Towards a non-perturbative calculation of the weak Hamiltonian Wilson coefficients

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Weak Effective Hamiltonian

Weak decays of hadrons: typical hadronic scale \( O(\Lambda_{QCD}) \) mediated by \( W \) boson

Example: \( c \to s u d \bar{d} \) only current-current operators (no penguins)

\[
\mathcal{H}_{\text{eff}} \propto G_F \sum_i C_i Q_i \quad \to \quad i = 1, 2 \text{ in our example}
\]

Long distance \( \langle Q_i \rangle \to \text{Lattice} \quad \text{Wilson Coefficients } C_i \to \text{PT} \)
Present calculations

$\Delta S = 1$ transitions:
- RBC/UKQCD calculations of $K \rightarrow \pi\pi$ ($I = 0$ and 2)
- Matrix elements up to 2 GeV from lattice [C.Kelly's talk]
- Running From 2 GeV up to $M_W$ perturbative
- Wilson Coefficients at $M_W$ not yet largest systematic errors

$\Delta B = 1$ transitions:
- Recent results B-decays [Fermilab+MILC '16]
- impact for more precise Wilson Coefficients

A lattice calculation can provide an all-order-in-$\alpha_s$ result
Perturbative results - I

By matching the full and effective theory at one loop in $\overline{\text{MS}}$:

\begin{align*}
C_1 &= -b_0 \alpha_s \log(M_W^2/\mu^2) \\
C_2 &= 1 + b_1 \alpha_s \log(M_W^2/\mu^2)
\end{align*}

$b_0, b_1$ positive coefficients

$\mu$ is the matching scale $\rightarrow$ large logs

$\downarrow$

Renormalization group improvement

[Buchalla, Buras, Lautenbacher '95]
Perturbative results - II

Initial conditions $C_1$ and $C_2$

$C_1(M_W) \approx 0.44\alpha_s(M_W)$

$C_2(M_W) = 1 + 0.15\alpha_s(M_W)$

Anomalous Dimension Matrix (ADM)

$U$ solution of RG equations

$\tilde{C}(\mu) = U(M_W, \mu)\tilde{C}(M_W)$

Resummation of large logs at scale $\mu$

Example: $\mu = 40$ GeV and $70$ GeV $< M_W < 90$ GeV:

$C_1$ varies by 40%, $C_2$ varies by less than 1% using 2-loop $\alpha_s$

Physical observable, e.g. $K \rightarrow \pi\pi$ amplitude (estimated) error from $C_1(M_W), C_2(M_W)$ around 3-5%.
Window problem

$\mu$ is the matching scale:

\[ aM_W \ll 1 \text{ for discretization effects} \]

\[ \mu \ll M_W \text{ for higher order operators} \]

\[ \mu \gg m, \mu L \gg 1 \text{ for infrared effects} \]

Present study is focused on unphysically small $m_W \approx 2 \text{ GeV}$
Renormalization scheme

With a momentum-subtraction scheme:

- perturbative calculations known to NLO
- off-shell external quark states with momentum $p$
  \[ \downarrow \]
  - gauge-dependent operators $O(p^2/M_W^2)$
  - exceptional scheme $\rightarrow$ chiral symmetry breaking effects

The limit $p^2/M_W^2 \rightarrow 0$:

- crucial to reduce some systematic uncertainties
- problematic for an exceptional scheme

**Two-step strategy**

1) matching at sufficiently small $\mu$ and 2) step-scale up to $M_W$
The calculation - I

Restriction to current-current diagrams (no penguins)

Green’s function \( G(Q_i) \)

\[ Q_1 = (\bar{s}_i c_j)_{V-A} (\bar{u}_j d_i)_{V-A} \]
\[ Q_2 = (\bar{s}_i c_i)_{V-A} (\bar{u}_j d_j)_{V-A} \]

Exceptional scheme:

\[ p_1 = -p_2 = p_3 = -p_4 \]

Compute amputated Green’s functions \( \Gamma(Q_i) \)

\[ P_1 = \delta_{ik} \delta_{jl} (\gamma_\mu \otimes \gamma_\mu + \gamma_\mu \gamma_5 \otimes \gamma_\mu \gamma_5) \]
\[ P_2 = \delta_{ij} \delta_{kl} (\gamma_\mu \otimes \gamma_\mu + \gamma_\mu \gamma_5 \otimes \gamma_\mu \gamma_5) \]

Define \( M_{ij} = P_j [\Gamma(Q_i)] \)

RI renormalization conditions \( M_{ik}^{RI} = Z_{ij}^{RI} M_{jk}^{bare} = M_{ik}^{tree} \)
The calculation - II

\[ G_{SM} = \left\langle W_{boson\ in\ Unitary\ gauge} \right\rangle \]

Use identical momentum configuration as before

Weak vertex factor \( \propto g \)

Compute the Amputated Green’s function \( \Gamma_{SM} \)

Define the vector \( W_i = P_i (\Gamma_{SM}) \)

RI renormalization conditions for the Wilson Coefficients

\[ G_F C_i^{RI} M_{ij}^{RI} \equiv W_j^{RI} \rightarrow C_i^{RI} = G_F^{-1} W_j^{RI} [M_{\text{tree}}]^{-1} \]

With \( G_F = \frac{g^2}{8M_W^2} \) and \( g \) the weak coupling constant
Step-scaling to $M_W$:

From invariance of $\langle \mathcal{H}_{\text{eff}} \rangle$

$$\vec{C}^T_{\text{RI}}(M_W)M^\text{RI}(M_W) = \vec{C}^T_{\text{RI}}(\mu)M^\text{RI}(\mu)$$

$$\downarrow$$

$$\Sigma(M_W, \mu) = Z^\text{RI}(\mu)[Z^\text{RI}(M_W)]^{-1}$$

$$\downarrow$$

$$\vec{C}^T_{M_W}(\mu) \equiv \vec{C}^T(\mu)\Sigma(M_W, \mu)$$

$\Sigma$ step-scaling function at finite $a$

$C_i$ at $M_W \approx 2$ GeV: chiral symmetry breaking effects reduced comparison with PT safer
Ensembles and methods

Two ensembles (different volumes):
\[ N_f = 2 + 1 \] Shamir Domain-Wall fermions
\[ a^{-1} \approx 1.7 \text{ GeV} \approx 0.11 \text{ fm} \]
\[ L \approx 1.8 \text{ AND } 2.6 \text{ fm} \]

zMobius acceleration to compute necessary propagators

RI scheme with external \( p \) between 0.5 and 1.7 GeV

Artificially small \( M_W \in [1.4, 2.1] \text{ GeV} \rightarrow 0.8 < aM_W < 1.2 \)

Current goal: preliminary study to investigate (some of) the systematic uncertainties and understand what kind of lattices are needed to safely accommodate all the relevant scales.
Step-scaling function

\[ \Sigma_{11}(m_W, \mu) \]

\[ \Sigma_{12}(m_W, \mu) \]

PT curve: NLO ADM, 2-loop \( \alpha_s, \Lambda^{(3)}_{\text{MS}} = 332(14) \text{ MeV} \) [Sommer’s talk]

Measured step-scaling function:
finite volume errors below 4-5 %
no evident deviations from PT at small momenta
discretization errors to be investigated
Wilson Coefficients $C_1$ and $C_2$

Blue bands: NLO PT $\vec{C}^{RI}(M_W)$

$$C_1 = O(\alpha_s), \quad C_2 = 1 + O(\alpha_s)$$

$$M_W = a^{-1} \sim 1.73 \text{ GeV}$$

$$\vec{C}^{T}_{M_W}(\mu) = \vec{C}^{T}(\mu) \Sigma(M_W, \mu)$$

$$\mu^2/M_W^2 \rightarrow 0 \text{ exact matching}$$

PT is in this limit
$M_W$ dependence - $C_1$

$L \approx 72 \text{ MeV}^{-1}$, $aM_W \in [0.8, 1.0, 1.2]$

Observation of about 15% effects due to $O(\mu^2/M_W^2)$
$M_W$ dependence - $C_2 - 1$

$L \approx 72 \text{ MeV}^{-1}$, $aM_W \in [0.8, 1.0, 1.2]$

Milder effects of $O(\mu^2/M_W^2)$ terms
Conclusions

Future plans:

- study non-exceptional schemes
- improve statistical uncertainty
- repeat the calculation at finer lattice spacings
- study extended basis

Thank you for your attention!