Towards a non-perturbative calculation of the weak Hamiltonian Wilson coefficients

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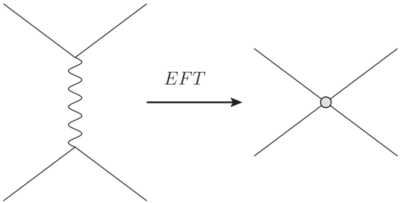
Weak Effective Hamiltonian

Weak decays of hadrons:

typical hadronic scale $O(\Lambda_{\mathrm{QCD}})$
mediated by W boson

$\rightarrow$ An effective theory of weak interactions

Example: $c \rightarrow s u \bar{d}$ only current-current operators (no penguins)



integrate out W boson generates four-quark vertices

higher order operators $O(1/M_{\mathrm{W}}^k)$

operator mixing in the EFT

$$\mathcal{H}_{\mathrm{eff}} \propto G_{\mathrm{F}} \sum_i C_i Q_i \quad \rightarrow \quad i = 1, 2 \text{ in our example}$$

Long distance $\langle Q_i \rangle \rightarrow$ Lattice

Wilson Coefficients $C_i \rightarrow$ PT

Present calculations

$\Delta S = 1$ transitions:

RBC/UKQCD calculations of $\mathrm{K} \to \pi\pi$ ($I = 0$ and 2)

Matrix elements up to $2\ \mathrm{GeV}$ from lattice [C.Kelly's talk]

Running From $2\ \mathrm{GeV}$ up to M_W perturbative

Wilson Coefficients at M_W not yet largest systematic errors

$\Delta B = 1$ transitions:

Recent results B-decays [Fermilab+MILC '16]

impact for more precise Wilson Coefficients

A lattice calculation can provide an all-order-in-α_s result

Perturbative results - I

[Buchalla, Buras, Lautenbacher '95]

By matching the full and effective theory at one loop in $\overline{\mathrm{MS}}$:

$$C_1 = -b_0 \alpha_s \log(M_{\mathrm{W}}^2/\mu^2)$$
$$C_2 = 1 + b_1 \alpha_s \log(M_{\mathrm{W}}^2/\mu^2)$$

b_0, b_1 positive coefficients

μ is the matching scale $\rightarrow$ large logs

$\downarrow$

Renormalization group improvement

Perturbative results - II

[Buchalla, Buras, Lautenbacher '95]

Initial conditions C1 and C2

$C_1(M_\mathrm{W}) \approx 0.44\alpha_s(M_\mathrm{W})$

$C_2(M_\mathrm{W}) = 1 + 0.15\alpha_s(M_\mathrm{W})$

Anomalous Dimension Matrix (ADM)

U solution of RG equations

$\vec{C}(\mu) = U(M_\mathrm{W}, \mu)\vec{C}(M_\mathrm{W})$

Resummation of large logs at scale μ

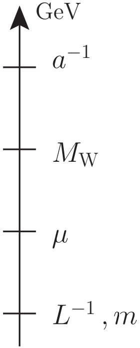
Example: $\mu = 40$ GeV and $70 \text{ GeV} < M_\mathrm{W} < 90$ GeV:

C_1 varies by 40%, C_2 varies by less than 1% using 2-loop α_s

Physical observable, e.g. $\mathrm{K} \to \pi\pi$ amplitude (estimated) error from $C_1(M_\mathrm{W}), C_2(M_\mathrm{W})$ around 3-5%.

Window problem

μ is the matching scale:



$aM_W \ll 1$ for discretization effects

$\mu \ll M_W$ for higher order operators

$\mu \gg m \, , \mu L \gg 1$ for infrared effects

Present study is focused on unphysically small $m_W \approx 2$ GeV

Renormalization scheme

With a momentum-subtraction scheme:

- pertubative calculations known to NLO
- off-shell external quark states with momentum p

  $\downarrow$

  gauge-dependent operators $O(p^2/M_{\mathrm{W}}^2)$
- exceptional scheme $\rightarrow$ chiral symmetry breaking effects

The limit $p^2/M_{\mathrm{W}}^2 \rightarrow 0$:

- crucial to reduce some systematic uncertainties
- problematic for an exceptional scheme

Two-step strategy

1) matching at sufficiently small μ and 2) step-scale up to M_{W}

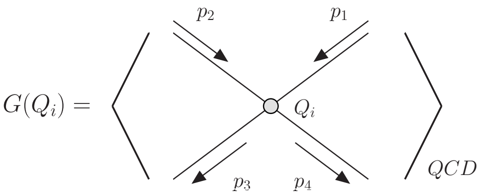
The calculation - I

Restriction to current-current diagrams (no penguins)

Green's function $G(Q_i)$

$Q_1 = (\bar{s}_i c_j)_{\mathrm{V-A}} (\bar{u}_j d_i)_{\mathrm{V-A}}$

$Q_2 = (\bar{s}_i c_i)_{\mathrm{V-A}} (\bar{u}_j d_j)_{\mathrm{V-A}}$



Exceptional scheme:

$p_1 = -p_2 = p_3 = -p_4$

Compute amputated Green's functions $\Gamma(Q_i)$

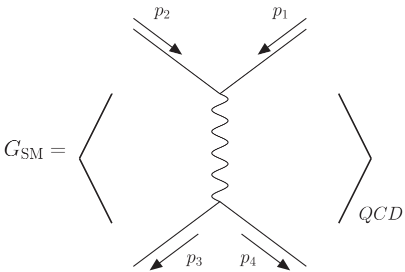
$$P_1 = \delta_{ik}\delta_{jl}(\gamma_\mu \otimes \gamma_\mu + \gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5)$$

$$P_2 = \delta_{ij}\delta_{kl}(\gamma_\mu \otimes \gamma_\mu + \gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5)$$

Define $M_{ij} = P_j[\Gamma(Q_i)]$

RI renormalization conditions $M_{ik}^{\mathrm{RI}} = Z_{ij}^{\mathrm{RI}} M_{jk}^{\mathrm{bare}} = M_{ik}^{\mathrm{tree}}$

The calculation - II



W boson in Unitary gauge

use identical momentum configuration as before

weak vertex factor $\propto g$

Compute the Amputated Green's function $\Gamma_{\rm SM}$

Define the vector $W_i = P_i(\Gamma_{\rm SM})$

RI renormalization conditions for the Wilson Coefficients

$$G_{\rm F} C_i^{\rm RI} M_{ij}^{\rm RI} \equiv W_j^{\rm RI} \quad \rightarrow \quad C_i^{\rm RI} = G_{\rm F}^{-1} W_j^{\rm RI} [M^{\rm tree}]_{ji}^{-1}$$

with $G_{\rm F} = \frac{g^2}{8M_{\rm W}^2}$ and g the weak coupling constant

The calculation - III

Step-scaling to M_{W}:

From invariance of $\langle \mathcal{H}_{\mathrm{eff}} \rangle$

$$\vec{C}^T_{\mathrm{RI}}(M_{\mathrm{W}}) M^{\mathrm{RI}}(M_{\mathrm{W}}) = \vec{C}^T_{\mathrm{RI}}(\mu) M^{\mathrm{RI}}(\mu)$$

$$\downarrow$$

$$\Sigma(M_{\mathrm{W}}, \mu) = Z^{\mathrm{RI}}(\mu)[Z^{\mathrm{RI}}(M_{\mathrm{W}})]^{-1}$$

$$\downarrow$$

$$\vec{C}^T_{M_{\mathrm{W}}}(\mu) \equiv \vec{C}^T(\mu) \Sigma(M_{\mathrm{W}}, \mu)$$

Σ step-scaling function at finite a

C_i at $M_{\mathrm{W}} \approx 2$ GeV:
- chiral symmetry breaking effects reduced
- comparison with PT safer

Ensembles and methods

Two ensembles (different volumes):

$N_{\rm f} = 2+1$ Shamir Domain-Wall fermions

$a^{-1} \approx 1.7$ GeV ≈ 0.11 fm

$L \approx 1.8$ AND 2.6 fm

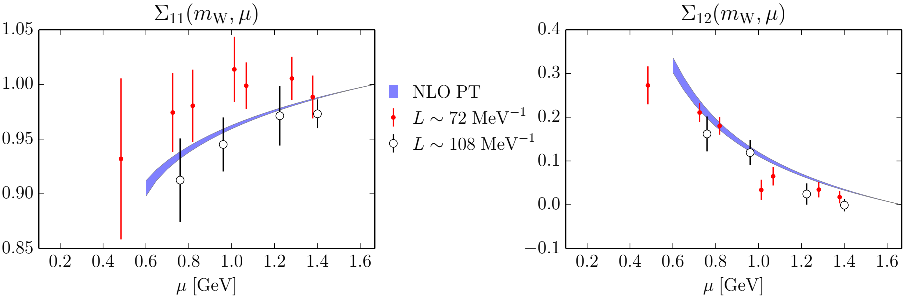
zMobius accelaration to compute necessary propagators

RI scheme with external p between 0.5 and 1.7 GeV

Artificially small $M_{\rm W} \in [1.4\,, 2.1]$ GeV $\to 0.8 < aM_{\rm W} < 1.2$

Current goal: preliminary study to investigate (some of) the systematic uncertainties and undestand what kind of lattices are needed to safely accomodate all the relevant scales.

Step-scaling function

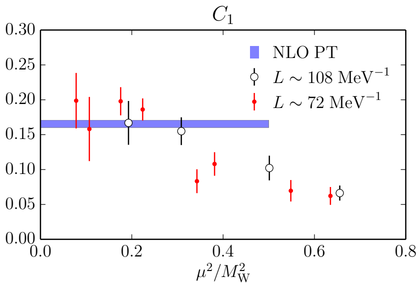


PT curve: NLO ADM, 2-loop α_s, $\Lambda^{(3)}_{\overline{\rm MS}} = 332(14)$ MeV [Sommer's talk]

Measured step-scaling function:

- finite volume errors below 4-5 %
- no evident deviations from PT at small momenta
- discretization errors to be investigated

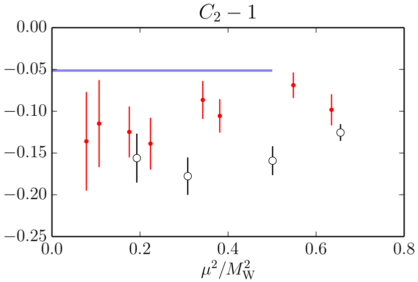
Wilson Coefficients C_1 and C_2



Blue bands: NLO PT $\vec{C}^{\mathrm{RI}}(M_\mathrm{W})$

$C_1 = O(\alpha_s)\,,\ C_2 = 1 + O(\alpha_s)$

$M_\mathrm{W} = a^{-1} \sim 1.73$ GeV



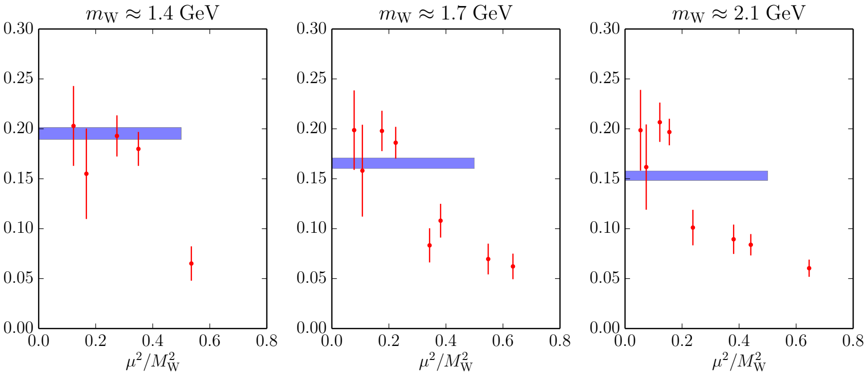
$\vec{C}^T_{M_\mathrm{W}}(\mu) = \vec{C}^T(\mu)\Sigma(M_\mathrm{W}, \mu)$

$\mu^2/M_\mathrm{W}^2 \to 0$ exact matching

PT is in this limit

M_{W} dependence - C_1

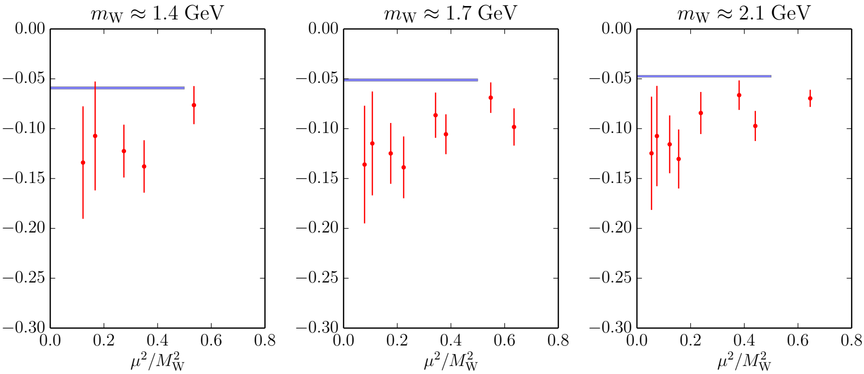
$L \approx 72\ \mathrm{MeV}^{-1}$, $aM_{\mathrm{W}} \in [0.8, 1.0, 1.2]$



Observation of about 15% effects due to $O(\mu^2/M_{\mathrm{W}}^2)$

M_{W} dependence - $C_2 - 1$

$L \approx 72\ \mathrm{MeV}^{-1}$, $aM_{\mathrm{W}} \in [0.8, 1.0, 1.2]$



Milder effects of $O(\mu^2/M_{\mathrm{W}}^2)$ terms

Conclusions

window in μ^2/M_{W}^2 with 1σ agreement with PT

Future plans:

- study non-exceptional schemes
- improve statistical uncertainty
- repeat the calculation at finer lattice spacings
- study extended basis

Thank you for your attention!