What we can learn from two-dimensional QCD-like theories at finite density

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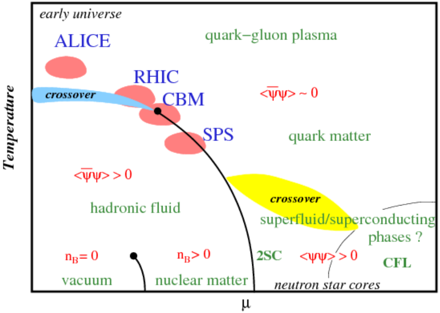


QCD sign problem at finite density:
Standard Monte-Carlo methods only for $\mu/T \leq 1$ applicable

- complex Langevin, Lefschetz thimbles, density-of-states approach
- hopping parameter expansion, strong coupling expansion
- isospin chemical potential, imaginary chemical potential
- functional methods (DSE, FRG)
- QCD-like theories

QCD-like theories

- Replace $SU(3)$ fundamental fermions by fermions in representation $\mathcal{R}$ of gauge group $\mathcal{G}$
- Wilson dirac operator: $D(\mu) = D(\mu; \mathcal{R}_{\mathcal{G}})$ with $\mu \in \mathbb{R}$

Additional (anti-)unitary symmetry

$$T\,D(\mu) = D^*(\mu)\,T\,, \quad T^*T = \pm\mathbb{1}\,, \quad T^\dagger T = \mathbb{1}$$

$$\det D(\mu) \det D(\mu) = \det D(\mu) \det D^*(\mu) = \det D(\mu) D^\dagger(\mu) \geq 0$$

$\Rightarrow$ no sign problem at baryon chemical potential

Partition function for baryon and isospin chemical potential

$$Z(\mu_{\mathrm{B}}) = Z(\mu_I)$$

Two flavour theory invariant under: $(u, d) \rightarrow (u, T\gamma_5 \bar{d}^{\mathsf{T}})$.

In this talk: $SU(2)$-QCD and G_2-QCD

$SU(2)$ gauge theory with fundamental fermions, $T = C\gamma_5 \otimes \sigma_2$

- 2 colors, 3 gluons
- only bound states with even quark number (only bosonic baryons)

$$n_q = 2 \sim \text{diquarks}(d) \sim q^T q$$

- second order deconfinement transition in gluodynamic

n_q	Particle	$d \leftrightarrow T\,\gamma_5\,\bar{d}^T$	Particle	n_q
0	η	$\leftrightarrow$	η	0
0	f	$\leftrightarrow$	f	0
0	π_0	$\leftrightarrow$	π_0	0
0	$\pi_\pm$	$\leftrightarrow$	$d^+_\pm$	2
0	$a_\pm$	$\leftrightarrow$	$d^-_\pm$	2

G_2 gauge theory with fundamental fermions, $T = C\gamma_5 \otimes \mathbb{1}$

- 7 colors, 14 gluons
- bound states with integer quark number (fermionic and bosonic baryons)

$$n_q = 1 \sim \text{Hybrid}(H) \sim qggg$$
$$n_q = 1 \sim \tilde{\Delta}, \tilde{N} \sim (\bar{q}q)q$$
$$n_q = 2 \sim \text{diquarks}(d) \sim q^T q$$
$$n_q = 3 \sim \Delta, N \sim (q^T q)q$$

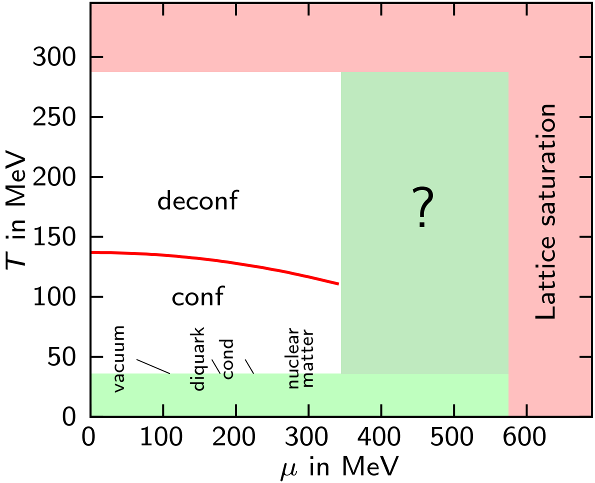
- gluodynamic very similar to $SU(3)$ (first order deconfinement transition)

n_q	Particle	$d \leftrightarrow T\gamma_5 \bar{d}^T$	Particle	n_q
1	H	$\leftrightarrow$	H	1
1	$\tilde{N}$	$\leftrightarrow$	N	3
1	$\tilde{\Delta}^{++,+,-}$	$\leftrightarrow$	$\tilde{\Delta}^{++,+,-}$	1
1	$\tilde{\Delta}^0$	$\leftrightarrow$	Δ^0	3
3	$\Delta^{++,+,-}$	$\leftrightarrow$	$\Delta^{++,+,-}$	3

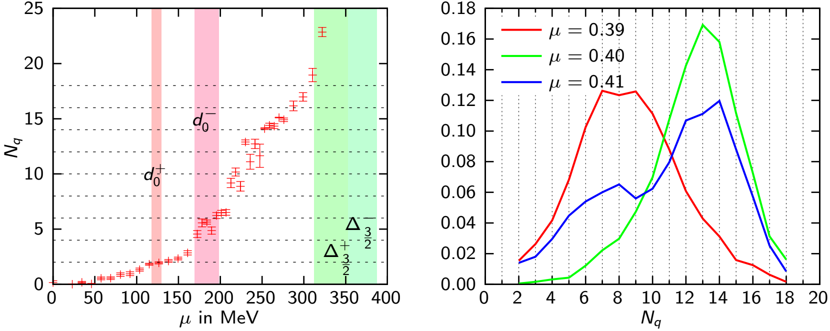
1. G_2-QCD in 4 dimensions
2. Free lattice fermions
3. Two-Color QCD in two dimensions
4. G_2-QCD in two dimensions

G_2-QCD in 4 dimensions

$N_f = 1$ G_2-QCD phase diagram with $m_{d_0^+} = 247$ MeV



$N_f = 1$ G_2-QCD phase diagram with $m_{d_0^+} = 247$ MeV



- Are these QCD-like theories similar to QCD with isospin chemical potential or to QCD with baryon chemical potential?
- What is the contribution of fermionic / bosonic baryons to the phase diagram?
- Simulations in 4 dimensions computationally very expensive

$\Rightarrow$ high precision simulations in 2 dimensions

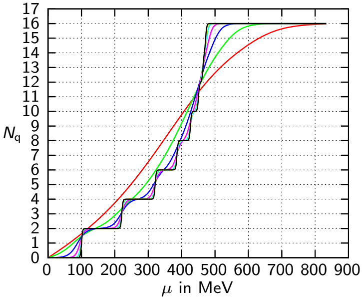
Free lattice fermions

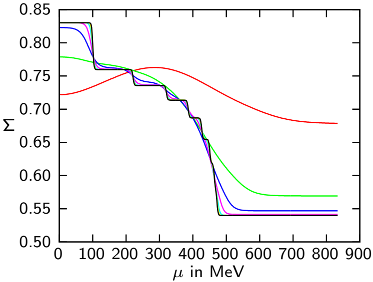
Ensemble with fixed particle number $k \mod N$

$$Z_N(k) = \frac{1}{N} \sum_{n=0}^{N-1} e^{-2\pi i k \frac{n}{N}} Z\left(\mu - \frac{2\pi i}{N} T n\right)$$

$$\Rightarrow Z_{\text{even}} = \frac{1}{2}\left(Z(\mu) + Z(\mu - i\pi T)\right)$$

Sum of ensembles with periodic and antiperiodic temporal boundary conditions





$N_t \times 16$ lattice with $N_t = 4 \ldots 128$

Two-Color QCD in two dimensions

Setup

- Two flavour $SU(2)$-QCD in $2d$
- $N_t \times 16$ lattice with $N_t = 2\ldots128$ at fixed β and κ
- Physical scale set by pion mass $m_\pi = 200$ MeV at $N_t = 32$

$\Rightarrow$ $a = 0.26(4)\,\text{fm} \sim 0.0013\,\text{MeV}^{-1}$

$\Rightarrow$ $T = 6\ldots385\,\text{MeV}$

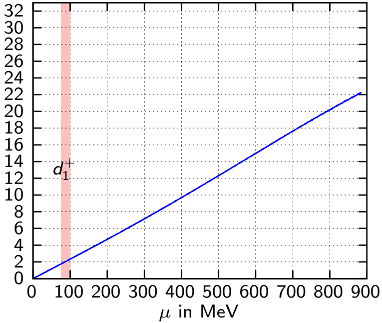
$\Rightarrow$ $\mu = 0\ldots885\,\text{MeV}$

$\Rightarrow$ diquark mass $m_{d_0^+} = 200\,\text{MeV}$

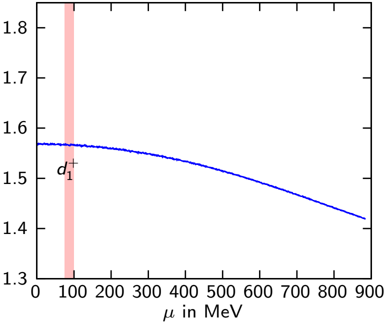
$\Rightarrow$ vector diquark mass $m_{d_1^+} = 177\,\text{MeV}$

$\Rightarrow$ a meson mass $m_a = 254\,\text{MeV}$

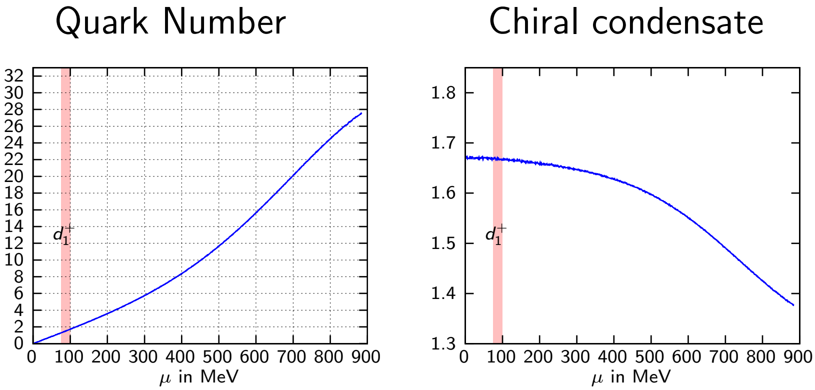
Quark Number



Chiral condensate



$T = 385$ MeV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μ in MeV
d1+
Chiral condensate
1.8
1.7
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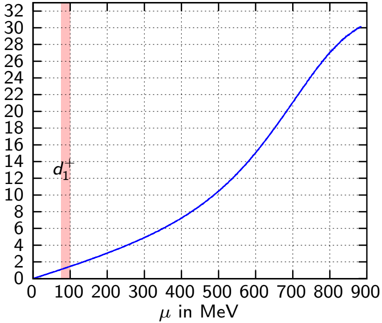
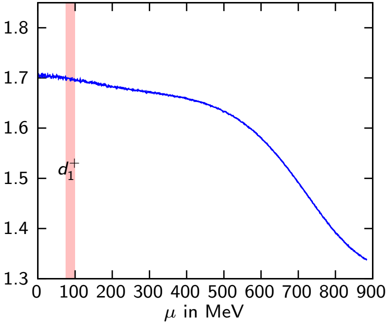
μ in MeV
d1+

$T = 192$ MeV

Quark Number

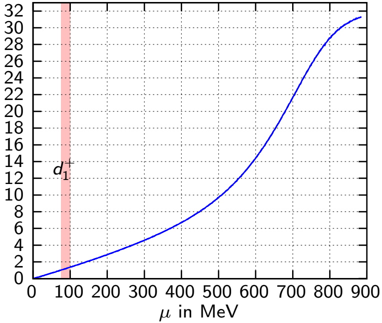


Chiral condensate

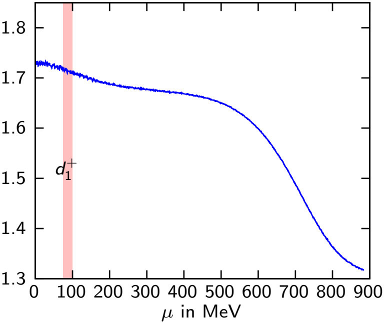


$T = 128$ MeV

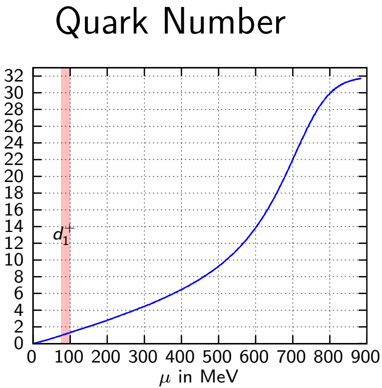
Quark Number



Chiral condensate



$T = 96$ MeV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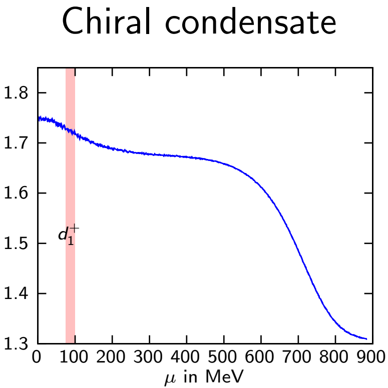
d_1^+
μ in MeV


Chiral condensate
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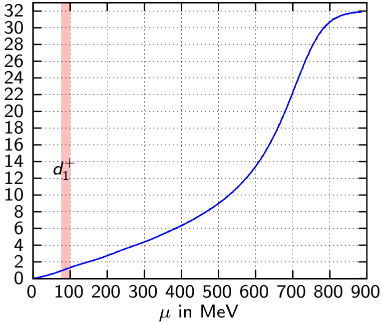
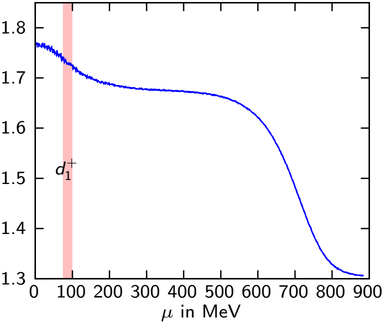
0 100 200 300 400 500 600 700 800 900
d_1^+
μ in MeV

$T = 77$ MeV

Quark Number

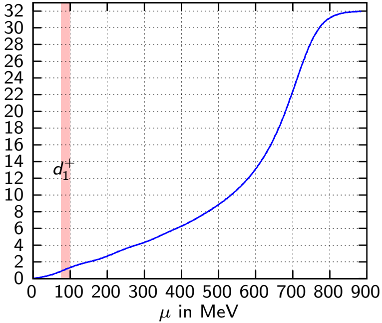


Chiral condensate

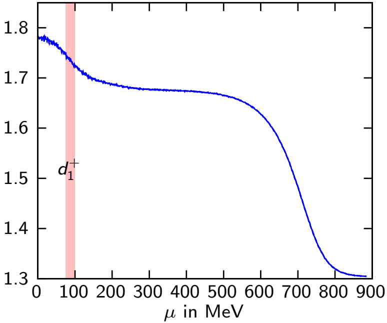


$T = 64$ MeV

Quark Number

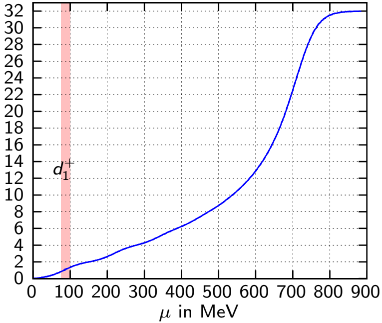


Chiral condensate

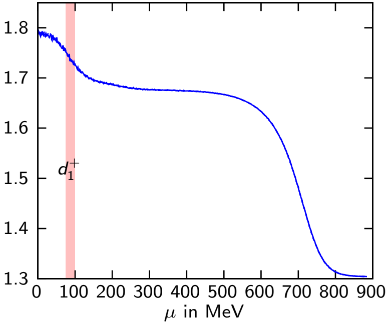


$T = 55$ MeV

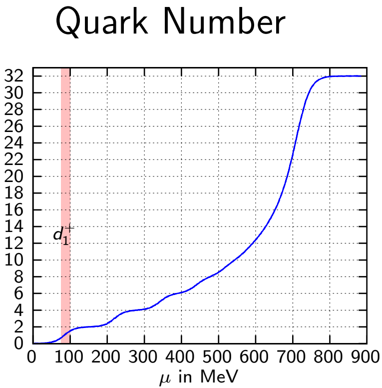
Quark Number



Chiral condensate



$T = 48$ MeV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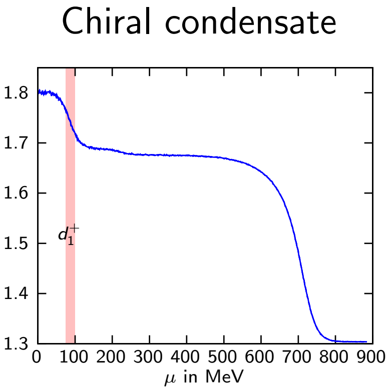
d_1^+
0 100 200 300 400 500 600 700 800 900
μ in MeV


Chiral condensate
1.8
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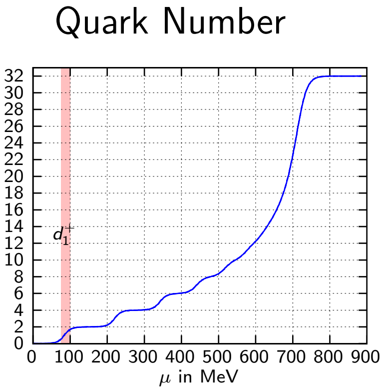
d_1^+
0 100 200 300 400 500 600 700 800 900
μ in MeV

$T = 32$ MeV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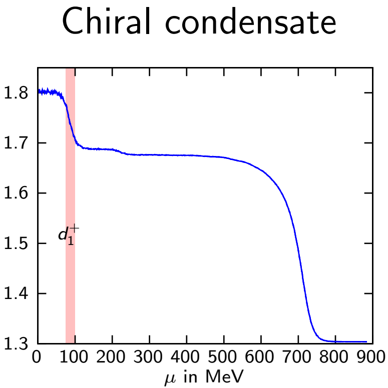
d_1^+
0 100 200 300 400 500 600 700 800 900
μ in MeV


Chiral condensate
1.8
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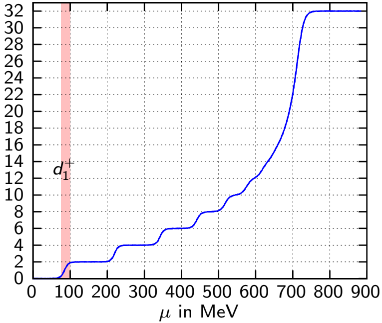
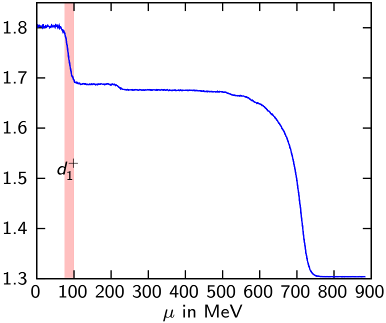
d_1^+
0 100 200 300 400 500 600 700 800 900
μ in MeV

$T = 24$ MeV

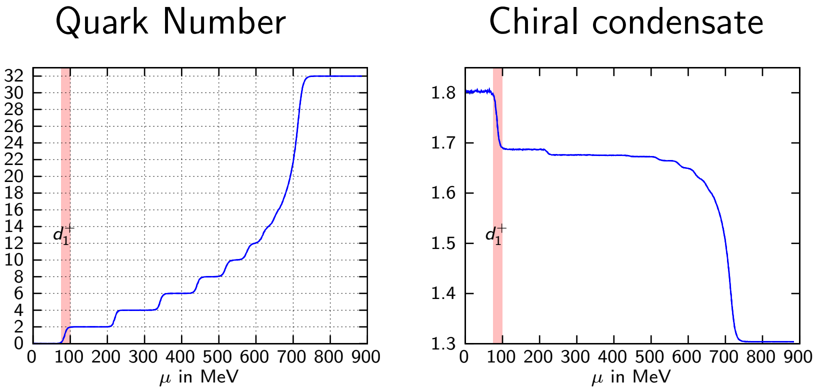
Quark Number



Chiral condensate



$T = 16$ MeV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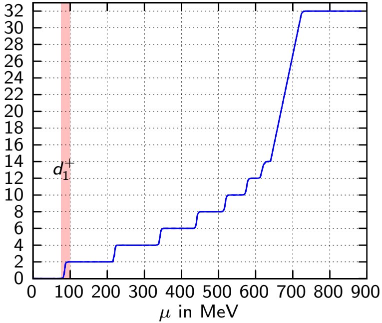
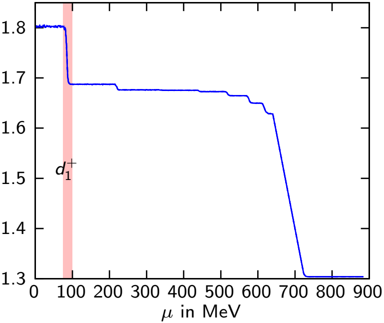
0 100 200 300 400 500 600 700 800 900
d_1^+
μ in MeV
Chiral condensate
1.8
1.7
1.6
1.5
1.4
1.3
0 100 200 300 400 500 600 700 800 900
d_1^+
μ in MeV

$T = 12$ MeV

Quark Number



Chiral condensate

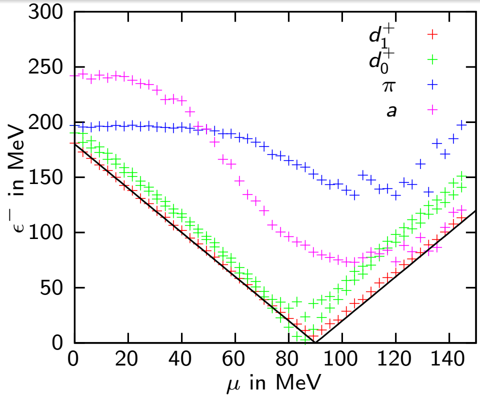


$T = 6$ MeV

Lattice correlation function for operator with quark number n_q

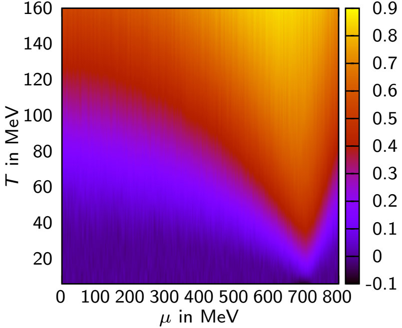
$$C(\mu, n_q) \sim a \exp^{-\epsilon^-(\mu,n_q)t} + b \exp^{\epsilon^+(\mu,n_q)t}$$

with $\quad \epsilon^+ = m(\mu) + n_q\mu \quad$ and $\quad \epsilon^- = m(\mu) - n_q\mu.$

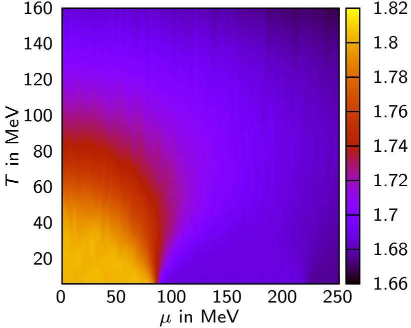


- ϵ^- obtained from fits to 2, 3 or 4 exponentials
- d_0^+ mass decreases close to the onset

Polyakov loop



Chiral condensate



- Phase diagram in $2d$ is resembling phase diagram in $4d$ (finite volume)
- also similar to QCD at isospin density as expected

G_2-QCD in two dimensions

Setup

- Two flavour G_2-QCD in $2d$
- $N_t \times 16$ lattice with $N_t = 2 \dots 64$ at fixed β and κ
- Physical scale set by pion mass $m_\pi = 200$ MeV at $N_t = 32$

$\Rightarrow$ $a = 0.16\,\mathrm{fm} \sim 0.0007\,\mathrm{MeV}^{-1}$

$\Rightarrow$ $T = 20 \dots 633\,\mathrm{MeV}$

$\Rightarrow$ $\mu = 0 \dots 757\,\mathrm{MeV}$

$\Rightarrow$ diquark mass $m_{d_0^+} = 262\,\mathrm{MeV}$

$\Rightarrow$ vector diquark mass $m_{d_1^+} = 194\,\mathrm{MeV}$

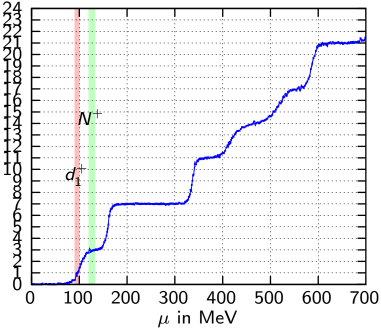
$\Rightarrow$ a meson mass $m_a = 262\,\mathrm{MeV}$

$\Rightarrow$ nucleon mass $m_{N^+} = 380\,\mathrm{MeV}$

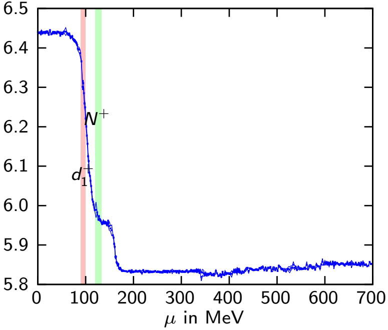
$\Rightarrow$ nucleon mass $m_{N^-} = 506\,\mathrm{MeV}$

$\Rightarrow$ hybrid mass $m_H \sim 440\,\mathrm{MeV}$

Quark Number

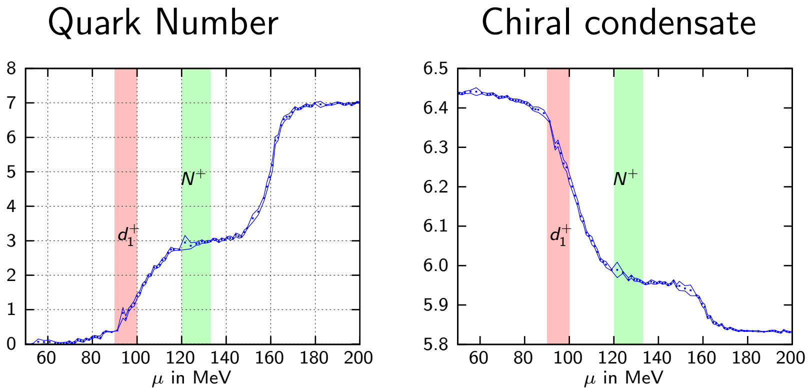


Chiral condensate



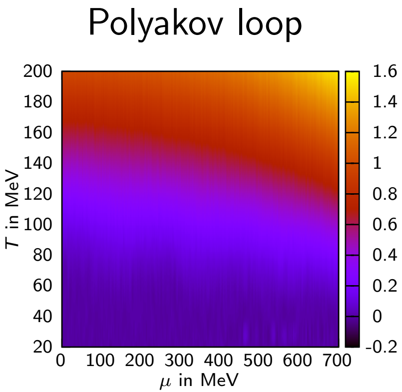
$T = 20$ MeV

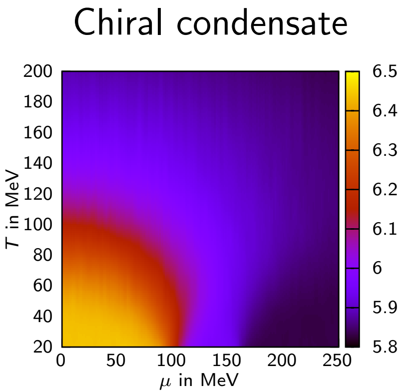
- very preliminary results indicate that nucleon / delta mass decreases above diquark onset



$T = 20$ MeV

- very preliminary results indicate that nucleon / delta mass decreases above diquark onset





- Phase diagram similar to phase diagram of two-color QCD, but contributions from fermionic and bosonic baryons

What can we learn from two-dimensional QCD-like theories at finite density?

- Phase diagrams for $SU(2)$ and G_2 in two dimensions very similar to each other
- In a finite volume they are also comparable to corresponding phase diagrams in 4 dimensions
- Behaviour at finite density qualitatively similar to free lattice fermions
- Mass dependence on chemical potential is a possible explanation for the observed discrepancies in 4 dimensions (possible onset of baryonic matter compared to baryon mass)

but probably it is very hard to obtain conclusive results for G_2-QCD in 4 dimensions because...

- we need very small temperatures to separate contributions from fermionic and bosonic baryons
- we face severe finite temperature effects
- we need large spatial lattices to decrease smallest spatial momenta in order to investigate diquark condensation or the nuclear matter phase