# Minimal Flavour Violation Scenarios

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– Coseners Forum *Heavy Flavour Physics* –
21.-22. June 2007

# Outline

# Introduction

Origin of Quark and Lepton Flavour in the SM remains a mystery!

- Only parametrization in terms of Yukawa matrices, yielding fermion masses and mixing angles after EWSB.

- In case of new physics at the TeV scale (→ hierarchy problem), and assuming generic new flavour couplings:

  *Why NP hasn't been observed in rare flavour decays?*

- Postulate additional constraints on NP flavour sector.
- Popular concept: Minimal Flavour Violation (MFV):

*New flavour couplings related to SM mixing angles and masses.*

[D'Ambrosio et al. '02, Ciuchini et al. '98, Buras et al. '00]

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## MFV does . . .

- provide additional suppression factors for NP flavour transitions, based on a symmetry principle.
- imply (SM-like) correlations between different flavour observables.
- give a systematic book-keeping device to parametrize flavour couplings in specific new-physics models.
  (reduction of free parameters in NP flavour sector)
- allow to formulate flavour violation within effective theory approach.

## MFV does not . . .

- represent a *theory* of flavour violation.
  (overall couplings in front of MFV structures still undetermined)
- *explain* the size of fermion masses and mixing angles.
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# Effective-Theory Framework for MFV

- MFV in the quark sector:
  - 1-Higgs-Doublet Models.
  - [Large tan $\beta$ effects in 2-Higgs Models.]

[D'Ambrosio et al., hep-ph/0207036]

- MFV in the lepton sector:
  - minimal field content (+ dim-5 LN-violating operator)
  - [extended field content (right-handed neutrino)]

[Cirigliano et al., hep-ph/0507001, hep-ph/0601111]

  - [from R-parity violating tri-linear coupling]

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# Quark flavour in the SM

- Flavour symmetry group that commutes with SM gauge group:

$$F = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$$

- Quark multiplets for three families transform as

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix} \sim (3,1,1)\,, \quad U_R \sim (1,3,1)\,, \quad D_R \sim (1,1,3)\,.$$

- SM Yukawa matrices $Y_U$, $Y_D$ break flavour symmetry $F$:
- Diagonalized by bi-unitary transformations: $\Rightarrow m_q^{\rm diag} = y_q\, v$
- $V_{\rm CKM} = V_{u_L}^\dagger V_{d_L}$ parameterizing the mismatch between the left-handed up- and down-quark rotations.

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# Yukawa matrices as Spurions

- Consider $Y_U$, $Y_D$ as spurion fields, transforming as

$$Y_U \sim (3, \bar{3}, 1)\,, \qquad Y_D \sim (3, 1, \bar{3})\,.$$

under flavour group $F = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$.

- Yukawa terms formally invariant under flavour group.
- Background values for $Y_U$ and $Y_D$ finally break $F$.
- Concept can be generalized to higher-dimensional operators!

Analogy: Treatment of quark mass matrix in ChPT . . .

# New Physics and Minimal Flavour Violation

- Consider SM as Effective Theory at the electroweak scale.
- Allow for higher-dimensional operators ,
  induced by NP effects above the electroweak scale.
- Assume that Flavour- *and* CP-symmetry breaking
  is still governed by SM spurions $Y_U$ and $Y_D$, only.

> NP contributions to flavour transitions suppressed
> by same factors of $V_{\rm CKM}$ and $m_Q$ as in the SM.

# Example: MFV in the MSSM

- New flavour structures arise from "soft SUSY-breaking" terms.
  - Squark mass matrices:

$$\begin{aligned}
\tilde{m}^2_{Q_L} &= \tilde{m}^2 \left( a_1 + b_1 \, Y_U Y_U^\dagger + b_2 \, Y_D Y_D^\dagger + \ldots \right) , \\
\tilde{m}^2_{U_R} &= \tilde{m}^2 \left( a_2 + b_5 \, Y_U^\dagger Y_U \right) , \\
\tilde{m}^2_{D_R} &= \tilde{m}^2 \left( a_3 + b_6 \, Y_D^\dagger Y_D \right)
\end{aligned}$$

  - Squark tri-linear couplings:

$$\begin{aligned}
A_U &= A \left( a_4 + b_7 \, Y_D^\dagger Y_D \right) \, Y_U , \\
A_D &= A \left( a_5 + b_8 \, Y_U^\dagger Y_U \right) \, Y_D ,
\end{aligned}$$

- Integrating out SUSY particles yields MFV structures at EW scale.

# Wolfenstein expansion

- Off-diagonal CKM elements suppressed by Wolfenstein-$\lambda \sim 0.2$.
- Quark masses small compared to EW scale. (except for $m_t$)

⇒ The more spurion insertions, the more suppression.

Exception: $t \to b$ transitions ($V_{tb} \sim 1$, $m_t \sim v$)

- Only need to consider minimal number of spurion insertions:
  - no insertion necessary for charged left-handed decays,
  - ...
  - 4 insertions necessary for neutral right-handed decays.

- All FCNCs with external down-type quarks controlled by

$$(\Delta^q_{LL})_{i\neq j} = (Y_U Y_U^\dagger)_{ij} = y_t^2\,(V^*_{\rm CKM})_{3i}\,(V_{\rm CKM})_{3j} \qquad (y_t = \mathcal{O}(1))$$

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# Example: $b \to s\gamma$

- Leading operators in weak effective Hamiltonian:

$$O_7 = \bar{s}_L \, \sigma_{\mu\nu} \, b_R \, F^{\mu\nu} \, , \qquad O_7' = \bar{s}_R \, \sigma_{\mu\nu} \, b_L \, F^{\mu\nu} \, .$$

- Construct FCNC $D_L$-$D_R$ transitions, invariant under $F$:

| | | | |
|---|---|---|---|
| $\bar{Q}_L' \, D_R'$ | | | not invariant |
| $\bar{Q}_L' \, Y_D \, D_R'$ | $\to$ | $y_D^{\rm diag} \, \bar{D}_L \, D_R$ | flavour-diagonal |
| $\bar{Q}_L' \, Y_U \, Y_U^\dagger \, Y_D \, D_R'$ | $\to$ | $\bar{D}_L \, V_{\rm CKM}^\dagger \, (y_U^{\rm diag})^2 \, V_{\rm CKM} \, y_D^{\rm diag} \, D_R$ | $\surd$ |
| $\bar{D}_R' \, Y_D^\dagger \, Y_U \, Y_U^\dagger \, Q_L'$ | $\to$ | $\bar{D}_R \, y_D^{\rm diag} \, V_{\rm CKM}^\dagger \, (y_U^{\rm diag})^2 \, V_{\rm CKM} \, D_L$ | $\surd$ |

  - FCNC require (at least) two CKM elements.
  - right-handed quarks suppressed by quark mass.

- $O_7$ pre-factor: $\sim \frac{m_b}{v} \, y_t^2 \, V_{ts}^* \, V_{tb}$ $\qquad O_7'$ pre-factor: $\sim \frac{m_s}{v} \, y_t^2 \, V_{ts}^* \, V_{tb}$

**The same flavour factors as from penguin diagrams in the SM !**

- No significant NP impact on CP asymmetries:
  - e.g. $A_{\rm CP}^{\rm dir,mix}$ for $B \to J/\psi K_s$ and $B \to \phi K_s$ as in SM
- NP contributions to different FCNC decays are related:
  - e.g. ratio of $b \to s\gamma$ and $b \to d\gamma$ still determined by $|V_{ts}/V_{td}|$
  - $\Delta M_{B_s}/\Delta M_{B_d}$ still given by $|V_{ts}/V_{td}|$
- NP contributions to charged decays insignificant:
  - e.g. $|V_{ub}|$ from $b \to u\ell\nu$
- Corrections to flavour observables smaller than in generic case

  $\Rightarrow$ NP scales in the TeV range still allowed.

# Phenomenological consequences of MFV (1-Higgs)

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# Phenomenological Signatures for beyond MFV ?

- Violation of MFV relations in FCNC:
  - $b \to s\gamma$ vs. $b \to d\gamma$ — SM, MFV $\surd$
  - $\Delta M_{B_d}$ vs. $\Delta M_{B_s}$ — SM, MFV $\surd$
  - $B \to J/\psi K_s$ vs. $B \to \phi K_s$ — ($\to$ nMFV)
  - ...
- Sizeable right-handed contributions to charged decays:
  - test of left-handedness in $b \to c\ell\nu$ — SM, MFV $\surd$
    from moment analysis of inclusive decay spectra
    [Mannel et al, hep-ph/0701054]
  - inconsistencies in $|V_{ub}/V_{cb}|$ measurements
    (exclusive vs. inclusive vs. $\sin 2\beta$) — (($\to$ nMFV))
  - ...

# Variations on MFV / Beyond MFV

- Extended Scalar Sector:
  - 2-Higgs models: $y_b \cdot \tan\beta \sim \mathcal{O}(1)$ [D'Ambrosio et al.]
  - MSSM with large $\mu$: $y_b \cdot |\mu| \sim \mathcal{O}(\tilde{m})$ [Buras et al.]
  - ...
- Extended Fermion Sector:
  - Littlest Higgs Models with $T$-parity [Buras et al., Hubisz et al.]
  - Froggatt-Nielsen scenarios
  - ...
- Extended Gauge Sector:
  - MFV and $SU(5)$ [Grinstein et al., hep-ph/0608123]
- Dominant couplings to 3rd generation [Agashe et al., hep-ph/0509117]

General framework would allow for several new spurion fields, with different transformations under flavour group, but still small expansion parameters in off-diagonal elements. [TF/Mannel 2006]

# MFV in the Lepton Sector (minimal field content)

# Lepton flavour in the (minimally extended) SM

- Flavour symmetry group: $F_{\text{lept.}} = SU(3)_{L_L} \times SU(3)_{E_R}$
- Lepton multiplets: $L_L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L \sim (3,1)\,, \quad E_R \sim (1,3)\,.$
- Charged-lepton Yukawas $Y_\ell \sim (3,\bar{3})$ break flavour symmetry.
- No neutrino Yukawas $\Rightarrow$ No lepton-mixing in the SM

## Including Neutrino Masses

- The combination $N = H^T \tau_2 L_L$ is a SM gauge singlet.

$\Rightarrow$ Gauge-invariant dim-5 operator:

$$\mathcal{L}_{\rm Maj} = \frac{1}{2\Lambda_{LN}} \left( N^T g\, N \right)$$

- Does not conserve lepton number.
- Generates neutrino Majorana mass terms of order $v^2/\Lambda_{\rm LN} \lll 1$

- New flavour matrix: (in basis where $Y_\ell$ is diagonal)

$$g = \frac{\Lambda_{\rm LN}}{v^2}\, U^*_{\rm PMNS}\, m_\nu^{\rm diag}\, U^\dagger_{\rm PMNS}$$

MLFV: Treat new flavour matrix as spurion field: $g \sim (\bar{6}, 1)$

## Uniqueness of MLFV ?

- Mechanism for neutrino mass generation unknown !
- Implementation of MFV in Lepton Sector not unambiguous !

see the discussion in [Davidson/Palorini, hep-ph/0607329]

# Lepton-flavour transitions induced by spurion $g$

- Consider all gauge-invariant operators of dim-6 (and higher)

  [Cirigliano et al., hep-ph/0507001]

- Bi-linear building blocks:

$$(\bar{L}\, g^\dagger g\, L)\,, \quad (\bar{L}\, g^\dagger g\, Y_\ell^\dagger\, R)\,, \quad (\bar{R}\, Y_\ell\, g^\dagger g\, Y_\ell^\dagger\, R)$$

$\Rightarrow$ e.g. radiative decays $\ell \to \ell'\gamma$ controlled by $y_\ell$ and

$$(g^\dagger g)_\tau^e \simeq \frac{\Lambda_{\rm LN}^2}{\sqrt{2}v^4}\left(-s_{12}c_{12}\,\Delta m^2_{\rm sol} \pm s_{13}e^{i\delta}\Delta m^2_{\rm atm}\right),$$

$$(g^\dagger g)_\tau^\mu \simeq \frac{\Lambda_{\rm LN}^2}{2v^4}\left(-c_{12}^2\,\Delta m^2_{\rm sol} \pm \Delta m^2_{\rm atm}\right),$$

$$(g^\dagger g)_\mu^e \simeq \frac{\Lambda_{\rm LN}^2}{\sqrt{2}v^4}\left(s_{12}c_{12}\,\Delta m^2_{\rm sol} \pm s_{13}e^{i\delta}\Delta m^2_{\rm atm}\right),$$

(insensitive to absolute neutrino mass scale and Majorana phases)

# Lepton-flavour transitions induced by spurion $g$

(continued)

- 4-lepton operators:

$$(\bar{L}\, g^\dagger g\, L)(\bar{L}L)\,, \quad (\bar{L}\, g^\dagger g\, L)(\bar{R}R)\,, \qquad \text{etc.}$$

  include the same flavour structures as radiative decays.

- independent flavour structure from:

$$(g)_{ij}(g^*)^{kl}\,(\bar{L}_k L^i)(\bar{L}_l L^j)$$

(corresponds to 27plet in $\bar{6} \otimes 6 = 1 + 8 + 27$)

- Sensitive to absolute neutrino mass scale and Majorana phases !

[Cirigliano/Grinstein, hep-ph/0601111]

# MLFV phenomenology

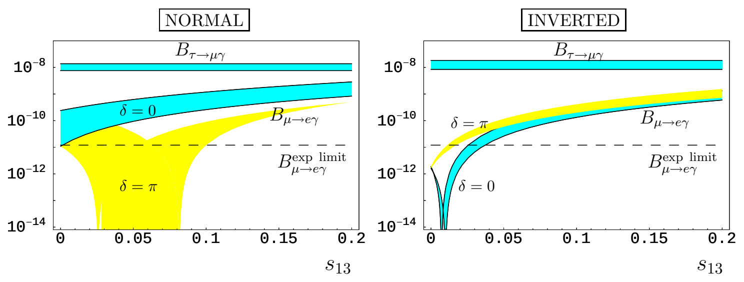
- dim-6 operators suppressed by $1/\Lambda_{\rm NP}^2$
- dominating flavour coefficients $|(g^\dagger g)_{ij}| \lesssim 1$
- Sizeable LFV effects possible for $\Lambda_{\rm NP}$ in the TeV range (i.e. $\Lambda_{\rm NP} \ll \Lambda_{\rm LN}$)

- radiative decays $\ell \to \ell' \gamma$ $\mapsto$
- decays into charged leptons $\tau \to 3\ell$ and $\mu \to 3\ell$ $\mapsto$
- hadronic decays $(\tau \to \mu \pi^0 \ldots)$, $\mu - e$ conversion in nuclei. [hep-ph/0601111]
- leptogenesis [hep-ph/0607068, hep-ph/0609067, hep-ph/0612262]

# Radiative decays

[Cirigliano et al., hep-ph/0507001]

- $s_{13} \leq 0.25$ and $\Delta m^2_{\rm atm} \gg \Delta m^2_{\rm sol}$ ,
- fix remaining parameters such that $B_{\tau\to\mu\gamma} = \frac{\Gamma(\tau\to\mu\gamma)}{\Gamma(\tau\to\mu\nu_\tau\bar\nu_\mu)} \sim 10^{-8}$

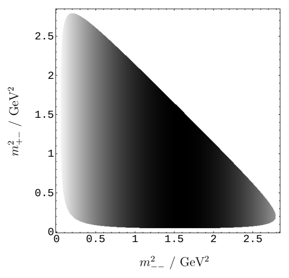



- $\mu \to e\gamma$ most sensitive probe of MLFV scenario.
- With existing bounds on $\mu \to e\gamma$ and for $s_{13} \gtrsim 0.1$, observation of $\tau \to \mu\gamma$ at near future experiments unlikely within MLFV.

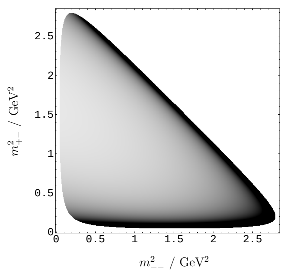
# Leptonic decays $\ell \to 3\ell'$

- two competing flavour structures: $(gg^\dagger)_{\ell\ell'}$ vs. $g_{\ell\ell'} g^*_{\ell'\ell'}$
  $\Rightarrow$ Study ratios $\Gamma[\ell \to 3\ell']/\Gamma[\ell \to \ell'\gamma]$. [hep-ph/0601111]
- 4-lepton operators vs. radiative decays $\Rightarrow$ study decay distributions.
  Example: Dalitz plots for $\tau^- \to \mu^-\mu^+\mu^-$ contributions:

$(\bar\mu_L \gamma^\mu \tau_L)(\bar\mu_L \gamma_\mu \mu_L)$




$\tau \to \mu\gamma^* \to 3\mu$




[Dassinger/TF/Mannel/Turczyk, preprint HEP-2007-06]

# Lessons

## Exploring NP flavour sector:

(1.) Establish NP in discovery channels

(2.) Test flavour precision data against MFV hypothesis:

either: disprove MFV (violation of correlations)
$\hookrightarrow$ Allow for new flavour structures (spurion fields).
$\hookrightarrow$ Flavour puzzle even more pronounced.

or: determine parameters within your favourite NP model for (1.)
$\hookrightarrow$ use MFV for book-keeping.