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Asymptotically Safe Gauge-Yukawa Theories and fRG

based on paper in progress with D. Litim

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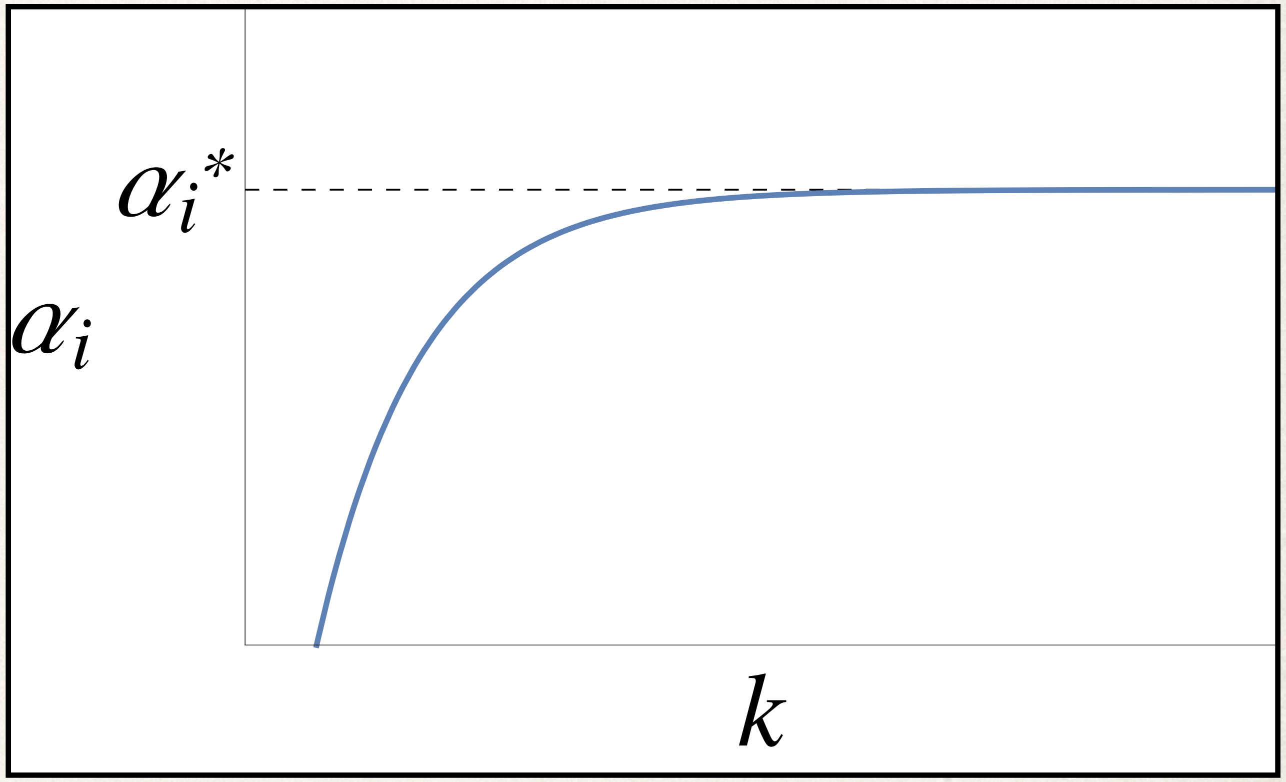
Asymptotic Safety

$$\beta_i = k\partial_k \alpha_i = A\alpha_i + B\alpha_i^2 + \cdots$$

$$\alpha_i* = 0 \qquad \alpha_i* = -\frac{A}{B}$$

- Asymptotic = in the UV
- Safe = no poles
- Existence of an interacting (non-zero) UV fixed point.
- Conjectured by Weinberg in 79′ for QG, and tested many times for various theories and truncations.



Ref: Litim, Sannino JHEP 1412 (2014) 178

Asymptotically Safe Gauge-Yukawa

$$L = \frac{1}{2}\mathrm{Tr}F^{\mu\nu}F_{\mu\nu} + \mathrm{Tr}(\bar{Q}i\gamma^{\mu}D_{\mu}Q) + y\mathrm{Tr}(\bar{Q}HQ) + \mathrm{Tr}(\partial_{\mu}H^{\dagger}\partial^{\mu}H) - V$$

$$V = \bar{\lambda}_1\mathrm{Tr}\left(H^{\dagger}H - \frac{1}{N_f}\mathrm{Tr}H^{\dagger}H\right)^2 + \bar{\lambda}_2(\mathrm{Tr}H^{\dagger}H)^2$$

- H is an N_f x N_f complex scalar matrix , where N_f is the number of fermion flavours. (not charged)
- SU(N_c) gauge fields.
- Q's are fermions in the fundamental representation.
- in Veneziano limit $\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$ $\quad 0 \leq \epsilon << 1$

$$\beta_g = \alpha_g \left(\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right)$$

$$\beta_y = \alpha_y \left((13 + 2\epsilon)\alpha_y - 6\alpha_g \right)$$

NLO

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2}$$

$$\Delta\beta_g^{(3)} = \alpha_g^2 \left(\left(\frac{701}{6} + \frac{53}{3}\epsilon - \frac{112}{27}\epsilon^2 \right)\alpha_g^2 - \frac{27}{8}(11 + 2\epsilon)^2 \alpha_g \alpha_y + \frac{1}{4}(11 + 2\epsilon)^2 (20 + 3\epsilon)\alpha_y^2 \right)$$

$$\Delta\beta_y^{(2)} = \alpha_y \left(\frac{20\epsilon - 93}{6}\alpha_g^2 + (49 + 8\epsilon)\alpha_g\alpha_y - \left(\frac{385}{8} + \frac{23}{2}\epsilon + \frac{\epsilon^2}{2} \right)\alpha_y^2 - (44 + 8\epsilon)\alpha_y\lambda_1 + 4\lambda_1^2 \right)$$

$$\beta_1 = -\alpha_y^2(2\epsilon + 11) + 4\alpha_y\lambda_1 + 8\lambda_1^2$$

$$\beta_2 = -\alpha_y^2(2\epsilon + 11) + 4\alpha_y\lambda_2 + 8\lambda_1^2 + 8\lambda_1\lambda_2 + 4\lambda_2^2$$

NNLO

- Computed in perturbation theory.
- Important note: Yukawa couples with scalars in 2-loop Yukawa beta function. And gauge couples indirectly via Yukawa in 3-loop level.

Ref: Litim, Sannino JHEP 1412 (2014) 178

$$\alpha_g* = 0.456140\,\epsilon + O(\epsilon^2)$$
$$\alpha_y* = 0.210526\,\epsilon + O(\epsilon^2)$$
$$\lambda_1* = 0.199781\,\epsilon + O(\epsilon^2)$$
$$\lambda_2* = 0.0625304\,\epsilon + O(\epsilon^2)$$

$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

- Fixed points are found by solving the beta functions for zero.
- Leading order terms are order epsilon.

Ref: Litim, Sannino JHEP 1412 (2014) 178

- Scaling exponents = - (eigenvalues of the stability matrix)
- They are the universal quantities that determine the phase structure. i.e. how does the RG flow approach the fixed point.
- Operators can have relevant, irrelevant or marginal directions depending on the sign of the scaling exponents.
- A given theory is predictive as long as it has a finite number of relevant directions.
- Negative eigenvalues: relevant directions , positive eigenvalues: irrelevant directions.

$$M_{ij} = \frac{\partial \beta_i}{\partial g_j}$$

$$eig(M_{ij}) = \theta$$

$$\theta_1 = -0.590643\epsilon^2 + O(\epsilon^3)$$
$$\theta_2 = 2.7368\epsilon + O(\epsilon^2)$$
$$\theta_3 = 4.03859\epsilon + O(\epsilon^2)$$
$$\theta_4 = 2.94059\epsilon + O(\epsilon^2)$$

Higher Dimension Operators

$$v_k(i_1, i_2) = u_k(i_1) + i_2 c_k(i_1)$$

$$u_k(i_1) = \sum_{j=2}^{N_i} \frac{(4\pi)^{2j-2} i_1^j \lambda_2 j - 2}{N_f^{2j-2}}$$

$$c_k(i_1) = \sum_{i=0}^{N_i} \frac{(4\pi)^{2j+2} i_1^j \lambda_2 j + 1}{N_f^{2j+1}}$$

$$i_1 = \mathrm{Tr}(h^\dagger h)$$

$$i_2 = \mathrm{Tr}\left(h^\dagger h - \frac{1}{N_f}(\mathrm{Tr} h^\dagger h)\right)^2$$

- We define a dimensionless, scale dependent potential that describe the scalar self interactions.

Motivations

- How do the higher dimensional operators in the scalar potential affect the fixed point structure?
- How many relevant/irrelevant operators do we have?
- What does the shape of the potential look like with the contribution of the higher dimensional operators?
- How does the functional methods/inclusion of the threshold effects due to massive modes affect the fixed point structure?

Functional Renormalisation Group

- Since all the higher order scalar self couplings have coupling constants with negative canonical mass dimensions, these are not perturbatively renormalisable => non-perturbative => fRG

- We use Wetterich equation which is an exact RG equation in the form of a 1-loop propagator, includes the contribution of all loop orders.

$$k\partial_k\Gamma_k = \frac{1}{2}\left(\mathrm{Tr}\frac{k\partial_k R_k}{\Gamma_k^{(2)} + R_k}\right)$$

Γ_k : Effective Average Action

$\Gamma_k^{(2)}$: The Hessian

$$R_k = (k^2 - q^2)\Theta(k^2 - q^2)$$

: The Regulator

Refs: C.Wetterich - Phys.Lett. B301 (1993) 90-94
D.Litim - Nucl.Phys.B631:128-158,2002

- We first take the wave function renormalisation Z=1, therefore we ignore the effects from the anomalous dimension.

- All the beta functions are computed from Wetterich equation by taking the appropriate derivatives.

- We compute the flow only for the scalar fields, where we add Yukawa contribution from perturbation theory.

- We confirm that the beta functions from the non-perturbative computation match the perturbative computation perfectly. And go on to calculate beyond quartic terms.

- Then we compute the fixed points by systematically solving beta functions one by one up to leading order in epsilon.

Coupling	Fixed Point
λ_1	$0.199781\,\epsilon$
λ_2	$0.0625304\,\epsilon$
λ_3	$0.442635\,\epsilon^3$
λ_4	$0.197829\,\epsilon^3$
λ_5	$-0.42182\,\epsilon^4$
λ_6	$-0.0912196\,\epsilon^4$
λ_7	$0.442354\,\epsilon^5$
λ_8	$0.0561861\,\epsilon^5$
λ_9	$-0.466105\,\epsilon^6$
λ_{10}	$-0.0389432\,\epsilon^6$
λ_{11}	$0.486798\,\epsilon^7$
λ_{12}	$0.0287923\,\epsilon^7$
λ_{13}	$-0.503072\,\epsilon^8$
λ_{14}	$-0.0221745\,\epsilon^8$

$$\lambda_i = \bar{\lambda}_i k^{-d_i}$$

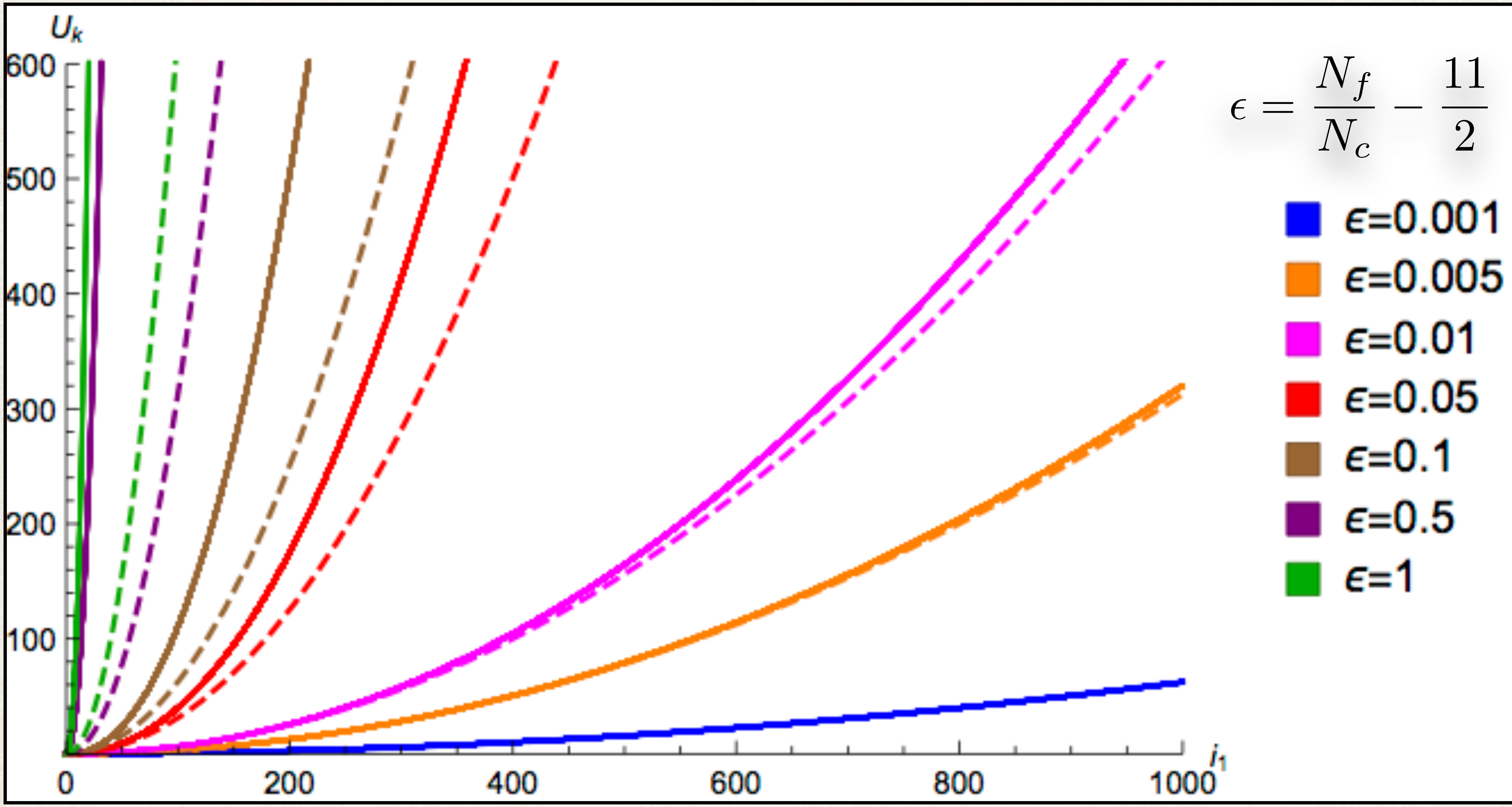
$$\beta_i = -d_i \lambda_i + \# \lambda_i^2 + \cdots$$

$$\beta_i = 0 \quad \& \quad d_i = 0 \qquad \Longrightarrow$$

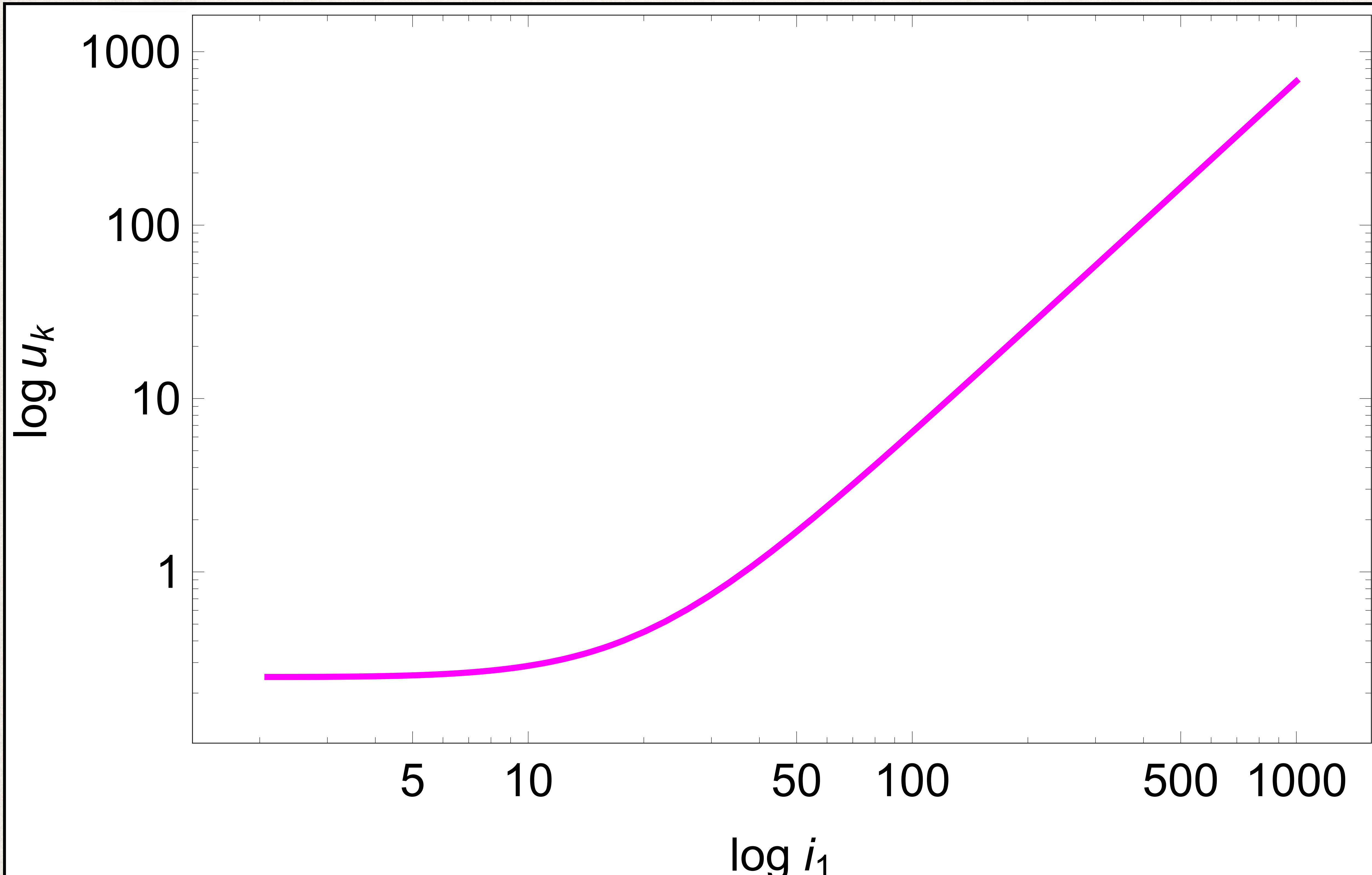
Leading order epsilon power in the fixed point will drop by half

The Global Effective Potential

- We compute the GEP by solving the Wetterich equation for a random potential without assuming an ansatz.
- LHS = 0 at the fixed point.
- 0 = RHS is a differential equation as a function of the potential and its derivatives.
- We plot the numerical result and compare it to the quartic truncation of the potential.


U_k
600
500
400
300
200
100
0
200
400
600
800
1000
i_1
$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$
ϵ=0.001
ϵ=0.005
ϵ=0.01
ϵ=0.05
ϵ=0.1
ϵ=0.5
ϵ=1

$\epsilon = 0.01$

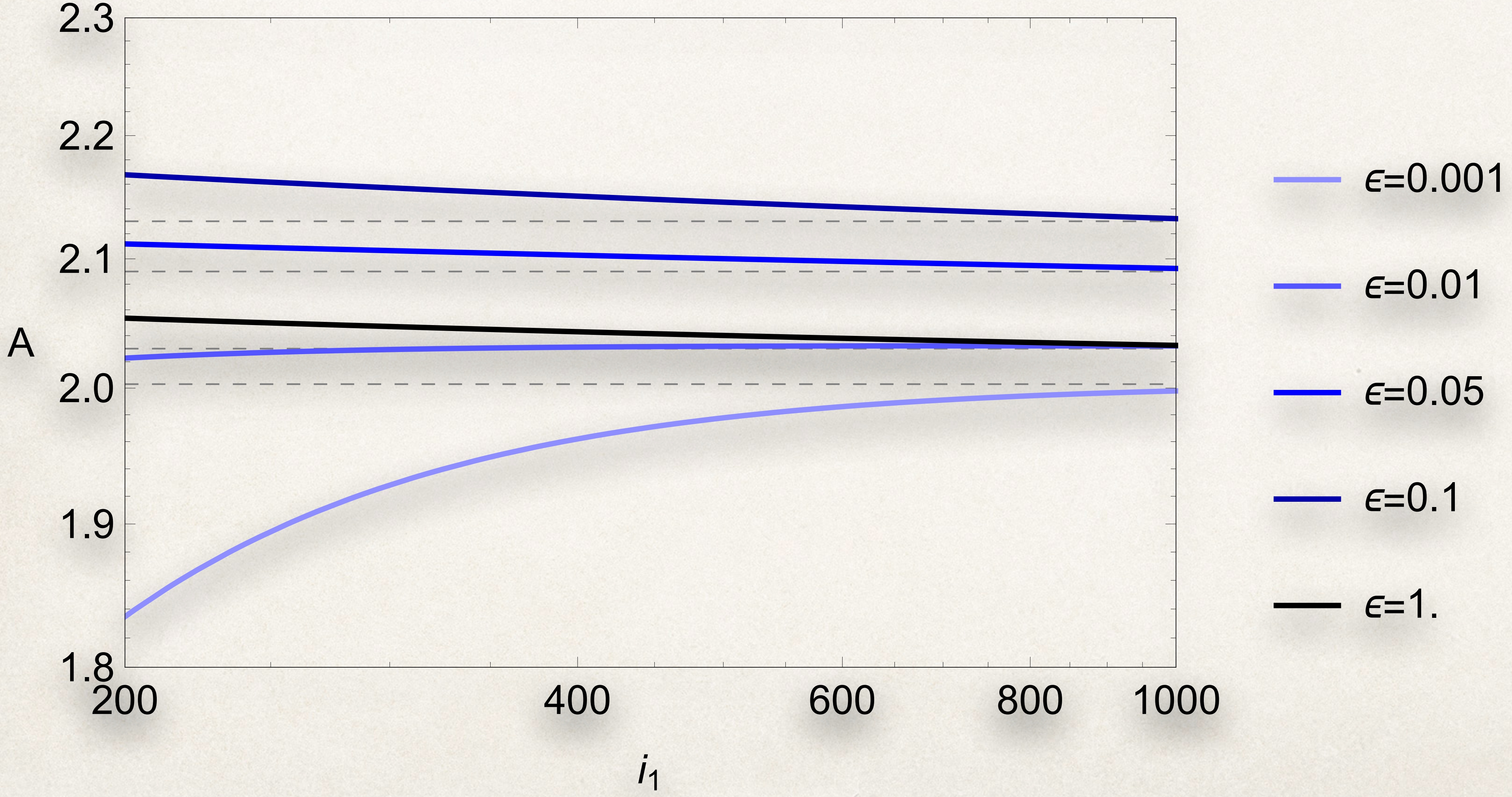


- We can compute the power law by using:

$$\log u_k = A \log i_1 + B$$

$$A \equiv \frac{1}{u_k} i_1 \partial_{i_1} u_k(i_1)$$

$$i_1 = \mathrm{Tr}(h^\dagger h)$$



Eigenvalues of the Stability Matrix

- are the universal quantities.

- **Bootstrap hypothesis:**

$$\alpha_i = \bar{\alpha}_i k^{-d_i}$$

$$\beta_i = -d_i \alpha_i + \text{quantum fluctuations}$$

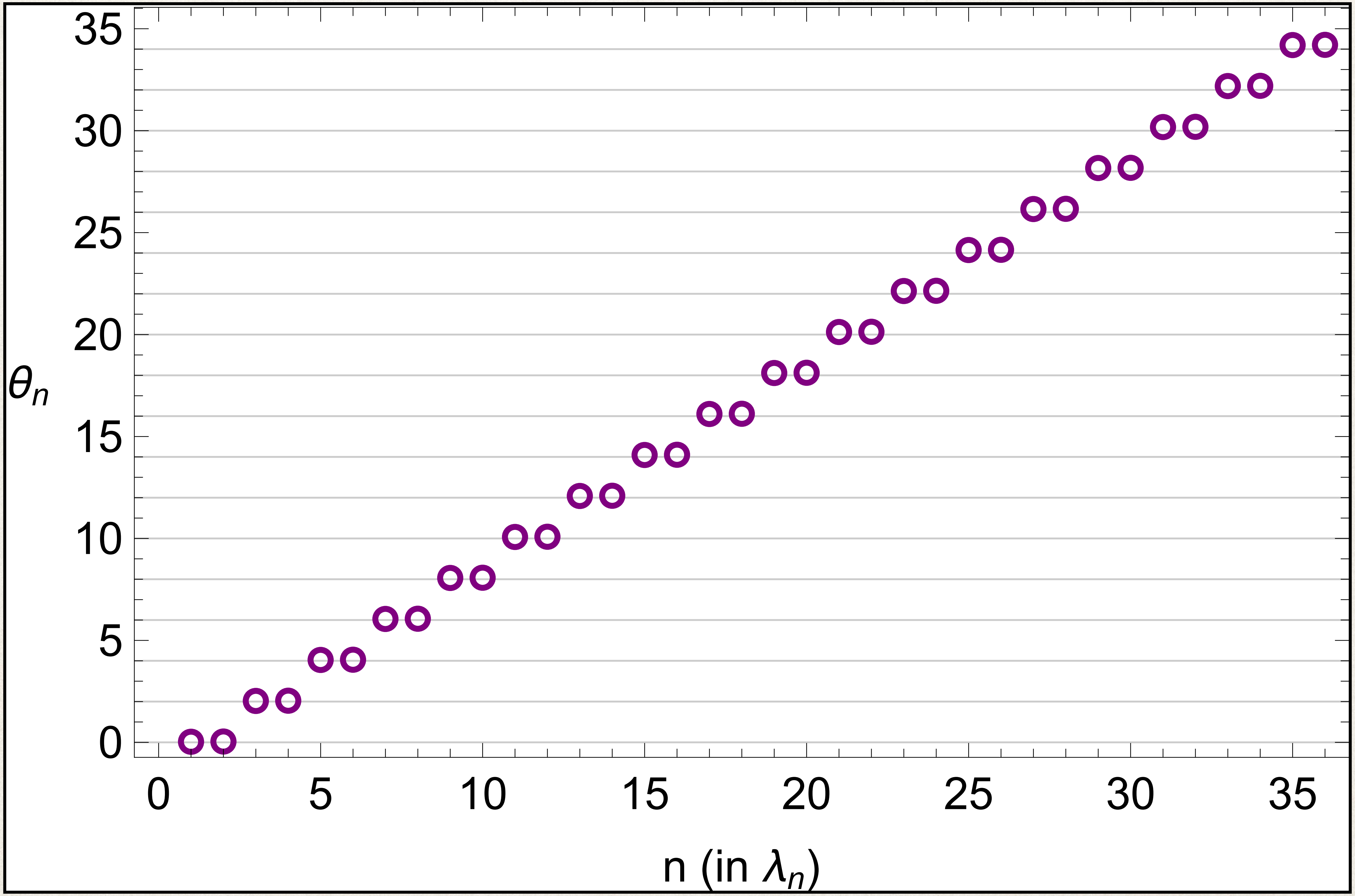
$$M_{ij} = \left.\frac{\partial \beta_i}{\partial \alpha_j}\right|_{\alpha_i^*}$$

$$\theta_i = -d_i + \text{correction from non-diagonal}$$

Ref: Falls, Litim, Nikolakopoulos, Rahmede - arXiv:1301.4191

Coupling	**Fixed Point**	**Eigenvalues of the Stability Matrix**
λ_1	$0.199781\,\epsilon$	$4.03859\,\epsilon$
λ_2	$0.0625304\,\epsilon$	$2.94059\,\epsilon$
λ_3	$0.442635\,\epsilon^3$	$2 + 3.14773\,\epsilon$
λ_4	$0.197829\,\epsilon^3$	$2 + 4.24573\,\epsilon$
λ_5	$-0.42182\,\epsilon^4$	$4 + 4.19698\,\epsilon$
λ_6	$-0.0912196\,\epsilon^4$	$4 + 5.29498\,\epsilon$
λ_7	$0.442354\,\epsilon^5$	$6 + 5.24622\,\epsilon$
λ_8	$0.0561861\,\epsilon^5$	$6 + 6.34422\,\epsilon$
λ_9	$-0.466105\,\epsilon^6$	$8 + 6.29546\,\epsilon$
λ_{10}	$-0.0389432\,\epsilon^6$	$8 + 7.39347\,\epsilon$
λ_{11}	$0.486798\,\epsilon^7$	$10 + 7.34471\,\epsilon$
λ_{12}	$0.0287923\,\epsilon^7$	$10 + 8.44271\,\epsilon$
λ_{13}	$-0.503072\,\epsilon^8$	$12 + 8.39395\,\epsilon$
λ_{14}	$-0.0221745\,\epsilon^8$	$12 + 9.49195\,\epsilon$

$\epsilon = 0.01$



Conclusions - Outlook

- Fixed points with irrelevant directions exist with the inclusion of the higher dimensional terms.
- Higher dimensional couplings are higher leading order in epsilon.
- The eigenvalues of the stability matrix are $-d_i + O(\epsilon)$ as expected. This satisfies the bootstrap hypothesis.
- Potential is stable at large field values. Next: Cosmological implications are to be checked.
- Potential's asymptotic behaviour is very close to a quartic potential.