New results for QCD at non-vanishing chemical potential from Taylor expansion

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Taylor expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(T,\mu_u,\mu_d,\mu_s) = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}(T) \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

with ($\hat{\mu} = \mu/T$)

$$\chi_{ijk} = \frac{\partial^i}{\partial \hat{\mu}_u^i} \frac{\partial^j}{\partial \hat{\mu}_d^j} \frac{\partial^k}{\partial \hat{\mu}_s^k} \left(\frac{p}{T^4}\right)|_{\vec{\mu}=0}$$

for instance

$$\chi_{200} = \frac{1}{VT^3} \left(\frac{1}{4} \left\langle \frac{\partial^2 \ln \det M_u}{\partial \hat{\mu}_u^2} \right\rangle + \frac{1}{16} \left\langle \left(\frac{\partial \ln \det M_u}{\partial \hat{\mu}_u} \right)^2 \right\rangle \right)$$

with

$$\frac{\partial^2 \ln \det M_u}{\partial \hat{\mu}_u^2} = \mathrm{tr}\left(M_u^{-1} \frac{\partial^2 M_u}{\partial \hat{\mu}_u^2} \right) - \mathrm{tr}\left(M_u^{-1} \frac{\partial M_u}{\partial \hat{\mu}_u} M_u^{-1} \frac{\partial M_u}{\partial \hat{\mu}_u} \right)$$

these can be computed on the lattice at $\vec{\mu} = 0$ ...

note: $c_{ijk} = 0$ for $i + j + k$ odd because of charge symmetry

... and combined to expansion in terms of μ_B, μ_S, μ_Q

$$\frac{p}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BSQ}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_S}{T}\right)^j \left(\frac{\mu_Q}{T}\right)^k$$

the generalized susceptibilities χ_{ijk}^{BQS} are related to

cumulants of fluctuations, $\delta N_X = N_X - \langle N_X \rangle$, of net quantum numbers $X = B, S, Q$, e.g.

$$\chi_2^X = \langle (\delta N_X)^2 \rangle$$

$$\chi_4^X = \langle (\delta N_X)^4 \rangle - 3\langle (\delta N_X)^2 \rangle^2$$

comparison with

HRG = Hadron Resonance Gas model: quite successful in describing hadron yields in HIC

$$\frac{p^{HRG}}{T^4} = \sum_{mesons} \ln Z_i^M(T, \mu_Q, \mu_S) + \sum_{baryons} \ln Z_i^B(T, \mu_B, \mu_Q, \mu_S)$$

$$\ln Z_i^{M/B} = \frac{d_i}{\pi^2} \left(\frac{m_i}{T}\right)^2 \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k^2} K_2(k\, m_i/T) \cosh(k(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S))$$

$$\simeq \frac{d_i}{\pi^2} \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \cosh(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S)$$

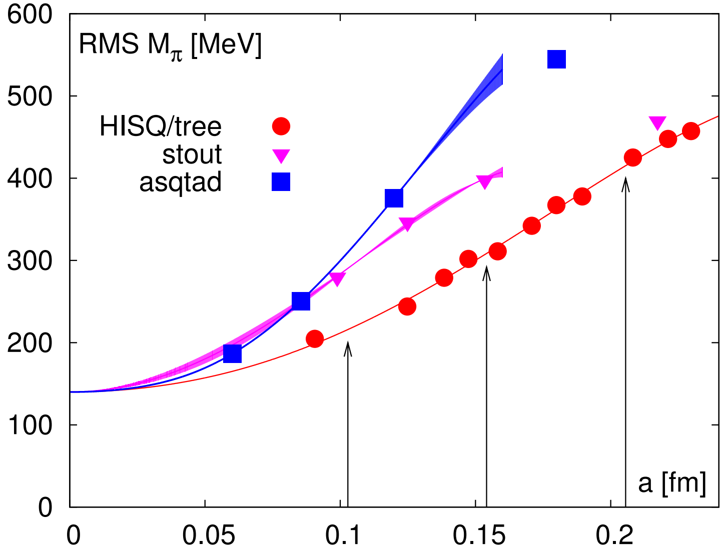
Boltzmann approximation: $k = 1$; recall $K_2(x) \sim e^{-x}$ $\rightarrow$ good except for pions

$\Rightarrow$ fluctuations as e.g.

$$\chi_n^B|_{\vec{\mu}=0} \sim \sum_{baryons\, i} B_i^n K_2(m_i/T) \simeq K_2(m_i/T)$$

Simulation parameters

- $N_F = 2+1$: two degenerate u/d quarks + strange quark
- along "line of constant physics" i.e. constant physical $m_K = 500$ MeV and constant m_π^G
- improved staggered action: **hisq**
- $m_\pi^G = 160,\ 140$ MeV corresponds to $m_{u,d}/m_s = 1/20, 1/27$
- $N_\sigma^3 \times N_\tau$ lattices with $N_\tau = 6, 8, 12, 16$, $N_\sigma = 4N_\tau$ note: $a = 1/(TN_\tau)$



staggered quarks:

16 pion operators in

8 different lattice reps.

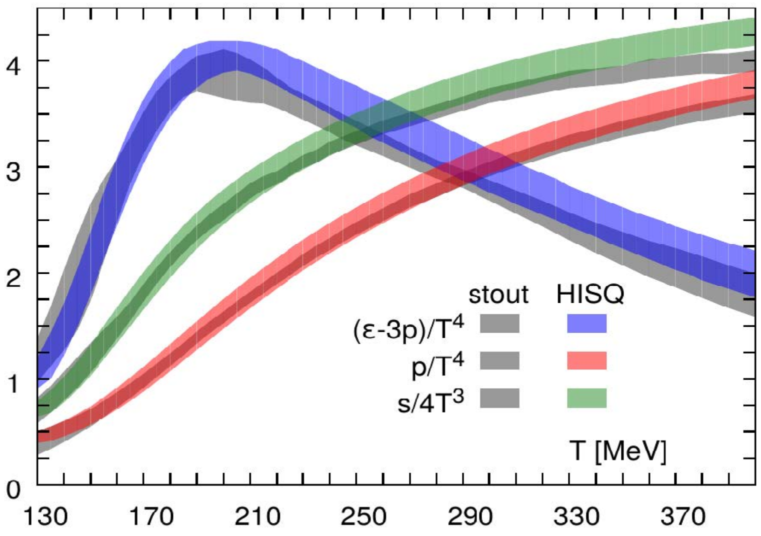
'taste violations'

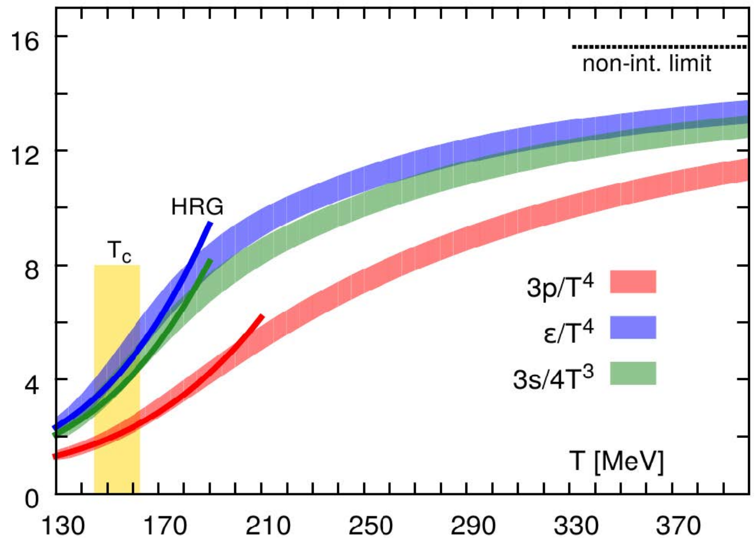
$$M_\pi^{\rm RMS} = \sqrt{\sum_{i=1}^{16} m_{\pi,i}^2}$$

at $T \simeq T_c$: **hisq** with much reduced taste violations:

Equation of State for (2+1) flavor QCD

first at $\boldsymbol{\mu_B = 0}$: pressure, entropy and energy density





- consistent between different lattice groups

– different actions
– continuum limit
– physical quark masses

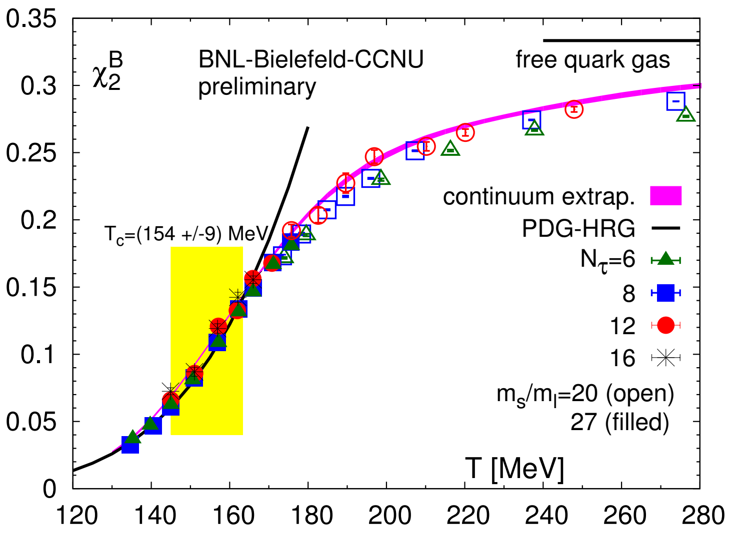
- comparison with HRG model

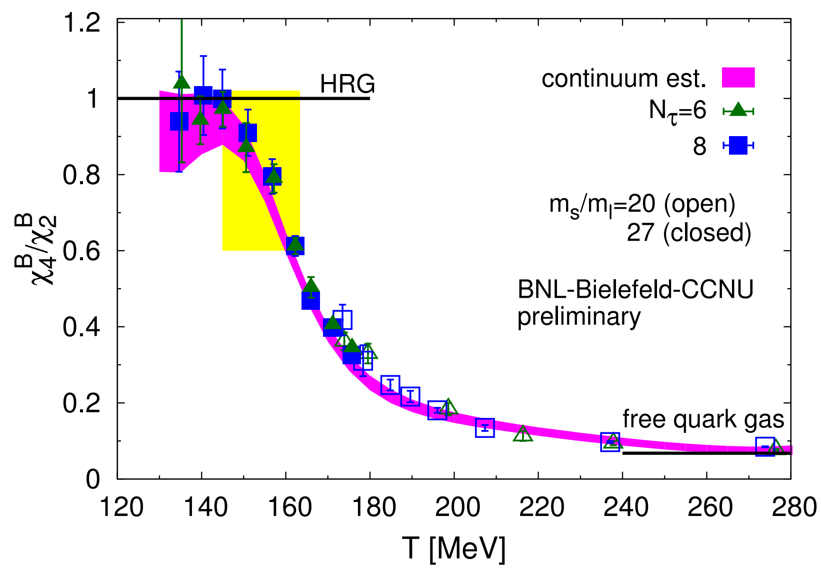
– up to crossover region:
QCD agrees quite well with HRG
– however: QCD systematically above HRG

next: $\boldsymbol{\mu_B} \neq 0$, but $\boldsymbol{\mu_Q = \mu_S = 0}$ $\Rightarrow$ $$\frac{p(T,\mu_B)}{T^4} = \sum_i \frac{1}{i!} \chi_{i00}^{BSQ}(T) \left(\frac{\mu_B}{T}\right)^i$$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{p(T,\mu_B)-p(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 \left[1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right]$$

variance kurtosis / variance : κ_B/σ_B^2

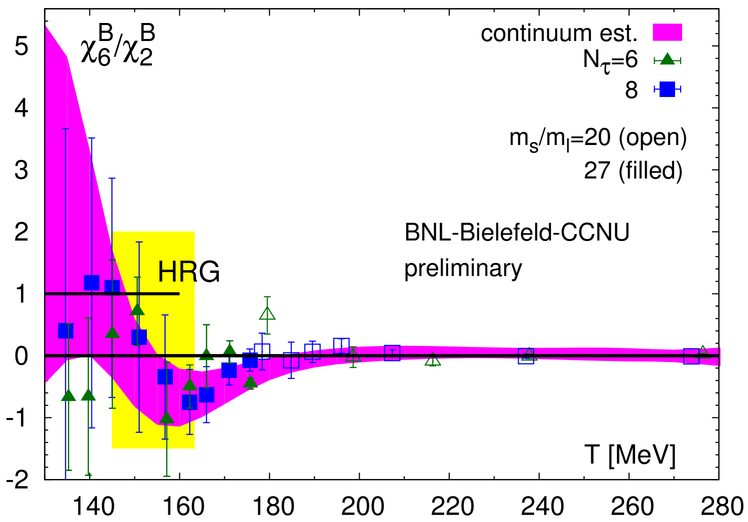


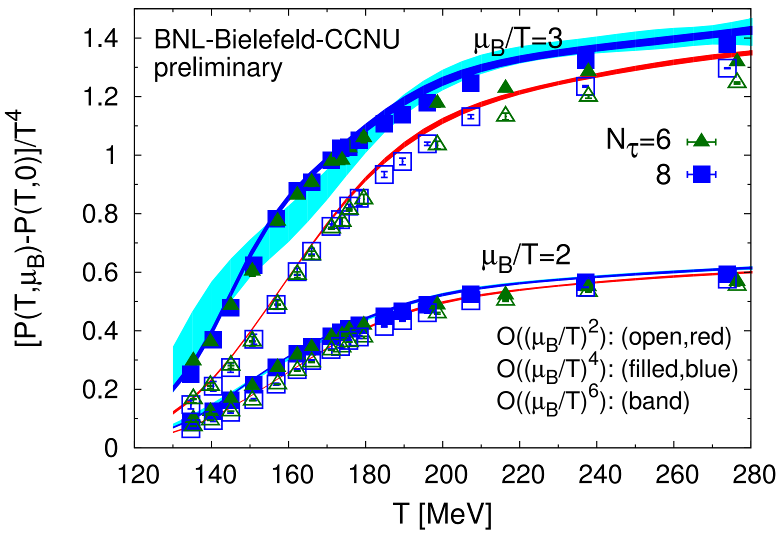


- leading and next to leading order agree well with HRG for $T < 150$ MeV
- but: in crossover region large deviations from HRG e.g. $\sim 40\%$ for $T \simeq 165$ MeV
- 4th order contribution rapidly becomes small at large T

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{p(T,\mu_B)-p(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2\left[1+\frac{1}{12}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2\right]$$

estimating next order correction $\quad \dfrac{1}{360}\dfrac{\chi_6^B}{\chi_2^B}\left(\dfrac{\mu_B}{T}\right)^4$





$\Rightarrow$ pressure well under control for $\mu_B/T \leq 2$

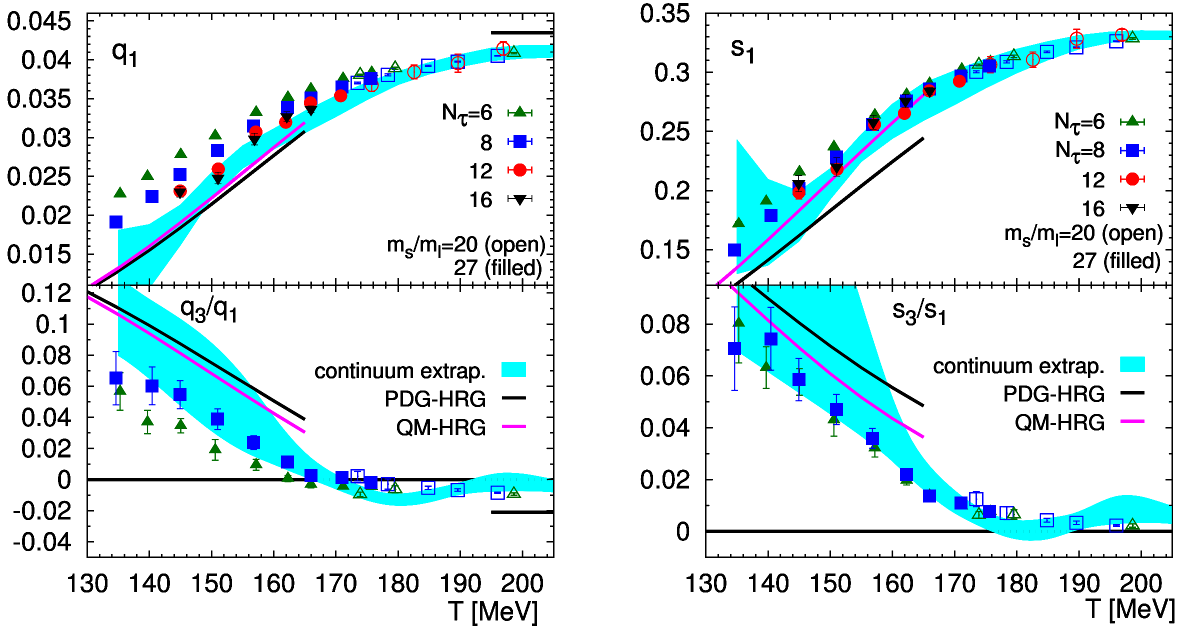
finally: all $\boldsymbol{\mu_B, \mu_Q, \mu_S \neq 0}$

- for instance: match experimental/initial conditions in heavy ion collision
  - strangeness neutrality: $\langle n_S \rangle = 0$
  - isospin asymmetry: $\langle n_Q \rangle = r \langle n_B \rangle$ with r = 0.4 for Au-Au and Pb-Pb

- expand in powers of μ_B, μ_Q, μ_S and solve for μ_Q, μ_S to given order $\Rightarrow$

$$\hat{\mu}_Q(T,\mu_B) = q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3$$
$$\hat{\mu}_S(T,\mu_B) = s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3$$

$\Rightarrow$ only 2 parameters T, μ_B



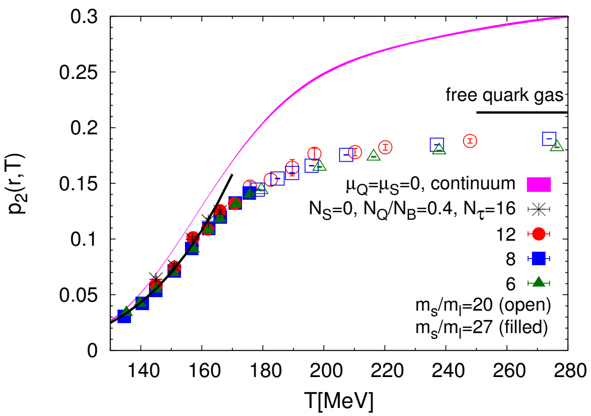
LO: continuum extrapolated

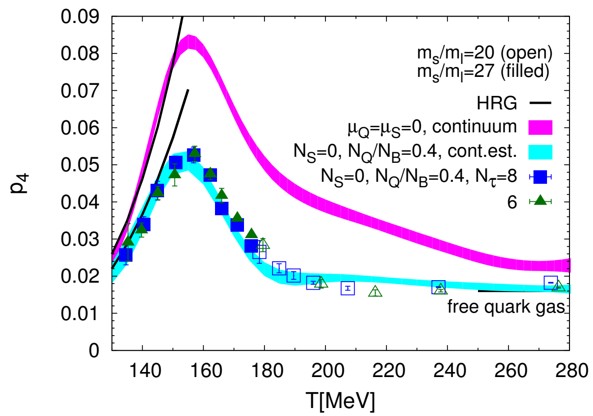
NLO: small cut-off dependence

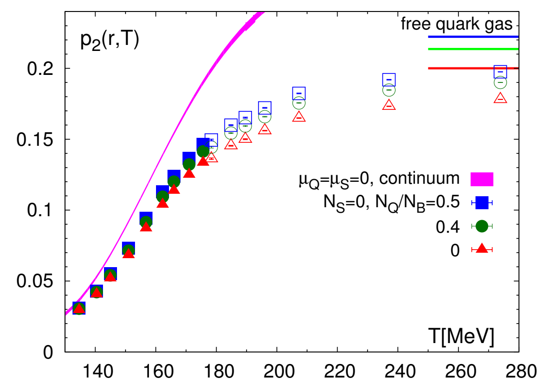
NLO corrections < 10 %

QCD AT $\mu_q \neq 0$ FROM TAYLOR EXPANSION

pressure coefficients
$$\frac{p(T,\mu_B)-p(T,0)}{T^4} = \frac{1}{2}p_2\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24}p_4\left(\frac{\mu_B}{T}\right)^4 + \ldots$$

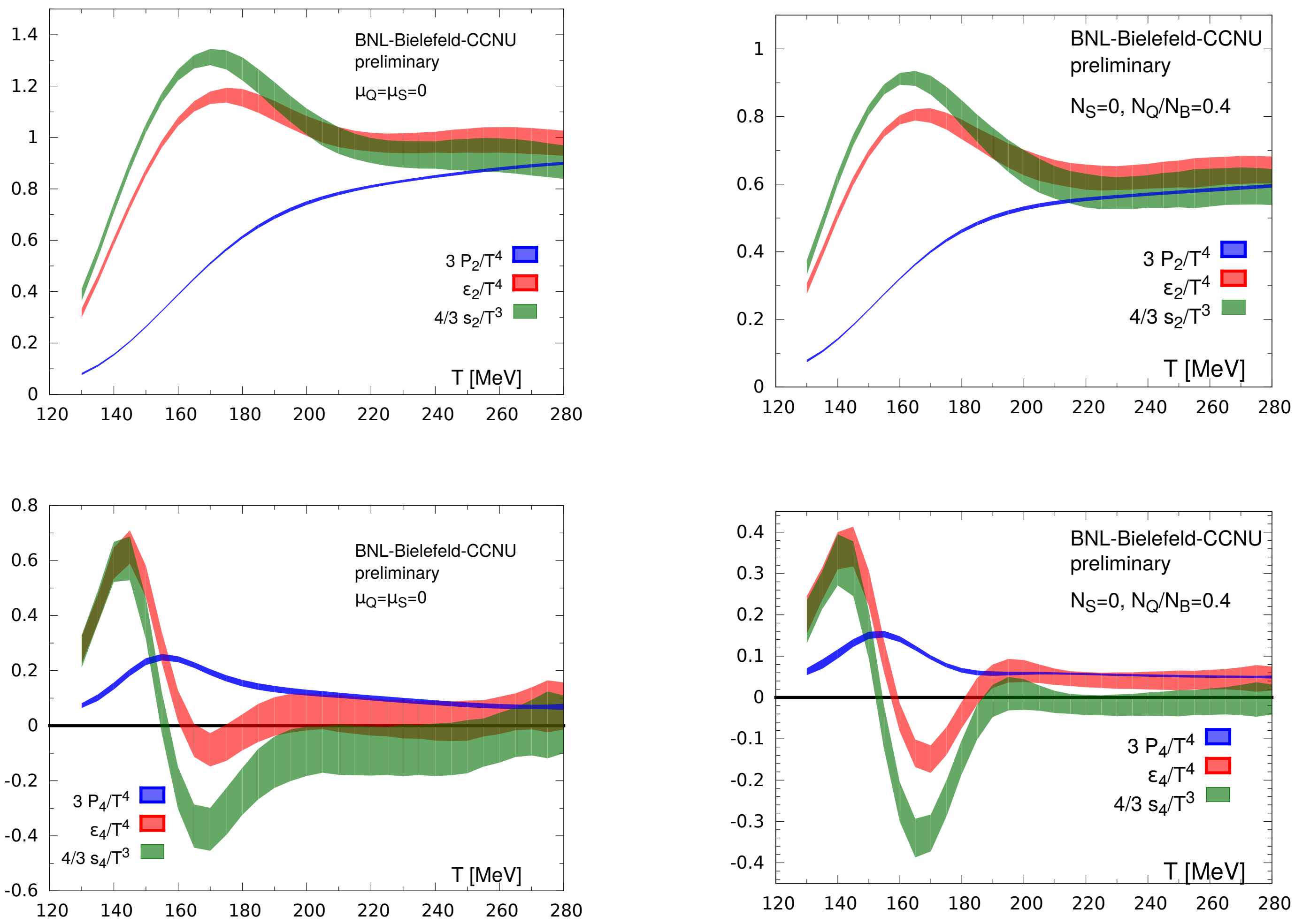






- considerable strangeness dependence
- mild isospin dependence

$$\frac{X(T,\mu_B)-X(T,0)}{T^4} = \frac{1}{2}X_2\left(\frac{\mu_B}{T}\right)^2 + \frac{1}{24}X_4\left(\frac{\mu_B}{T}\right)^4 + \ldots$$



$\mu_Q = \mu_S = 0$

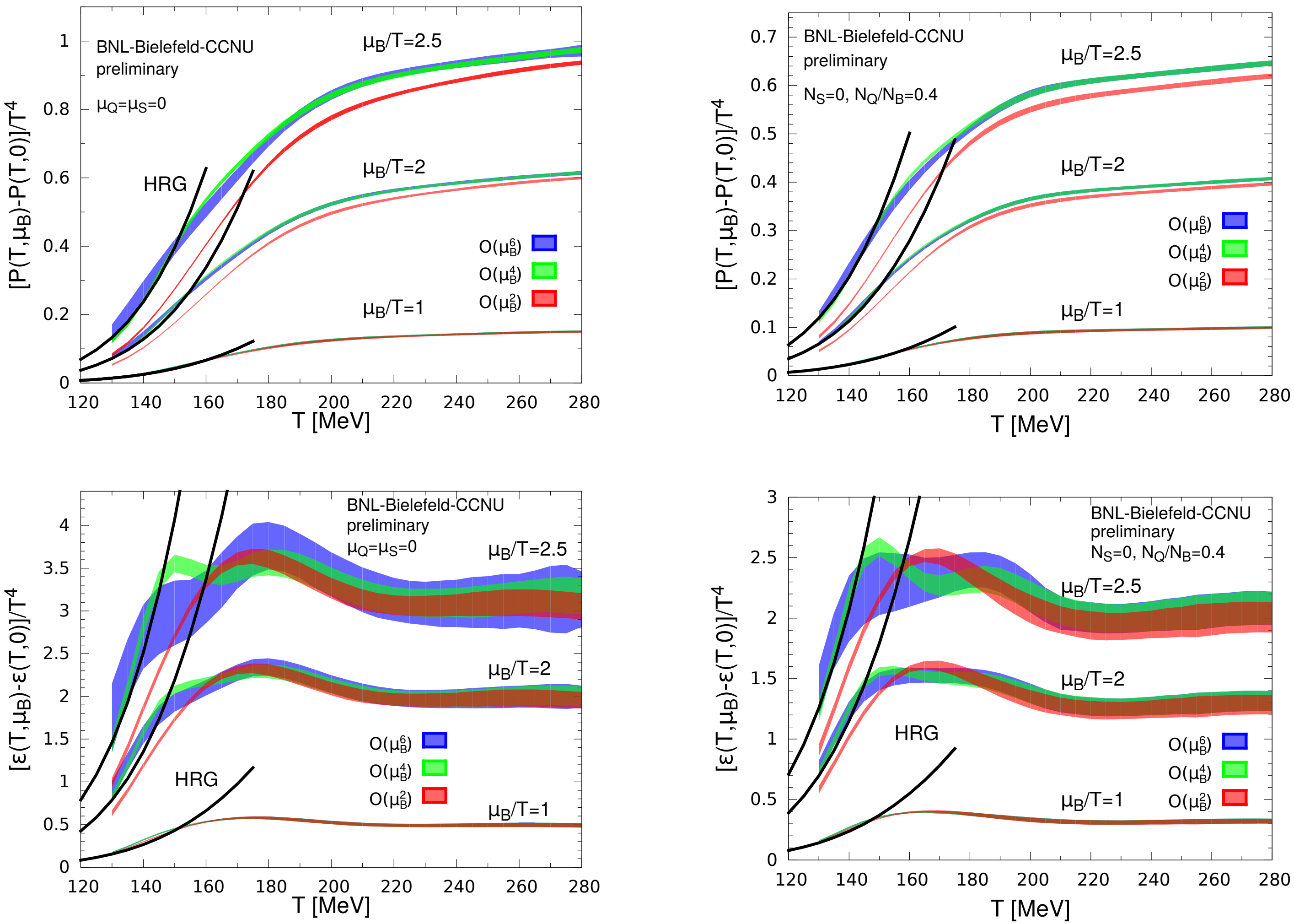
$n_S = 0$ and $n_Q/n_B = 0.4$

QCD AT $\mu_q \neq 0$ FROM TAYLOR EXPANSION

finally

pressure

energy



$\mu_Q = \mu_S = 0$

$n_S = 0$ and $n_Q/n_B = 0.4$

Summary

- used Taylor coefficients for **Equation of State** at $\mu_B \neq 0$
  - at $\mu_S = 0 = \mu_Q$ as well as
  - at $\mu_S \neq 0 \neq \mu_Q$ especially at $n_S = 0$ and $n_Q/n_B = 0.4$ (RHIC, LHC)
- expansion under control up to $\mu_B/T \lesssim 2$ $(\sqrt{s_{NN}} \gtrsim 12$ GeV)