

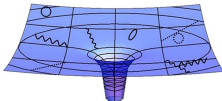
# Spectroscopy of two dimensional $N=2$ Super Yang Mills theory

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# Motivation

Many interesting non-perturbative aspects of SUSY

confinement, massless composite fermions,  
spontaneous chiral symmetry breaking,...

Our focus:  $N=2$  SYM in  $d=2$

dimensional reduction of  $N=1$   $d=4$  SYM  
gauge group  $SU(2)$

[DESY-Münster Collaboration]

Restoration of SUSY via parameter fine-tuning

# SUSY on the lattice

## Problem

$\{Q^I, (Q^I)^\dagger\} \sim P^\mu \rightarrow$  SUSY must be broken on the lattice  
or: Leibniz rule is violated  $\rightarrow$  Action not invariant under SUSY

two possible solutions

Accidental Symmetry

Preserve subset of SUSY

# Model

## N=2 SYM in d=2

### Reducible Model

$$\mathcal{L} = \frac{1}{g^2} \text{tr} \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\lambda} \Gamma_{\mu} D^{\mu} \lambda - \frac{1}{2} D_{\mu} \phi_m D^{\mu} \phi^m + \frac{1}{2} \bar{\lambda} \Gamma_{1+m} [\phi^m, \lambda] + \frac{1}{4} [\phi_m, \phi_n] [\phi^m, \phi^n] \right\}$$

### Field Content

- $\lambda$  Majorana fermion with four spin indices
- $\Gamma_{\mu}$  four dimensional Gamma matrices
- $A_{\mu}$  two dimensional gauge fields
- $\phi^m$  two scalars

# Model

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### Wilson Fermions

- introduce Wilson term for  $\lambda$
- preserves the R symmetry
- flat directions in the scalar potential

# Fine-tuning

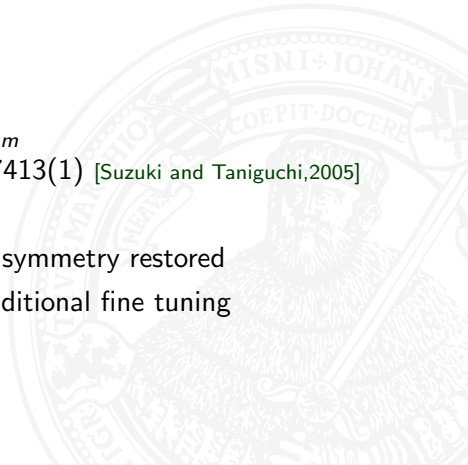
## General Considerations

### Loop Expansion

- super renormalizable
- only relevant operator  $m_s^2 \phi^m \phi_m$   
continuum limit:  $m_s^2 = 0.3297413(1)$  [Suzuki and Taniguchi,2005]

### Chiral Limit

- in the continuum limit: chiral symmetry restored
- improve lattice results with additional fine tuning
- introduce the operator  $m_F \bar{\lambda} \lambda$



# Fine-tuning

## Ward Identities

- the originating operator is

$$\text{Tr}_c \left\{ \bar{\lambda}_b(x) (\gamma_{\mu\nu})_a^b F^{\mu\nu}(x) \right\}$$

### Bosonic Ward Identity

$$\beta \langle S_B \rangle = -\frac{3}{8} \left\langle \frac{i}{2} \bar{\lambda}(x) \not{D} \lambda(x) \right\rangle = 4.5V$$

- problematic for fine-tuning
- we use the Ward Identity

$$\beta \left\langle \frac{1}{2} [\phi_1, \phi_2]^2 \right\rangle = \frac{1}{4} \left\langle \frac{i}{2} \bar{\lambda} \Gamma_2 [\phi_1, \lambda] + \frac{i}{2} \bar{\lambda} \Gamma_3 [\phi_2, \lambda] \right\rangle$$

# Lattice setup and Continuum Limit

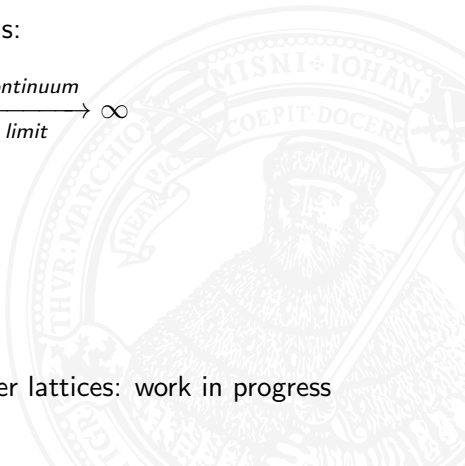
## Continuum Limit

coupling constant in two dimensions:

$$\beta = \frac{N_c}{a^2 g^2} \xrightarrow[\text{limit}]{\text{Continuum}} \infty$$

## Lattice setup

- RHMC algorithm
- Symanzik improved action
- Wilson fermions
- 16x16 and 32x32 lattices, larger lattices: work in progress
- $\beta = 1.5, 2.0, 2.5, 3.0$



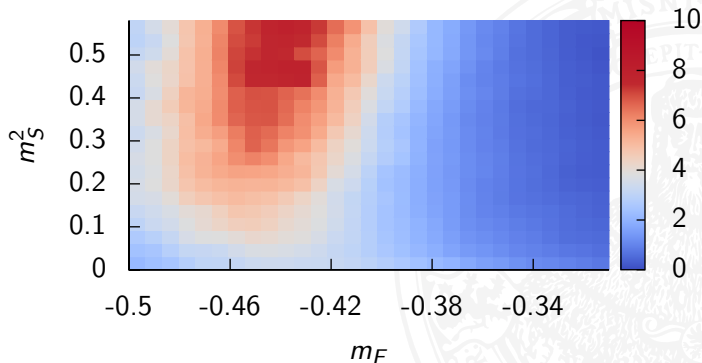


# Chiral Limit

## Parameter space

### Chiral Susceptibility

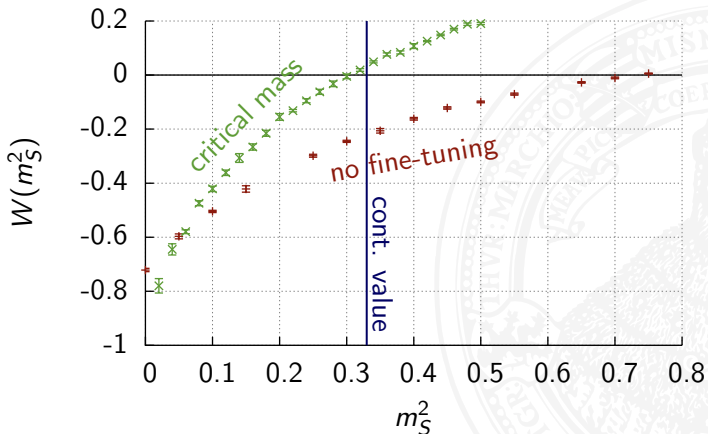
$16 \times 16$  lattice



# Chiral Limit

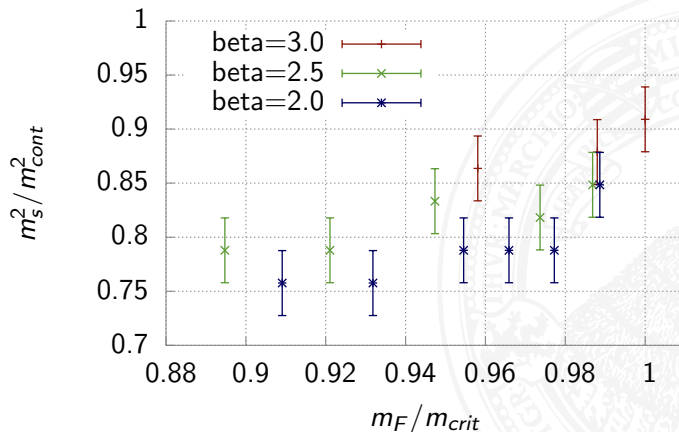
## Ward Identity

16 × 16 lattice



# Chiral Limit Ward Identity

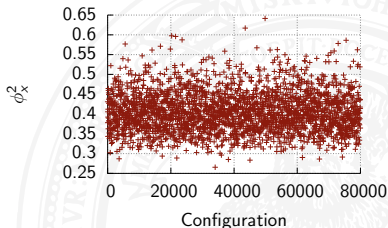
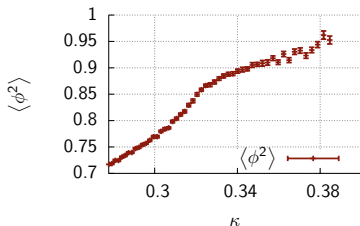
$32 \times 32$  lattice



# Flat directions

## $\langle \phi^2 \rangle$ Measurement

Data for  $m_s^2 = 0$



**Figure:** On the left: value and variance of  $\langle \phi^2 \rangle$  depending on  $\kappa$ . On the right: example data for  $\kappa = 0.3125$ , chiral limit:  $\kappa_{crit} \approx 0.31$

# Mass spectrum

## Expectations from four dimensions

### Veneziano and Yankielowicz

particle	spin	name
$\bar{\lambda}\gamma_5\lambda$	0	a- $\eta'$
$\bar{\lambda}\lambda$	0	a- $f_0$
$F_{\mu\nu}\Sigma^{\mu\nu}\lambda$	$\frac{1}{2}$	gluino-gluonball

[Veneziano and Yankielowicz, 1982]

### Farra, Gabadadze and Schwetz

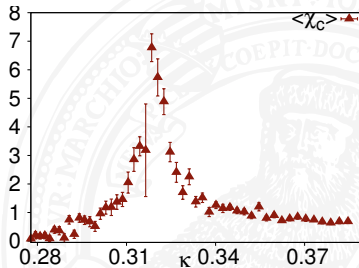
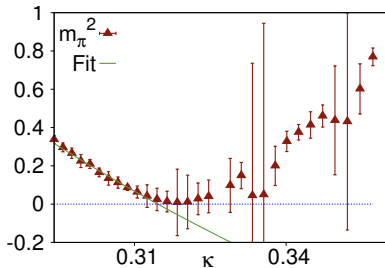
particle	spin	name
$F_{\mu\nu}\epsilon^{\sigma\rho}F^{\sigma\rho}$	0	$0^-$ -gluonball
$F_{\mu\nu}F^{\mu\nu}$	0	$0^+$ -gluonball
$F_{\mu\nu}\Sigma^{\mu\nu}\lambda$	$\frac{1}{2}$	gluino-gluonball

[Farra, Gabadadze and Schwetz, 1998]

# Mass spectrum

## Pion

- gaugino mass related to pion mass  $m_g \sim m_\pi^2$  [Donini et al. 1998]
- connected part of  $\eta' \sim \pi$  [Veneziano and Yankielowicz, 1982]



**Figure:** Pion mass squared with  $\kappa_{crit} \approx 0.315$ (left) and chiral Susceptibility with  $\kappa_{crit} \approx 0.320$ (right)

# Meson Masses

$16 \times 16$

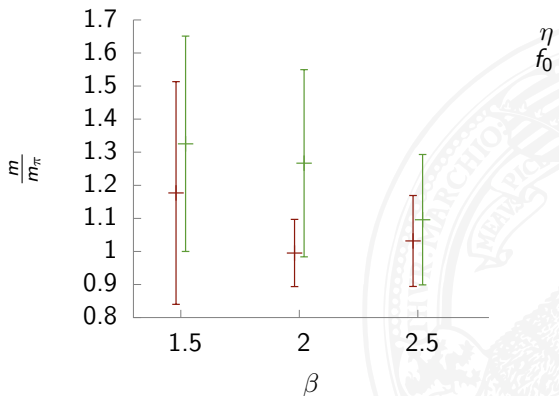


Figure: Ground states for three different  $\beta$

# Glue Ball Correlator

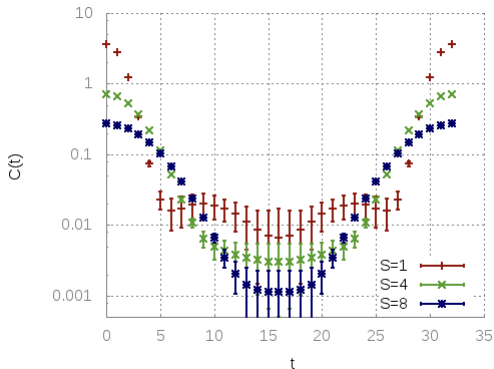


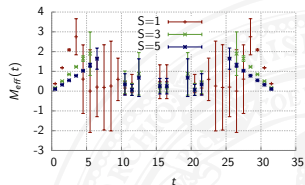
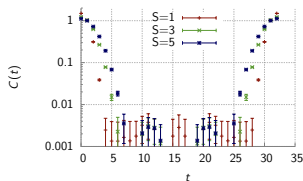
Figure: Glue Ball Correlator for different Smearing parameters

- Smearing Steps  $N_S$
- Smearing Parameter  $\rho_S$
- in Graph:  
 $S = N_S \cdot \rho_S$

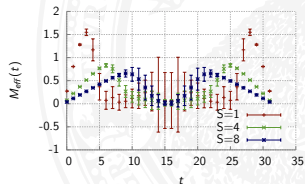
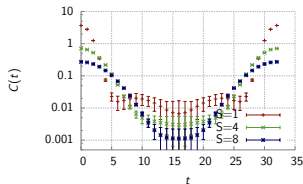


# Comparison Glue Balls

## pure Gauge Theory



## N=2 SYM



# Summary

## Conclusion

- We simulated  $N=1$  SYM dimensional reduced to two dimensions on the lattice in the chiral limit
- probably mass degenerate Meson ground states
- probably massless Glue Balls
- other parts of the super multiplet: work in progress

## Outlook

- perform the continuum limit
- increase lattice size
- check further Ward identities

