Vector Boson Scattering:

A Phenomenological Perspective

HiggsTools Summer School

Valle d'Aosta – July 2015

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outline

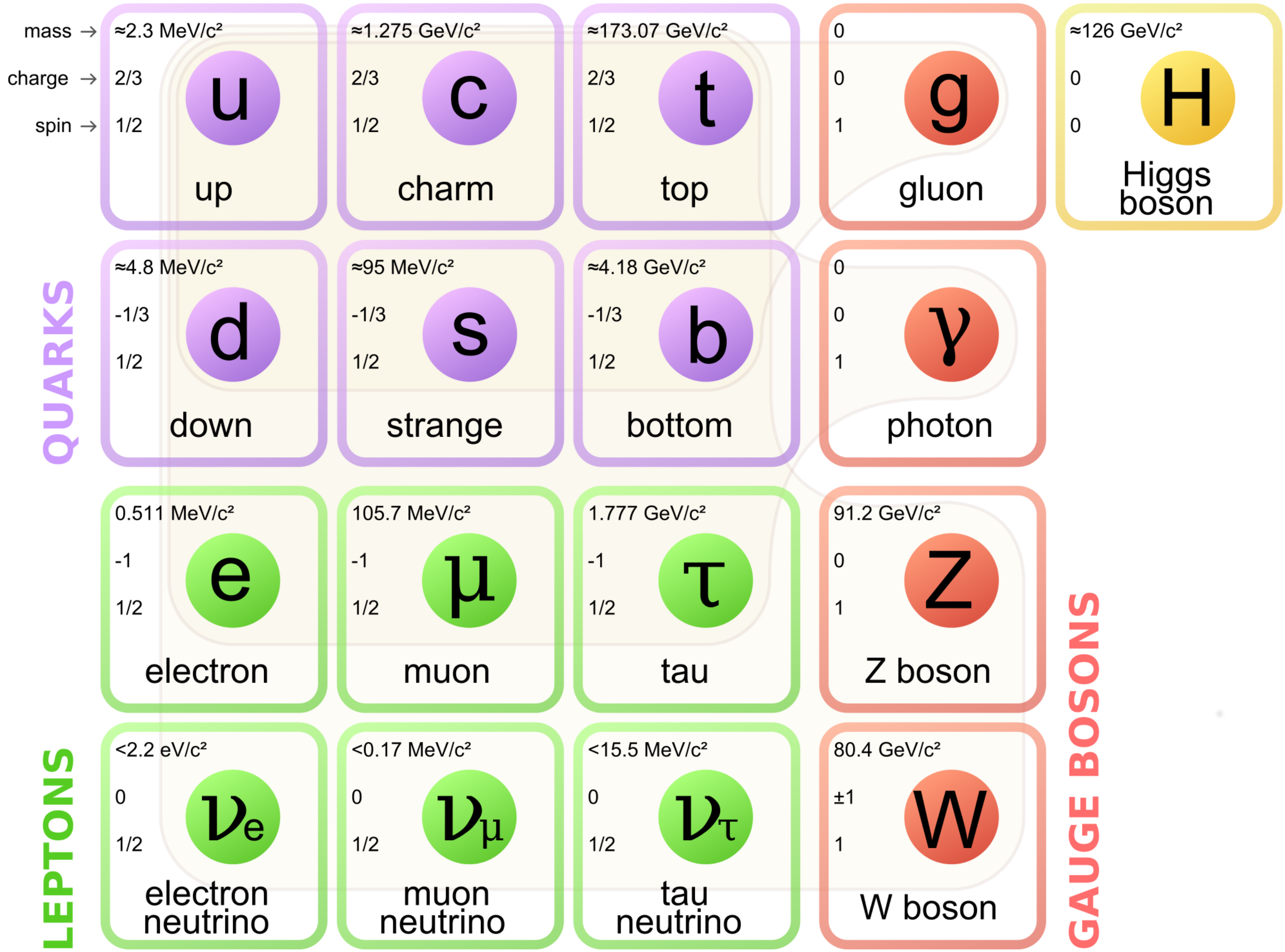
vector boson scattering:

- theoretical concepts & techniques
- phenomenological results
- the quest for more realistic predictions

Higgs production via vector boson fusion:

- motivation: a super-clean environment
- precise predictions and unexpected features
- omipresent: backgrounds

the 21st century picture of elementary particles



spontaneous breaking of local gauge symmetry

basic concept:

gauge boson sector of the SM: $\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}}$

gauge fields ($W^{\pm}$, Z, γ)

extra massive, neutral, scalar field

– full Lagrangian invariant
– vacuum state not invariant under electroweak symmetry

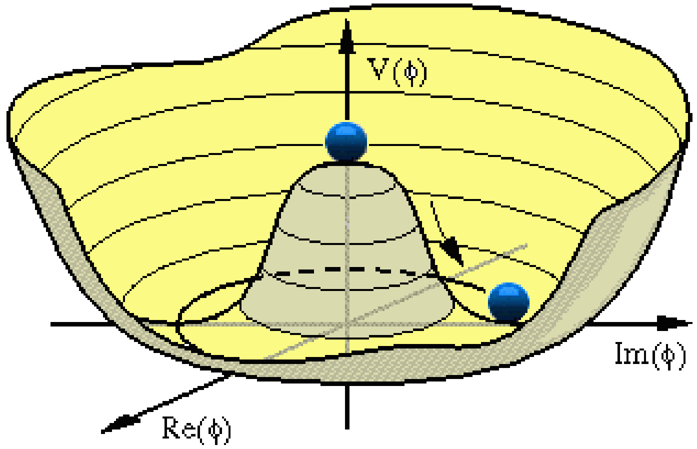
☞ symmetry is spontaneously broken!

spontaneous symmetry breaking within the SM

complex scalar field $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

with self interaction potential

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \frac{\lambda}{4}(\phi^\dagger \phi)^2, \; \lambda > 0$$



crucial point: $\mu^2 > 0$

$V(\phi)$ minimal for $|\phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$

specific choice $\phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ breaks gauge invariance spontaneously

the Englert-Brout-Higgs-Hagen-Guralnik-Kibble mechanism

recall Goldstone's theorem:

each spontaneously broken symmetry
gives rise to one massless Goldstone boson

Standard Model: complex scalar doublet
with four degrees of freedom

photon remains massless

three (real) longitudinal polarization states of massive $W^{\pm}$ and Z bosons

one component real scalar particle: ☞ "Higgs boson"

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Standard Model: complex scalar doublet
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photon remains massless ✔

three (real) longitudinal polarization states of massive $W^\pm$ and Z bosons ✔

one component ☞ real scalar particle: "Higgs boson" ???

the Higgs: tasks for the community

✔ detect *"Higgs boson"* and determine M_H

✧ investigate properties of the new particle carefully

determination of

couplings, charge, spin, CP quantum numbers

necessary to reveal

SM, SUSY, or something completely different?

↓

full, quantitative understanding of

most promising search channels required

from experiment and theory

the Higgs: only one part of the full picture

going beyond the Higgs boson:

are observations fully consistent with the SM picture of electroweak symmetry breaking?

vector boson fusion (VBF) &
vector boson scattering (VBS)

Standard Model:

- important production mode for the Higgs boson
- sensitive to Higgs couplings and CP properties

beyond the Standard Model:

sensitive to the mechanism of electroweak symmetry breaking

$\updownarrow$

strongly interacting weak sector, new resonances, . . . ?

the big advantage:

- ✔ experimentally clean signature
- ✔ perturbatively well under control

before the Higgs discovery

Standard Model:
couplings and parameters
strongly constrained

free parameter: M_H

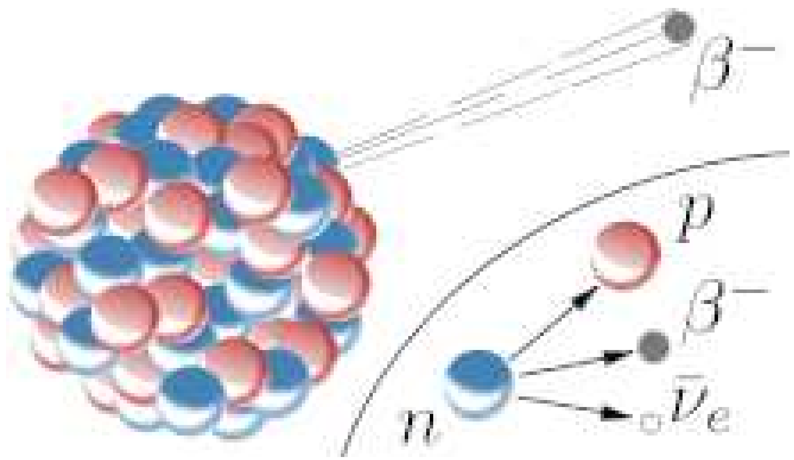
still:
theory & experiment imposed
variety of bounds on Higgs mass . . .

. . . such as the perturbative unitarity limit

perturbative unitarity

Fermi's model of four-fermion interaction (1933):

- four fermions interact directly at one vertex
- explanation of radioactive beta decay ($n \to p\, e^- \bar{\nu}_e$)
- can also be applied to other reactions,
  such as $\mu^- \to e^- \bar{\nu}_e \nu_\mu$
- works fine at low energies



perturbative unitarity

Fermi's model of four-fermion interaction (1933):

- can be applied to $\nu_\mu e^- \to \nu_e \mu^-$
- works fine at low energies
- but: cross section grows with energy $\to$ unitarity is violated

$$\mathcal{M}(\nu_\mu e^- \to \nu_e \mu^-) = \frac{G_F s}{2\sqrt{2}\pi^2}$$

$s = E_{c.m.s}$ ... center-of-mass energy squared,

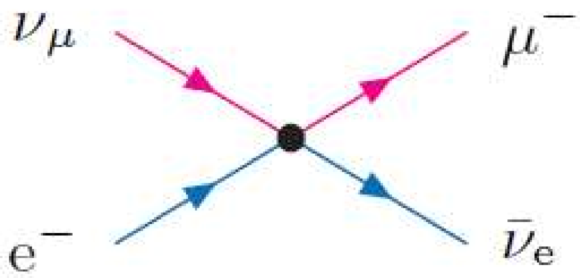
$G_F = 1.16637 \times 10^{-5}$ GeV^2 ... Fermi constant

perturbative unitarity

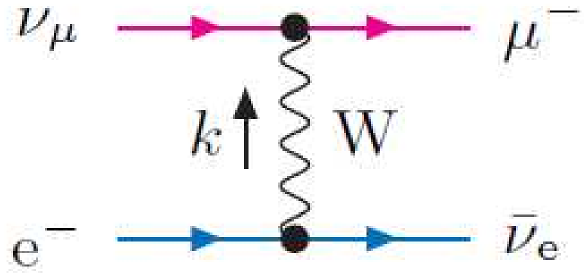
- four-fermion interaction: cross section grows with energy

$$\mathcal{M}(\nu_\mu e^- \to \nu_e \mu^-) = \frac{G_F s}{2\sqrt{2}\pi^2}$$

- unitarity violation can be cured by masssive mediator (W boson)



☞

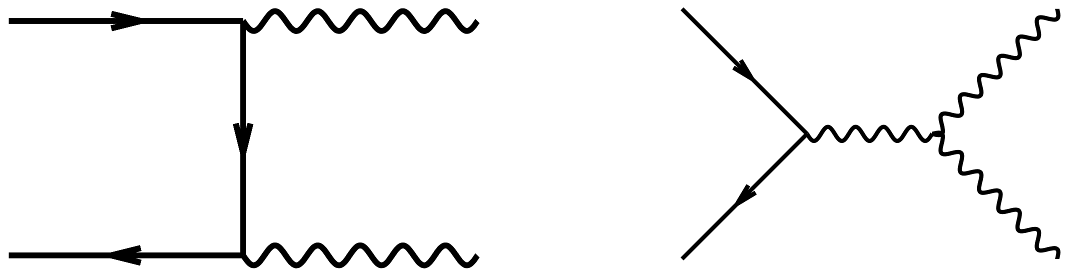


perturbative unitarity

still encounter unitarity violations, e.g. in $e^+e^- \to W^+W^-$

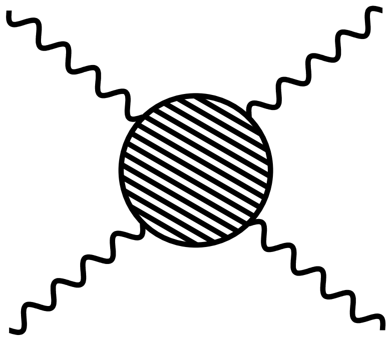
$\to$ need neutral gauge boson with the "right" gauge coupling structure

requirement of unitarity provides info on structure of the theory



perturbative unitarity

can we employ the requirement of unitarity in processes with massive gauge bosons to constrain the weak sector?



most sensitive to the mechanism of electroweak symmetry breaking:

longitudinal modes of the $W^\pm$ and Z bosons

$\rightarrow$ consider longitudinal gauge boson scattering:

$$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$$

$W_L^+ W_L^- \to W_L^+ W_L^-$

momenta in center-of-mass system:

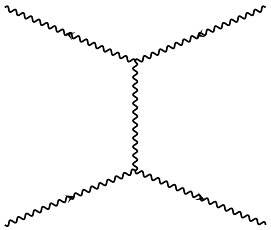
$$\begin{aligned} p_1 &= (E, 0, 0, +p)\,, \\ p_2 &= (E, 0, 0, -p)\,, \\ p_3 &= (E, 0, +p\sin\theta, +p\cos\theta)\,, \\ p_4 &= (E, 0, -p\sin\theta, -p\cos\theta)\,. \end{aligned}$$

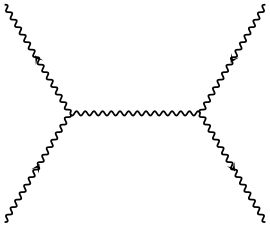
polarization vectors:

$$\begin{aligned} \varepsilon_L(p_1) &= \left(\frac{p}{M_W}, 0, 0, +\frac{E}{m_W}\right)\,, \\ \varepsilon_L(p_2) &= \left(\frac{p}{M_W}, 0, 0, -\frac{E}{m_W}\right)\,, \\ \varepsilon_L(p_3) &= \left(\frac{p}{M_W}, 0, +\frac{E}{m_W}\sin\theta, +\frac{E}{m_W}\cos\theta\right)\,, \\ \varepsilon_L(p_4) &= \left(\frac{p}{M_W}, 0, -\frac{E}{m_W}\sin\theta, -\frac{E}{m_W}\cos\theta\right)\,. \end{aligned}$$

$W_L^+ W_L^- \to W_L^+ W_L^-$

diagrams with triple-gauge couplings:

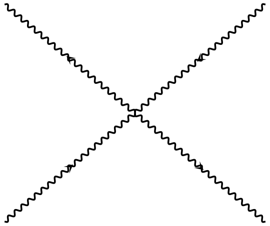




$+ \cdots$

$$T^{VVV} = g_W^2 \left\{ \frac{p^4}{M_W^4} \left[3 - 6\cos\theta - \cos^2\theta\right] + \frac{p^2}{M_W^2} \left[\frac{9}{2} - \frac{11}{2}\cos\theta - 2\cos^2\theta\right] \right\}$$

diagram with quartic gauge coupling:

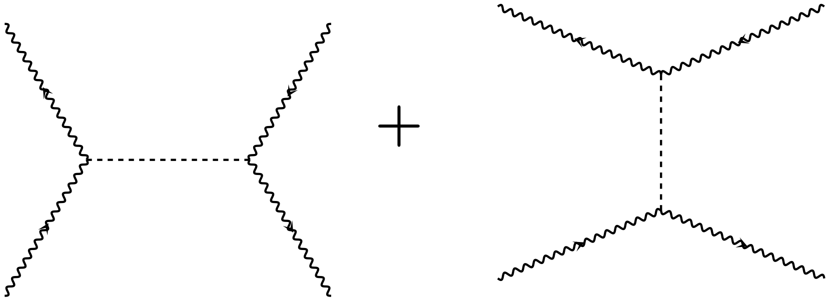


$$T^{VVVV} = g_W^2 \left\{ \frac{p^4}{M_W^4} \left[-3 + 6\cos\theta + \cos^2\theta\right] + \frac{p^2}{M_W^2} \left[-4 + 6\cos\theta + 2\cos^2\theta\right] \right\}$$

$$T^{VVV} + T^{VVVV} = g_W^2 \left\{ \frac{p^4}{M_W^4} \cdot 0 + \frac{p^2}{M_W^2} \left[\frac{1}{2} + \frac{1}{2}\cos\theta\right] \right\}$$

$W_L^+ W_L^- \rightarrow W_L^+ W_L^-$

diagrams with Higgs boson:



$$T^H = g_W^2 \left\{ \frac{p^2}{M_W^2} \left[-\frac{1}{2} - \frac{1}{2}\cos\theta \right] - \frac{M_H^2}{4M_W^2} \left[\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right] \right\}$$

sum of all contributions:

$$T^{VVV+VVVV+H} = -g_W^2 \frac{M_H^2}{4M_W^2} \left[\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right], \text{with } s = 4E^2$$

well-behaved at high energies ($s \rightarrow \infty$)

problems with unitarity can arise, if M_H becomes too large;

$\rightarrow$ can derive bound on M_H from unitarity requirement

$W_L^+ W_L^- \to W_L^+ W_L^-$

partial-wave expansion in terms of Legendre polynomials P_J:

$$T(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta) \,,$$

angular momentum

partial-wave amplitude

$\cos\theta = 1 + 2t/s$

can write total cross section as

$$\sigma = 16\pi \frac{1}{s} \sum_J (2J+1) |a_J(s)|^2$$

$W_L^+ W_L^- \to W_L^+ W_L^-$

partial-wave expansion in terms of Legendre polynomials P_J:

$$T(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta) \, ,$$

divergent behavior $\leftrightarrow$ $J = 0, 1, 2$ partial waves

$$a_0 \;=\; -\frac{G_F M_H^2}{8\pi\sqrt{2}} \left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s} \ln\left(1 + \frac{s}{M_H^2}\right)\right]$$

high-energy limit: $$a_0 \underset{s \gg M_H^2}{\longrightarrow} -\frac{G_F M_H^2}{4\pi\sqrt{2}}$$

$W_L^+ W_L^- \to W_L^+ W_L^-$

high-energy limit: $a_0 \underset{s \gg M_H^2}{\longrightarrow} -\frac{G_F M_H^2}{4\pi\sqrt{2}}$

partial-wave unitarity condition:

$$|a_0| \leq 1 \quad \Longrightarrow \quad \frac{G_F M_H^2}{4\pi\sqrt{2}} \leq 1$$

upper bound on the Higgs mass:

$$M_H^2 \leq \frac{4\pi\sqrt{2}}{G_F} \lesssim 1.5 \text{ TeV}^2$$

$V_L V_L \to V_L V_L$

refinement via additional channels $(Z_L Z_L, HZ_L, HH)$:

$$M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F} \right)^{1/2} \lesssim 1\ \mathrm{TeV}$$

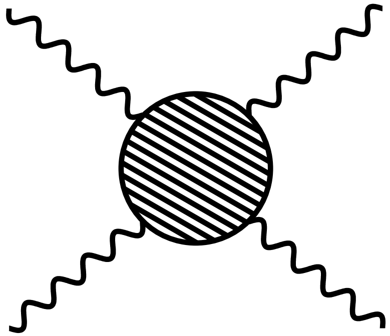
- ☞ weak interactions remain weak at all energies
- ☞ perturbation theory reliable everywhere

alternative:

- ✥ weak interactions become strong at the TeV scale
- ✥ perturbation theory breaks down

$\to$ new physics effects

$V_L V_L \to V_L V_L$



alternative:

- weak interactions become strong at the TeV scale
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$\to$ new physics effects

implications of unitarity in VBS

historic speculations:

- heavy Higgs boson?

  challenging: in TeV range $\Gamma_H \sim M_H$

- no Higgs boson?

  - strong interactions of V_L modes?
  - resonances in TeV range?
  - spectacular signals ?

implications of unitarity in VBS

historic speculations:

- heavy Higgs boso[illegible]

  c[illegible]en[illegible] in TeV range $\Gamma_H \sim M_H$

- no Higg[illegible]

  - st[illegible]g inte[illegible]tions of V_L modes?
  - resonances in TeV range?
  - spectacular signals ?

. . . OUTDATED!!

approaches to VBS today

experimental fact: $M_H = 125$ GeV $\ll 1$ TeV

☞ BSM effects expected to be small

in this kinematic range

- specific models with one or more Higgs boson(s)
- model-independent effective analysis

based on particle content of the SM:

$$\mathcal{L}_{\text{eff}} = \sum \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)} = \mathcal{L}_{\text{SM}} + \sum \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \ \dots$$

effective operator approach

$$\mathcal{L}_{\rm eff} = \sum \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)} = \mathcal{L}_{\rm SM} + \sum \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \ \ldots$$

- operators constructed to obey all symmetries of the theory
- choice of operator parameterization is not unique
  (most useful basis depends on process)
- equations of motion relate differerent operators
- generically EFT operators yield contributions of order $\mathcal{O}(s/\Lambda^2)$
- approach valid at scales far below new physics ($E \ll \Lambda$)
  at large scales, expect violations of unitarity
  (can be cured by form factors, but introduce arbitrariness)

effective operator approach

$$\mathcal{L}_{\text{eff}} = \sum \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)} = \mathcal{L}_{\text{SM}} + \sum \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \ldots$$

- operators constructed to obey all symmetries of the theory
- choice of operator parameterization is not unique
  (most useful basis depends on process)
- equations of motion relate different operators
- generically EFT operators yield contributions of order $\mathcal{O}(s/\Lambda^2)$
- approach valid at scales far below new physics ($E \ll \Lambda$)
  at large scales, expect violations of unitarity
  (can be cured by form factors, but introduce arbitrariness)

for details see lectures by Aneesh Manohar and Fabio Maltoni

effective operator approach

model-independent effective analysis

based on particle content of the SM:

$$\mathcal{L}_{\text{eff}} = \sum \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)} = \mathcal{L}_{\text{SM}} + \sum \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \ \ldots$$

higher-dim operators may give rise to

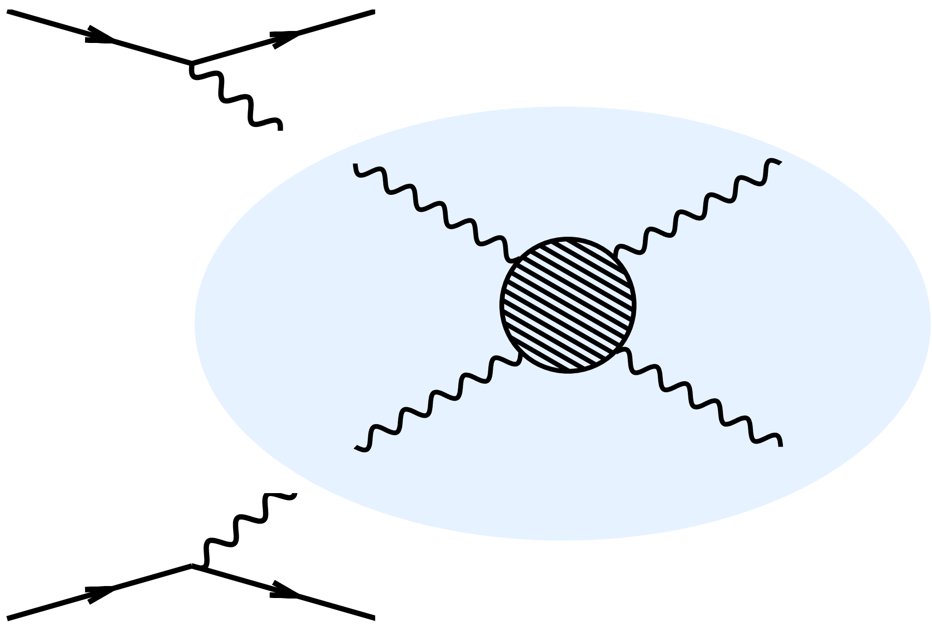
anomalous triple and quartic gauge couplings

☞ precision in theory and experiment needed to identify small deviations from SM

vector boson scattering at colliders

the real word: how can we access weak boson scattering processes at high energies experimentally?

vector boson scattering in e^+e^- collisions?



simplistic approach:

consider heavy gauge bosons as
effective constituents of a fermion

"effective vector boson approximation"

effective vector boson approximation (EVBA)

$$\sigma_{\rm pp} \sim f_{V_1/f_1} \otimes f_{V_2/f_2} \otimes \hat{\sigma}_{V_1 V_2 \to V_3 V_4}$$

probabilities for finding boson with longitudinal momentum fraction x in a fermion depend on its polarization:

$$P_T(x, p_T) = \frac{g_W^2}{16\pi^2} \frac{1 + (1-x)^2}{x} \frac{p_T^3}{[(1-x)m_V^2 + p_T^2]^2}$$

$$P_L(x, p_T) = \frac{g_W^2}{16\pi^2} \frac{1 - x}{x} \frac{2(1-x)m_V^2 p_T}{[(1-x)m_V^2 + p_T^2]^2}$$

effective vector boson approximation (EVBA)

$$\sigma_{\text{pp}} \sim f_{V_1/f_1} \otimes f_{V_2/f_2} \otimes \hat{\sigma}_{V_1 V_2 \to V_3 V_4}$$

probabilities for finding boson with longitudinal momentum fraction x in a fermion depend on its polarization:

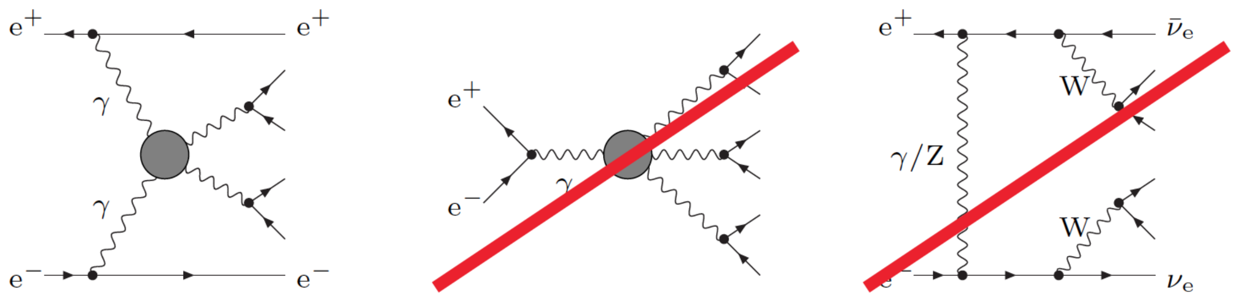
transverse modes:

- suppressed for $p_T \ll M_W$
- enhanced in central scattering region for $p_T > M_W$

☞ kinematic features can be used to define cuts that favor longitudinal components

EVBA in QED

analogy: Weizsäcker-Williams approximation for “photon in fermion”



photon flux collinear to $e^{\pm}$ universally enhanced by $\alpha \ln(s/m_e^2)$

$$\sigma_{\mathrm{ee}} \sim P_{e \to e\gamma^\star} \otimes P_{e \to e\gamma^\star} \otimes \hat{\sigma}_{\gamma\gamma \to VV}$$

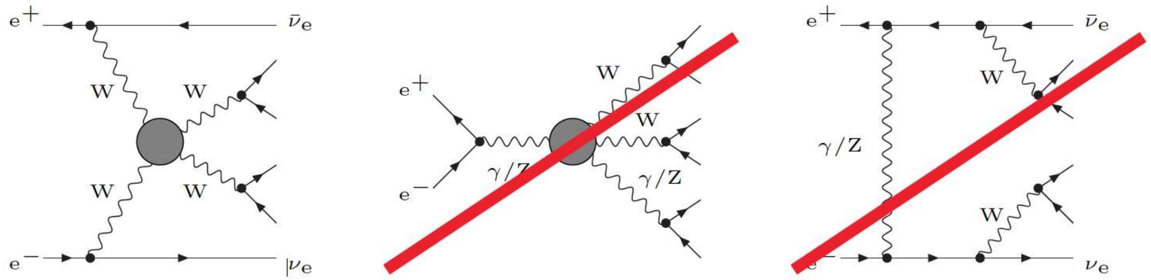
approximations:

✘ neglect irreducible background diagrams (keep only $\gamma\gamma \to VV$)
✘ project photon off-shellness to $q_\gamma^2 = 0$
✘ approach intrinsically restricted to contributions $\sim \ln(s/m_e^2)$

☞ expect quality to improve with energy

EVBA with massive bosons

[c.f. Chanowitz; Kane et al. (1984); Dawson (1985); Kuss, Spiesberger (1995), . . .]



W flux collinear to $e^\pm$ universally enhanced by $\alpha \ln(s/M_W^2)$

$$\sigma_{\mathrm{ee}} \sim P_{e\to\nu W^\star} \otimes P_{e\to\nu W^\star} \otimes \hat{\sigma}_{WW\to VV}$$

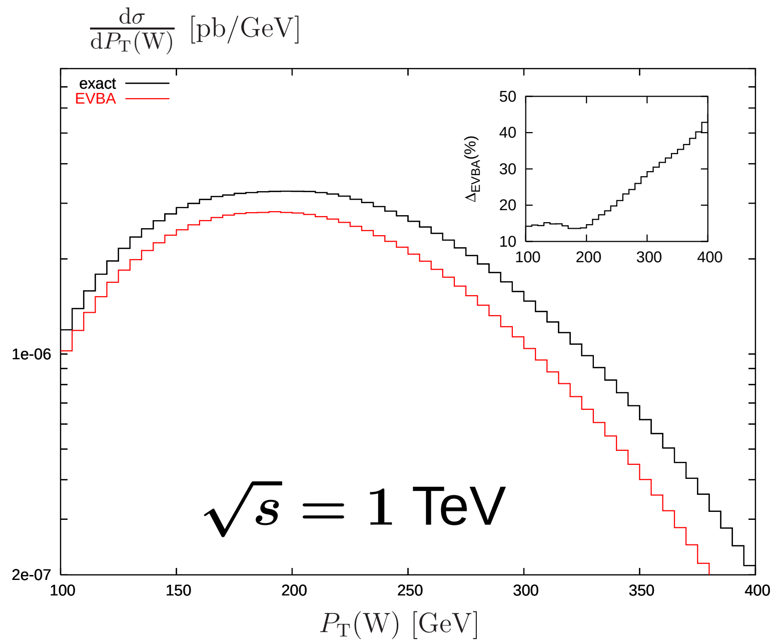
approximations:

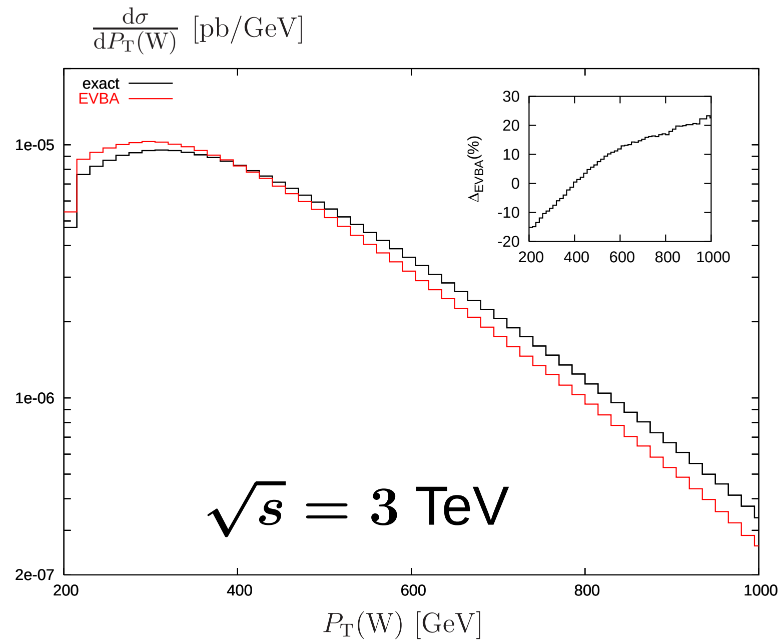
- ✘ neglect irreducible background diagrams (keep only $WW \to VV$)
- ✘ project W off-shellness $q_W^2 < 0$ to $q_W^2 = M_W^2$
- ✘ approach restricted to contributions $\sim \ln(s/M_W^2), s/M_W^2$

☞ not expected to work well unless $\sqrt{s} \gg 1$ TeV

EVBA versus full calculation in e^+e^- collisions

$e^+e^- \to W^+W^-\nu_e\bar{\nu}_e$ at high energies *[Accomando et al. (2006)]*

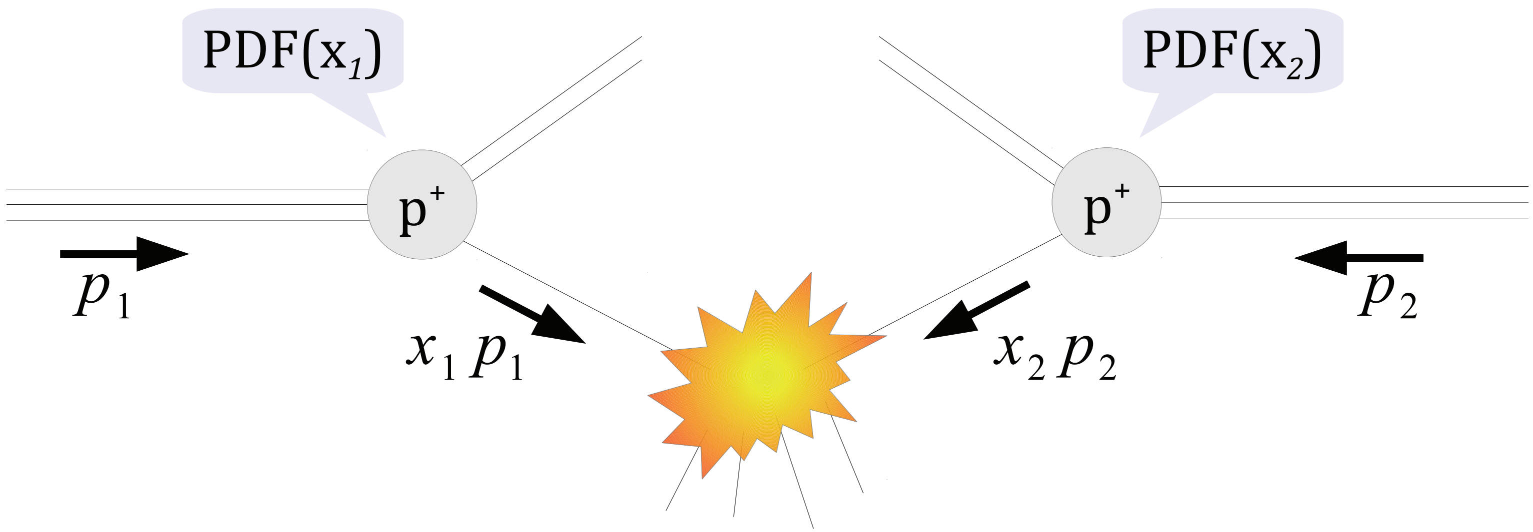




- ✘ quality of EVBA improves with increasing energy
- ✘ still uncertainties of several tens of percent

☞ EVBA may provide qualitative estimates, but no precise results!

hadron-hadron collision

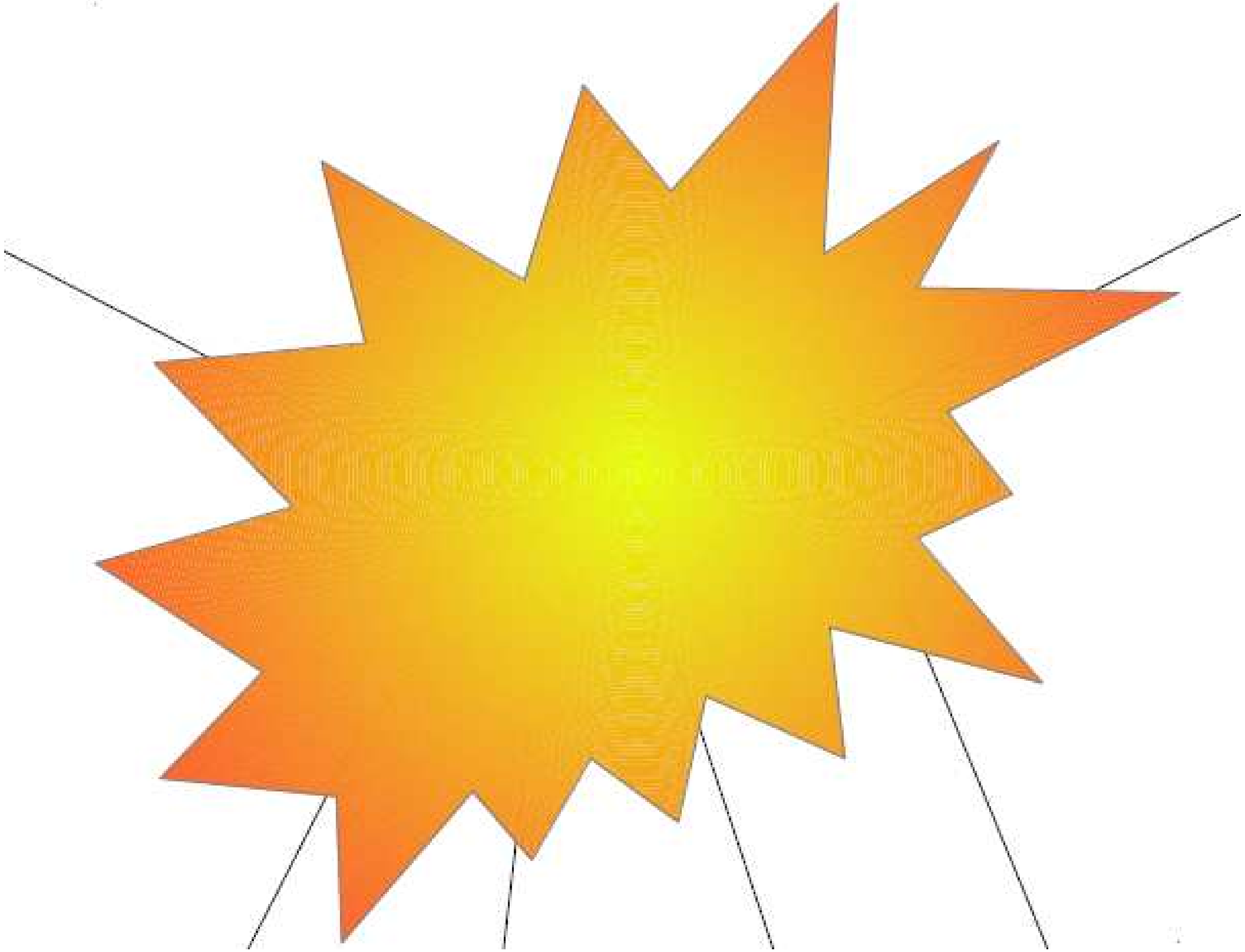


foundation for predictive power of perturbative QCD:

long-distance structure of hadrons
can be separated from
hard partonic scattering

☞ factorization

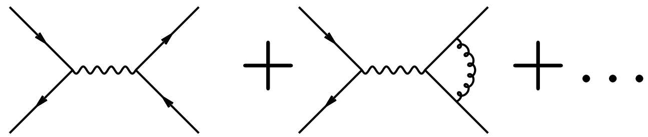
hadron-hadron collision?



hard scattering: the perturbative approach

QCD @ high energies: (ideally) series expansion in α_s

$$\sigma = \sum_{n=n_0}^{N} \alpha_s^n \sigma^{(n)} + \mathcal{O}(\alpha_s^{N+1})$$



truncation at fixed order α_s^N ($\rightarrow$ LO, NLO, ...)

order N provided by theoretician *("# of loops")* depends on:

- complexity of the problem
  - kinematic properties of the reaction
  - multiplicity of the final state *("# of legs")*
  - mass scales of involved particles
  - ...
- accuracy which can be achieved in experiment
- computational skills of the perturbationist

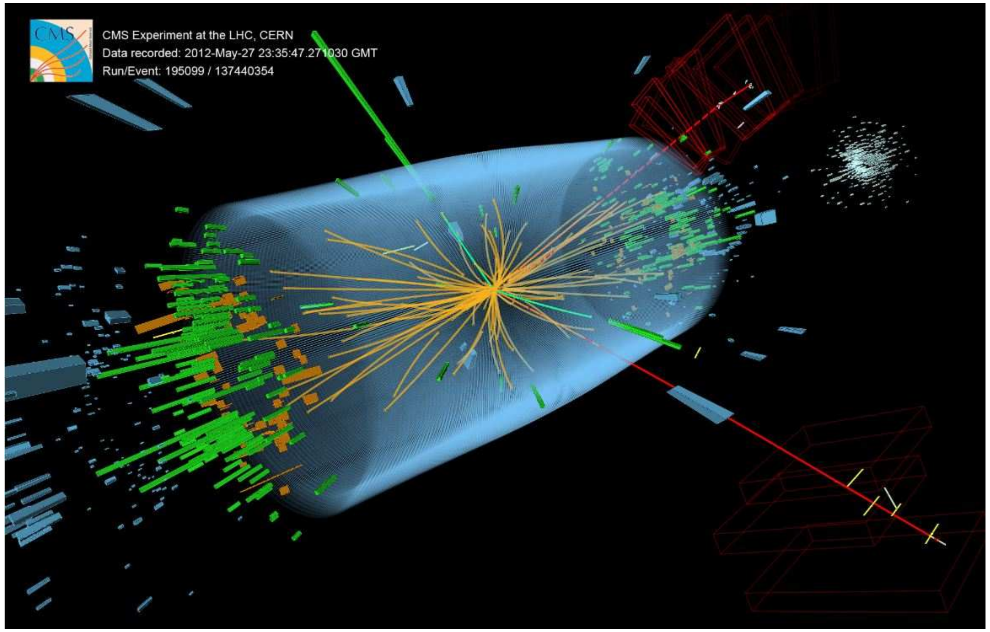
loops and legs at the LHC

legs

realistic description of complex final state

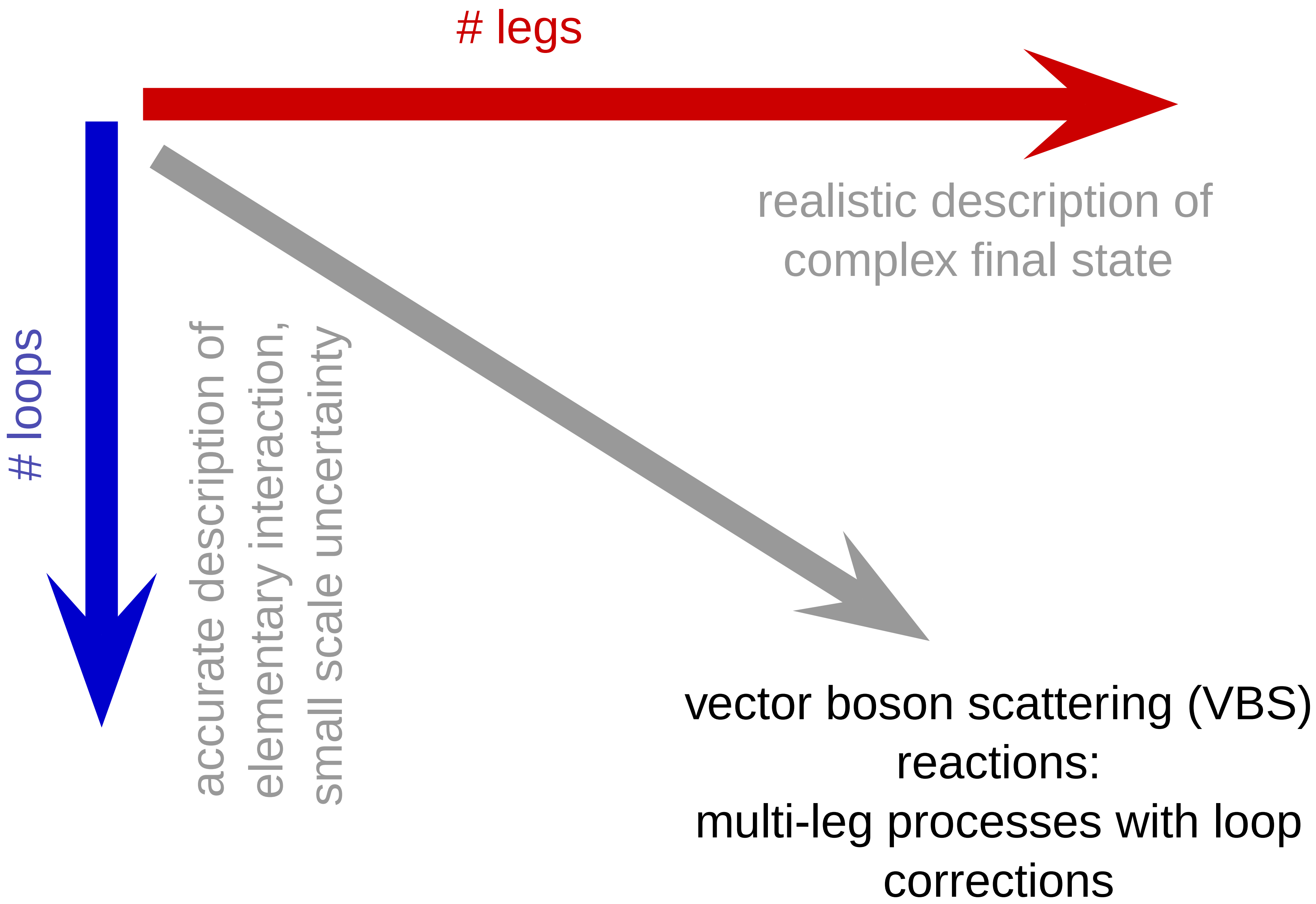
loops

accurate description of elementary interaction, small scale uncertainty

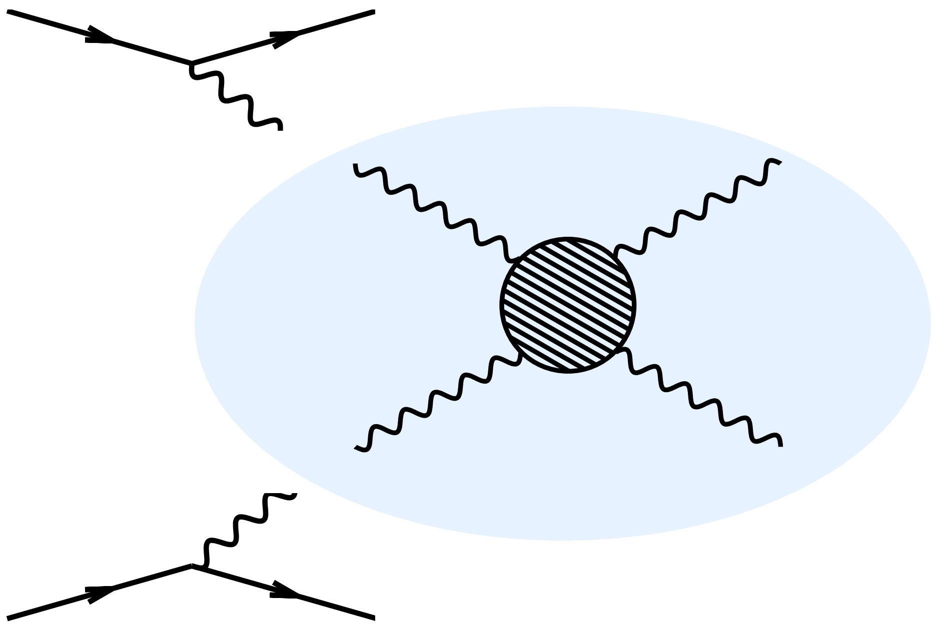


Higgs event recorded by CMS

loops and legs at the LHC: an example



vector boson scattering in pp collisions?



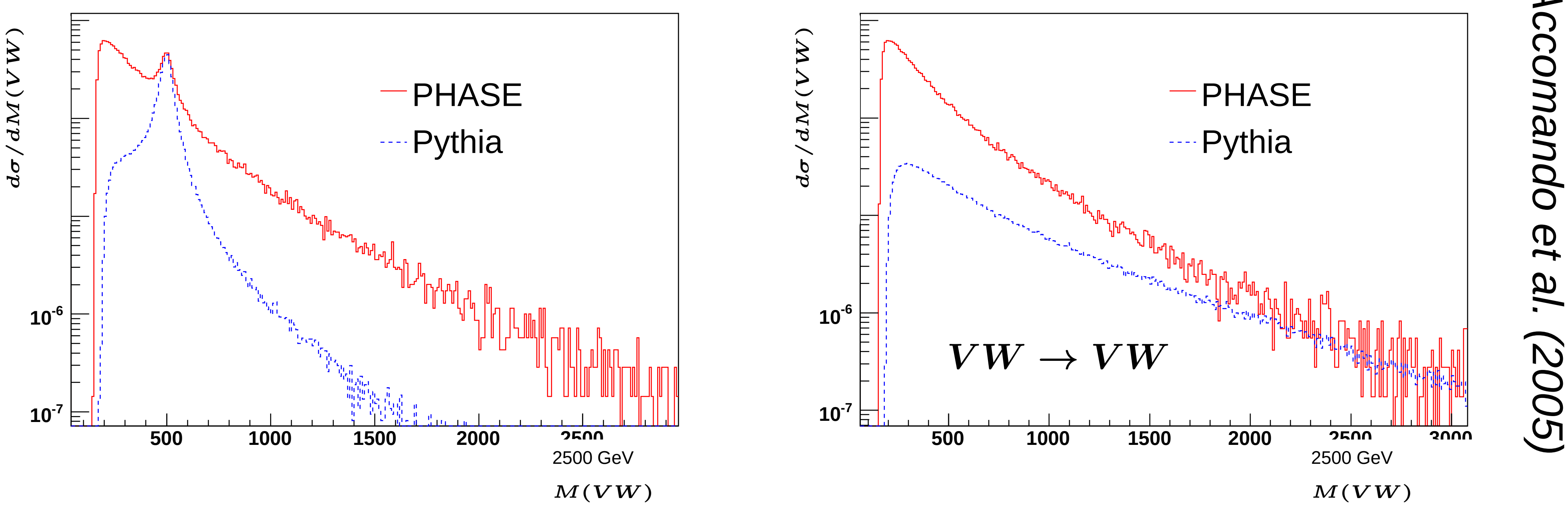
effective vector boson approximation:

analogous to the EVBA in e^+e^- collisions:

consider heavy gauge bosons as
effective constituents of a fermion

EVBA: quality of the approximation at the LHC

useful for studying qualitative features, but agreement with full parton-level calculation only within a factor of 2



PHASE:
off-shell effects fully considered

PYTHIA:
EWA with longitudinal vector bosons

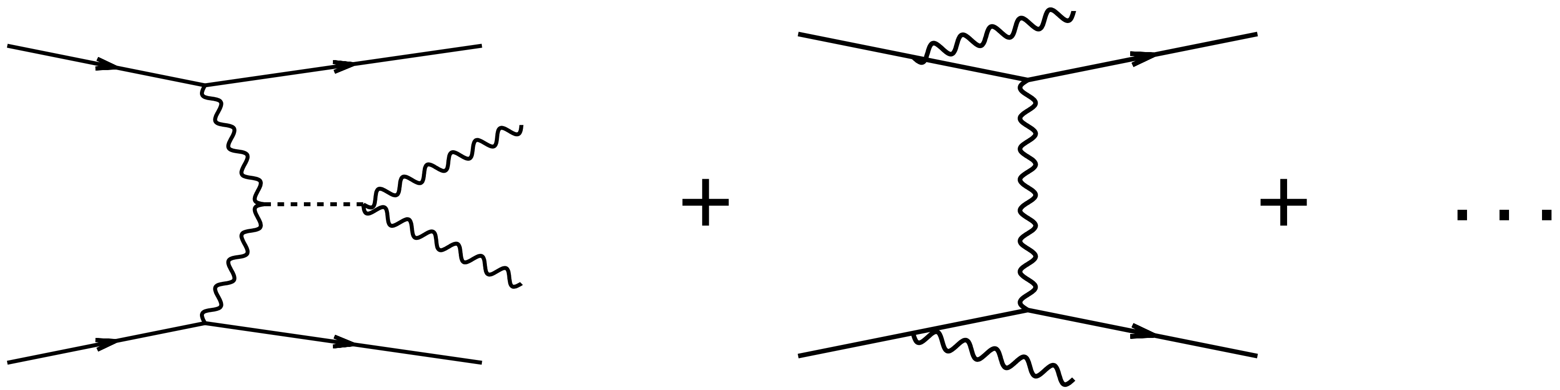
beyond the EVBA: full matrix elements

essential improvement:

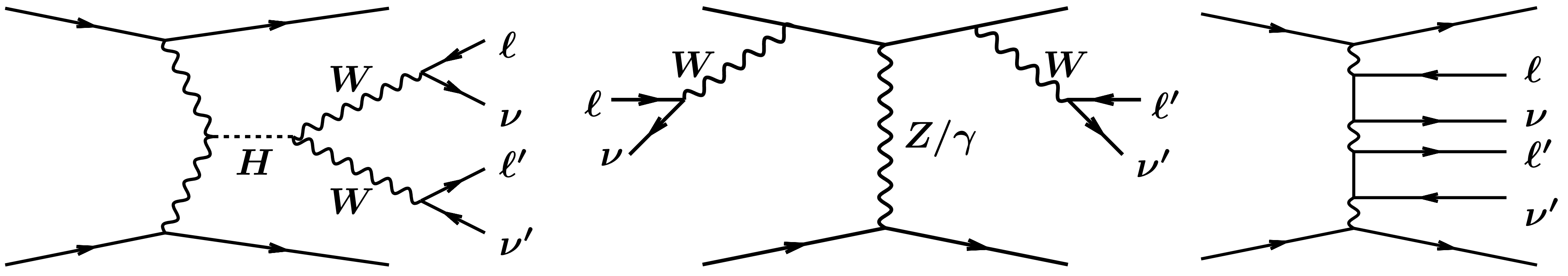
full calculation of matrix elements for $qq \to qqVV$

instead of

convolution of the $VV \to VV$ scattering amplitudes with distribution functions for the weak bosons inside the quarks



$pp \to VVjj$: vector boson scattering in the Standard Model

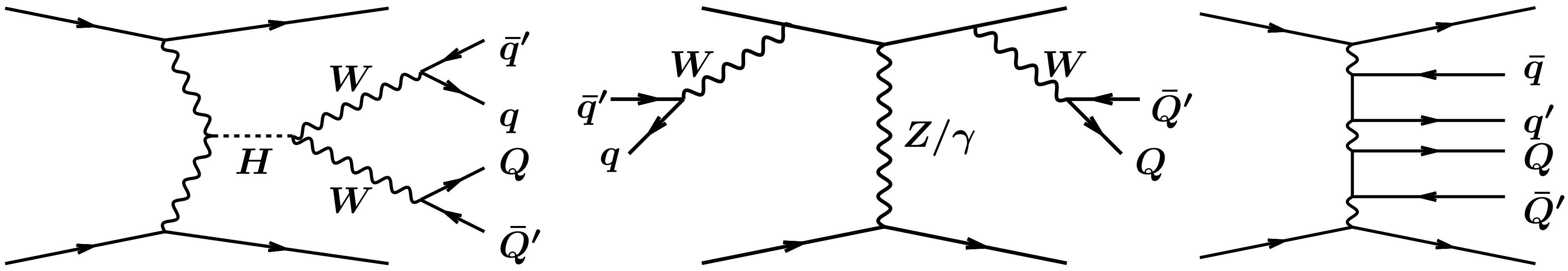


experiment: don't observe $VVjj$ final state, but
hadronic or leptonic decay products

$4\text{leptons} + jj$

low statistics
clean signature

$pp \to VVjj$: vector boson scattering in the Standard Model



experiment: don't observe $VVjj$ final state, but hadronic or leptonic decay products

$\mathbf{4jets} + jj$

high statistics
large backgrounds

$\mathbf{4leptons} + jj$

low statistics
clean signature

how does a calculation proceed in practice?

- leading-order calculation and Monte-Carlo programs `PHASE,PHANTOM`

Accomando, Ballestrero, Maina et al. (2005 ff.)

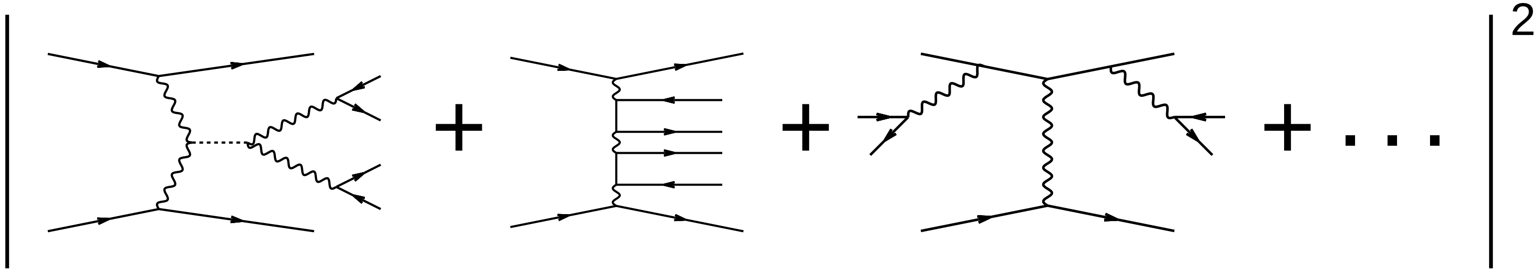
☞ tree-level part of an NLO calculation

Bozzi, Oleari, Zeppenfeld, B.J. (2006 ff.)

the leading order

let's focus on $pp \to jje^+\nu_e\mu^-\bar{\nu}_\mu$ (short: "$pp \to jjW^+W^-$")

need to compute numerical value for

$|\mathcal{M}_B|^2 =$ 

at each generated phase space point in 4 dim (finite)

. . . altogether 92 diagrams for CC, 181 diagrams for NC processes (and even more for $pp \to jjZZ$)

matrix elements can be computed numerically using amplitude techniques

evaluation of Feynman diagrams

need to evaluate

$$\sum_{\textbf{helicities}} |\mathcal{M}|^2 = \sum_{\textbf{helicities}} (\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \ldots) \cdot (\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \ldots)^\star$$

amplitude techniques:

evaluate $\mathcal{M} = (\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \ldots)$ first numerically for specific helicities of external particles, then square it!

fast numerical programs and many implementations available, e.g.

approach proposed by *Hagiwara, Zeppenfeld (1986,1989)*:

implemented in `HELAS` *(Murayama et al., 1992)*

employed by `MadGraph` *(Stelzer et al., 1994ff)*

amplitude techniques

basic approach of `HELAS/MadGraph`:

– each phase space point

$\longrightarrow$ numerical values of external 4-momenta p_i^μ, k_i^μ

– polarization vectors $\varepsilon^\mu(k,\lambda)$ and spinors $u(p,\sigma)$

$\simeq$ complex 4-arrays

– products like

$$\not\varepsilon \frac{1}{\not p - \not k - m} u(p,\lambda)$$

of momenta, polarization vectors, spinors, and γ^μ-matrices are computed via numerical 4×4 matrix multiplication

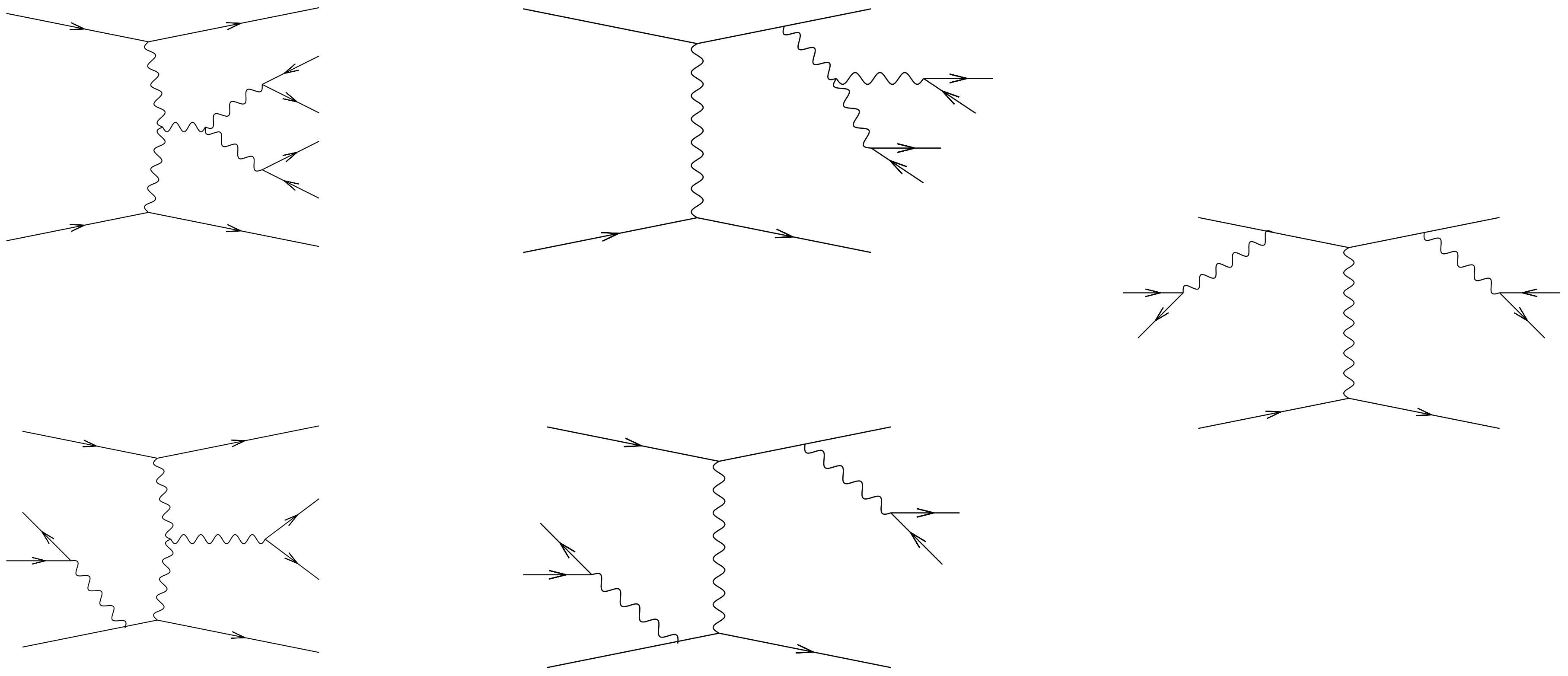
☞ perfect for LO amplitudes

(results completely finite)

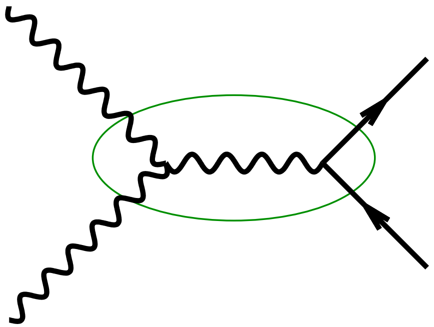
helicity amplitudes

$pp \to jjVV$: need helicity amplitudes

for five different topologies:



finite width effects

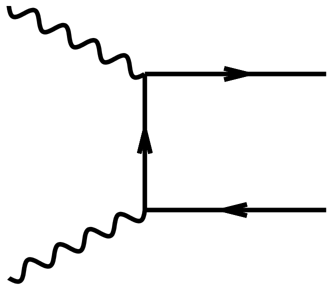


resonant diagrams: need

$$\frac{1}{q^2 - M_V^2} \rightarrow \frac{1}{q^2 - M_V^2 + iM_V\Gamma_V}$$

in s-channel vector-boson propagators

but: how should non-resonant graphs be treated?



naive implementation: violation of EW gauge invariance $\rightarrow$ handle with care!

complex mass scheme *(Denner et al.)*:

$M_V^2 \rightarrow M_V^2 - iM_V\Gamma_V$ in propagators and couplings

finite width effects

our approach: modified complex mass scheme

replace $\quad M_V^2 \to M_V^2 - i M_V \Gamma_V$

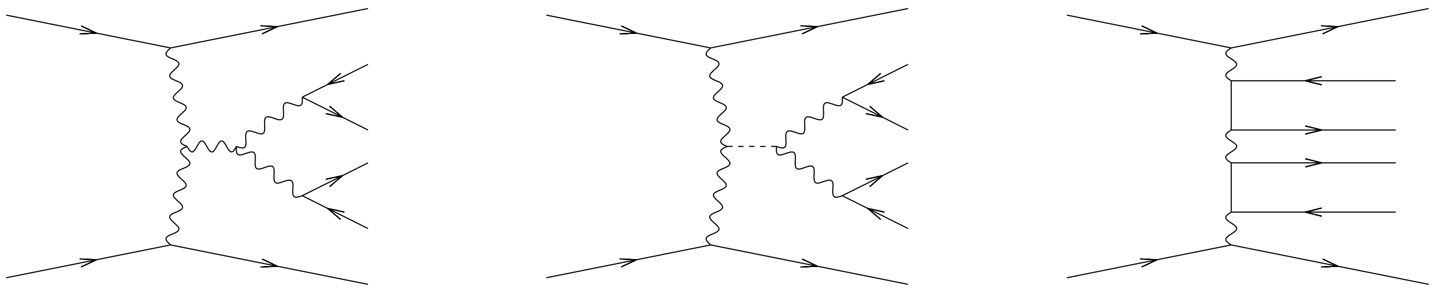
in all weak boson propagators, but not in couplings,

i.e. keep real value for $\sin\theta_W^2 = 1 - \frac{M_W^2}{M_Z^2}$

↓

- easy to implement (cf. `MadGraph`)
- preserves em. gauge invariance
- check for $pp \to jjV$: ambiguity $\lesssim 0.5\ \%$

. . . all t-channel diagrams that contribute
to $e^+\nu_e\mu^-\bar{\nu}_\mu$ in the final state

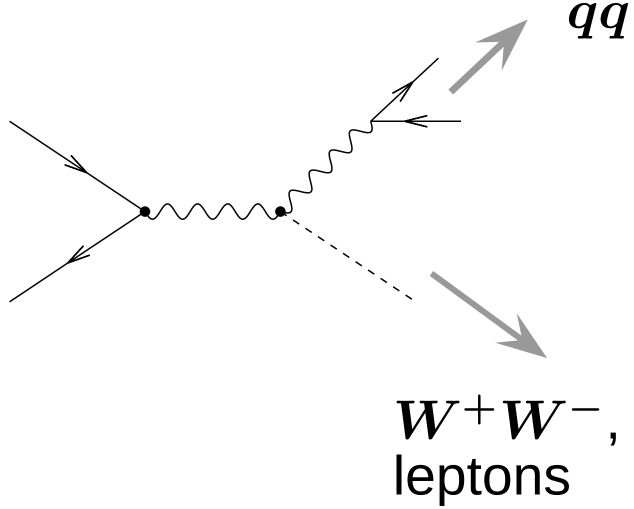


leptons not necessarily produced
via $W^+ \to e^+\nu_e$ and $W^- \to \mu^-\bar{\nu}_\mu$
(non-resonant diagrams considered)

not included

· interference effects from diagrams obtained by interchanging identical initial- or final-state (anti)quarks

· identical flavor annihilation processes with subsequent decay into quarks and similar contributions like



not included

· interference effects from diagrams obtained by interchanging identical initial- or final-state (anti)quarks

· identical flavor annihilation processes with subsequent decay into quarks and similar contributions

neglected terms strongly suppressed in PS region where VBF can be observed experimentally

(require two widely separated quark jets of large invariant mass)

. . . both contributions on sub-percent level for related VBF processes (checked)

Oleari, Zeppenfeld (2003); Ciccolini et al. (2007); Andersen et al. (2007); Bredenstein et al. (2008)

impact of neglected contributions

no full calculation for EW $VVjj$ production available, but careful analysis for related case of EW Hjj production *[Ciccolini et al. (2007)]*

σ [fb]	120 GeV	150 GeV	120 GeV	150 GeV
LO	5943(1)	4331(1)	1876.3(5)	1589.8(4)
NLO	5872(2)	4202(2)	1665(1)	1407.5(8)
LO, s	1294.4(2)	639.4(1)	0.0025	0.0015
NLO, s	1582.1(4)	769.4(2)	9.45(1)	5.21(1)
LO, t/u-int	-9.2	-5.6	-0.12	-0.091
NLO, t/u-int	-27.6	-9.4	-0.75	0.17

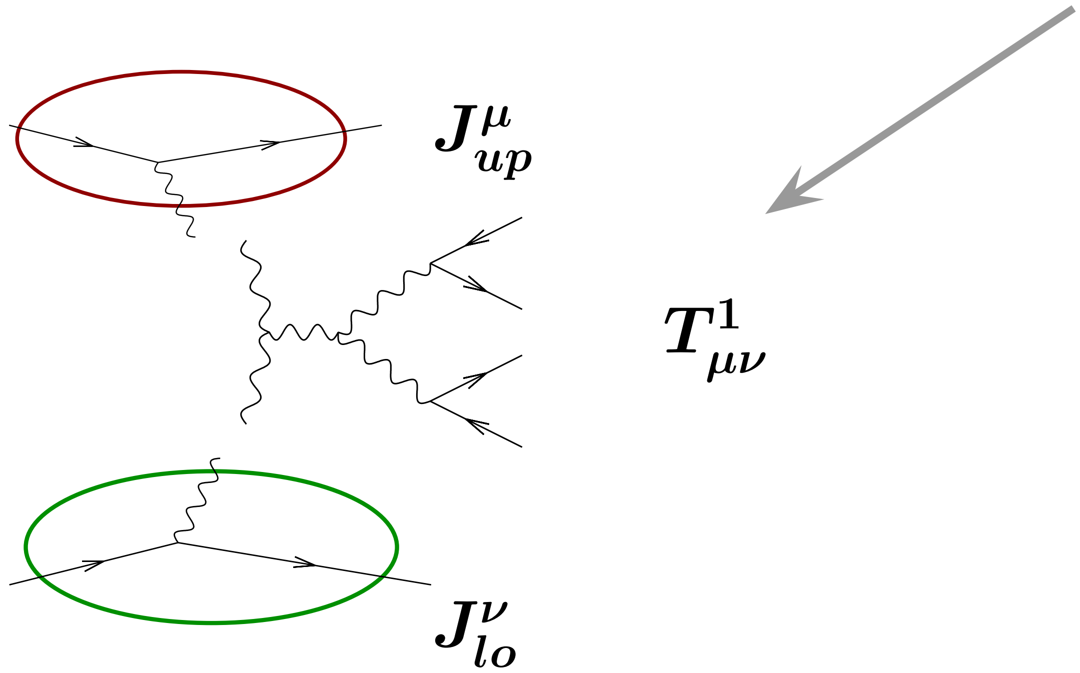
no cuts

VBF cuts

practical implementation

- step I: compute benchmark result for $qq \to qq\, e^+\nu_e\mu^-\bar{\nu}_\mu$ with reference code generated by MadGraph

  + reliable, automatized code generation
  – all diagrams are calculated from scratch, regardless of repeated sub-structures



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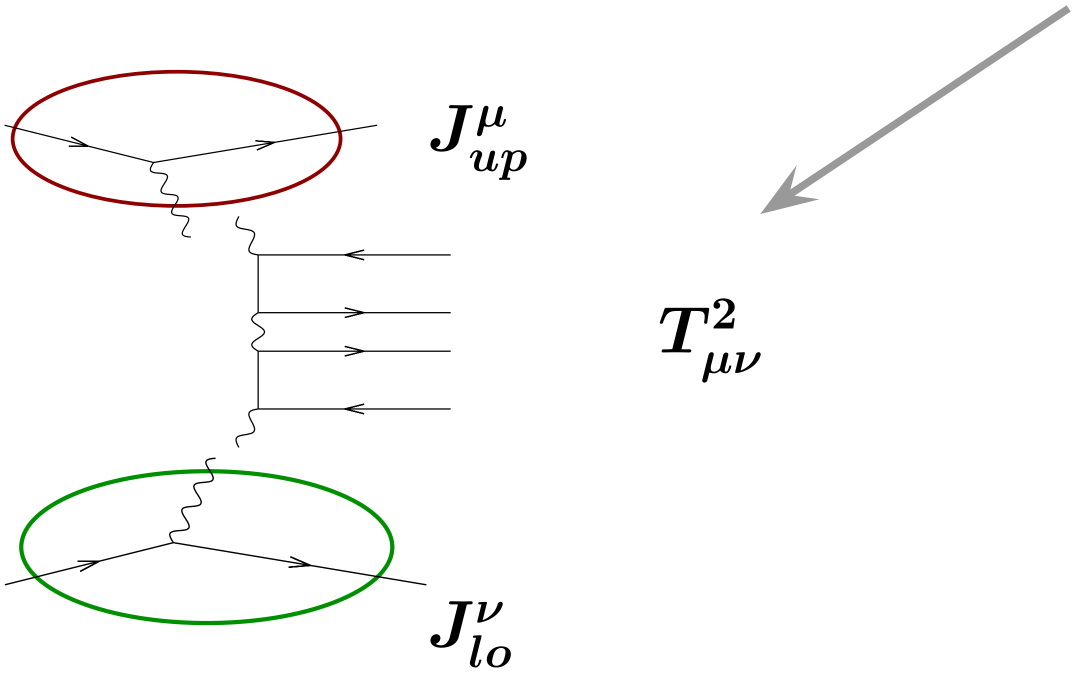
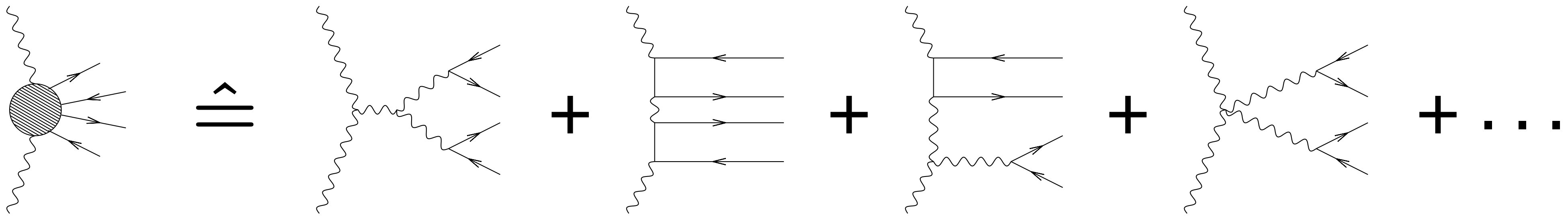


practical implementation

- step II: speed up code by pre-calculating leptonic tensors for individual topologies, e.g. $T_{\mu\nu}$



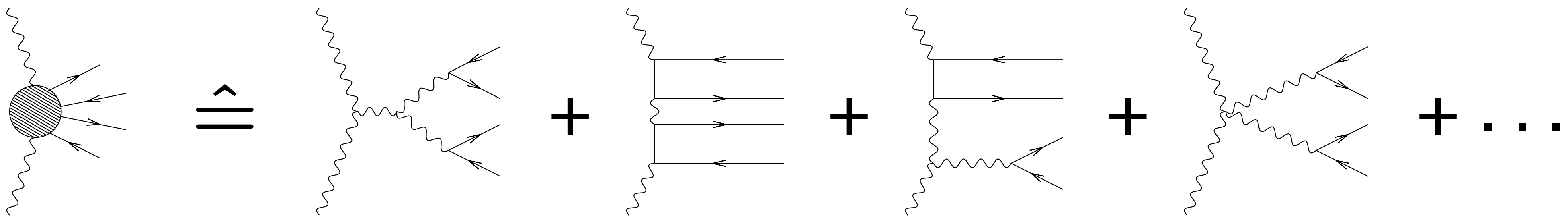
with the help of Helas/MadGraph

· generate sub-process amplitudes with MadGraph and adapt to our needs (e.g. by replacing external polarization tensors with vector boson propagators)

· remaining currents are calculated by hand-made code and contracted

practical implementation

- step II: speed up code by pre-calculating leptonic tensors for individual topologies, e.g. $T_{\mu\nu}$

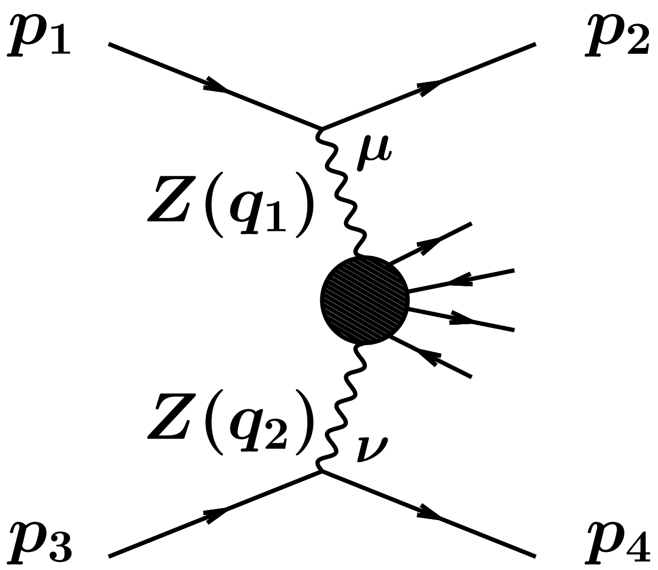


calculate each building block only once per phase space point

modular structure $\rightarrow$ can also be used for real emission contributions

implementation of "new physics" straightforward

practical implementation: example



for NC process $uc \to uc\, e^+\nu_e\mu^-\bar{\nu}_\mu$ need:

$$J^\mu_{up} = -\frac{ig}{\cos\theta_w}\bar{u}(p_2)\gamma^\mu\left(g_V^f + g_A^f\gamma^5\right)u(p_1)$$

$$J^\nu_{lo} = -\frac{ig}{\cos\theta_w}\bar{u}(p_4)\gamma^\mu\left(g_V^f + g_A^f\gamma^5\right)u(p_3)$$

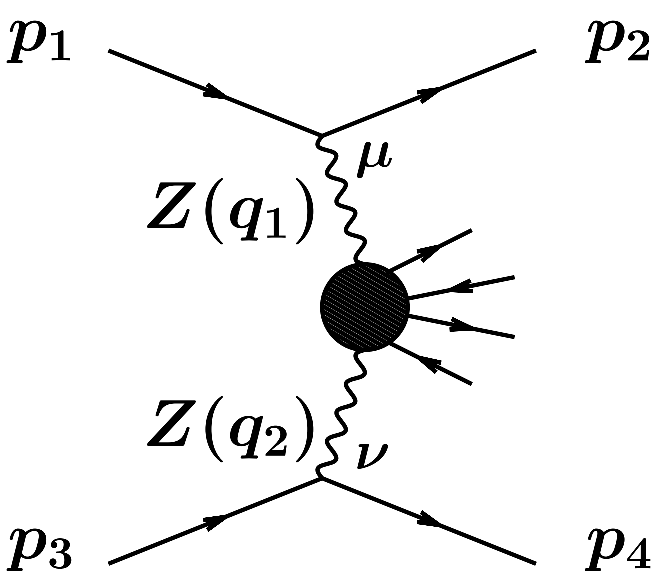
☞ $\mathcal{M}_i = J^\mu_{up} J^\nu_{lo} T_{\mu\nu}$

J^μ_{up}, J^ν_{lo} . . . compute according to *(Hagiwara, Zeppenfeld 1986)* once per phase space point and store

$T_{\mu\nu}$ . . .

- call `MadGraph` for $ZZ \to e^+\nu_e\mu^-\bar{\nu}_\mu$;
- replace $\varepsilon_Z(q_i)$ by Z-propagator; remove polarization sums
- compute tensor coefficients for each $\mu, \nu = 0, \ldots 3$ separatly

practical implementation: example



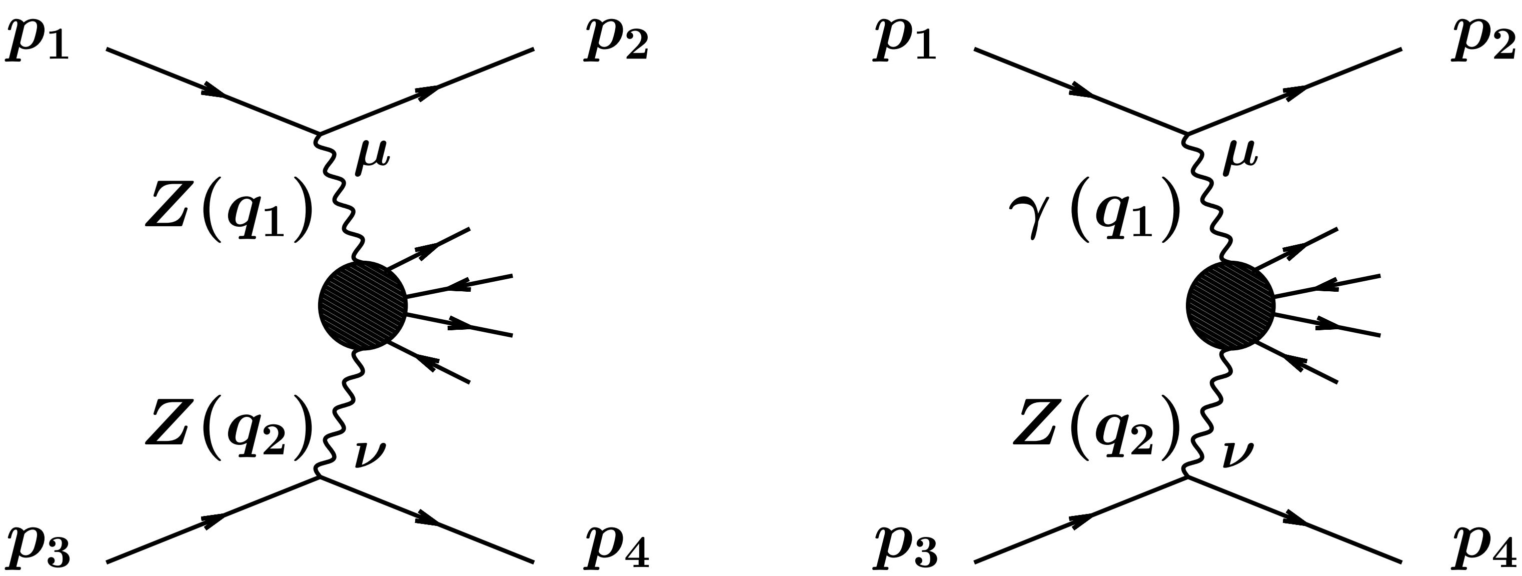
perform contraction with currents explicitly

```
do mu = 0,3
   do nu = 0,3
     M = jup(mu)*jlo(nu)*T(mu,nu)
   enddo
enddo
```

once per phase space point only
(instead of once for each sub-diagram of $\mathcal{M}_i$)

increasingly important as number of diagrams with similar structure increases
(c.f. $pp \to jjZZ$ production and real emission contributions)

practical implementation: example

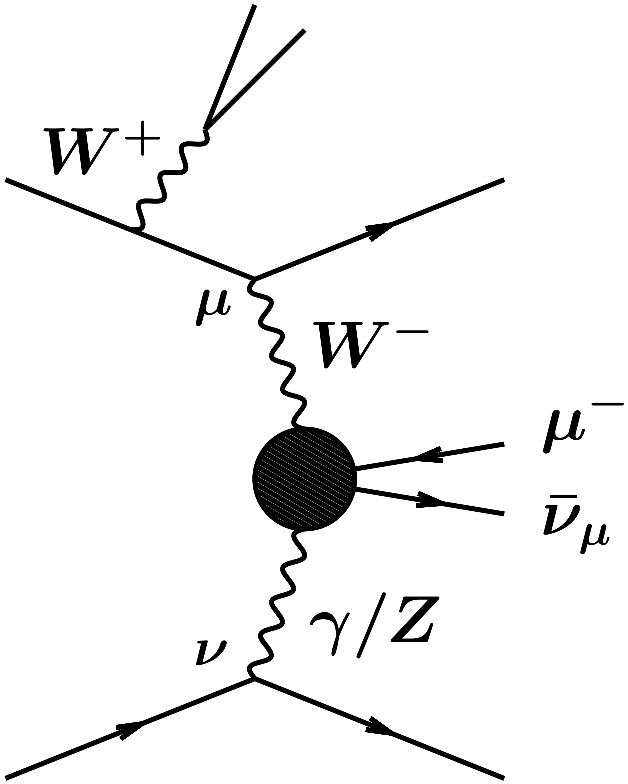


extra: don't compute each sub-process from scratch, but obtain, e.g., $\gamma Z \to e^+\nu_e\mu^-\bar{\nu}_\mu$ from $ZZ \to e^+\nu_e\mu^-\bar{\nu}_\mu$ by

- replacing $Zf\bar{f}$ by $\gamma f\bar{f}$ vertices
(attention: no photon-neutrino coupling)

- replacing massive Z with massless γ propagators

practical implementation: examples (II)



same process ($uc \to uc\, e^+\nu_e\mu^-\bar{\nu}_\mu$), different topology

even more building blocks:

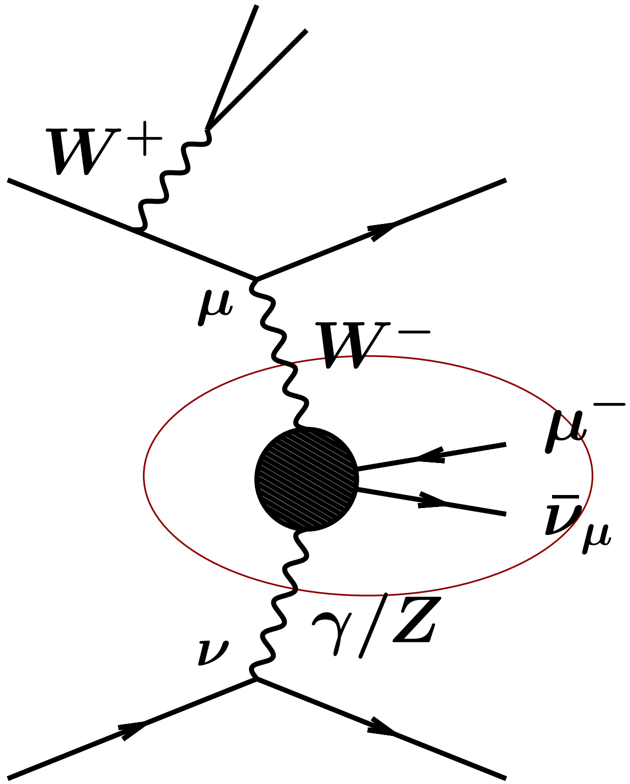
need $\mathcal{M}_i = J^\mu_{up,eff} J^\nu_{lo}\, T^{WV}_{\mu\nu}$

proceed as before:

- compute $T^{WZ}_{\mu\nu}$ by using `MadGraph`: generate $W^-Z \to \mu^-\bar{\nu}_\mu$ and adapt
- modify to obtain tensor for $W^-\gamma \to \mu^-\bar{\nu}_\mu$

- take (stored) current J^ν_{lo} from previous example
- compute "effective polarization vector" for $W^+ \to e^+\nu_e$
- compute "effective current" $J^\mu_{up,eff}$ for upper line

practical implementation: examples (II)



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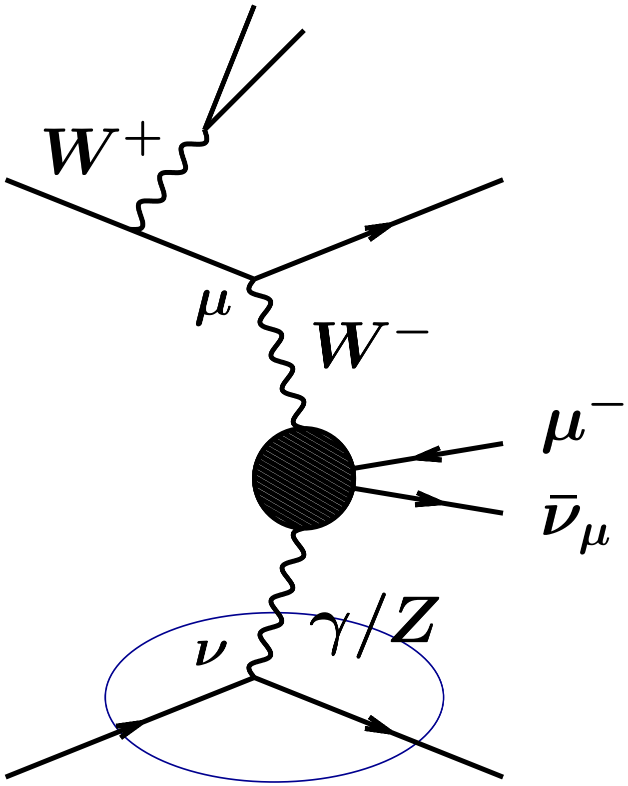

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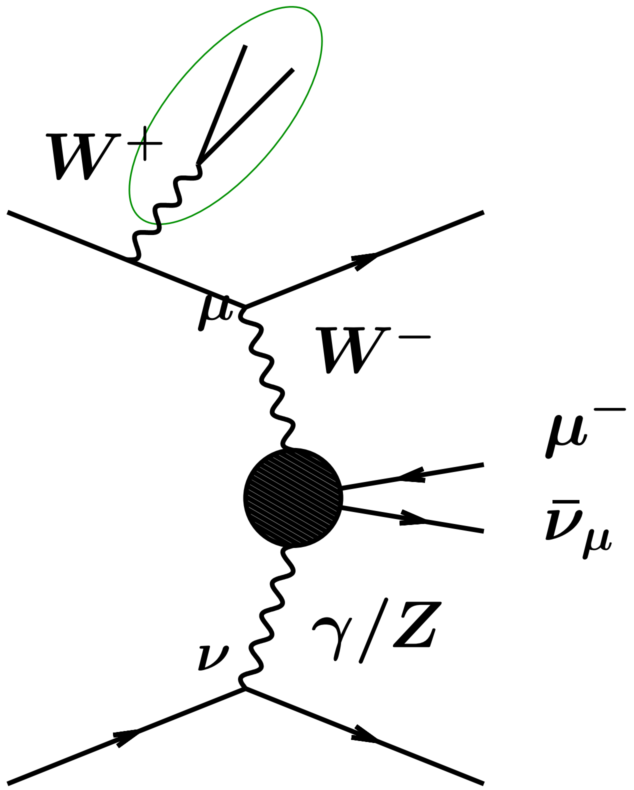

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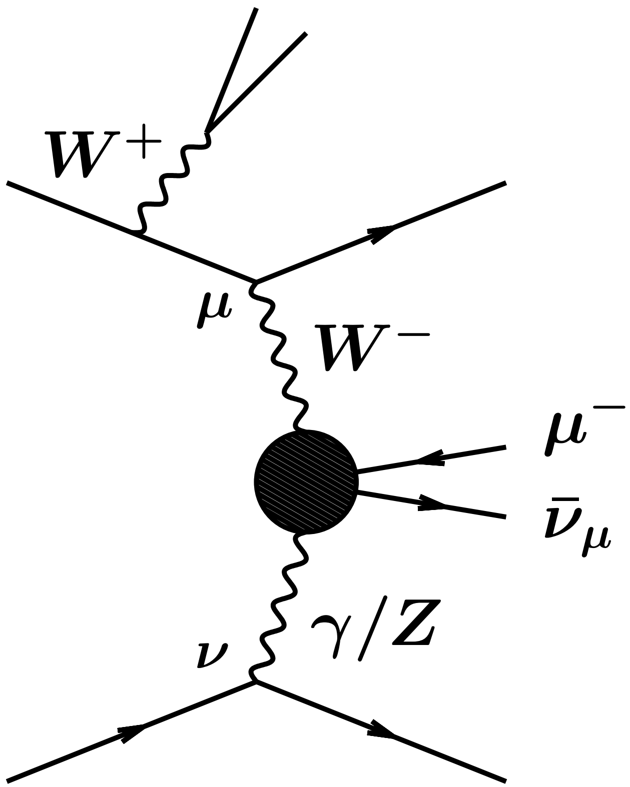
practical implementation: examples (II)



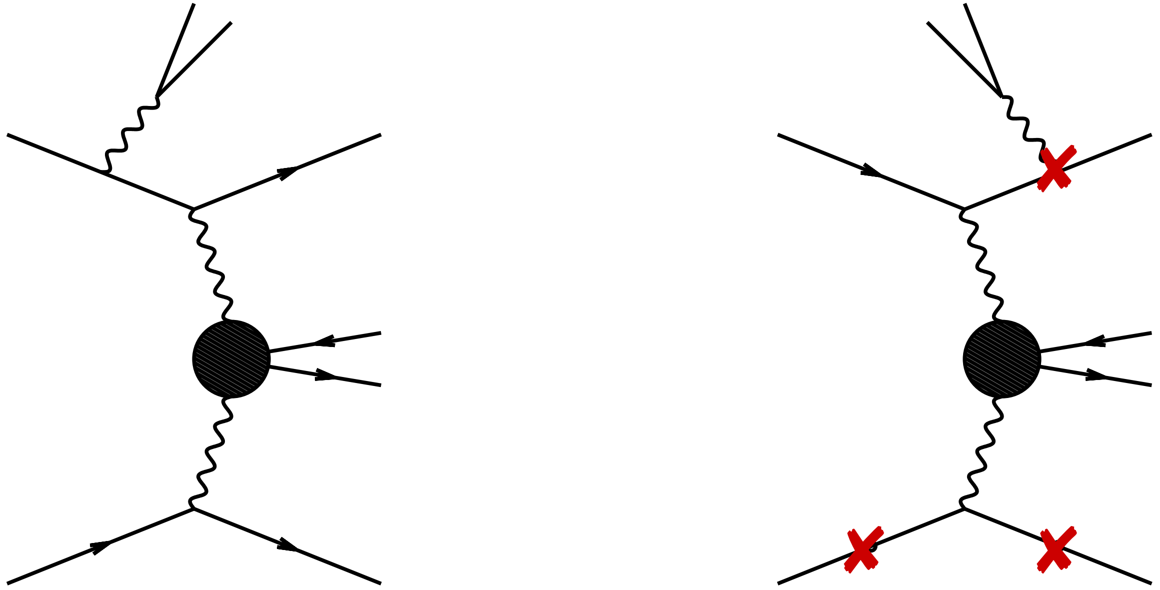
put all pieces together (including suitable couplings)

$$\mathcal{M}_i = J^\mu_{up,eff} J^\nu_{lo} \left(c_Z T^{WZ}_{\mu\nu} + c_\gamma T^{W\gamma}_{\mu\nu} \right)$$

practical implementation: examples (II)

the good news: can recycle all building blocks (tensors, polarization vectors, currents) for lots of other diagrams within one sub-process

same entities emerge for various flavor combinations and crossed processes



such recycling is used to a very small extent by `MadGraph/MadEvent` (within each sub-process and esp. for different sub-processes)

efficiency

- ✘ employ MadGraph for computing first reference result
but: repeated calculation of similar diagrams makes code extremely slow

(2006: about one month CPU time on a single Linux PC for
$\Delta\sigma/\sigma \approx 0.2\%$
for WW and even more in ZZ-case)

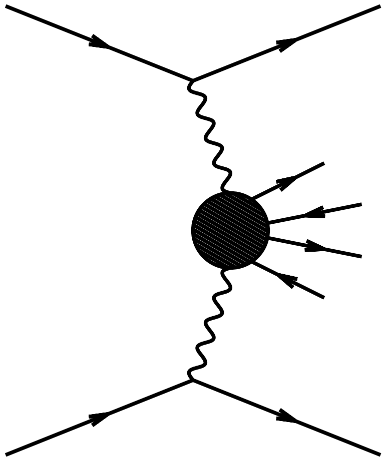
- ✘ high statistics needed especially for kinematic distributions

- ✘ pre-calculate leptonic tensors (for full NLO program)

  ☞ gain speed-up of factor 70 for full code

- ✘ valuable check: comparison to result obtained with MadGraph

extra feature: new physics



nice extra: implementation of new interactions in leptonic tensors straightforward, e.g.

extend SM to effective theory by adding additional terms:

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_i \frac{f_i^5}{\Lambda} \mathcal{O}_i^5 + \sum_i \frac{f_i^6}{\Lambda^2} \mathcal{O}_i^6 + \ldots$$

f_i^5, f_i^6 . . . dimensionless coupling constants

Λ . . . "new physics" scale

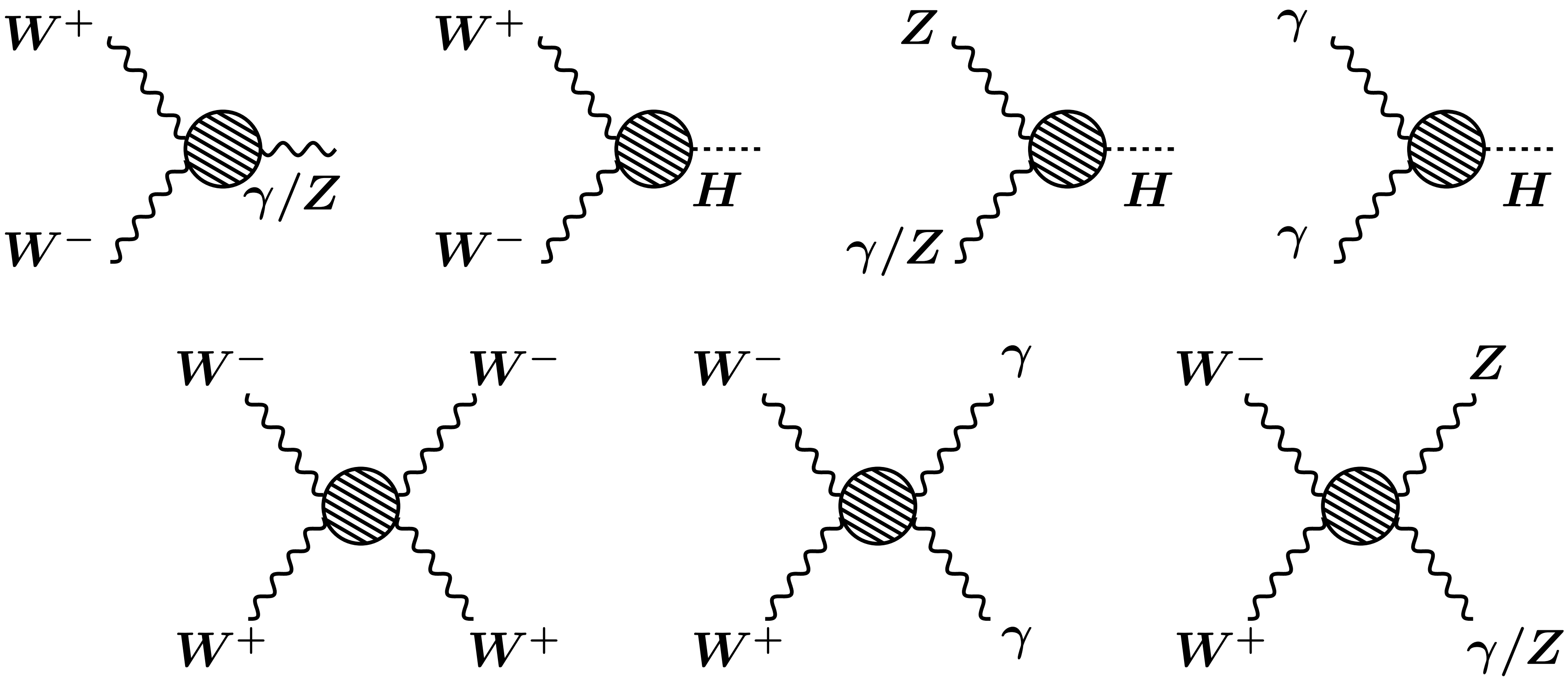
$\mathcal{O}_i^5$ . . . not SU(2) and Lorentz invariant

☞ consider only dimension 6 operators $\mathcal{O}_i^6$

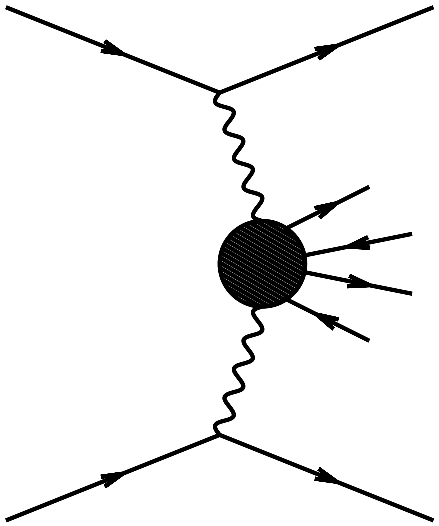
extra feature: new physics

- consider only dim 6 operators which can be built from scalar and vector fields
- derive new Feynman rules from Lagrangian

☞ additional contributions to 3- and 4-boson vertices:



extra feature: new physics



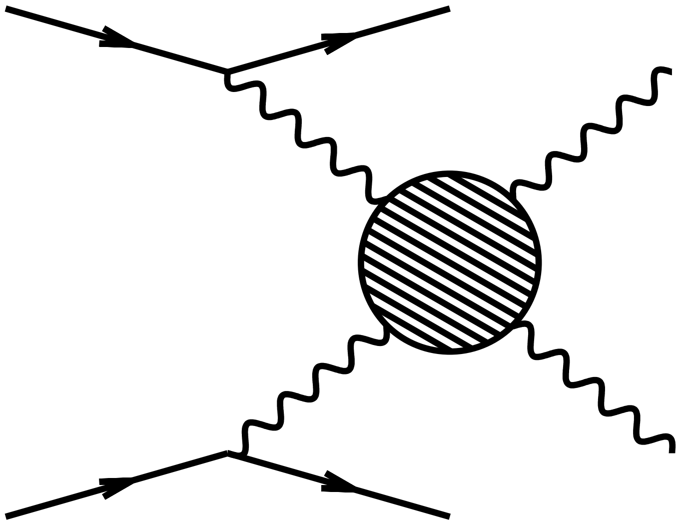
implementation in code:

- use new Feynman rules to compute leptonic tensors
(replace original `Helas` routines by hand-made tensors)

- fermionic pieces don't change

- perform checks: Lorentz and gauge invariance, charateristic high energy behavior

more new interactions in the gauge boson sector



can we distinguish signatures of
SM-type Higgs mechanism from other
scenarios of EW symmetry breaking?

↓

comprehensive analysis of
signal and backgrounds needed

cf. Bagger et al. (1993, 1995); . . .
Englert, Worek, Zeppenfeld, B. J. (2008)

☞ minimize backgrounds with respect to signal
☞ maximize number of surviving signal events

framework: signal

consider two “prototype” scenarios for the VBF signal:

- SM with heavy Higgs boson ($M_H = 1$ TeV, $\Gamma_H = 0.5$ TeV)

  naive estimate of strongly coupled sector with
  scalar, iso-scalar resonance at the TeV scale

- Warped Higgsless model with extra vector resonances

$$\begin{aligned} (m_{W_2} &= 700 \text{ GeV}, \; \Gamma = 13.7 \text{ GeV}, \\ m_{Z_2} &= 695 \text{ GeV}, \; \Gamma = 18.7 \text{ GeV}, \\ m_{Z_3} &= 718 \text{ GeV}, \; \Gamma = 6.4 \text{ GeV}) \end{aligned}$$

the Warped Higgsless model

consider gauge boson sector of Randall-Sundrum scenario with one compactified extra dimension and AdS_5 metric

$$ds^2 = \frac{R^2}{y^2}\left\{g_{\mu\nu}dx^\mu dx^\nu - dy^2\right\}$$

$$R \leq y \leq R'$$

R → Planck brane

R' → TeV brane

5-dim gauge fields decompose under unbroken 4-dim Lorentz group

$A_M(x,y) = (A_\mu, A_5)$ = 4-dim vectors $\oplus$ 4-dim scalar

bulk gauge fixing ➠ A_5 becomes longitudinal component of A_μ

the Warped Higgsless model

boundary conditions along extra dimension
[Csáki, Grojean, Murayama, Pilo, Terning]

➠ Kaluza-Klein decomposition of the gauge fields

$$W_\mu(x,y) = \sum_k \psi_k^{(W)} W_\mu^{(k)}(x)$$

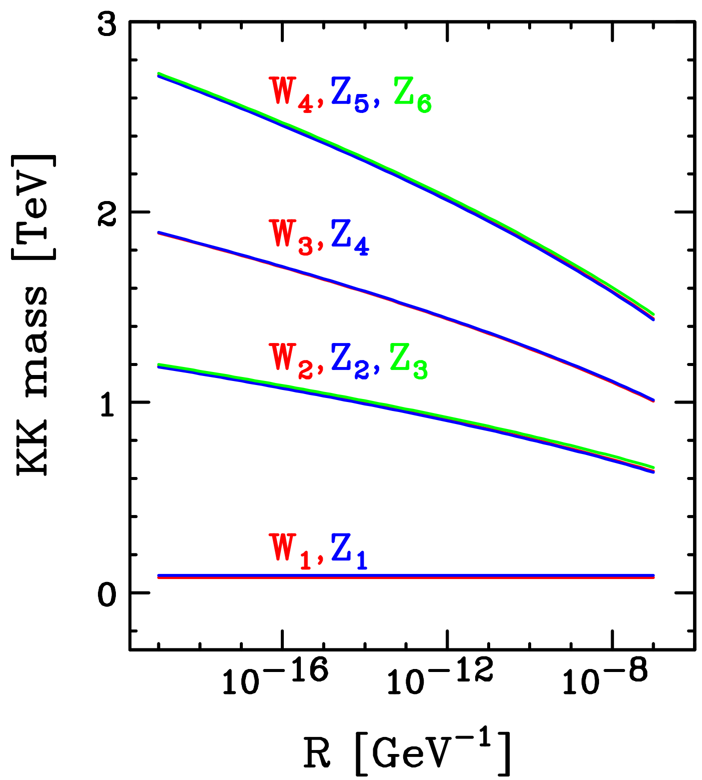
$$Z_\mu(x,y) = \sum_k \psi_k^{(Z)} Z_\mu^{(k)}(x)$$

$k = 0$: photon

$k = 1$: SM Z, $W^\pm$

$k > 1$: KK Z_k, $W_k^\pm$

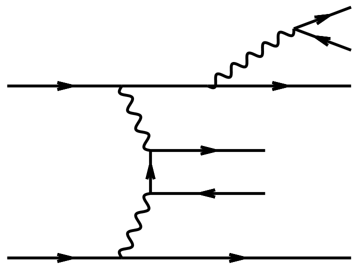
model fully determined by R



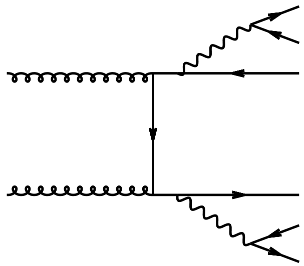
framework: backgrounds

backgrounds to the strongly interacting gauge boson signal in the heavy Higgs (HH) and Kaluza-Klein (KK) scenarios:

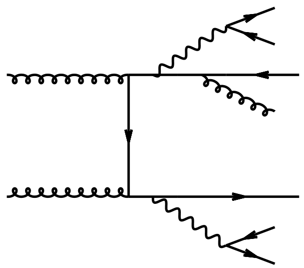
EW $VVjj$ production



QCD $VVjj$ production



$t\bar{t} + \text{jets}$ production (with $t \to Wb$)



leptonic cuts

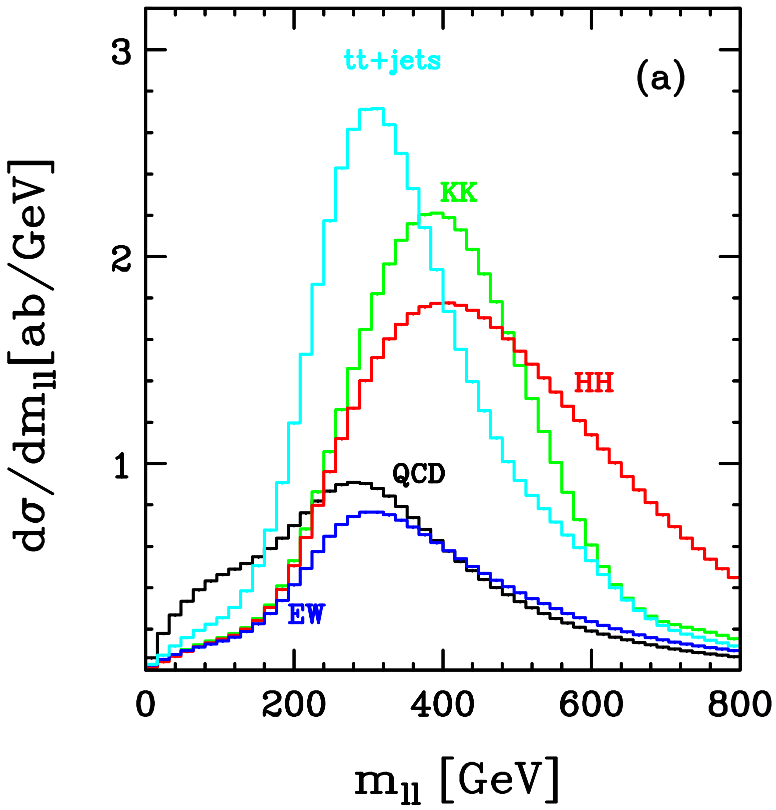
in contrast to backgrounds, signal processes feature energetic leptons of high p_T and large invariant mass

details of leptonic cuts depend on decay channel

$\underline{pp \to W^+W^-jj}$:

$p_{T\ell} > 100$ GeV
$\Delta p_T(\ell\ell) > 250$ GeV

$\min(m_{\ell j}) > 180$ GeV



inclusive cuts, VBF cuts, CJV & b-veto
$p_{T\ell} > 100$ GeV $\min(m_{\ell j}) > 180$ GeV

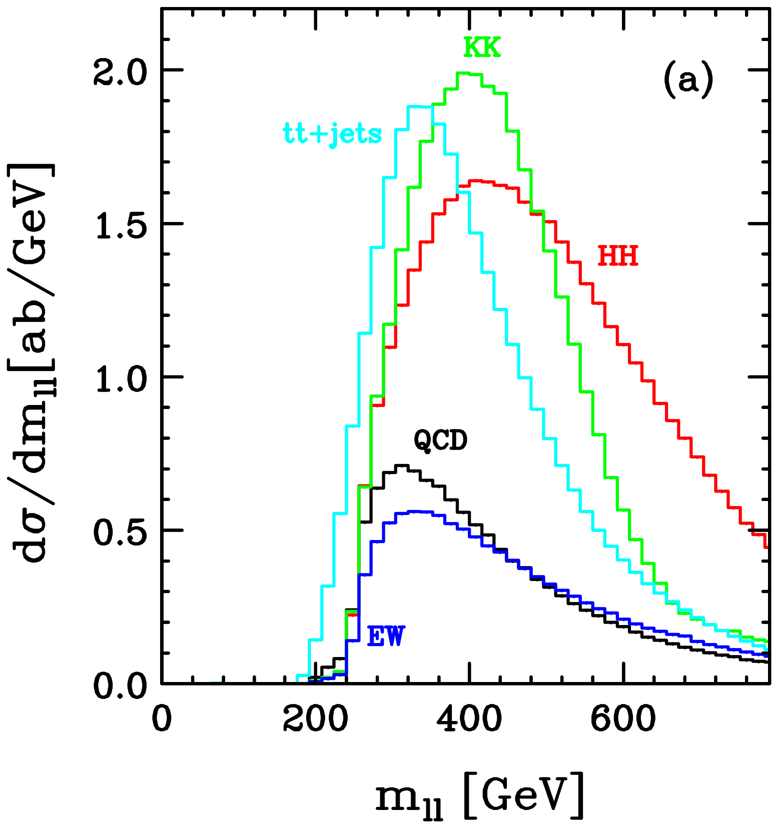
leptonic cuts

in contrast to backgrounds, signal processes feature energetic leptons of high p_T and large invariant mass

details of leptonic cuts depend on decay channel

$\underline{pp \to W^+W^-jj}$:

$$p_{T\ell} > 100 \text{ GeV}$$
$$\Delta p_T(\ell\ell) > 250 \text{ GeV}$$
$$m_{\ell\ell} > 200 \text{ GeV}$$
$$\min(m_{\ell j}) > 180 \text{ GeV}$$



...final level of cuts

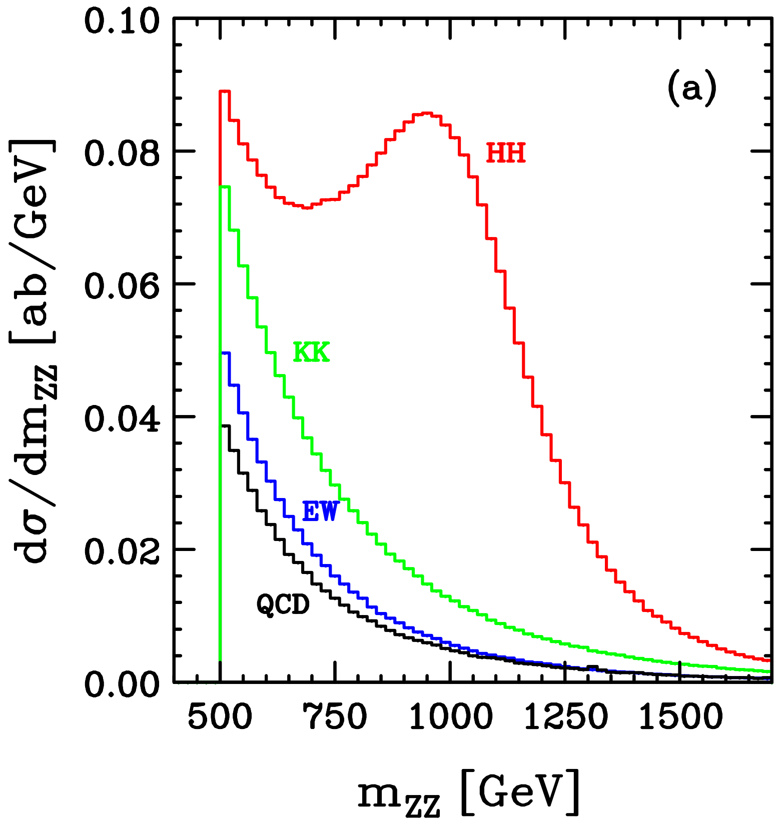
results of SEWSB analysis: scalar resonance

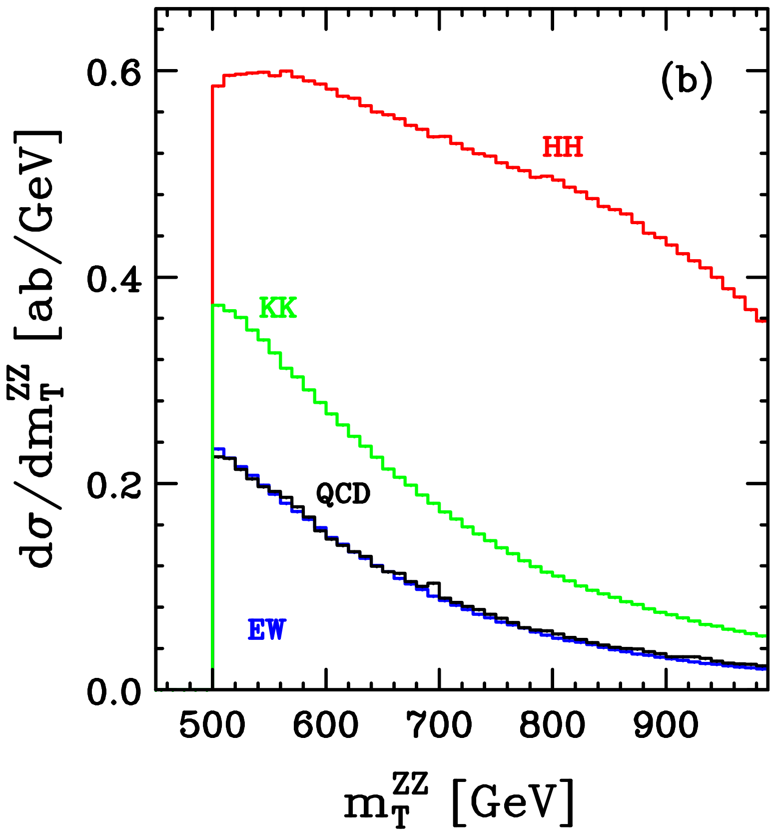
$$\sigma_S = \sigma_{SM}(m_H = 1\text{TeV}) - \sigma_{SM}(m_H = 100\text{GeV})$$

Process	σ_S	σ_B	S/B	$S/\sqrt{B}$	N_{signal}	$N_{\text{bkgd.}}$
$ZZjj \to 4\ell\, jj$ $ZZjj \to 2l2\nu\, jj$ W^+W^-jj	0.048 0.27 0.51	0.021 0.10 0.78	2.2 2.7 0.6	5.7 14.8 10.0	14 81 153	6 30 234
$W^\pm Zjj$	0.031	0.386	0.1	0.9	9	116

final level of cuts & integrated luminosity . . . 300 fb^{-1}

results: scalar resonance





$ZZjj \to 4\ell jj$

$ZZjj \to 2\ell 2\nu jj$

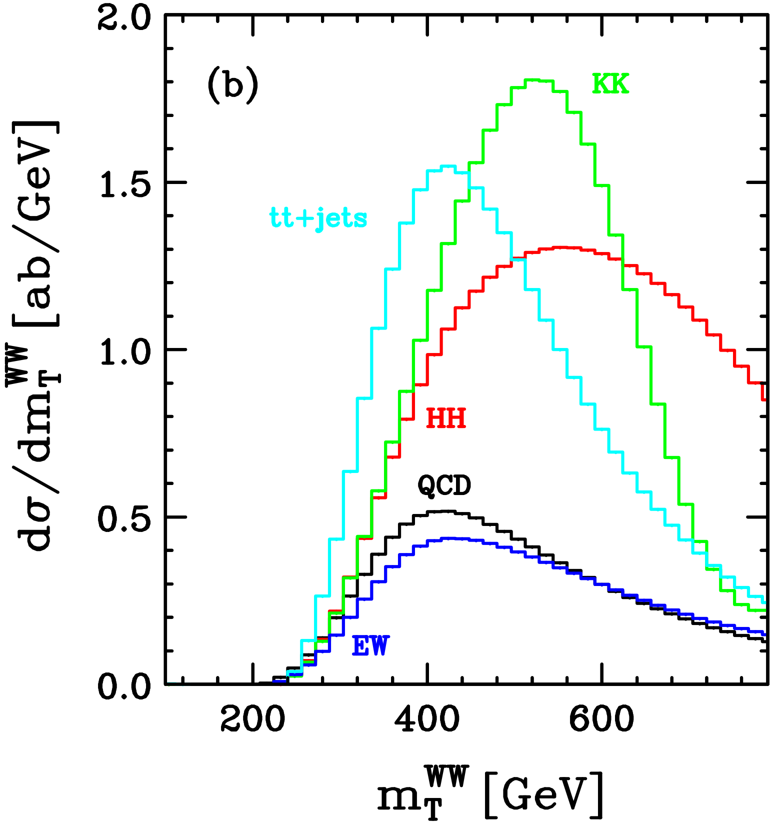
result of SEWSB analysis: vector resonance

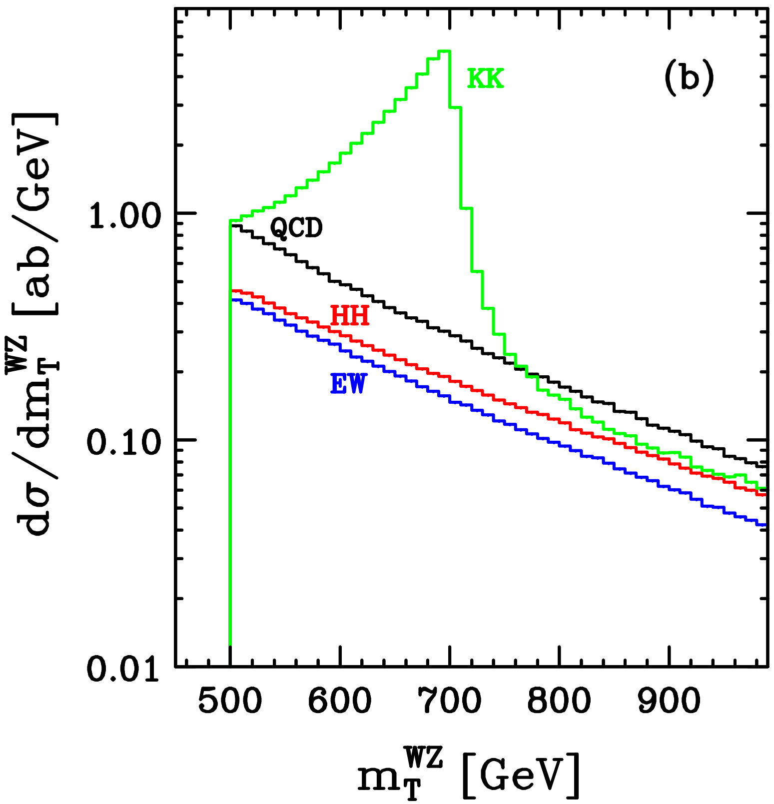
$$\sigma_S = \sigma_{KK} - \sigma_{SM}(m_H = 100\text{GeV})$$

Process	σ_S	σ_B	S/B	$S/\sqrt{B}$	N_{signal}	$N_{\text{bkgd.}}$
$W^{\pm}Zjj$ $W^{+}W^{-}jj$	0.68 0.40	0.39 0.78	1.7 0.5	18.9 7.9	204 120	117 234
$ZZjj \to 4\ell\, jj$ $ZZjj \to 2\ell 2\nu\, jj$	0.009 0.05	0.021 0.10	0.4 0.5	1.1 2.7	3 15	6 30

final level of cuts & integrated luminosity . . . 300 fb^{-1}

results: vector resonance

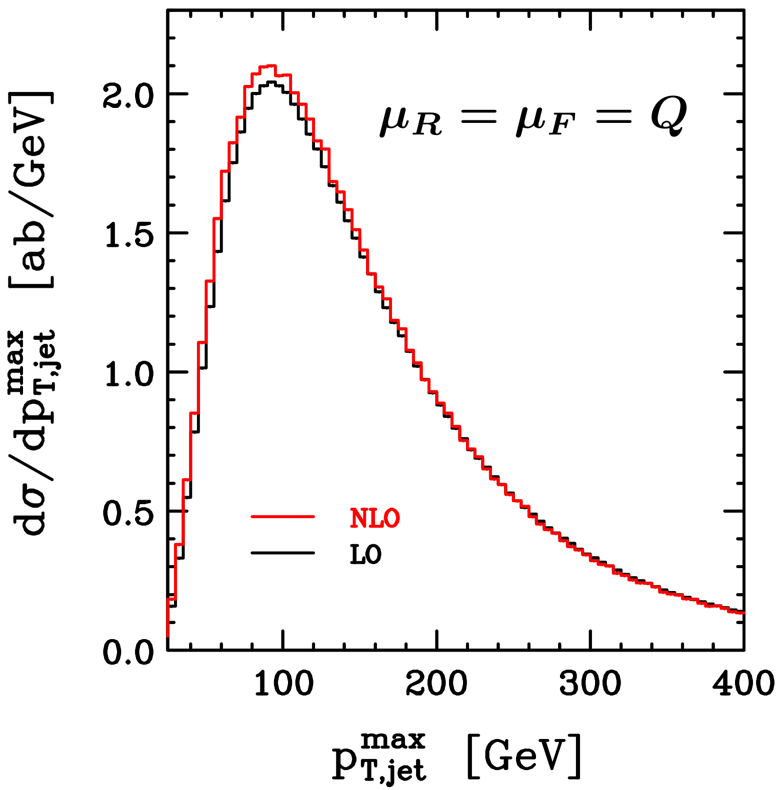


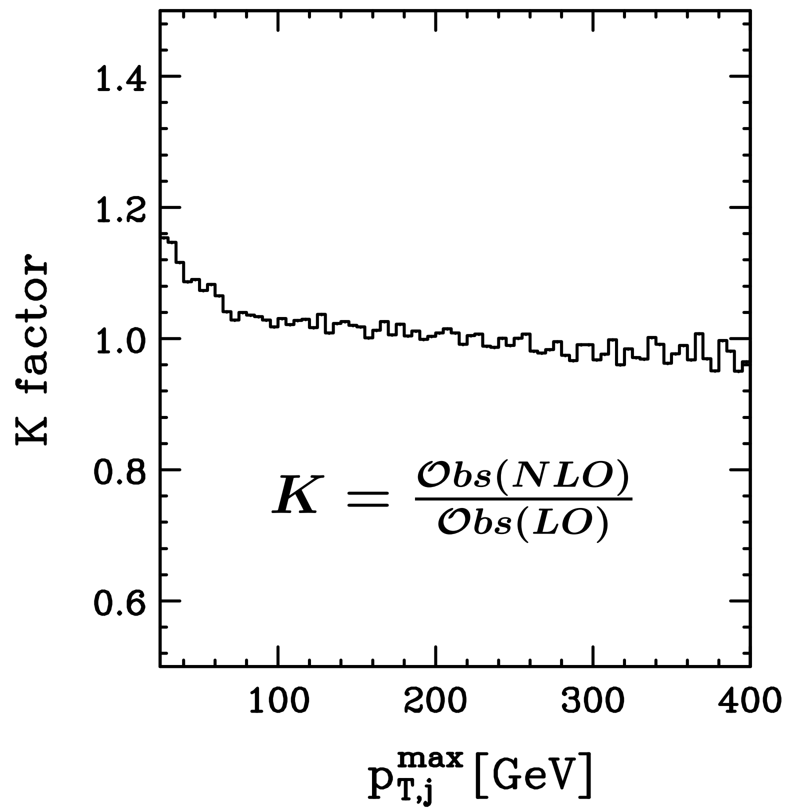


W^+W^-jj

W^+Zjj

impact of NLO-QCD corrections





NLO-QCD corrections always in the few-percent range

. . . more precision . . .

the next-to-leading order:

- real emission
- subtraction terms
- virtual corrections

some complications at NLO

obvious: meaningful observables

↓

theoretical prediction: finite result

but: how is finite result obtained in practice?

generally: perturbative calculation beyond LO
$\longrightarrow$ singularities encountered in intermediate steps

even though they will eventually cancel,
divergencies need to be treated properly
throughout!

regularization

☞ regularization needed to manifest singularities in intermediate steps of a calculation

dimensional regularization:

dimension of space-time $d = 4 \to d = 4 - 2\varepsilon$

$$\int_0^\infty \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2)^n} \to \int_0^\infty \frac{d^d q}{(2\pi)^d} \frac{1}{(q^2)^n}$$

$\varepsilon > 0 \ldots$ UV regulator, $\varepsilon < 0 \ldots$ IR regulator

Dirac algebra has to be performed in d dimensions

divergencies $\to$ poles in ε

✔ preserves Lorentz and gauge invariance

cancelation of divergencies at NLO

UV divergencies
$\updownarrow$
renormalization of
α_s at scale μ_r

collinear singularities
$\updownarrow$
factorization
at scale μ_f

soft singularities
$\updownarrow$
cancel in sum of
virtual and real
emission contributions

sum of all real and
virtual contributions to
well-defined
observable:

finite

cancelation of divergencies at NLO

intermediate steps: regularize all divergencies by $d \to 4 - 2\varepsilon$

collinear singularities
$\updownarrow$
factorization
at scale μ_f

soft singularities
$\updownarrow$
cancel in sum of virtual and real emission contributions

sum of all real and virtual contributions to well-defined observable:

finite for $\varepsilon \to 0$

cancelation of divergencies at NLO

cancelation of ε poles can be performed explicitly in analytical calculation, but how can divergencies be handled in numerical calculation?

collinear singularities
$\updownarrow$
factorization
at scale μ_f

soft singularities
$\updownarrow$
cancel in sum of virtual and real emission contributions

sum of all real and virtual contributions to well-defined observable:

finite for $\varepsilon \to 0$

cancelation of divergencies at NLO

typical NLO QCD calculation up to 1990ies:

- compute $|\mathcal{M}_{NLO}|^2$ analytically in d dimensions
- perform phase-space integration analytically in d dim (considering polarization, cuts etc.)
- cancel matching poles in real emission and virtual contributions
- set $\varepsilon \to 0$ and convolute $d\hat{\sigma}$ with PDFs numerically for $d = 4$

cancelation of divergencies at NLO

procedure perfect for processes with only a few particles and minimal set of cuts (e.g., total cross sections):

- poles cancelled analytically
  $\rightarrow$ no delicate numerical cancelations needed

- resulting code fast and efficient

but:

- complete calculation has to be performed analytically in d dim (Dirac algebra can become very complicated; γ^5 problem ...)
- PS integration can be done explicitly for “simple” reactions only
- implementation of cuts for realistic distributions hard

cancelation of divergencies at NLO

basic idea of modern approaches:

- treat only minimal part of full calculation analytically (pieces containing divergencies are computed in process-independent way)
- finite contributions are treated with Monte-Carlo methods

two types of algorithm to handle divergencies numerically:

- phase space slicing
- subtraction method

actual implementation may depend on authors, but basic concepts are general

subtraction method

introduce local counterterm which cancels divergencies before integration

↓

numerically stable

first applied in $e^+e^- \to 3$ jets

Ellis, Ross, Terrano (1981)

in process-specific manner

generalized to arbitrary reactions with massless partons

by *Catani, Seymour (hep-ph/9605323)*

extended to massive case by

Catani, Dittmaier, Seymour, Trocsanyi (hep-ph/0201036)

dipole subtraction

needed: $$\sigma^{NLO} \equiv \int d\sigma^{NLO} = \int_{m+1} d\sigma^{R} + \int_{m} d\sigma^{V}$$

IR divergent

☞regularize in $d = 4 - 2\varepsilon$ dim

introduce local counterterm $d\sigma^A$ with same singularity structure as $d\sigma^R$:

$$\sigma^{NLO} = \underbrace{\int_{m+1} \left[d\sigma^{R} - d\sigma^{A}\right]}_{\text{finite}} + \int_{m+1} d\sigma^{A} + \int_{m} d\sigma^{V}$$

dipole subtraction

$$\boldsymbol{\sigma}^{NLO} = \int_{m+1} \left[d\boldsymbol{\sigma}^{R} - d\boldsymbol{\sigma}^{A} \right] \Big|_{\varepsilon=0} + \int_{m} d\boldsymbol{\sigma}^{V} + \int_{m+1} d\boldsymbol{\sigma}^{A}$$

↓

integrate over one-parton PS analytically

explicitly cancel poles & then set $\varepsilon \to 0$

↓

$$\boldsymbol{\sigma}^{NLO} = \int_{m+1} \left[d\boldsymbol{\sigma}^{R}_{\varepsilon=0} - d\boldsymbol{\sigma}^{A}_{\varepsilon=0} \right] + \int_{m} \left[d\boldsymbol{\sigma}^{V} + \int_{1} d\boldsymbol{\sigma}^{A} \right]_{\varepsilon=0}$$

dipole subtraction: ingredients

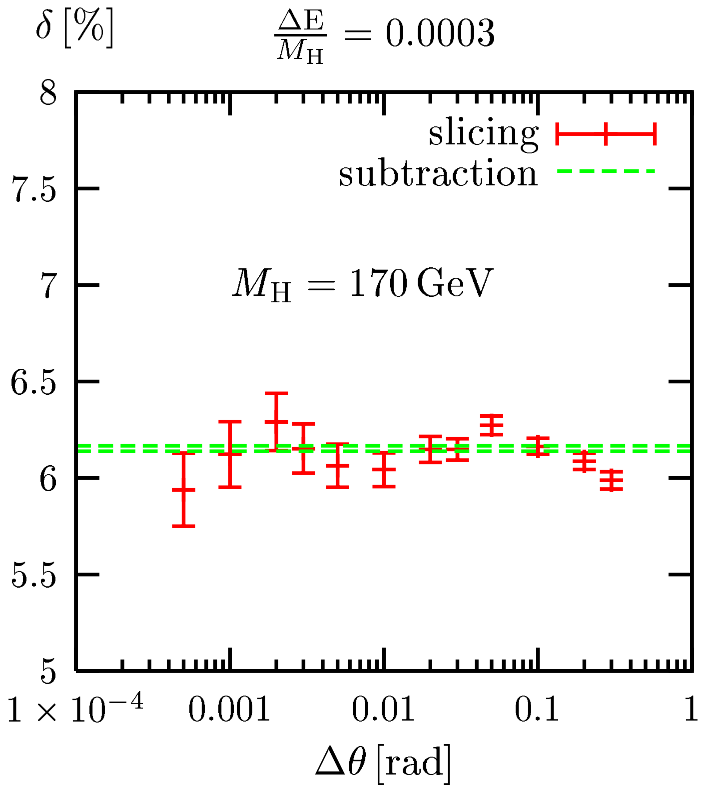
- real emission contribution $d\sigma^R$ in four dimensions
- one-loop contribution $d\sigma^V$ in d dimensions
- counterterm $d\sigma^A$ that matches singular behavior of $d\sigma^R$ independently of particular jet observable and can be integrated analytically over the one-parton PS in d dim

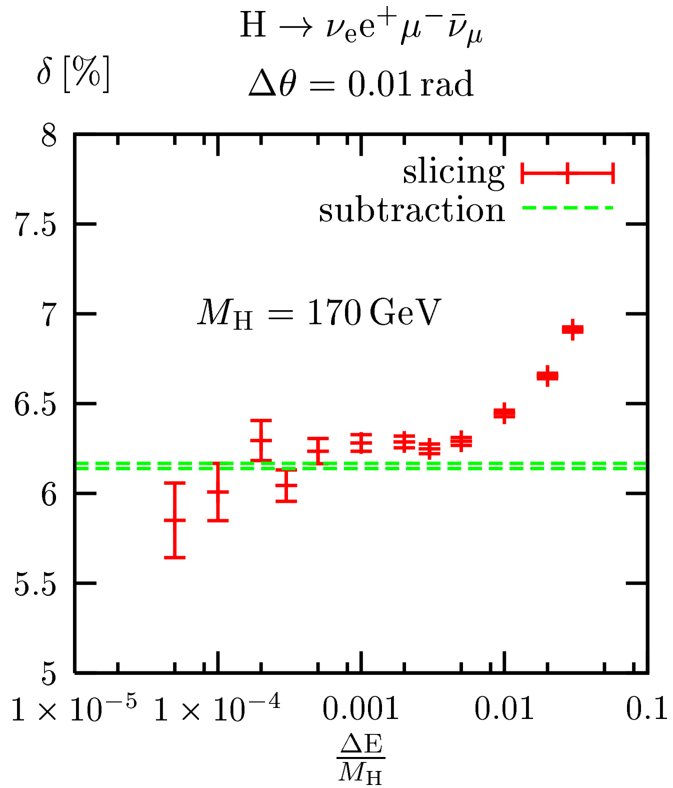
factorized dipole formula proposed by *Catani & Seymour* :

$$d\sigma^A = \sum_{dipoles} d\sigma^B \otimes dV_{dipole}$$

Monte Carlo methods: a comparison

phase space slicing and subtraction techniques are in priciple equivalent, but are they in practice?





taken from *Bredenstein, Denner, Dittmaier, Weber,*
“ Precise predictions for the Higgs-boson decay
$\boldsymbol{H \to WW/ZZ \to}$ 4 *leptons”, hep-ph/0604011*

return to process of interest

☞ sketch elements of NLO-QCD calculation for

$$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu jj$$

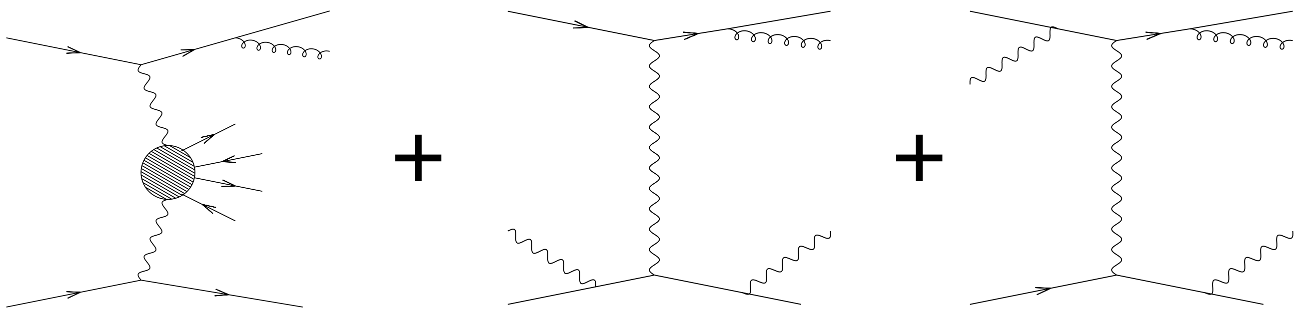
via electroweak boson exchange

Bozzi, Oleari, Zeppenfeld, B.J. (2006 ff.)

real emission contributions

Catani & Seymour: for $d\sigma^R$ we need numerical value for

$$|\mathcal{M}_R|^2 = \Big|$$



$$+ \ldots \Big|^2$$

at each generated phase space point in 4 dimensions

↓

can apply same (numerical) amplitude techniques as at LO

keep in mind: kinematics different from LO
($2 \to 7$ instead of $2 \to 6$ particles)

real emission contributions

the main problem: large number of diagrams

e.g. real emission corrections to LO sub-process $ud \to ud\, e^+\nu_e\mu^-\bar{\nu}_\mu$:

- attach extra gluon in all possible ways (yields $ud \to ud\, g\, e^+\nu_e\mu^-\bar{\nu}_\mu$) (obtain 836 graphs out of 181 at LO)
- perform all possible crossings $\to$ need to consider also gluon initiated processes like $ug \to ud\bar{d}\, e^+\nu_e\mu^-\bar{\nu}_\mu$

without optimization code extremely slow!

real emission contributions

the solution: apply speed-up tricks developed at LO

here even more effective, since leptonic tensors are not affected by extra gluon $\rightarrow$ building blocks are used even more frequently

side remark: $\mathcal{M}_R$ for real emission contributions to

$$pp \rightarrow jjW^+W^-$$

$\Updownarrow$

LO contribution $\mathcal{M}_B$ for $pp \rightarrow jjjW^+W^-$

(can use `MadGraph` for reference)

subtracted $(n+1)$-parton contribution

$$\sigma^{NLO}_{(n+1)} = \int_{n+1} \left[d\sigma^{R}_{\varepsilon=0} - d\sigma^{A}_{\varepsilon=0} \right]$$

$$= \int d\mathrm{PS}_{(n+1)}(p_1, \ldots, p_{n+1})$$

$$\times \left[|\mathcal{M}_R|^2 \mathcal{F}_J^{(n+1)}(p_1, \ldots, p_{n+1}) - |\mathcal{M}_A|^2 \mathcal{F}_J^{(n)}(\tilde{p}_1, \ldots \tilde{p}_n) \right]$$

cut functions defined on $(n+1)$ and n parton phase spaces

soft and collinear limits:

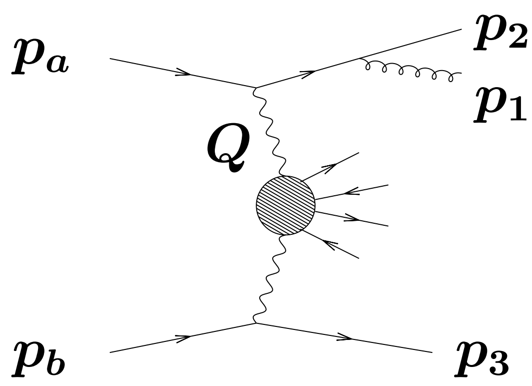
$$\{p_1, \ldots, p_{n+1}\} \to \{\tilde{p}_1, \ldots, \tilde{p}_n\}$$

$$\mathcal{F}_J^{(n+1)} \to \mathcal{F}_J^{(n)}$$

dipoles and integrated dipoles

continuous interpolation
between
soft and collinear
gluon radiation:

$$\sim \frac{x^2 + z^2}{(1-x)(1-z)} |\mathcal{M}_B(\tilde{p})|^2$$



analytical integration over
one-particle phase space:

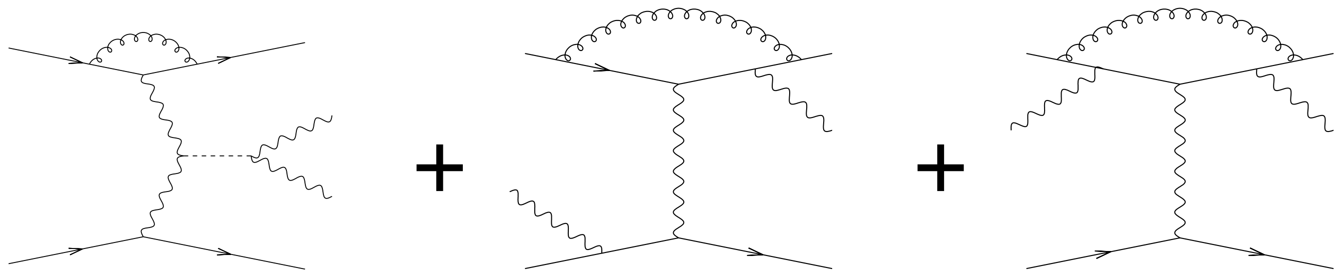
$$\sim |\mathcal{M}_B(p)|^2 \left[\frac{2}{\varepsilon^2} + \frac{3}{\varepsilon} + \text{const.}\right]$$

Catani, Seymour (1996)

$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^R_{\varepsilon=0} - d\sigma^A_{\varepsilon=0}\right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A\right]_{\varepsilon=0}$$

virtual corrections

…interference of LO diagrams with

$\mathcal{M}_V =$  $+ \ldots$

2-parton kinematics (like LO)

in VBF: no color exchange between

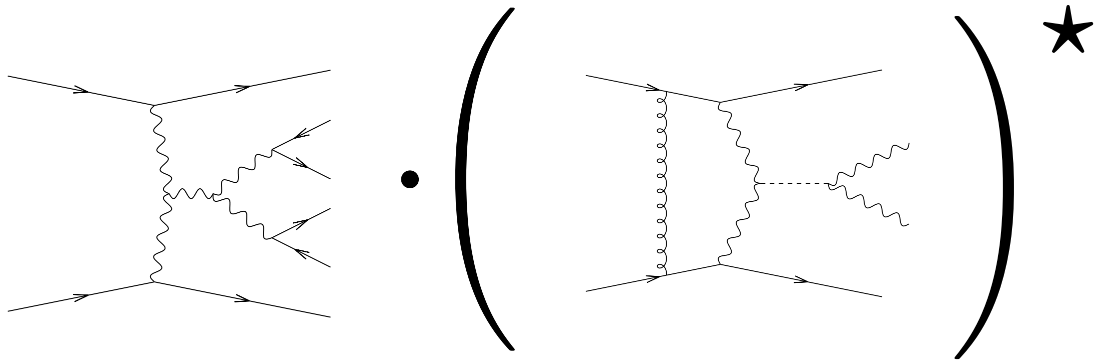
upper / lower quark line at $\mathcal{O}(\alpha_s)$

↓

need radiative corrections to single quark line only

virtual corrections: color structure

…interference of gluon-exchange with LO diagrams



color factor:

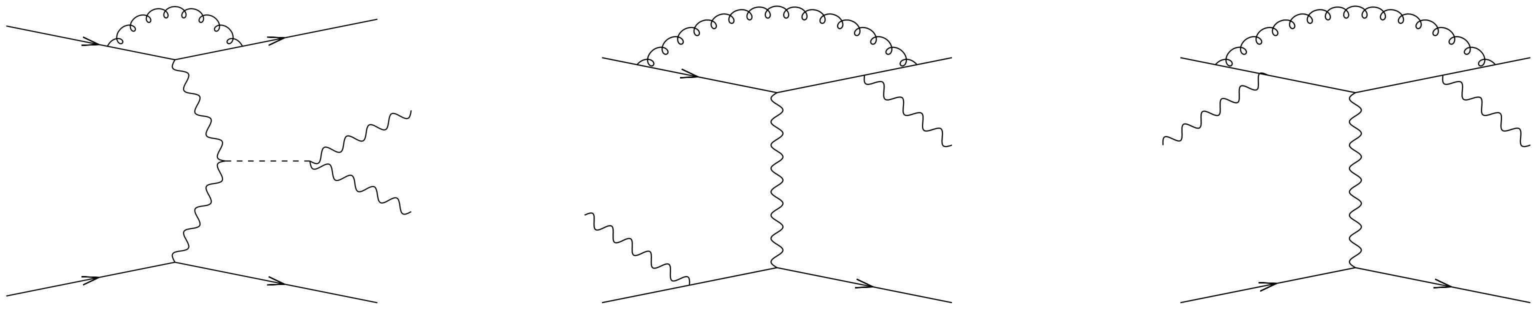
$$\sim \ \mathbf{Tr}[\boldsymbol{T^a}] \cdot \mathbf{Tr}[\boldsymbol{T^a}] = \mathbf{0}$$

non-vanishing contributions only, if
t-channel diagram interferes with u-channel diagram

such contributions are kinematically very strongly suppressed
(typically t- and u-type configurations not large at the same time)

virtual corrections

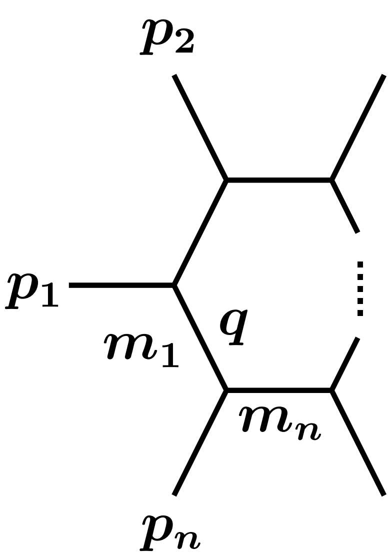
split virtual corrections into classes depending on the number of gauge bosons attached to a quark line:



need to compute tensor integrals with up to three / four / five internal propagators

loop integrals

in any loop calculation we encounter tensor integrals of type



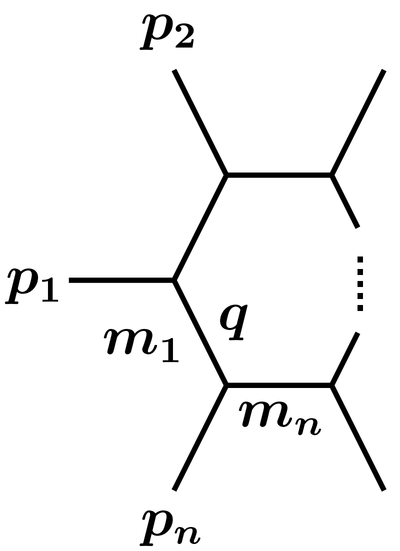
$$T_{\mu_1 \ldots \mu_m}(p_1, \ldots, p_n; m_1, \ldots, m_n) = \int \frac{d^d q}{i\pi^2} \frac{q_{\mu_1} \ldots q_{\mu_m}}{D_1 D_2 \ldots D_n}$$

with

$$\begin{aligned} D_1 &= q^2 - m_1^2 + i\epsilon \\ D_2 &= (q + p_1)^2 - m_2^2 + i\epsilon \\ &\ldots \\ D_n &= (q + \ldots + p_{n-1})^2 - m_n^2 + i\epsilon \end{aligned}$$

loop integrals

in any loop calculation we encounter tensor integrals of type



$$T_{\mu_1 \ldots \mu_m}(p_1, \ldots, p_n; m_1, \ldots, m_n) = \int_0^\infty \frac{d^d q}{i\pi^2} \frac{q_{\mu_1} \ldots q_{\mu_m}}{D_1 D_2 \ldots D_n}$$

nomenclature:

scalar integrals with

$$n = 1, 2, 3, 4, 5, \ldots$$

↓

$$A_0, B_0, C_0, D_0, E_0, \ldots$$

and analogous for tensor integrals:

$$A_\mu, B_\mu, B_{\mu\nu}, \ldots$$

tensor integrals

...calculable from scalar integrals by Passarino-Veltman reduction

$$T^{\{0,\mu,\mu\nu,\ldots\}}(p_1,\ldots) \;=\; \int \frac{d^dq}{i\pi^2}\,\frac{\{1,q^\mu,q^\mu q^\nu,\ldots\}}{D_1\ldots D_n}$$

bubbles :

$$\begin{aligned} B^\mu &= p_1^\mu B_1 \\ B^{\mu\nu} &= p_1^\mu p_1^\nu B_{21} + g^{\mu\nu} B_{22} \end{aligned}$$

triangles :

$$\begin{aligned} C^\mu &= p_1^\mu C_{11} + p_2^\mu C_{12} \\ C^{\mu\nu} &= p_1^\mu p_1^\nu C_{21} + p_2^\mu p_2^\nu C_{22} + \{p_1 p_2\}^{\mu\nu} C_{23} + g^{\mu\nu} C_{24} \\ C^{\mu\nu\rho} &= p_1^\mu p_1^\nu p_1^\rho C_{31} + p_2^\mu p_2^\nu p_2^\rho C_{32} + \{p_1 p_1 p_2\}^{\mu\nu\rho} C_{33} \\ &\quad + \{p_1 p_2 p_2\}^{\mu\nu\rho} C_{34} + \{p_1 g\}^{\mu\nu\rho} C_{35} + \{p_2 g\}^{\mu\nu\rho} C_{36} \end{aligned}$$

tensor integrals

boxes:

$$D^{\mu} = p_1^{\mu} D_{11} + p_2^{\mu} D_{12} + p_3^{\mu} D_{13}$$

$$\begin{aligned} D^{\mu\nu} &= p_1^{\mu} p_1^{\nu} D_{21} + p_2^{\mu} p_2^{\nu} D_{22} + p_3^{\mu} p_3^{\nu} D_{23} + \{p_1 p_2\}^{\mu\nu} D_{24} \\ &\quad + \{p_1 p_3\}^{\mu\nu} D_{25} + \{p_2 p_3\}^{\mu\nu} D_{26} + g^{\mu\nu} D_{27} \end{aligned}$$

$$\begin{aligned} D^{\mu\nu\rho} &= p_1^{\mu} p_1^{\nu} p_1^{\rho} D_{31} + p_2^{\mu} p_2^{\nu} p_2^{\rho} D_{32} + p_3^{\mu} p_3^{\nu} p_3^{\rho} D_{33} + \{p_1 p_1 p_2\}^{\mu\nu\rho} D_{34} \\ &\quad + \{p_1 p_1 p_3\}^{\mu\nu\rho} D_{35} + \{p_1 p_2 p_2\}^{\mu\nu\rho} D_{36} + \{p_1 p_3 p_3\}^{\mu\nu\rho} D_{37} \\ &\quad + \{p_2 p_2 p_3\}^{\mu\nu\rho} D_{38} + \{p_2 p_3 p_3\}^{\mu\nu\rho} D_{39} + \{p_1 p_2 p_3\}^{\mu\nu\rho} D_{310} \\ &\quad + \{p_1 g\}^{\mu\nu\rho} D_{311} + \{p_2 g\}^{\mu\nu\rho} D_{312} + \{p_3 g\}^{\mu\nu\rho} D_{313} \end{aligned}$$

scalar coefficients D_{ij} depend on B_0, C_0, D_0

tensor integrals

example:

$$B_\mu(p) = p_\mu B_1(p) = \int \frac{d^d q}{i\pi^2} \frac{q_\mu}{q^2(q+p)^2}$$

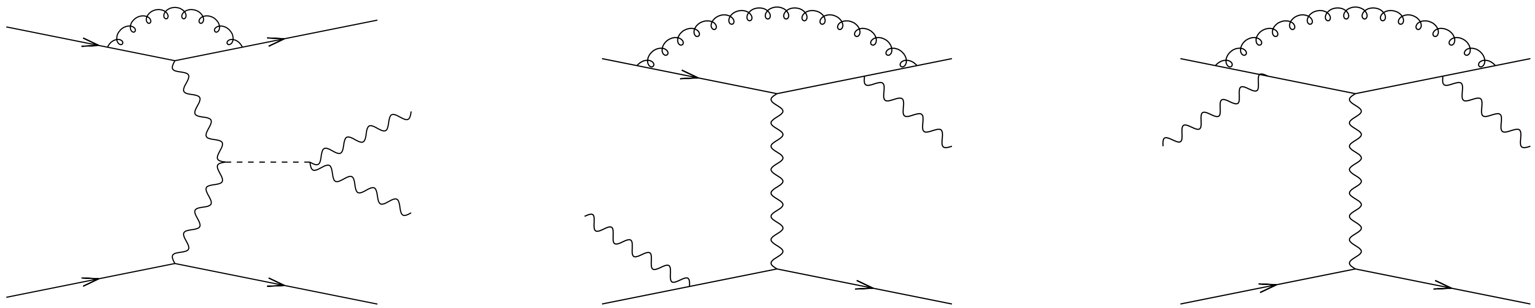
compute B_1 by suitable contractions:

$$\begin{aligned} p^\mu B_\mu(p) = p^2 B_1(p) &= \int \frac{d^d q}{i\pi^2} \frac{p \cdot q}{q^2(q+p)^2} \\ &= \int \frac{d^d q}{i\pi^2} \frac{1}{2} \frac{\left[(p+q)^2 - p^2 - q^2\right]}{q^2(q+p)^2} \\ &= \frac{1}{2}\left[A(0) - A(0) - p^2 B_0\right] \end{aligned}$$

$$\longrightarrow \quad B_1 = -\frac{1}{2} B_0$$

tensor integrals

reminder: for $\mathcal{M}_V$ need loop integrals of type

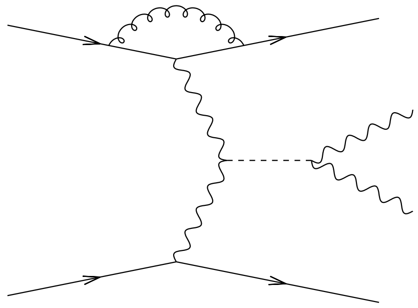


application of Passarino-Veltman tensor reduction

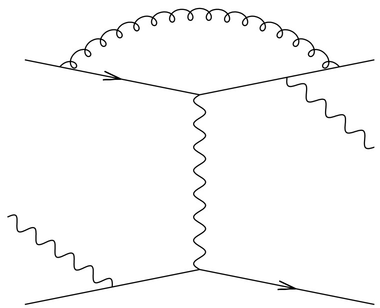
“straightforward” for bubbles, triangles, and boxes

virtual corrections

after “some” algebra find:



$$\sim \mathcal{M}_B F(Q) \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + c_{\text{virt}} \right]$$



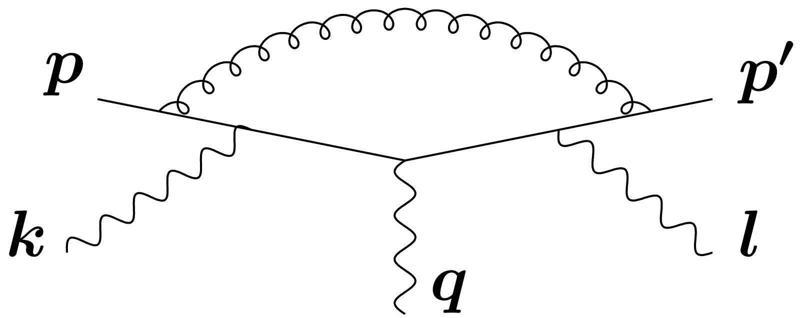
$$\sim \mathcal{M}_B F(Q) \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + c_{\text{virt}} \right] + \tilde{\mathcal{M}}_V^B$$

finite piece $\tilde{\mathcal{M}}_V^B$ is expressed by

finite parts of tensor coefficients B_{ij}, C_{ij}, D_{ij}

pentagon contributions

$$\mathcal{M}_5 = \varepsilon_\mu(k)\varepsilon_\nu(l)j_\rho(q)P^{\mu\nu\rho}(p,k,q,l)$$



planar configurations with linearly dependent momenta

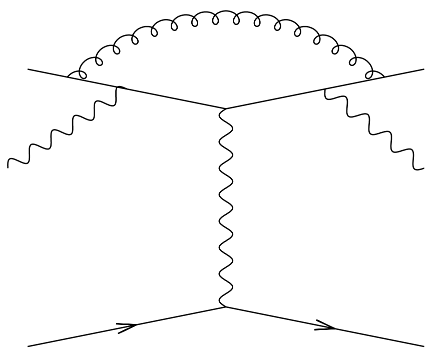
$\rightarrow$ trouble with Passarino-Veltman reduction

but: singularity unphysical!

express in terms of lower-rank tensor integrals if possible

virtual corrections

similar to simpler contributions end up with



$$\sim \mathcal{M}_B \, F(Q) \left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + c_{\text{virt}} \right] + \tilde{\mathcal{M}}_V^P$$

finite piece $\tilde{\mathcal{M}}_V^P$ is expressed by finite parts

of tensor integrals B_{ij}, C_{ij}, D_{ij}, E_{ij}

pentagon contributions

further improvement by gauge invariant decomposition:

$$\varepsilon_\mu(k) \to \varepsilon'_\mu(k) = \varepsilon_\mu(k) - \beta\, k_\mu$$

use $k_\mu \mathcal{E}^{\mu\nu\rho}(p,k,q,l) = \mathcal{D}^{\nu\rho}(p,k+q,l)$

$$\begin{aligned}\mathcal{M}_5 &= \left[\varepsilon'_\mu(k) + \beta\, k_\mu\right] \varepsilon_\nu(l)\, j_\rho(q)\, \mathcal{E}^{\mu\nu\rho}(p,k,q,l) \\ &= \varepsilon'_\mu(k)\; \varepsilon_\nu(l)\, j_\rho(q)\, \mathcal{E}^{\mu\nu\rho}(p,k,q,l) \\ &\quad +\beta\; \varepsilon_\nu(l)\, j_\rho(q) \mathcal{D}^{\nu\rho}(p,k+q,l)\end{aligned}$$

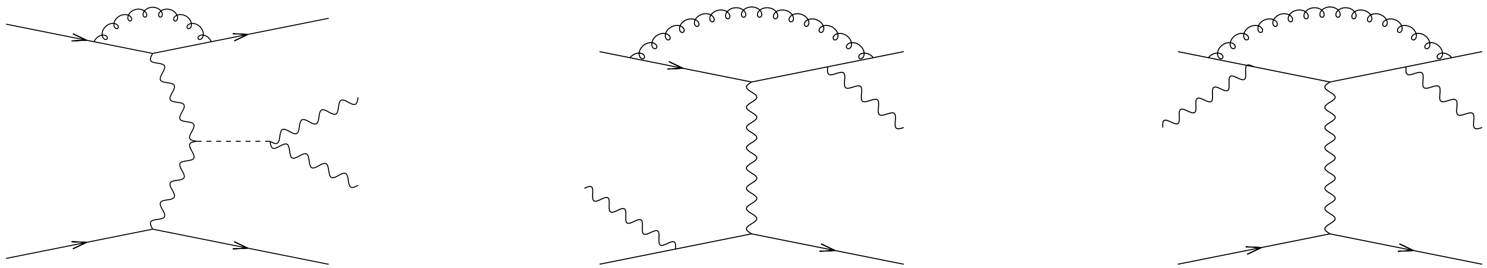
proper choice of $\beta \to$ remaining “true” pentagon small

box-type contributions numerically stable

remaining pentagons: more sophisticated tensor reduction

[Denner, Dittmaier (2002,2005)]

virtual corrections



split adaptive VEGAS integration:

- leading order contributions
- finite parts of virtual contributions:
  - pieces proportional to Born
  - box type contributions
  - pentagon type contributions
- real emission contributions and subtraction terms

☞ can adjust Monte-Carlo accuracy for each piece separately

verification

checks

to ensure reliability of calculation: perform some checks!

✔ comparison of LO and real emission amplitudes with MadGraph:

compare numerical value of $\mathcal{M}_B$ and $\mathcal{M}_R$ for each sub-process at every generated phase space point

keep in mind: $\mathcal{M}_R$ for $qq \to qqW^+W^-$ corresponds to $\mathcal{M}_B$ for $qq \to qqgW^+W^-$

$\to$ generation with `MadGraph` possible

expect agreement at 10^{-10} level

checks

✔ check subtraction:

in soft / collinear limits expect $d\sigma^R \to d\sigma^A$
(non-singular contributions become sub-dominant)

generate events in singular regions

$\to d\sigma^R$ approaches $d\sigma^A$ as

two partons become collinear ($p_i \cdot p_j \to 0$) or

gluon becomes soft ($E_g \to 0$)

checks

✔ QCD gauge invariance of real emission contributions demands:

$$\mathcal{M}_R = \varepsilon_\mu(p_g)\, \mathcal{M}_R^\mu = [\varepsilon_\mu(p_g) + \beta\, p_{g\,\mu}]\, \mathcal{M}_R^\mu$$

↓

expect $p_{g\,\mu}\mathcal{M}_R^\mu = 0$

replace $\varepsilon_\mu(p_g)$ throughout with $p_{g\mu}$

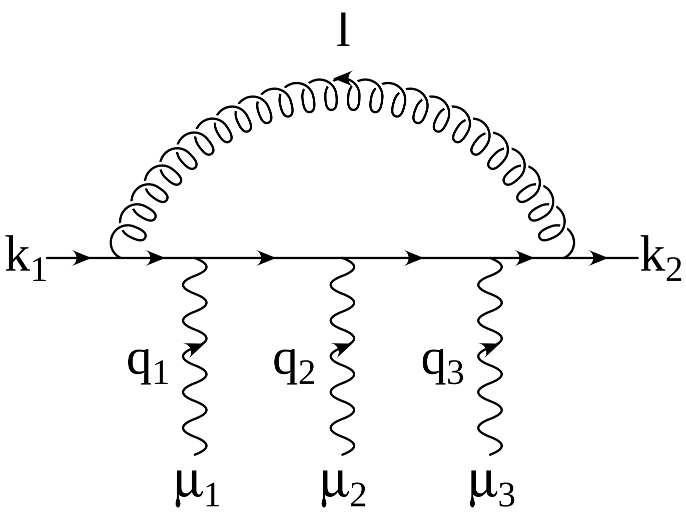
expected relation fulfilled within numerical accuracy of the program

checks

✔ EW gauge invariance of virtual contributions

recall: pentagon loop

$$\mathcal{E}_{\mu_1\mu_2\mu_3}(k_1, q_1, q_2, q_3)$$
$$= \int \frac{d^d l}{(2\pi)^d} \gamma^\alpha \frac{1}{\not{l} + \not{k}_1 + \not{q}_{123}} \gamma_{\mu_3} \frac{1}{\not{l} + \not{k}_1 + \not{q}_{12}} \gamma_{\mu_2}$$
$$\times \frac{1}{\not{l} + \not{k}_1 + \not{q}_1} \gamma_{\mu_1} \frac{1}{\not{l} + \not{k}_1} \gamma_\alpha \frac{1}{l^2}$$



$$q_{12} = q_1 + q_2$$
$$q_{123} = q_1 + q_2 + q_3$$

checks

contracting $\mathcal{E}_{\mu_1\mu_2\mu_3}(k_1, q_1, q_2, q_3)$ with external momentum

$\rightarrow$ combination of boxes:

$$
\begin{aligned}
q_1^{\mu_1}\mathcal{E}_{\mu_1\mu_2\mu_3} &= \mathcal{D}_{\mu_2\mu_3}(k_1, q_1+q_2, q_3) - \mathcal{D}_{\mu_2\mu_3}(k_1+q_1, q_2, q_3) \\
q_2^{\mu_2}\mathcal{E}_{\mu_1\mu_2\mu_3} &= \mathcal{D}_{\mu_1\mu_3}(k_1, q_1, q_2+q_3) - \mathcal{D}_{\mu_1\mu_3}(k_1, q_1+q_2, q_3) \\
q_3^{\mu_3}\mathcal{E}_{\mu_1\mu_2\mu_3} &= \mathcal{D}_{\mu_1\mu_2}(k_1, q_1, q_2) - \mathcal{D}_{\mu_1\mu_2}(k_1, q_1, q_2+q_3)
\end{aligned}
$$

and analogously for the sum of all virtual contributions along a quark line

check that after contraction with q_{W^+} or q_{W^-} only box-type contributions to $qq \rightarrow qqW^+W^-$ remain (no pentagons left)

checks

✔ produce two independent codes
(in our case: done for neutral current amplitudes)

↓

find agreement within numerical accuracy of the two `fortran` programs

EW $VVjj$ production at NLO-QCD

NLO-QCD calculation
including off-shell effects and decay correlations
for VBF production of

$$pp \to W^+W^-jj, ZZjj, W^\pm Zjj\ , W^\pm W^\pm jj$$

G. Bozzi, C. Oleari, D. Zeppenfeld, B. J. (2006-2009)

A. Denner, L. Hosekova, S. Kallweit (2012)

and $pp \to HHjj$

Figy (2008), Baglio et al. (2013)

available in `VBFNLO` Monte Carlo program and stand-alone codes

vbfnlo

`vbfnlo` is a fully flexible parton level Monte Carlo for processes with electroweak bosons at NLO-QCD in the SM and beyond

included processes:

- various weak vector boson fusion processes
- double and triple weak boson production processes
- double weak boson production processes
  in association with a hard jet
- Higgs production via gluon fusion
  in association with two jets

`http://www-itp.particle.uni-karlsruhe.de/~vbfnloweb`

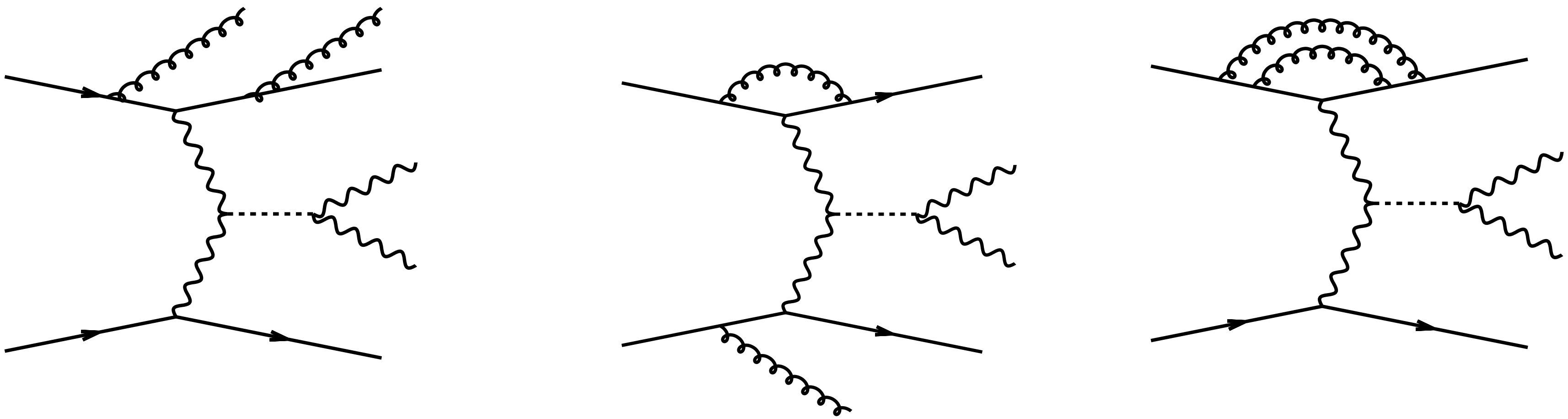
vbfnlo: features

- cross sections and distributions at NLO-QCD accuracy
- NLO-EW corrections to VBF Hjj production
- arbitrary selection cuts
- various choices for factorization and renormalization scales
- LO predictions for all processes with one extra jet
- interface to LHAPDF $\longrightarrow$ any currently available PDF set
- LO: event files in Les Houches Accord (LHA) format
- MSSM: SUSY parameters input via standard SLHA file
- various BSM features:
  anomalous couplings, Kaluza-Klein models, . . .

. . . even more precision . . .

- the next-to-next-leading order (NNLO) in QCD
- NLO electroweak (EW) corrections
- mixed QCD-EW effects

NNLO QCD ?



various contributions:

- double real emission squared: $|\boldsymbol{\mathcal{M}}_{RR}|^2$
- mixed real-virtual contributions: $\boldsymbol{\mathcal{M}}_{RV} \cdot \boldsymbol{\mathcal{M}}_R{}^\star$
- one-loop virtuals squared: $|\boldsymbol{\mathcal{M}}_V|^2$
- two-loop virtuals interfered with Born: $\boldsymbol{\mathcal{M}}_{VV} \cdot \boldsymbol{\mathcal{M}}_B{}^\star$

requires computation of two-loop pentagon diagrams

$\longrightarrow$ forbidding complexity with today's technology

EW corrections

for details:

Stefan Dittmaier's lecture

EW corrections: generic features

naive expectation:

$\alpha \sim \alpha_s^2 \rightarrow$ NLO EW $\sim$ NNLO QCD ?

but: systematic enhancements possible, e.g.:

- kinematic effects
- photon emission $\longrightarrow$ mass-singular logs, e.g. $\frac{\alpha}{\pi} \ln\left(\frac{Q}{m_\mu}\right)$
- high energies $\longrightarrow$ EW Sudakov logs, e.g. $\frac{\alpha}{\pi} \ln^2\left(\frac{Q}{M_W}\right)$

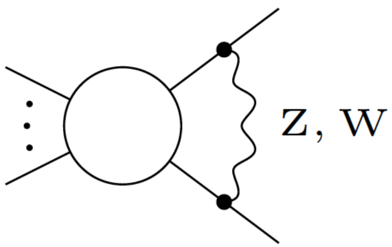
EW corrections: Sudakov logarithms

typical $\mathbf{2} \to \mathbf{2}$ process: at high energy
EW corrections enhanced by large logs

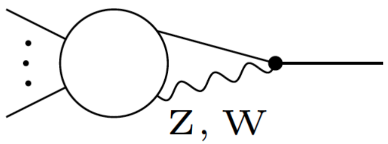
$$\ln^2\left(\frac{Q^2}{M_W^2}\right) \sim 25 \text{ @ energy scale of 1 TeV}$$

universal origin of leading EW logs:

mass singularities in virtual corrections related to external lines



soft and collinear virtual gauge bosons: $\to$ double logs



soft or collinear virtual gauge bosons: $\to$ single logs

EW corrections: Sudakov logarithms

compare to QED / QCD:

IR singularities of virtuals canceled
by real-emission contributions

electroweak bosons massive

$\rightarrow$ real radiation experimentally distinguishable

non-Abelian charges of W, Z are open

$\rightarrow$ Bloch-Nordsieck theorem not applicable

M. Ciafaloni, P. Ciafaloni, Comelli; Beenakker, Werthenbach; Denner, Pozzorini; Kühn et al., Baur; . . .

EW effects in PDFs

consistent calculation at NLO EW requires PDFs including
$\mathcal{O}(\alpha)$ corrections and new photon PDF

MRST2004QED: first PDF set with $\mathcal{O}(\alpha)$ corrections

NNPDF2.3QED (2013): NNPDF set with $\mathcal{O}(\alpha)$ corrections

- currently best PDF prediction at (N)NLO QCD + NLO QED
- PDF samples for error estimate provided
- photon PDF fitted to DIS and Drell-Yan data $(10^{-5} \lesssim x \lesssim 10^{-1})$
  (note lack of experimental information for large x)

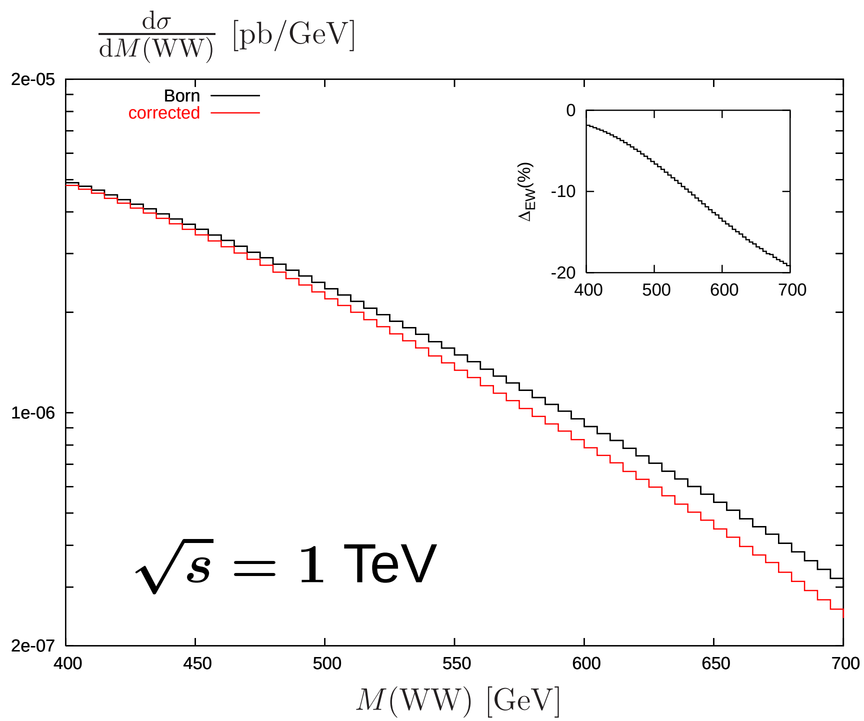
$VVjj$ production: electroweak corrections

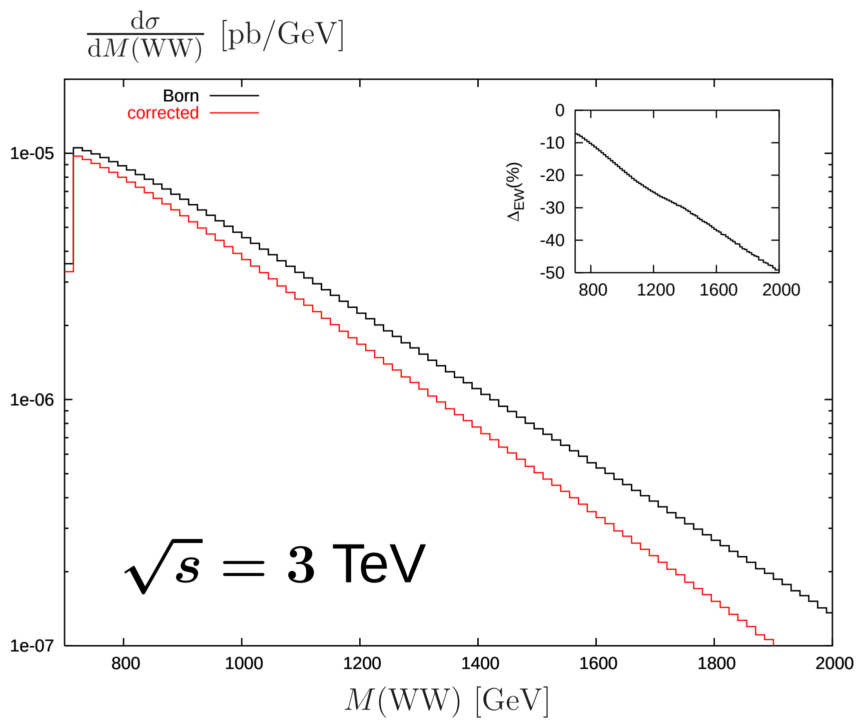
very tough – no calculations available to date for $pp \to VVjj$

related case of $e^+e^- \to \nu_e\bar{\nu}_e W^+W^-$:

dominant EW corrections can be as large as 50% in TeV range

[Accomando, Denner, Pozzorini (2006)]





technical details: summary

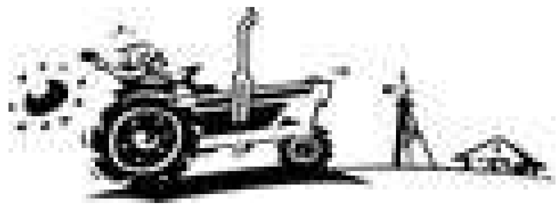
went into the technical details of an

NLO-QCD calculation for $pp \to jj\,VV$:

- general framework and approximations
- speed optimization of matrix element computation
- possible implementation of “new physics”
- handling of divergencies in NLO-Monte Carlo program
- loop contributions
- gauge invariance tricks and checks

applications

. . . we are now in a position to apply our knowledge . . .



theoretical uncertainty

for judging the reliability of our predictions we should estimate
the theoretical uncertainties associated with it

- bugs in code: performed lots of checks . . .
- numerical instabilities
- PDF uncertainties:
  c.f. 3.5% uncertainty for related VBF reactions ($pp \to Hjj$)
- effect of neglected contributions:
  - interference effects
  - s-channel vector boson production
  - neglected higher order contributions
  - . . .
- dependence on unphysical renormalization
  and factorization scales

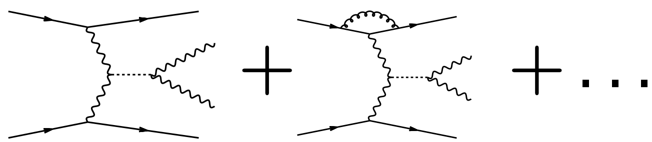
theoretical uncertainty

observable: does not depend on unphysical scales μ
(e.g. renormalization scale)

theoretical prediction: typically depends on unphysical scales

QCD @ high-energy colliders: (ideally) series expansion in α_s

$$\sigma = \sum_{n=n_0}^{N} \alpha_s^n \sigma^{(n)} + \mathcal{O}(\alpha_s^{N+1})$$



truncation at fixed order α_s^N ($\rightarrow$ LO, NLO, . . .)

theoretical uncertainty

truncation $\rightarrow$ residual dependence on unphysical scale μ:

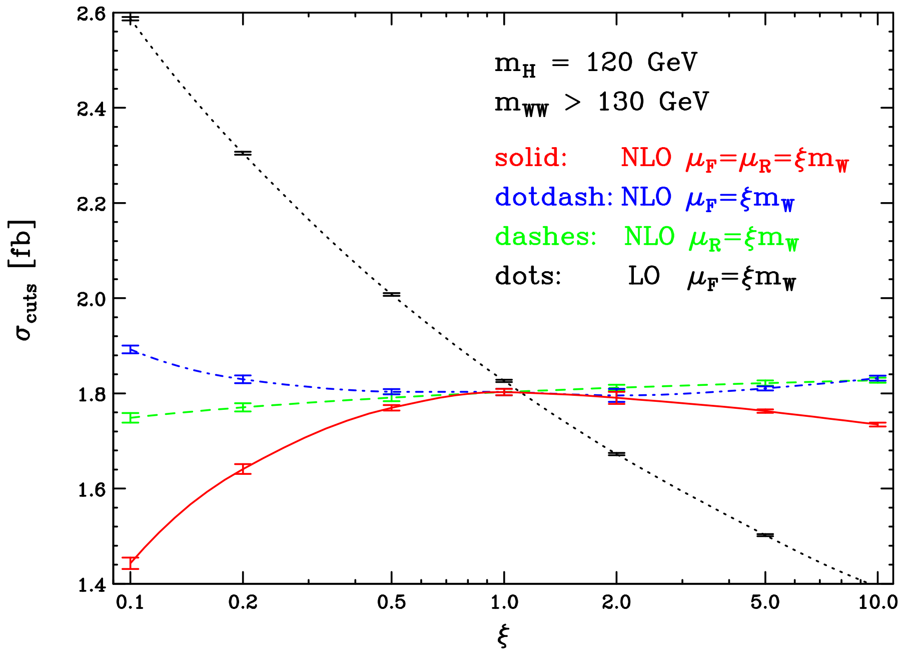
$$\mu \frac{d}{d\mu}\sigma = \mu \frac{d}{d\mu}\left(\sum_{n=n_0}^{N} \alpha_s^n \sigma^{(n)} + \sum_{n=N+1}^{\infty} \alpha_s^n \sigma^{(n)}\right) = 0$$

for perturbative fixed-order prediction, this implies

$$\mu \frac{d}{d\mu} \underbrace{\sum_{n=n_0}^{N} \alpha_s^n \sigma^{(n)}}_{\text{calculated}} = -\mu \frac{d}{d\mu} \underbrace{\sum_{n=N+1}^{\infty} \alpha_s^n \sigma^{(n)}}_{\text{disregarded}}$$

➠ scale dependence $\sim$ measure for reliability of perturbative prediction

scale uncertainty

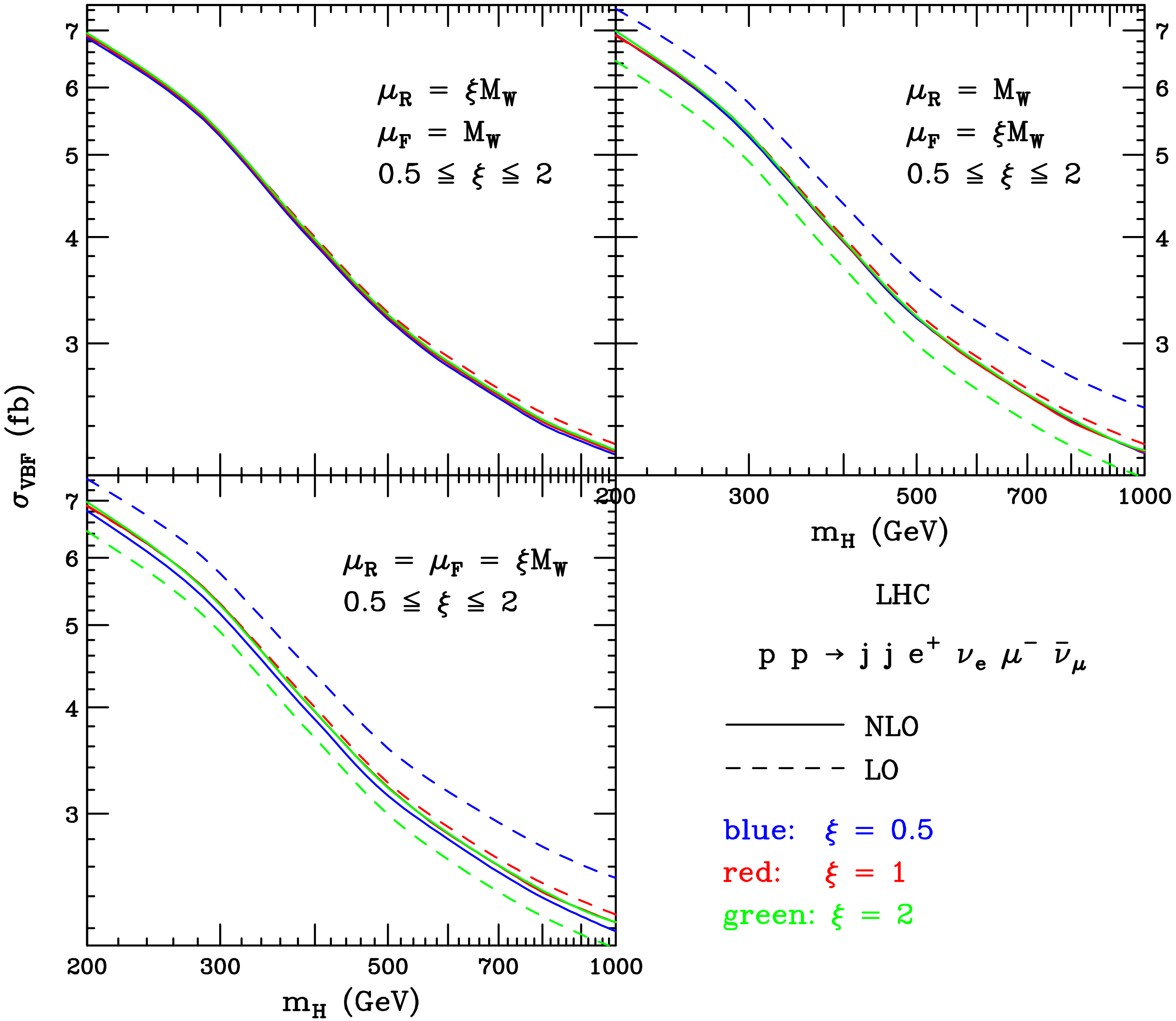


choose $\mu_R = \xi M_W$ and $\mu_F = \xi M_W$ with variable ξ

LO: no control on scale

NLO: scale dependence strongly reduced

scale uncertainty



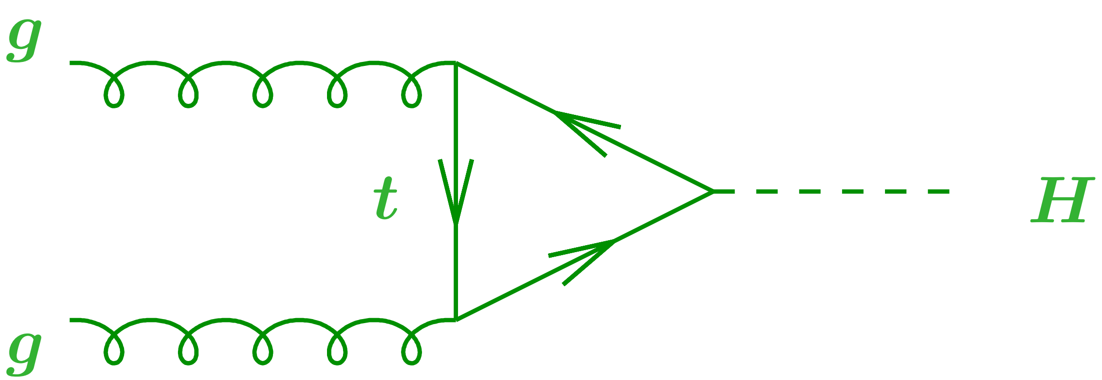
…and its effects on $\sigma_{cuts}(m_H)$

gluon fusion

how does this performance compare to gluon fusion?

reminder: $gg \to H$

. . . largest Higgs production cross section at LHC



gluon fusion at NLO QCD

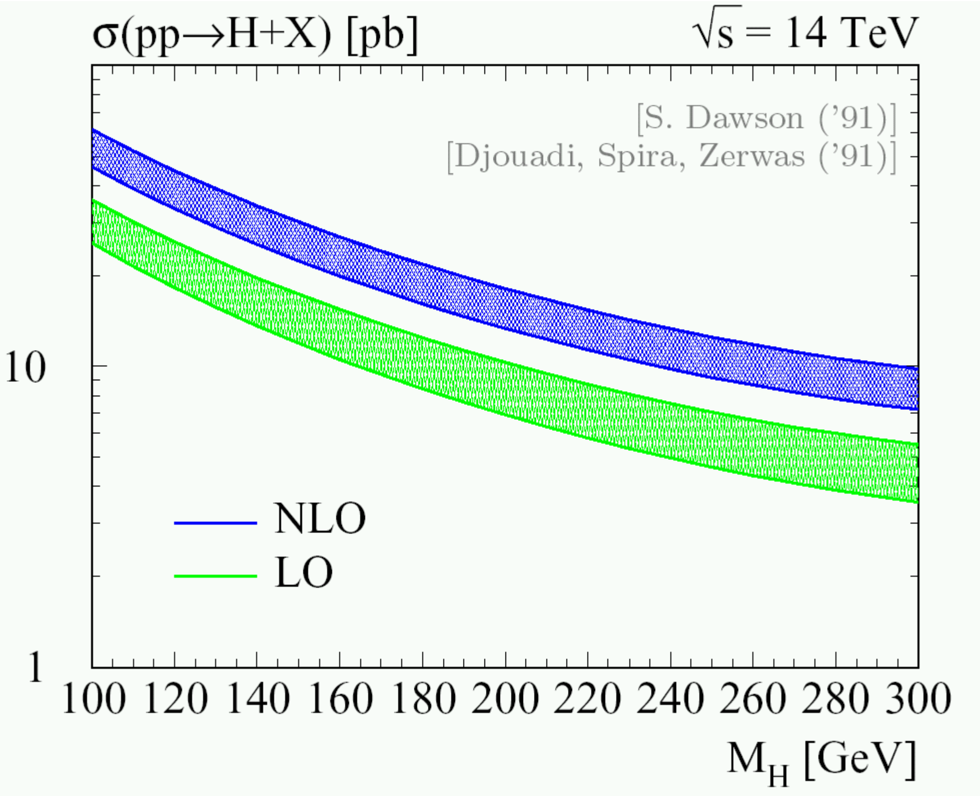
NLO-QCD calculation reveals:

large corrections with

$$\frac{\sigma^{\mathrm{NLO}}}{\sigma^{\mathrm{LO}}} \sim 1.7 \div 2.0$$

Dawson (1991)

Djouadi, Spira, Graudenz, Zerwas (1991,1993)



gluon fusion at NNLO QCD

convergence properties

and uncertainties

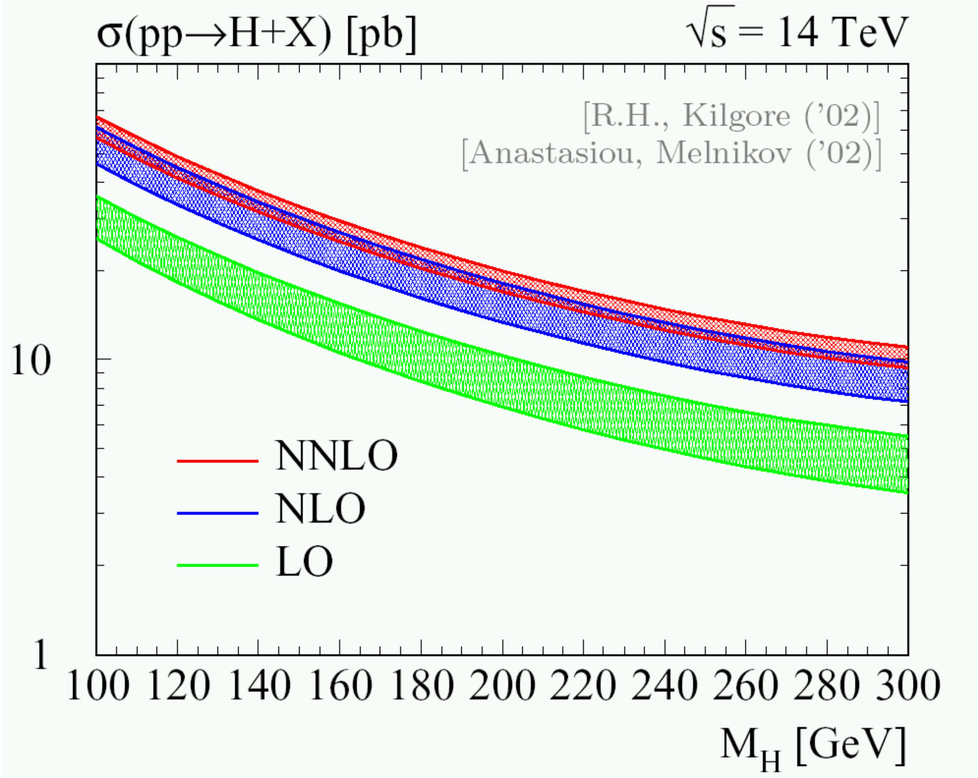
much improved at NNLO QCD

- in $m_{\text{top}} \to \infty$ approximation:

Harlander, Kilgore (2002)

Ravindran, Smith, van Neerven (2003)

Anastasiou, Melnikov (2002)



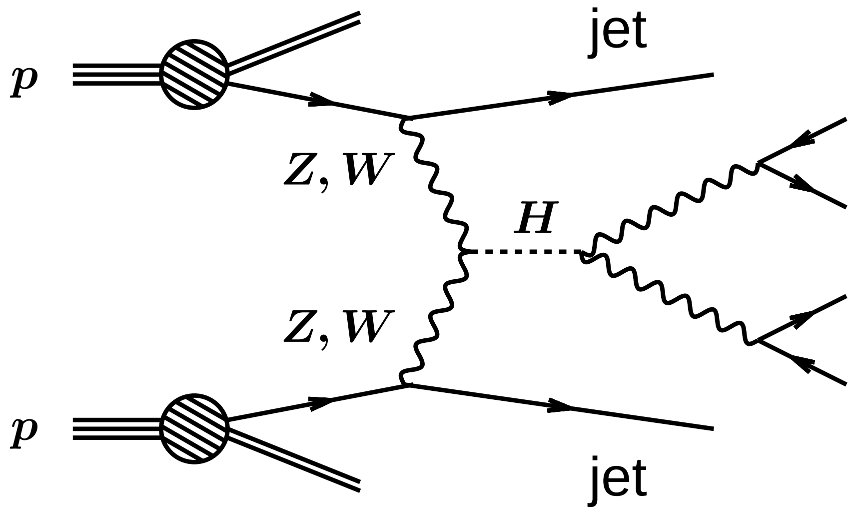
note: more accurate predictions available as of now

gluon fusion at higher orders

for details:

Claude Duhr’s lecture

VBF / VBS event topology



suppressed color exchange between quark lines gives rise to

- little jet activity in central rapidity region
- scattered quarks $\longrightarrow$ two forward tagging jets (energetic; large rapidity)
- decay products of the weak-boson system typically between tagging jets

applications

focus on $pp \to e^+\nu\mu^-\bar{\nu} + 2\,\mathrm{jets}$ at the LHC

use k_T algorithm, CTEQ6 partons distributions, and apply typical VBF cuts:

tagging jets	$p_{Tj} \geq 20$ GeV, $\lvert y_j \rvert \leq 4.5$ $\Delta y_{jj} = \lvert y_{j1} - y_{j2} \rvert > 4,\ y_{j1} \times y_{j2} < 0$ $M_{jj} > 600$ GeV
charged leptons	$p_{T\ell} \geq 20$ GeV, $\lvert \eta_\ell \rvert \leq 2.5,\ \Delta R_{j\ell} \geq 0.4$ $y_{j,min} < \eta_\ell < y_{j,max}$

applications

total $pp \to W^+W^- + 2\,\mathrm{jet}$ cross section

resonant $H \to WW$ contribution
(around m_H)

WW continuum
(above W-pair treshold)

separate by invariant mass cut

$$M_{WW} = \sqrt{(p_e + p_{\nu_e} + p_\mu + p_{\nu_\mu})^2} > m_H + 10\ \mathrm{GeV}$$

(m_H below W-pair treshold)

applications

focus on WW-continuum in the following

by setting

$m_H = 120$ GeV and $M_{WW} > 130$ GeV

↑

separate by invariant mass cut

$$M_{WW} = \sqrt{(p_e + p_{\nu_e} + p_\mu + p_{\nu_\mu})^2} > m_H + 10 \text{ GeV}$$

(m_H below W-pair treshold)

results

parton-level Monte Carlo program:
can calculate cross sections and kinematic
distributions

often more interesting than
inclusive cross sections

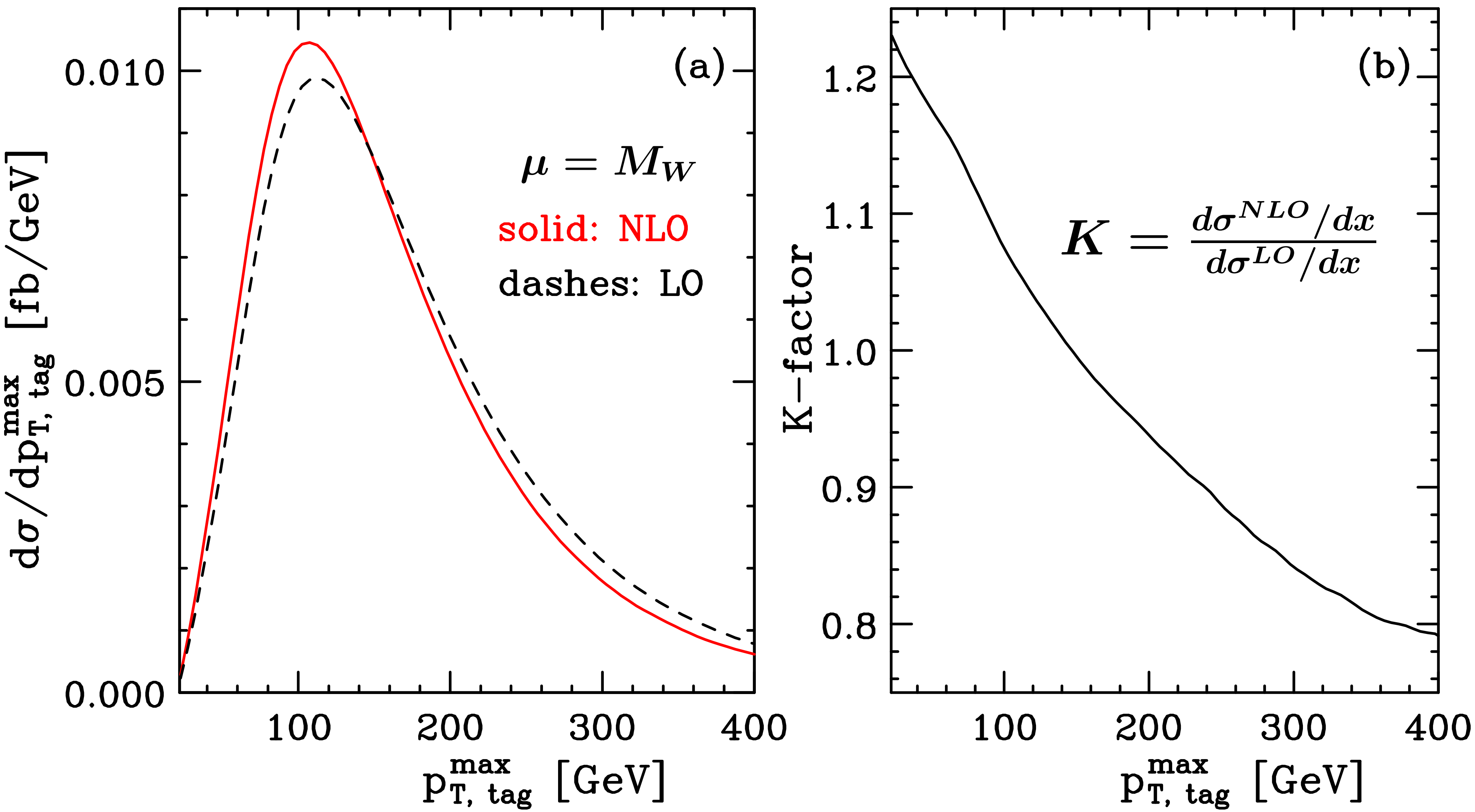
$\updownarrow$

simplify separation of signal
from backgrounds

estimate for importance of NLO
contributions:
dynamical K-factor

$$K(x) = \frac{d\sigma_{NLO}/dx}{d\sigma_{LO}/dx}$$

distributions



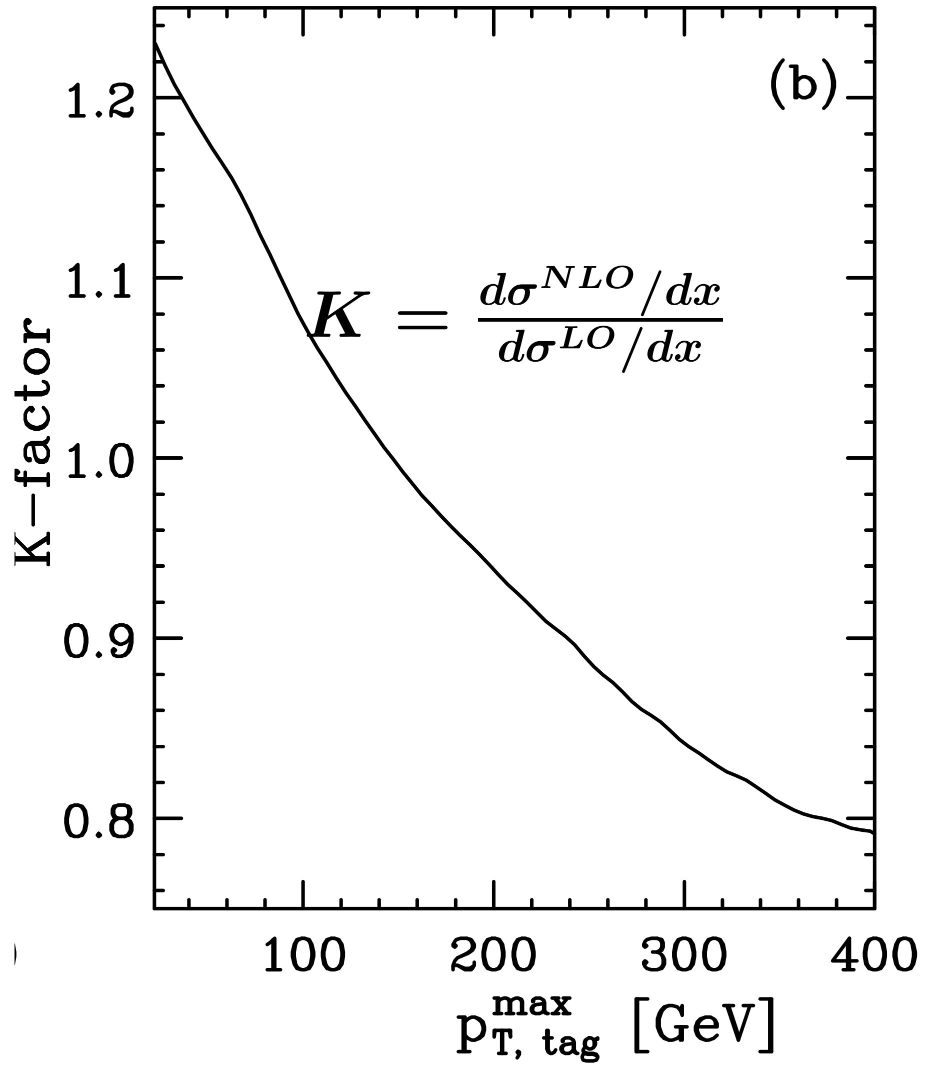
distributions

note: significant change in shape at NLO

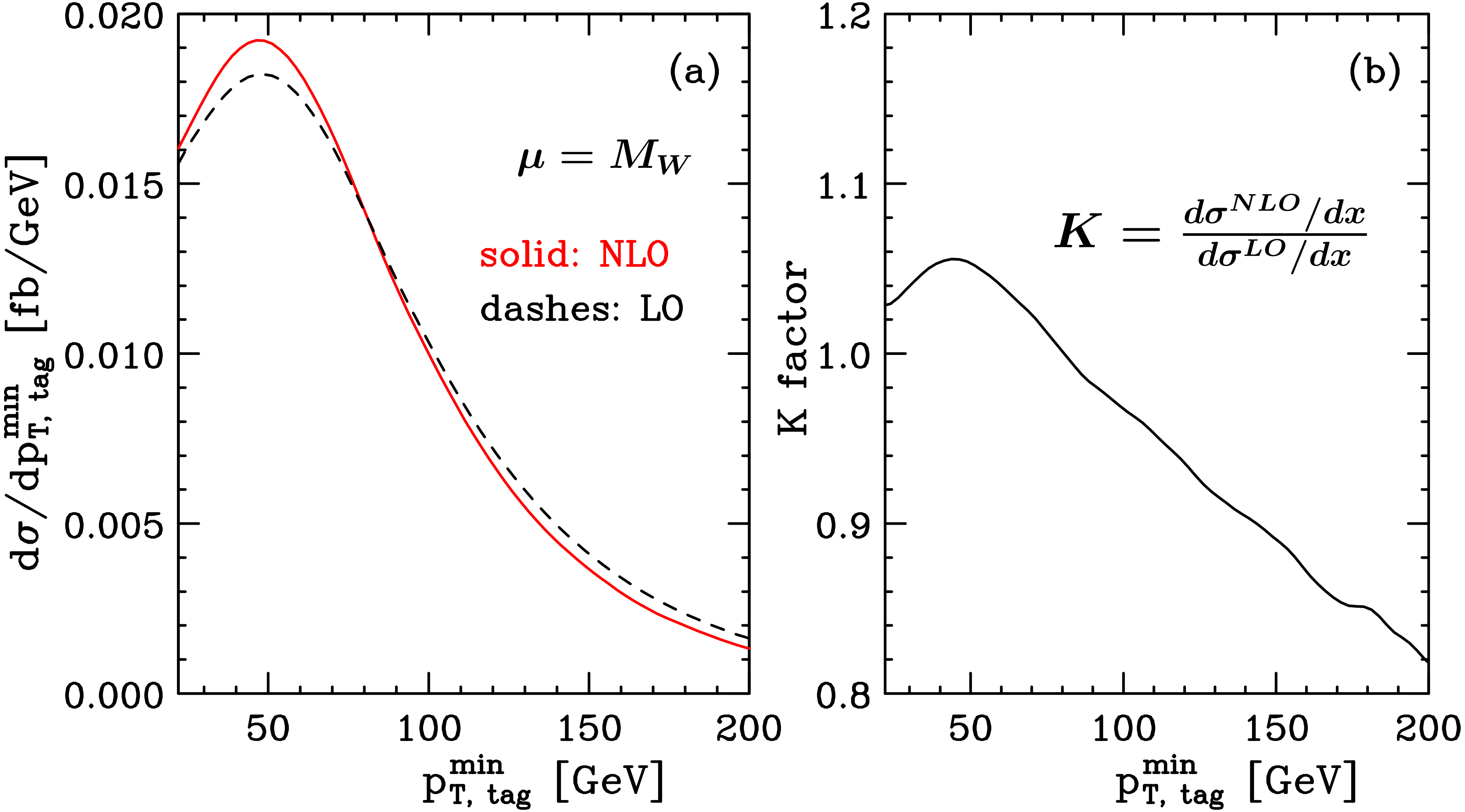
extra gluon in real emission contributions

↓

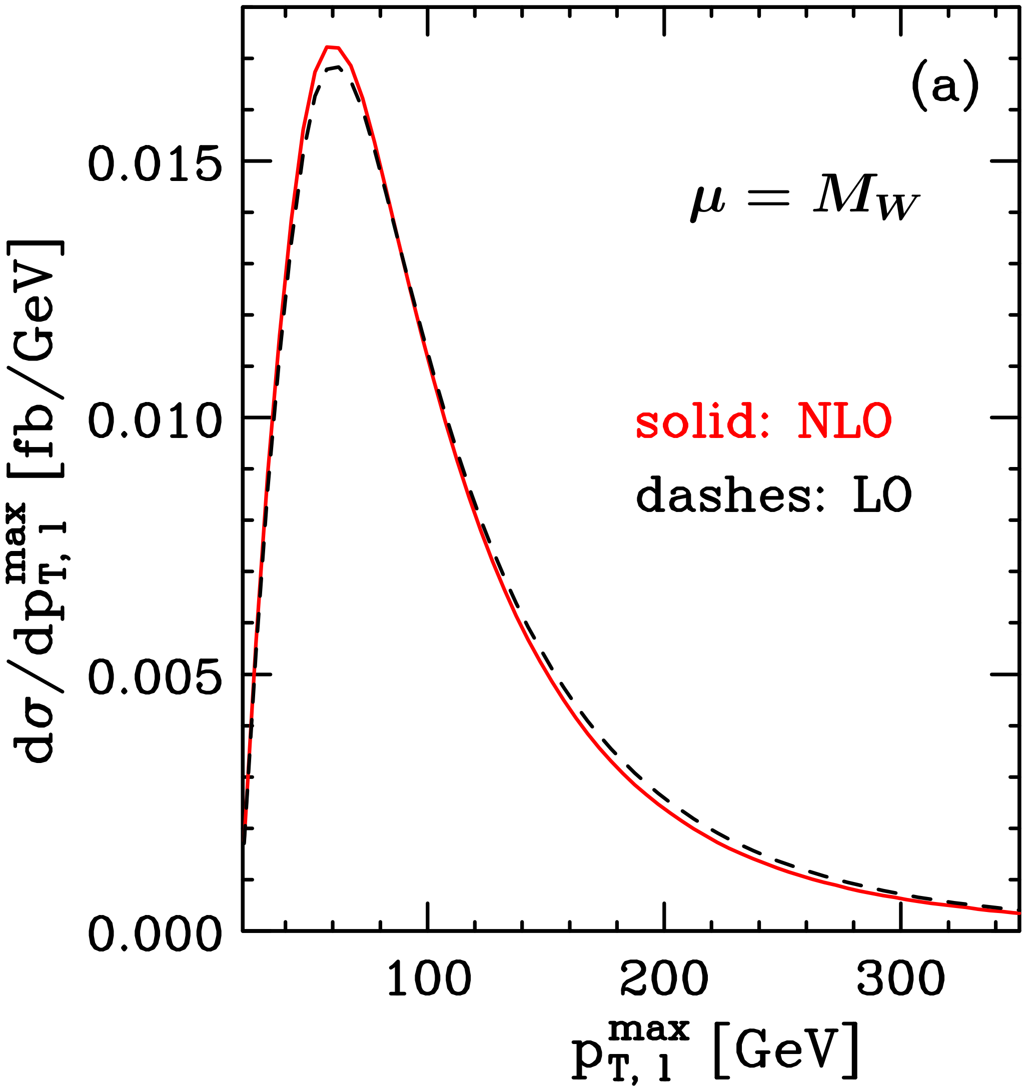
quarks which build tag-jets carry lower transverse momenta

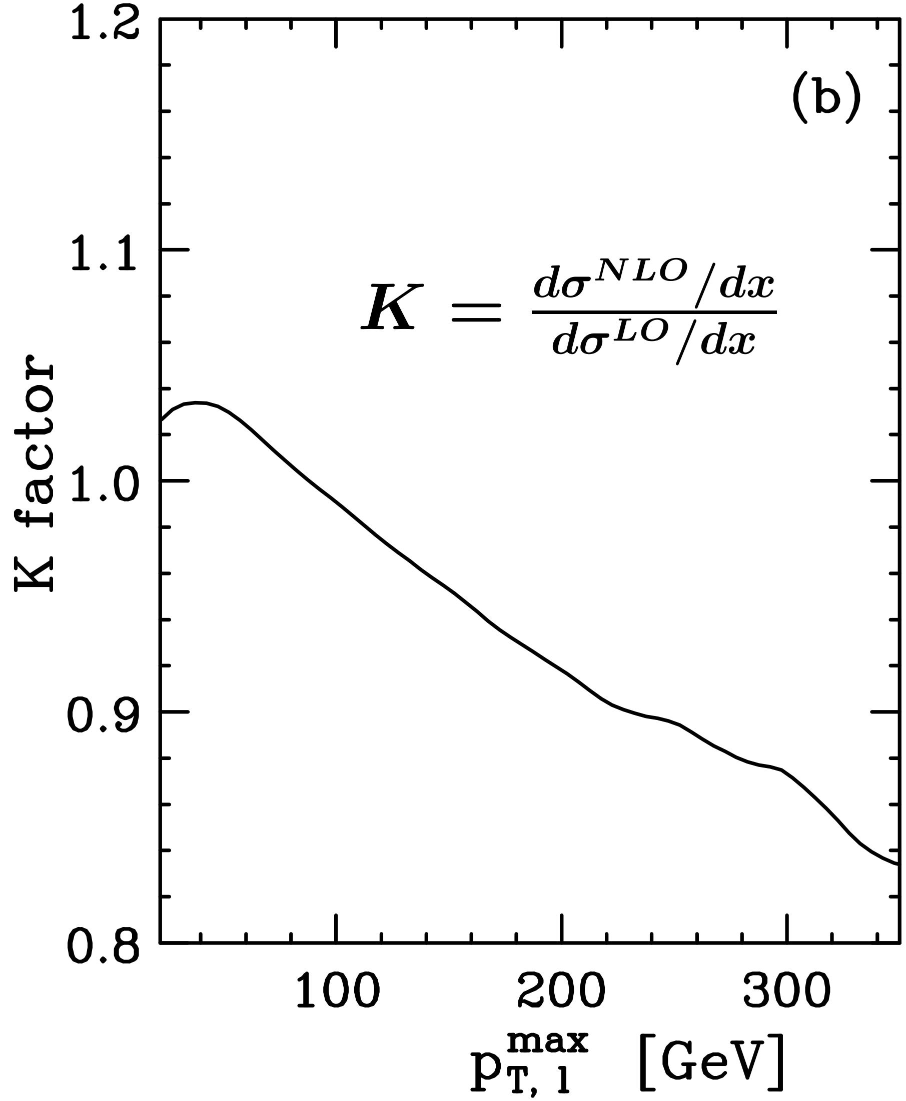


distributions



distributions





more processes . . .

so far: considered $pp \to jj\, e^+\nu_e\mu^-\bar{\nu}_\mu$

("EW $W^+W^-\,jj$ production")

but methods we developed also applicable to related processes with different leptonic final states

↓

in the following also consider

- $pp \to jj\, e^+e^-\mu^+\mu^-$ and $pp \to jj\, e^+e^-\nu_\mu\bar{\nu}_\mu$
  ("EW $ZZ\,jj$ production")

- $pp \to jj\, e^+\nu_e\mu^+\mu^-$ and $pp \to jj\, e^-\bar{\nu}_e\mu^+\mu^-$
  ("EW $W^+Z\,jj$ and $W^-Z\,jj$ production")

$pp \to ZZ\,jj$

✔ clean final state for $pp \to \ell^+\ell^-\ell'^+\ell'^-\,jj$
(all leptons can be detected)

✘ small branching ratios $Z \to$ leptons:

$$BR\,(W \to \ell_i\nu_i) \sim 10.8\%$$
$$BR\,(Z \to \ell_i^+\ell_i^-) \sim 3.3\%$$
$$BR\,(Z \to \nu_i\bar{\nu}_i) \sim 6.6\%$$

$\to$ cross sections small: $\sigma_{ZZ} \ll \sigma_{WW}$

work-around: consider $pp \to \ell^+\ell^-\nu\bar{\nu}\,jj$

[more difficult to reconstruct from experiment,
but larger BR and x-sec]

$pp \to ZZ\,jj$

charged leptons couple to Z bosons and photons
$\to$ need to consider Z and γ intermediate states

↓

large number of diagrams:

	NC (4ℓ)	CC (4ℓ)	NC ($2\ell\,2\nu$)	CC ($2\ell\,2\nu$)
LO	579	241	225	120
NLO	2892	1236	1156	612

c.f. encountered 836 graphs for NC real emission contributions in W^+W^- case

$pp \rightarrow ZZjj$

- helicities of final state charged leptons not fixed

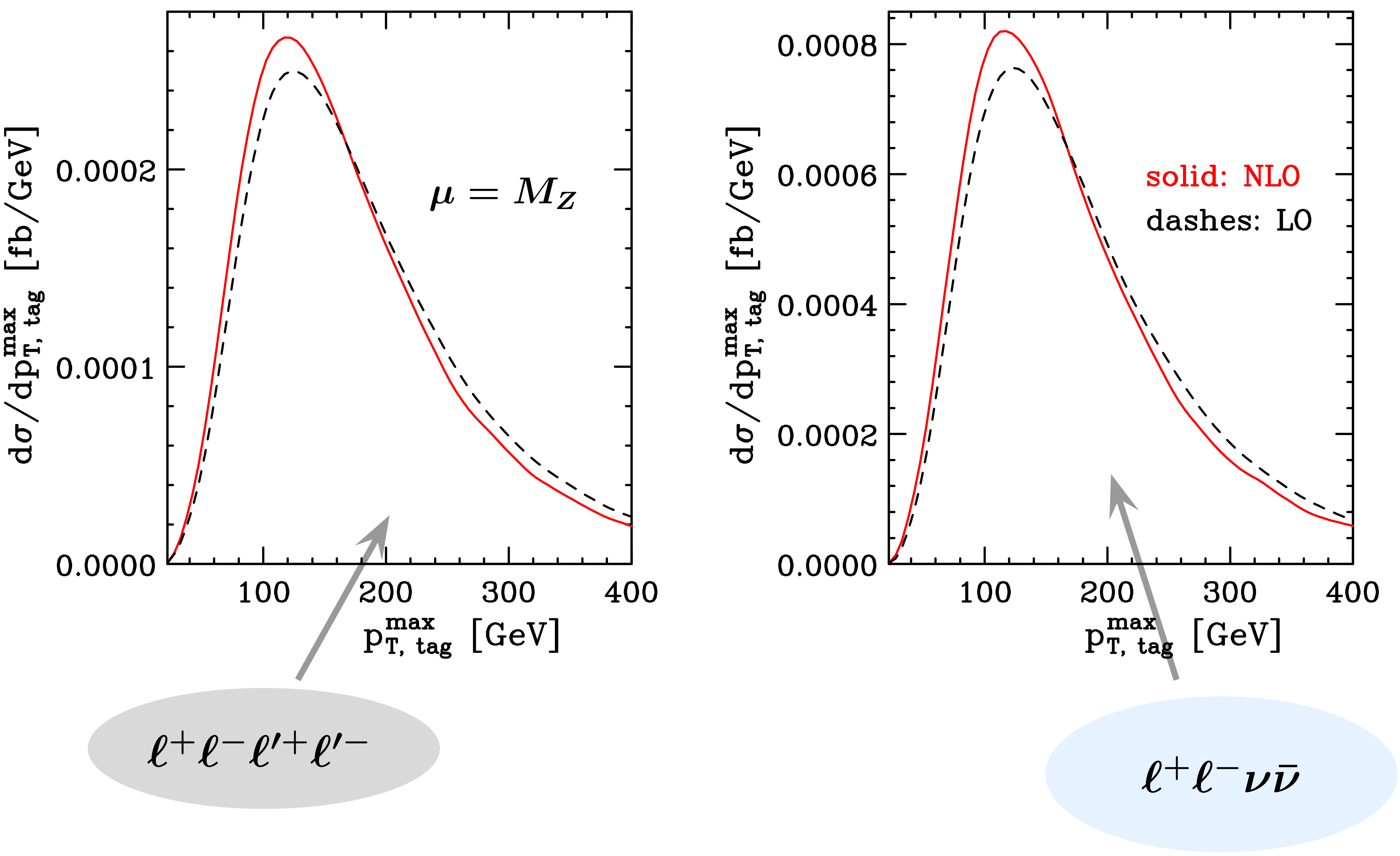
  ↓

  need to keep helicities in leptonic tensors

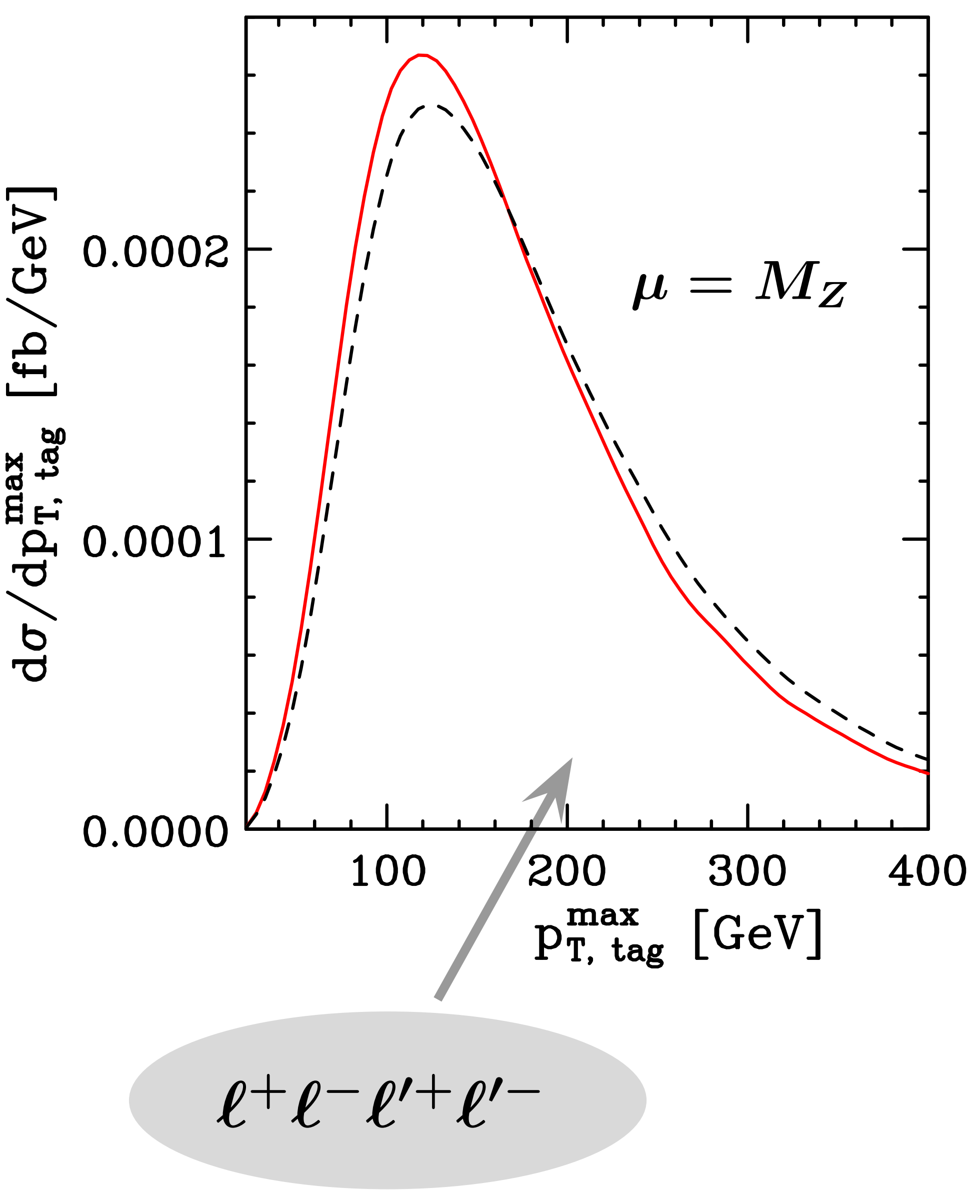
  perform explicit or random helicity summation

- other features very similar to $pp \rightarrow W^+W^-jj$ case

distributions: $pp \to ZZjj$

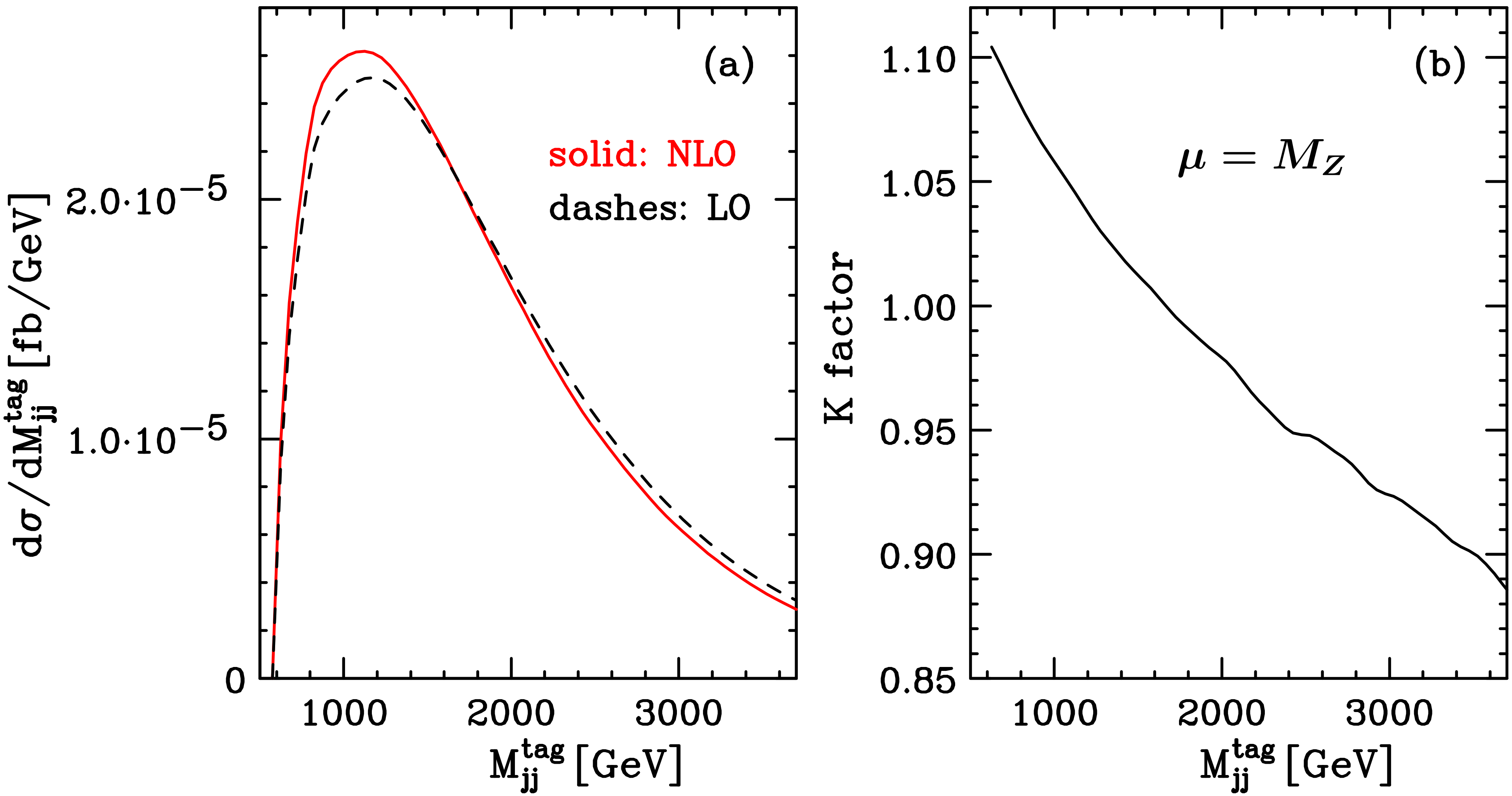


distributions: $pp \to ZZjj$



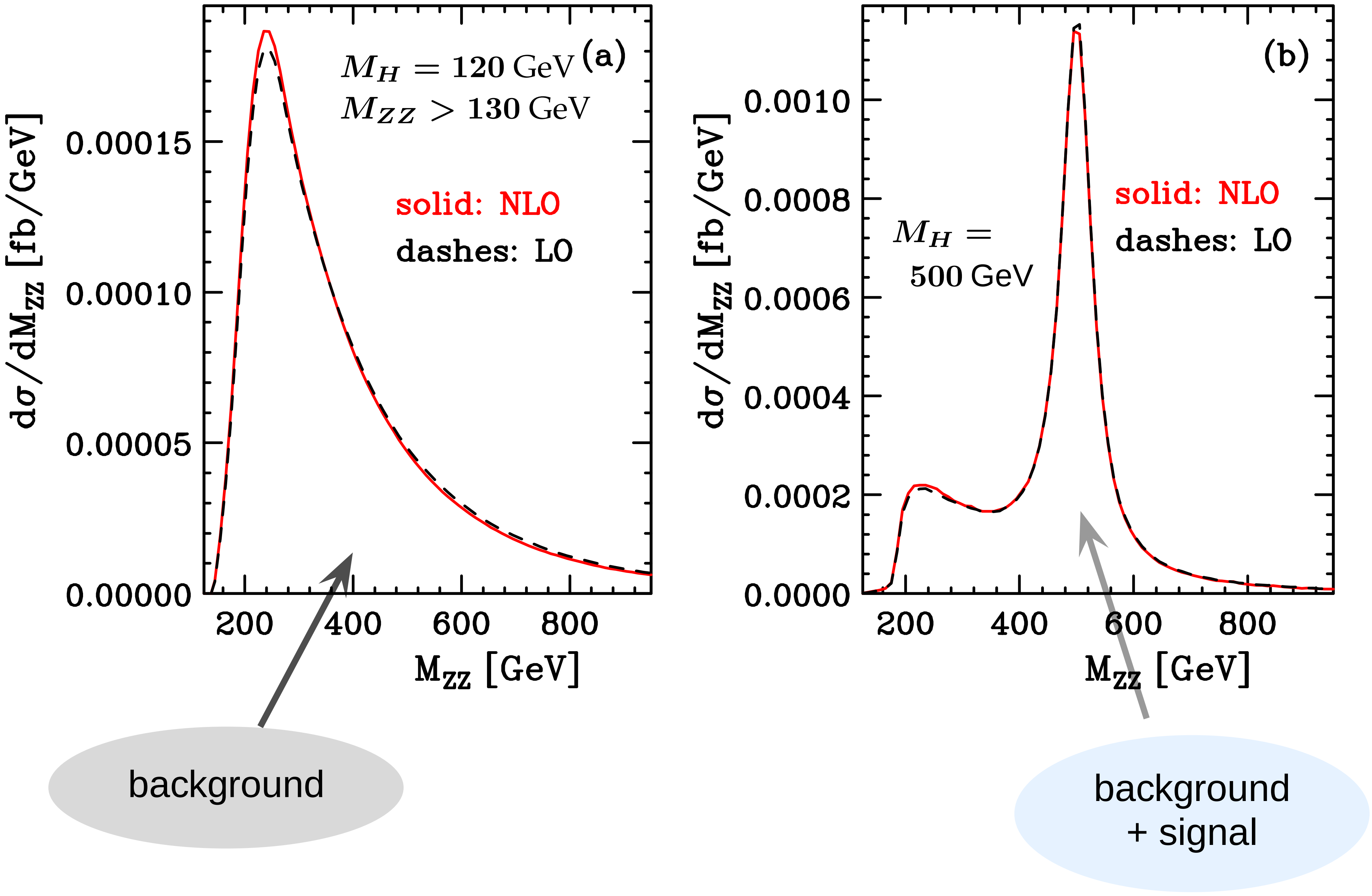
K-factor for p_T-distributions behaves completely analogous to $pp \to W^+W^- jj$ case

distributions: $pp \to jj\, e^+e^-\mu^+\mu^-$



signature typical for VBF $\to$ helps to suppress backgrounds,
e.g., from gluon fusion of QCD $VVjj$ production

distributions: $pp \to \ell^+\ell^-\ell'^+\ell'^- jj$



distributions: $pp \to \ell^+\ell^-\ell'^+\ell'^- \, jj$

reminder:

$$M_{ZZ} = \sqrt{(p_{\ell^+} + p_{\ell^-} + p_{\ell'^+} + p_{\ell'^-})^2}$$

- observable very sensitive to light Higgs boson:
  pronounced resonance behavior for $m_H \lesssim 800$ GeV

- for $m_H \sim 1$ TeV: peak diluted ($\Gamma_H \sim 500$ GeV)
  $\to$ signal distributed over wide range in M_{ZZ}

more VBS: $W^\pm Z\,jj$ and $W^\pm W^\pm\,jj$ production

calculation proceeds in the same way as for
$pp \to W^\pm W^\mp\,jj$ and $pp \to ZZ\,jj$

same-sign gauge boson pair production:

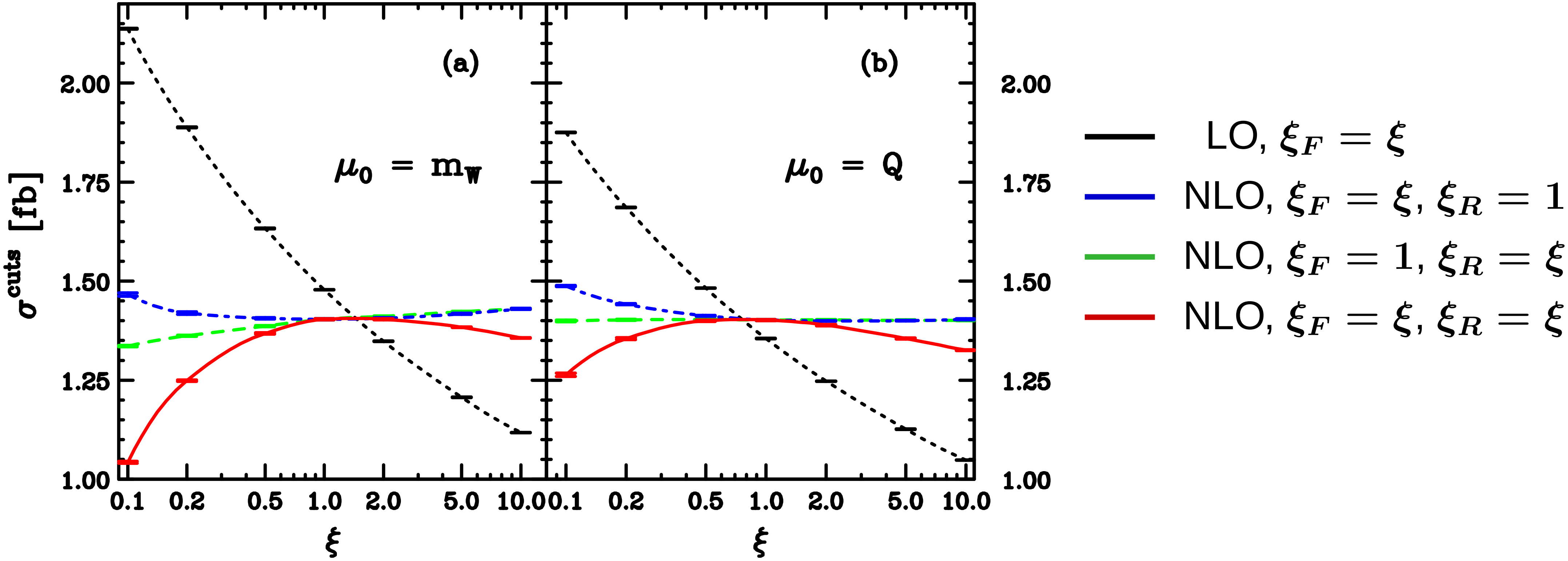
- distinct signature: two same-sign leptons
  plus missing energy and two jets
- low QCD backgrounds
- important background to double-parton scattering
- background for new-physics scenarios
  (R-parity violating SUSY models,
  doubly charged Higgs bosons, . . .)

scale uncertainty: $pp \to W^+W^+jj$

choose default scale $\mu_0 = m_W$ or $\mu_0 = Q$

set $\mu_R = \xi_R \mu_0$ and $\mu_F = \xi_F \mu_0$, with variable ξ

Oleari, Zeppenfeld, B. J. (2009)

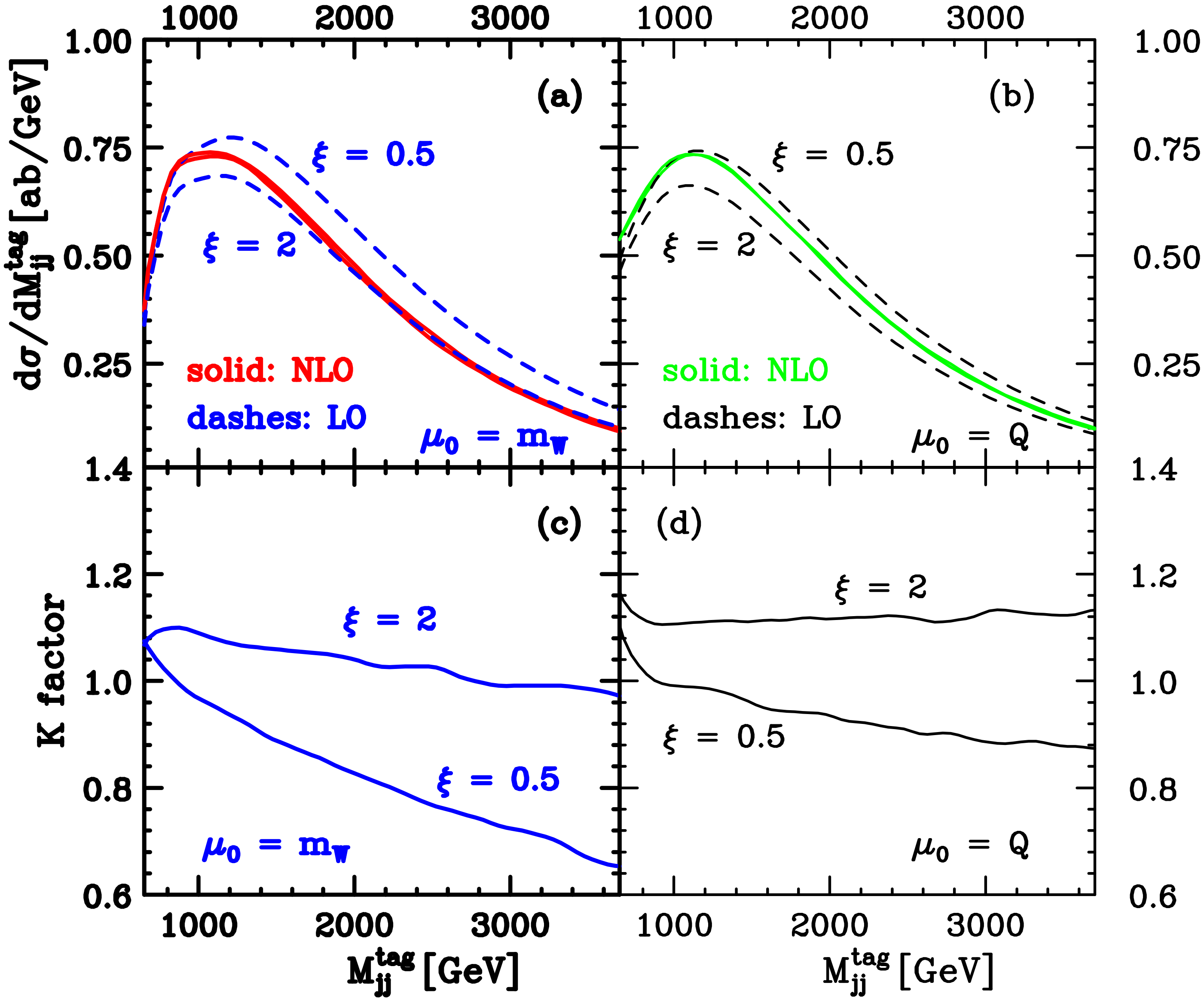


LO: no control on scale

NLO QCD: scale dependence strongly reduced

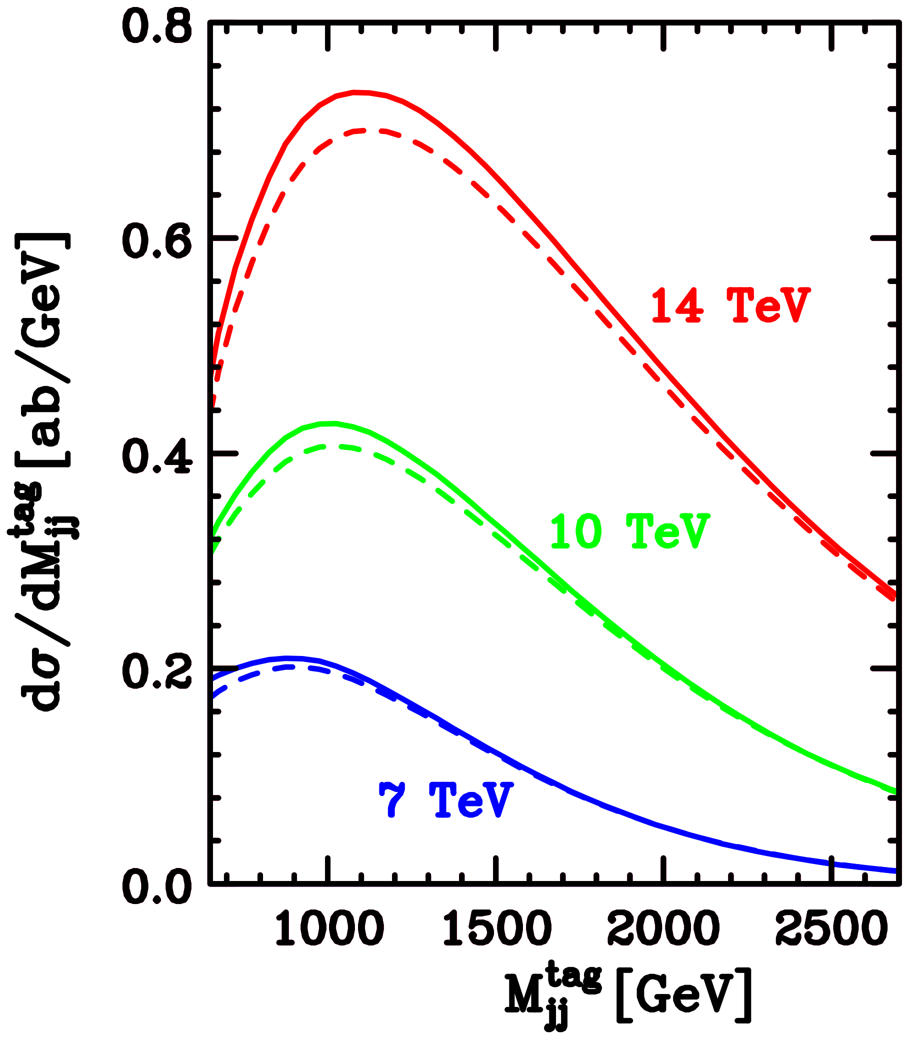
scale dependence: $pp \to W^+W^+jj$

Oleari, Zeppenfeld, B. J. (2009)



energy dependence: $pp \to W^+W^+jj$

Oleari, Zeppenfeld, B. J. (2009)



impact of NLO-QCD corrections
for different collider energies

VBS results: summary

- ✔ obtained numerical results at NLO-QCD for various weak boson scattering processes

  (focusing on fully leptonic final states)

- ✔ all reactions under excellent control perturbatively

  (moderate K-factors and small scale dependencies at NLO)

- ✔ shape of some distributions changes noticeably at NLO

  (advantageous: dynamical scale choice)