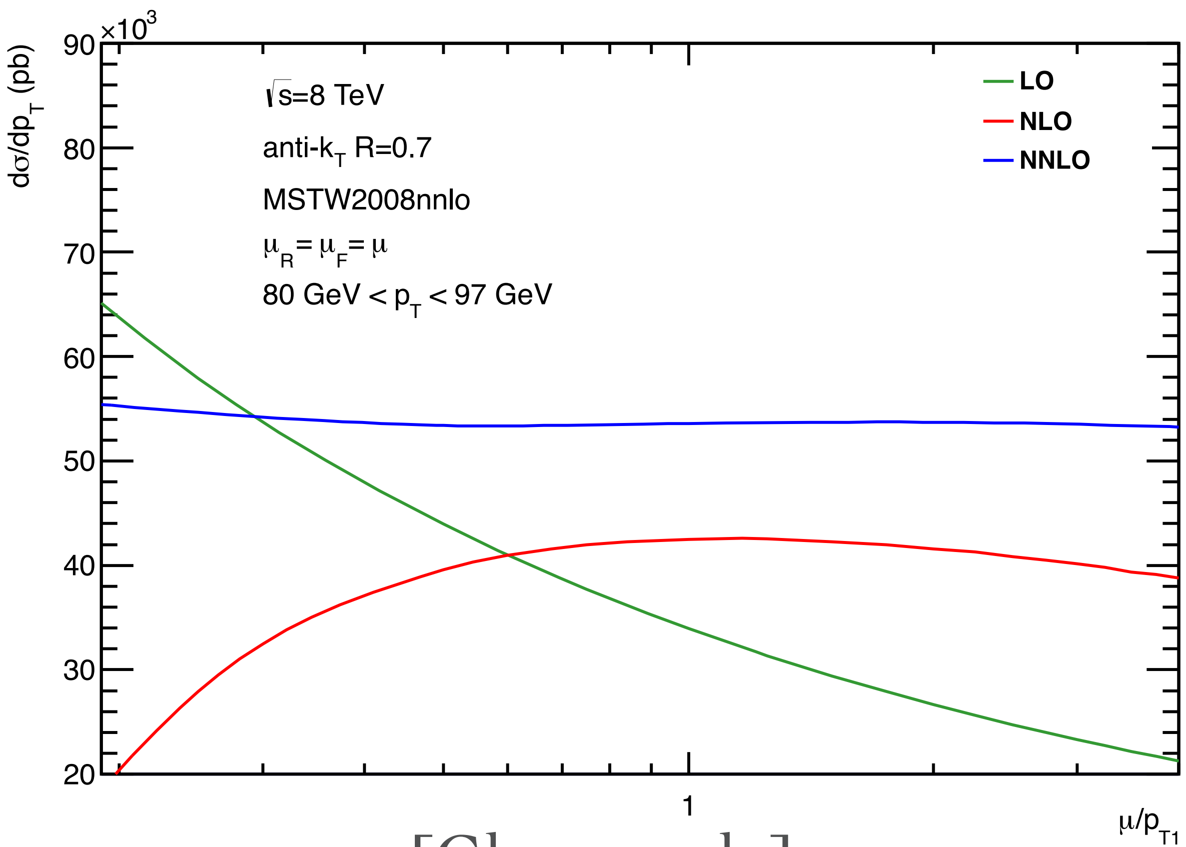
QCD @ higher orders



QCD @ higher orders



[Czakon, Fiedler, Mitov]



[Gluons only]

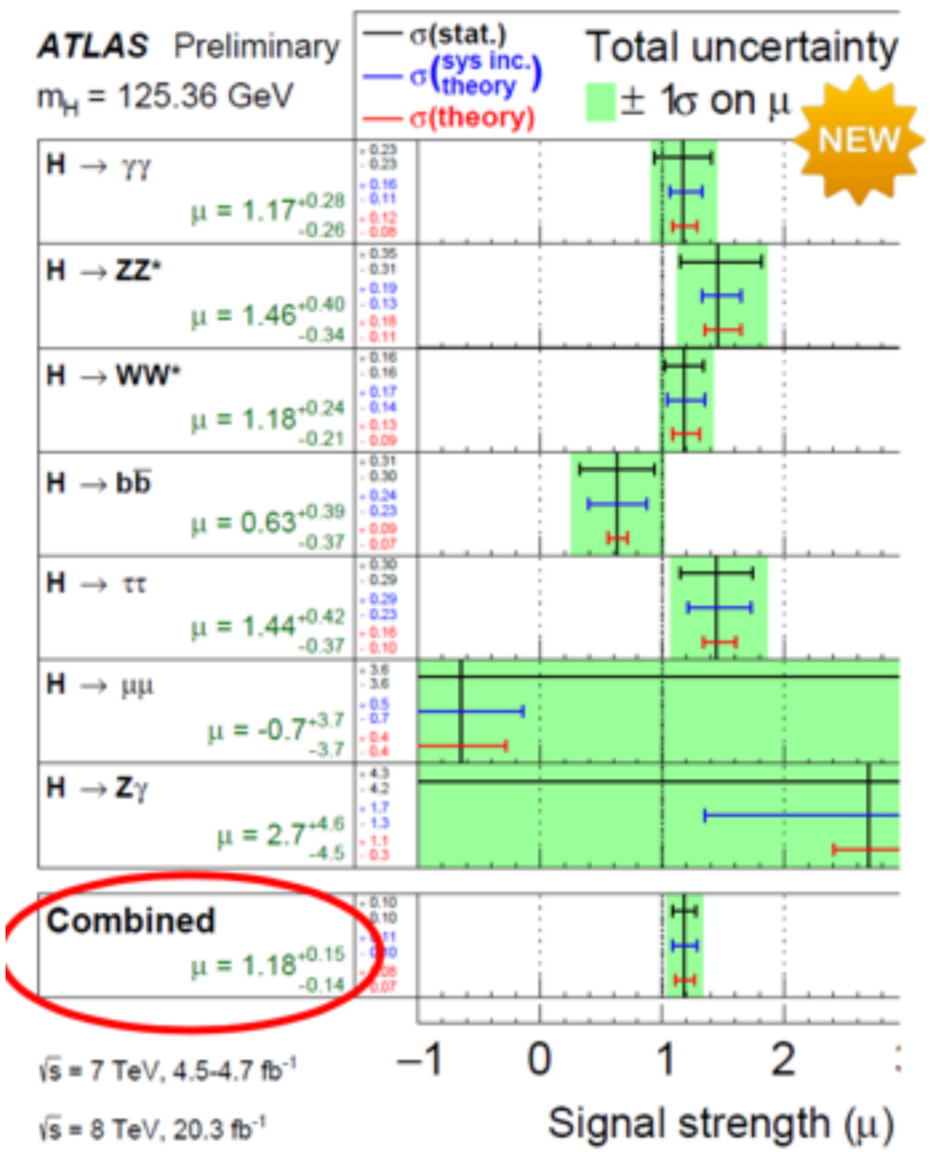
[Currie, Glover, Gehrmann, Gehrmann-de Ridder, Pries, Wells]

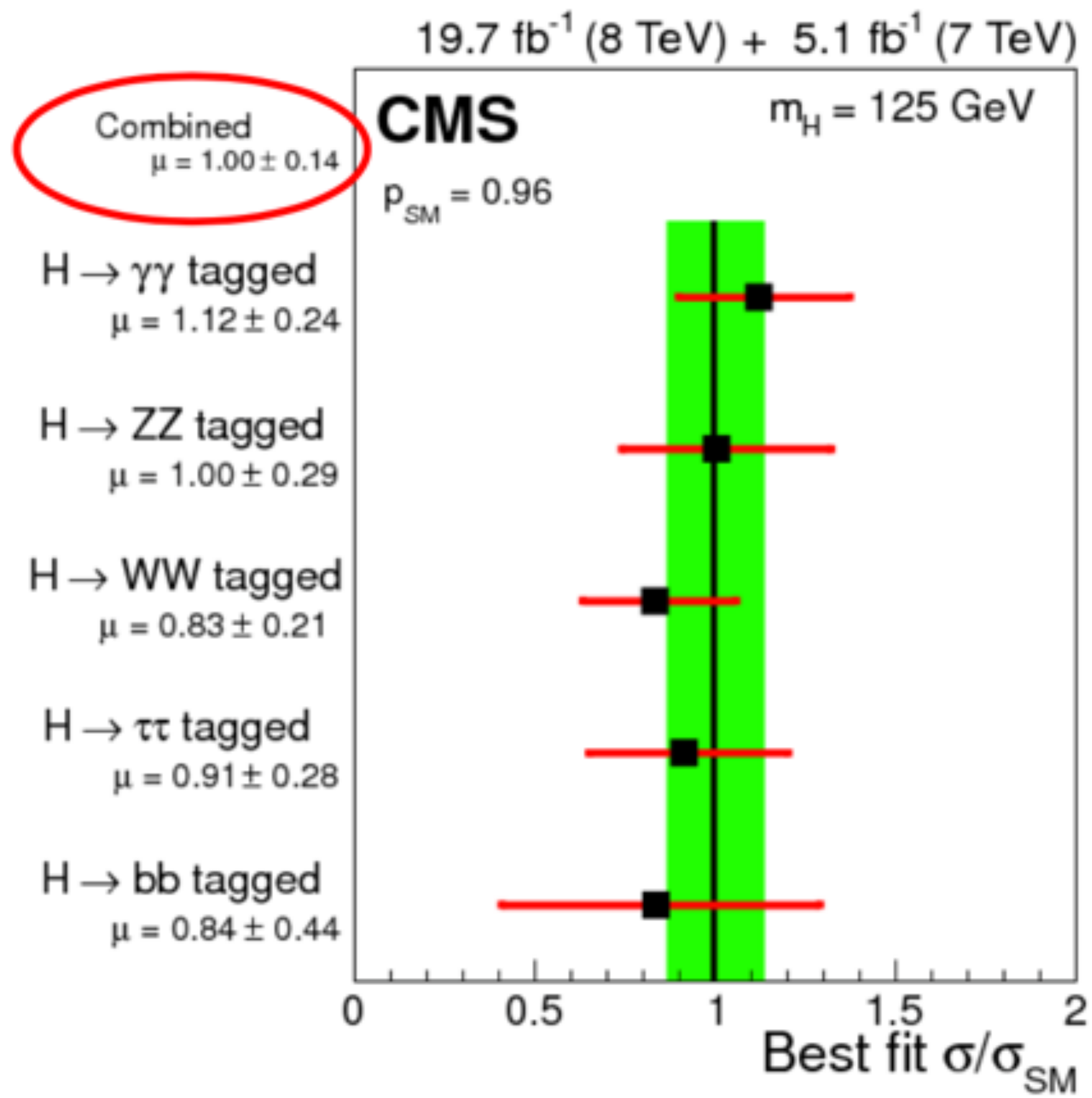
QCD for Higgs physics

QCD for Higgs physics

- So far all considerations were generic and apply to arbitrary LHC processes.
- The biggest success of the LHC Run I was the discovery of a resonance which looks very much like the SM Higgs boson.
- Studying the properties of this new particle is of outmost importance for Run II.
    - ➡ Coupling measurements.
    - ➡ Total and differential cross sections.
    - ➡ SM vs. BSM?
- Aim: Use the concepts of the 1st part of the lecture to make precise predictions for SM Higgs physics at the LHC.

Higgs physics at the LHC





$$\mu_{ATLAS} = 1.18^{+0.15}_{-0.14}$$

$$\mu_{CMS} = 1.00 \pm 0.14$$

$$\text{stat.} = {}^{+0.10}_{-0.10}$$

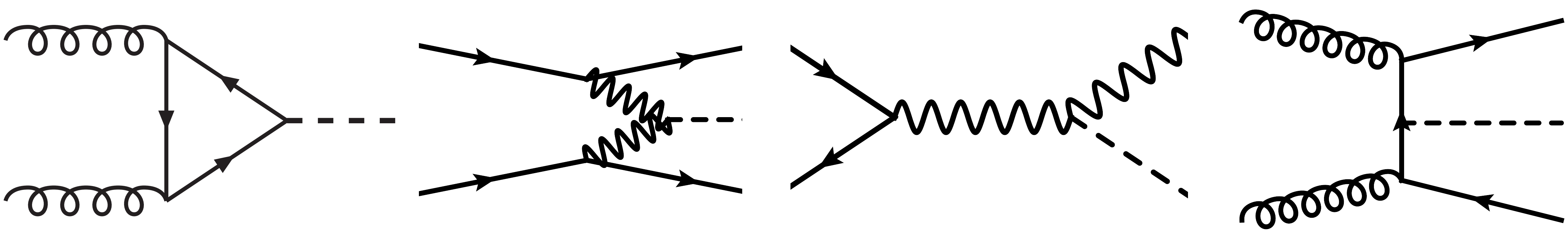
$$\text{sys. (inc. theo.)} = {}^{+0.11}_{-0.10}$$

$$\text{theory} = {}^{+0.08}_{-0.07}$$

[M. Dührssen @ Moriond EW 2015]

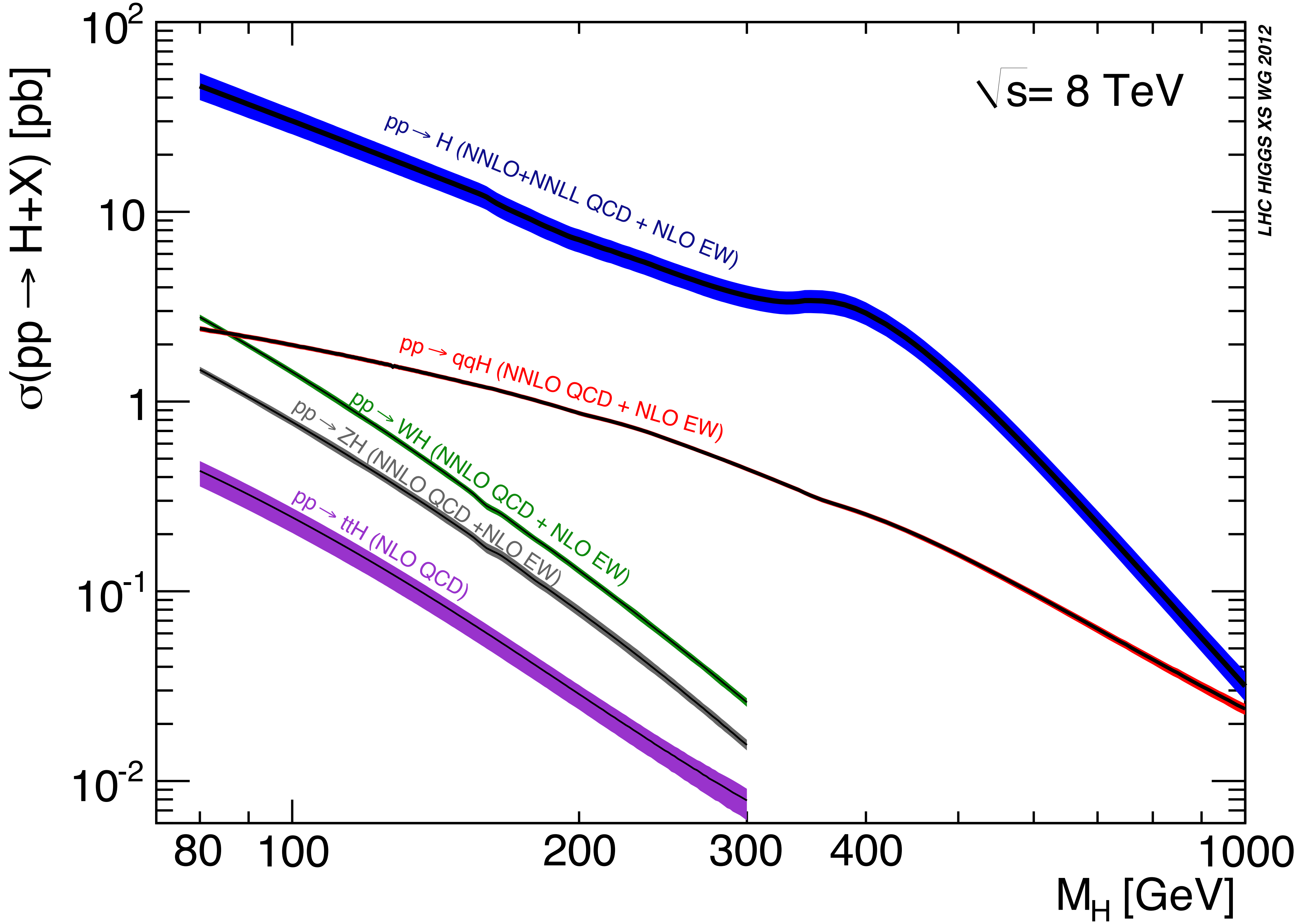
Higgs physics at the LHC

- Higgs-boson production modes at the LHC:



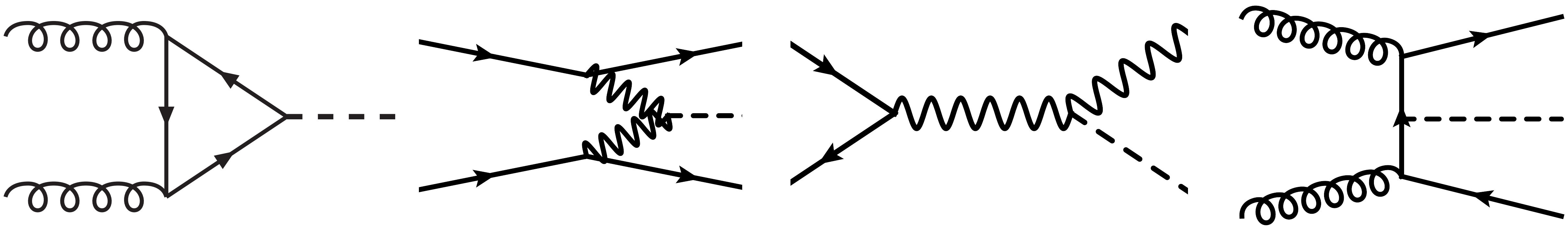
Gluon fusion | VBF | Higgs strahlung | TTH

Higgs physics at the LHC



Higgs physics at the LHC

- Higgs-boson production modes at the LHC:

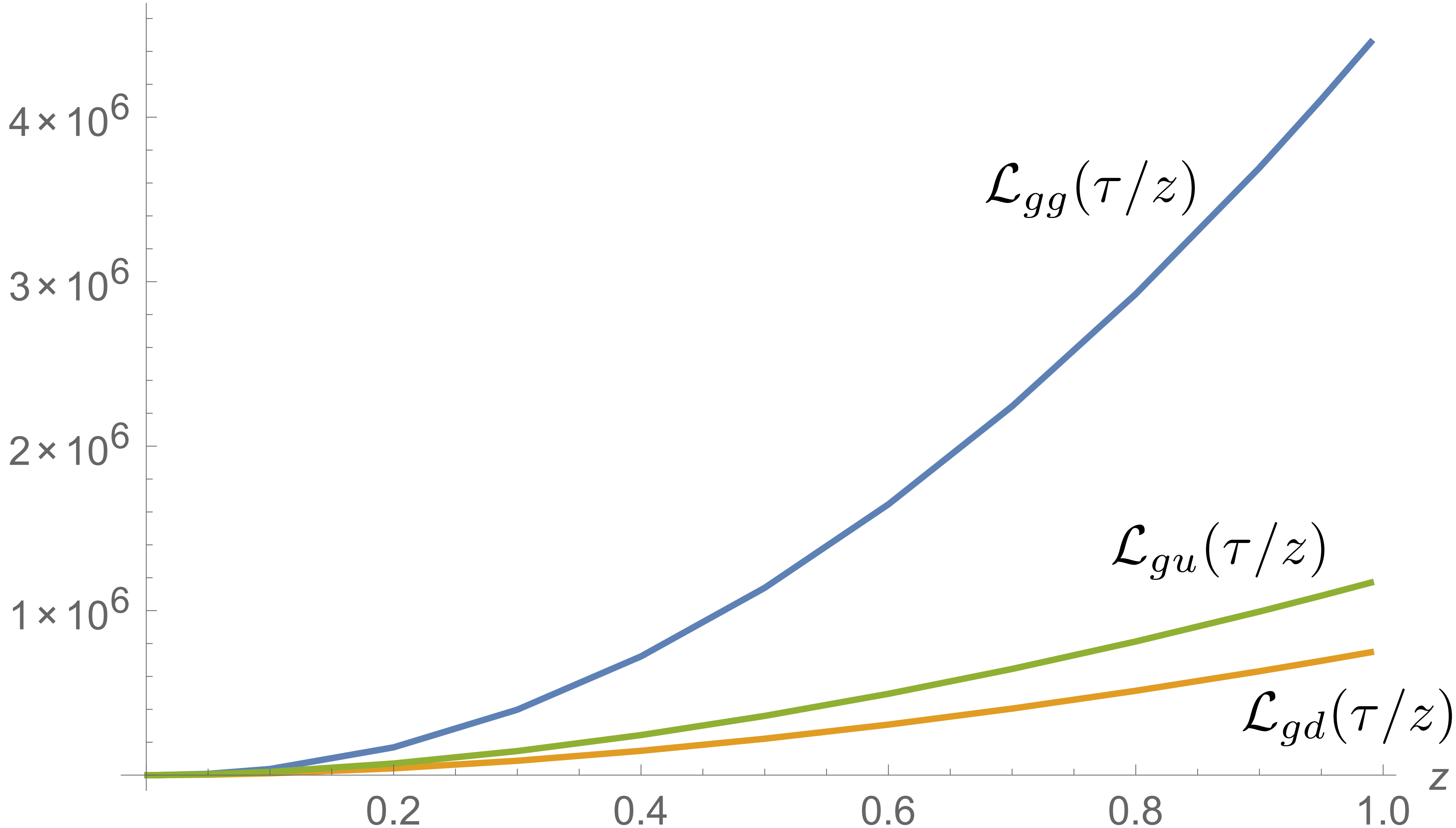


Gluon fusion VBF Higgs strahlung TTH

- Gluon fusion dominates, followed by VBF.

Higgs physics at the LHC

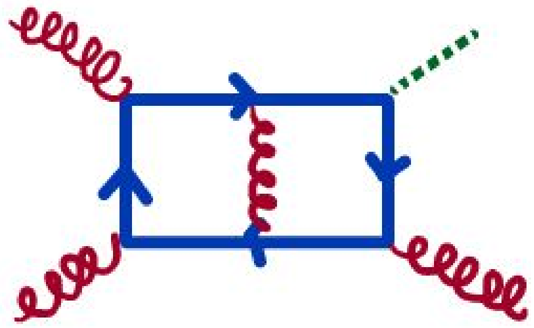
$$\sigma = \tau \sum_{ij} \int_{\tau}^{1} \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) \frac{\hat{\sigma}_{ij}(z)}{z} \qquad z = \frac{m_H^2}{\hat{s}} \qquad \tau = \frac{m_H^2}{S} \simeq 10^{-4}$$



The gluon fusion process

The gluon fusion cross section

- Gluon-fusion is a loop-induced process.
  - ➡ LO is one loop.
  - ➡ NLO is two loops.
  - ➡ etc.
- At NNLO, need double box with top-quark loop!
  - ➡ Currently unknown.



- Luckily, the Higgs boson is lighter than the top-pair threshold.
  - ➡ Try to integrate out the top quark and work with an effective theory

The large mt limit

$$\hat{\sigma}_{gg}(\hat{s}, m_H^2, m_t^2, \alpha_s) = \sum_{\ell=2}^{\infty} \sum_{k=1}^{\infty} \frac{\alpha_s^\ell}{m_t^k} \hat{\sigma}_{\ell,k}(\hat{s}, m_H^2) \qquad m_H^2 \ll 4m_t^2$$

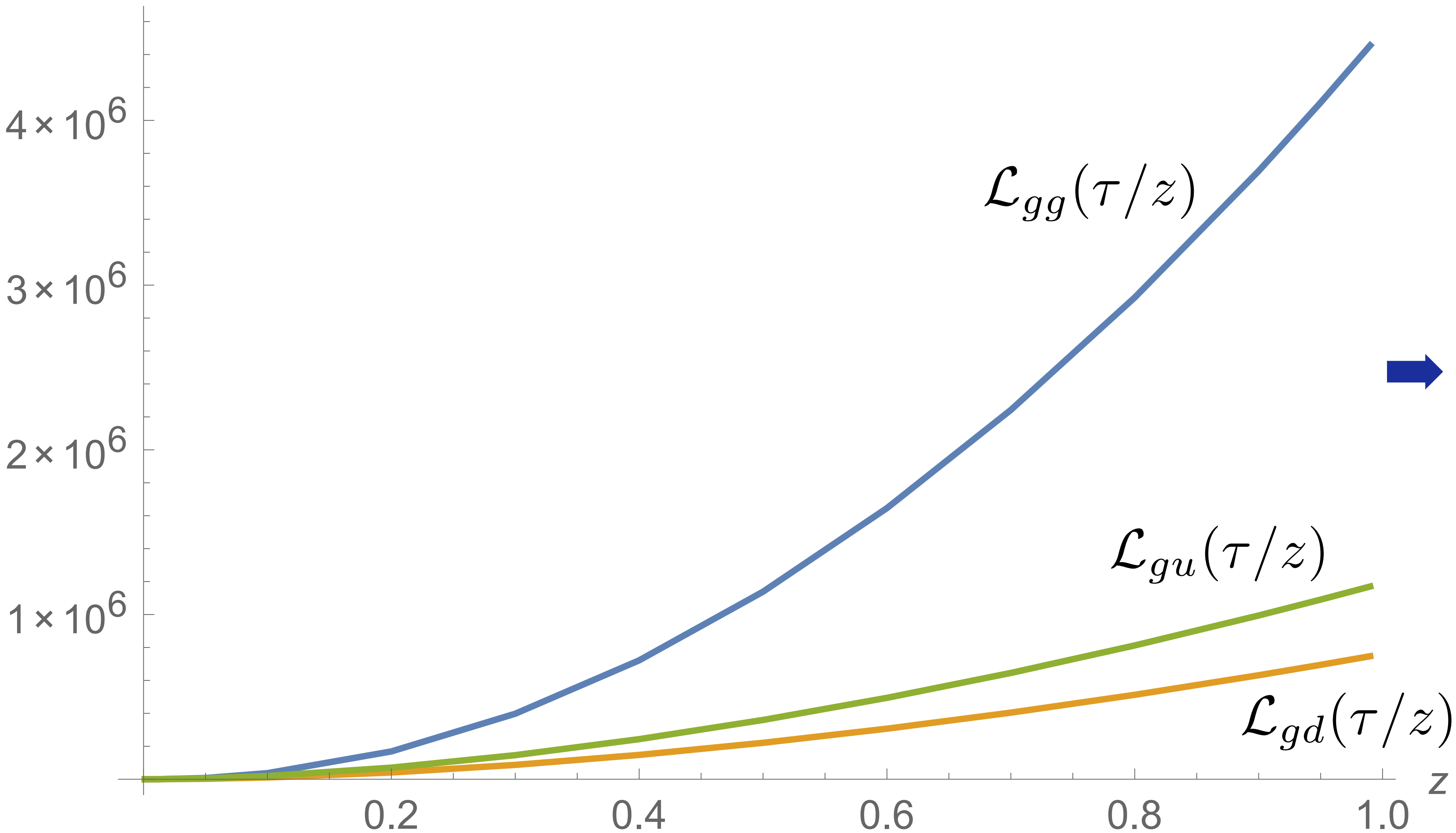
- Effective field theory approach:

$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 \, H \, G_{\mu\nu}^a \, G_a^{\mu\nu} + \ldots$$

- How well can this work?
  - ➡ Higgs mass is not that much below top mass.
- Real radiation could spoil this naive picture:
  - ➡ Hard gluon emissions at $4m_t^2 \simeq \hat{s} > m_H^2$ beyond leading order!

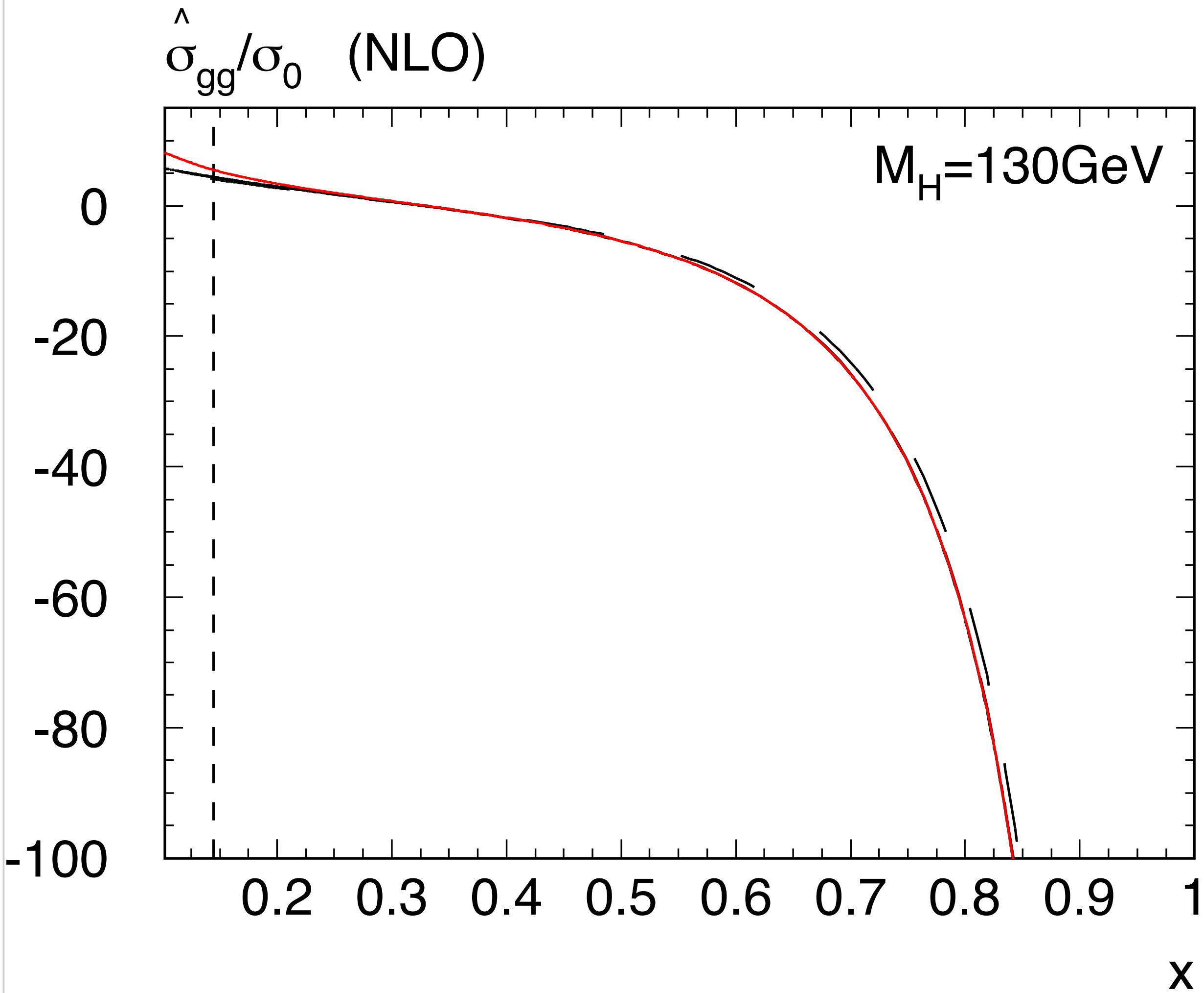
The large mt limit

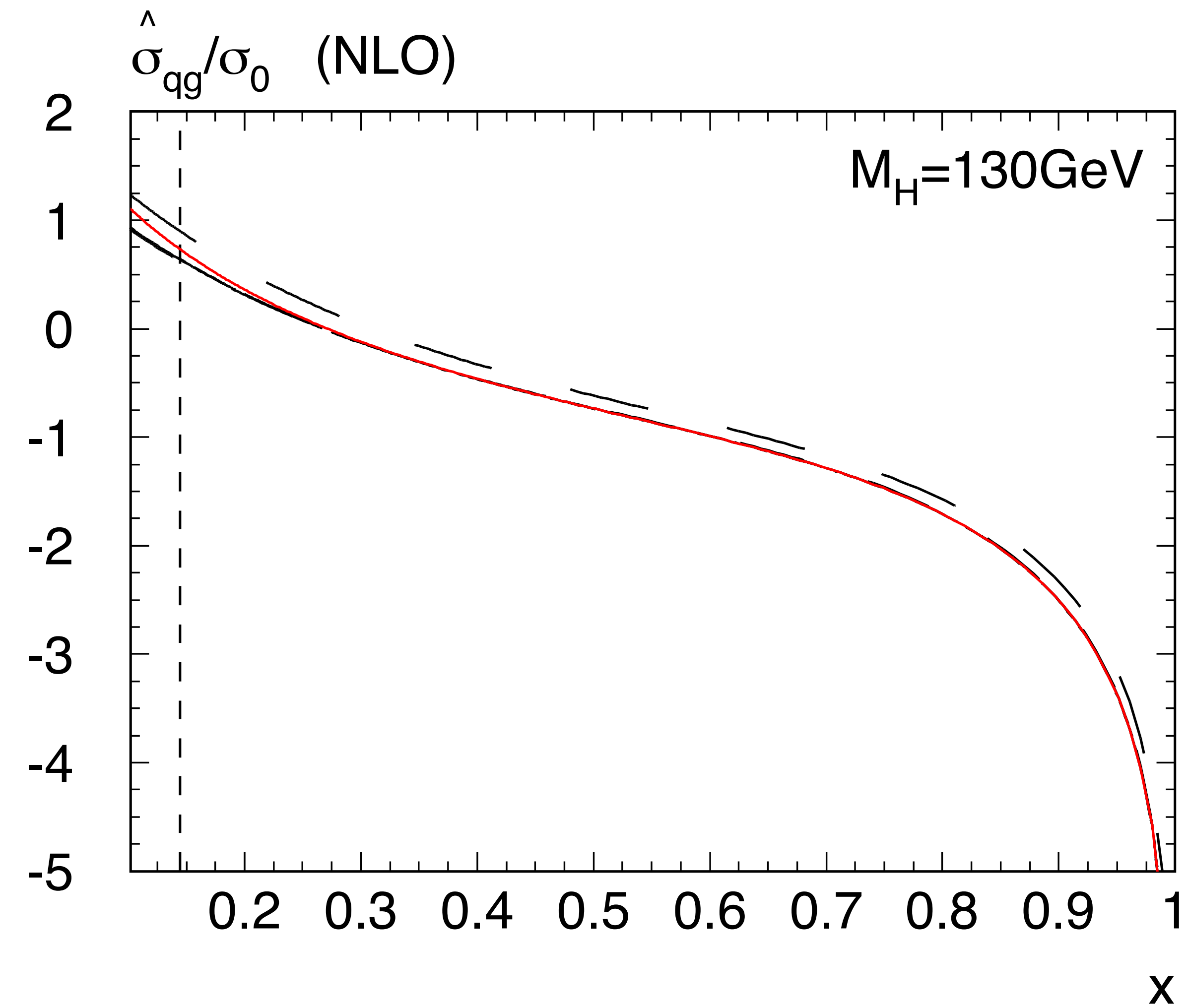
$$\sigma = \tau \sum_{ij} \int_\tau^1 \frac{dz}{z} \mathcal{L}_{ij}(\tau/z) \frac{\hat{\sigma}_{ij}(z)}{z} \qquad z = \frac{m_H^2}{\hat{s}} \qquad \tau = \frac{m_H^2}{S} \simeq 10^{-4}$$



- ➡ Main contribution from region where $z \simeq 1$.
- ➡ Physically: production at threshold + emission of soft partons.

The large mt limit





The gluon fusion cross section

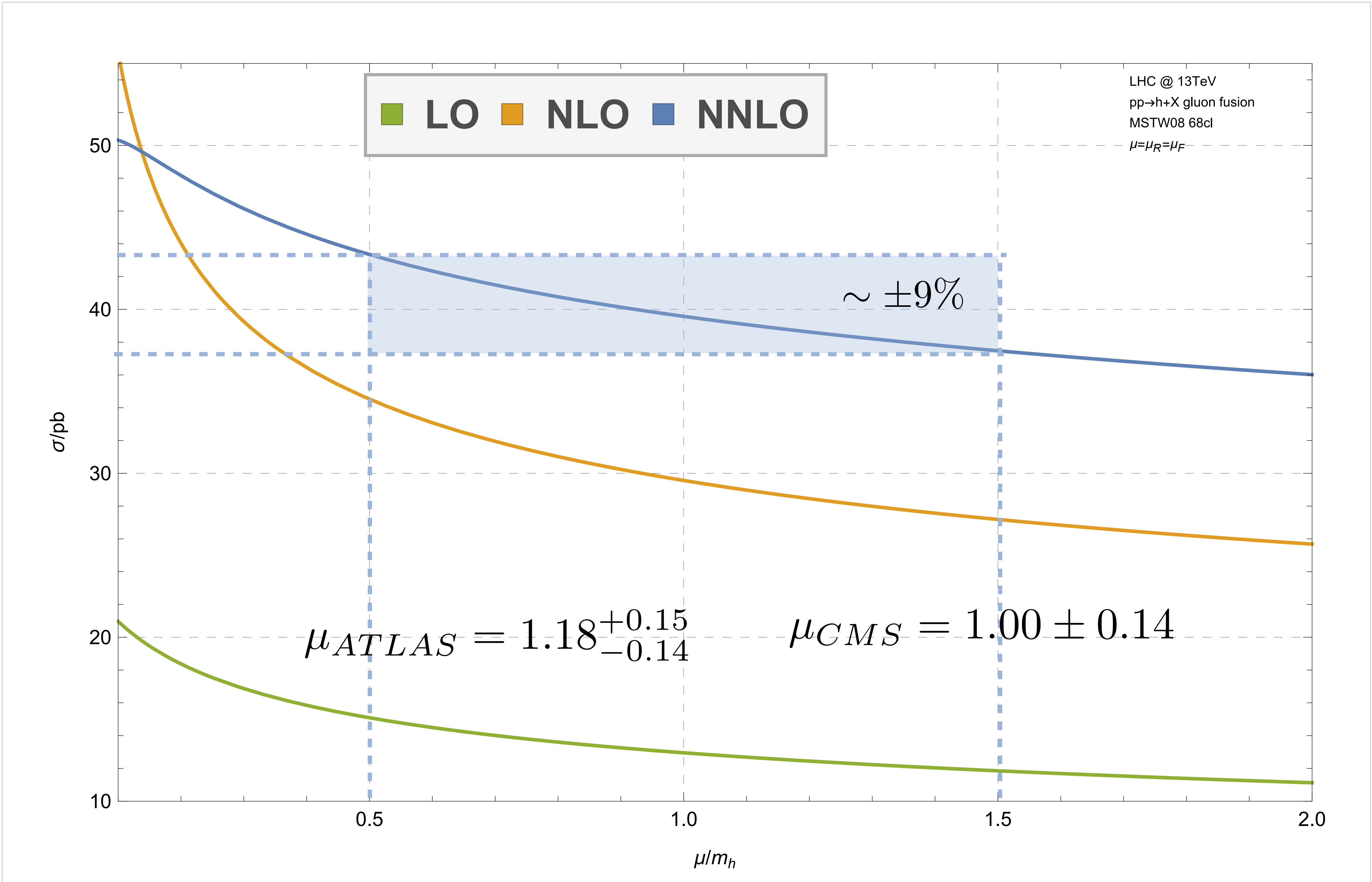
- The inclusive gluon-fusion cross section was computed
  - ➡ at LO and NLO. [Dawson; Djouadi, Graudenz,
  - ➡ at NNLO in the large mt EFT, including 1/mt corrections.

    [Harlander, Kilgore; Anastasiou, Melnikov; Ravindran, Smith, van Neerven
    Harlander, Mantler, Marzani, Ozeren]
  - ➡ at N3LO in the large mt EFT.

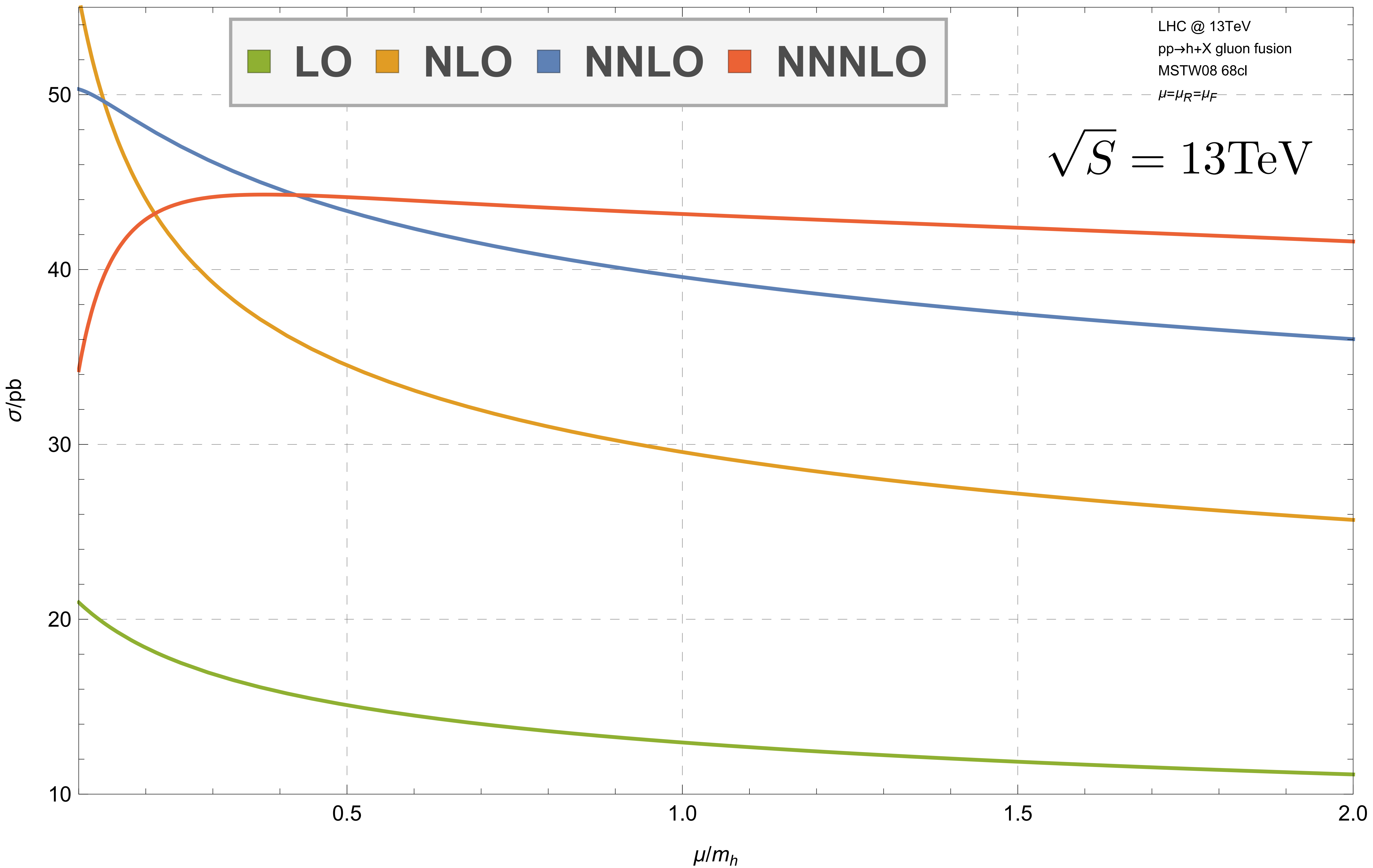
    [Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger
- 1/mt corrections at NNLO were found to be very small.

  [Harlander, Ozeren; Pak, Rogal, Steinhauser; Ball, Del Duca, Marzani, Forte, Vicini; Harlander, Mantler, Marzani, Ozeren]
- What motivated such a high order computation?

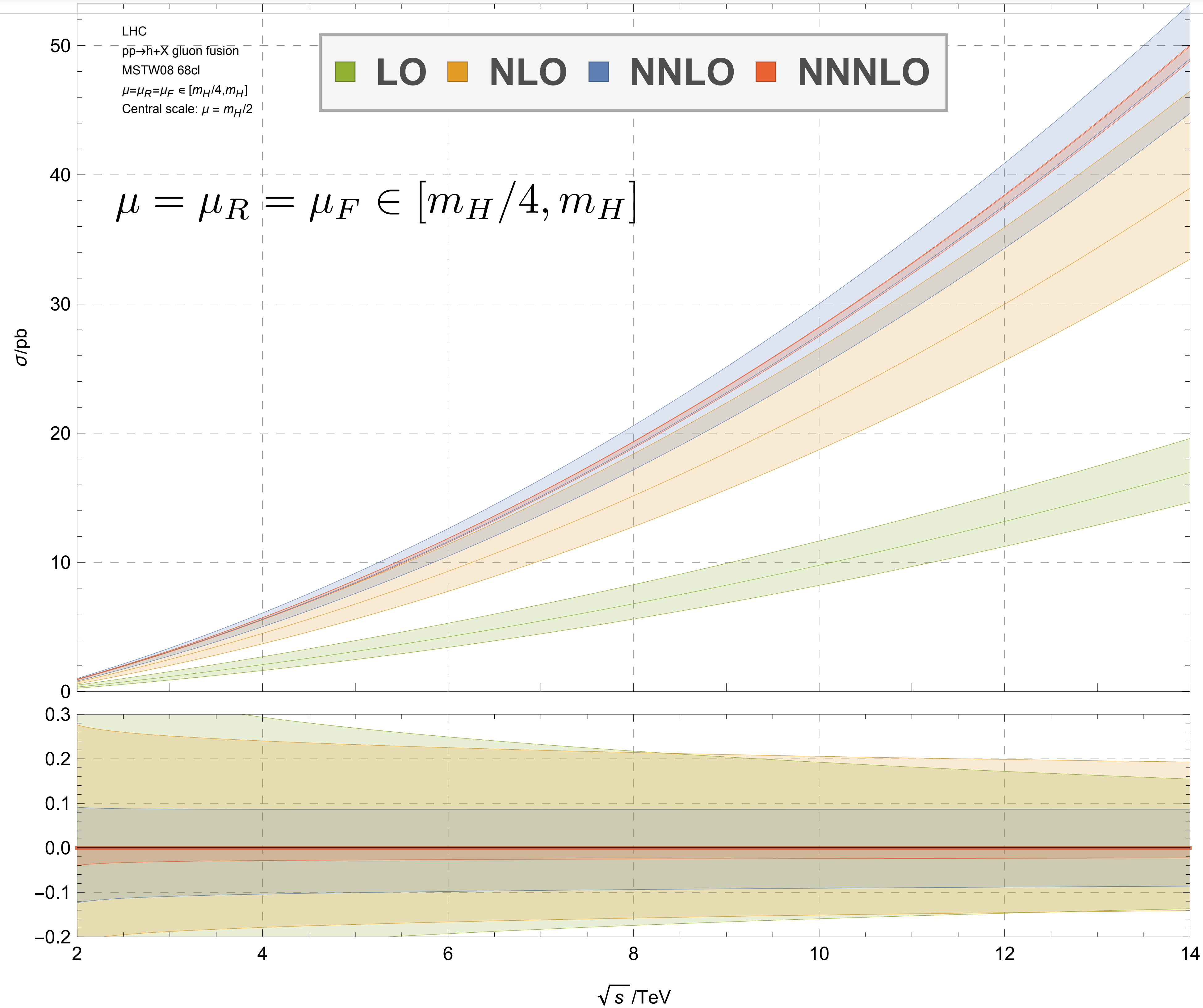
The gluon fusion cross section



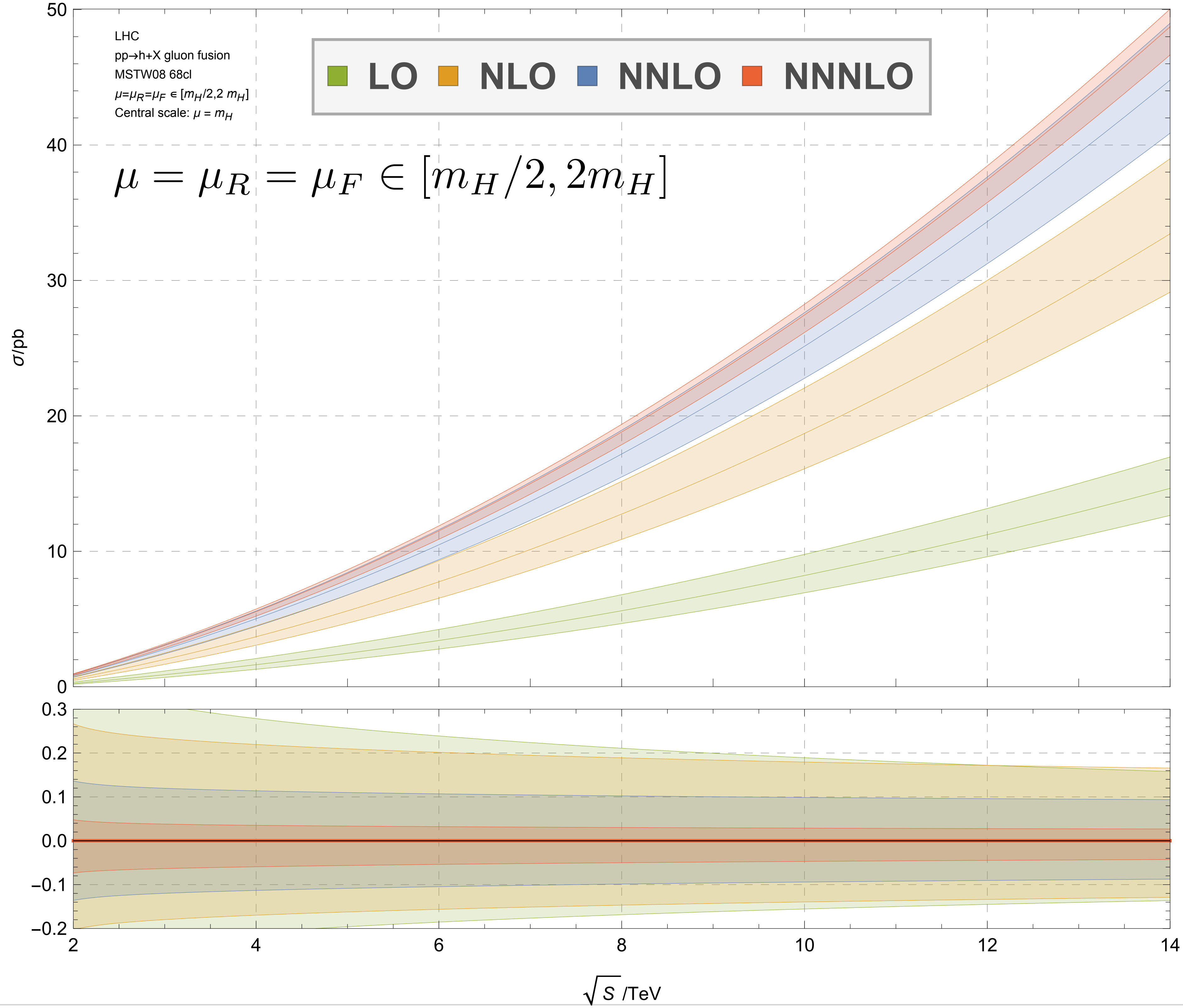
Scale variation



Energy variation



Energy variation

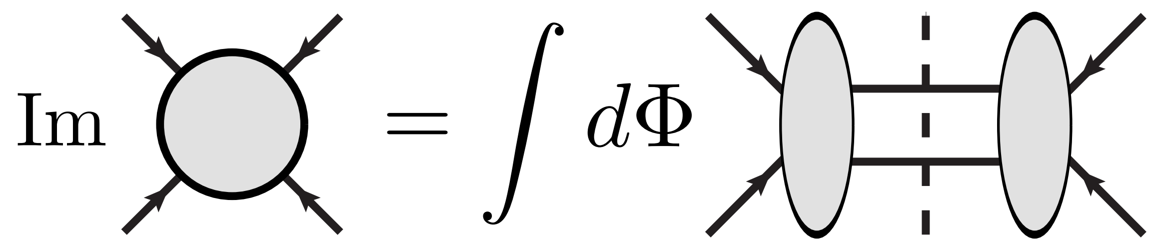


NNLO and N3LO

- How to perform such a high-order computation..?
- In principle: Can proceed in exactly the same way as at for LO and NLO discussed in the first part of the lecture.
  - ➡ Parametrise D-dimensional phase space.
  - ➡ IR singularities show up as poles in epsilon.
- In practise: This is not feasible, because we have to deal with multi-body phase space integrals.
  - ➡ Phase space parametrisations are not really suitable for analytic integration
  - ➡ Need some new technology

Reverse - Unitarity

- Optical theorem:



  ➡ Discontinuities of amplitudes are phase-

- Discontinuities of loop integrals are given by Cutkosky's rule:

$$\frac{1}{p^2 - m^2 + i\varepsilon} \to \delta_+(p^2 - m^2) = \delta(p^2 - m^2)\,\theta(p^0)$$

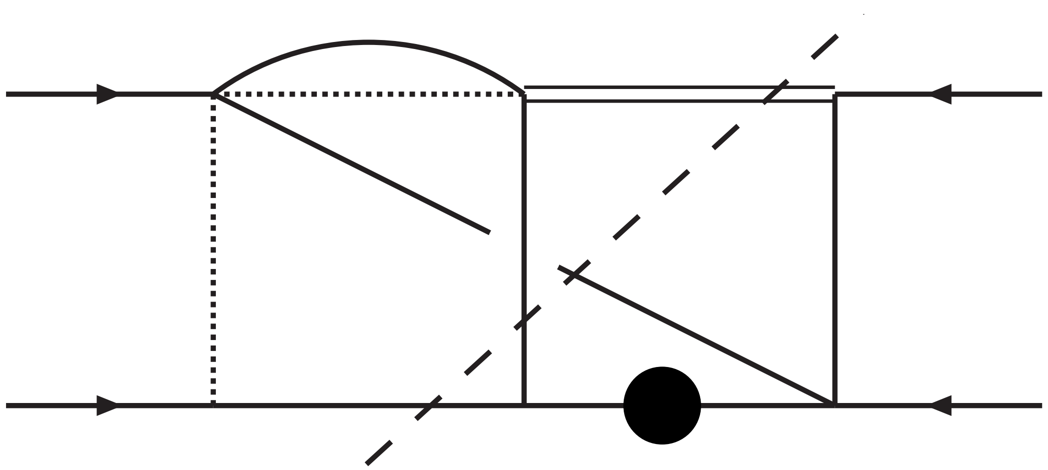
- Read optical theorem 'backwards': inclusive phase-integrals as unitarity cuts of loop integrals.

  [Anastasiou, Melnikov; Anastasiou, Dixon, Melnikov, Petriello]

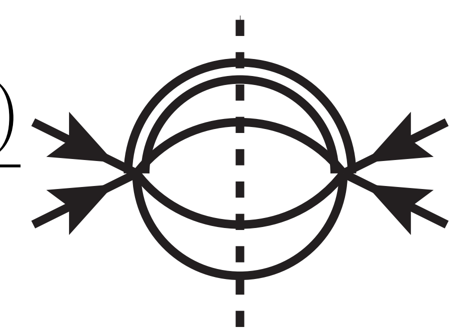
  ➡ Makes phase-space integrals accessible to loop technology!

IBPs and master integrals

- Loop integrals are not independent, but they are related by various relations.
  - ➡ Integration-by-parts identities (IBPs). [Chetyrkin, Tkachov]
- IBPs can be solved algorithmically. [Laporta]
  - ➡ All integrals are linear combination of a small set of master integrals.



$$= -\frac{(\epsilon-1)(2\epsilon-1)(3\epsilon-2)(3\epsilon-1)(6\epsilon-5)(6\epsilon-1)}{\epsilon^4(\epsilon+1)(2\epsilon-3)}$$



Differential equations

- We can use reverse-unitarity to differentiate with respect to the Higgs mass:

$$\frac{\partial}{\partial m_H^2}\delta_+(p_H^2 - m_H^2) \to \frac{\partial}{\partial m_H^2}\frac{1}{p_H^2 - m_H^2} = \frac{1}{(p_H^2 - m_H^2)^2}$$

- Can use IBP relations to reduce back to master integrals.
  - ➡ Master integrals satisfy a system of 1st order ODEs.

$$\frac{\partial}{\partial \bar{z}}\vec{I} = A(\bar{z},\epsilon)\,\vec{I} \qquad \bar{z} = 1 - z$$

- Boundary conditions are given by the soft limit $\bar{z} \to 0$.
  - ➡ Limits of Feynman integrals can be obtained from momentum space expansions and expansion by regions.

[Beneke, Smirnov]

The threshold expansion

- Solving the differential equations can still be very tough!
  - ➡ Huge system of coupled differential equations!
- We know that the cross section is dominated by $z \to 1$.
  - ➡ Approximated the cross section by a series around $z = 1$.

$$\frac{\hat{\sigma}_{ij}(z)}{z} = \hat{\sigma}^{SV}\,\delta_{ig}\,\delta_{jg} + \sum_{N=0}^{\infty} \hat{\sigma}_{ij}^{(N)}\,\bar{z}^N$$

- The coefficients in the expansion are not constants, but they are polynomials in $\log(1-z)$.
  - ➡ At N3LO: $\hat{\sigma}_{ij}^{(N)} = \sum_{k=0}^{5} c_{ijk}^{(N)} \log^k(1-z)$
- The first term is called the soft-virtual term and is distribution-valued:
  - ➡ At N3LO: $\hat{\sigma}^{SV} = a\,\delta(1-z) + \sum_{k=0}^{5} b_k \left[\frac{\log^k(1-z)}{1-z}\right]_+$

The threshold expansion

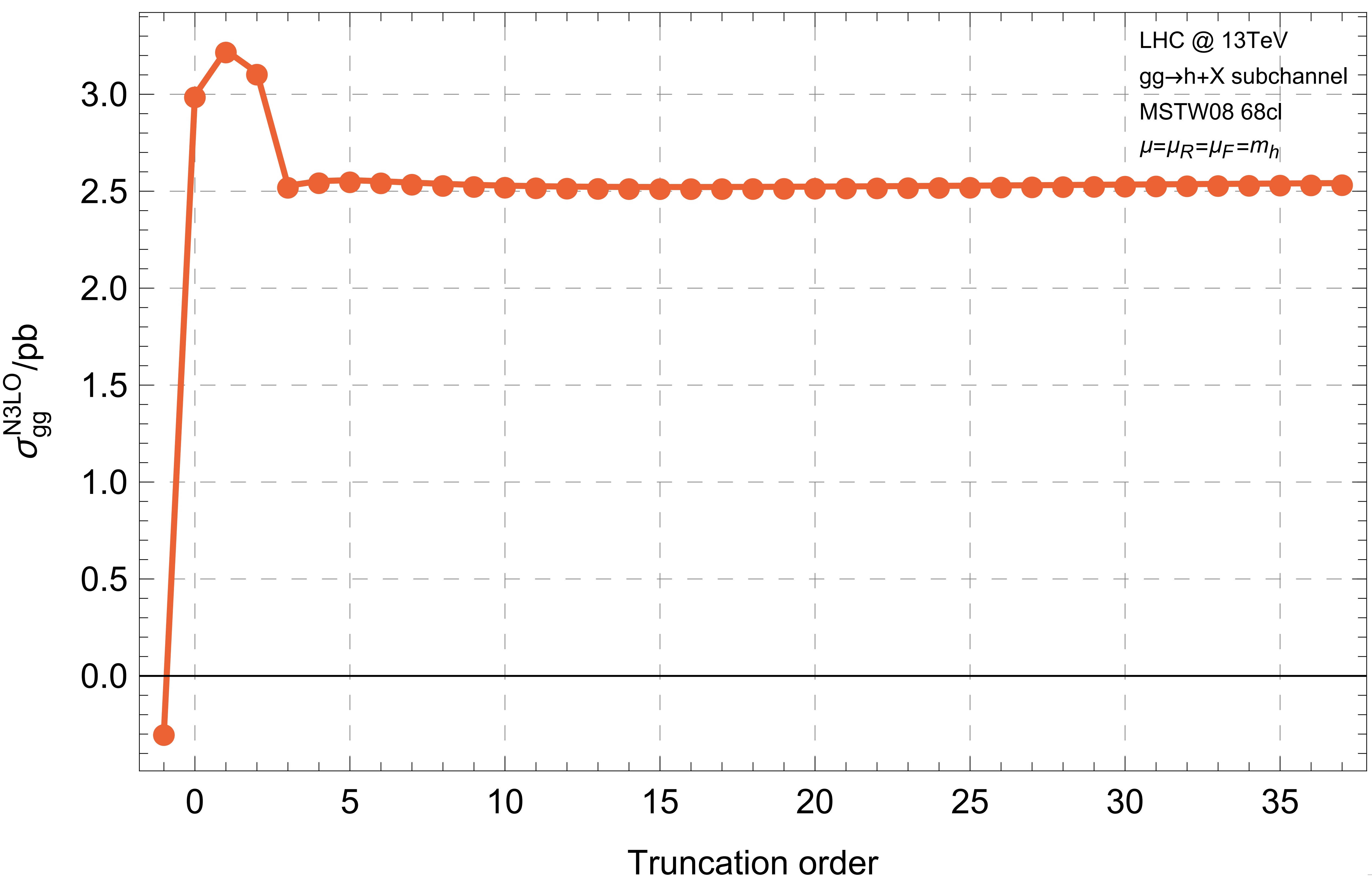
- One can compute the master integrals as an expansion around threshold.
  - ➡ Single-emission contributions can be computed exactly.

- Remaining contributions can be obtained by
  - ➡ Writing an ansatz for each master integral

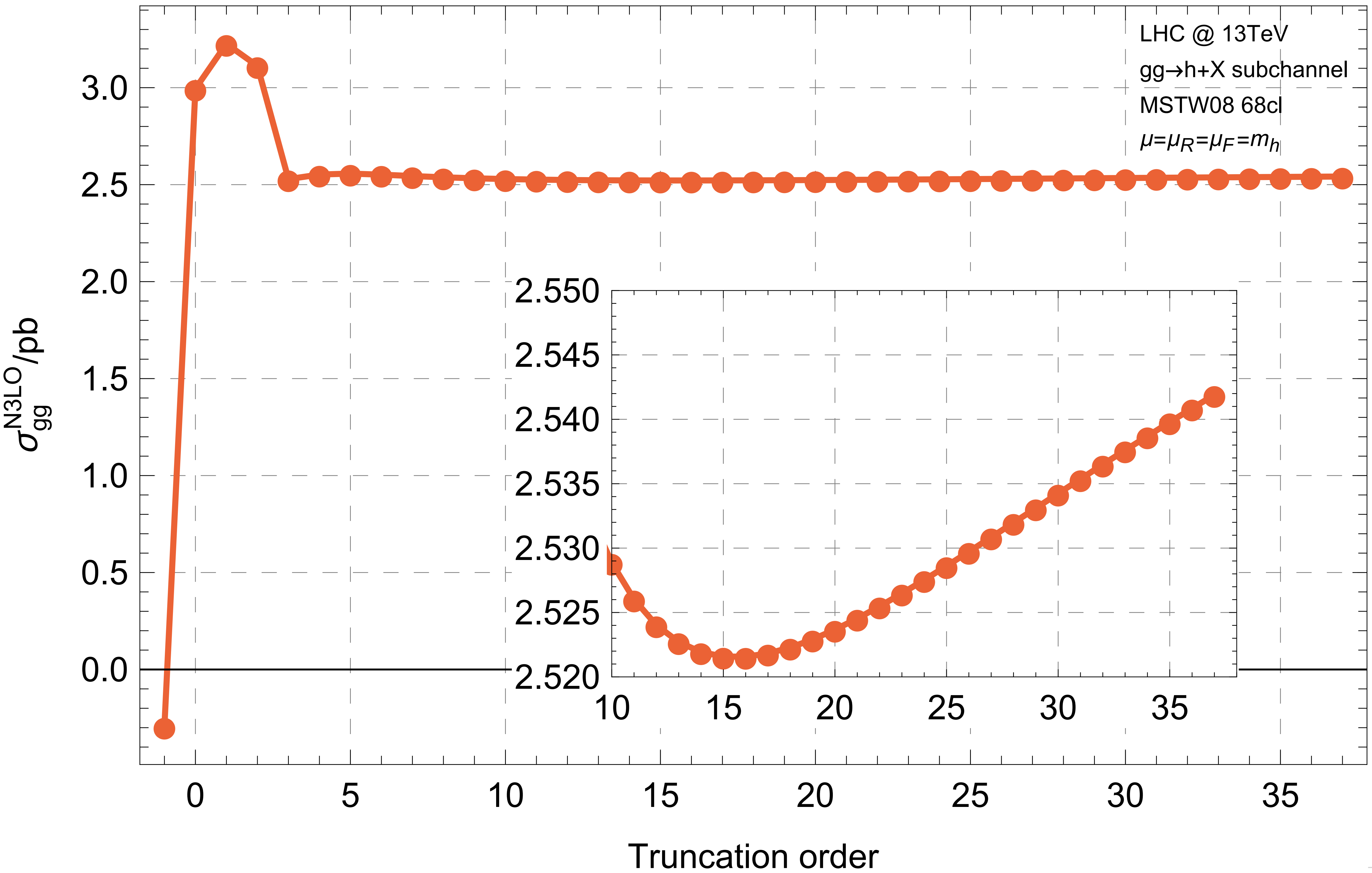
$$M_i = \sum_j \sum_{k=2}^{6} c_{ijk} (1-z)^{(j-k\epsilon)}$$

  - ➡ Insert ansatz into differential equations.
  - ➡ Solve a huge linear system for the coefficients.

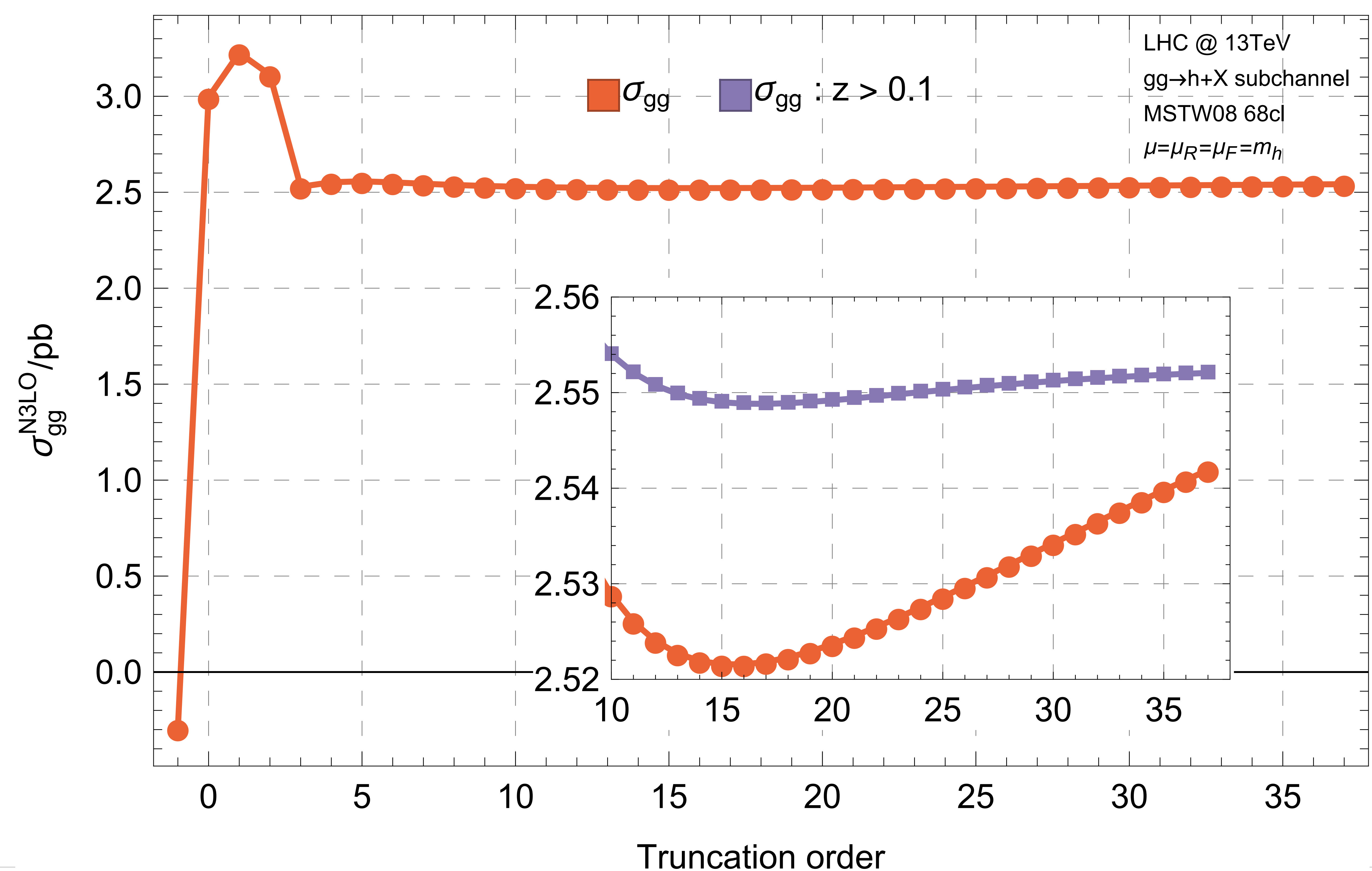
Threshold expansion



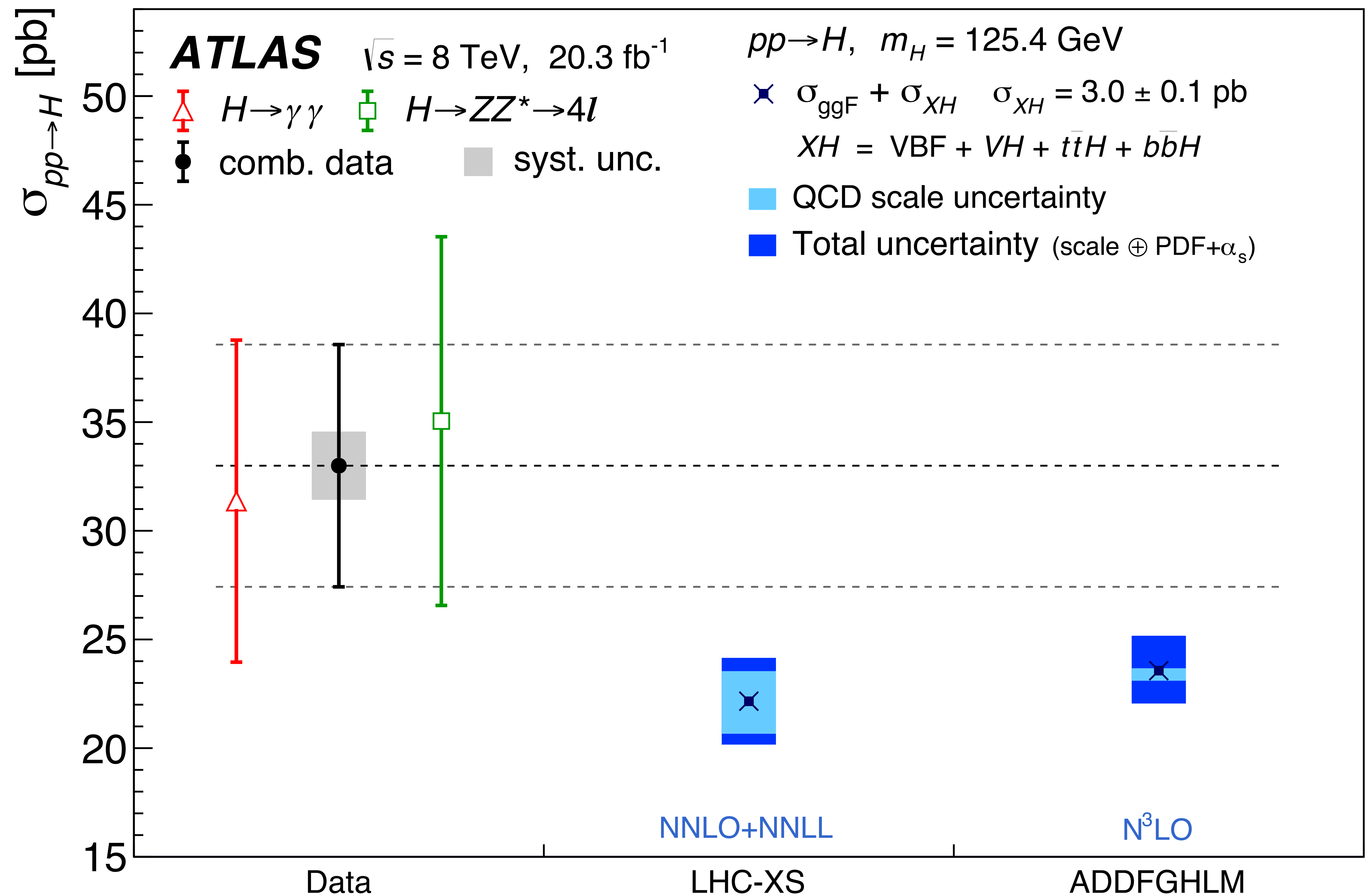
Threshold expansion



Threshold expansion



Scale vs. PDF uncertainty



Scale vs. PDF uncertainty

	CT14	MMHT2014	NNPDF3.0	CT10
8 TeV	$18.66^{+2.1\%}_{-2.3\%}$	$18.65^{+1.4\%}_{-1.9\%}$	$18.77^{+1.8\%}_{-1.8\%}$	$18.37^{+1.7\%}_{-2.1\%}$
13 TeV	$42.68^{+2.0\%}_{-2.4\%}$	$42.70^{+1.3\%}_{-1.8\%}$	$42.97^{+1.9\%}_{-1.9\%}$	$42.20^{+1.9\%}_{-2.5\%}$

[CTEQ collaboration

Threshold resummation

Threshold logarithms

- The cross section is dominated by $z \to 1$.

$$\frac{\hat{\sigma}_{ij}(z)}{z} = \hat{\sigma}^{SV}\,\delta_{ig}\,\delta_{jg} + \sum_{N=0}^{\infty} \hat{\sigma}_{ij}^{(N)}\,\bar{z}^N$$

$$\hat{\sigma}^{SV} = a\,\delta(1-z) + \sum_{k=0}^{5} b_k \left[\frac{\log^k(1-z)}{1-z}\right]_+$$

- At each order in perturbation theory there are 'large logarithms' (plus distributions) that we might want to resum.
- It is possible to resum threshold logarithms to all orders, but we need to go to Mellin space:

$$\hat{\sigma}(N) = \int_0^1 dz\, z^{N-1}\,\hat{\sigma}(z) \qquad\qquad \hat{\sigma}(z) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i}\, z^{-N}\,\hat{\sigma}(N)$$

Mellin space

- Let us consider threshold resummation in QED.
  - ➡ Consider $q\,\bar{q} \to Z + n$ soft gluons.
- Using eikonal Feynman rules, one can see that the eikonal matrix element factorises:

$$\mathcal{M}_n^{\text{eik.}} = \frac{1}{n!}\left[\mathcal{M}_1^{\text{eik.}}\right]^n$$

- The phase space contains a delta function $\delta(z - z_1 \ldots z_n)$
  - ➡ Spoils naive factorisation.
- Phase space factorises in Mellin space:

$$\int_0^1 dz\, z^{N-1}\,\delta(z - z_1 \ldots z_n) = z_1^{N-1} \ldots z_n^{N-1}$$

- Then:

$$\sum_{n=0}^{\infty} \frac{1}{n}\left[\mathcal{M}_n^{\text{eik.}}(N)\right]^n = \exp\left[\mathcal{M}_n^{\text{eik.}}(N)\right]$$

Threshold resummation

- The $z \to 1$ limit corresponds to $N \to \infty$.

$$\int_0^1 dz\, z^{N-1} \left[\frac{\log^k(1-z)}{1-z}\right]_+ = \frac{(-1)^k}{k+1} \log^{k+1} N + \mathcal{O}(1/N)$$

- One can show that in this limit the cross section in Mellin space takes the form

$$\hat{\sigma}_{gg} = g_0(a_s) \exp\left[\frac{1}{a_s} g_1(\lambda) + g_2(\lambda) + a_s\, g_3(\lambda) + \ldots\right] + \mathcal{O}(1/N)$$

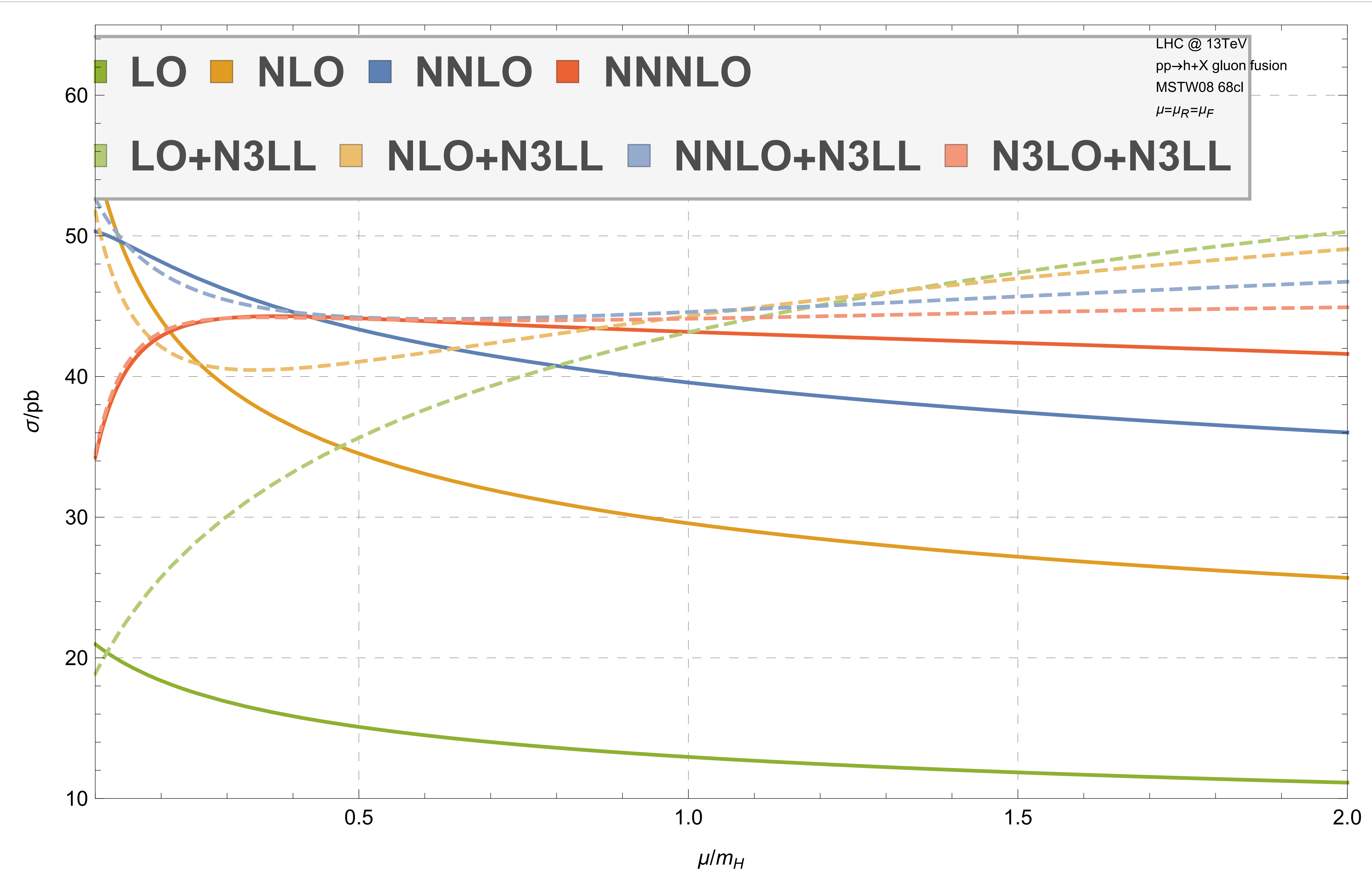
LL NLL NNLL $\lambda = a_s \log N$

- The function in the exponent are known up to i=4 (N4LL), up to one constant.

Threshold resummation

$$
\begin{array}{lllllll}
1 + \alpha_s L^2 & + \alpha_s^2 L^4 & + \alpha_s^3 L^6 & + \alpha_s^4 L^8 & + \dots & & \text{LL} \\
\;\;+ \alpha_s L & + \alpha_s^2 L^3 & + \alpha_s^3 L^5 & + \alpha_s^4 L^7 & + \dots & & \text{NLL} \\
\;\;+ \alpha_s & + \alpha_s^2 L^2 & + \alpha_s^3 L^4 & + \alpha_s^4 L^6 & + \dots & & \text{NLL}' \\
 & + \alpha_s^2 L & + \alpha_s^3 L^3 & + \alpha_s^4 L^5 & + \dots & & \text{NNLL} \\
 & + \alpha_s^2 & + \alpha_s^3 L^2 & + \alpha_s^4 L^4 & + \dots & & \text{NNLL}' \\
 & & + \alpha_s^3 L & + \alpha_s^4 L^3 & + \dots & & \text{N}^3\text{LL} \\
 & & + \alpha_s^3 & + \alpha_s^4 L^2 & + \dots & & \text{N}^3\text{LL}' \\
 & & & + \alpha_s^4 L & + \dots & & \text{N}^4\text{LL} \\
 & & & + \alpha_s^4 & + \dots & & \text{N}^4\text{LL}' \\
 & & & & + \dots & & \text{N}^5\text{LL}
\end{array}
$$

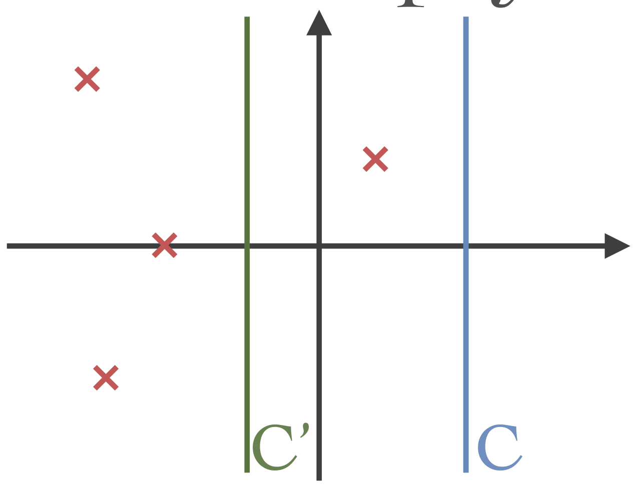
N3LL threshold resummation



Uncertainties (1)

- We need to go from Mellin-space back to the 'physical space'.

$$f(z) = \int_C \frac{dN}{2\pi i} \tau^{-N} f(N)$$



  - ➡ Left most pole corresponds to Landau pole!
  - ➡ Inverse transform exists order by order, but series diverges (asymptotic series).

- There are different prescriptions to deal with this:

  - ➡ Minimal prescription.
  - ➡ Borel prescription.

Uncertainties (2)

- There is an ambiguity what to exponentiate:

$$\hat{\sigma}_{gg} = g_0(a_s)\,\exp\left[\frac{1}{a_s}\,g_1(\lambda) + g_2(\lambda) + a_s\,g_3(\lambda) + \ldots\right]$$

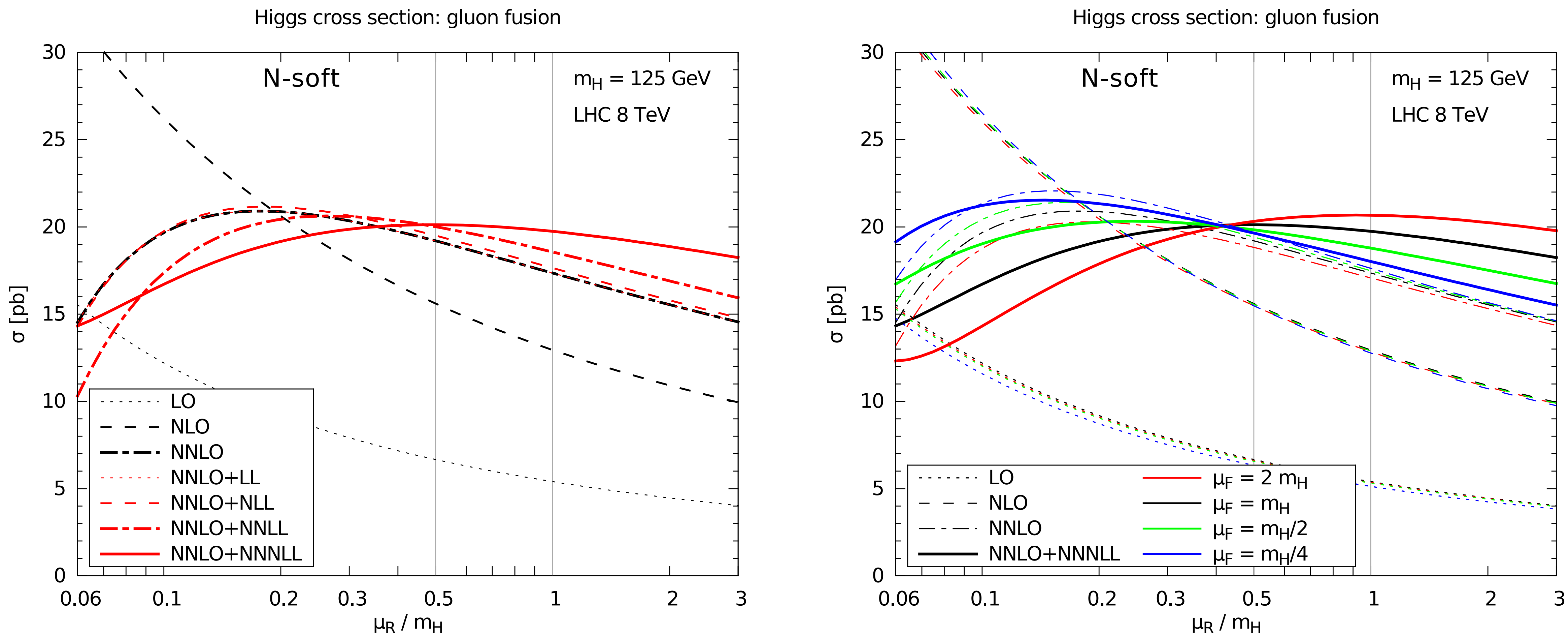
$$= \tilde{g}_0(a_s)\,\exp\left[\frac{1}{a_s}\,\tilde{g}_1(\lambda) + \tilde{g}_2(\lambda) + a_s\,\tilde{g}_3(\lambda) + \ldots\right]$$

$$\tilde{g}_0(a_s) = g_0(a_s)\,e^{f(a_s)} \qquad \tilde{g}_i(\lambda) = g_i(\lambda) - f_i \qquad f(a_s) = \sum_{k=0}^{\infty} a_s^k\,f_k$$

- If I truncate the perturbative series in the exponent and in the hard coefficient g_0, I get different answers!

➡ Can produce different results for fixed log-accuracy

Uncertainties (2)



Uncertainties (3)

- There is an ambiguity which logarithms to resum:

$$\hat{\sigma}_{gg} = g_0(a_s)\,\exp\left[\frac{1}{a_s}\,g_1(\lambda) + g_2(\lambda) + a_s\,g_3(\lambda) + \ldots\right] + \mathcal{O}(1/N)$$

$$\lambda = a_s\,\log N$$

$$\log(1+N) = \log N + \mathcal{O}(1/N)$$

$$\left(1+\frac{1}{N}\right)\log N = \log N + \mathcal{O}(1/N)$$

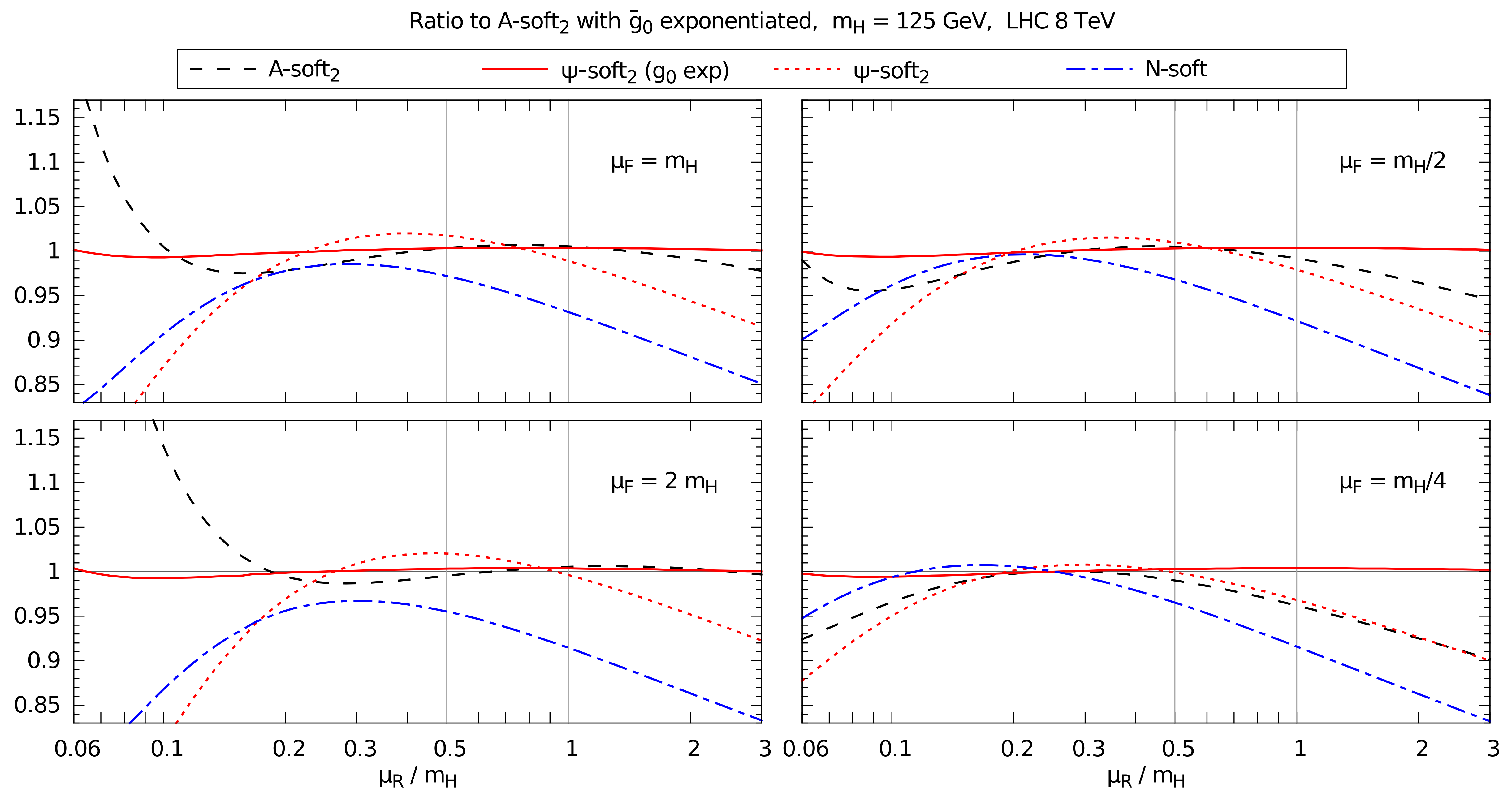
$$\psi(N) = \log N + \mathcal{O}(1/N) \qquad\qquad \psi(N) = \frac{d}{dN}\log\Gamma(N)$$

$$\ldots$$

- All these choices are formally equivalent, but can lead to different predictions.

➡ Some choices motivated by, e.g., analyticity.

Uncertainties (3)

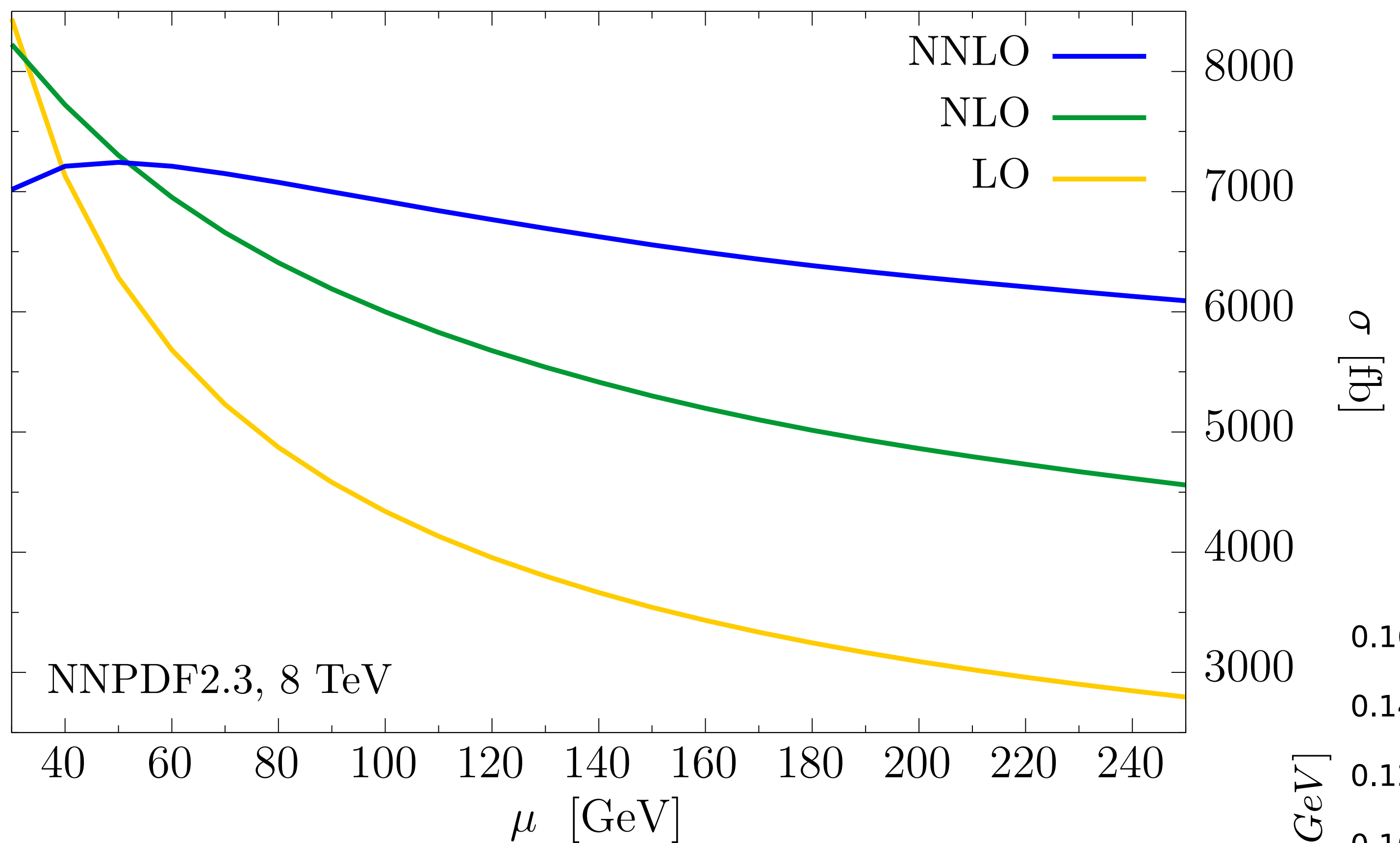


Beyond the inclusive large-mt limit

H+j @ NNLO

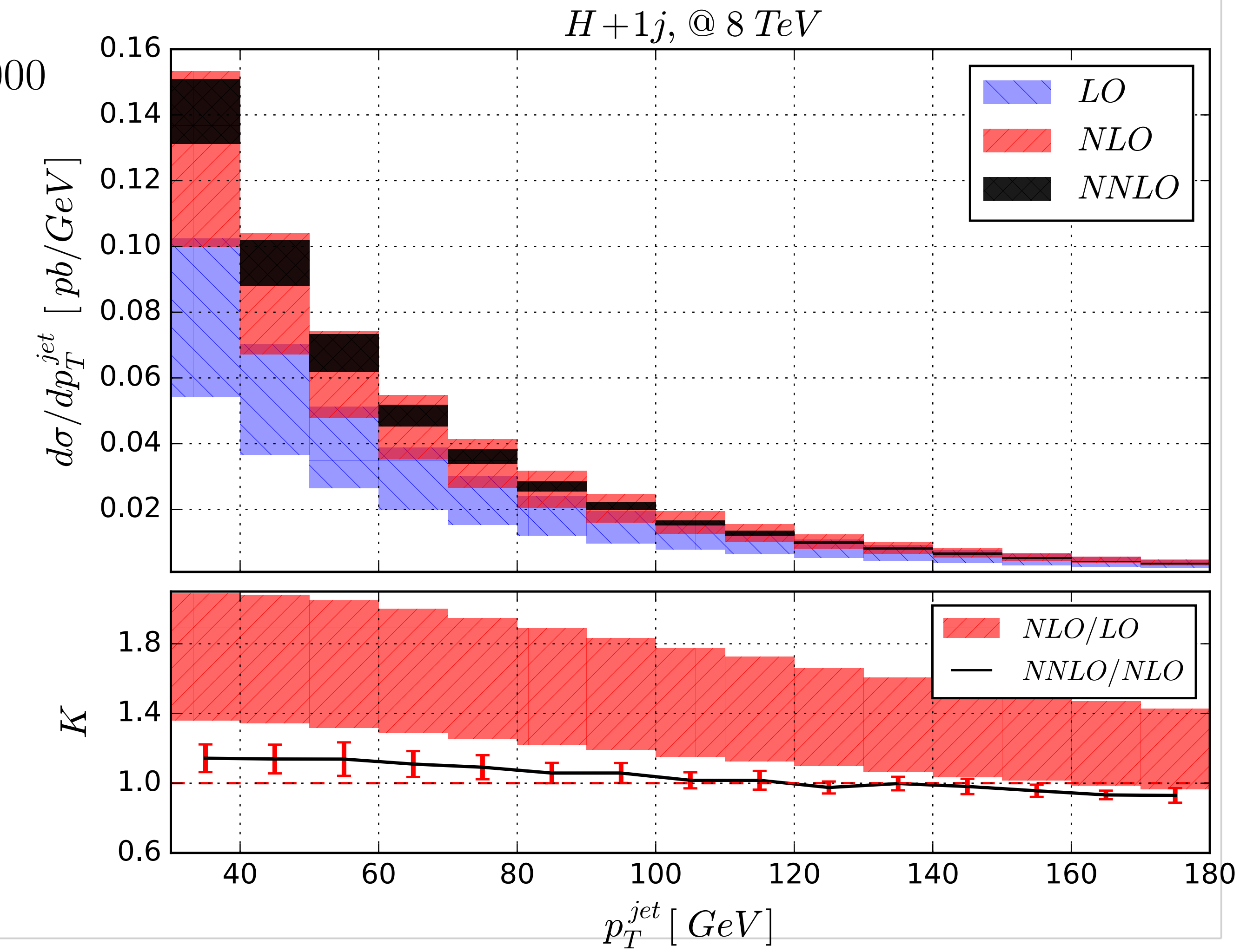
- We now even have fully differential distributions for H+j at NNLO!
- Three independent computations:
  - ➡ Stripper. [Boughezal, Caola, Melnikov, Petriello, Schulze
  - ➡ N-jettiness subtraction. [Boughezal, Focke, Giele, Liu, Petriello
  - ➡ Antenna subtraction. [Chen, Gehrmann, Glover, Jaquier
- Computation is done in the large-mt limit.
- Fully differential!
  - ➡ Allows for arbitrary cuts on the final state.

H+j @ NNLO



[Boughezal, Caola, Melnikov, Petriello, Schulze

[Boughezal, Focke, Giele, Liu, Petriello



Beyond large mt

- We expect the large-mt approximation to break down for large pT's!
  - ➡ There is a new scale, which may be as large as the top mass!
- The full NLO corrections to H+j are currently beyond reach.
  - ➡ Requires the computation of double boxes with top-quark loop.
- The same reasoning applies to double (triple, etc) Higgs production.
  - ➡ Here the large-mt is not even supposed to work for the total cross section.

Beyond large mt

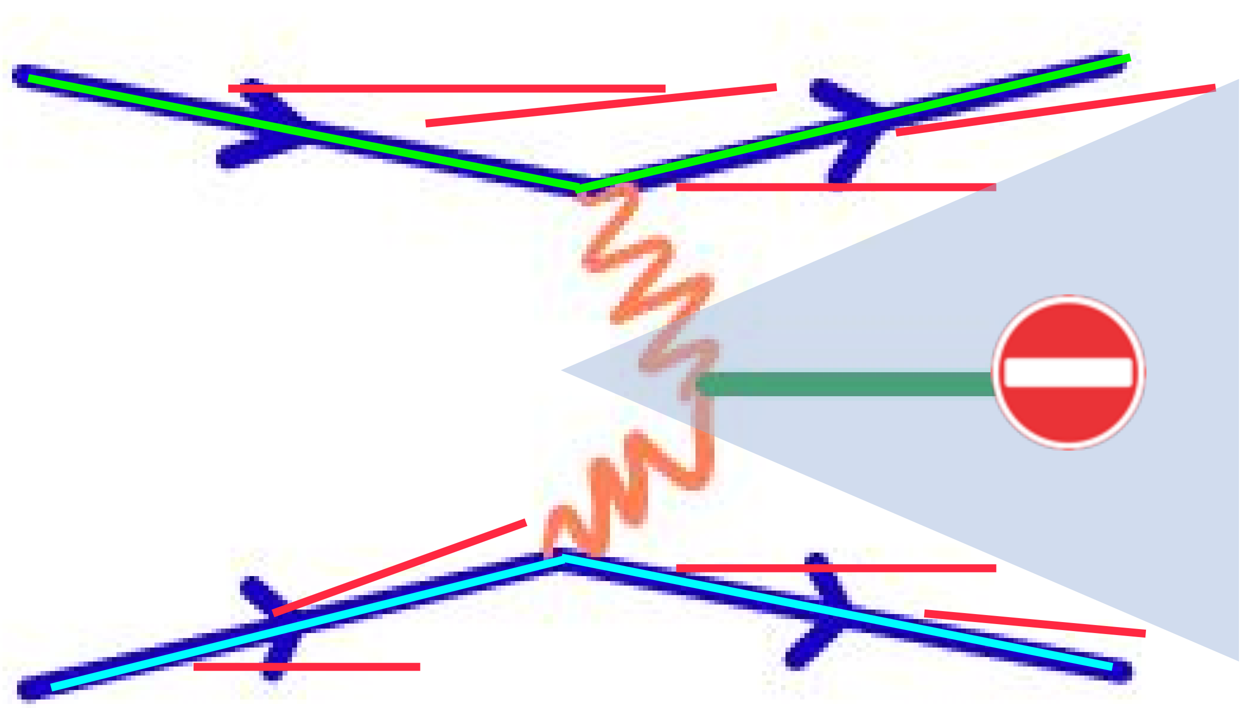
HH production in gluon-gluon fusion at 14 TeV		Cross section [fb]
LO	HEFT	$19.2^{+35.2+2.8\%}_{-24.3-2.9\%}$
	FT, $\Gamma_t = 0$ GeV	$23.2^{+32.3+2.0\%}_{-22.9-2.3\%}$
	FT, $\Gamma_t = 1.5$ GeV	$22.7^{+32.3+2.0\%}_{-22.9-2.3\%}$

[Maltoni, Vryonidou, Zaro

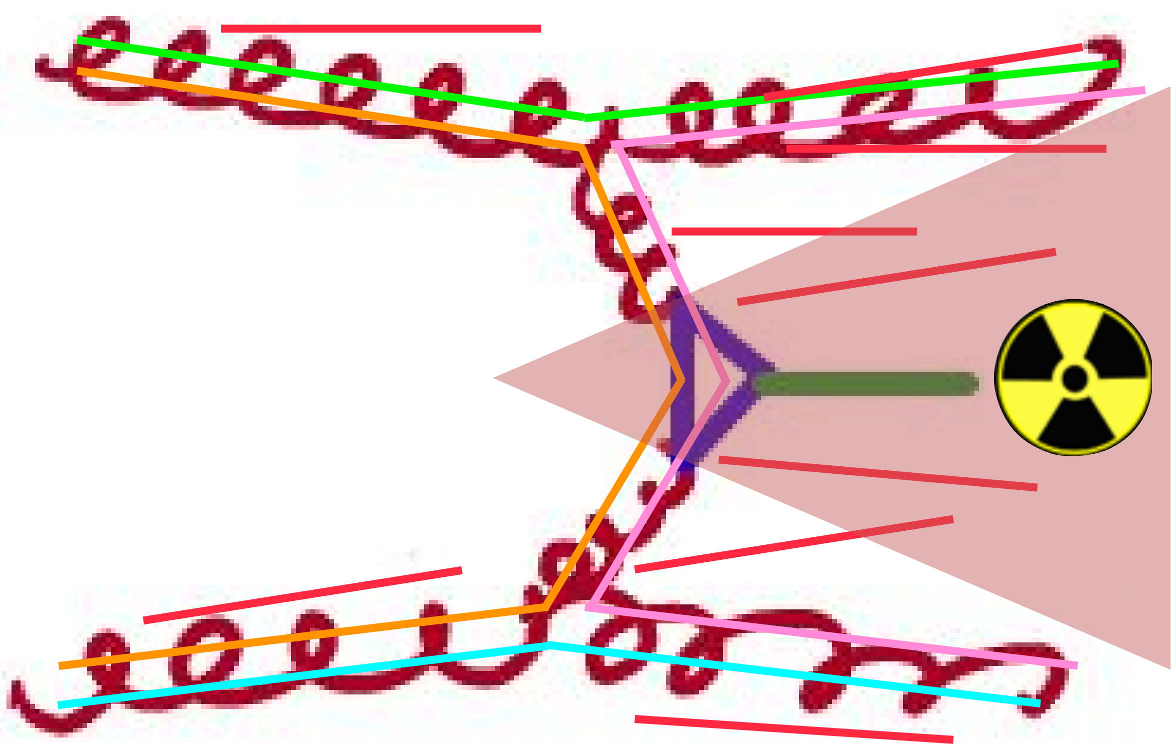
Vector-boson fusion

Vector-boson-fusion

- If we want to probe the coupling of the Higgs boson to gauge bosons, then ggF is not adequate.
- It is more advantageous to use the VBF process in this case.



Singlet exchange



Octet exchange

- However, VBF is 'buried' underneath ggF!

Vector-boson-fusion

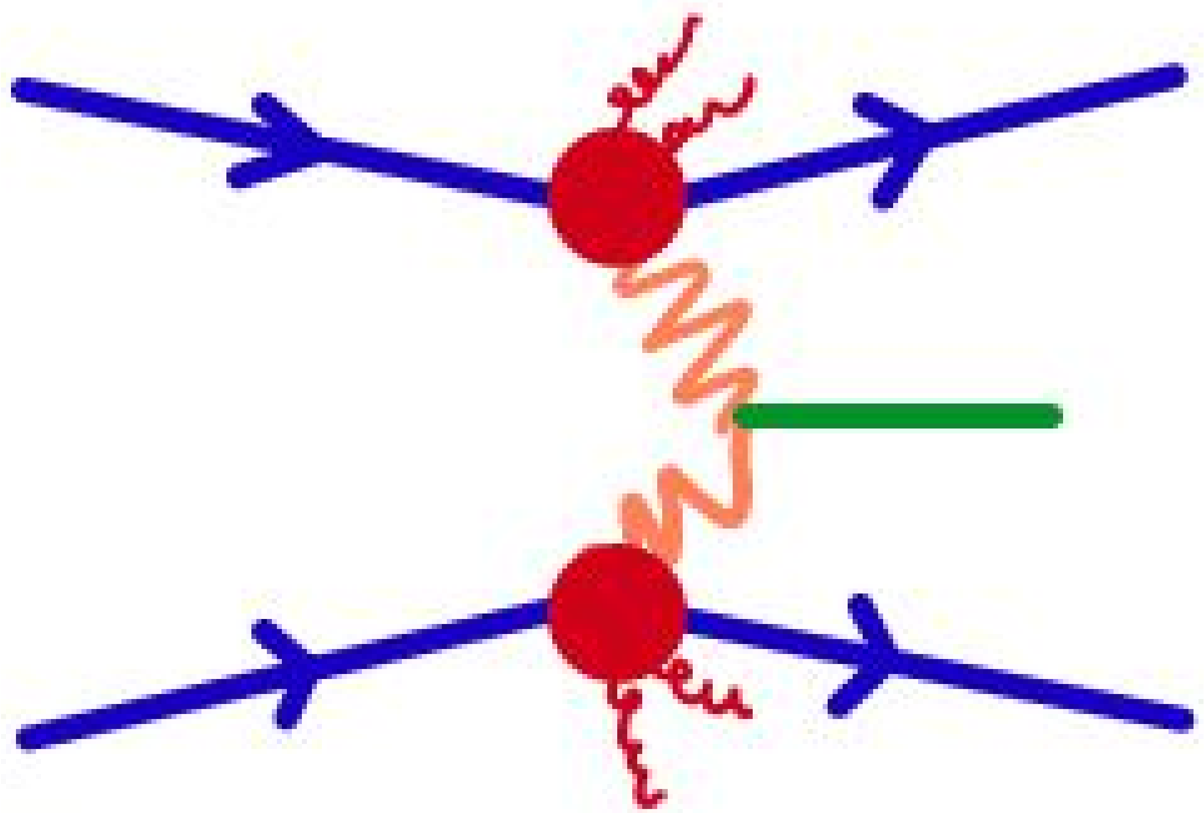
- We can suppress the contribution from ggF by requiring 2 forward jets, and low hadronic activity in the central detector.
- Typical VBF cuts:
  - ➡ At least to jets with $p_T > 25\text{GeV}$.
  - ➡ The two hardest jets satisfy

$$|y| < 4.5 \qquad \Delta y_{j_1 j_2} < 4.5 \qquad M^2_{j_1 j_2} > (600\text{GeV})^2$$

- Imposing VBF cuts, one can reduce the ggF contamination to ~10%.
- NNLO corrections to VBF would require the computation of pentaboxes with massive propagators.
  - ➡ Beyond the reach of current technology.

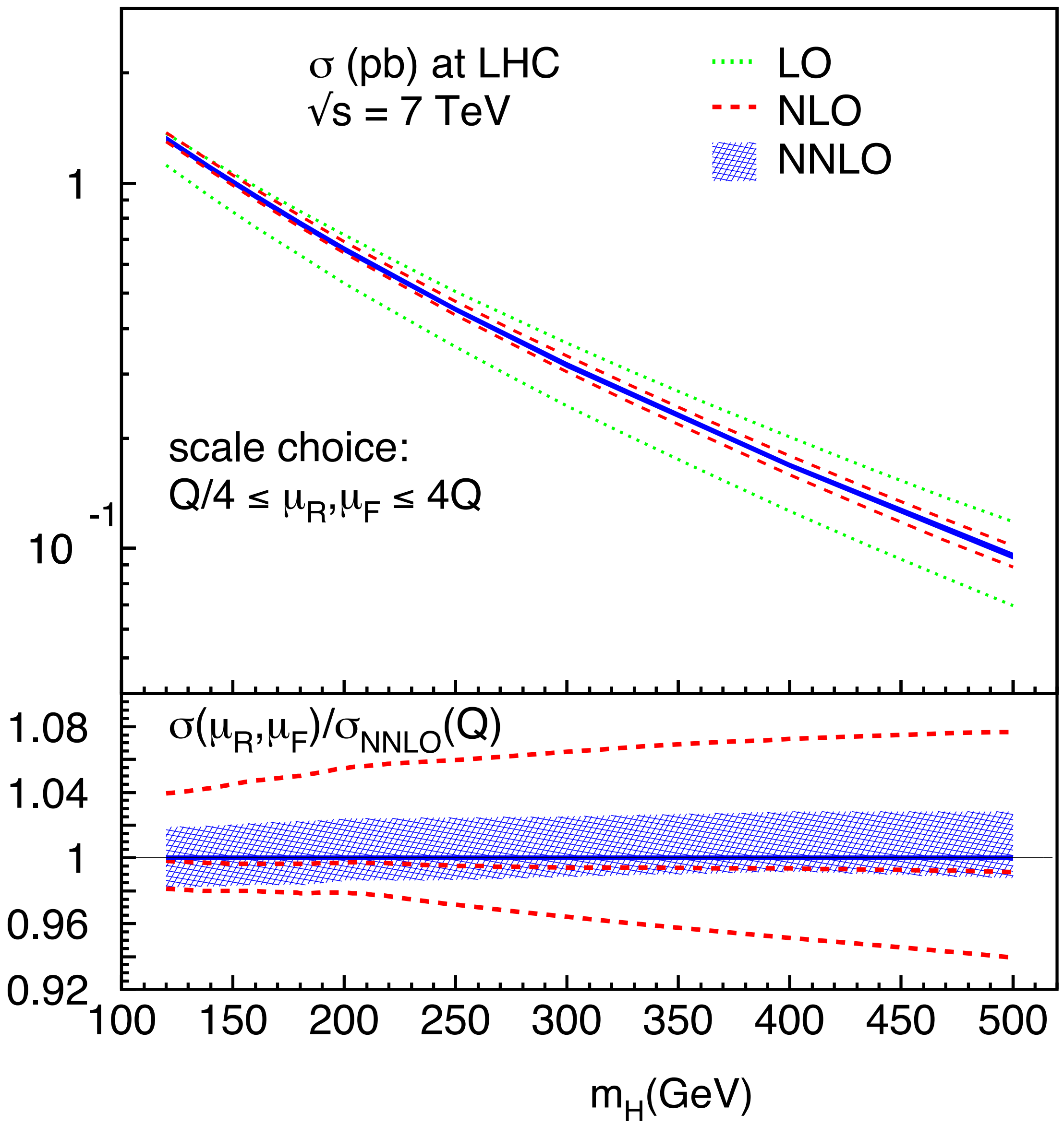
Structure function approach

- Assume that there is no colour exchange at all between the upper and lower lines
  - ➡ Exact at LO and NLO.
  - ➡ Beyond NLO, non-factorisable diagrams are suppressed by colour, and by kinematics (angular ordering)



- QCD corrections completely factorise into DIS form factors!

Inclusive cross section



$\sqrt{S} = 7$ TeV			
Higgs mass	LO	NLO	NNLO
120	$1.235^{+0.131}_{-0.116}$	$1.320^{+0.054}_{-0.022}$	$1.324^{+0.025}_{-0.024}$
160	$0.857^{+0.121}_{-0.099}$	$0.915^{+0.046}_{-0.016}$	$0.918^{+0.019}_{-0.015}$
200	$0.614^{+0.106}_{-0.082}$	$0.655^{+0.038}_{-0.012}$	$0.658^{+0.015}_{-0.010}$
300	$0.295^{+0.070}_{-0.049}$	$0.314^{+0.022}_{-0.010}$	$0.316^{+0.008}_{-0.004}$
400	$0.156^{+0.045}_{-0.030}$	$0.166^{+0.013}_{-0.007}$	$0.167^{+0.005}_{-0.001}$

[Bolzoni, Maltoni, Moch, Zaro

- Small remaining Scale uncertainty (~1-2%)!

Differential cross section

- Recently, the differential NNLO cross section in the structure function approach was obtained. [Cacciari, Dreyer, Karlberg, Salam, Zanderighi
  - ➡ Can apply VBF cuts!
  - ➡ Method: Combine inclusive computation with H+3j computation from POWHEG.

	$\sigma^{(\text{no cuts})}$ [pb]	$\sigma^{(\text{VBF cuts})}$ [pb]
LO	$4.032^{+0.057}_{-0.069}$	$0.957^{+0.066}_{-0.059}$
NLO	$3.929^{+0.024}_{-0.023}$	$0.876^{+0.008}_{-0.018}$
NNLO	$3.888^{+0.016}_{-0.012}$	$0.826^{+0.013}_{-0.014}$
	~1%	~5-6%

Differential cross section