HIGGS IN GLUON FUSION
UNCERTAINTIES

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HIGGS COUPLINGS LUMLEY CASTLE, OCTOBER 12, 2015

WHAT DID WE KNOW IN THE YEAR 2000?

- DGLAP SPLITTING FUNCTIONS KNOWN UP TO NLO
  NNLO SPLITTING FUNCTIONS PUBLISHED IN 2002-2004
  (Moch, Vermaseren, Vogt)

- ONLY INCLUSIVE TOTAL CROSS SECTIONS WITH COLORLESS FINAL STATES (DRELL-YAN, HIGGS) KNOWN AT NNLO
  RAPIDITY DISTRIBUTIONS COMPUTED TO NNLO IN 2003 (HIGGS), 2004 (DY) (Anastasiou, Melnikov, Petriello)

- PDF UNCERTAINTIES ESTIMATED BY COMPARING DIFFERENT PDF SETS
  FIRST GOBAL PDF SETS WITH UNCERTAINTIES PUBLISHED 2002
  (CTEQ, MRST)

- EW CORRECTIONS MOSTLY AVAILABLE FOR LEPTON COLLIDER PROCESSES
  EW CORRECTIONS TO HIGGS PRODUCTION 2004-2006
  (Aglietti, Degrassi et al., Passarino et al., Dittmaier et al.)

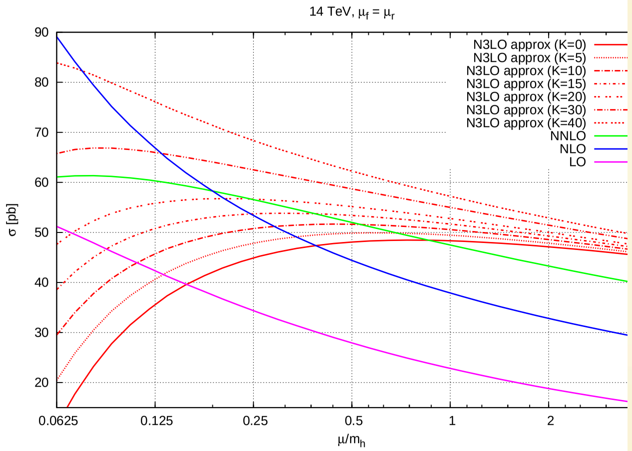
WHAT ARE THE UNCERTAINTIES NOW?

- HIGHER ORDERS
  - PERTURBATIVE STABILITY AND DIVERGENCE
  - ESTIMATING THE UNKNOWN
- ELECTROWEAK CORRECTIONS
  - THE PHOTON PDF
  - MIXED QCD-EW CORRECTIONS
- PDFs (& α_s)
  - PDF UNCERTAINTIES TODAY AND TOMORROW
  - PDFs BEYOND NNLO & THEORETICAL UNCERTAINTIES

HIGHER ORDERS AND RESUMMATION

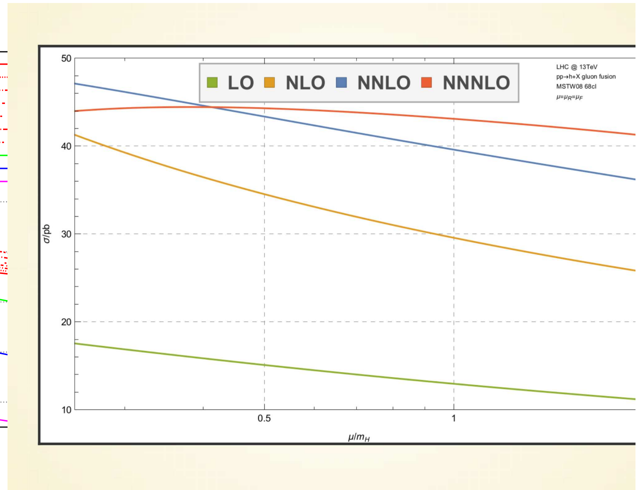
HIGHER ORDERS:
WHAT DIS WE LEARN FROM THE N^{3}LO COMPUTATION?

SCALE DEP VS N^{3}LO SIZE



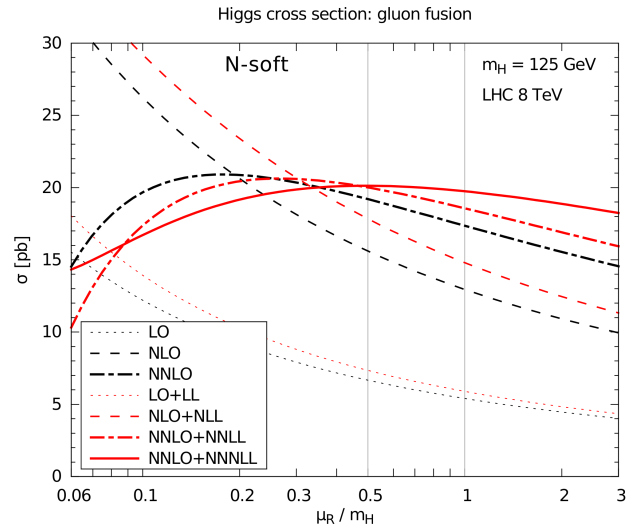
(Bühler, Lazopoulos, 2013)

EXACT N^{3}LO



(Anastasiou et al, 2015)

UNRESUMMED VS RESUMMED EXPANSION



(Bonvini, Marzani, 2014)

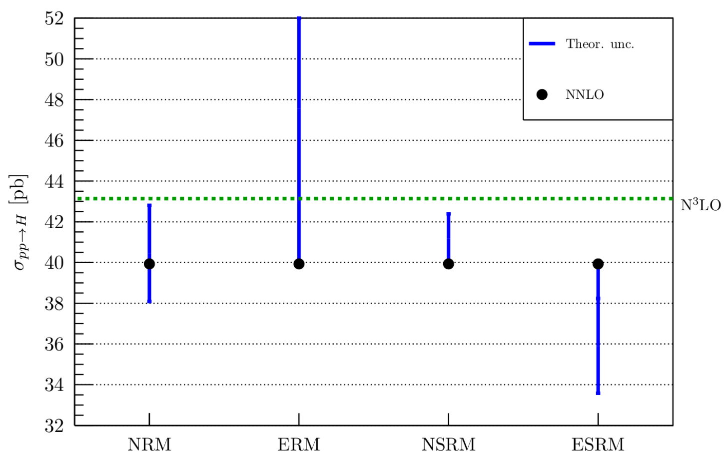
- THE N^{3}LO RESULT IS AT THE SCALE-STABILITY POINT:
  THE PERTURBATIVE EXPANSION IS WELL-BEHAVED
- THRESOLD RESUMMED VALUE & PROFILE CLOSER TO N^{3}LO
  SOFT RESUMMATION ACCELERATES CONVERGENCE

WHEN WILL THE EXPANSION START DIVERGING?

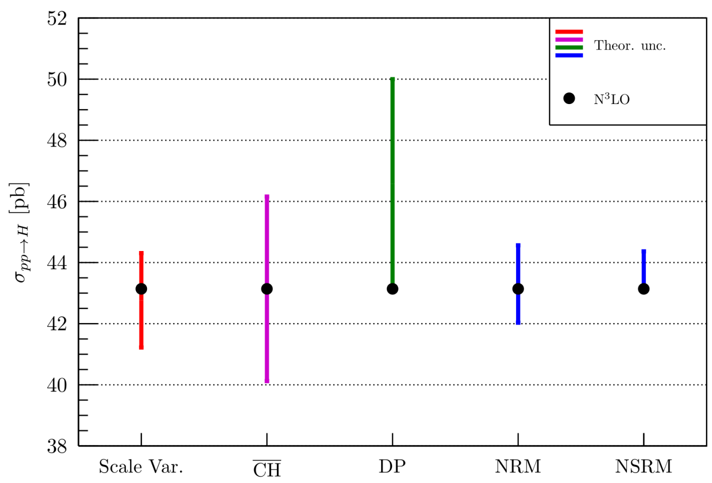
- THE QCD PERTURBATIVE EXPANSION IS ASYMPTOTIC:
  DO WE HAVE TO WORRY ABOUT ITS DIVERGENCE?
- CAN WE USE THE ASYMPTOTIC SUM AS A
  MEANS TO ESTIMATE THE THEORETICAL UNCERTAINTY?

HIGGS IN GLUON FUSION:

NNLO UNCERTAINTY VS TRUE N^3LO
VARIOUS MODELS



NNNLO UNCERTAINTY



(s.f., Isgrò, in preparation)

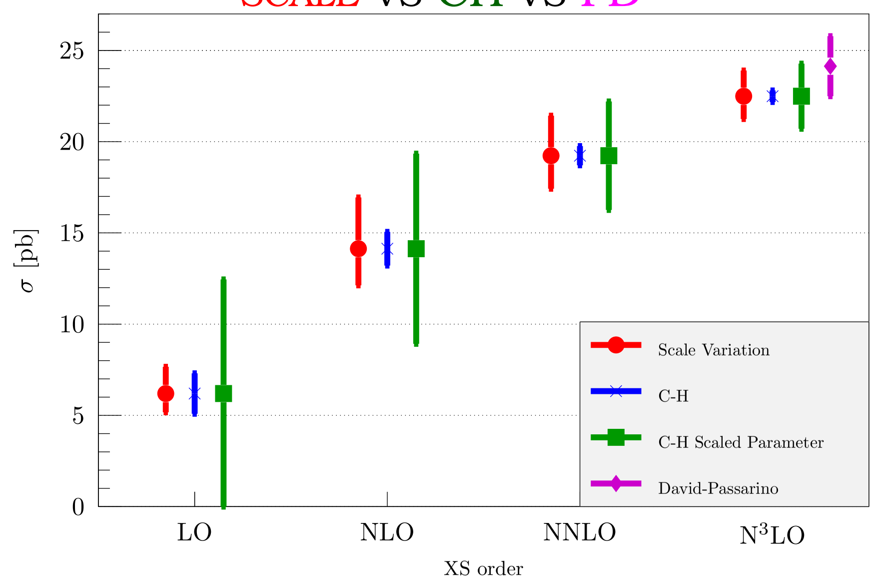
- ORDER OF THE DIVERGENCE: LANDAU POLE $\Rightarrow n \sim 70$ (COEFFNS START GROWING FOR $n > \bar{n} = 33$); RENORMALONS $\Rightarrow n \sim 30$ ($\bar{n} = 13$)
- ASSUME ASYMPTOTIC SUM GOVERNED BY RENORMALONS AFTER SUBTRACTING SOFT CONTRIBUTION (NSRM) $\Rightarrow$ REASONABLE RESULTS AT NNLO
- BECOMES MORE RELIABLE AS ORDER INCREASES

HOW DO WE ESTIMATE MISSING HIGHER ORDERS?

- SCALE VARIATION IS A CRUDE METHOD WHICH OFTEN FAILS
- CACCIARI-HOUDEAU: LOOK AT THE BEHAVIOUR OF THE PERTURBATIVE EXPANSION
- PASSARINO-DAVID: TRY TO DEFINE SUM OF THE SERIES BASED ON SERIES ACCELERATION METHODS
- BETTER UNDERSTANDING AS MORE PROCESSES KNOWN

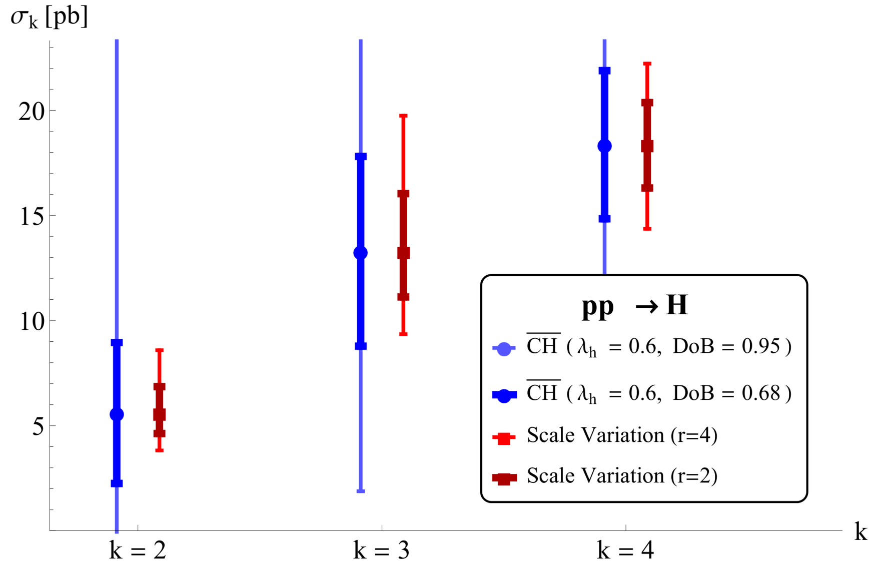
HIGGS IN GLUON FUSION:

SCALE VS CH VS PD



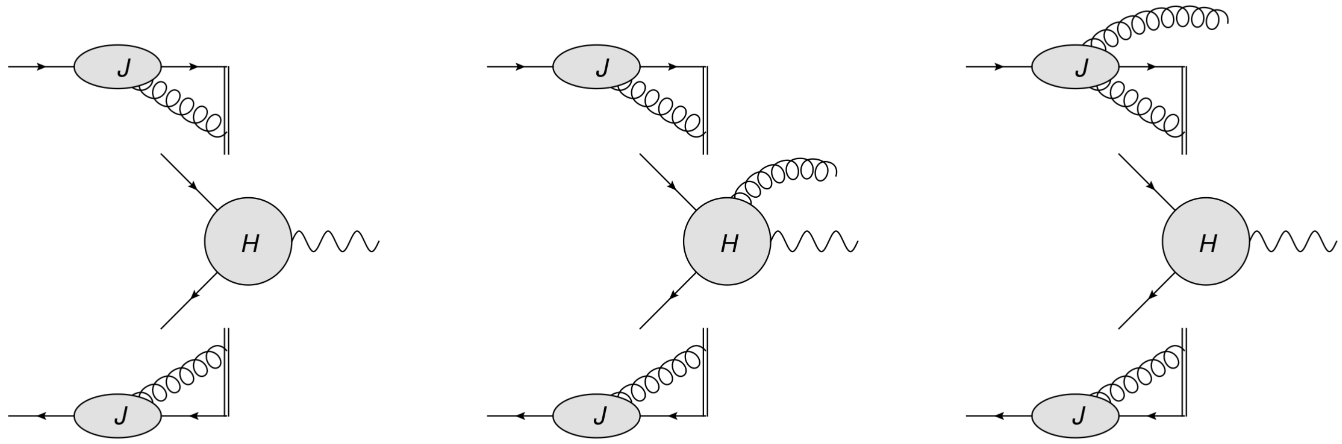
(s.f., Isgrò, Vita, 2014)

DEP ON DOB AND ON SCALE RANGE



(Bagnaschi, Cacciari et al., 2015)

IMPROVING THE SOFT EXPANSION
BEYOND THE LEADING POWER



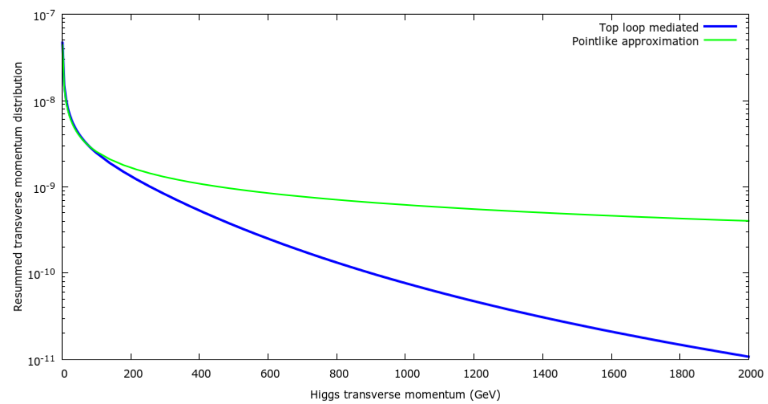
(a) (b) (c)

- IMPRESSIVE PROGRESS IN THE ORGANIZATION OF RESUMMATION BEYOND THE EIKONAL LEVEL: NEXT-TO-EIKONAL $\frac{ln^k N}{N}$; NEXT-TO-NEXT-TO-EIKONAL $\frac{ln^k N}{N^2}$ ETC (Laenen, Magnea, C.White et al, 2012-2105)
- "METHOD OF REGIONS": SEPARATE INTEGRATION REGIONS OVER MOMENTA THROUGH APPROPRIATE SCALING
- COMBINE WITH FACTORIZATION TO CLASSIFY EMISSIONS FROM INTERNAL BLOBS, DRESSED WITH RADIATION FROM EXTERNAL LINES

IMPROVING HIGH ENERGY RESUMMATION
BEYOND INCLUSIVE OBSERVABLES

- CURRENTLY KNOWN TO LL FOR TOTAL CROSS SECTIONS (HIGGS, DRELL-YAN, HEAVY QUARKS)
- RAPIDITY DISTRIBUTIONS KNOWN FOR HIGGS (Caola, SF, Marzani 2011)

HIGGS IN GLUON FUSION: LLx p_t SPECTRUM
POINTLIKE VS. EXACT



FOR $p_t \gtrsim m_t$
EXACT DEVIATES FROM POINTLIKE

(Muselli, SF, Vita, in preparation)

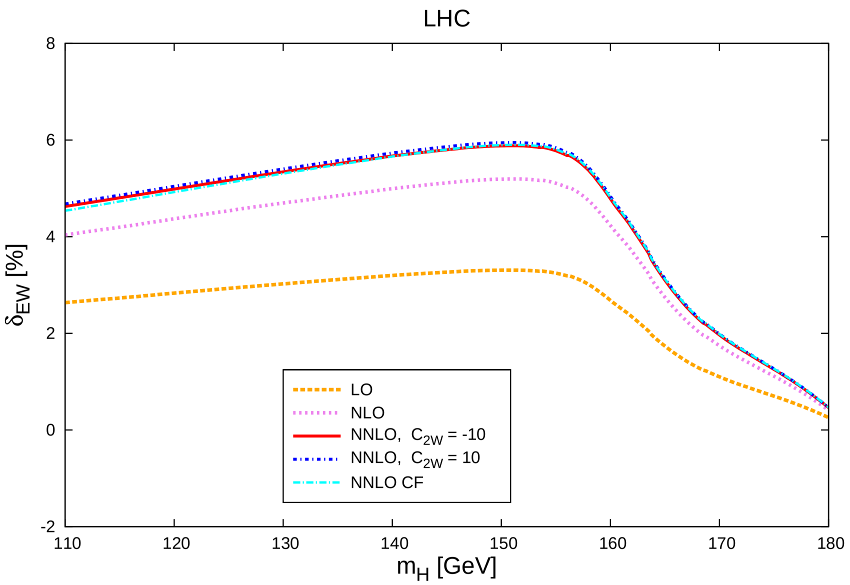
- p_T DISTRIBUTIONS :
  - CAN STUDY SENSITIVITY TO m_t
  - BEHAVIOUR OF p_t RESUMMATION IN REGION $m_b < p_t \ll m_h$

ELECTROWEAK CORRECTIONS

ELECTROWEAK CORRECTIONS

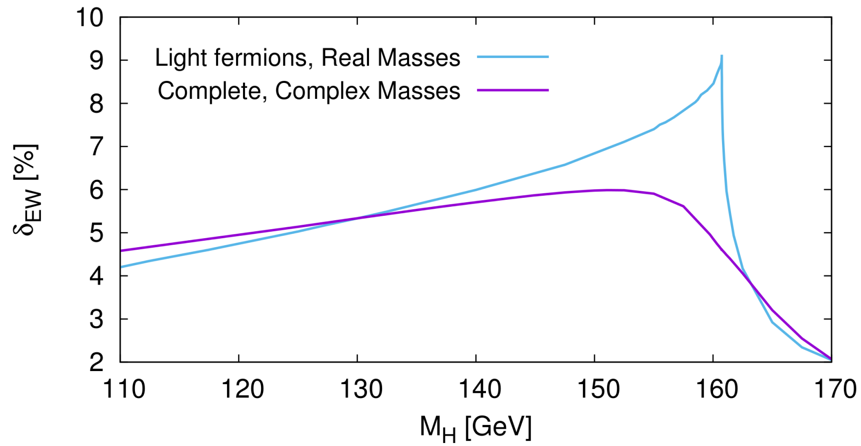
- EW CORRECTIONS HAVE A SIZABLE IMPACT ON THE GLUON FUSION XSECT $\sim 5\%$
- CURRENTLY INCLUDED USING COMPLETE FACTORIZATION:
  IF ONLY APPLIED TO LO (PARTIAL FACTORIZATION), CHANGES UP TO 3%
- WILSON COEFFICIENT COMPUTED TO $O(\lambda_{\rm EW}\alpha_s)$ IN EFT
  (EXPANSION IN $\frac{m_H}{2m_t}$, $\frac{m_h}{m_W}$) (Anastasiou, Boughezal, Petriello 2008)
  100% VIOLATION OF FACTORIZATION NOT ENOUGH TO PRODUCE SIGNIFICANT EFFECT
- FINITE m_t, m_W CORRECTIONS PROBABLY SMALL
- CAN WE REALLY TRUST FACTORIZATION TO $O(\lambda_{\rm EW}\alpha_s^2)$?

DEPENDENCE OF FACTORIZATION



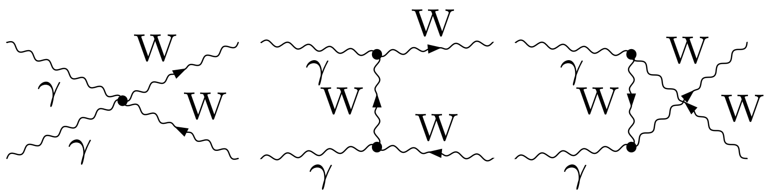
(Anastasiou, Boughezal, Petriello 2008)

ACCURACY OF EFT



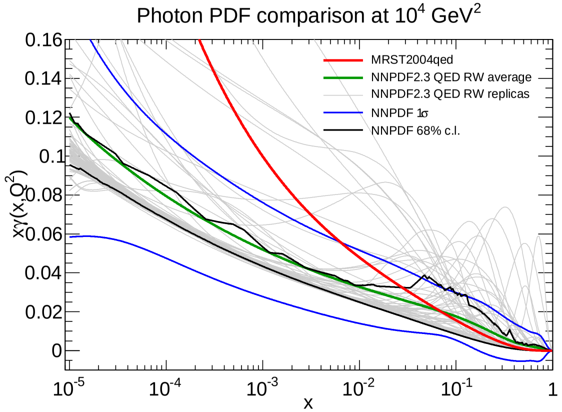
(Uccirati, Les Houches 2015)

QED PDFs



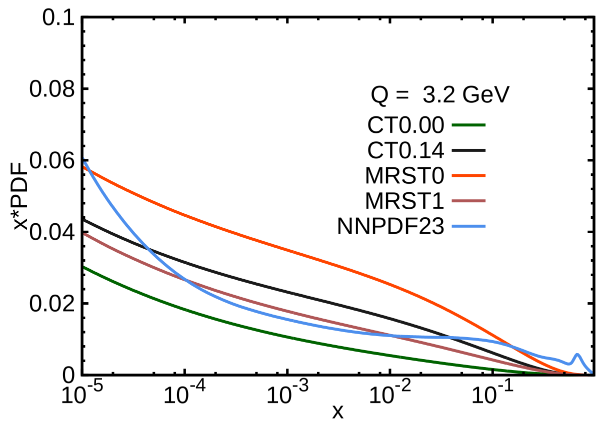
- NNPDF2.3 QED PHOTON PDF DETERMINED FROM FIT TO LHCb LOW-MASS DY, ATLAS W PRODUCTION AND HIGH-MASS (Carrazza et al. 2013)
- PREVIOUS MRST2004QED (MODEL); RECENT CT14QED (MODEL+ CONSTRAINTS FROM ISOLATED PHOTON IN DIS FROM HERA) (Schmidt, Pumplin, Stump, Yuan, 2015)
- RELEVANT FOR W PAIR PRODUCTION: PHOTON-INDUCED COMPARABLE TO QUARK-INDUCED FOR LARGE INVARIANT MASS

γ: NNPDF2.3QED VS MRST2004



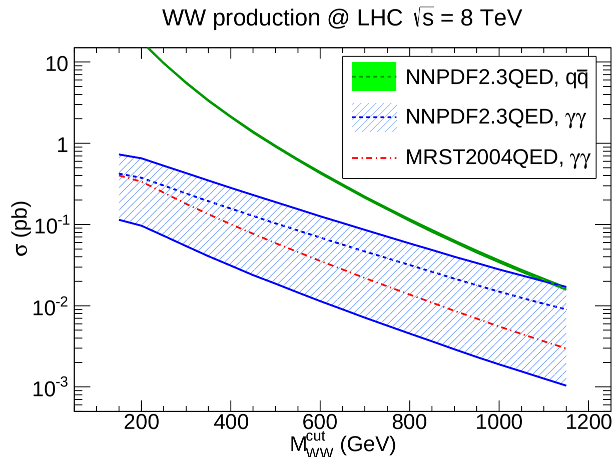
NNPDF2.3QED

γ: NNPDF2.3QED VS MRST2004 VS CT14QED



CT14QED

W PAIR PRODN.: $\bar{q}q$ VS $\gamma\gamma$

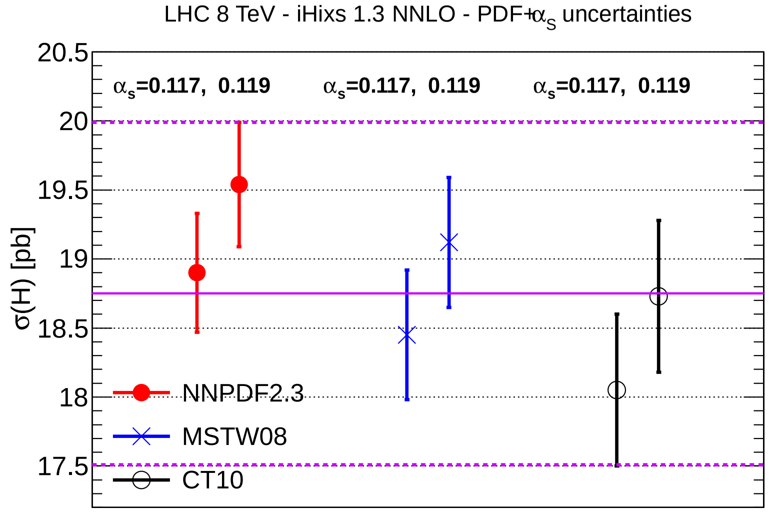


(NNPDF2.3QED)

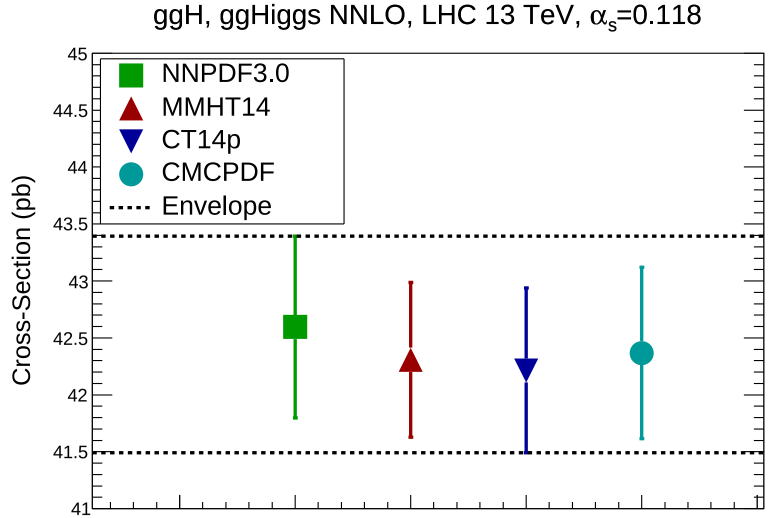
PDFs (AND α_s)

PDFs: RECENT PROGRESS

2012



2015

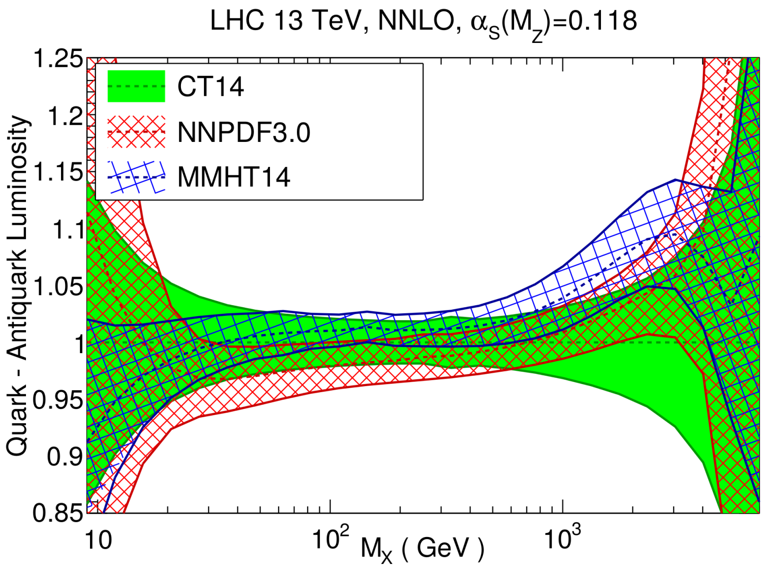


- MAJOR UPGRADES FROM ALL GLOBAL FITTING GROUPS: NNPDF2.3→3.0 (10/2015); MSTW08 → MMHT14 (12/2015); CT10 ⇒ CT14 (06/2015)
- PDF UNCERTAINTY ON HIGGS PRODUCTION DOWN TO ABOUT 2% ENVELOPE NO LONGER NECESSARY ⇒ NEW PDF4LHC PRESCRIPTION
- HOW AND WHY DID IT HAPPEN?

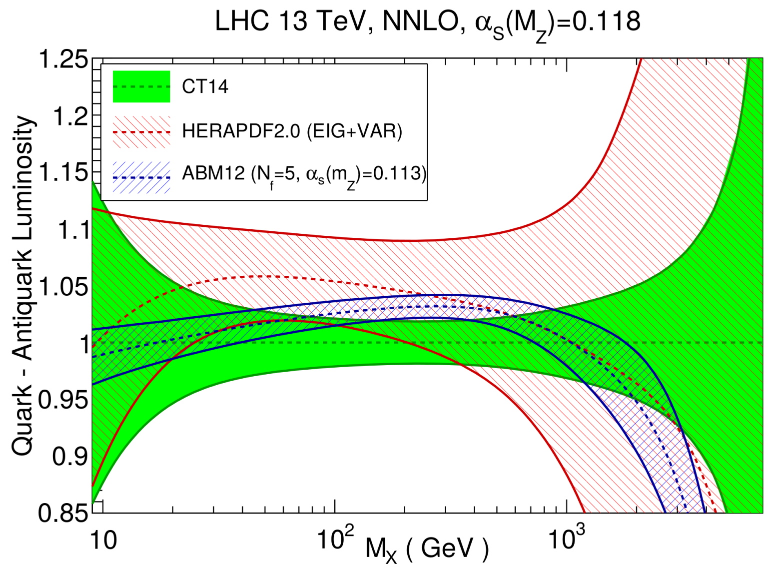
PARTON LUMINOSITIES

QUARK-ANTIQUARK

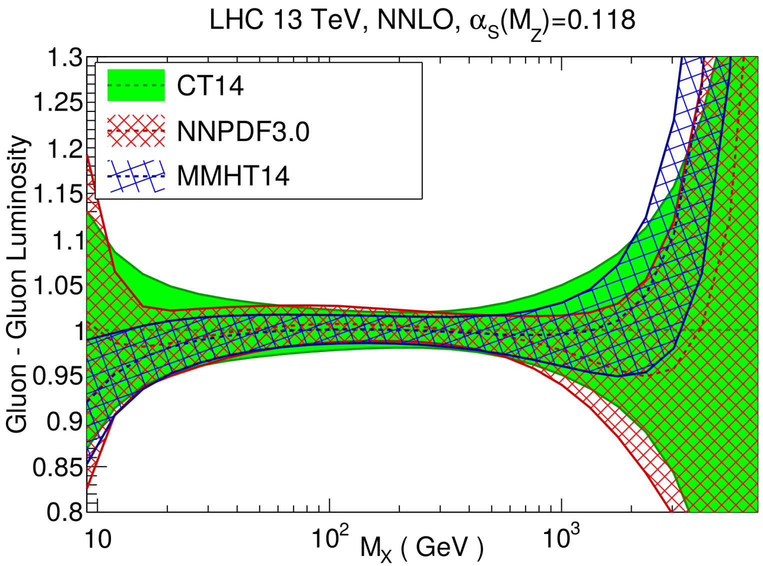
GLOBAL DATASET

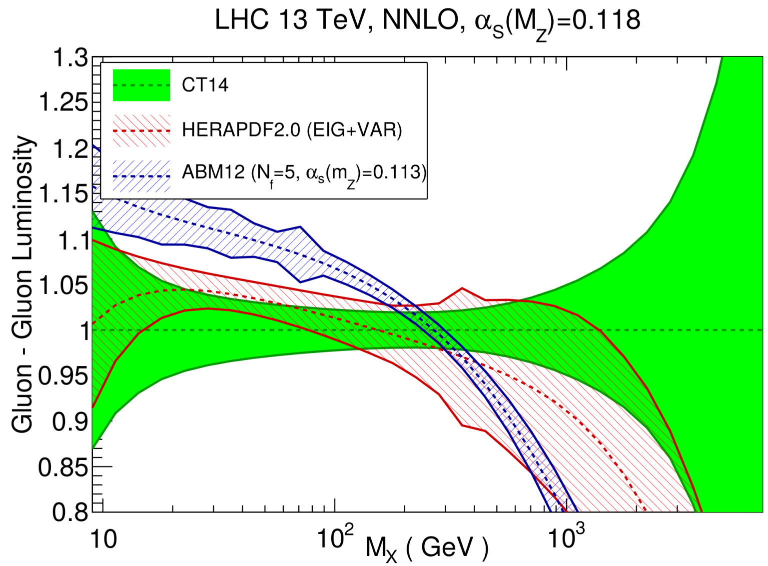


REDUCED DATASET



GLUON-GLUON



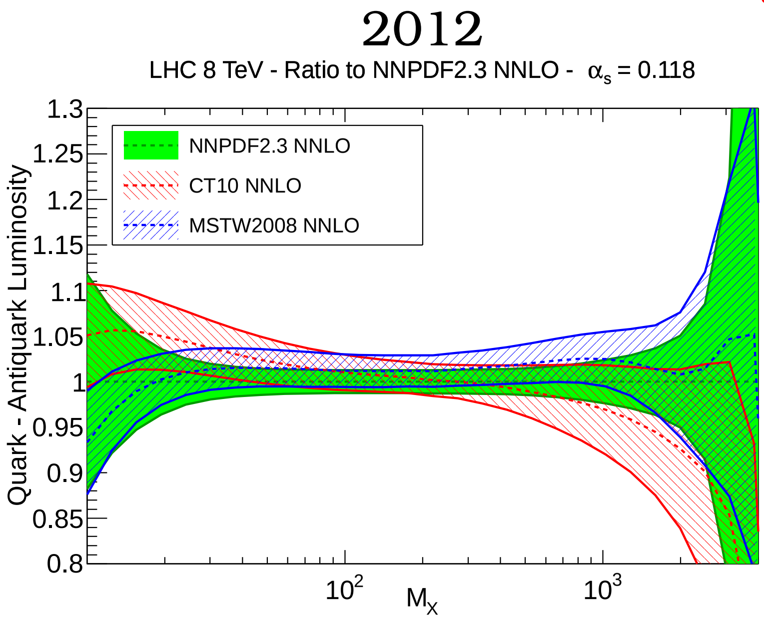


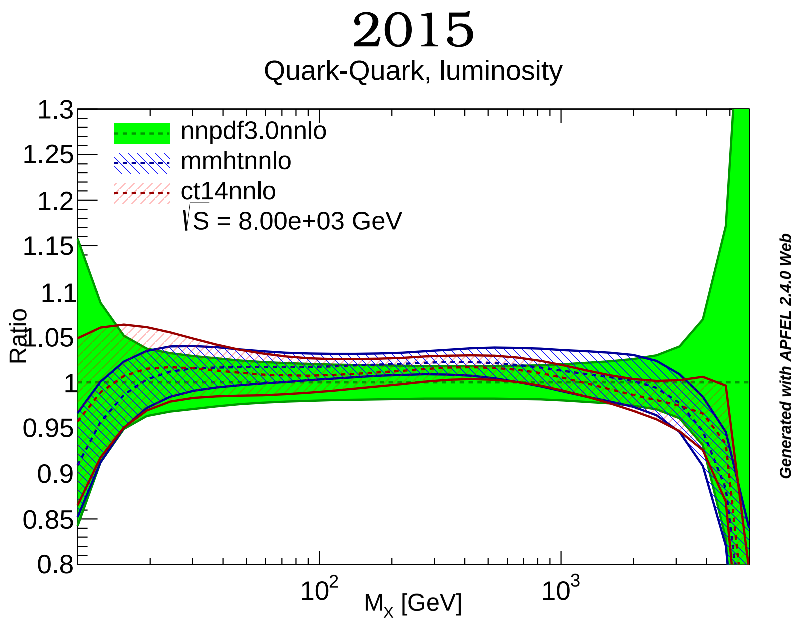
- GLOBAL FITS AGREE WELL
- FITS BASED ON REDUCED DATASET HAVE EITHER LARGE UNCERTAINTIES, OR SHOW SIZABLE DEVIATIONS

PARTON LUMINOSITIES

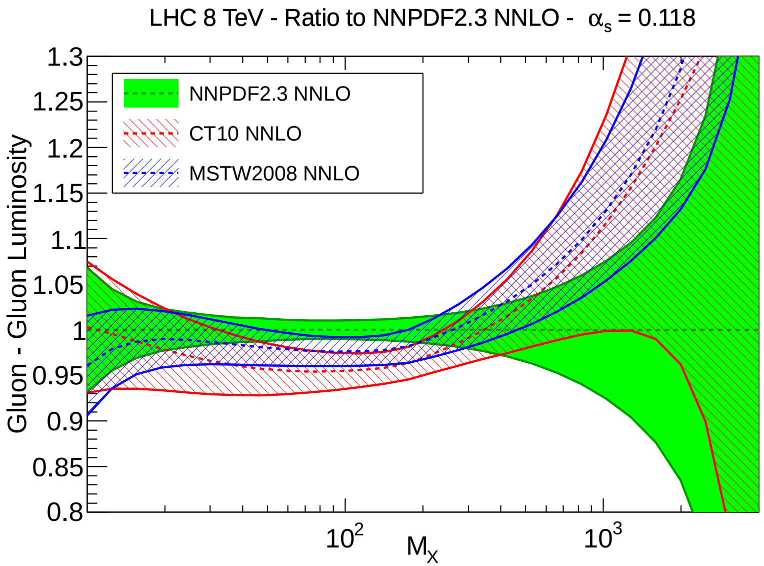
RECENT PROGRESS

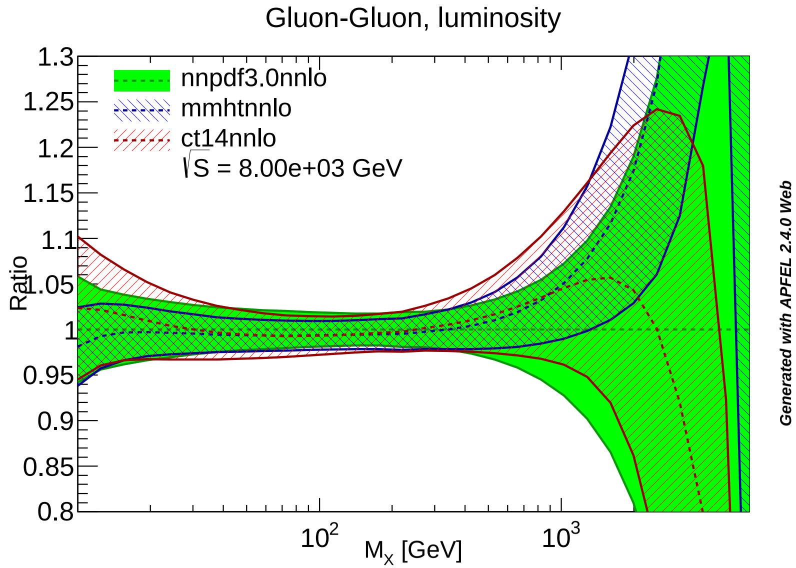
QUARK-QUARK





GLUON-GLUON





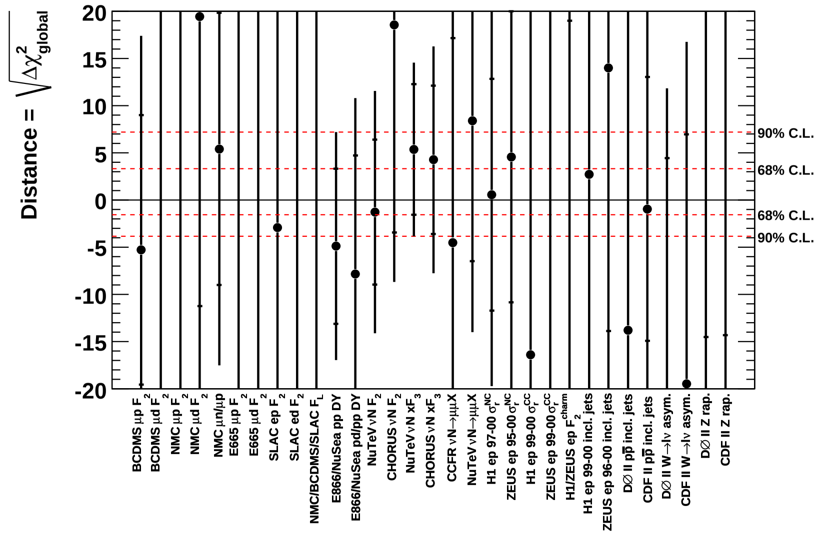
PDF UNCERTAINTIES

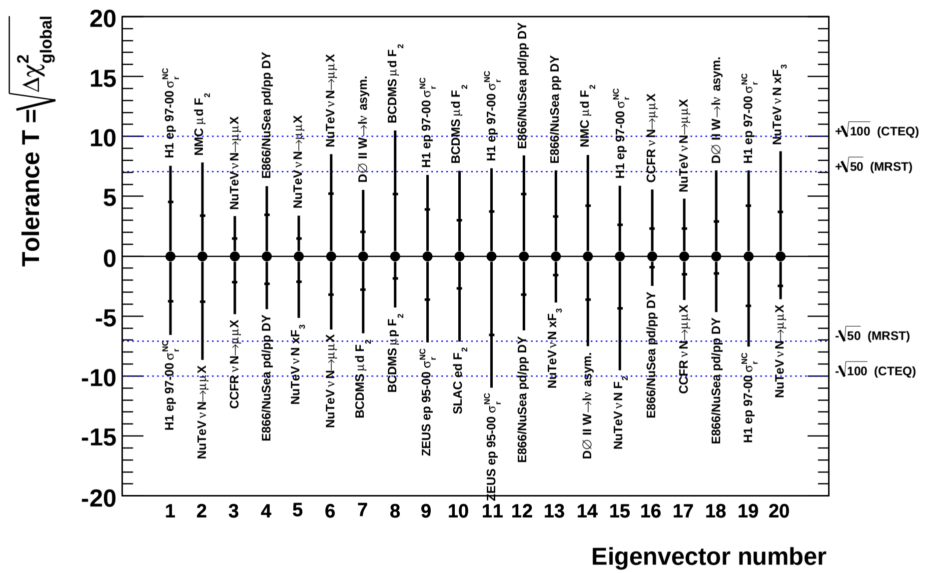
- Q: WHY ARE PDF UNCERTAINTIES ON GLOBAL FITS OF SIMLAR SIZE?
  - SIMILAR DATASETS
  - BUT DIFFERENT PROCEDURES
- A: UNCERTAINTY TUNING

TOLERANCE (MMHT-CT)

GLOBAL MSTW TOLERANCE

MSTW TOLERANCE PLOT FOR 13TH EIGENVEC.





- (MSTW/MMHT) FOR EACH EIGENVECTOR IN PARAMETER SPACE DETERMINE CONFIDENCE LIMIT FOR THE DISTRIBUTION OF BEST-FITS OF EACH EXPERIMENT
- RESCALE $\Delta\chi^2 = T$ INTERVAL SUCH THAT CORRECT CONFIDENCE INTERVALS ARE REPRODUCED
- SIMILAR PROCEDURE ADOPTED BY CTEQ

PDF UNCERTAINTIES

- Q: WHY ARE PDF UNCERTAINTIES ON GLOBAL FITS OF SIMLAR SIZE?
  - SIMILAR DATASETS
  - BUT DIFFERENT PROCEDURES
- A: UNCERTAINTY TUNING

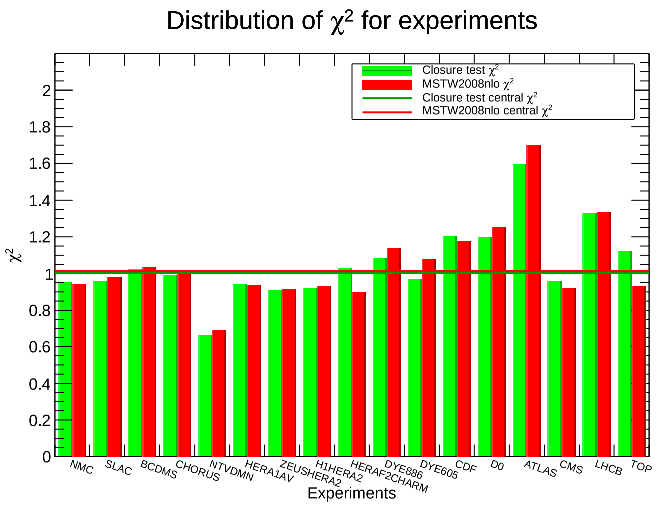
CLOSURE TESTS (NNPDF)

BASIC IDEA

- ASSUME PDFS KNOWN: GENERATE FAKE EXPERIMENTAL DATA
- CAN DECIDE DATA UNCERTAINTY (ZERO, OR AS IN REAL DATA, OR . . .)
- FIT PDFS TO FAKE DATA
- CHECK WHETHER FIT REPRODUCES UNDERLYING "TRUTH":
  - CHECK WHETHER TRUE VALUE GAUSSIANLY DISTRIBUTED ABOUT FIT
  - CHECK WHETHER UNCERTAINTIES FAITHFUL
  - CHECK STABILITY
    (INDEP. OF METHODOLOGICAL DETAILS)

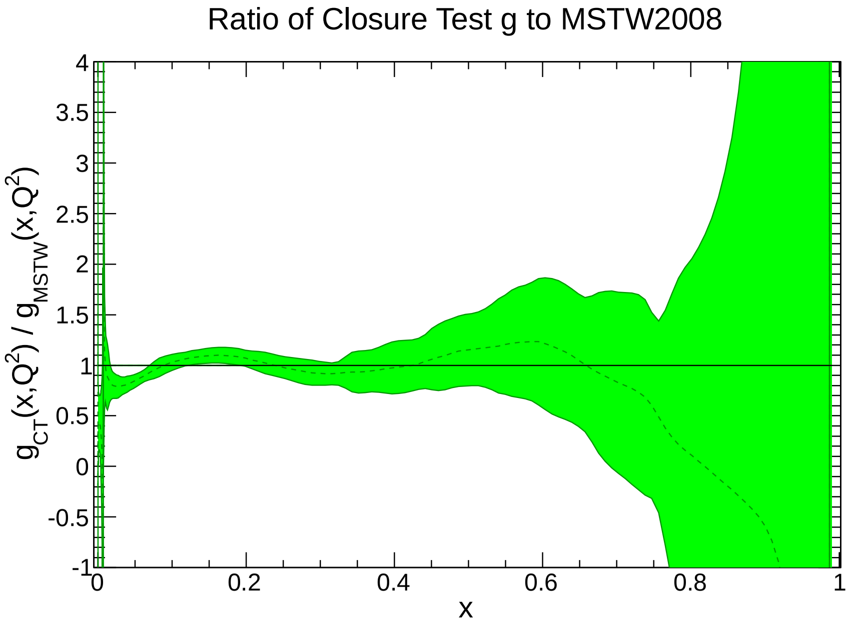
CLOSURE TEST RESULTS (NNPDF3.0)

CENTRAL VALUES AND UNCERTAINTIES

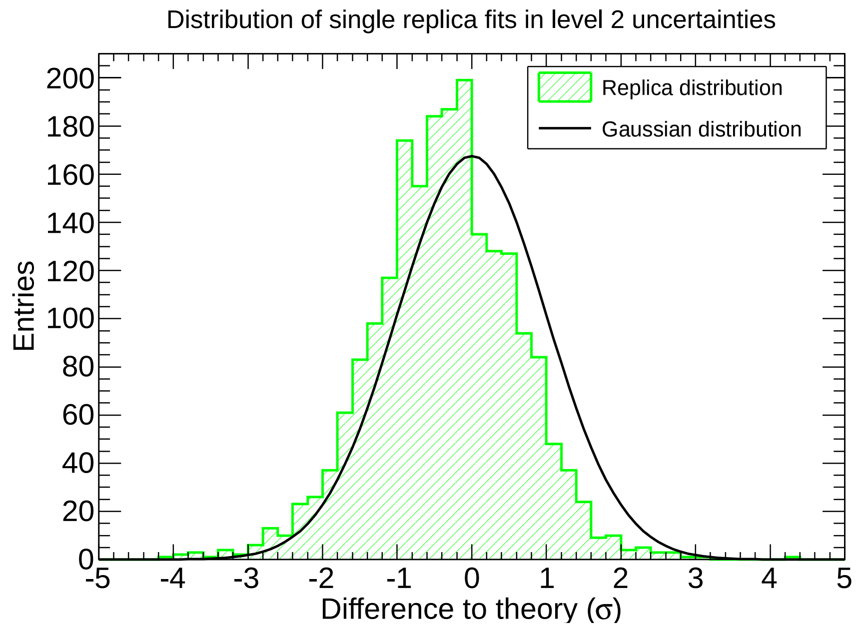


- CENTRAL VALUES: COMPARE FITTED VS. "TRUE" χ^2 BOTH FOR INDIVIDUAL EXPERIMENTS & TOTAL DATASET FOR TOTAL $\Delta\chi^2 = 0.001 \pm 0.003$
- UNCERTAINTIES: DISTRIBUTION OF DEVIATIONS BETWEEN FITTED AND "TRUE" PDFs SAMPLED AT 20 POINTS BETWEEN 10^{-5} AND 1 FIND 0.699% FOR ONE-SIGMA, 0.948% FOR TWO-SIGMA C.L.

THE GLUON: FITTED/"TRUE"



NORM. DISTRIBUTION OF DEVIATIONS



PROGRESS

- Q: WHAT HAS DRIVEN DRIVES THE IMPROVED AGREEMENT OF GLOBAL FITS
  - SIMILAR DATASETS
  - BUT DIFFERENT PROCEDURES
- A: DATA+METHODOLOGY

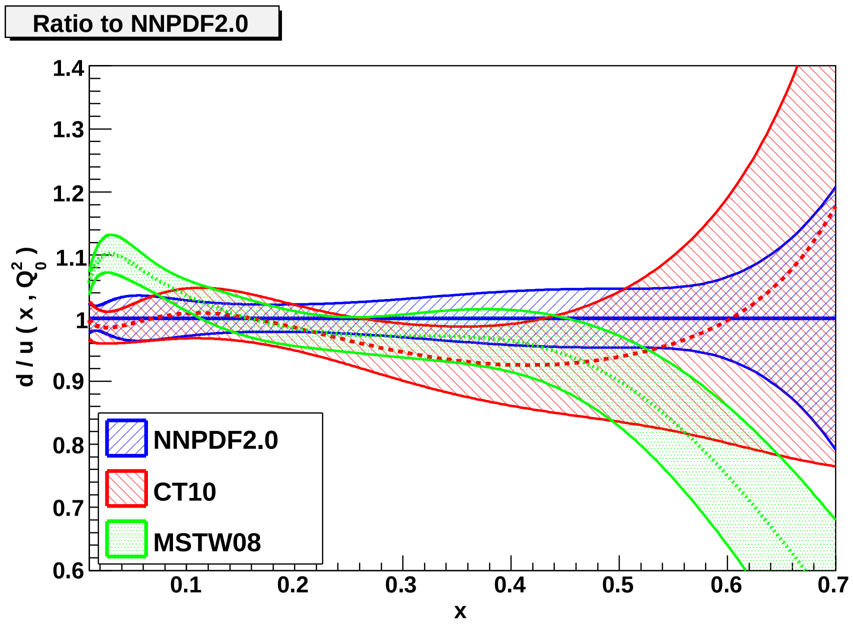
METHODOLOGY

	NNPDF3.0	MMHT14	CT14
No. of fitted PDFs	7	7	6
Parametrization	neural nets	$x^a(1-x)^b\times$ Chebyschev	$x^a(1-x)^b\times$Bernstein
Free parameters	259	37	30-35
Uncertainties	Replicas	Hessian	Hessian
Tolerance	None	Dynamical	Dynamical
Closure test	✔	✗	✗
Reweighting	replicas	eigenvectors	eigenvectors

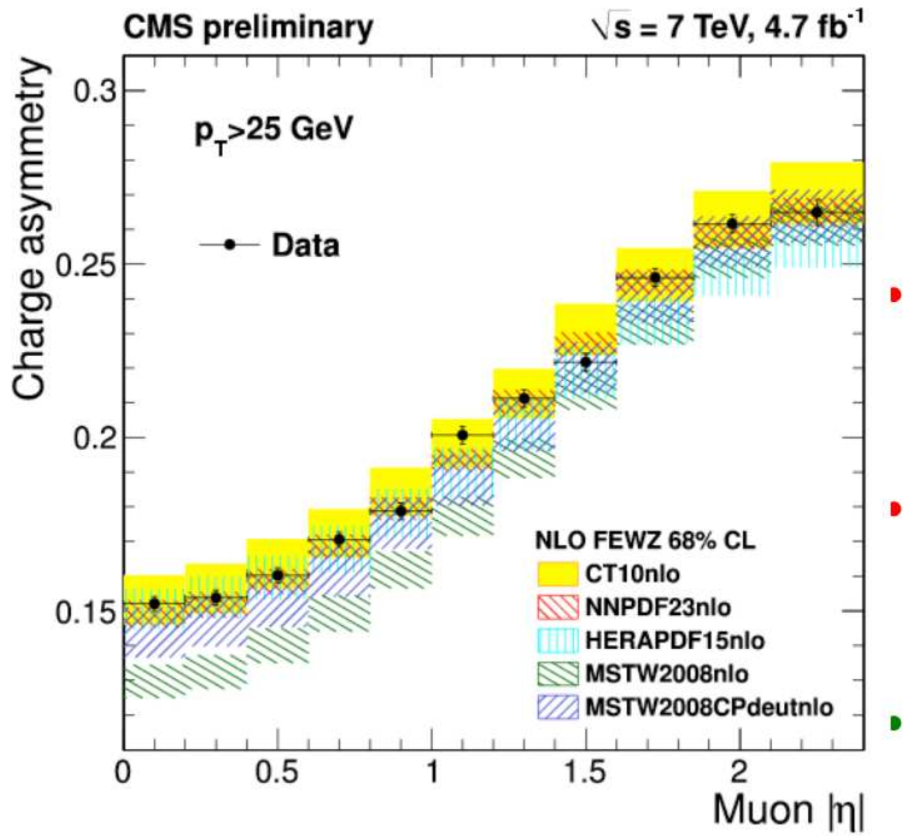
- MMHT, CT10 larger # of parms., orthogonal polynomials
- NNPDF closure test

DATA ⇒ METHODOLOGY
MSTW/MMHT: THE d/u RATIO

THE d/u RATIO



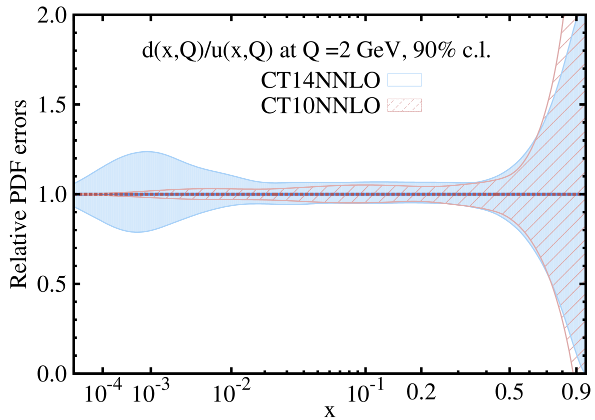
THE CMS W ASYMMETRY



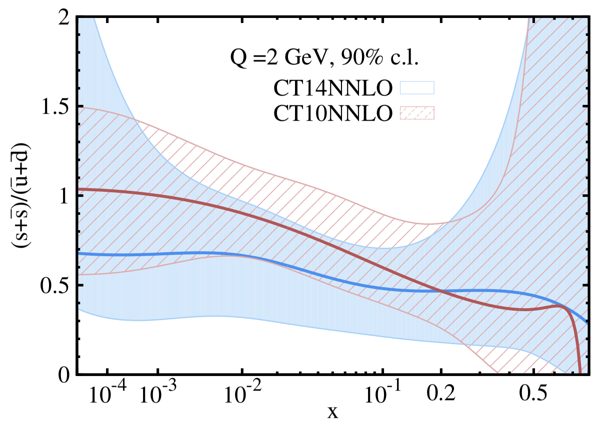
- LONG-STANDING DISCREPANCY IN THE d/u RATIO BETWEEN MSTW AND OTHER GLOBAL FITS
- RESOLVED BY W ASYMMETRY DATA
- EXPLAINED BY INSUFFICIENTLY FLEXIBLE PDF PARAMETRIZATION ⇒ FIXED IN MSTW08DEUT/MMHT

DATA ⇒ METHODOLOGY
CT14: EXTENDED PARAMETRIZATION

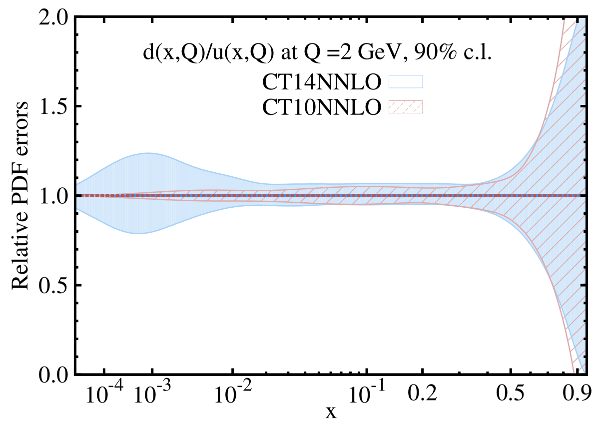
d/u RATIO



$\bar{d}/\bar{u}$ RATIO



$(s+\bar{s})/(\bar{u}+\bar{d})$ RATIO



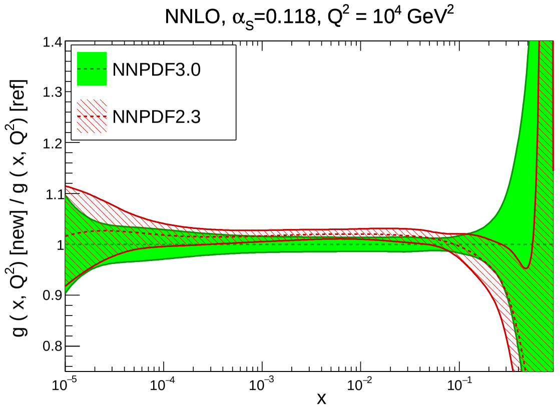
- D0 RUN II CHARGE-ASYMMETRY DATA (NOT INCLUDED BY OTHERS) AFFECT u/d
- LHC W/Z DATA IMPACT STRANGENESS
- SOMEWHAT INCREASED UNCERTAINTIES DUE TO MORE FLEXIBLE FUNCTIONAL FORM

DATA ⇒ METHODOLOGY
NNPDF: 3.0 VS. 2.3

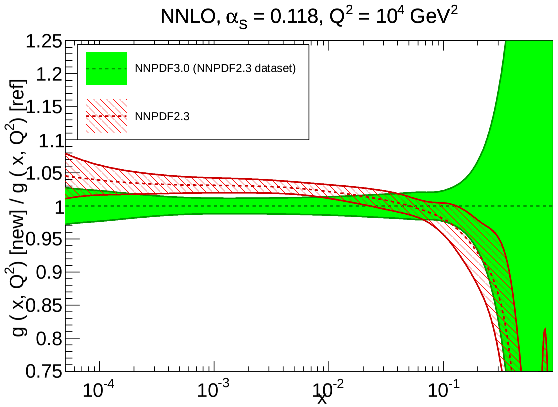
- REPEAT THE 3.0 FIT BUT WITH 2.3 DATASET
- COMPARE WITH 2.3 DATA & METHODOLOGY; 2.3 DATA BUT 3.0 METHODOLOGY; 3.0 DATA & METHODOLOGY

THE GLUON DISTRIBUTION

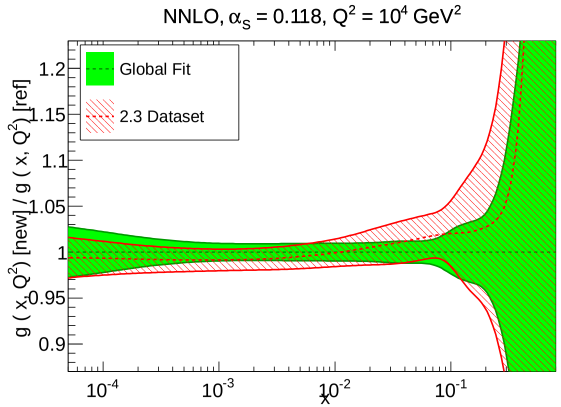
DEFAULT 3.0 VS DEFAULT 2.3



DEFAULT 2.3 VS. 2.3 DATA



DEFAULT 3.0 VS 2.3 DATA



- THE METHODOLOGY HAS A DOMINANT EFFECT ON THE GLUON
- EFFECT OF DATA SUBDOMINANT,
  LARGER ON OTHER COMBINATIONS (VALENCE)

THEORETICAL UNCERTAINTIES ON PDFS:

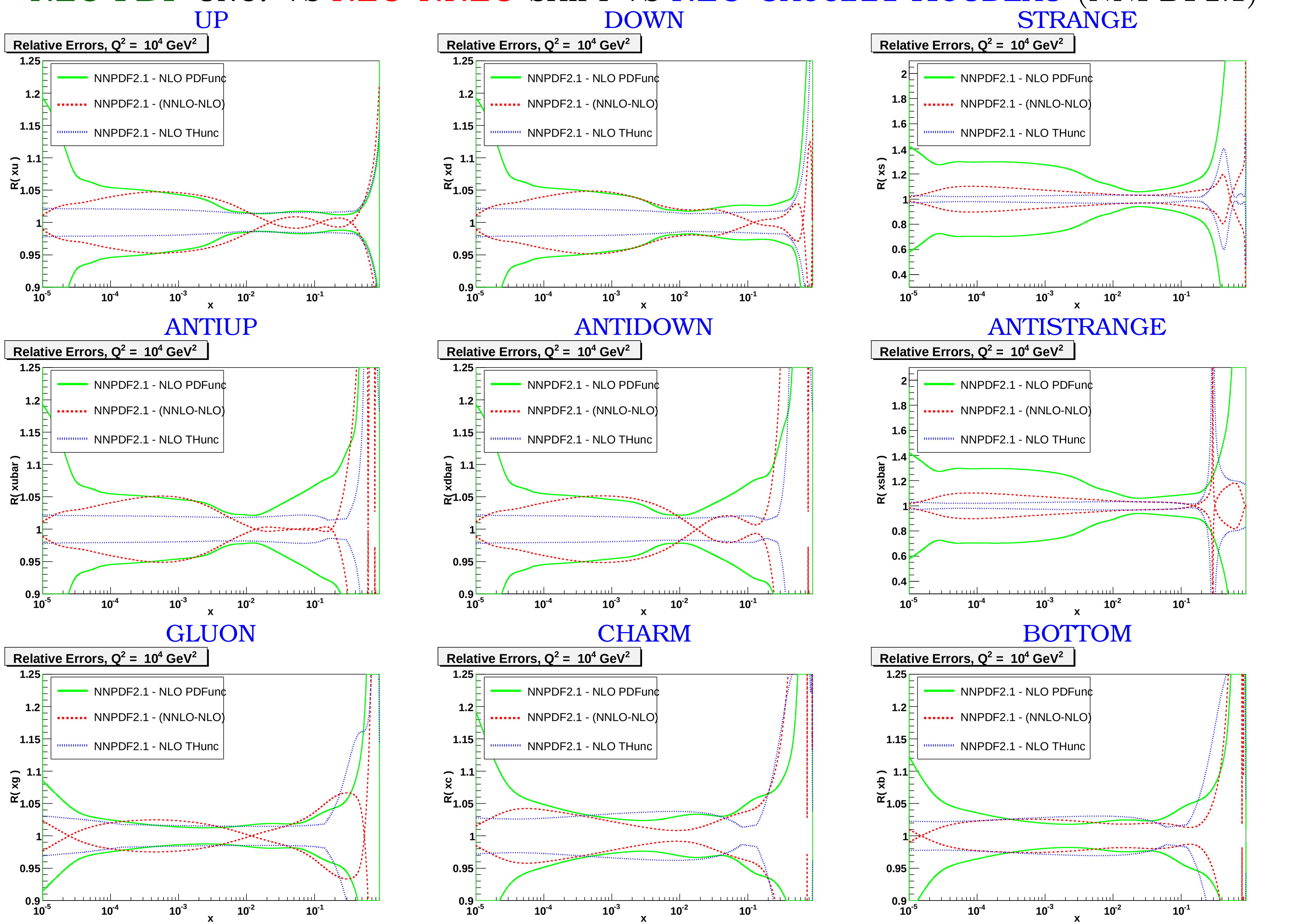
- PDFS ARE DETERMINED BY COMPARING TO DATA THEORY AT SOME FINITE ORDER
- AFFECTED BY THEORETICAL UNCERTAINTY JUST LIKE HARD CROSS-SECTIONS
- CURRENTLY NOT INCLUDED IN "PDF UNCERTAINTY"
  (ACCOUNTS ONLY FOR DATA & METHODOLOGY)
  $\Rightarrow$ IS IT GOOD ENOUGH FOR 2% ACCURACY?

CAN WE ESTIMATE THEM?

- SCALE VARIATION DIFFICULT: CORRELATED BETWEEN PROCESSES?
  HOW DOES IT CORRELATE WITH PROCESSES IN WHICH PDFS ARE USED?
- AT NLO: WE KNOW THE SHIFT W.R. TO NNLO
- AT NNLO: LOOK AT THE BEHAVIOUR OF THE PERTURBATIVE EXPANSION
  (CACCIARI-HOUDEAU)

THEORETICAL UNCERTAINTIES

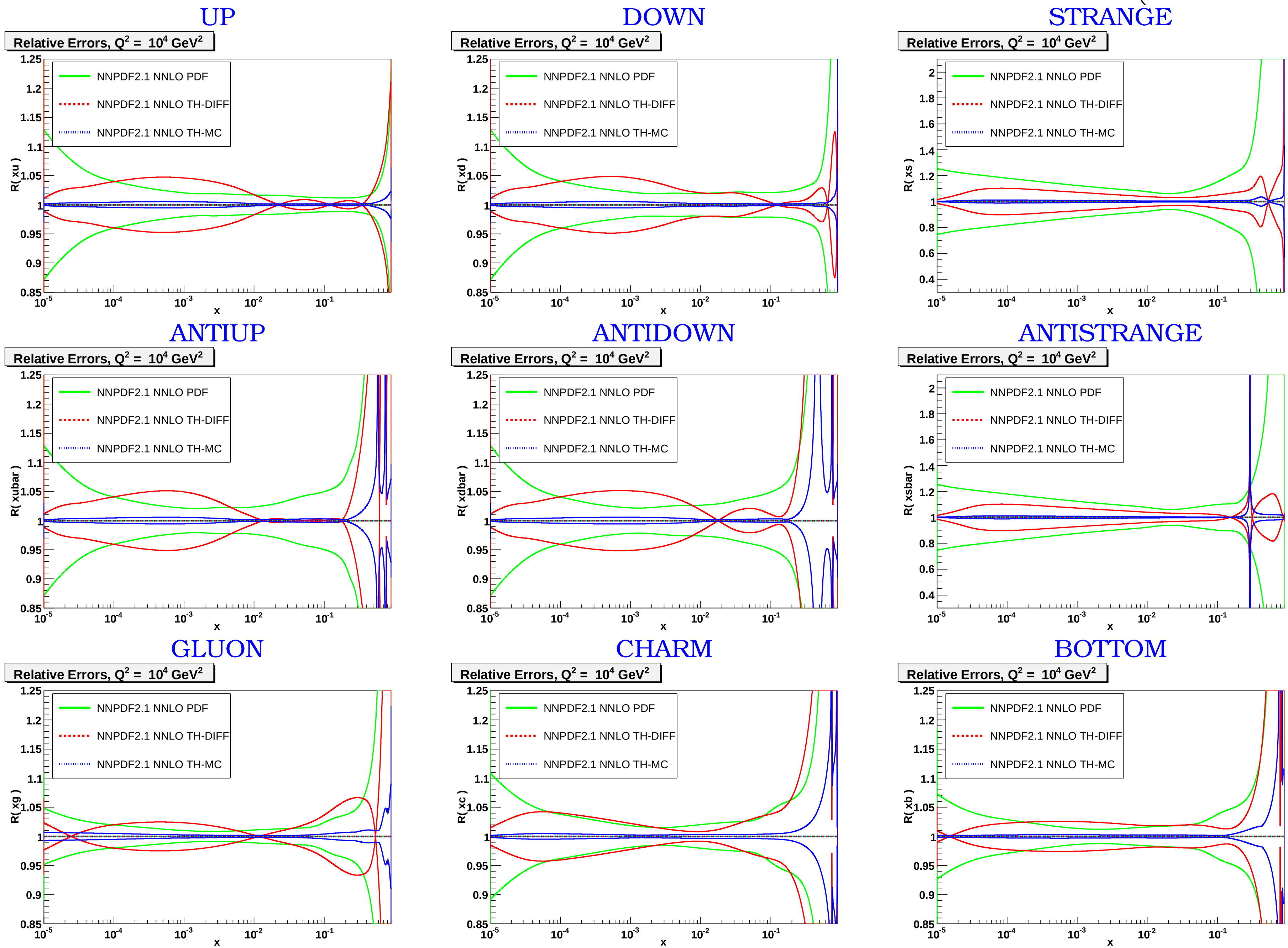
NLO PDF unc. vs NLO-NNLO shift vs NLO Cacciari-Houdeau (NNPDF2.1)



THEORETICAL UNCERTAINTY ON NLO PDF IS ORDER 5% $\Rightarrow$ COMPARABLE TO PDF UNCERTAINTY

THEORETICAL UNCERTAINTIES

NNLO PDF unc. vs NLO-NNLO shift vs NNLO Cacciari-Houdeau (NNPDF2.1)



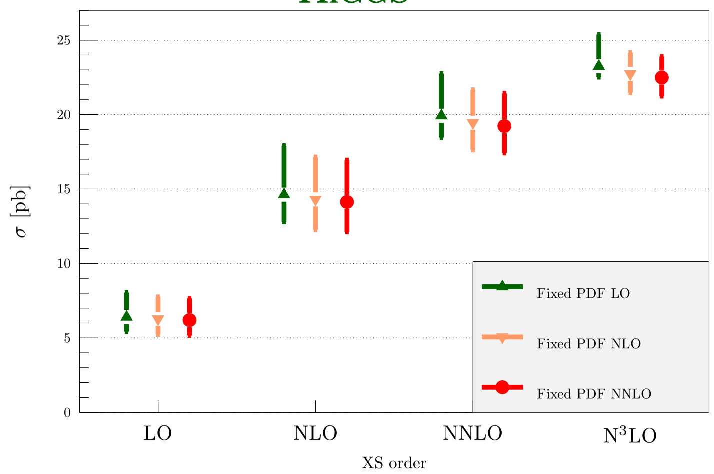
Theoretical uncertainty on NNLO PDF is order 1% ⇒ smaller than PDF uncertainty

N^{3}LO PDFs:

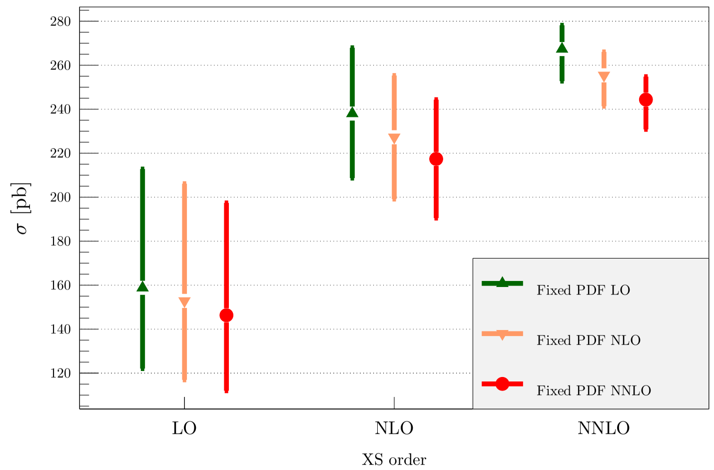
- NEEDED AT THE 1% ACCURACY LEVEL
- IMPACT OF N^{3}LO DEPENDS ON PROCESS:
  - HIGGS GLUON FUSION: PERTURBATIVE DEP. OF PDF NEGLIGIBLE IN COMPARISON TO MATRIX ELEMENT $\Rightarrow$ N^{3}LO NOT NEEDED
  - TOP: PERTURBATIVE DEP. OF PDF SMALLER, BUT NOT NEGLIGIBLE IN COMPARISON TO MATRIX ELEMENT, ANTICORRELATED TO IT $\Rightarrow$ N^{3}LO NECESSARY

SCALE UNCERTAINTY & DEP. ON PERTURBATIVE ORDER

HIGGS



TOP



(s.f., Isgrò, Vita, 2014)

WHEN WILL WE HAVE THEM?

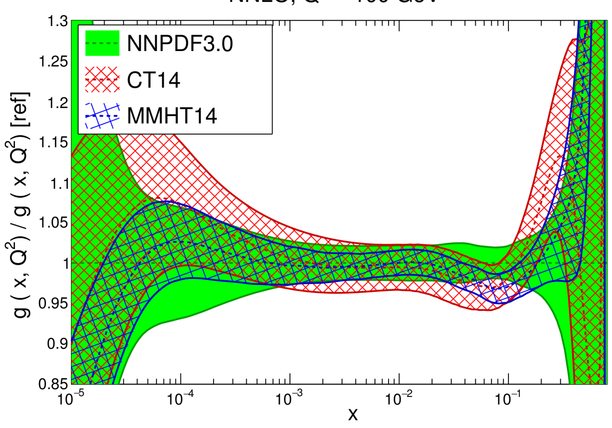
- N^{3}LO DIS COEFFICIENT FUNCTIONS KNOWN
- BOTTLENECK: N^3LO ANOMALOUS DIMENSIONS
- ANOMALOUS DIMENSIONS: LO: 1974; NLO: 1981; NNLO: 2004; N^{3}LO: 2030?

PDFS AT LHC RUN II

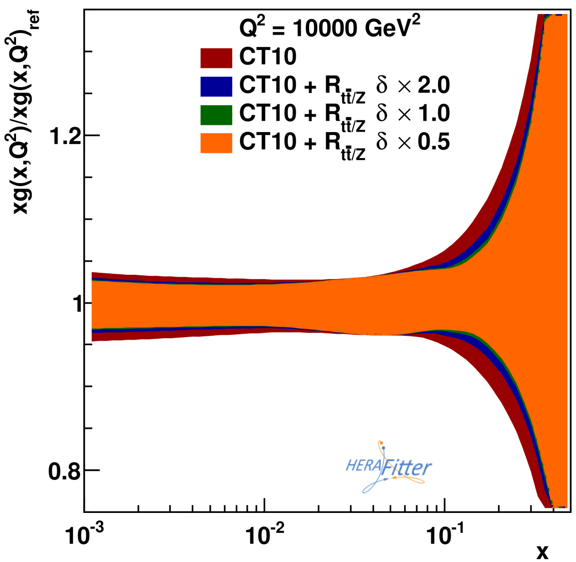
- DATA AT HIGHER CM ENERGY & INFO ON CORRELATION TO LOW ENERGY
  → EXTENDED KINEMATIC COVERAGE & REDUCED SYSTEMATICS
- POSSIBLY REDUCED STAT. UNCERTAINTIES
- EXPECT REDUCTION IN MODEL DEPENDENCE
- MODERATE REDUCTION IN UNCERTAINTY
- FURTHER IMPROVEMENTS DUE TO THEORY
  EXAMPLE: GLUON FROM NNLO Z p_T DISTRIBUTION

THE GLUON

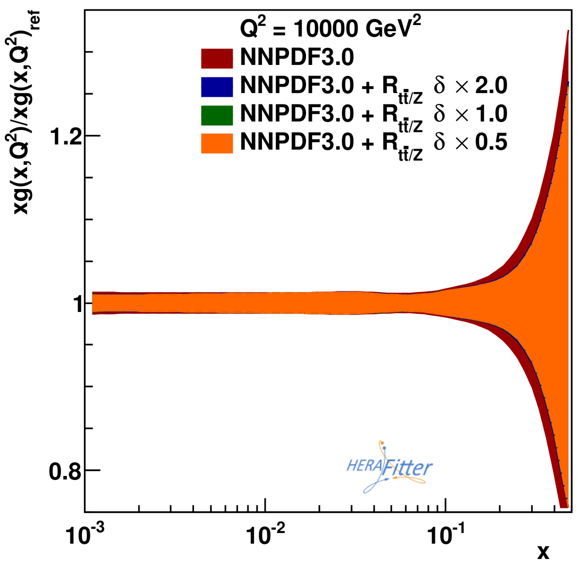
NOW



CTEQ AFTER RUN II



NNPDF AFTER RUN II



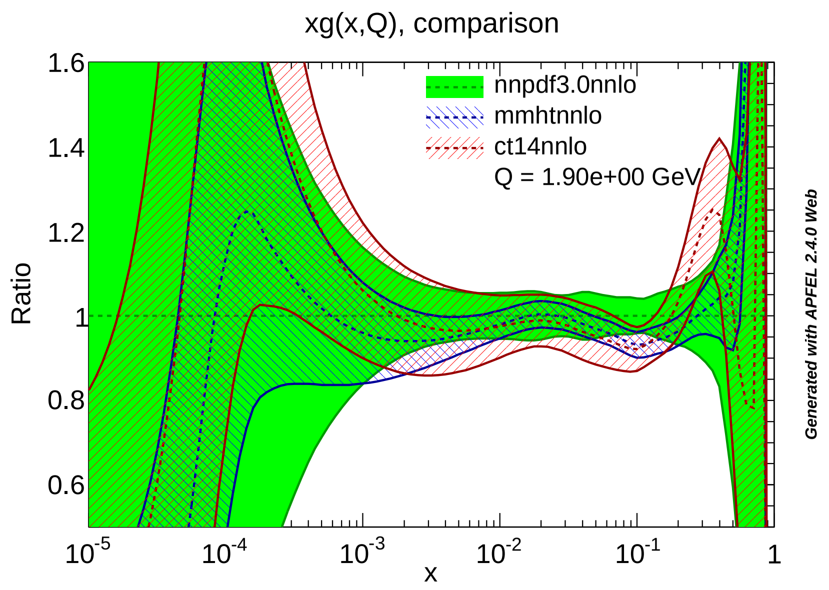
(PDF4LHC: 1507.00556)

PDFS AT THE LHEC?

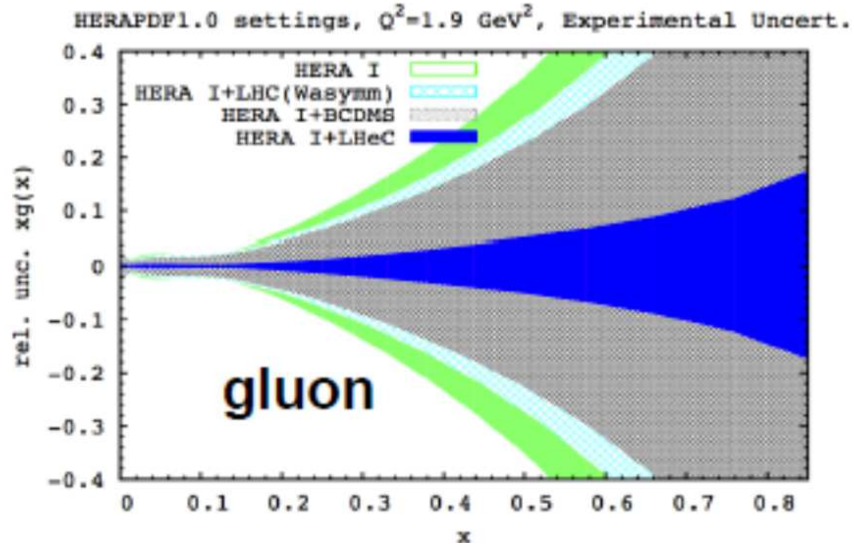
- ESPECIALLY IF CORRELATION INFO AVAILABLE, LHC RUN II CAN LIKELY
  - REMOVE SYSTEMATIC BIAS & DISCREPANCIES
  - BRING DOWN PDF UNCERTAINTIES TO $\sim 4\%$ LEVEL
- VERY DIFFICULT TO REDUCE UNCERTAINTIES FURTHER AT A HADRON COLLIDER
- HIGH-ENERGY DEEP-INELASTIC DATA COULD LEAD TO PDFS AT 1% LEVEL

THE GLUON AGAIN

NOW



AT THE LHEC



(A. Cooper-Sarkar & Voica Radescu, 2015)

THE VALUE OF α_s

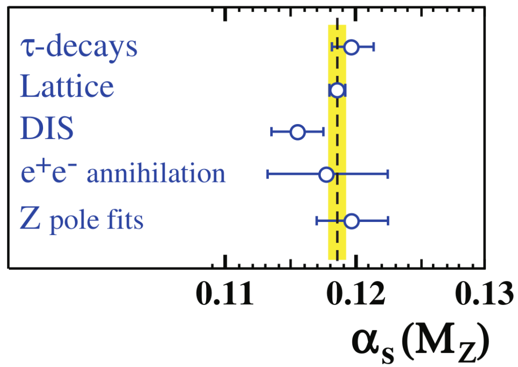
PDG VALUE (AUGUST 2014): $\alpha_s(M_Z) = 0.1185 \pm 0.0006$

- LATTICE UNCERTAINTY CURRENTLY ESTIMATED BY FLAG (arXiv:1310.8555) TO BE TWICE THE PDG VALUE (± 0.0012)
  (IF PDG WERE TO ADOPT THIS, COMBINED UNCERTAINTY LIKELY TO DOUBLE)
- IT IS AN AN AVERAGE OF AVERAGES
- SOME SUB-AVERAGES (E.G. DIS) INCLUDE MUTUALLY INCONSISTENT VALUES
- SOME SUB-AVERAGES (E.G. τ OR JETS) INCLUDE DETERMINATIONS WHICH DIFFER FROM EACH OTHER BY EVEN FOUR-FIVE σ
- AVERAGING THE TWO MOST RELIABLE VALUES (GLOBAL EW FIT & τ, BOTH N^{3}LO, NO DEP. ON HADRON STRUCTURE) GIVES

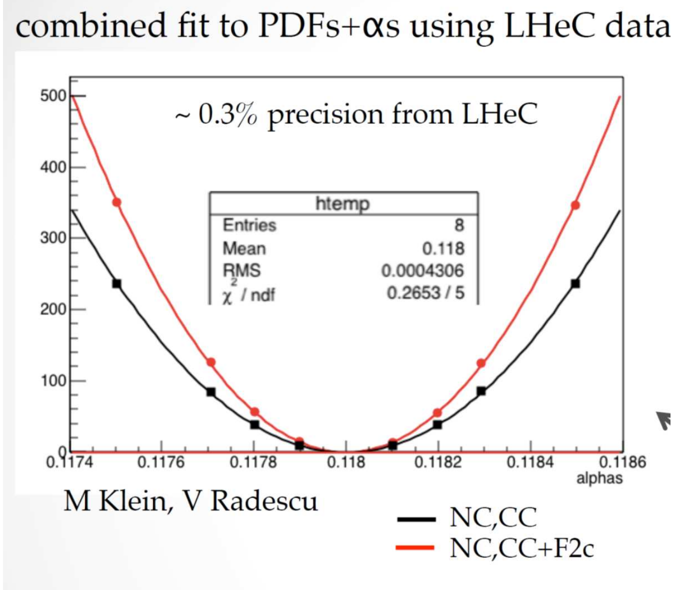
$$\alpha_s = 0.1196 \pm 0.0010$$

- LITTLE PROGRESS FOR MANY YEARS: PDG 1998-2006 $\Delta\alpha_s(M_Z) = 0.002$; PDG 2010-2014 $\Delta\alpha_s(M_Z) = 0.0006 \div 0.0008$ (CHANGE OF AUTHOR)
- UNLIKELY TO CHANGE - SHOULD DETERMINE α_s FROM HIGGS IN GLUON FUSION?

α_s DETERMINATIONS IN PDG



α_s AT THE LHEC



- COULD BE DETERMINED ACCURATELY AT THE LHEC OR AT A NEUTRINO FACTORY!

SUMMARY

- PERTURBATIVE EXPANSION STABLE, DIVERGENCE VERY FAR
- NEED BETTER WAYS OF ESTIMATING MISSING HIGHER ORDERS: RESUMMATION MAY PROVIDE NEEDED INFORMATION
- PDF UNCERTAINTIES DUE TO DATA/METHODOLOGY BROADLY CONSISTENT & UNDER CONTROL IN GLOBAL FITS
- THEORETICAL UNCERTAINTIES ON PDFs MUST BE ESTIMATED IN ORDER TO CONTROL UNCERTAINTIES AT PERCENT LEVEL

HOW DO WE ESTIMATE THE AMOUNT OF OUR IGNORANCE?

[21δ] ἐντεῦθεν οὖν τούτῳ τε ἀπηχθόμην καὶ πολλοῖς τῶν παρόντων: πρὸς ἐμαυτὸν δ' οὖν ἀπιὼν ἐλογιζόμην ὅτι τούτου μὲν τοῦ ἀνθρώπου ἐγὼ σοφώτερός εἰμι: κινδυνεύει μὲν γὰρ ἡμῶν οὐδέτερος οὐδὲν καλὸν κἀγαθὸν εἰδέναι, ἀλλ' οὗτος μὲν οἴεταί τι εἰδέναι οὐκ εἰδώς, ἐγὼ δέ, ὥσπερ οὖν οὐκ οἶδα, οὐδὲ οἴομαι: ἔοικα γοῦν τούτου γε σμικρῷ τινι αὐτῷ τούτῳ σοφώτερος εἶναι, ὅτι ἃ μὴ οἶδα οὐδὲ οἴομαι εἰδέναι. ἐντεῦθεν ἐπ' ἄλλον ᾖα τῶν ἐκείνου δοκούντων σοφωτέρων εἶναι καί

Plato. Platonis Opera, ed. John Burnet. Oxford University Press. 1903.

I am wiser than this man; for neither of us really knows anything fine and good, but this man thinks he knows something when he does not, whereas I, as I do not know anything, do not think I do either. I seem, then, in just this little thing to be wiser than this man at any rate, that what I do not know I do not think I know either.

EXTRAS

THE NEW PDF4LHC PRESCRIPTION

- PERFORM MONTE CARLO COMBINATION OF UNDERLYING PDF SETS
- SETS ENTERING THE COMBINATION MUST SATISFY COMMON REQUIREMENTS
- DELIVER A SINGLE COMBINED PDF SET THROUGH SUITABLE TOOLS

MONTE CARLO VS. HESSIAN DELIVERY

- MONTE CARLO: A SET OF PDF REPLICAS IS DELIVERED;
  QUANTITIES COMPUTED FOR EACH REPLICA: CENTRAL VALUE IS THE MEAN, UNCERTAINTY IS STANDARD DEVIATION
- HESSIAN: A CENTRAL SET AND ERROR SETS ARE DELIVERED;
  CENTRAL SET PROVIDES CENTRAL PREDICTION, UNCERTAINTY IS THE SUM IN QUADRATURE OF ERROR DEVIATIONS

TREATMENT OF α_s

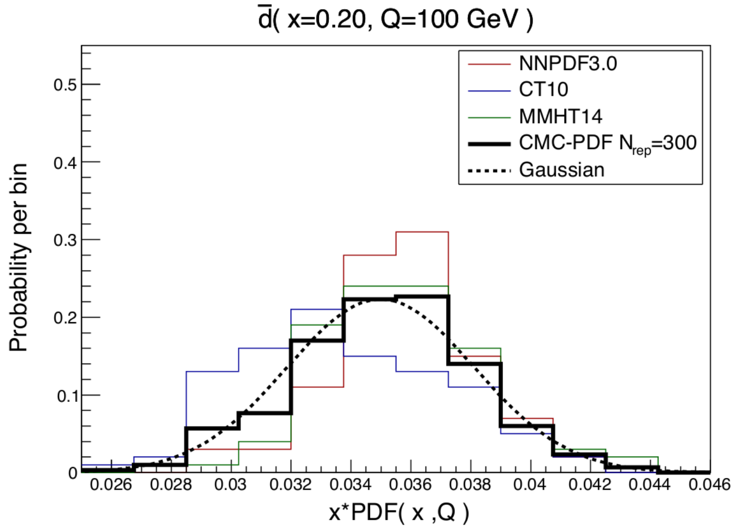
- PDFS ARE DELIVERED FOR EACH VALUE OF α_s
- PDF AND α_s UNCERTAINTIES TO BE KEPT SEPARATE
  (SEE BELOW FOR α_s UNCERTAINTY)

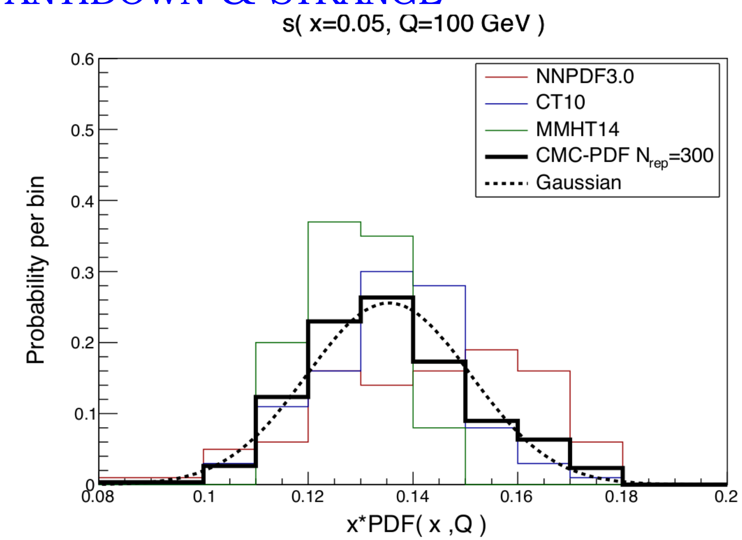
MONTE CARLO COMBINATION

(Watt, S.F., 2010-2013)

- CONVERT ALL SETS INTO MONTE CARLO
- HESSIAN SETS CAN BE CONVERTED BY PERFORMING MONTE CARLO IN PARAMETER SPACE (Watt, Thorne, 2012)
- COMBINE MONTE CARLO REPLICAS INTO SINGLE SET

COMBINED MC SETS FOR ANTIDOWN & STRANGE



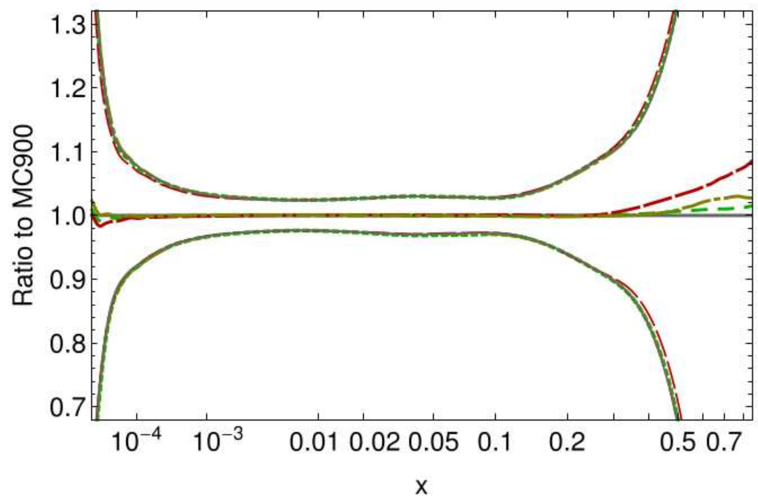


CURRENT COMBINED SET

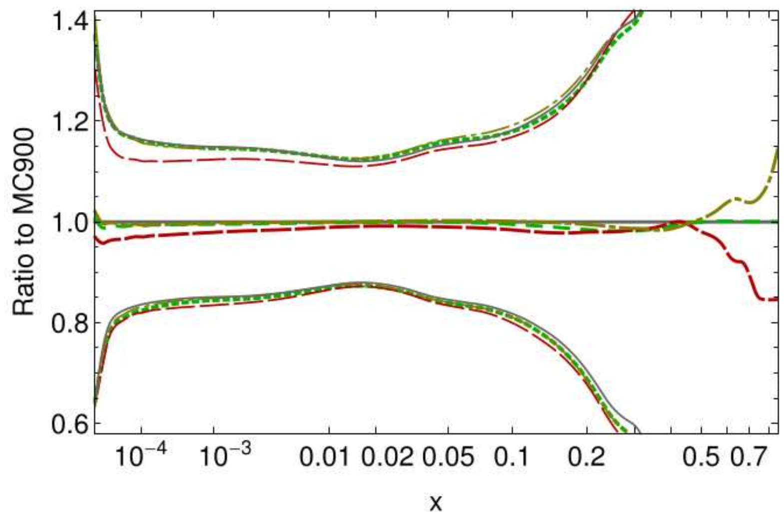
- INCLUDES CT14, MMHT, NNPDF3.0
- 900 REPLICAS (300 FOR EACH SET) ENSURE PRECENTAGE ACCURACY ON ALL QUANTITIES

300, 900, 1800 REPLICAS
(RATIO TO 900)

gluon



strange

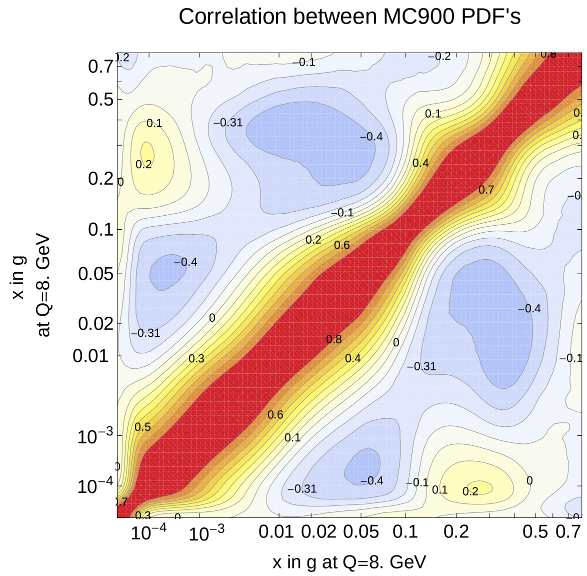


TOOLS FOR DELIVERY

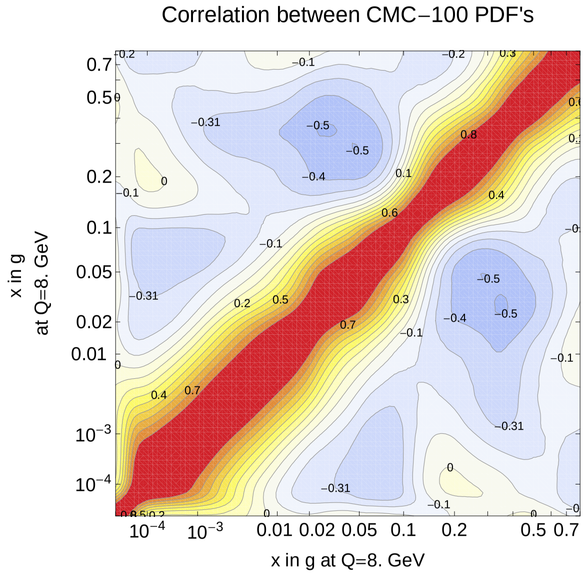
- COMBINED MC SET IS LARGE (900 REPLICAS)
- HESSIAN DELIVERY OFTEN DESIRABLE (PDF UNCERTAINTIES AS NUISANCE PARAMETERS)
- CMC (1504.06469): GA COMPRESSION OF MC SET TO A SMALLER SET WITH MINIMAL INFORMATION LOSS (900 $\rightarrow$ 40-100 REPLICAS)
- META-PDFS (1401.0013) OR MC-H (1505.06736) PDFS: HESSIAN REPRESENTATION OBTAINED BY
  REFITTING MC REPLICAS WITH FUNCTIONAL FORM (META), OR
  REPRESENTING THEM ON LINEAR BASIS OF REPLICAS (MC-H)
- VALIDATION AND BENCHMARKING SUCCESFUL

GLUON-GLUON CORRELATION

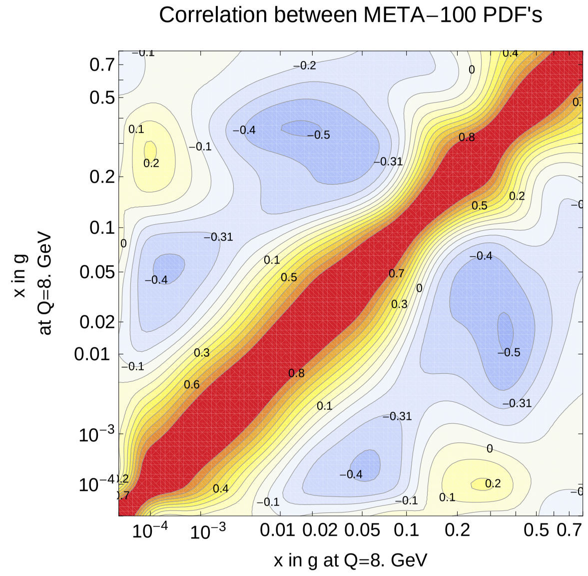
STARTING COMBINED SET



COMPRESSED MC SET



META-PDF SET



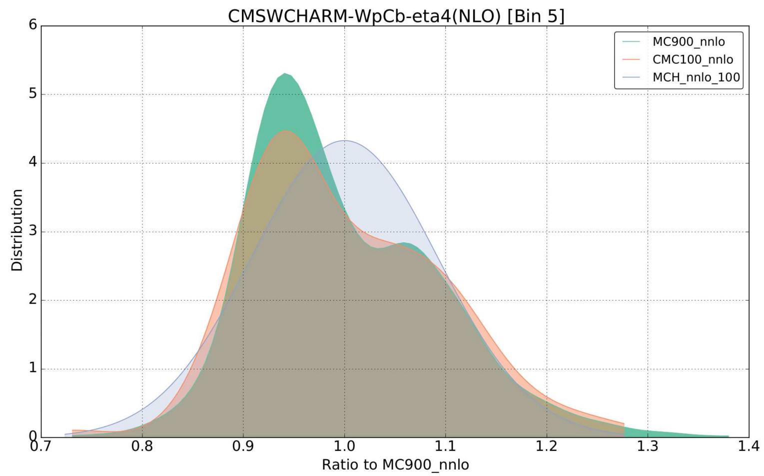
MONTECARLO PDFs: NONGAUSSIAN BEHAVIOUR

DEVIATION FROM GAUSSIANITY OBSERVED

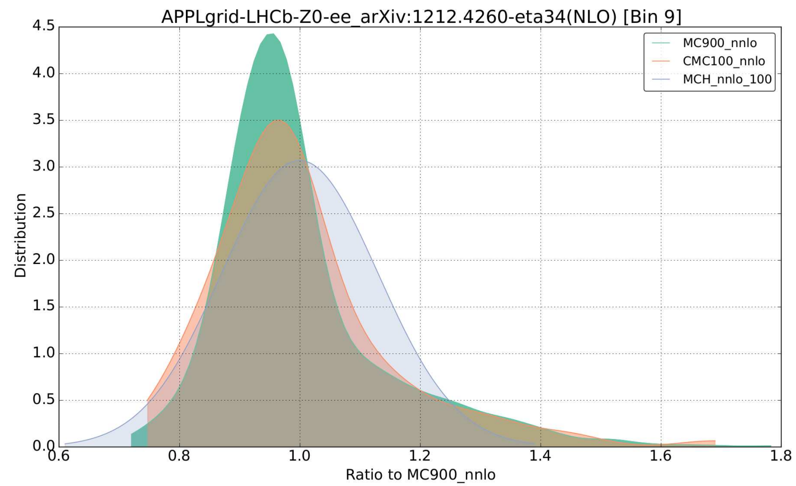
E.G. AT LARGE x, DUE TO LARGE UNCERTAINTY & POSITIVITY BOUNDS
(MAY BE RELEVANT FOR SEARCHES)

MONTE CARLO COMPARED TO HESSIAN

CMS $W + c$ production



LHCb electrons



(Carrazza, Latorre, Rojo, Watt, 2015)

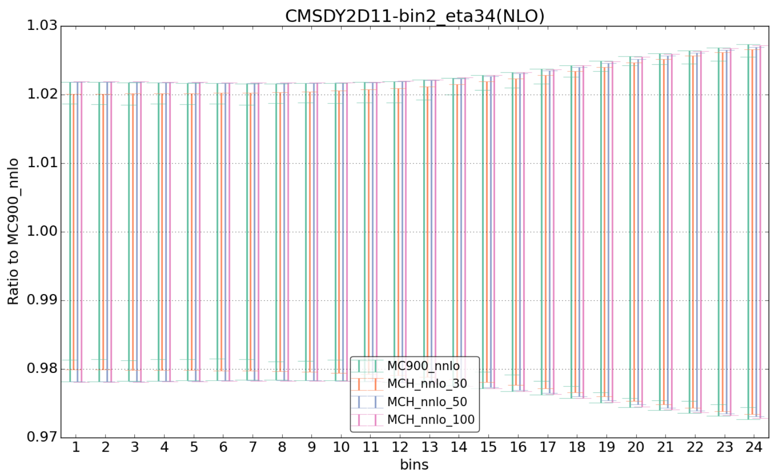
- MONTE CARLO PDFs REPRODUCE NONGAUSSIAN, HESSIAN FAIL
- COMPRESSION EFFECTIVELY REPRODUCES NONGAUSSIAN SHAPES
- PROBLEM DOES NOT ARISE FOR BULK OF DATA $\Rightarrow$ HESSIAN ADEQUATE!

HESSIAN PDFs: HIGH ACCURACY

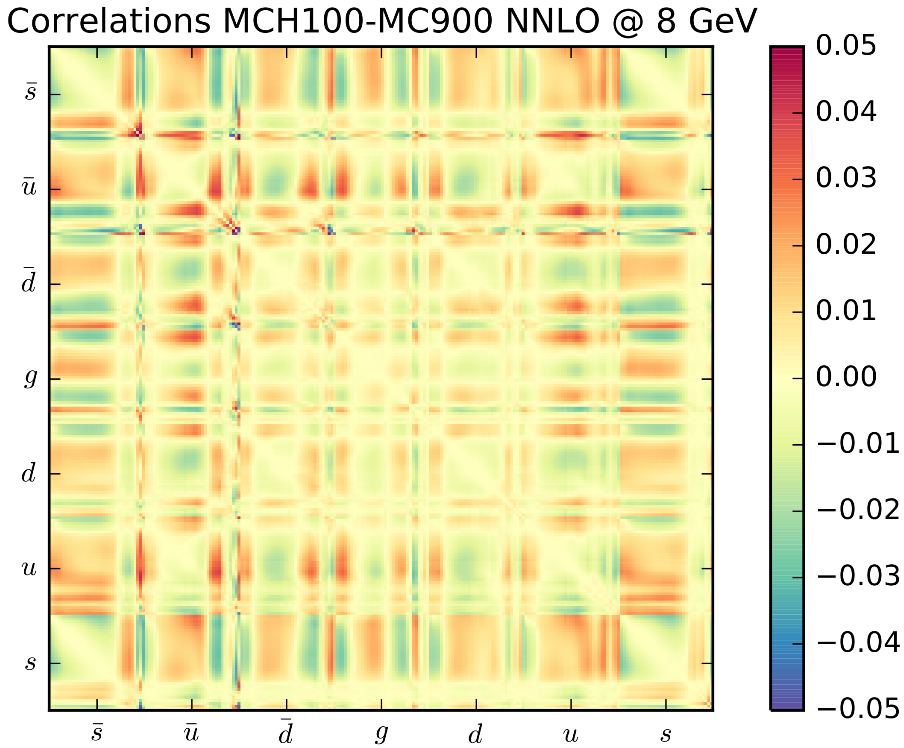
- FOR STANDARD CANDLES OR HIGGS PRODUCTION, GAUSSIAN APPROXIMATION ACCURATE
- HESSIAN REPRESENTATION EXTREMELY ACCURATE WITH 100 ERROR SETS

HESSIAN: FROM 30 TO 100 SETS VS MC PRIOR

CMS double diff DY



PDF correlations
(deviation from exact)



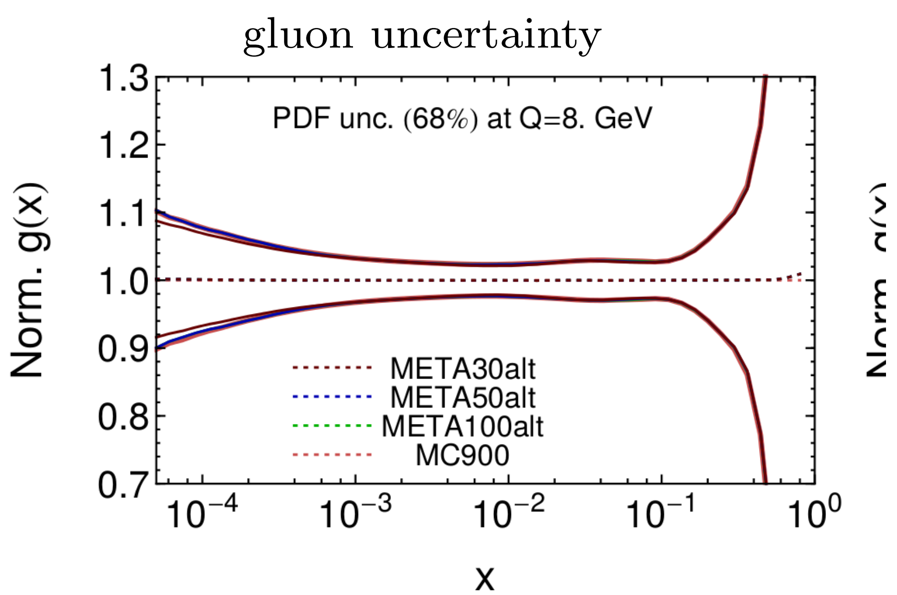
(Carrazza, Kassabov, S.F., Latorre, Rojo, 2015)

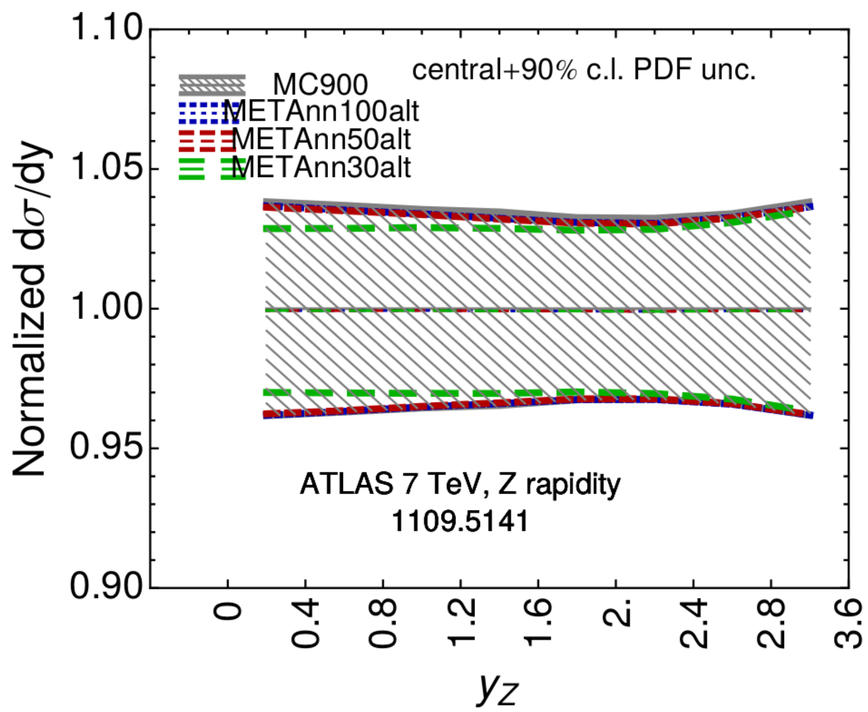
- UNCERTAINTIES PERFECTLY REPRODUCED
- CORRELATION MATRIX OF PDFs TO PERCENT ACCURACY

HESSIAN PDFs: FAST IMPLEMENTATION

- FOR ACCEPTANCE CALCULATIONS OR EXTRAPOLATION FROM FIDUCIAL TO FULL, SOME LOSS OF ACCURACY IS TOLERABLE
- SMALL NUMBER OF NUISANCE PARAMETERS OFTEN DESIRABLE
- HESSIAN REPRESENTATION WITH 30 ERROR SETS ADEQUATE (ESPECIALLY IN DATA REGION)

HESSIAN: FROM 100 TO 30 SETS VS MC PRIOR



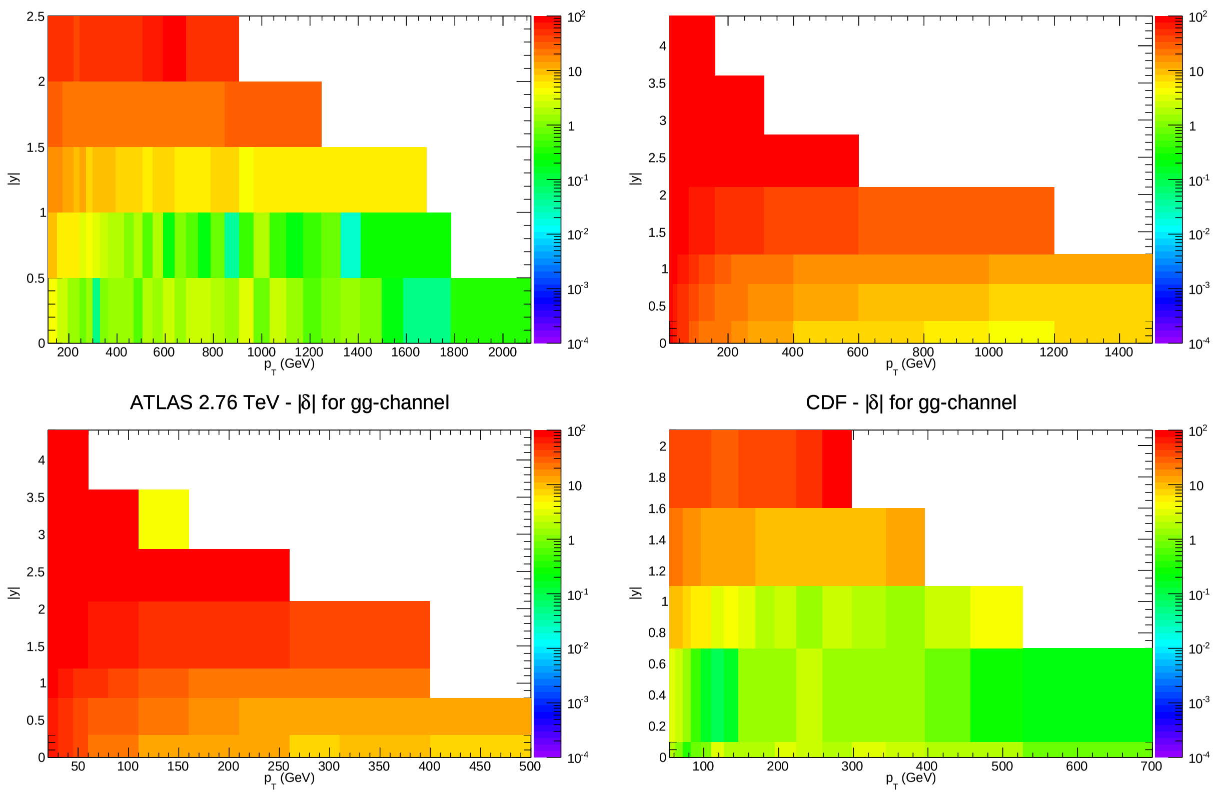


(Nadolsky, Gao, 2014)

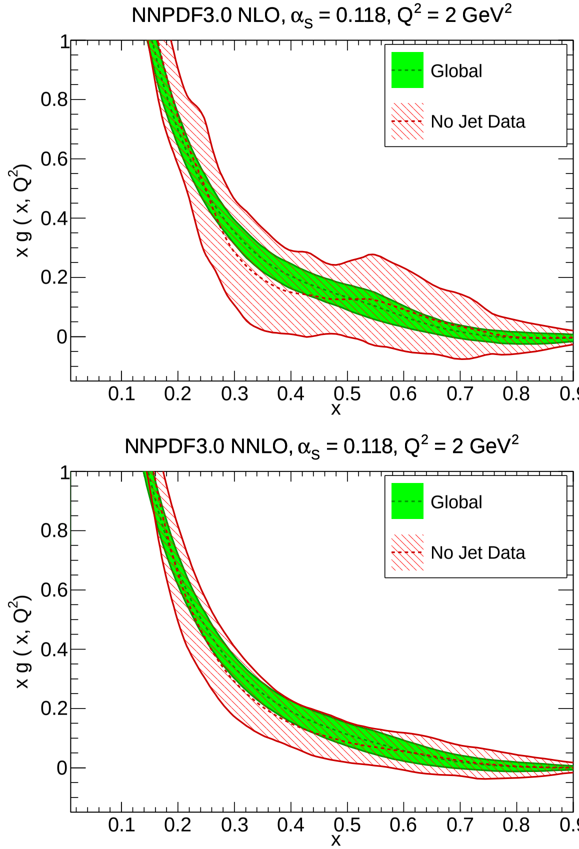
- PDFs REASONABLY WELL REPRODUCED IN DATA REGION
- UNCERTAINTIES ON OBSERVABLES REPRODUCED TO BETTER THAN 10%, USUALLY MUCH BETTER

JET DATA AT NLO AND NNLO

PERCENTAGE ACCURACY OF THRESHOLD APP.
IN GLUON CHANNEL



IMPACT OF THE JET DATA:
NLO&NNLO



- EXACT NNLO CORRECTIONS NOW KNOWN IN GLUON CHANNEL (GEHRMANN, GLOVER, DE RIDDER, PIRES, 2014)
- THRESHOLD APPROX AVAILABLE IN ALL CHANNELS (DE FLORIAN ET AL, 2014)
- ONLY ACCURATE AT HIGH p_T, CENTRAL RAPIDITY, LARGER JET RADIUS (CARRAZZA & PIRES, 2014)
- AT NNLO RETAIN ONLY BINS SUCH THAT ACCURACY BETTER THAN 10% CORRECTION THERE SMALLER THAN 15% $\Rightarrow$ ATLAS ($R = 0.4$) MOSTLY CUT OUT
- IMPACT OF JET DATA STILL SIZABLE

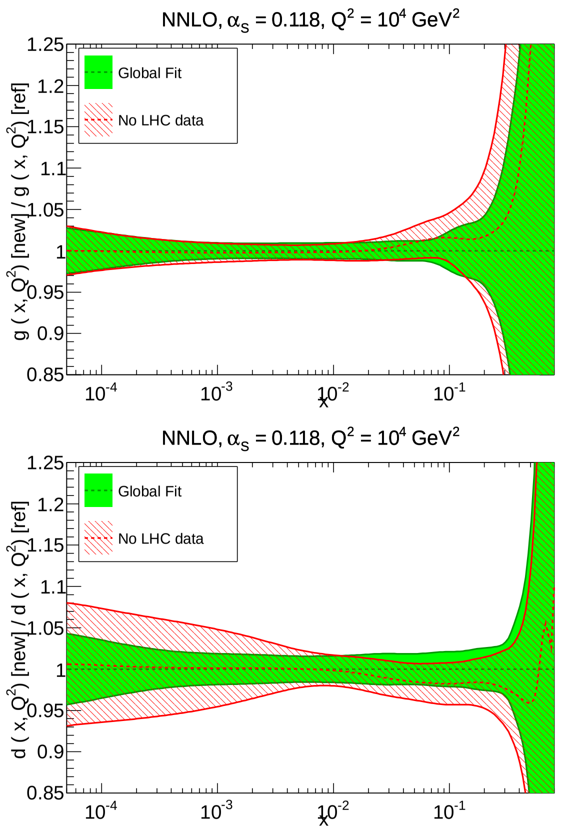
THE IMPACT OF LHC (AND HERA) DATA

- OVERALL MEASURE OF IMPACT:
  $\phi \Rightarrow$ FIT UNCERTAINTY/DATA UNCERTAINTY
- HERA-II IMPACT SIZABLE
- IMPACT OF LHC DATA MODERATE BUT VISIBLE
- IMPACT OF CMS OR ATLAS COMPARABLE TO (MODERATE) IMPACT OF NON-LHC, NON-HERA DATA

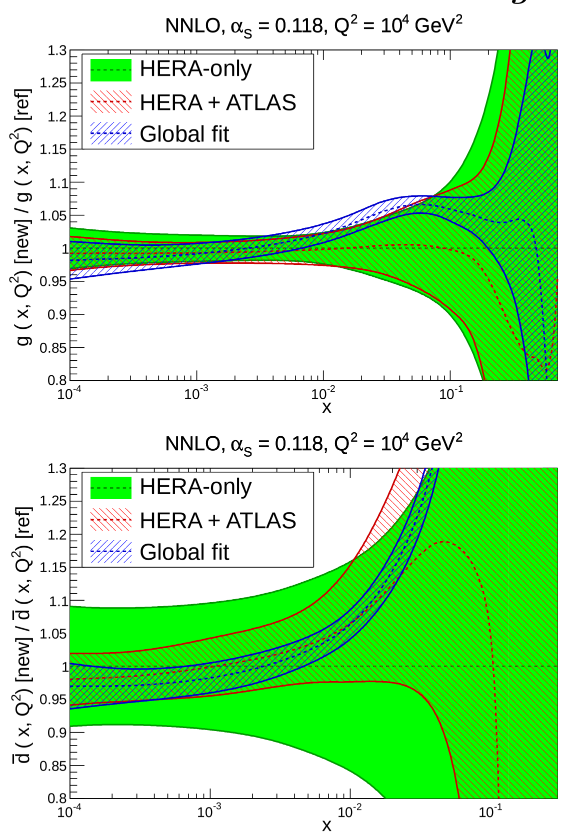
FRACTIONAL UNCERTAINTY

Dataset	φ_{χ^2} NLO	φ_{χ^2} NNLO
Global	0.291	0.302
HERA-I	0.453	0.439
HERA all	0.375	0.343
HERA+ATLAS	0.391	0.318
HERA+CMS	0.315	0.345
Conservative	0.422	0.478
no LHC	0.312	0.316

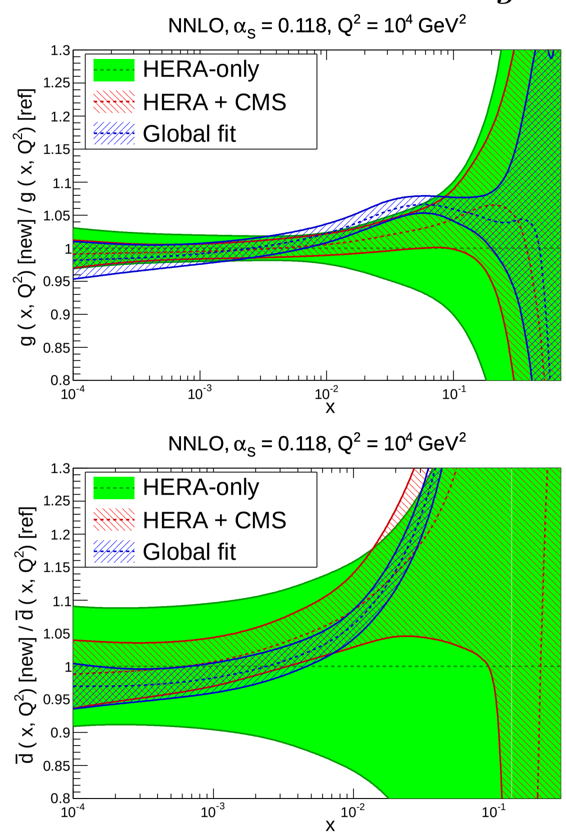
GLOBAL VS NO LHC: g& d



GLOBAL VS HERA+ATLAS: g& $\bar{d}$



GLOBAL VS HERA+CMS: g& $\bar{d}$

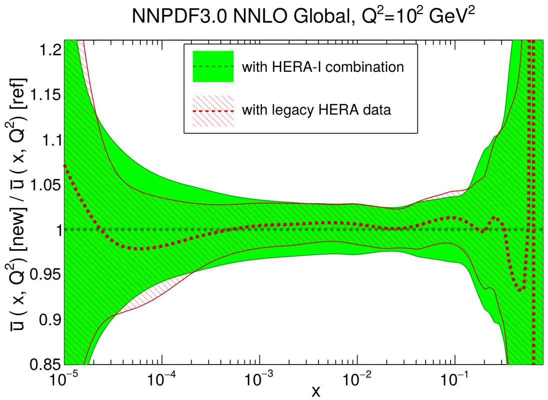


IMPACT OF THE COMBINED HERA DATA

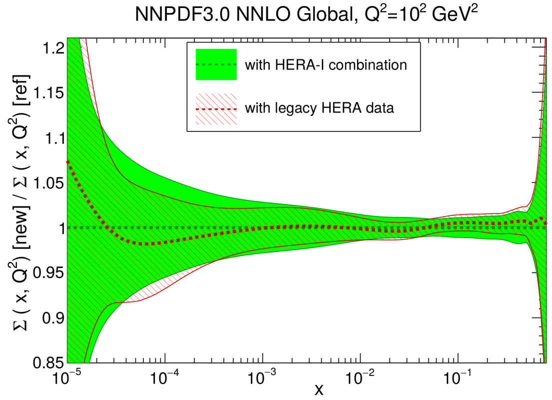
- REMOVE ALL HERA-II DATA FROM NNPDF3.0 FIT
- REPLACE COMBINED HERA-I+ SEPARATE ZEUS, H1 DATA WITH COMBINED HERA I+II IN NNPDF3.0

GLOBAL FIT WITH ONLY HERA I VS COMBINED HERA I+II

ANTIUP

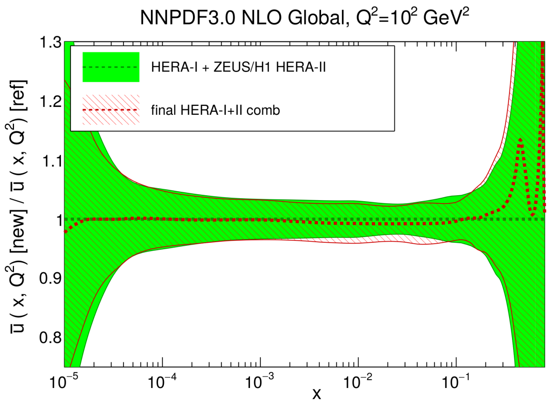


QUARK SINGLET

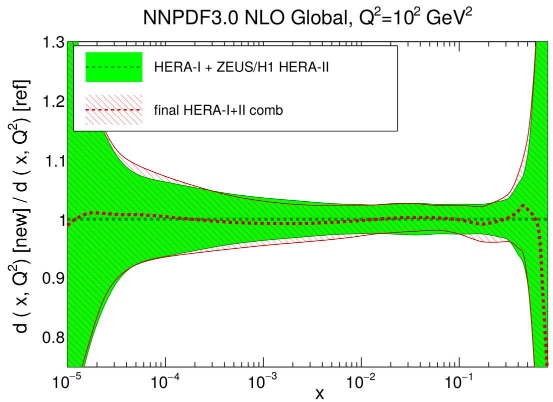


GLOBAL FIT WITH HERA I+UNCOMBINED HERA II VS COMBINED HERA I+II

ANTIUP



DOWN



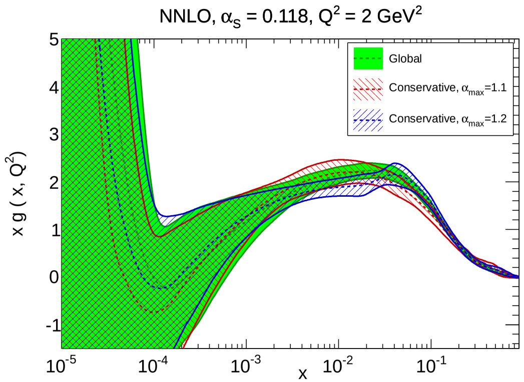
- IMPACT OF HERA-II ON GLOBAL FIT MODERATE
- DIFFERENCE BETWEEN COMBINED & UNCOMBINED EXTREMELY SMALL

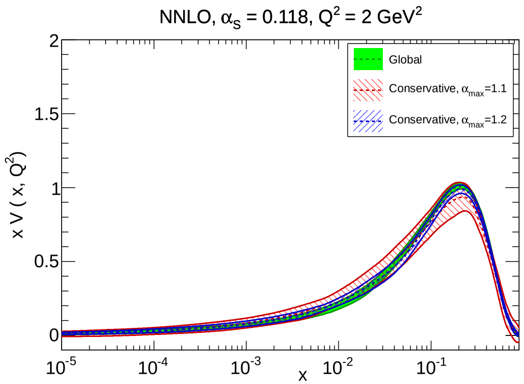
CONSERVATIVE PARTONS

- RESCALE ALL UNCERTAINTIES $\sigma \to \alpha\sigma$: $\chi^2 \to \chi^2/\alpha^2$ FOR A GIVEN EXPERIMENT
- DETERMINE PROBABILITY DISTRIBUTION $P(\alpha)$ (USING BAYES)
- DISCARD ALL EXPERIMENT FOR WHICH $P(\alpha)$ PEAKS WELL ABOVE ONE
  TWO OUT OF MEDIAN, MODE, MEAN, GREATER THAN $\alpha_{threshold}$
- $\chi^2 = 1.29$ FOR NNLO GOBAL, BECOMES $\chi^2 = 1.16$ FOR $\alpha_{threshold} = 1.3$, $\chi^2 = 1.10$ FOR $\alpha_{threshold} = 1.2$, $\chi^2 = 1.01$ FOR $\alpha_{threshold} = 1.1$, BUT CONSIDERABLE DETERIORATION OF UNCERTAINTIES

CONSERV. VS. DEFAULT

GLUON AND VALENCE



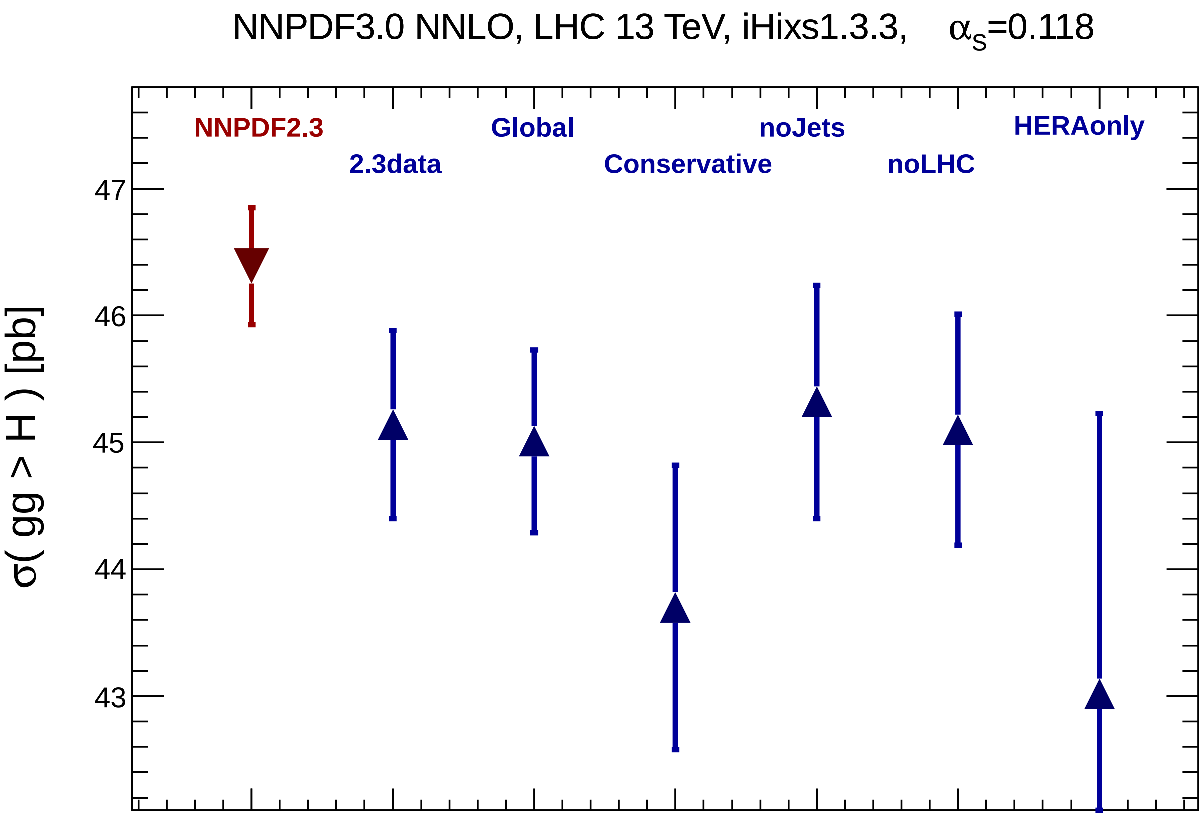


α PEAK FOR EXPERIMENTS DISCARDED IN CONS. FIT WHEN INCLUDED OR EXCLUDED FROM FIT

	NNLO global fit			NNLO cons. fit $\alpha_{max} = 1.1$		
Experiment	mean	mode	median	mean	mode	median
NMC $\sigma^{NC,p}$	1.27	1.26	1.27	1.50	1.45	1.48
SLAC	1.13	1.09	1.12	1.61	1.37	1.48
BCDMS	1.20	1.19	1.20	2.02	1.86	1.92
CHORUS	1.10	1.09	1.09	2.55	1.69	2.32
ZEUS HERA-II	1.25	1.24	1.25	1.38	1.33	1.36
H1 HERA-II	1.35	1.34	1.34	1.51	1.47	1.49
HERA σ^c_{NC}	1.14	1.11	1.13	1.13	1.09	1.12
E886 p	1.15	1.14	1.15	2.18	1.62	2.03
CDF Z rapidity	1.39	1.32	1.36	1.56	1.40	1.50
CDF Run-II k_t jets	1.15	1.12	1.14	1.25	1.18	1.22
ATLAS W, Z 2010	1.17	1.12	1.15	1.38	1.25	1.32
ATLAS high-mass DY	1.00	1.34	1.63	1.63	1.19	1.45
CMS W muon asy	1.60	1.40	1.53	2.90	2.48	2.81
CMS $W+c$ total	1.50	1.09	1.33	1.85	1.37	1.67
CMS $W+c$ ratio	2.00	1.39	1.69	2.12	1.58	1.94
CMS 2D DY 2011	1.28	1.27	1.28	1.29	1.28	1.29
LHCb	1.20	1.12	1.17	1.58	1.22	1.48

IMPACT OF INDIVIDUAL DATASETS

HIGGS IN GLUON FUSION



REQUIREMENTS FOR INCLUSION

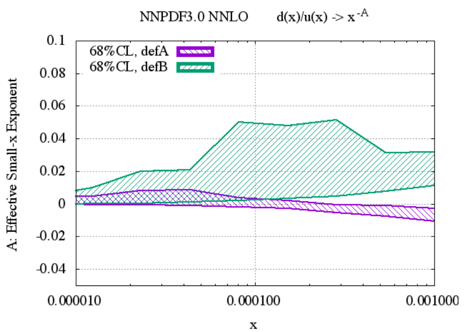
PDFs to be included in the combination must satisfy requirements which make them compatible
document in PDF4LHC April 2015 indico
some listed here (see document for full list):

- provide results at NLO and NNLO, with NNLO α_s, evolution benchmarked against HOPPET or QCDNUM
- separate treatment of α_s as discussed
- use a general-mass variable flavor number scheme with up to 5 flavors
- current technology: fits must be based on approximately common, global set of data
  request to relax this (HERAPDF)

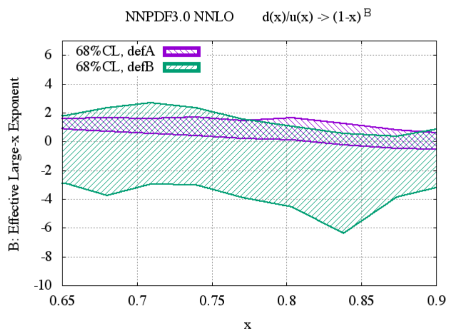
ASYMPTOTIC EXPONENTS

- IN NNPDF APPROACH, PDF BEHAVIOUR DETERMINED BY DATA
- AS $x \to 0$, PDF GOES AS $x^{-\alpha}$, AS $x \to 1$, PDF GOES AS $(1-x)^{\beta}$
- DEFINE EFFECTIVE EXPONENTS $\alpha \equiv -\frac{\ln f_i(x)}{\ln 1/x}$, $\beta \equiv \frac{\ln f_i(x)}{\ln(1-x)}$ (DEF A)
  OR $\alpha \equiv \frac{d}{d\ln x} \ln f$, $\beta \equiv \frac{d}{d\ln(1-x)} \ln f$

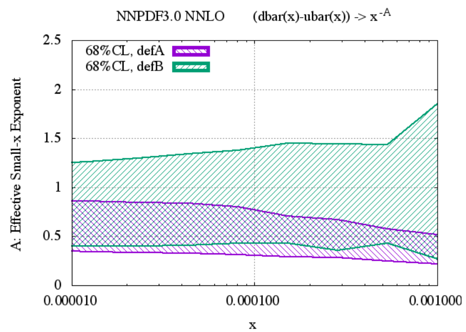
$d/u\ \alpha$



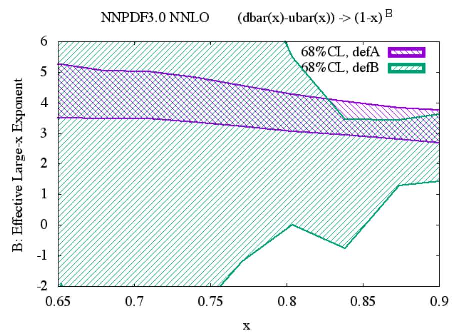
$d/u\ \beta$



$\bar{d}-\bar{u}\ \alpha$



$\bar{d}-\bar{u}\ \beta$

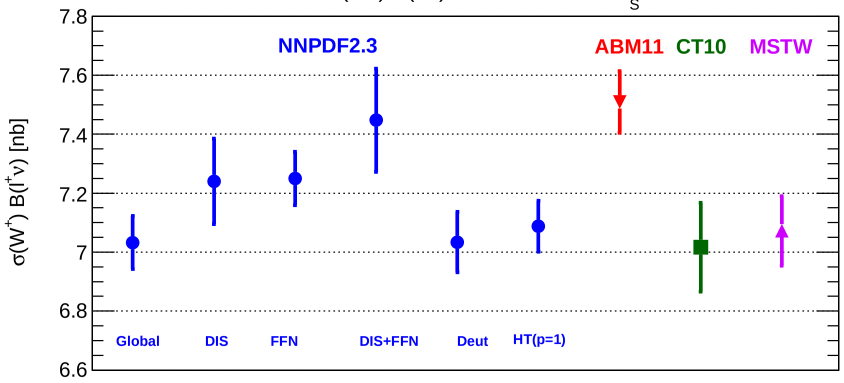


- d/u: $\alpha \sim 0$ TO GOOD APPROX; $-1 \lesssim \beta \lesssim 2$
- $\bar{d}-\bar{u}$: $0.5 \lesssim \alpha \lesssim 1.5$ (REGGE: 0.5); $3.5 \lesssim \beta \lesssim 4.5$
- REGGE BEHAVIOUR AT SMALL x SUPPORTED BY DATA,
  COUNTING RULES CANNOT BE VERIFIED ACCURATELY

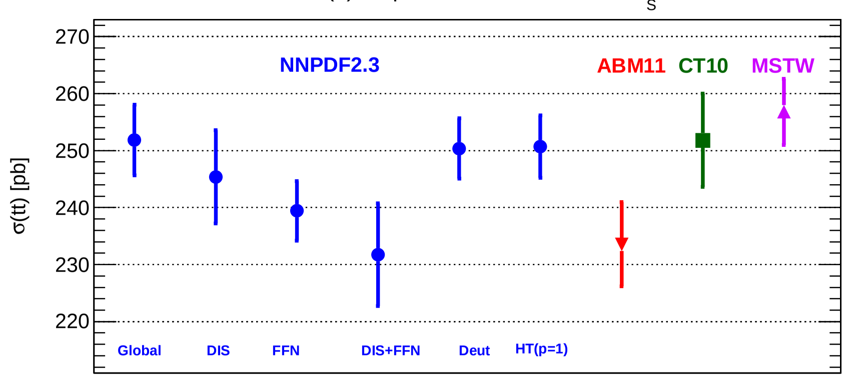
FFN PDFs

- SOME PDF SETS ADOPT A FFN SCHEME (ABM, JR)
- ABM ALSO INCLUDES HIGHER TWIST & NUCLEAR CORRECTIONS
- ALSO, ABM MOSTLY BASED ON DIS DATA
  (ONLY HADRONIC DATA IS ITEMIZE FIXED-TARGET DY)
- WHAT IS THE RELATIVE SIZE OF ALL THESE EFFECTS?
- NNPDF WITH FFN &DIS DATA SET AGREES WITH ABM;
  HIGHER TWIST & NUCLEAR CORRRECTIONS HAVE SMALL & LOCALIZED EFFECT;
  SIMILAR RESULTS FOUND BY MSTW AND CTEQ

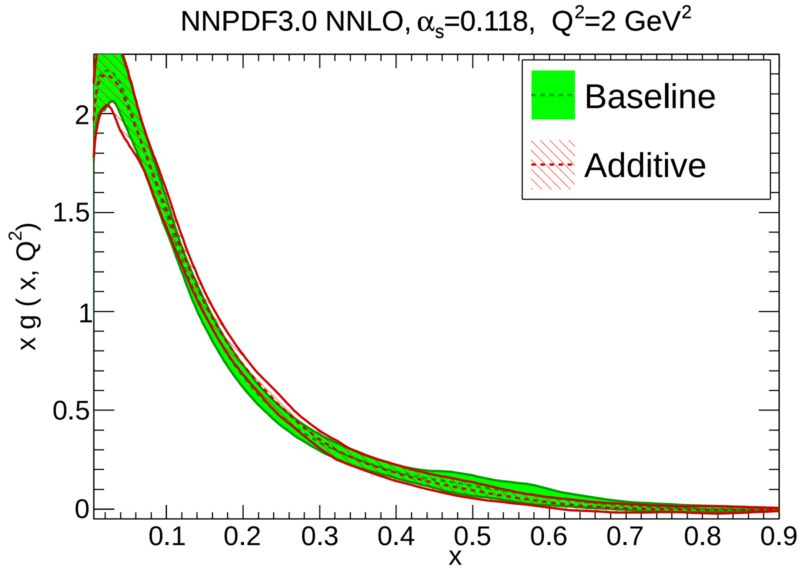
W^+

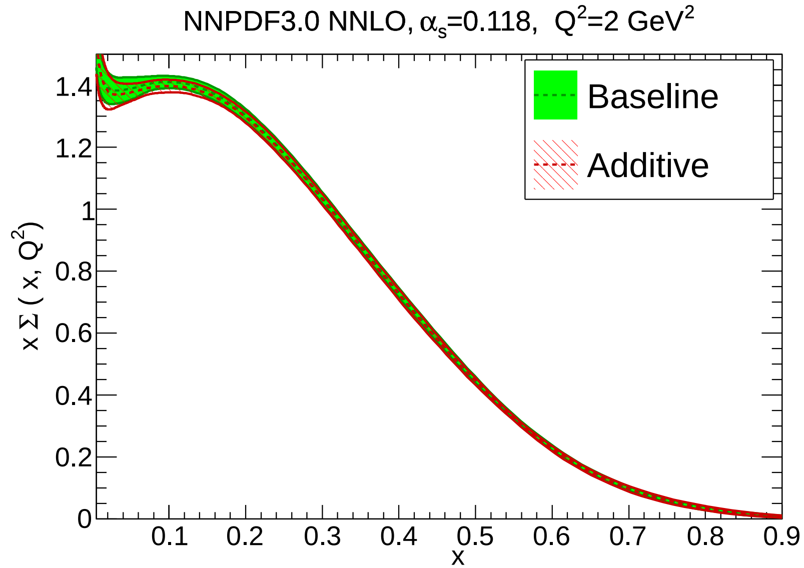


TOP



MULTIPLICATIVE VS. ADDITIVE UNCERTAINTIES





- NORMALIZATION UNCERTAINTY PROPORTIONAL TO MEASURED VALUE $\Rightarrow$ MULTIPLICATIVE
- WHAT ABOUT OTHER UNCERTAINTIES?
  IN COLLIDER MOSTLY MULTIPLICATIVE (A. Glazov)
- IN NNPDF3.0 DEFAULT ALL UNCERTAINTIES MULTPLICATIVE FOR HERA, TEVATRON, LHC
- REPEAT FIT WITH ALL UNCERTAINTIES ADDITIVE (AS IN NNPDF2.3)
- t_0 METHOD USED FOR MULTIPLICATIVE UNCERTAINTIES
- NEGLIGIBLE CHANGE SEEN

THE VALUE OF α_s

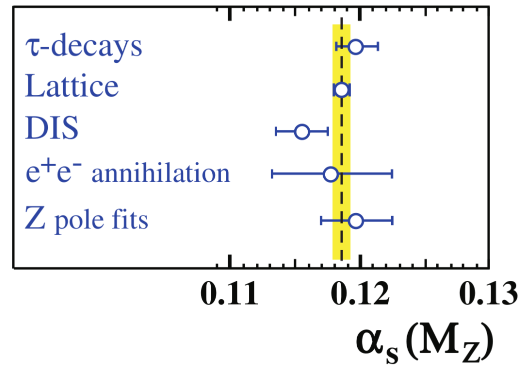
PDG VALUE (AUGUST 2014): $\alpha_s(M_Z) = 0.1185 \pm 0.0006$

- LATTICE UNCERTAINTY CURRENTLY ESTIMATED BY FLAG (arXiv:1310.8555) TO BE TWICE THE PDG VALUE (± 0.0012)
  (IF PDG WERE TO ADOPT THIS, COMBINED UNCERTAINTY LIKELY TO DOUBLE)
- IT IS AN AN AVERAGE OF AVERAGES
- SOME SUB-AVERAGES (E.G. DIS) INCLUDE MUTUALLY INCONSISTENT VALUES
- SOME SUB-AVERAGES (E.G. τ OR JETS) INCLUDE DETERMINATIONS WHICH DIFFER FROM EACH OTHER BY EVEN FOUR-FIVE σ
- AVERAGING THE TWO MOST RELIABLE VALUES (GLOBAL EW FIT & τ, BOTH N^{3}LO, NO DEP. ON HADRON STRUCTURE) GIVES

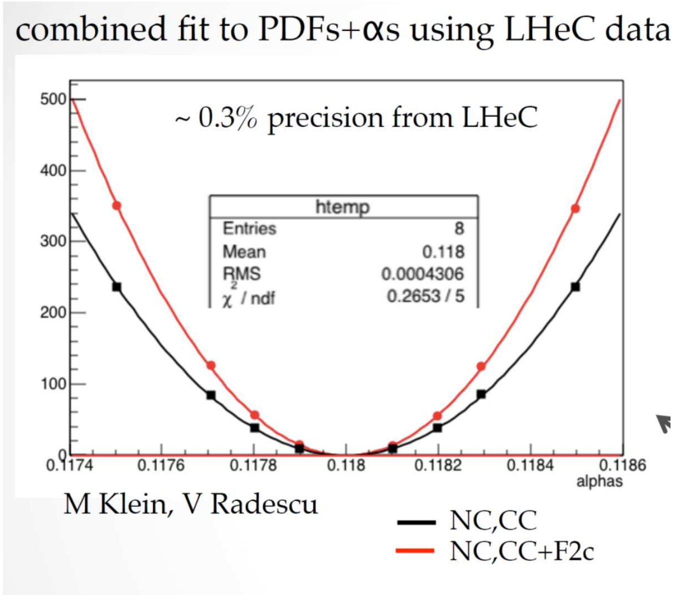
$$\alpha_s = 0.1196 \pm 0.0010$$

- LITTLE PROGRESS FOR MANY YEARS: PDG 1998-2006 $\Delta\alpha_s(M_Z) = 0.002$; PDG 2010-2014 $\Delta\alpha_s(M_Z) = 0.0006 \div 0.0008$ (CHANGE OF AUTHOR)
- UNLIKELY TO CHANGE - SHOULD DETERMINE α_s FROM HIGGS IN GLUON FUSION?

α_s DETERMINATIONS IN PDG

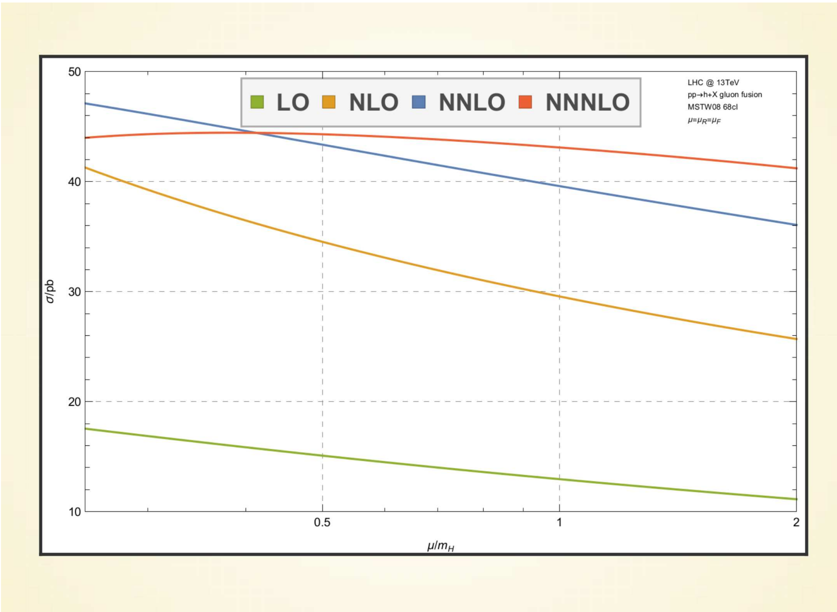


α_s AT THE LHEC

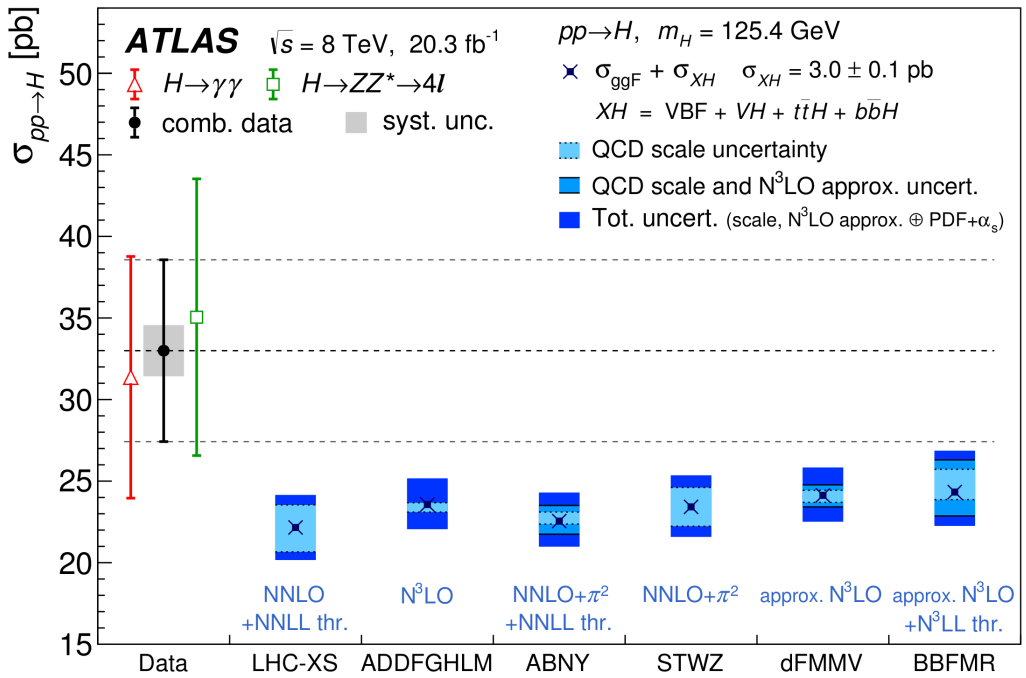


- COULD BE DETERMINED ACCURATELY AT THE LHEC OR AT A NEUTRINO FACTORY!

Higgs in gluon fusion:
EXACT N^{3}LO!



(Anastasiou et al, 2015)



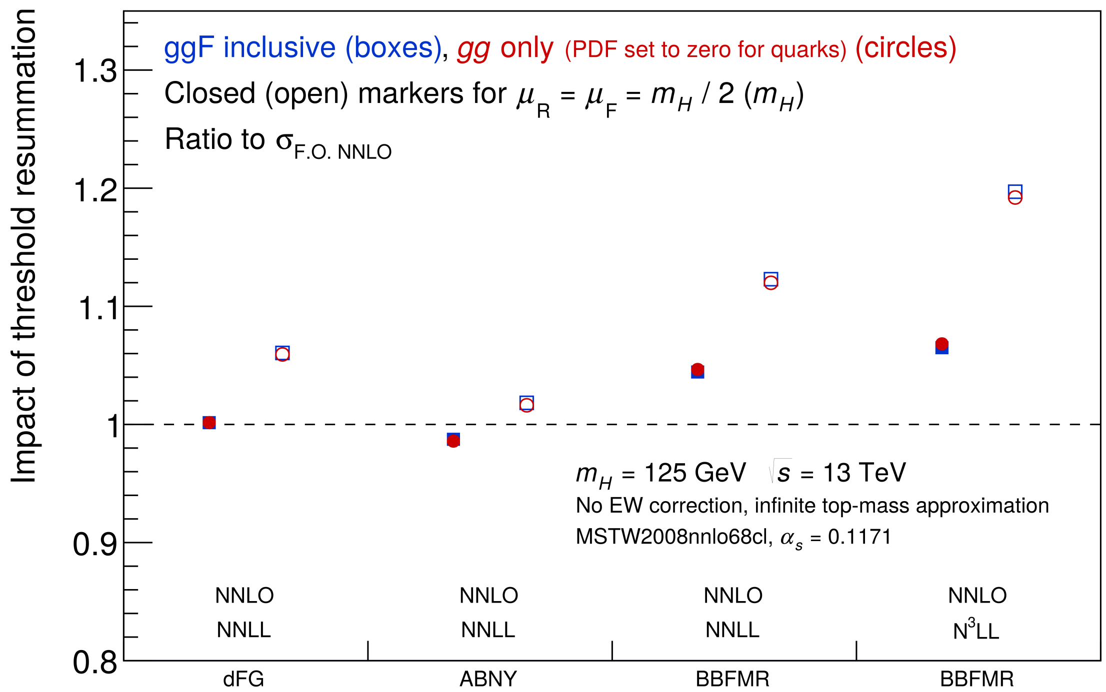
(ATLAS, 2015)

- EXACT N^{3}LO PUBLISHED IN PRELIMINARY FORM, OBTAINED AS SERIES EXPANSION ABOUT THE SOFT LIMIT, IN POINTLIKE APPROX
- HAS THE SERIES CONVERGED?
- IN GOOD AGREEMENT WIH RESUMMATION

HIGGS IN GLUON FUSION: RESUMMATION AMBIGUITIES

- LARGE SUBLEADING TERMS $\Rightarrow$ LARGE AMBIGUITIES
- SCET RESUMMATION SMALLER, DUE TO $1/N$ TERMS (Bonvini, SF, Ridolfi, Rottoli)
- RESUMMATION EFFECTIVELY AMOUNTS TO APPROXIMATE HIGHER ORDERS

RESUMMED/UNRESUMMED RATIO (NO CONST. EXPONENTIATION)



note k factor computed wr to NNLO at respective scale

- De Florian, Grazzini (DFG) (HXSWG REFERENCE) RESUM $\ln N$
- Becher, Neubert et al. (ABNY): SCET (z SPACE) APPROACH TO NNLL (REALLY N^3LL*)
- Ball et al. (BBFMR): RESUM $\ln N$, DIFFERS FROM DFG BECAUSE OF 'ANALYTIC' RESUMMATION (correct small-N singularities when expanded to finite order)
- BBFMR: N^3LL ALSO AVAILABLE

GLOBAL FITS: PROGRESS

- MMHT (successor of MSTW08) December 2014
- CT14 June 2015
- for all, progress in methodology & dataset

METHODOLOGY

	NNPDF3.0	MMHT14	CT14
No. of fitted PDFs	7	7	6
Parametrization	neural nets	$x^a(1-x)^b\times$ Chebyschev	$x^a(1-x)^b\times$Bernstein
Free parameters	259	37	30-35
Uncertainties	Replicas	Hessian	Hessian
Tolerance	None	Dynamical	Dynamical
Closure test	✔	✗	✗
Reweighting	replicas	eigenvectors	eigenvectors

- MMHT, CT10 larger # of parms., orthogonal polynomials
- NNPDF closure test

DATASET

	NNPDF3.0	MMHT14	CT14(PREL)
SLAC P,D DIS	✔	✔	✗
BCDMS P,D DIS	✔	✔	✔
NMC P,D DIS	✔	✔	✔
E665 P,D DIS	✗	✔	✗
CDHSW NU-DIS	✗	✗	✔
CCFR NU-DIS	✗	✔	✔
CHORUS NU-DIS	✔	✔	✗
CCFR DIMUON	✗	✔	✔
NuTeV DIMUON	✔	✔	✔
HERA I NC,CC	✔	✔	✔
HERA I CHARM	✔	✔	✔
H1,ZEUS JETS	✗	✔	✗
H1 HERA II	✔	✗	✗
ZEUS HERA II	✔	✗	✗
E605 & E866 FT DY	✔	✔	✔
CDF & D0 W ASYM	✗	✔	✔
CDF & D0 Z RAP	✔	✔	✔
CDF Run-II JETS	✔	✔	✔
D0 Run-II JETS	✗	✔	✔
ATLAS HIGH-MASS DY	✔	✔	✔
CMS 2D DY	✔	✔	✗
ATLAS W,Z RAP	✔	✔	✔
ATLAS W PT	✔	✔	✗
CMS W ASY	✔	✔	✔
CMS W +C	✔	✗	✗
LHCb W,Z RAP	✔	✔	✔
ATLAS JETS	✔	✔	✔
CMS JETS	✔	✔	✔
TTBAR TOT XSEC	✔	✔	✗
TOTAL NLO	4276	2996	3248
TOTAL NNLO	4078	2663	3045

OUTLOOK: NNPDF3.1

NEW DATA (MINIMAL SET)

- HERA-II COMBINED DATA arxiv:1506.06042
- D0 W ASYMMETRY arXiv:1412.2862
- ATLAS LOW-MASS DY arXiv:1404.1212
- ATLAS PROMPT PHOTON arXiv:1311.1440
- ATLAS $W + c$ arXiv:1402.6263
- ATLAS Z p_t arXiv:1406.3660
- ATLAS $t\bar{t}$ RAPIDITY arXiv:1407.0371
- ATLAS INCLUSIVE JETS 7 TEV 5 FB^{-1} arXiv:1410.8857
- CMS DOUBLE-DIFFERENTIAL 8 TEV arXiv:1412.1115
- CMS Z p_t arXiv:1504.03511
- LHCB $W \to \mu\nu$ RAP. arxiv:1408.4354

COMBINING INFORMATION

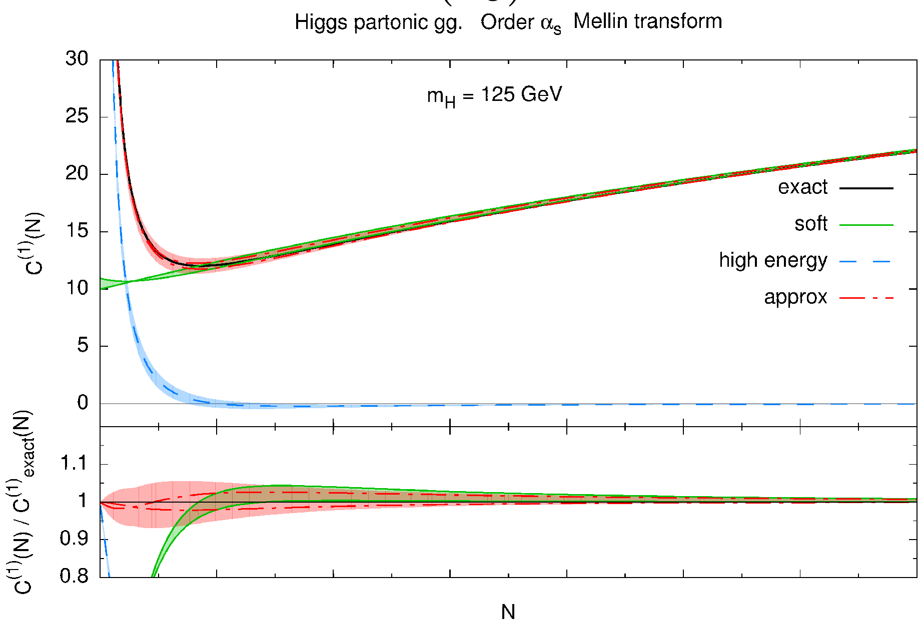
$$C^{(k)}_{\rm APPROX}(N) = C^{(k)}_{\rm SOFT}(N) + C^{(k)}_{\rm H.E.}(N)$$

NOTHING ELSE TO DO: SINGULARITIES TAKE CARE OF THEMSELVES

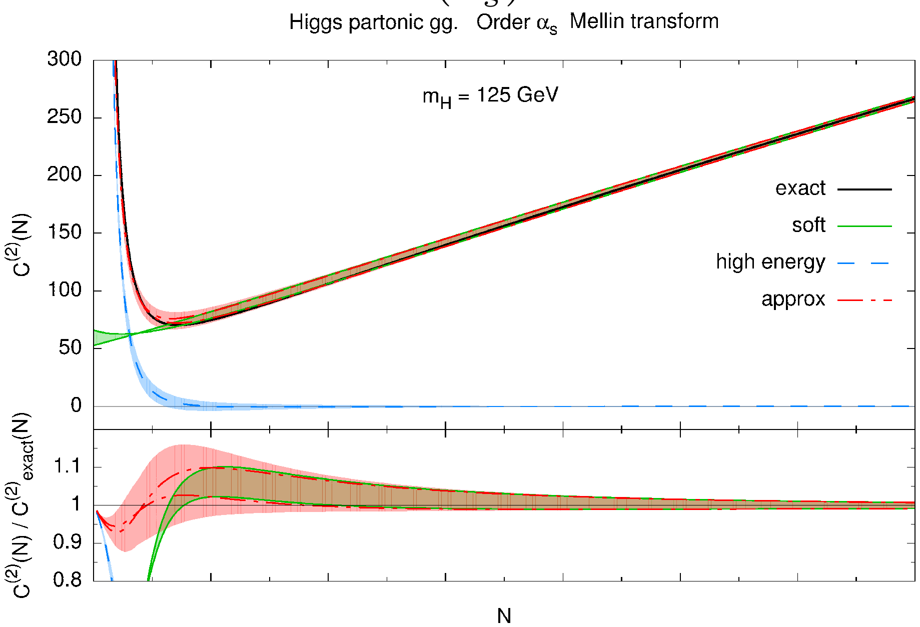
NLO AND NNLO

EXACT+SOFT+HIGH ENERGY+APPROX.

$O(\alpha_s)$



$O(\alpha_s^2)$



NOTE FOR $N \sim 1.2$ WHERE SMALL DISCRAPANCY WITH NNLO IS OBSERVED, “EXACT” IS BASED ON AN UNRELIABLE APPROX.

THE PIECES OF THE PUZZLE

https://twiki.cern.ch/twiki/bin/view/LHCPhysics/CERNYellowReportPageAt8TeV#gluon_gluon_Fusion_Process

$$m_h = 125 \text{ GeV}; \qquad \sigma = 19.27 \text{ pb}^{+7.2}_{-7.8} \text{ (SCALE) } ^{+7.5}_{-6.9} \text{ (PDF+}\alpha_s\text{)}$$

$$\mu_r = \mu_F = m_H$$

(NOTE: IF $m_H = 125.5 \rightarrow \sigma = 19.12$ pb)

(De Florian, Grazzini, 2012)

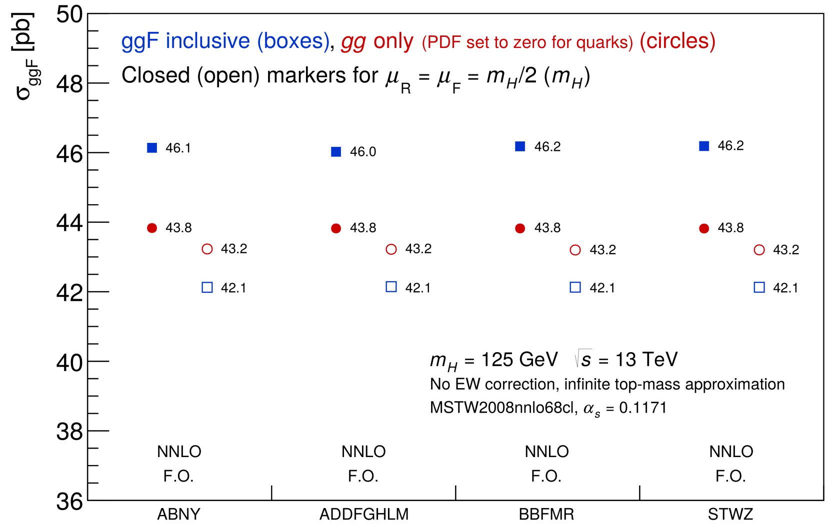
- NNLO+NNLL
- TOP MASS DEP, AND BOTTOM AND CHARM CONTRIBUTIONS AT NLO+NLL
  DECREASE $\sim 1.5\%$ DUE TO TOP+BOTTOM (MOSTLY BOTTOM)+
  FURTHER $\sim 1\%$ DUE TO CHARM
- NLO ELECTROWEAK CORRECTIONS (COMPLETE FACTORIZATION)
  INCREASE $\sim 5\%$ (WOULD CHANGE BY A FEW PERCENT WITH
  PARTIAL FACTORIZATION)
- COMPLEX-POLE (OFFP)
  FINITE-WIDTH BELOW $\sim 1\%$ FOR LIGHT HIGGS
- NO MIXED QCD-EW AND REAL EW RADIATION
  BELOW $\sim 1\%$

BENCHMARKING

THE GROUPS INVOLVED:

- **ABNY** Ahrens, Becher, Neubert, Yang → NNLO+NNLL RESUMMED (SCET)
- **STWZ** Stewart, Tackmann, Walsh, Zuberi → NNLO WITH EXP. CONSTANTS
- **dFMMV** de Florian, Mazzitelli, Moch, Vogt → APPROXIMATE N^3LO
- **BBFMR** Ball, Bonvini, SF, Marzani, Ridolfi → APPROXIMATE N^3LO+N^3LL
- **ADDFGLHM** Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Lazopoulos, Herzog, Mistlberger → PARTIAL N^3LO

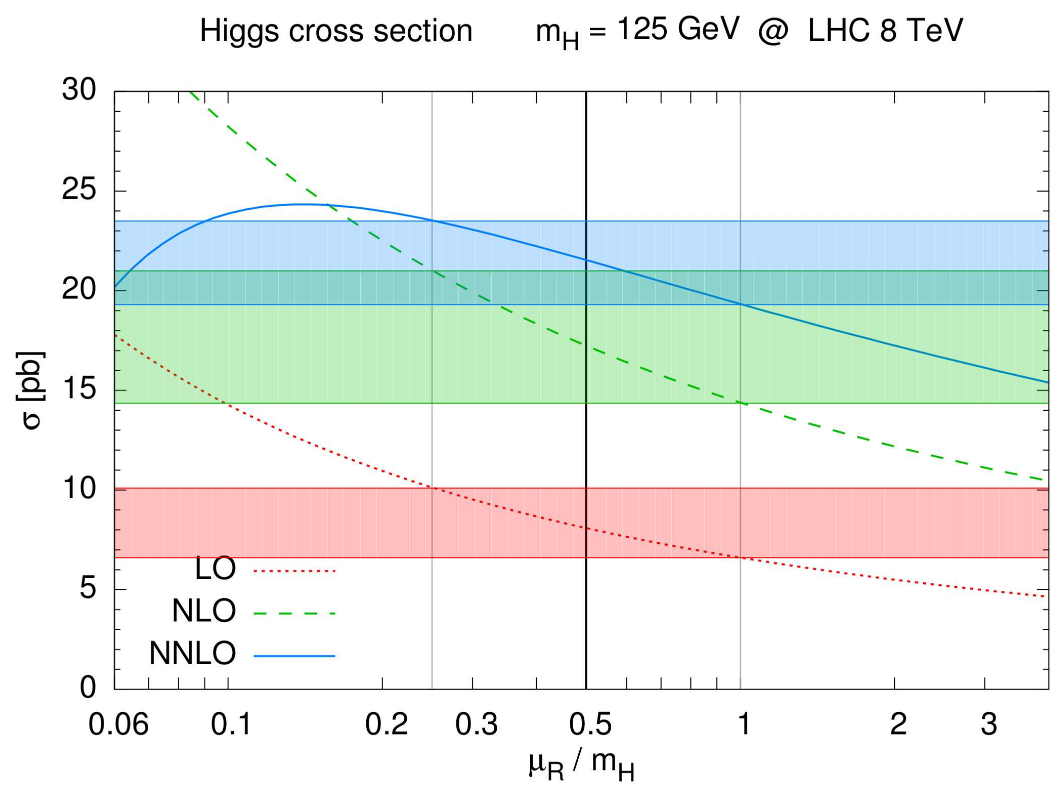
THE NNLO CROSS-SECTION



COMMON SETTINGS FOR PDFs, EW, HW, ETC ADOPTED;
NOT MEANT TO BE REALISTIC, FOCUS ON SIZE OF QCD CORRECTIONS

BEYOND NNLO

LO, NLO AND NNLO



- THE PERTURBATIVE SERIES FOR $gg \to H$ CONVERGES SLOWLY
- SCALE VARIATION UNDERESTIMATES SHIFT TO NEXT ORDER
- FACTORIZATION SCALE DEPENDENCE NEGLIGIBLE, ONLY RENORMALIZATION SCALE DEPENDENCE SIGNIFICANT

QUESTIONS

- DOES RESUMMATION IMPROVE THE FIXED-ORDER RESULT? HOW AMBIGUOUS IS IT?
- CAN WE CONSTRUCT A REASONABLE APPROXIMATION TO N^3LO BASED ON CURRENT KNOWLEDGE?

RESUMMATION

$$\sigma(N, m_H^2) \equiv \int_0^1 d\tau\, \tau^{N-2}\sigma(\tau, m_H^2); \quad z = \frac{M_h^2}{s} \qquad \left(\frac{\ln^{k-1}(1-x)}{1-x}\right)_+ \Leftrightarrow \ln^k N + O(\ln^{k-1} N) + O\left(\frac{1}{N}\right)$$

$$\sigma_{\rm res}(N, \alpha_s) = \sigma_0 g_0(\alpha_s) \exp\left[\frac{1}{\alpha_s} g_1(\alpha_s \ln N) + g_2(\alpha_s \ln N) + \alpha_s g_3(\alpha_s \ln N) + \ldots\right];$$

$$g_0(\alpha_s) = 1 + \alpha_s g_{0,1} + \alpha_s^2 g_{0,2} + \mathcal{O}(\alpha_s^3);$$

N^{k}LL RESUMMATION: ALL g_i WITH $i < k+1$, ALL g_{0j} WITH $j < k$
TO ORDER α_s^n, $\ln^m N$ PREDICTED WITH $2(n-k) \leq m \leq 2n$

in SCET literature, this is called N^{k}LL', at N^{k}LL one less power of log: $2(n-k)+1 \leq m \leq 2n$

AMBIGUITIES

- SHOULD g_0 SIT IN THE EXPONENT, ENTIRELY OR IN PART?

  NOTE $g_{0,1}$ NATURALLY PART OF g_{i+2}

  $\Rightarrow$ DIFFERENCE SUBLEADING!

- HOW DOES ONE FIX SUBLEADING TERMS?

  $\ln N$, $\ln N + 1$, $\psi_0(N)$ (MELLIN OF $\left(\frac{1}{1-x}\right)_+$) BEHAVE IN THE SAME WAY AS $N \to \infty$, UP TO $O(1)$ &

  $O(1/N)$ TERMS

  $\Rightarrow$ DIFFERENCE SUBLEADING!

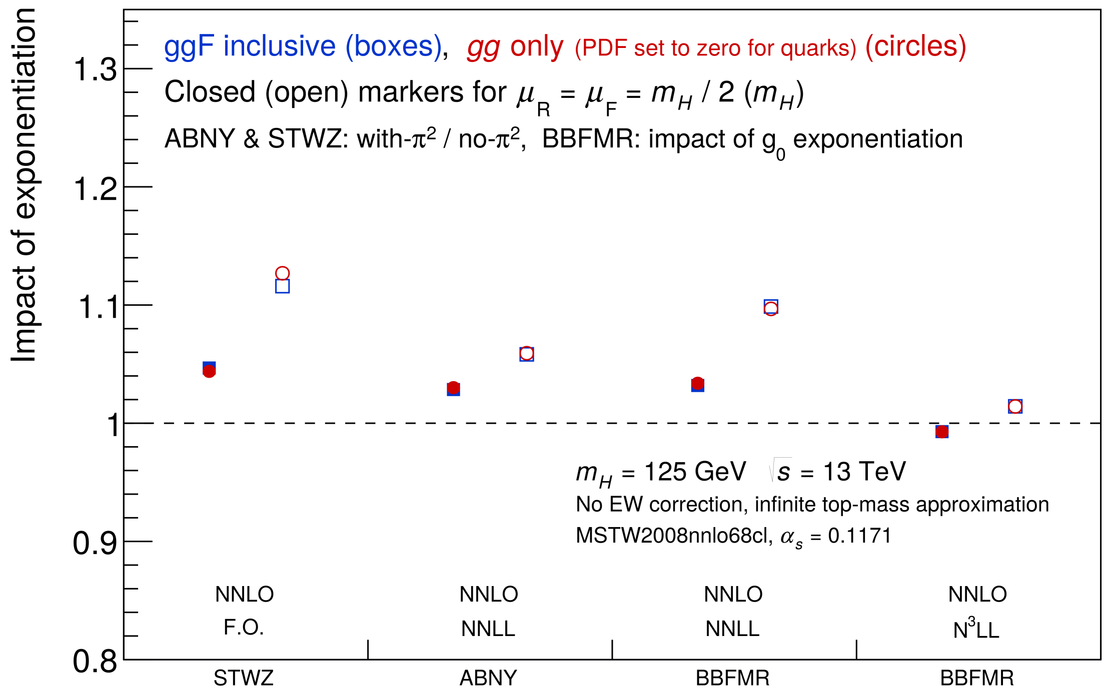
  NOTE IN SCET APPROACH, RESUMMATION NATURALLY IN z SPACE $\Rightarrow$ SUBLEADING

  DIFFERENCES

CONSTANT EXPONENTIATION

- IN SCET APPROACH, π^2 EXPONENTIATION W. IMAGINARY SCALE:
  STWZ: ADDED TO FIXED NNLO; ABNY: ADDED TO NNLO+NNLL
- IN DQCD RESUMMATION DIS-SCHEME CONSTANT EXPONENTIATION
  BBFMR: BOTH NNLO+NNLL AND NNLO+N^3LL AVAILABLE

EXPONENTIATED/UNEXPONENTIATED RATIO (NO RESUMMATION)



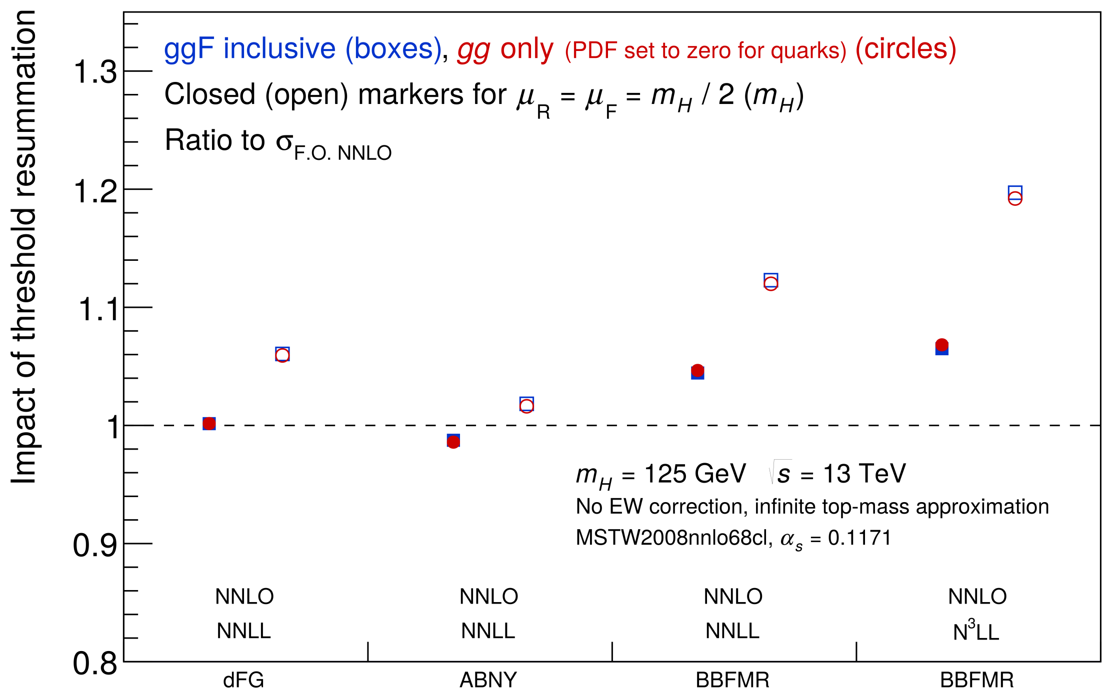
note k factor computed wr to NNLO at respective scale

- IMPACT SMALLER FOR $\mu_R = m_H/2$ THAN FOR $\mu_R = m_H$
- IMPACT SMALLER AT RESUMMED THAN AT FIXED ORDER
- IMPACT SMALLER AT N^3LL THAN AT NNLL

CHOICE OF LOGS

- DFG: CURRENT REFERENCE $\mu_R = m_H$
- ABNY: SCET (z SPACE) APPROACH TO NNLL (REALLY N^{3}LL*, OR SCET-N^{3}LL)
- BBFMR: DIFFERS FROM DFG BECAUSE OF 'ANALYTIC' RESUMMATION
  (correct small-N singularities when expanded to finite order)
- BBFMR: N^{3}LL ALSO AVAILABLE

RESUMMED/UNRESUMMED RATIO (NO CONST. EXPONENTIATION)



note k factor computed wr to NNLO at respective scale

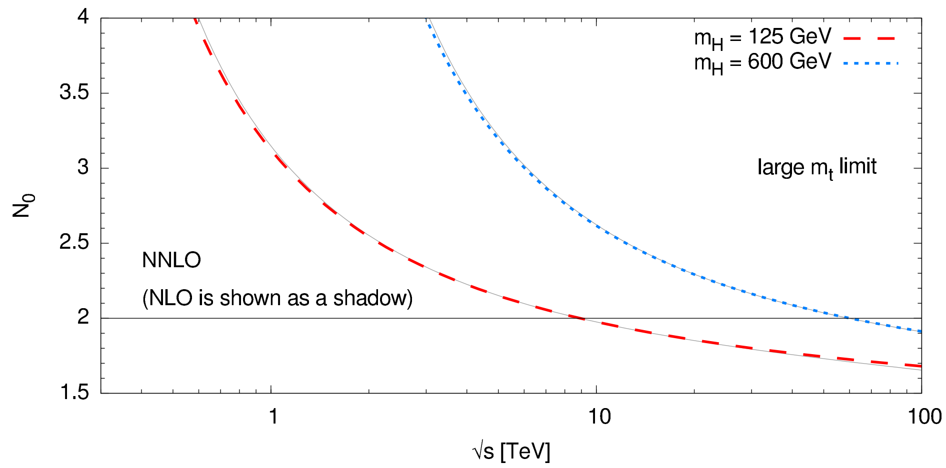
- IMPACT SMALLER FOR $\mu_R = m_H/2$ THAN FOR $\mu_R = m_H$
- z-SPACE: LITTLE EFFECT, ANALYTIC: LARGER EFFECT
- IMPACT LARGER AT N^{3}LL THAN AT NNLL

FROM RESUMMATION TO N^{3}LO

TECHNICAL INTERLUDE

- PARTONIC CROSS SECTIONS $\hat\sigma(z,\alpha_s)$ ARE DISTRIBUTIONS, $0 \le z = \frac{M_h^2}{s} \le 1$
- THEIR MELLIN TRANSFORMS $\hat\sigma(N,\alpha_s)$ ARE ORDINARY FUNCTIONS
- FOR GIVEN m_H^2 AND s ONLY ONE "SADDLE" N VALUE CONTRIBUTES
- BONUS: HAD. XSCT PRODUCT OF PARTONIC TIMES LUMI: $\sigma(N,m_H^2) = \hat\sigma\left(m_H^2,\alpha_s\right)\mathscr{L}(N)$

SADDLE VS COLLIDER ENERGY



Q: WHY ARE THERE LARGE RESUMMATION AMBIGUITIES?

A: BECAUSE $\beta_0\alpha_s \ln 2 \approx 0.04 \ll 1$:RESUMMATION IS PERTURBATIVE!

IN ALL FLAVOURS OF RESUMMATION DISCUSSED ABOVE
BETWEEN 2/3 AND 3/4 OF THE NNLL OR N^{3}LL RESUMMATION
COMES FROM N^{3}LO,
ALL REMAINING INFINITE ORDERS CONTRIBUTING $\sim 1 \div 2\%$
RESUMMATION IS AMBIGUOUS BECAUSE N^{3}LO IS APPROXIMATE!

N^3LO: WHAT DO WE KNOW

THE SOFT EXPANSION

ABOUT $N \to \infty$ OR $z = 1$

$$\hat\sigma^{(3)}(N) = c_{06}\ln^6 N + c_{05}\ln^5 N + \cdots + c_{02}\ln^2 N + c_{01}\ln N + c_{00} + \qquad \text{SOFT (S)}$$
$$. \qquad + \frac{1}{N}\left(c_{15}\ln^5 N + c_{14}\ln^4 N + \cdots + c_{10}\right) + \qquad \text{NEXT-TO-SOFT (NS)}$$
$$. \qquad + \frac{1}{N^2}\left(c_{25}\ln^5 N + c_{24}\ln^4 N + \cdots + c_{20}\right) + \ldots \qquad \text{NNS}$$

$$\hat\sigma^{(3)}(z) = c'_{06}\left(\frac{\ln^5(1-z)}{1-z}\right)_+ + \cdots + c'_{01}\left(\frac{1}{1-z}\right)_+ + c'_{00}\delta(1-z) + \qquad \text{SOFT (S)}$$
$$. \qquad + \left(c'_{15}\ln^5(1-z) + c'_{14}\ln^4(1-z) + \cdots + c'_{10}\right) + \qquad \text{NEXT-TO-SOFT (NS)}$$
$$. \qquad + (1-z)\left(c'_{25}\ln^5(1-z) + +c'_{24}\ln^4(1-z) + \cdots + c'_{20}\right) + \ldots \qquad \text{NNS}$$

NSPACE c COEFFICIENTS DETERMINE THE z SPACE c' AND CONVERSELY:

- N-SPACE N^k SOFT $\Leftrightarrow$ z-SPACE N^k SOFT
- N-SPACE N^kLL $\Leftrightarrow$ z-SPACE N^kLL
- MELLIN OF z SPACE N^k SOFT EQUALS N-SPACE N^k SOFT PLUS N^{k+1} TERMS
- MELLIN OF z SPACE N^kLL EQUALS N-SPACE N^kLL SOFT PLUS N^{k+1}LL TERMS

N^{3}LO: WHAT DO WE KNOW

THE SOFT EXPANSION
ABOUT $N \to \infty$ OR $z = 1$

$$\begin{aligned}\hat\sigma^{(3)}(N) &= c_{06}\ln^6 N + c_{05}\ln^5 N + \cdots + c_{02}\ln^2 N + c_{01}\ln N + c_{00} + && \text{SOFT (S)}\\ &+ \frac{1}{N}\left(c_{15}\ln^5 N + c_{14}\ln^4 N + \cdots + c_{10}\right) + && \text{NEXT-TO-SOFT (NS)}\\ &+ \frac{1}{N^2}\left(c_{25}\ln^5 N + c_{24}\ln^4 N + \cdots + c_{20}\right) + \ldots && \text{NNS}\end{aligned}$$

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$\mathrm{N^3LO}$: WHAT DO WE KNOW

THE SOFT EXPANSION

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N^{3}LO: WHAT DO WE KNOW

THE HIGH-ENERGY EXPANSION

SMALL N POLES OR ABOUT $z=0$

rightmost singularity at $N=1$:

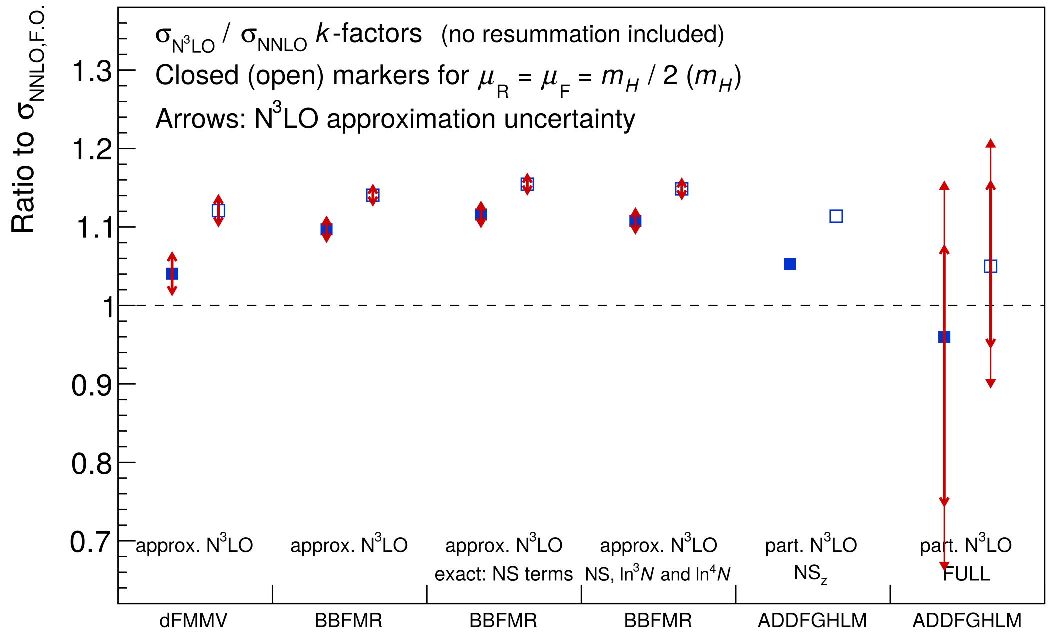
$$\begin{aligned}
\hat\sigma^{(3)}(N) &= \left(d_{13}\frac{1}{(N-1)^3}+d_{12}\frac{1}{(N-1)^2}+d_{11}\frac{1}{(N-1)}\right)+ && \text{LEADING LOG } x \text{ (LL}x\text{)}\\
. \quad &+\left(d_{06}\frac{1}{N^6}+\cdots+d_{01}\frac{1}{N}\right)+ && \text{NLL}x\\
. \quad &+\left(d_{-16}\frac{1}{(N+1)^6}+\cdots+d_{-11}\frac{1}{N+1}\right)++\dots && \text{NNLL}x\\
\hat\sigma^{(3)}(z) &= \left(d_{13}\frac{\ln^2 z}{z}+d_{12}\frac{\ln z}{z}+d_{11}\frac{1}{z}\right)+ && \text{LEADING LOG } x \text{ (LL}x\text{)}\\
. \quad &+\left(d_{06}\ln^6 z+\cdots+d_{01}\ln z\right)+ && \text{NLL}x\\
. \quad &+z\left(d_{-16}\ln^6 z+\cdots+d_{-11}\ln z\right)+\dots && \text{NNLL}x
\end{aligned}$$

- AS ABOVE, z-SPACE $\Leftrightarrow$ N-SPACE, ORDER BY ORDER (LOG & POWER), UP TO SUBLEADING TERMS
- ONLY d_{13} KNOWN EXACTLY FROM BFKL+SMALL x RESUMMATION
- CLASS OF DOMINANT (?) CONTRIBUTIONS TO d_{12}, d_{11} KNOWN FROM NL BFKL
- ALL SUBLEADING SINGS UNKNOWN

APPROXIMATE N^{3}LO

- DFMMV: N-SPACE APPROXIMATE NS; NOW NS KNOWN EXACTLY: SMALL EXPECTED CHANGE
- BBFMR: ALL KNOWN SINGULARITIES (SOFT & HIGH ENERGY):
  NS AND BEYOND APPROXIMATELY DETERMINED
  $\to$ IMPROVED WITH EXACT NS $\to$ IMPROVED WITH EXACT $\ln^4 N$, $\ln^3 N$ (SMALL CHANGES)
- ADDFGHLM: ALL KNOWN z SPACE TERMS (FULL) $\to$ PURE z SPACE NS ALSO SHOWN

N^{3}LO/NNLO k-FACTOR



note k factor computed wr to NNLO at respective scale
UNCERTAINTIES (ARROWS)

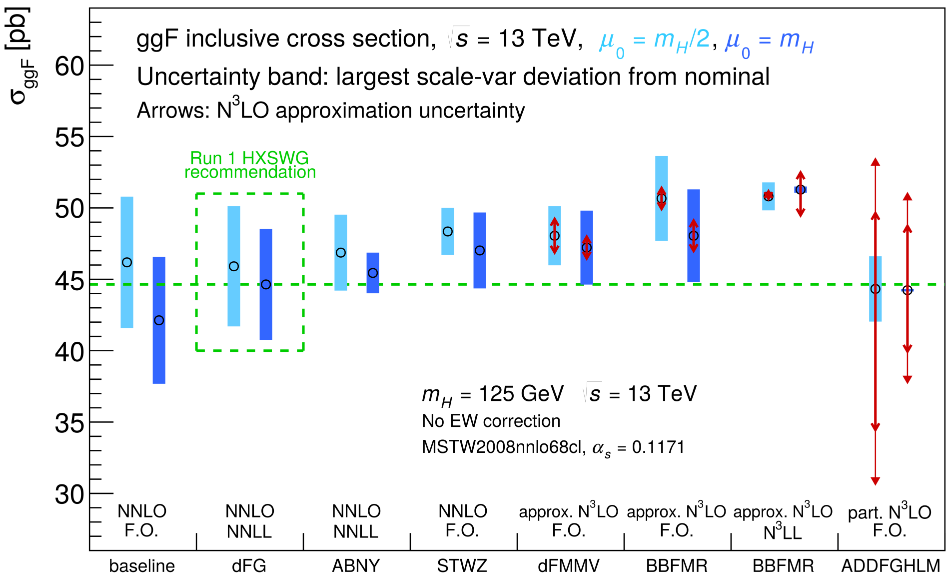
- DFMMV: S-NS BAND
- BBFMR: ESTIMATE OF SUBLEADING SINGULARITIES VALIDATED BY KNOWN ORDERS
- ADDFGHLM: ENVELOPE OF SCALE VAR. OF NS-FULL DIFFERENCE AT m_H, $\times$ 1.5 OR 1

SIZE OF UNCERTAINTY HOTLY DEBATED!!

WHAT IS THE CROSS-SECTION?

- STWZ: DIFFERS FROM NNLO BY CONST. EXPONENTIATION
- ABNY: DIFFERS FROM NNLO+NNLL (REF) BY CONST. EXPONENTIATION & DIFFERENT RESUMMATION
- DFMMV: APPROXIMATE N^3LO BASED ON APPROXIMATE N-SPACE NS; ARROWS: S-NS DIFFERENCE
- BBFMR: APPROXIMATE N^3LO BASED ON ANALYTIC APPROX (INCLUDING NS+BFKL); ARROWS: ESTIMATED MISSING SINGS+ FACT SCALE VARIATION FROM MISSING QUARK CHANNELS; $\to$ N^3LO+N^3LL BEST PREDICTION
- ADDFGHLM: ALL KNOWN z SPACE TERMS; ARROWS: ENVELOPE OF SCALE VARIATION OF NS-FULL DIFFERENCE

THE GLUON FUSION CROSS-SECTION



all numbers given with benchmark settings (K-factor to dFG very stable)
band is seven-point fact. and ren. scale variation

Q: WHEN (AND WHY) IS SOFT RESUMMATION USEFUL?

- PARTONIC CROSS SECTIONS $\hat\sigma(z,\alpha_s)$ ARE DISTRIBUTIONS, $0 \le z = \frac{M_h^2}{s} \le 1$, CONVOLUTED WITH LUMINOSIITY TO GIVE HADRONIC CROSS-SECTION
- THEIR MELLIN TRANSFORMS $\hat\sigma(N,\alpha_s)$ ARE ORDINARY FUNCTIONS
- FOR GIVEN m_H^2 AND s ONLY ONE “SADDLE” N VALUE CONTRIBUTES; POSITION OF SADDLE DETERMINED ESSENTIALLY BY PDF
  $\Rightarrow$ PDF TELLS YOU WHICH FRACTION OF HADRON MOMENTUM GOES INTO PARTONIC PROCESS

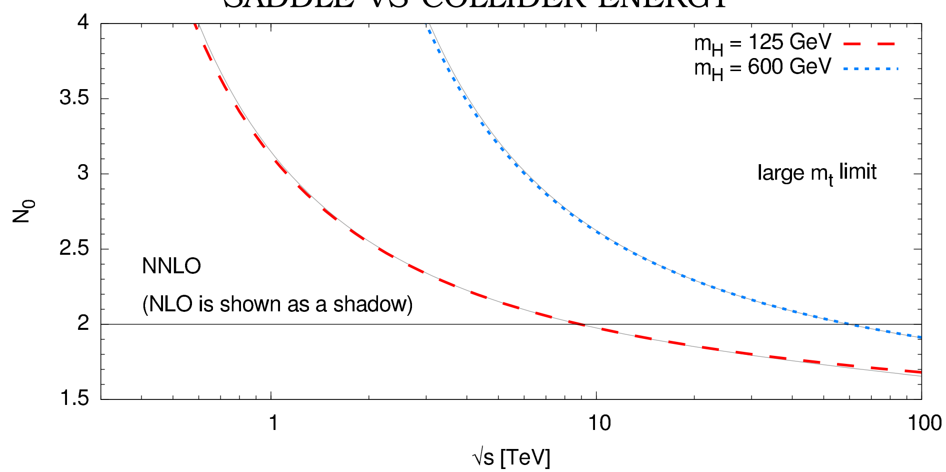
A: WHEN SADDLE N IS LARGE ENOUGH

- SOFT RESUMMATION INCLUDES TO ALL ORDERS $\alpha_s \ln(1-z)$, $z = \frac{M_X^2}{s} \Leftrightarrow \alpha_s \ln N$
- SOFT $\leftrightarrow z \to 1 \leftrightarrow N \to \infty$.

HIGGS IN GLUON FUSION

- RESUMMATION CURRENTLY INCLUDED IN HXSWG RECOMMENATION: NNLO+NNLL
- BUT $N_{\mathrm{saddle}} \sim 2$, $\Rightarrow$ RESUMMATION SUMS UP $\beta_0 \alpha_s \ln 2 \approx 0.04 \ll 1$
- WHAT IS “LARGE ENOUGH”??

SADDLE VS COLLIDER ENERGY



(Bonvini, SF, Ridolfi, 2012)

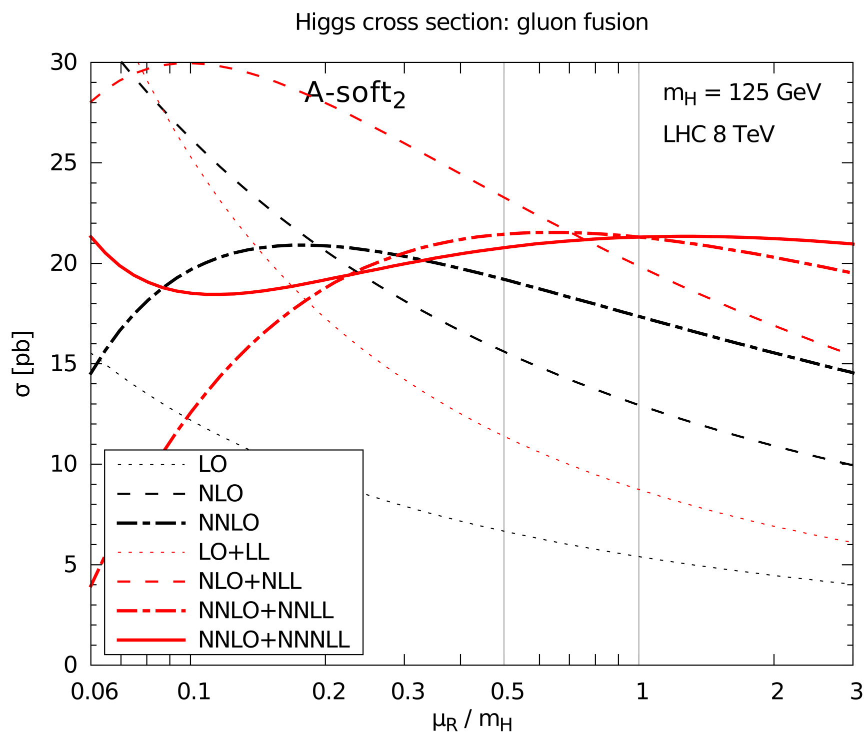
Higgs in gluon fusion:
SHOULD RESUMMATION HELP?

NO?

$$
\begin{aligned}
M\left[\eta_{gg}^{(3)}\right](N) \simeq\ & 36\log^6 N && (\to 0.0013\%)\\
&+170.679\ldots\log^5 N && (\to 0.0226\%)\\
&+744.849\ldots\log^4 N && (\to 0.2570\%)\\
&+1405.185\ldots\log^3 N && (\to 1.0707\%)\\
&+2676.129\ldots\log^2 N && (\to 4.0200\%)\\
&+1897.141\ldots\log N && (\to 5.1293\%)\\
&+1783.692\ldots && (\to 8.0336\%)\\
&+108\frac{\log^5 N}{N} && (\to 0.0105\%)\\
&+615.696\ldots\frac{\log^4 N}{N} && (\to 0.1418\%)\\
&+2036.407\ldots\frac{\log^3 N}{N} && (\to 0.9718\%)\\
&+3305.246\ldots\frac{\log^2 N}{N} && (\to 2.9487\%)\\
&+3459.105\ldots\frac{\log N}{N} && (\to 5.2933\%)\\
&+703.037\ldots\frac{1}{N} && (\to 1.7137\%).
\end{aligned}
$$

(Anastasiou et al, 2015)

YES!



Bonvini and Marzani, 2014

- "LEADING" LOGS DO NOT PROVIDE THE DOMINANT CONTRIBUTION
- EXPANSION IN POWERS OF $1/N$ CONVERGES VERY SLOWLY
- RESUMMED EXPANSION CONVERGES FASTER THAN UNRESUMMED ONE
- SCALE DEPENDENCE ALWAYS SMALLER AT RESUMMED LEVEL

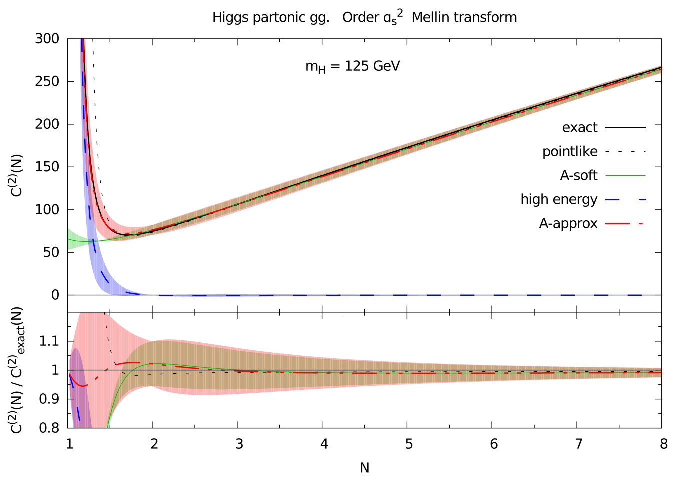
ANALYTIC STRUCTURE IN N SPACE

- MELLIN-SPACE PARTONIC CROSS SECTIONS ARE ANALYTIC FUNCTIONS OF N
- AS $N \to \infty$ THEY BEHAVE AS POWERS OF $\ln N$

- FINITE-N SINGULARITIES ARE ISOLATED POLES ON THE REAL AXIS FOR INTEGER $N \leq 1$

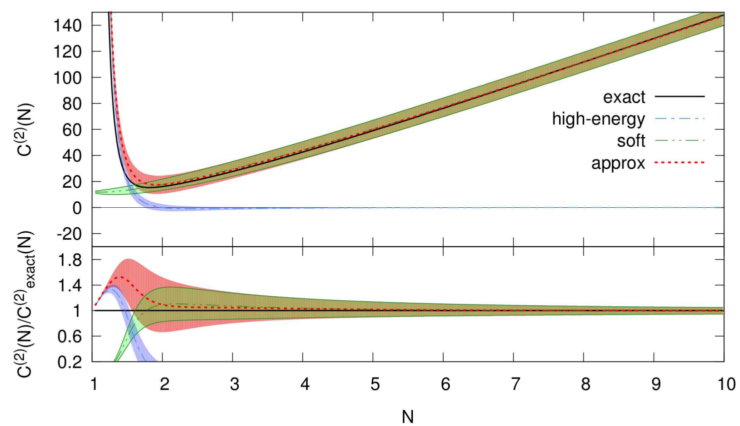
- RECONSTRUCT FROM SINGULARITIES, WITHOUT INTRODUCING SPURIOUS CUTS EXAMPLE: LARGE-N $\ln N \Rightarrow \psi(N)$.

HIGGS IN GLUON FUSION



(Ball et al., 2013)

TOP PAIRS



(Muselli et al., 2015)

WHAT DO WE KNOW?

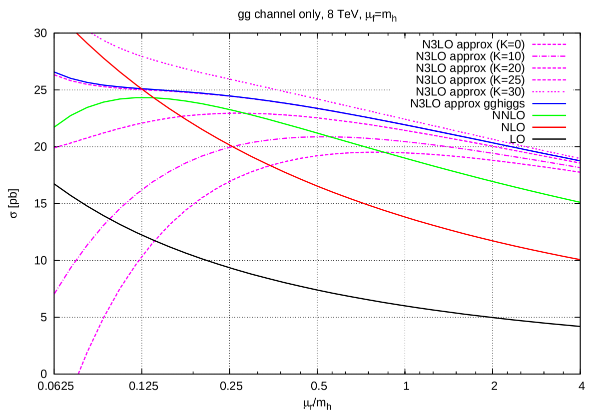
- EIKONAL OR SOFT EXPANSION: AS $N \to \infty$ (I.E. $z \to 1$)
  $\hat{\sigma}^{(3)}(N) = c_{06} \ln^6 N + c_{05} \ln^5 N + \cdots + c_{02} \ln^2 N + c_{01} \ln N + c_{00} + \frac{1}{N}\left(c_{15} \ln^5 N + c_{14} \ln^4 N + \cdots + c_{10}\right) + \frac{1}{N^2}\left(c_{24} \ln^4 N + \cdots + c_{20}\right) + \ldots$
  CURRENT KNOWLEDGE: EIKONAL, UP TO NNLL FOR MANY DIFFERENTIAL DISTRIBUTIONS

- BFKL OR HIGH-ENERGY EXPANSION: AS N DECREASES (I.E. $z \to 0$)
  RIGHTMOST SINGULARITY AT $N = 1$:
  $\hat{\sigma}^{(3)}(N) = \left(d_{13} \frac{1}{(N-1)^3} + d_{12} \frac{1}{(N-1)^2} + d_{11} \frac{1}{(N-1)}\right) + \left(d_{06} \frac{1}{N^6} + \cdots + d_{01} \frac{1}{N}\right) + \ldots$
  CURRENT KNOWLEDGE: LL, FOR SEVERAL TOTAL CROSS SECTIONS; ONLY GGHIGGS RAPDITY DISTRIBUTION

APPROXIMATE HIGHER ORDERS?

HIGGS GLUON FUSION: SCALE DEP

APPROX VS. GENERAL



(Bühler, Lazopoulos, 2013)

- AT NNLO METHOD WORKS WELL
- FOR GGHIGGS, FROM SCALE VARIATION RESULT SEEMS IN GOOD AGREEMENT WITH EXACT (AWAITING FOR FULL COMPARISON)
- N^4LO COMPUTATIONS?
- IMPROVING BOTH THE SOFT AND THE BFKL EXPANSION?