Friedrich-Schiller-Universität Jena

Four-Fermion Theories with Exact Chiral Symmetry in Three Dimensions

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Quantum and Gravitational Fields

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Outline

What are Four-Fermion Theories?

QFTs of fermions with 4th power of fermion fields as interaction:

$$\mathcal{L} = \bar{\psi}_j \left(\partial\!\!\!/ + m\right) \psi_j + \sum_\alpha \frac{g_\alpha^2}{2N_f} \left(\bar{\psi}_j \Gamma_\alpha \psi_j\right)^2 \qquad j = 1, \ldots, N_f$$

Main Models

Thirring 1958, soluble fermionic theory in 2D
$\Gamma_\alpha = \gamma_\mu$

Nambu & Jona-Lassinio 1961, dynamical mass generation in 4D
$\Gamma_1 = \mathbb{1}, \Gamma_2 = i\gamma_5$

Gross & Neveu 1974, asymptotic freedom, chiral symmetry breaking in 2D
$\Gamma = \mathbb{1}$

What is the Thirring model?

QFT with N_f flavours of massless fermions with current interaction

Euclidean Lagrangian

$$\mathcal{L} = \bar{\psi}_j \partial\!\!\!/ \psi_j + \frac{g^2}{2N_f} \sum_{\mu=1}^{3} \left(\bar{\psi}_j \gamma^\mu \psi_j\right)^2 \qquad j = 1, \ldots, N_f$$

- 3-dimensional euclidean spacetime
- representation of Clifford algebra:

  irreducible 2-dimensional (later in this talk)

  reducible 4-dimensional (now)

Motivation

Similarity to QED_3 and possible applications in superconductors, graphene, ...

Why is the Thirring model interesting?

Symmetries

- chiral symmetry, generated by $\{\mathbb{1}, \gamma_4, \gamma_5, i\gamma_4\gamma_5\}$
- flavour symmetry

Result: $U(N_f, N_f)$, can be spontaneously broken to $U(N_f) \otimes U(N_f)$
$\Rightarrow$ chiral condensate $\langle \bar{\psi}\psi \rangle \neq 0$

Chiral Symmetry Breaking

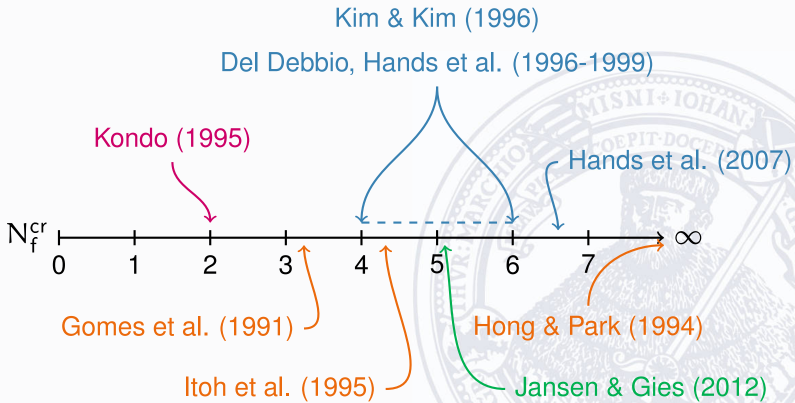
"$N_f = 0.5$" irreducible representation for $N_{f,irr} = 1$ corresponds to Gross-Neveu model with chiral symmetry breaking

$N_f \to \infty$ no chiral symmetry breaking

$\Rightarrow$ There is a N_f^{cr} where chiral behaviour changes.

What is the value of N_f^{cr}?

Results for N_f^{cr} from Schwinger-Dyson equations, $\frac{1}{N_f}$-expansion, functional renormalization group, Lattice simulation with staggered fermions:



Chiral Symmetry on the Lattice

Nielsen-Ninomiya Theorem

It is not possible to have a chiral, local and translational invariant Dirac operator with correct continuum limit without doublers.

older results: staggered fermions with mass

- mass breaks symmetry explicitly
- still doublers and no full chiral symmetry
- symmetry correct in the continuum limit?

our approach: SLAC derivative [fine for non-gauge theories: Bergner et al. arXiv:0705.2212; Wozar, Wipf arXiv:1107.3324]

- in momentum space: multiplication by $i\gamma^{\mu}p_{\mu}$
- exact chiral symmetry
- not local: need to do Fourier transformation

Problems

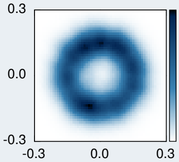
Technical Problems

- Chiral condensate always zero due to exact chiral symmetry and integration over fermions.
- Peak in susceptibility may indicate lattice artefact phase.

Coupling to Global Model [PoS(LATTICE 2015)050]

- Can obtain nice histograms of $\langle \bar{\psi}\psi \rangle$.
- Hard to recover Thirring model.

$\Rightarrow$ No reliable conclusion regarding N_f^{cr}, but likely $N_f^{cr} \leqslant 2$.



Scaning Larger Theory-Space

Why Coupled Four-Fermion Interactions?

- Easy to study chiral symmetry breaking in Gross-Neveu model. $\Rightarrow$ Gain new insights into Thirring model and the larger theory space by coupling there two models.
- Test predictions from functional renormalization group.

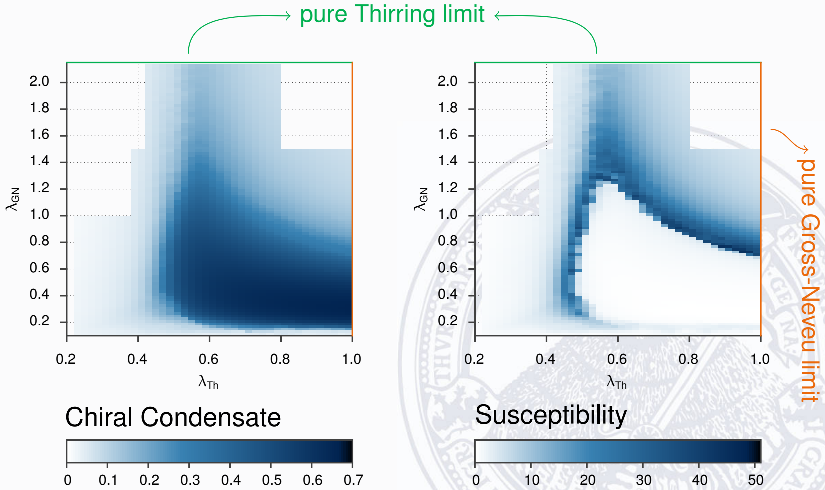
Interaction with $i\gamma_4\gamma_5$

$\left(\bar{\psi} i\gamma_4\gamma_5\psi\right)^2$ is interesting because

- same symmetry (and problems) as Thirring, while
- expected to be Gross-Neveu-like, corresponds to irreducible Gross-Neveu model.

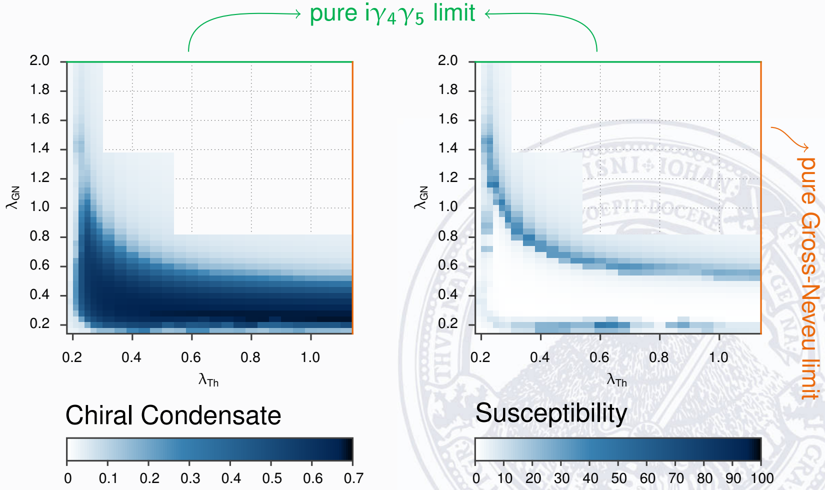
Gross-Neveu and Thirring

$\lambda = \frac{1}{2g^2}$, Lattice 8x7x7, $N_f = 1$



Gross-Neveu and $i\gamma_4\gamma_5$

$\lambda = \frac{1}{2g^2}$, Lattice 12x11x11, $N_f = 1$



Gross-Neveu and $i\gamma_4\gamma_5$

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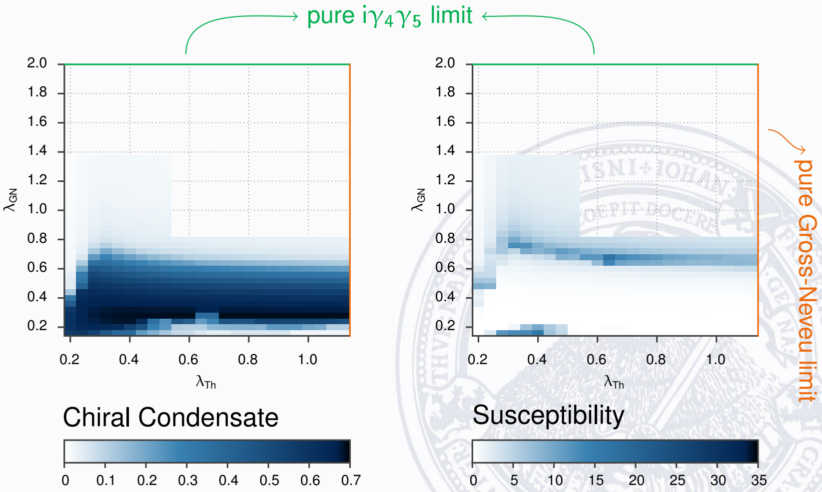
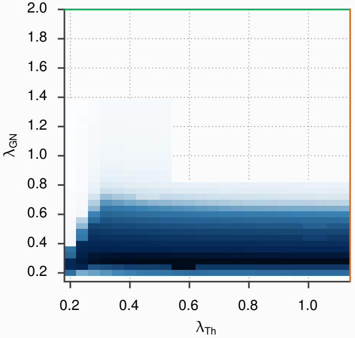
$\lambda = \frac{1}{2g^2}$, Lattice 12x11x11, $N_f = 2$



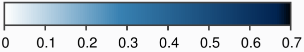
Gross-Neveu and $i\gamma_4\gamma_5$

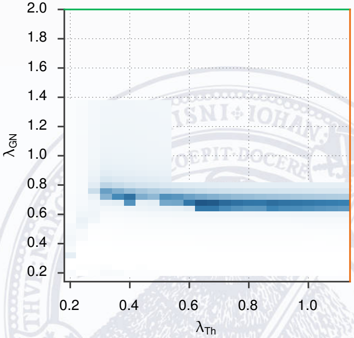
$\lambda = \frac{1}{2g^2}$, Lattice 12x11x11, $N_f = 3$

→ pure $i\gamma_4\gamma_5$ limit ←

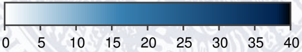


Chiral Condensate





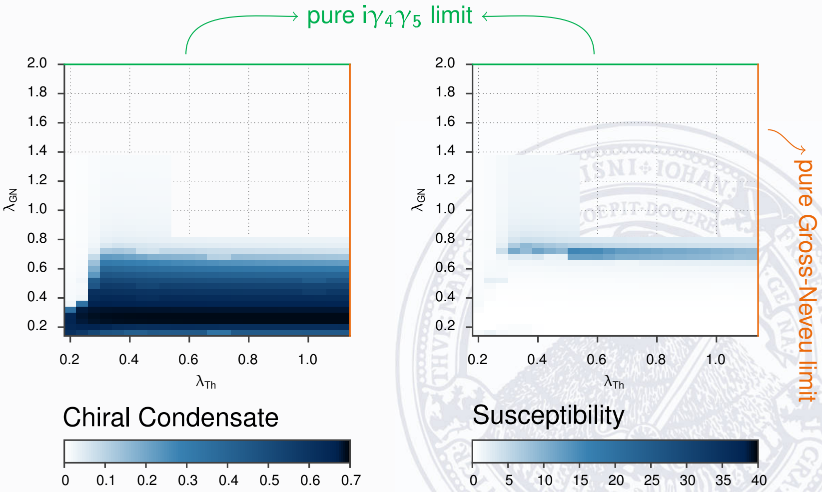
Susceptibility



pure Gross-Neveu limit

Gross-Neveu and $i\gamma_4\gamma_5$

$\lambda = \frac{1}{2g^2}$, Lattice 12x11x11, $N_f = 4$



Fierz Identity (1) for Thirring Interaction

Goal: rewrite interaction term, use irreducible representation:
2-component spinors χ^a, $a = 1, \dots, 2N_f \coloneqq N_{f,irr}$

$$\left(\bar{\chi}^a \sigma_\mu \chi^a\right) \left(\bar{\chi}^b \sigma^\mu \chi^b\right) = -\left(\bar{\chi}^a \chi^a\right) \left(\bar{\chi}^b \chi^b\right) - 2 \left(\bar{\chi}^a \chi^b\right) \left(\bar{\chi}^b \chi^a\right) \quad (1)$$

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Lagrangian after Hubbard-Stratonovich Transformations

ϕ real scalar field, T^{ab} hermitian, traceless matrix field

$$\mathcal{L}_1 = \bar{\chi}_a \underbrace{\left[\left(\not{\partial} + \phi\right)\delta^{ab} + T^{ab}\right]}_{=D_1^{ab}} \chi_b + \frac{N_{f,irr}}{4g^2} T_{ab} T^{ba} + \frac{N_{f,irr}^2}{2g^2(2 + N_{f,irr})} \phi^2$$

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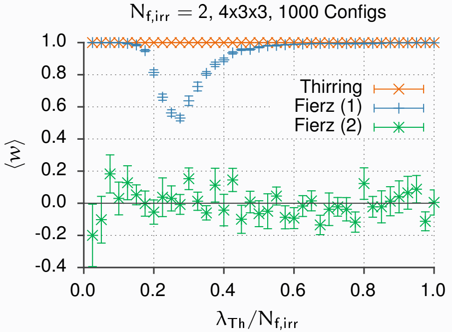
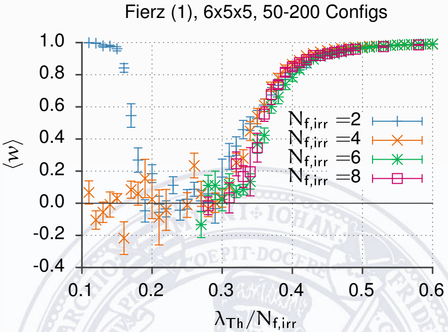
D_1^{ab} has no special properties $\Rightarrow$ complex eigenvalues

Sign Problem after Fierz Rearrangement





Thirring has no sign problem

Fierz (1) has a complex phase with $\langle w \rangle := \left\langle e^{-i\, \mathrm{Im} S} \right\rangle \approx 0$ for $\lambda_{\mathrm{Th}} \in [0.2, 0.3]$

Fierz (2) another identity, imaginary eigenvalues, switches sign on nearly every update $\Rightarrow \langle w \rangle \approx 0\ \forall \lambda_{\mathrm{Th}}$

Fermion Bag Approach

Current approach to get informations about chiral symmetry breaking and solving the sign problem of Fierz (1):

$\Rightarrow$ Introduce a spin field $k_{xi}^{ab} \in \{0, 1\}$, integrate fermions, T^{ab} and ϕ:

Final Partition Sum

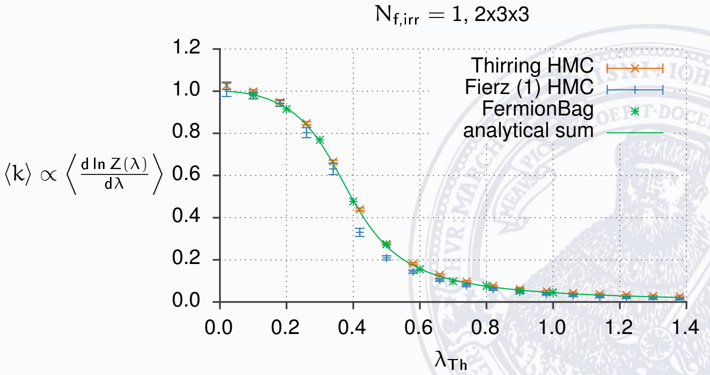
$$Z(\lambda) \propto \sum_{k} (-\lambda)^{-\frac{k}{2}} \det(i\partial\!\!\!/[k]) 2^{\tilde{n}_x^2} \prod_{x} f(n_x^1, n_x^2)$$

- $\partial\!\!\!/[k]$ is the SLAC operator matrix with columns and rows deleted corresponding to k.
- n_x^1, n_x^2 and $\tilde{n}_x^2$ count certain entries of k_{xi}^{ab}
- $f(a, b)$ is a product of gamma- and confluent hypergeometric functions

Simulation Results

Simple Metropolis simulation for $N_{f,irr} = 1$:

- Agreement with analytical calculation and original formulation.
- Deviations for Fierz(1) (sign problem!).



The End

- The Thirring model shows chiral symmetry breaking for $\mathrm{N_f} < \mathrm{N_f^{cr}}$, for which there are many different predictions.
- Simulations with multiple four-fermion couplings may allow new insights.
- We get a sign problem after Fierz transformation.

- Fermion bag approach may solve the sign problem and allow access to the chiral condensate.

Thank you for your attention!