Extraction of the bare form factors for semi-leptonic B_s decays

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The goal

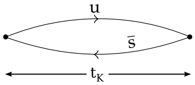
- Extract the bare ground-state matrix elements for static HQET $\mathrm{B_s} \to \mathrm{K}\ell\nu$ decay at $\approx 2\%$ precision.

$$h_{\parallel}^{\mathrm{stat,bare}} = (2m_{\mathrm{B_s}})^{-1/2} \langle \mathrm{K}(p_\mathrm{K}) | V^0(0) | \mathrm{B_s}(0) \rangle = \varphi_0^{(0,0)} \sqrt{2E_\mathrm{K}^{(0)}},$$

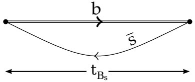
$$p_\mathrm{K}^k h_{\perp}^{\mathrm{stat,bare}} = (2m_{\mathrm{B_s}})^{-1/2} \langle \mathrm{K}(p_\mathrm{K}) | V^k(0) | \mathrm{B_s}(0) \rangle = \varphi_k^{(0,0)} \sqrt{2E_\mathrm{K}^{(0)}}$$

- Correlation functions (three smearings for $\mathrm{B_s}$, one for Kaon):

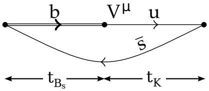
$$\mathcal{C}^{\mathrm{K}}(t_\mathrm{K}) \cong \sum_m (\kappa^{(m)})^2 e^{-E_\mathrm{K}^{(m)} t_\mathrm{K}},$$



$$\mathcal{C}_{ij}^{\mathrm{B_s}}(t_{\mathrm{B_s}}) \cong \sum_n \beta_i^{(n)} \beta_j^{(n)} e^{-E_{\mathrm{B_s}}^{(n)} t_{\mathrm{B_s}}},$$



$$\mathcal{C}_{\mu,i}^{\mathrm{B_s}\to\mathrm{K}}(t_\mathrm{K}, t_{\mathrm{B_s}}) \cong \sum_{m,n} \kappa^{(m)} \varphi_\mu^{(m,n)} \beta_i^{(n)} e^{-E_\mathrm{K}^{(m)} t_\mathrm{K} - E_{\mathrm{B_s}}^{(n)} t_{\mathrm{B_s}}}$$



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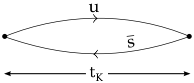
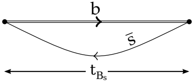
$$p_K^k h_{\perp}^{\mathrm{stat,bare}} = (2m_{B_s})^{-1/2}\langle K(p_K)|V^k(0)|B_s(0)\rangle = \varphi_k^{(0,0)}\sqrt{2E_K^{(0)}}$$

- Correlation functions (three smearings for B_s, one for Kaon):

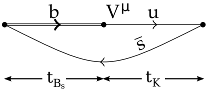
$$\mathcal{C}^K(t_K) \cong \sum_m^{N_K} (\kappa^{(m)})^2 e^{-E_K^{(m)} t_K},$$



$$\mathcal{C}_{ij}^{B_s}(t_{B_s}) \cong \sum_n^{N_{B_s}} \beta_i^{(n)}\beta_j^{(n)} e^{-E_{B_s}^{(n)} t_{B_s}},$$



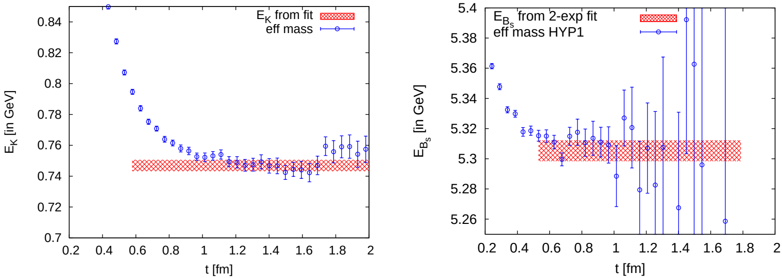
$$\mathcal{C}_{\mu,i}^{B_s\to K}(t_K, t_{B_s}) \cong \sum_{m,n}^{N_K, N_{B_s}} \kappa^{(m)}\varphi_\mu^{(m,n)}\beta_i^{(n)} e^{-E_K^{(m)} t_K - E_{B_s}^{(n)} t_{B_s}}$$



The goal cntd.

- CLS $N_f = 2$ lattices:
  - Three ensembles for the continuum limit @ fixed q^2
  - All the plots shown here are for the finest lattice N6, $48^3 \times 96$, $a = 0.0483(4)$ fm, $m_\pi = 340$ MeV
  - $ap^k_{\mathrm{K}} = \frac{2\pi}{L}\delta_{k1}$ on N6, twisted b.c. for equal p_{K} on other ensembles
- We use stochastic sources with full-time dilution for measurements $\Rightarrow$ all $t_{\mathrm{K}}, t_{\mathrm{B_s}}$ accessible.
- Especially the $\mathrm{B_s}$-sector (treated in HQET) problematic due to large contamination by excited states + signal-to-noise problem.

The goal cntd.



Effective energies of $\mathcal{C}^{K}$ and $\mathcal{C}^{B_s}$, ensemble N6. Both plots are scaled to show the same y-axis span. Red bands show the results of two-exp fits.

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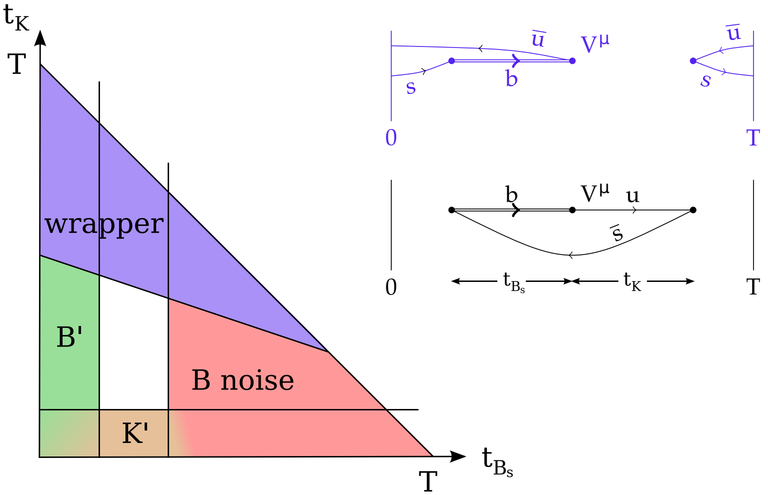
Combined fits

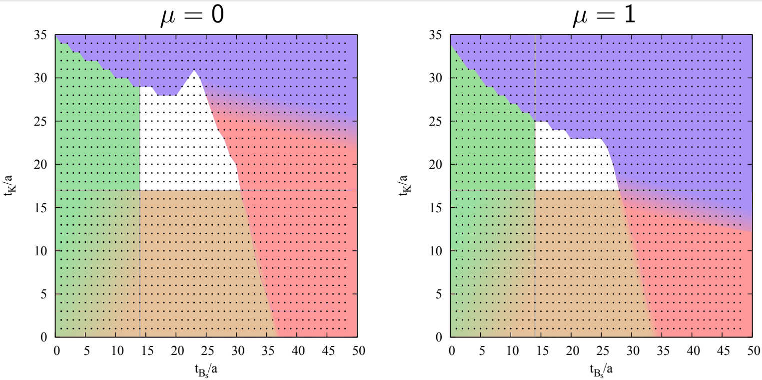
- Strategy: do a combined non-linear fit to both two-point and three-point functions, to get energies, amplitudes and the form factors.
- In practice $N_{\mathrm{B_s}} = 3$ and $N_{\mathrm{K}} = 1$ or 2 needed for stable fit results.
- Many parameters ($\approx$20 parameters to $\mathcal{O}(10^3)$ data points) $\Rightarrow$ good starting values:
  1. Find $E_{\mathrm{K}}^{(m)}$ and $\kappa^{(m)}$ from $\mathcal{C}^{\mathrm{K}}$,
  2. From $\mathcal{C}_{ij}^{\mathrm{B_s}}$, find $E_{\mathrm{B_s}}^{(n)}$ with GEVP and feed them to fit for $\beta_i^{(n)}$,
  3. Find a region in $\mathcal{C}_{\mu,i}^{\mathrm{B_s}\to\mathrm{K}}$ free of wrap-around states coming from finite T,
  4. Do a linear fit for $\varphi_{\mu}^{(m,n)}$.
- Feed these as starting values to the (uncorrelated) combined fit. Analyze stability w.r.t. fit ranges.

Fit ranges, finite T effects



Selection of data points for fitting in $\mathcal{C}_{\mu}^{B_s \to K}(t_K, t_{B_s})$, schematic plot

Fit ranges, finite T effects



Selection of data points for fitting in $\mathcal{C}_\mu^{\mathrm{B_s}\to\mathrm{K}}(t_\mathrm{K}, t_{\mathrm{B_s}})$, ensemble N6

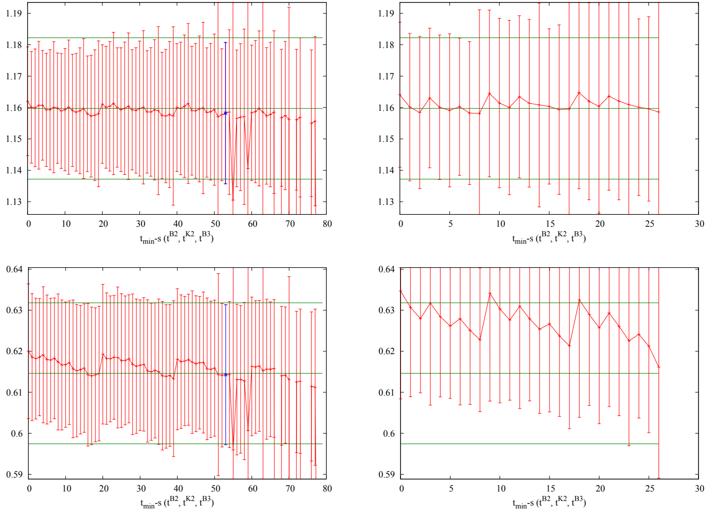
- $t_\mathrm{min}^\mathrm{K3} \approx 0.82$ fm (for $N_\mathrm{K} = 1$), $t_\mathrm{min}^\mathrm{B3} \approx 0.67$ fm ($N_{\mathrm{B_s}} = 3$)
- Wrapper criterion limits the relative wrapper contribution to $1/3$ of the statistical noise, noise criterion is SNR ≥ 10.
- Wrapper contribution larger for φ_k than φ_0.

Combined fit stability w.r.t. $t_{min}^{B2}, t_{min}^{K3}, t_{min}^{B3}$

$(N_K, N_{B_s}) =$ (1, 3) (2, 3)

$\varphi_0^{(0,0)}$

$\varphi_1^{(0,0)}$



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Ratios

- A ratio should exponentially converge to $\varphi_\mu^{(0,0)}$. Many different ratios can be defined ($\tau = t_{\mathrm{K}} + t_{\mathrm{B_s}}$):

$$\mathcal{R}^{\mathrm{I}}_{\mu,i}(t_{\mathrm{K}}, t_{\mathrm{B_s}}) = \frac{\mathcal{C}^{\mathrm{B_s}\to\mathrm{K}}_{\mu,i}(t_{\mathrm{K}}, t_{\mathrm{B_s}})}{\left[\mathcal{C}^{\mathrm{K}}(\tau)\mathcal{C}^{\mathrm{B_s}}_{ii}(\tau)\right]^{1/2}} e^{\left(E^{\mathrm{eff}}_{\mathrm{B_s}}(\tau) - E^{\mathrm{eff}}_{\mathrm{K}}(\tau)\right)\frac{t_{\mathrm{B_s}} - t_{\mathrm{K}}}{2}}$$

$$\mathcal{R}^{\mathrm{II}}_{\mu,i}(t_{\mathrm{K}}, t_{\mathrm{B_s}}) = \frac{\mathcal{C}^{\mathrm{B_s}\to\mathrm{K}}_{\mu,i}(t_{\mathrm{K}}, t_{\mathrm{B_s}})}{\left[\mathcal{C}^{\mathrm{K}}(t_{\mathrm{K}})\mathcal{C}^{\mathrm{B_s}}_{ii}(t_{\mathrm{B_s}})\right]^{1/2}} e^{E^{\mathrm{eff}}_{\mathrm{B_s}}(\tau)\frac{t_{\mathrm{B_s}}}{2} + E^{\mathrm{eff}}_{\mathrm{K}}(\tau)\frac{t_{\mathrm{K}}}{2}}$$

$$\mathcal{R}^{\mathrm{III}}_{\mu,i}(t_{\mathrm{K}}, t_{\mathrm{B_s}}) = \frac{\mathcal{C}^{\mathrm{B_s}\to\mathrm{K}}_{\mu,i}(t_{\mathrm{K}}, t_{\mathrm{B_s}})}{\mathcal{N}^{\mathrm{K}}\mathcal{C}^{\mathrm{K}}(t_{\mathrm{K}})\,\mathcal{N}^{\mathrm{B_s}}_{i}\mathcal{C}^{\mathrm{B_s}}_{ii}(t_{\mathrm{B_s}})}$$

$$\mathcal{R}^{\mathrm{f}}_{\mu,i}(t_{\mathrm{K}}, t_{\mathrm{B_s}}) = \frac{\mathcal{C}^{\mathrm{B_s}\to\mathrm{K}}_{\mu,i}(t_{\mathrm{K}}, t_{\mathrm{B_s}})}{\left[\mathcal{C}^{\mathrm{K}}(\tau)\mathcal{C}^{\mathrm{B_s}}_{ii}(\tau)\right]^{1/2}} \left[\frac{\mathcal{C}^{\mathrm{K}}(t_{\mathrm{B_s}})\mathcal{C}^{\mathrm{B_s}}_{ii}(t_{\mathrm{K}})}{\mathcal{C}^{\mathrm{K}}(t_{\mathrm{K}})\mathcal{C}^{\mathrm{B_s}}_{ii}(t_{\mathrm{B_s}})}\right]^{1/2}$$

- A priori, none of them is clearly superior.
- In practice, statistical errors & convergence vastly differ.

Ratios

- A ratio should exponentially converge to $\varphi_\mu^{(0,0)}$. Many different ratios can be defined ($\tau = t_\mathrm{K} + t_{\mathrm{B_s}}$):

$$\mathcal{R}^\mathrm{I}_{\mu,i}(t_\mathrm{K}, t_{\mathrm{B_s}}) = \frac{\mathcal{C}^{\mathrm{B_s}\to\mathrm{K}}_{\mu,i}(t_\mathrm{K}, t_{\mathrm{B_s}})}{\left[\mathcal{C}^\mathrm{K}(\tau)\mathcal{C}^{\mathrm{B_s}}_{ii}(\tau)\right]^{1/2}} e^{(E^\mathrm{eff}_{\mathrm{B_s}}(\tau) - E^\mathrm{eff}_\mathrm{K}(\tau))\frac{t_{\mathrm{B_s}} - t_\mathrm{K}}{2}}$$

$$\mathcal{R}^\mathrm{f}_{\mu,i}(t_\mathrm{K}, t_{\mathrm{B_s}}) = \frac{\mathcal{C}^{\mathrm{B_s}\to\mathrm{K}}_{\mu,i}(t_\mathrm{K}, t_{\mathrm{B_s}})}{\left[\mathcal{C}^\mathrm{K}(\tau)\mathcal{C}^{\mathrm{B_s}}_{ii}(\tau)\right]^{1/2}} \left[\frac{\mathcal{C}^\mathrm{K}(t_{\mathrm{B_s}})\mathcal{C}^{\mathrm{B_s}}_{ii}(t_\mathrm{K})}{\mathcal{C}^\mathrm{K}(t_\mathrm{K})\mathcal{C}^{\mathrm{B_s}}_{ii}(t_{\mathrm{B_s}})}\right]^{1/2}$$

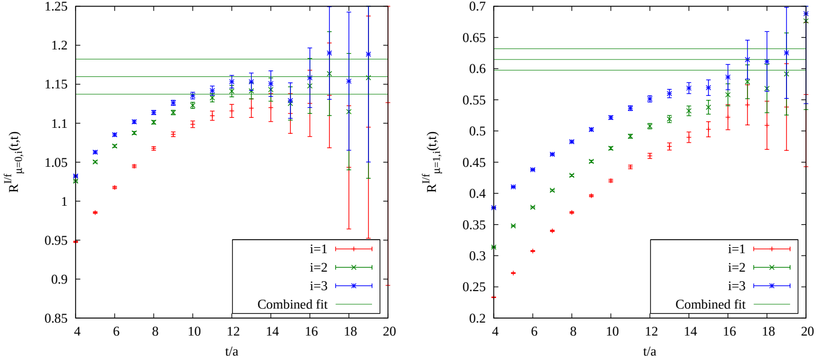
Ratios

- A ratio should exponentially converge to $\varphi_\mu^{(0,0)}$. Many different ratios can be defined ($\tau = t_{\mathrm{K}} + t_{\mathrm{B_s}}$):

Particularly simple form for $t_{\mathrm{K}} = t_{\mathrm{B_s}} = t$:

$$\mathcal{R}^{\mathrm{I/f}}_{\mu,i}(t,t) = \frac{\mathcal{C}^{\mathrm{B_s}\to\mathrm{K}}_{\mu,i}(t,t)}{\left[\mathcal{C}^{\mathrm{K}}(\tau)\mathcal{C}^{\mathrm{B_s}}_{ii}(\tau)\right]^{1/2}}$$

Ratios



Ratio $\mathcal{R}^{\mathrm{I/f}}_{\mu,i}$ as function of $t = t_{\mathrm{K}} = t_{\mathrm{B_s}}$. Large contribution from excited states visible.

Ratios

- A ratio should exponentially converge to $\varphi_\mu^{(0,0)}$. Many different ratios can be defined ($\tau = t_{\mathrm{K}} + t_{\mathrm{B_s}}$):

  Particularly simple form for $t_{\mathrm{K}} = t_{\mathrm{B_s}} = t$:

$$\mathcal{R}^{\mathrm{I/f}}_{\mu,i}(t,t) = \frac{\mathcal{C}^{\mathrm{B_s}\to\mathrm{K}}_{\mu,i}(t,t)}{\left[\mathcal{C}^{\mathrm{K}}(\tau)\mathcal{C}^{\mathrm{B_s}}_{ii}(\tau)\right]^{1/2}}$$

- Rather late onset of plateaux – one possible solution: fit functional form, including excited state.
- But: convergence $\sim e^{-\Delta E^{\mathrm{min}} t}$. Can we do better?

Summed ratios

- Better suppresion of excited states may be provided by summing the ratios:

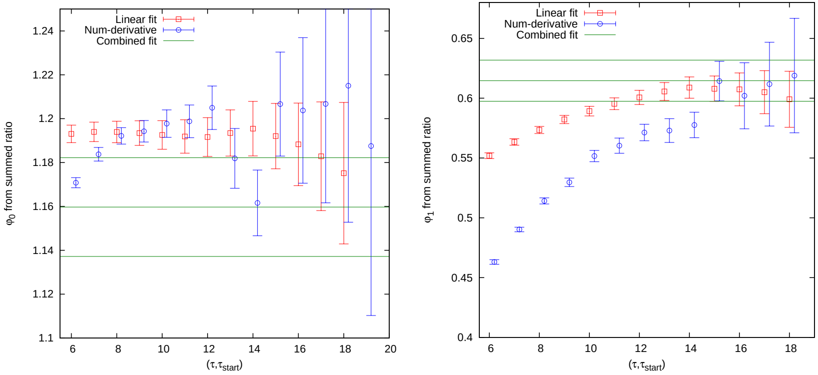
$$\mathcal{M}^{X,\mathrm{eff}}_{\mu,i}(\tau) = -\partial_\tau a\, \mathcal{S}^X_{\mu,i}(\tau), \text{ where } \mathcal{S}^X_{\mu,i}(\tau) = \sum_{t_{\mathrm{B_s}}} \mathcal{R}^X_{\mu,i}(\tau - t_{\mathrm{B_s}}, t_{\mathrm{B_s}})$$

- Improved convergence $\sim \tau \Delta E^{\mathrm{min}} e^{-\Delta E^{\mathrm{min}}\tau}$ holds for $\mathcal{R}^{\mathrm{I}}$

  [Maiani et al. 1987, Capitani et al. 2010, Bulava et al. 2010,2011]

- One can extract $\varphi^{(0,0)}_\mu$ from numerical derivative or linear fit to $\mathcal{S}_\mu(\tau) = \varphi_\mu \tau + C$

Summed ratios



Comparison of $\varphi_\mu^{(0,0)}$ extracted from summed ratio $\mathcal{M}^I_{\mu,3}(\tau)$ and the combined fit. The fit is done in a fit range τ_{start} to $32a$.

Summed ratios

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- One can extract $\varphi^{(0,0)}_\mu$ from numerical derivative or linear fit to $\mathcal{S}_\mu(\tau) = \varphi_\mu \tau + C$
- Approach to the plateaux clearly improved

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Summary & Outlook

- Full-time dilution of stochastic propagators
  $\Rightarrow$ Access to all values of t_{K} and $t_{\mathrm{B_s}}$
  $\Rightarrow$ Good control over systematics from excited states and finite T
- Combined fits: stable results with $N_{\mathrm{B_s}} = 3$; starting values from 2pt functions and linear fits to 3pt functions
- Ratios: good statistical signal for selected ratios, but late onset of plateaux
- Summed ratios: improved plateaux convergence, in agreement with theory

Outlook:

- $1/m$ insertions, non-perturbative HQET parameters, more values of q^2
- Chiral extrapolation $\Rightarrow \mathrm{B} \to \pi$