Renormalization of three-quark operators for baryon distribution amplitudes

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Reminder

- Definition of leading twist octet DAs

$$
\begin{aligned}
\Phi_{\pm}^{B\neq\Lambda}(x_1,x_2,x_3) &= \tfrac{1}{2}\Big([V-A]^B(x_1,x_2,x_3) \pm [V-A]^B(x_3,x_2,x_1)\Big)\\
\Pi^{B\neq\Lambda}(x_1,x_2,x_3) &= T^B(x_1,x_3,x_2)\\
\Phi_{+}^{\Lambda}(x_1,x_2,x_3) &= +\sqrt{\tfrac{1}{6}}\Big([V-A]^\Lambda(x_1,x_2,x_3) + [V-A]^\Lambda(x_3,x_2,x_1)\Big)\\
\Phi_{-}^{\Lambda}(x_1,x_2,x_3) &= -\sqrt{\tfrac{3}{2}}\Big([V-A]^\Lambda(x_1,x_2,x_3) - [V-A]^\Lambda(x_3,x_2,x_1)\Big)\\
\Pi^{\Lambda}(x_1,x_2,x_3) &= \sqrt{6}\,T^\Lambda(x_1,x_3,x_2)
\end{aligned}
$$

- Expansion in terms of shape parameters

$$
\begin{aligned}
\Phi_{+}^{B} &= 120x_1x_2x_3\big(\varphi_{00}^B\mathcal{P}_{00} + \varphi_{11}^B\mathcal{P}_{11} + \dots\big) & f^B &= \varphi_{00}^B\\
\Phi_{-}^{B} &= 120x_1x_2x_3\big(\varphi_{10}^B\mathcal{P}_{10} + \dots\big) & &\\
\Pi^{B\neq\Lambda} &= 120x_1x_2x_3\big(\pi_{00}^B\mathcal{P}_{00} + \pi_{11}^B\mathcal{P}_{11} + \dots\big) & f_T^{B\neq\Lambda} &= \pi_{00}^B\\
\Pi^{\Lambda} &= 120x_1x_2x_3\big(\pi_{10}^\Lambda\mathcal{P}_{10} + \dots\big) & &
\end{aligned}
$$

Renormalization overview

$$\text{Lattice} \xrightarrow{\text{nonpert. ren.}} \text{RI}'/\text{SMOM} \xrightarrow{\text{continuum PT}} \overline{\text{MS}}$$

- Bare lattice values have to be renormalized
- In the end we should be able to give our results in the popular continuum $\overline{\text{MS}}$ scheme
- This scheme cannot be implemented directly on the lattice
- We use a nonperturbative RI$'$/SMOM scheme for the lattice renormalization
- We use continuum perturbation theory to convert from RI$'$/SMOM to $\overline{\text{MS}}$

A tale of two schemes: $\overline{\mathsf{MS}}$ and RI′/SMOM

- $\overline{\mathsf{MS}}$ requires the use of dimensional regularization
- perform loop integral calculations in $4-2\epsilon$ dimensions
- expand in ϵ and subtract divergencies order-by-order, usually in the shape of

$$G^{\overline{\mathsf{MS}}}(p,\mu) = \left(1 - \frac{\alpha_s(\mu)}{3\pi}\xi\left(\frac{1}{\epsilon} - \gamma_E + \ln(4\pi)\right)\right)G^{\mathsf{bare}}(p)$$

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- RI′ is a class of regularization invariant renormalization schemes
- defined through a renormalization condition at a specific renormalization point, e.g.,

$$1 \stackrel{!}{=} \frac{1}{12p^2}\,\mathrm{Tr}\left[-i\not{p}\left(G^{\text{RI}'}\right)^{-1}(p,\mu)\right]_{p^2=\mu^2}$$

- SMOM: Symmetric renormalization point
- Better behaved than exceptional momentum configurations

RI′/SMOM renormalization procedure

- Renormalization point

$$p_{\mathsf{SMOM}}:\quad p_1^2 = p_2^2 = p_3^2 = (p_1+p_2)^2 = (p_1+p_3)^2 = (p_1+p_2+p_3)^2 = \mu^2$$

- Amputated four-point vertex function

$$\Lambda(\mathcal{O}|p_1,p_2,p_3)^{fgh}_{\alpha\beta\gamma} = \int dx_1 dx_2 dx_3\, e^{i\sum_j p_j\cdot x_j}\,\epsilon^{ijk}\left\langle \mathcal{O}(0)\bar{f}^i_{\alpha'}(x_1)\bar{g}^j_{\beta'}(x_2)\bar{h}^k_{\gamma'}(x_3)\right\rangle$$
$$\times\, G_2^{-1}(p_1)_{\alpha'\alpha} G_2^{-1}(p_2)_{\beta'\beta} G_2^{-1}(p_3)_{\gamma'\gamma}$$

RI′/SMOM renormalization procedure

- Renormalization point

$$p_{\mathsf{SMOM}}: \quad p_1^2 = p_2^2 = p_3^2 = (p_1 + p_2)^2 = (p_1 + p_3)^2 = (p_1 + p_2 + p_3)^2 = \mu^2$$

- Amputated four-point vertex function

$$\Lambda(\mathcal{O}|p_1, p_2, p_3)^{fgh}_{\alpha\beta\gamma} = \int dx_1 dx_2 dx_3 \, e^{i\sum_j p_j \cdot x_j} \epsilon^{ijk} \left\langle \mathcal{O}(0) \bar{f}^i_{\alpha'}(x_1) \bar{g}^j_{\beta'}(x_2) \bar{h}^k_{\gamma'}(x_3) \right\rangle \\ \times G_2^{-1}(p_1)_{\alpha'\alpha} G_2^{-1}(p_2)_{\beta'\beta} G_2^{-1}(p_3)_{\gamma'\gamma}$$

- Renormalization condition

$$\left(L^{\mathsf{S}}\right)_{mn} = \sum_i \sum_{\substack{f,g,h \\ \alpha,\beta,\gamma}} \Lambda^{\mathsf{S}}\left(\mathcal{O}^{(i)}_m \middle| p_{\mathsf{SMOM}}\right)^{fgh}_{\alpha\beta\gamma} \left(\Lambda^{\mathsf{Born}}\left(\mathcal{O}^{(i)}_n \middle| p_{\mathsf{SMOM}}\right)^{fgh}_{\alpha\beta\gamma}\right)^*$$

$$\mathbb{1} \stackrel{!}{=} L^{\mathsf{RI}} \left(L^{\mathsf{Born}}\right)^{-1}$$

- Which operators can mix?

Operator classification: $\overline{\mathrm{H}(4)}$

- Euclidean spacetime has continuous $\mathrm{O}(4)$ symmetry
- For fermions on a lattice this is broken down to the spinorial hypercubic group $\overline{\mathrm{H}(4)}$ (a discrete double cover group with 768 elements)
- Five irreducible spinorial representations: $\tau_1^{\underline{4}}$, $\tau_2^{\underline{4}}$, $\tau^{\underline{8}}$, $\tau_1^{\underline{12}}$, $\tau_2^{\underline{12}}$
- Classify baryon operators by decomposing direct products
- E.g. for three quarks: $\tau_1^{\underline{4}} \otimes \tau_1^{\underline{4}} \otimes \tau_1^{\underline{4}} = 5\tau_1^{\underline{4}} \oplus \tau^{\underline{8}} \oplus 3\tau_1^{\underline{12}}$
- Operators belonging to different representations cannot mix

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- Operators belonging to different representations cannot mix

	no derivatives dimension 9/2	1 derivative dimension 11/2	2 derivatives dimension 13/2
$\tau_1^{\underline{4}}$	$\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5$	$\dots$	$\mathcal{O}_{DD1}, \mathcal{O}_{DD2}, \mathcal{O}_{DD3}, \dots$
$\tau_2^{\underline{4}}$			$\mathcal{O}_{DD4}, \mathcal{O}_{DD5}, \mathcal{O}_{DD6}, \dots$
$\tau^{\underline{8}}$	$\mathcal{O}_6$	$\mathcal{O}_{D1}, \dots$	$\mathcal{O}_{DD7}, \mathcal{O}_{DD8}, \mathcal{O}_{DD9}, \dots$
$\tau_1^{\underline{12}}$	$\mathcal{O}_7, \mathcal{O}_8, \mathcal{O}_9$	$\mathcal{O}_{D2}, \mathcal{O}_{D3}, \mathcal{O}_{D4}, \dots$	$\mathcal{O}_{DD10}, \mathcal{O}_{DD11}, \mathcal{O}_{DD12}, \mathcal{O}_{DD13}, \dots$
$\tau_2^{\underline{12}}$		$\mathcal{O}_{D5}, \mathcal{O}_{D6}, \mathcal{O}_{D7}, \mathcal{O}_{D8}$	$\mathcal{O}_{DD14}, \mathcal{O}_{DD15}, \mathcal{O}_{DD16}, \mathcal{O}_{DD17}, \mathcal{O}_{DD18}, \dots$

Operator classification: $\overline{\mathrm{H}(4)}$

- In the continuum operators with different number of derivatives do not mix
- On the lattice there is an additional dimensionful quantity: the lattice spacing a
- Schematically: $Dqqq$ can now mix with $\frac{1}{a}qqq$
- Inverse power of a dependence is very problematic in the continuum limit $a \to 0$
- We can avoid these problems by only using operators where this cannot happen
- Are these operators enough to measure all interesting quantities?

	no derivatives dimension 9/2	1 derivative dimension 11/2	2 derivatives dimension 13/2
τ_1^4	$\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5$	...	$\mathcal{O}_{DD1}, \mathcal{O}_{DD2}, \mathcal{O}_{DD3}, \ldots$
τ_2^4			$\mathcal{O}_{DD4}, \mathcal{O}_{DD5}, \mathcal{O}_{DD6}, \ldots$
τ^8	$\mathcal{O}_6$	$\mathcal{O}_{D1}, \ldots$	$\mathcal{O}_{DD7}, \mathcal{O}_{DD8}, \mathcal{O}_{DD9}, \ldots$
τ_1^{12}	$\mathcal{O}_7, \mathcal{O}_8, \mathcal{O}_9$	$\mathcal{O}_{D2}, \mathcal{O}_{D3}, \mathcal{O}_{D4}, \ldots$	$\mathcal{O}_{DD10}, \mathcal{O}_{DD11}, \mathcal{O}_{DD12}, \mathcal{O}_{DD13}, \ldots$
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- $\mathcal{O}_1$, $\mathcal{O}_2$: higher twist, LLL/RRR $\Rightarrow \lambda_2^B$
  can be combined to form Dosch current
- $\mathcal{O}_3$, $\mathcal{O}_4$, $\mathcal{O}_5$: higher twist, LLR/RRL $\Rightarrow \lambda_1^B$, λ_T^Λ
  can be combined to form Ioffe current
- $\mathcal{O}_7$, $\mathcal{O}_8$, $\mathcal{O}_9$: leading twist, LLR/RRL $\Rightarrow f^B$, $f_T^{B\neq\Lambda}$
  correspond to $\mathcal{V}+\mathcal{A}$, $\mathcal{V}-\mathcal{A}$ and $\mathcal{T}$, respectively
- $\mathcal{O}_{D5}$, $\mathcal{O}_{D6}$, $\mathcal{O}_{D7}$: leading twist, LLR/RRL $\Rightarrow \varphi^B_{00,(1)}$, φ^B_{11}, φ^B_{10}, $\pi^{B\neq\Lambda}_{00,(1)}$, $\pi^{B\neq\Lambda}_{11}$, π^Λ_{10}
  correspond to first moments of $\mathcal{V}+\mathcal{A}$, $\mathcal{V}-\mathcal{A}$ and $\mathcal{T}$, respectively

Operator classification: $\mathcal{S}_3$

- Under $\mathrm{SU}(3)$ quark fields transform according to the fundamental representation "3"
- For three-quark operators we have the decomposition $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$
- The renormalization is defined in the chiral limit and thus in the limit of exact $\mathrm{SU}(3)$, where operators belonging to different representations cannot mix
- Consider a local three-quark operator with separate flavor, spinor and color structures

$$F^{fgh} S^{\alpha\beta\gamma} \epsilon^{ijk} f^i_\alpha(0) g^j_\beta(0) h^k_\gamma(0)$$

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$$F^{fgh} S^{\alpha\beta\gamma} \epsilon^{ijk} f^i_\alpha(0) g^j_\beta(0) h^k_\gamma(0)$$

- For the renormalization consider only structures that transform according to $\mathcal{S}_3$
- The $\mathrm{SU}(3)$ multiplets correspond to the three irreducible representations of $\mathcal{S}_3$:
  - decuplet $\rightarrow$ totally symmetric $\rightarrow$ trivial representation
  - singlet $\rightarrow$ totally antisymmetric $\rightarrow$ signum representation
  - 2 octets $\rightarrow$ mixed symmetric & mixed antisymmetric $\rightarrow$ two-dim. representation

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$$F_d^{f_{\pi(1)} f_{\pi(2)} f_{\pi(3)}} = F_d^{f_1 f_2 f_3} \qquad S_d^{\alpha_{\pi(1)} \alpha_{\pi(2)} \alpha_{\pi(3)}} = S_d^{\alpha_1 \alpha_2 \alpha_3}$$

$$F_s^{f_{\pi(1)} f_{\pi(2)} f_{\pi(3)}} = \mathrm{sgn}(\pi) F_s^{f_1 f_2 f_3} \qquad S_s^{\alpha_{\pi(1)} \alpha_{\pi(2)} \alpha_{\pi(3)}} = \mathrm{sgn}(\pi) S_s^{\alpha_1 \alpha_2 \alpha_3}$$

$$F_{o,t}^{f_{\pi(1)} f_{\pi(2)} f_{\pi(3)}} = \sum_{t'=1}^{2} \Big[T(\pi)\Big]_{t't} F_{o,t'}^{f_1 f_2 f_3} \qquad S_{o,t}^{\alpha_{\pi(1)} \alpha_{\pi(2)} \alpha_{\pi(3)}} = \sum_{t'=1}^{2} \Big[T(\pi)\Big]_{t't} S_{o,t'}^{\alpha_1 \alpha_2 \alpha_3}$$

Renormalization matrix (Nucleon)

For the nucleon we have the following renormalization pattern:

- A simple multiplicative renormalization for f^N

$$\left(f^N\right)^{\overline{\mathsf{MS}}} = Z^{\mathscr{O} f} \left(f^N\right)^{\mathsf{lat}}$$

- A 2×2 mixing matrix for the higher twist couplings λ_1^N and λ_2^N
- A 3×3 mixing matrix for the three first moments

$$\begin{pmatrix} \varphi^N_{00,(1)} \\ \sqrt{2}\varphi^N_{11} \\ \sqrt{2}\varphi^N_{10} \end{pmatrix}^{\overline{\mathsf{MS}}} = \begin{pmatrix} Z^{\mathscr{O}\varphi}_{11} & Z^{\mathscr{O}\varphi}_{12} & Z^{\mathscr{O}\varphi}_{13} \\ Z^{\mathscr{O}\varphi}_{21} & Z^{\mathscr{O}\varphi}_{22} & Z^{\mathscr{O}\varphi}_{23} \\ Z^{\mathscr{O}\varphi}_{31} & Z^{\mathscr{O}\varphi}_{32} & Z^{\mathscr{O}\varphi}_{33} \end{pmatrix} \begin{pmatrix} \varphi^N_{00,(1)} \\ \sqrt{2}\varphi^N_{11} \\ \sqrt{2}\varphi^N_{10} \end{pmatrix}^{\mathsf{lat}}$$

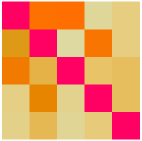
Renormalization matrix (full beauty)

- The general case is more complicated due to the additional moments of Π^B
- Renormalization of f^B is not necessarily multiplicative
- Decuplet and singlet renormalization factors are now also relevant for octet baryons

$$\begin{pmatrix} f^{B\neq\Lambda} \\ f_T^{B\neq\Lambda} \end{pmatrix}^{\overline{\mathsf{MS}}} = \frac{1}{3} \begin{pmatrix} Z^{\mathscr{O}f} + 2Z^{\mathscr{D}f} & 2Z^{\mathscr{O}f} - 2Z^{\mathscr{D}f} \\ Z^{\mathscr{O}f} - Z^{\mathscr{D}f} & 2Z^{\mathscr{O}f} + Z^{\mathscr{D}f} \end{pmatrix} \begin{pmatrix} f^B \\ f_T^B \end{pmatrix}^{\mathsf{lat}}$$

$$\begin{pmatrix} \varphi_{00,(1)}^{B\neq\Lambda} \\ \pi_{00,(1)}^{B\neq\Lambda} \\ \sqrt{2}\varphi_{11}^{B\neq\Lambda} \\ \sqrt{2}\pi_{11}^{B\neq\Lambda} \\ \sqrt{2}\varphi_{10}^{B\neq\Lambda} \end{pmatrix}^{\overline{\mathsf{MS}}} = \frac{1}{3} \begin{pmatrix} Z_{11}^{\mathscr{O}\varphi} + 2Z_{11}^{\mathscr{D}\varphi} & 2Z_{11}^{\mathscr{O}\varphi} - 2Z_{11}^{\mathscr{D}\varphi} & Z_{12}^{\mathscr{O}\varphi} + 2Z_{12}^{\mathscr{D}\varphi} & 2Z_{12}^{\mathscr{O}\varphi} - 2Z_{12}^{\mathscr{D}\varphi} & 3Z_{13}^{\mathscr{O}\varphi} \\ Z_{11}^{\mathscr{O}\varphi} - Z_{11}^{\mathscr{D}\varphi} & 2Z_{11}^{\mathscr{O}\varphi} + Z_{11}^{\mathscr{D}\varphi} & Z_{12}^{\mathscr{O}\varphi} - Z_{12}^{\mathscr{D}\varphi} & 2Z_{12}^{\mathscr{O}\varphi} + Z_{12}^{\mathscr{D}\varphi} & 3Z_{13}^{\mathscr{O}\varphi} \\ Z_{21}^{\mathscr{O}\varphi} + 2Z_{21}^{\mathscr{D}\varphi} & 2Z_{21}^{\mathscr{O}\varphi} - 2Z_{21}^{\mathscr{D}\varphi} & Z_{22}^{\mathscr{O}\varphi} + 2Z_{22}^{\mathscr{D}\varphi} & 2Z_{22}^{\mathscr{O}\varphi} - 2Z_{22}^{\mathscr{D}\varphi} & 3Z_{23}^{\mathscr{O}\varphi} \\ Z_{21}^{\mathscr{O}\varphi} - Z_{21}^{\mathscr{D}\varphi} & 2Z_{21}^{\mathscr{O}\varphi} + Z_{21}^{\mathscr{D}\varphi} & Z_{22}^{\mathscr{O}\varphi} - Z_{22}^{\mathscr{D}\varphi} & 2Z_{22}^{\mathscr{O}\varphi} + Z_{22}^{\mathscr{D}\varphi} & 3Z_{23}^{\mathscr{O}\varphi} \\ Z_{31}^{\mathscr{O}\varphi} & 2Z_{31}^{\mathscr{O}\varphi} & Z_{32}^{\mathscr{O}\varphi} & 2Z_{32}^{\mathscr{O}\varphi} & 3Z_{33}^{\mathscr{O}\varphi} \end{pmatrix} \begin{pmatrix} \varphi_{00,(1)}^{B\neq\Lambda} \\ \pi_{00,(1)}^{B\neq\Lambda} \\ \sqrt{2}\varphi_{11}^{B\neq\Lambda} \\ \sqrt{2}\pi_{11}^{B\neq\Lambda} \\ \sqrt{2}\varphi_{10}^{B\neq\Lambda} \end{pmatrix}^{\mathsf{lat}}$$

- Structure of renormalization matrix:



Renormalization matrix in the $\mathrm{SU}(3)$ limit

- Different renormalization matrices for $B \neq \Lambda$ and $B = \Lambda$

$$\begin{pmatrix} \varphi_{00,(1)}^{B\neq\Lambda} \\ \pi_{00,(1)}^{B\neq\Lambda} \\ \sqrt{2}\varphi_{11}^{B\neq\Lambda} \\ \sqrt{2}\pi_{11}^{B\neq\Lambda} \\ \sqrt{2}\varphi_{10}^{B\neq\Lambda} \end{pmatrix}^{\overline{\mathrm{MS}}} = \frac{1}{3}\begin{pmatrix} Z_{11}^{\mathscr{O}\varphi} + 2Z_{11}^{\mathscr{D}\varphi} & 2Z_{11}^{\mathscr{O}\varphi} - 2Z_{11}^{\mathscr{D}\varphi} & Z_{12}^{\mathscr{O}\varphi} + 2Z_{12}^{\mathscr{D}\varphi} & 2Z_{12}^{\mathscr{O}\varphi} - 2Z_{12}^{\mathscr{D}\varphi} & 3Z_{13}^{\mathscr{O}\varphi} \\ Z_{11}^{\mathscr{O}\varphi} - Z_{11}^{\mathscr{D}\varphi} & 2Z_{11}^{\mathscr{O}\varphi} + Z_{11}^{\mathscr{D}\varphi} & Z_{12}^{\mathscr{O}\varphi} - Z_{12}^{\mathscr{D}\varphi} & 2Z_{12}^{\mathscr{O}\varphi} + Z_{12}^{\mathscr{D}\varphi} & 3Z_{13}^{\mathscr{O}\varphi} \\ Z_{21}^{\mathscr{O}\varphi} + 2Z_{21}^{\mathscr{D}\varphi} & 2Z_{21}^{\mathscr{O}\varphi} - 2Z_{21}^{\mathscr{D}\varphi} & Z_{22}^{\mathscr{O}\varphi} + 2Z_{22}^{\mathscr{D}\varphi} & 2Z_{22}^{\mathscr{O}\varphi} - 2Z_{22}^{\mathscr{D}\varphi} & 3Z_{23}^{\mathscr{O}\varphi} \\ Z_{21}^{\mathscr{O}\varphi} - Z_{21}^{\mathscr{D}\varphi} & 2Z_{21}^{\mathscr{O}\varphi} + Z_{21}^{\mathscr{D}\varphi} & Z_{22}^{\mathscr{O}\varphi} - Z_{22}^{\mathscr{D}\varphi} & 2Z_{22}^{\mathscr{O}\varphi} + Z_{22}^{\mathscr{D}\varphi} & 3Z_{23}^{\mathscr{O}\varphi} \\ Z_{31}^{\mathscr{O}\varphi} & 2Z_{31}^{\mathscr{O}\varphi} & Z_{32}^{\mathscr{O}\varphi} & 2Z_{32}^{\mathscr{O}\varphi} & 3Z_{33}^{\mathscr{O}\varphi} \end{pmatrix} \begin{pmatrix} \varphi_{00,(1)}^{B\neq\Lambda} \\ \pi_{00,(1)}^{B\neq\Lambda} \\ \sqrt{2}\varphi_{11}^{B\neq\Lambda} \\ \sqrt{2}\pi_{11}^{B\neq\Lambda} \\ \sqrt{2}\varphi_{10}^{B\neq\Lambda} \end{pmatrix}^{\mathrm{lat}}$$

$$\begin{pmatrix} \varphi_{00,(1)}^{\Lambda} \\ \sqrt{2}\varphi_{11}^{\Lambda} \\ \sqrt{2}\varphi_{10}^{\Lambda} \\ \sqrt{2}\pi_{10}^{\Lambda} \end{pmatrix}^{\overline{\mathrm{MS}}} = \frac{1}{3}\begin{pmatrix} 3Z_{11}^{\mathscr{O}\varphi} & 3Z_{12}^{\mathscr{O}\varphi} & Z_{13}^{\mathscr{O}\varphi} & 2Z_{13}^{\mathscr{O}\varphi} \\ 3Z_{21}^{\mathscr{O}\varphi} & 3Z_{22}^{\mathscr{O}\varphi} & Z_{23}^{\mathscr{O}\varphi} & 2Z_{23}^{\mathscr{O}\varphi} \\ 3Z_{31}^{\mathscr{O}\varphi} & 3Z_{32}^{\mathscr{O}\varphi} & Z_{33}^{\mathscr{O}\varphi} + 2Z^{\mathscr{S}\varphi} & 2Z_{33}^{\mathscr{O}\varphi} - 2Z^{\mathscr{S}\varphi} \\ 3Z_{31}^{\mathscr{O}\varphi} & 3Z_{32}^{\mathscr{O}\varphi} & Z_{33}^{\mathscr{O}\varphi} - Z^{\mathscr{S}\varphi} & 2Z_{33}^{\mathscr{O}\varphi} + Z^{\mathscr{S}\varphi} \end{pmatrix} \begin{pmatrix} \varphi_{00,(1)}^{\Lambda} \\ \sqrt{2}\varphi_{11}^{\Lambda} \\ \sqrt{2}\varphi_{10}^{\Lambda} \\ \sqrt{2}\pi_{10}^{\Lambda} \end{pmatrix}^{\mathrm{lat}}$$

- In the $\mathrm{SU}(3)$ symmetric limit $\pi_{00,(1)} \to \varphi_{00,(1)}$, $\pi_{11} \to \varphi_{11}$ and $\pi_{10} \to \varphi_{10}$
  $\Rightarrow$ The renormalization matrix for all baryons is reduced to

$$\begin{pmatrix} \varphi_{00,(1)}^{\star} \\ \sqrt{2}\varphi_{11}^{\star} \\ \sqrt{2}\varphi_{10}^{\star} \end{pmatrix}^{\overline{\mathrm{MS}}} = \begin{pmatrix} Z_{11}^{\mathscr{O}\varphi} & Z_{12}^{\mathscr{O}\varphi} & Z_{13}^{\mathscr{O}\varphi} \\ Z_{21}^{\mathscr{O}\varphi} & Z_{22}^{\mathscr{O}\varphi} & Z_{23}^{\mathscr{O}\varphi} \\ Z_{31}^{\mathscr{O}\varphi} & Z_{32}^{\mathscr{O}\varphi} & Z_{33}^{\mathscr{O}\varphi} \end{pmatrix} \begin{pmatrix} \varphi_{00,(1)}^{\star} \\ \sqrt{2}\varphi_{11}^{\star} \\ \sqrt{2}\varphi_{10}^{\star} \end{pmatrix}^{\mathrm{lat}}$$

Test of the renormalization programme

- In the continuum

$$\mathrm{DA}_{lmn} = \int [dx]\, x_1^l x_2^m x_3^n \mathrm{DA}(x_1,x_2,x_3) = \int [dx]\, (x_1+x_2+x_3) x_1^l x_2^m x_3^n \mathrm{DA}(x_1,x_2,x_3)$$
$$= \mathrm{DA}_{(l+1)mn} + \mathrm{DA}_{l(m+1)n} + \mathrm{DA}_{lm(n+1)}$$

- On the lattice

$$\varphi^B_{00,(1)} \propto [V-A]^B_{100} + [V-A]^B_{010} + [V-A]^B_{001} \stackrel{?}{=} [V-A]^B_{000} \propto f^B$$
$$\pi^{B\neq\Lambda}_{00,(1)} = T^B_{100} + T^B_{010} + T^B_{001} \stackrel{?}{=} T^B_{000} = f^B_T$$

Test of the renormalization programme

- In the continuum

$$\mathrm{DA}_{lmn} = \int [dx]\, x_1^l x_2^m x_3^n \mathrm{DA}(x_1,x_2,x_3) = \int [dx]\,(x_1+x_2+x_3) x_1^l x_2^m x_3^n \mathrm{DA}(x_1,x_2,x_3)$$
$$= \mathrm{DA}_{(l+1)mn} + \mathrm{DA}_{l(m+1)n} + \mathrm{DA}_{lm(n+1)}$$

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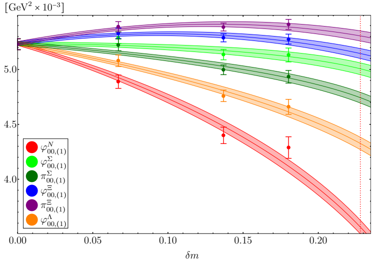
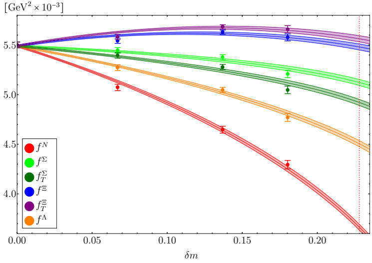
$$\varphi^B_{00,(1)} \propto [V-A]^B_{100} + [V-A]^B_{010} + [V-A]^B_{001} \stackrel{?}{=} [V-A]^B_{000} \propto f^B$$
$$\pi^{B\neq\Lambda}_{00,(1)} = T^B_{100} + T^B_{010} + T^B_{001} \stackrel{?}{=} T^B_{000} = f^B_T$$





Test of the renormalization programme

- In the continuum

$$\mathrm{DA}_{lmn} = \int [dx]\, x_1^l x_2^m x_3^n \mathrm{DA}(x_1,x_2,x_3) = \int [dx]\, (x_1+x_2+x_3) x_1^l x_2^m x_3^n \mathrm{DA}(x_1,x_2,x_3)$$
$$= \mathrm{DA}_{(l+1)mn} + \mathrm{DA}_{l(m+1)n} + \mathrm{DA}_{lm(n+1)}$$

- On the lattice

$$\varphi^B_{00,(1)} \propto [V-A]^B_{100} + [V-A]^B_{010} + [V-A]^B_{001} \stackrel{?}{=} [V-A]^B_{000} \propto f^B$$
$$\pi^{B\neq\Lambda}_{00,(1)} = T^B_{100} + T^B_{010} + T^B_{001} \stackrel{?}{=} T^B_{000} = f^B_T$$

- Numerical results

	$\varphi^N_{00,(1)}/f^N$	$\varphi^\Sigma_{00,(1)}/f^\Sigma$	$\varphi^\Xi_{00,(1)}/f^\Xi$	$\varphi^\Lambda_{00,(1)}/f^\Lambda$	$\pi^\Sigma_{00,(1)}/f^\Sigma_T$	$\pi^\Xi_{00,(1)}/f^\Xi_T$
$\overline{\text{MS}}$	0.988(35)	0.971(17)	0.963(13)	0.971(23)	0.965(17)	0.967(13)
bare	0.842(30)	0.827(14)	0.820(11)	0.827(19)	0.822(14)	0.824(11)

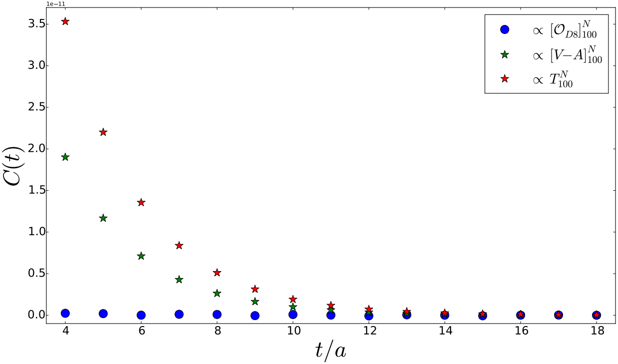
Summary and outlook

- We carried out a nonperturbative renormalization in a RI′/SMOM scheme, followed by a conversion to $\overline{\text{MS}}$ applying continuum perturbation theory at one-loop accuracy
- Constructed operators to make full use of the underlying symmetry groups
- First study to incorporate the full $\mathrm{SU}(3)$ baryon octet
- Renormalization matrices for the normalizations and the first moments of the leading twist DAs (and also for all higher twist normalization constants)
- Renormalization for the second moments of the leading twist DAs will be available for use in future studies

Backup slides

What about $\mathcal{O}_{D8}$?

- $\mathcal{O}_{D8}$: leading twist, LLL/RRR $\Rightarrow$???
- Built from Dirac structures corresponding to twist 4 distribution amplitude Ξ_4
- Operator is non-zero but octet baryon matrix elements are zero
- Verified in the continuum and on the lattice



Conversion to $\overline{\mathsf{MS}}$

- Renormalization condition

$$\left(L^{\mathsf{S}}\right)_{mn} = \sum_{i} \sum_{\substack{f,g,h \\ \alpha,\beta,\gamma}} \Lambda^{\mathsf{S}}\left(\mathcal{O}_m^{(i)} \middle| p_{\mathsf{SMOM}}\right)_{\alpha\beta\gamma}^{fgh} \left(\Lambda^{\mathsf{Born}}\left(\mathcal{O}_n^{(i)} \middle| p_{\mathsf{SMOM}}\right)_{\alpha\beta\gamma}^{fgh}\right)^{*}$$

$$\mathbb{1} \stackrel{!}{=} L^{\mathsf{RI}}\left(L^{\mathsf{Born}}\right)^{-1}$$

- Define conversion matrix

$$\Lambda^{\overline{\mathsf{MS}}}\left(\mathcal{O}_m^{(i)} \middle| p_{\mathsf{SMOM}}\right) = C_{mm'} \Lambda^{\mathsf{RI}}\left(\mathcal{O}_{m'}^{(i)} \middle| p_{\mathsf{SMOM}}\right)$$

- Calculate conversion matrix

$$\left(L^{\overline{\mathsf{MS}}}\left(L^{\mathsf{Born}}\right)^{-1}\right)_{mn} = C_{mm'}\left(L^{\mathsf{RI}}\left(L^{\mathsf{Born}}\right)^{-1}\right)_{m'n} = C_{mm'}\mathbb{1}_{m'n} = C_{mn}$$