K-K-Scattering at Maximal Isospin from $N_f = 2 + 1 + 1$ Twisted Mass Lattice QCD

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Why Low Energy Scattering Lengths?

- Many particles in standard model resonances created in scattering processes
- κ-Meson from π-K-scattering
- Complications: different Isospin channels, several quark flavours, ...
  - Tue, 5:10 pm: L. Liu, π-π-scattering at $I = 0$
  - Tue, 2:00 pm: M. Werner, ρ-resonance from π-π at $I = 1$
- Simpler: K-K-scattering to learn about strange quarks
- Take analysis procedure from π-π-scattering at $I = 2$[1]

[1] ETMC, J. High Energ. Phys. (2015) 2015:109

Scattering Parameters

- Definition of scattering length a_0 (phaseshift δ_0, scattering momentum q):

$$\lim_{q \to 0} q \cot \delta_0(q) = -\frac{1}{a_0}$$

- Two particles in finite box ⇒ wave functions overlap ⇒ energy of total system gets shifted

$$\delta E_{KK}^{I=1} = \left(E_{KK}^{I=1} - 2M_K \right)$$

- Expansion of δE in terms of a_0 and L^{-1} by Lüscher[2]

$$\delta E_{KK}^{I=1} = -\frac{4\pi a_0}{M_K L^3} \left[1 + c_1 \frac{a_0}{L} + c_2 \left(\frac{a_0}{L} \right)^2 \right] + O(L^{-6})$$

[2] M.Lüscher, Comm. Math. Phys. Volume 105, Number 2 (1986), 153-188.

Correlation Functions

- Kaon interpolating fields with $\vec{p} = \vec{0}$

$$K(t) = \sum_{\vec{x}} \bar{\psi}_s(\vec{x}, t)\gamma_5\psi_u(\vec{x}, t)$$

$$\mathcal{O}_{KK}(t) = K(t)K(t)$$

- 2-pt correlation function $C_K(t) = \langle K(t)K^\dagger(0)\rangle \to M_K$

- 4-pt correlation function $C_{KK}(t) = \langle \mathcal{O}_{KK}(t)\mathcal{O}^\dagger_{KK}(0)\rangle \to E_{KK}$

Extracting Energies

- Get energies from fits to correlation functions
- 2 challenges for $C_{KK}(t)$:

1. at early times: excited states shift plateaus to late times
   $\Rightarrow$ Stochastic Laplacian Heaviside Smearing[3]
2. at late times: temporally constant thermal states spoil plateaus
   $\Rightarrow$ Shifted ratio[4] $R(\tilde{t})$

$$R(t + a/2) = \frac{C_{KK}(t) - C_{KK}(t+a)}{C_K^2(t) - C_K^2(t+a)}$$

[3] C. Morningstar, et al.: Phys. Rev. D 83 (2011) 114505
[4] Feng, Jansen, Renner: Phys. Lett. B684 (2010) 268-274

Lattice Setup

- Gauge Configurations: ETMC $N_f = 2 + 1 + 1$ WtmLQCD at maximal twist
- Sea quarks: Wilson twisted mass dynamical fermions
- Valence quarks: Osterwalder-Seiler fermions for flavour s:
  - Avoid flavour mixing in heavy doublet
  - Can correct for small mismatches in sea quark action

Ensemble parameters

- 3 lattice spacings with each 3 strange quark masses
- Up to 5 light quark masses per lattice spacing
  $\Rightarrow M_\pi \in (280, 500)$MeV
- 2 lattice volumes on 2 of 3 lattice spacings
  $\Rightarrow L \in (2, 4)$fm
- About 300 Configurations per ensemble

$\Rightarrow$ Sound basis for chiral and continuum extrapolation

Contact with Physical World

- Have 3 strange quark mass parameters $a\mu_s$ available
- Relation between continuum and lattice:

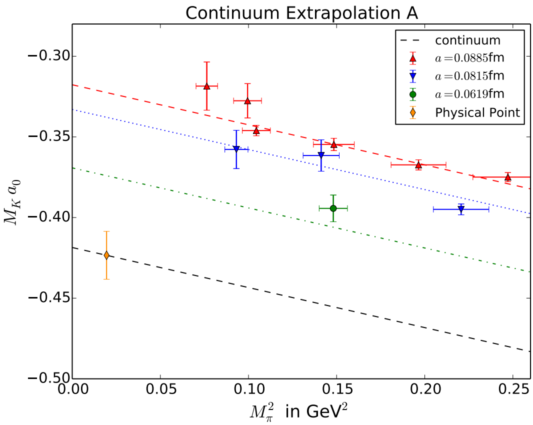
$$m_s = \frac{(a\mu_s)}{Z_P a}, \quad M_K^{\text{phys}} = \frac{(aM_K)^{\text{lat}}}{a}$$

- Renormalization Constant[5] Z_P, lattice spacing a
- Consistency check by fixing m_s in two ways
  - A: $r_0^2(M_K^2 - 0.5M_\pi^2)$
  - B[5]: (renormalised) strange quark mass μ_s.
- strange quark mass set to its physical value using M_K (light quark mass using M_π)

[5] ETMC: Nucl.Phys. B887 (2014) 19-68

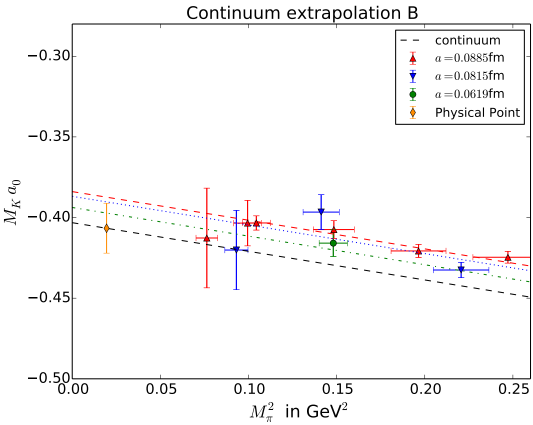
A: Fixing m_s with $r_0^2(M_K^2 - 0.5M_\pi^2)_{\text{phys}}$

Fit formula: $M_K a_0 = p_0 M_\pi^2 + p_1 a^2 + p_2$



B: Fixing m_s with μ_s

Fit formula: $M_K a_0 = p_0 M_\pi^2 + p_1 a^2 + p_2$



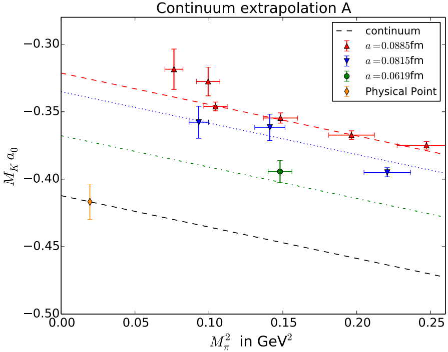
Fit-Results A & B

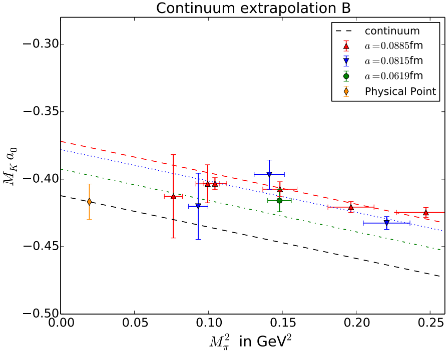
	Method A	Method B
$(M_K a_0)_{\text{ext}}$	$-0.42(1)$	$-0.41(2)$
χ^2/dof	13.16/7	6.27/7
p-val	0.07	0.5

- Fit-Results at physical point agree well
- Observe sizeable discretisation effects

$\Rightarrow$ Try combined fit of both Methods

Simultaneous Fit of A & B





Results

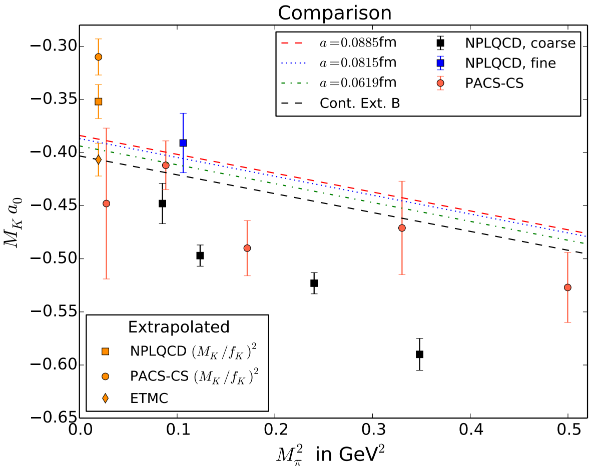
- Combined Fitresults

$(M_K a_0)_{\text{phys}}$	χ^2/dof	p-val
$-0.42(1)$	$25.72/16$	0.06

- Comparison

Method	$(M_K a_0)_{\text{phys}}$
A: $M_K^2 - 0.5M_\pi^2$	$-0.42(1)$
B: $M_{K,\text{phys}}^2$	$-0.41(2)$
p-value weighted	$-0.41(2)(^{+0}_{-114})$
Combined	$-0.42(1)$

Comparison with NPLQCD[6] and PACS-CS[7]



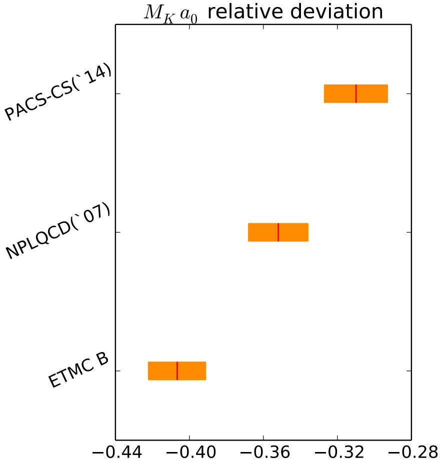
[6] S. Beane, et al. Phys. Rev. D 77, 094507 (2008)
[7] K. Sasaki, et al. Phys. Rev. D 89, 054502 (2014)

Summary & Challenges

- Calculated scattering length of K-K-scattering at maximum isospin
- Investigated 2 ways to fix m_s
- Used 3 lattice spacings and 10 values of M_π
- Arrive at: $(M_K a_0)_{\text{phys}} = -0.42(1)$
- Raw data agree with PACS-CS and NPLQCD within errors
- Well equipped to address π-K-scattering in isospin channels $I = 1/2, 3/2$

Thank you

Comparison with NPLQCD[8] and PACS-CS[9]

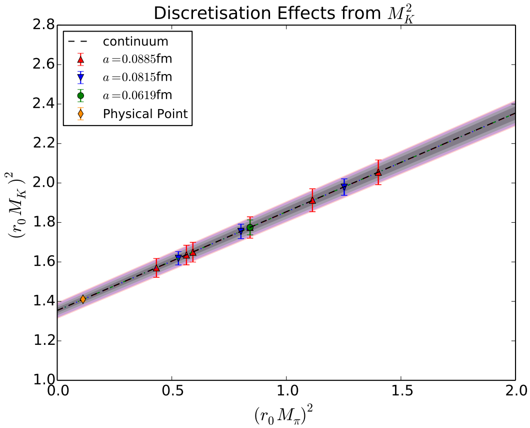


- No direct agreement
- PACS-CS and NPLQCD use χ-PT in (M_K/f_K)
- Dependance of a not directly taken into account

[8] S. Beane, et al. Phys. Rev. D 77, 094507 (2008)
[9] K. Sasaki, et al. Phys. Rev. D 89, 054502 (2014)

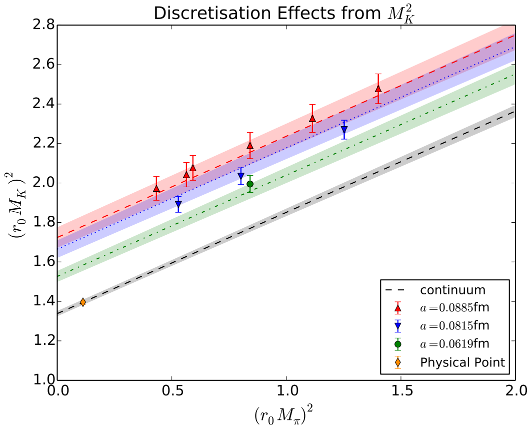
Discretisation Effects from M_K in Method A

Fit formula: $(r_0 M_K)^2 = p_0 (r_0 M_\pi)^2 + p_1 (\frac{a}{r_0})^2 + p_2$



Discretisation Effects from M_K in Method B

Fit formula: $(r_0 M_K)^2 = p_0 (r_0 M_\pi)^2 + p_1 (\frac{a}{r_0})^2 + p_2$



Stochastic Laplacian Heaviside Smearing

- Cut off eigenspectrum of lattice Laplace Operator to reduce excited state contributions
- Noise reduction in correlation functions by diluting quark propagators
- Store and Reuse all-to-all quark propagators once they are computed
- Useful for scenarios with many operators
- Needs: careful tuning, serious amount of computing time, already done[1]

[1] ETMC, J. High Energ. Phys. (2015) 2015:109

Stochastic Laplacian Heaviside Smearing

- Laplacian-Heaviside smearing of quark fields for excited states reduction

$$\tilde{\psi}_{a,\alpha}(x) = \mathcal{S}_{ab}(x,y)\psi(y)_{b,\alpha}$$

- $\mathcal{S}$ given by truncating eigenvalues of smeared spatial Laplace operator Δ on the lattice

$$\mathcal{S} = \Theta(\sigma^2 + \Delta)$$
$$\Delta = V_\Delta \Theta(\sigma^2 + \Lambda_\Delta) V_\Delta^\dagger$$
$$\Rightarrow \mathcal{S} \approx V_\mathcal{S} V_\mathcal{S}^\dagger$$

- Use $\mathcal{S}$ together with diluted randomvectors $P^{(b)}\rho$ to estimate quark propagators

Ensemble Parameters

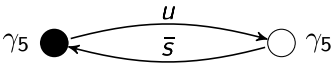
ensemble	β	$a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	$(L/a)^3 \times T/a$	N_{conf}
$A30.32$	1.90	0.0030	0.150	0.190	$32^3 \times 64$	280
$A40.24$	1.90	0.0040	0.150	0.190	$24^3 \times 48$	404
$A40.32$	1.90	0.0040	0.150	0.190	$32^3 \times 64$	250
$A60.24$	1.90	0.0060	0.150	0.190	$24^3 \times 48$	314
$A80.24$	1.90	0.0080	0.150	0.190	$24^3 \times 48$	306
$A100.24$	1.90	0.0100	0.150	0.190	$24^3 \times 48$	312
$B35.32$	1.95	0.0035	0.135	0.170	$32^3 \times 64$	250
$B55.32$	1.95	0.0055	0.135	0.170	$32^3 \times 64$	311
$B85.24$	1.95	0.0085	0.135	0.170	$32^3 \times 64$	296
$D45.32sc$	2.10	0.0045	0.0937	0.1077	$32^3 \times 64$	301

Bare Strange Quark Masses

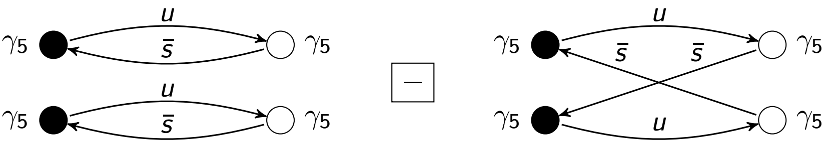
β	Z_P	$a\mu_s$
1.90	0.529(7)	0.01850 0.02250 0.02464
1.95	0.509(4)	0.01600 0.01860 0.02100
2.10	0.516(2)	0.01300 0.01500 0.01800

Diagrams

- 2-pt Correlation function



- 4-pt Correlation function



Discretisation Effects from Match to Meson Masses

Ensemble	$(r_0 M_\pi)^2$	$(r_0 M_K)^2$
A30.32	0.4328(0)	1.5698(477)
A40.32	0.5639(0)	1.6349(496)
A40.24	0.5922(0)	1.6493(500)
A60.24	0.8414(0)	1.7746(541)
A80.24	1.1138(0)	1.9126(580)
A100.24	1.4013(0)	2.0542(626)
B25.32t	0.3898(0)	1.5481(326)
B35.32	0.5287(0)	1.6186(340)
B55.32	0.8017(0)	1.7549(368)
B85.24	1.2525(0)	1.9793(417)
D45.32	0.8415(0)	1.7749(380)

Data Values from Match to Meson Masses

Ensemble	$(r_0 M_\pi)^2$	$M_K a_0$
A30.32	0.4328(14)	−0.3185(149)
A40.32	0.5639(22)	−0.3276(106)
A40.24	0.5922(43)	−0.3461(32)
A60.24	0.8414(44)	−0.3547(38)
A80.24	1.1138(46)	−0.3673(31)
A100.24	1.4013(44)	−0.3749(28)
B25.32t	0.3898(37)	−0.3176(209)
B35.32	0.5287(25)	−0.3578(119)
B55.32	0.8017(22)	−0.3615(98)
B85.24	1.2525(49)	−0.3949(34)
D45.32	0.8415(42)	−0.3943(83)

Data Values from Matching to $m_{s,\text{ren}}$

Ensemble	$(r_0 M_\pi)^2$	$M_K a_0$
A30.32	0.4328(14)	−0.4172(323)
A40.32	0.5639(22)	−0.4070(147)
A40.24	0.5922(43)	−0.4060(46)
A60.24	0.8414(44)	−0.4099(57)
A80.24	1.1138(46)	−0.4232(42)
A100.24	1.4013(44)	−0.4270(38)
B25.32t	0.3898(37)	−0.2975(381)
B35.32	0.5287(25)	−0.4237(257)
B55.32	0.8017(22)	−0.3992(110)
B85.24	1.2525(49)	−0.4350(49)
D45.32	0.8415(42)	−0.4174(86)

Discretisation Effects from Match to $m_{s,\mathrm{ren}}$

Ensemble	$(r_0 M_\pi)^2$	$(r_0 M_K)^2$
A30.32	0.4328(14)	1.9919(605)
A40.32	0.5639(22)	2.0617(626)
A40.24	0.5922(43)	2.0967(636)
A60.24	0.8414(44)	2.2097(673)
A80.24	1.1138(46)	2.3468(713)
A100.24	1.4013(44)	2.4978(760)
B25.32t	0.3898(37)	1.8746(395)
B35.32	0.5287(25)	1.9099(401)
B55.32	0.8017(22)	2.0526(431)
B85.24	1.2525(49)	2.2889(482)
D45.32	0.8415(42)	2.0125(430)

Thermal state cancellation[10]

- Define shifted Ratio $R(\tilde{t})$ for cancelling thermal states

$$R(t + a/2) = \frac{C_{KK}(t) - C_{KK}(t+a)}{C_K^2(t) - C_K^2(t+a)}$$

- δE now is fitparameter of

$$R(t + a/2) = A_R \left(\cosh\left(\delta E_{KK} t'\right) + \sinh\left(\delta E_{KK} t'\right) \coth\left(2M_K t'\right)\right)$$
$$t' = t + \frac{a}{2} - \frac{N_t}{2}$$

[10] Feng, Jansen, Renner: Phys. Lett. B684 (2010) 268-274

Dilution Schemes

- Each $P^{(b)}$ combines dilution in Time, Dirac space and LapH space
- Statistical errors of correlation functions
  - Random vectors $\propto \frac{1}{\sqrt{N_R}}$
  - Dilution vectors $\propto \frac{1}{N_D}$

  $\Rightarrow$ Find balance between N_R and N_D for best signal in dependence of number of inversions
- Inversions: typically between 1500 and 2500 per configuration

Discretisation Effects

	M1	M2	Physical
$(r_0 M_K)^2_{\text{ext.}}$	1.4103(91)	1.4053(104)	1.4058
χ^2/dof	0.00/8	5.3/8	
p-val	1.00	0.73	
p_0	0.500(16)	0.519(18)	
p_1	0	11(1)	
p_2	1.354(3)	1.347(11)	