|Vus| from inclusive strange τ decay data and lattice HVPs

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outline

- Introduction
  Inclusive tau decay experiment
  Finite energy sum rule and |Vus| determination

- Lattice HVPs and tau decay

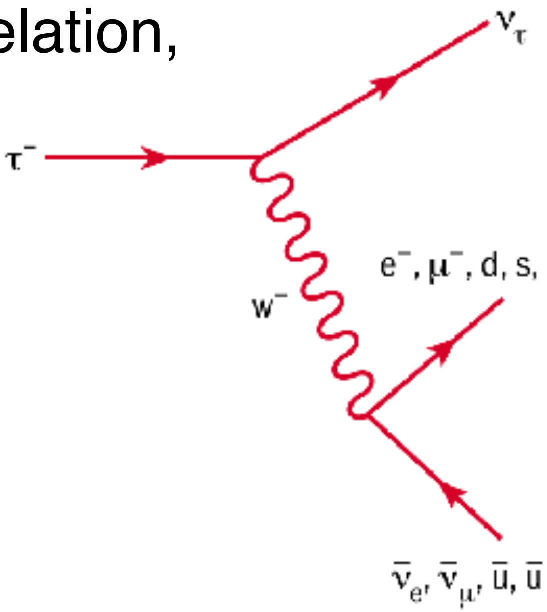
- Result of |Vus|

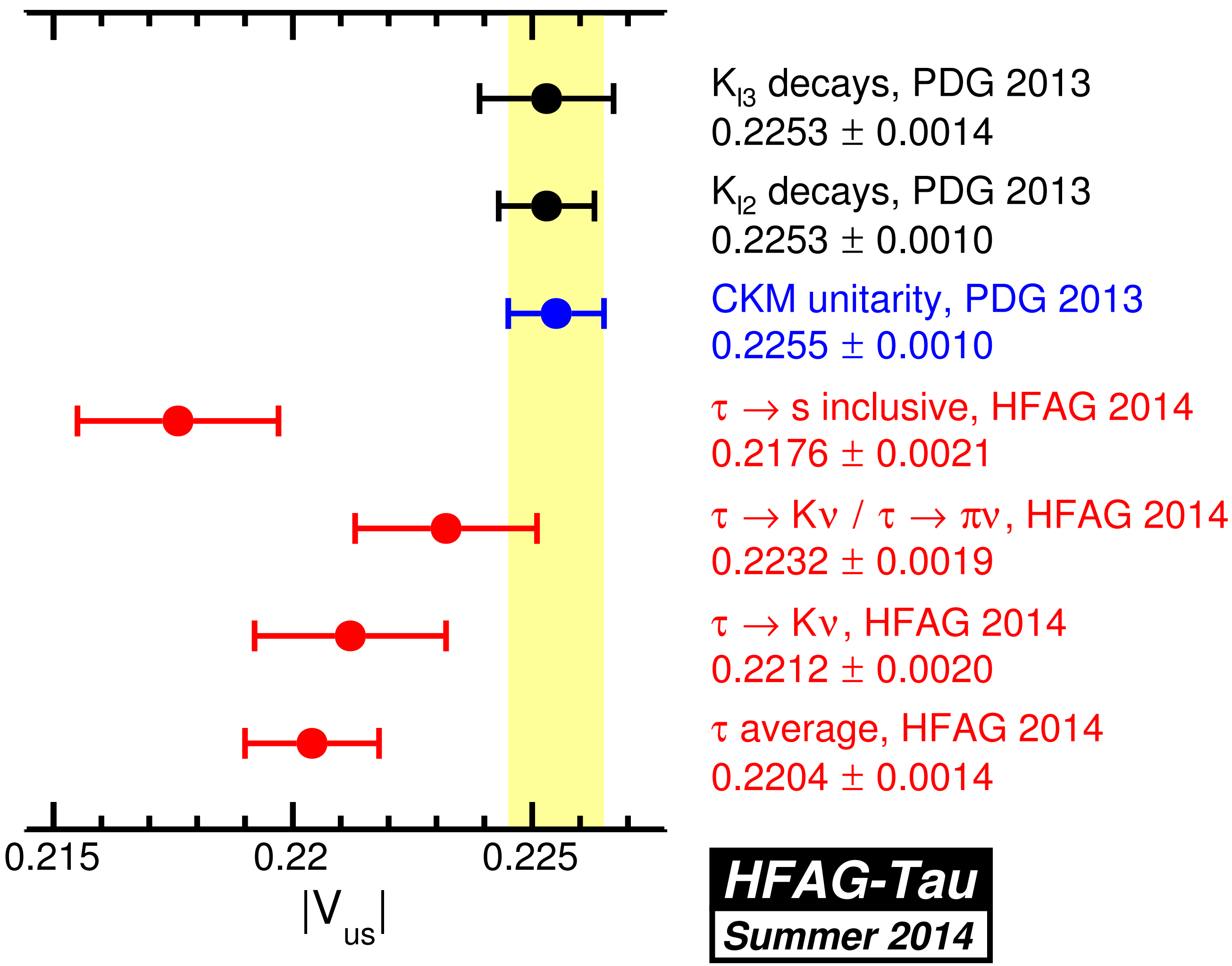
- Summary

Intruduction

- Lattice QCD calculation can apply to the exclusive modes:
  $f\pi$, fK: $K \to \pi$
- How about inclusive hadronic decay?
  We use τ inclusive Kaon decay experiments -> $|V_{us}|$ determination

- Using optical theorem and dispersion relation,
  τ decay differential cross section
  (τ hadronic decay/τ leptonic decay)
  and the hadronic vacuum polarization
  (HVP) function are related.
  -> We can use lattice HVP calculations.





- |Vus| from inclusive τ decay -> 3 σ deviation from CKM unitarity
- pQCD and high order OPE -> problematic uncertainties?

This work

- We would like to propose an alternative method to calculate |Vus| from the inclusive τ decay.

- By combing both the lattice data and pQCD, we could expect more precise determination of |Vus|.
- As a result, pQCD uncertainty can be suppressed.

Our strategy

- Using a different type of the weight function w(s) which has residues

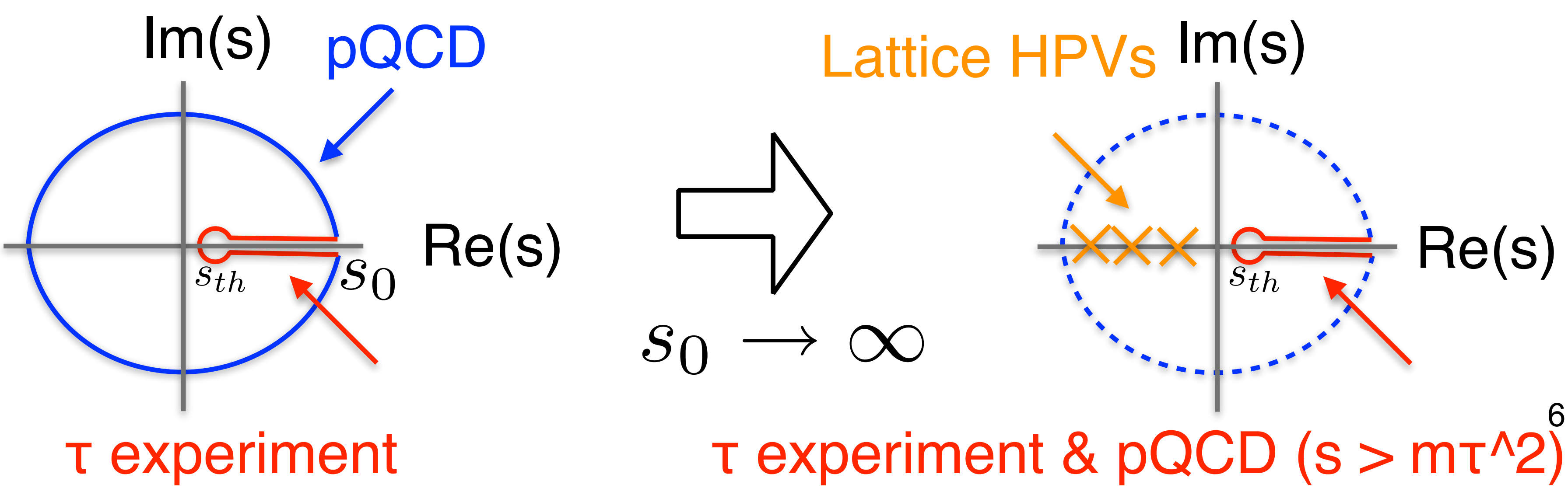
$$\omega(s) = \frac{1}{(s+Q_1^2)(s+Q_2^2)\cdots(s+Q_N^2)}$$

and taking S0 -> ∞,

$$\int_0^\infty \rho(s)\omega(s)ds = \sum_k^N \mathrm{Res}\left(\Pi(-Q_k^2)\omega(-Q_k^2)\right)$$

LHS … Experimental data and pQCD

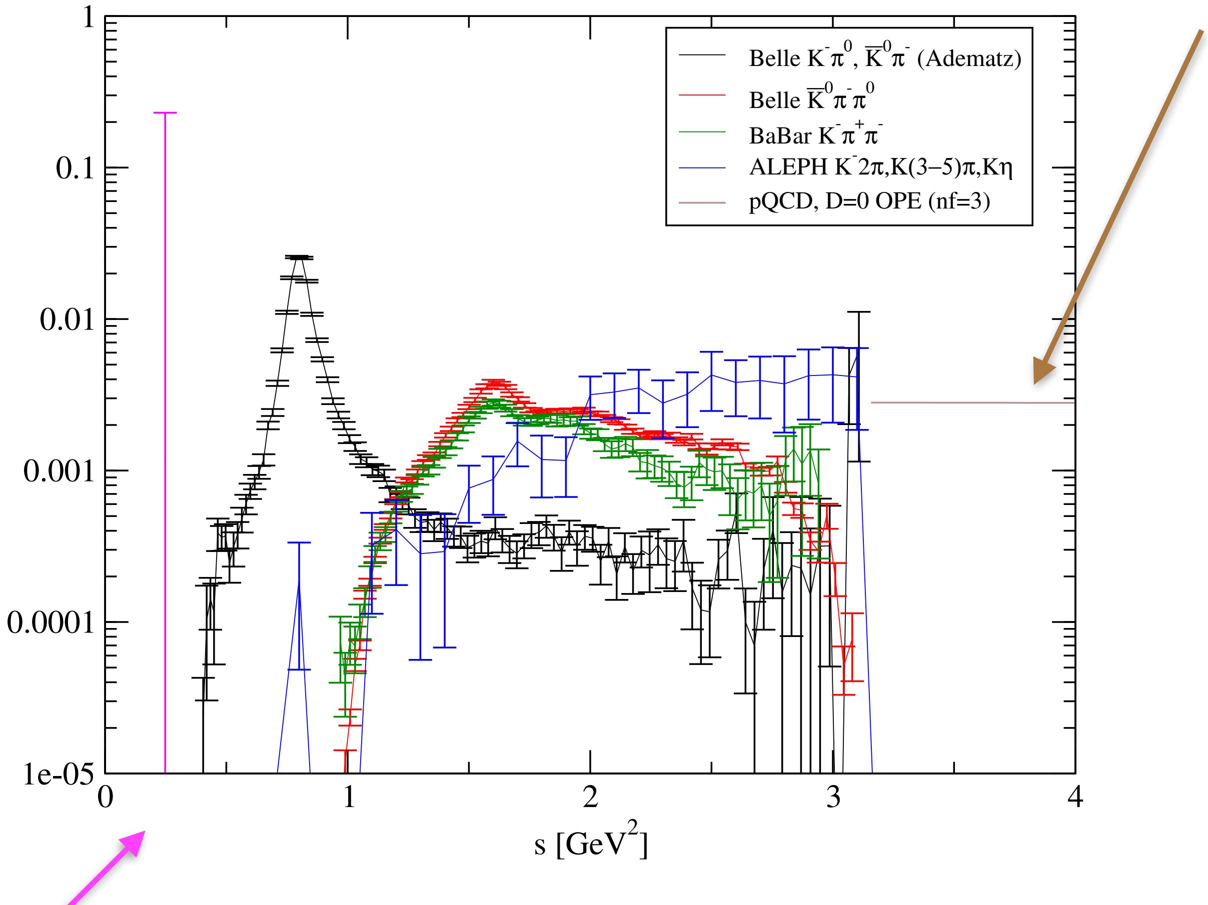
RHS … Lattice HPVs Π(Q) at Euclidean momentum region



τ inclusive decay experiment

$$\rho(s) \equiv |V_{us}|^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\Pi^1(\mathrm{s}) + \mathrm{Im}\Pi^0(\mathrm{s}) \right]$$

To compare with experiments,
a conventional value of |Vus|=0.2253 is used



For K pole, we assume a delta function form, whose coefficient is obtained from the experimental value of K-> μ decay width

$$\delta(s - m_k^2) 0.0012299(46) \ \sim 2 f_k^2 |V_{us}|^2$$

Weight function

- we use pole-type weight function;

$$\omega(s) = \prod_{k}^{N} \frac{1}{(s+Q_k^2)}, \quad (Q_k^2 > 0)$$

(Number of poles: N)

For convergence of contour integral,

a weight function with N ≧ 3 is required, which can suppress

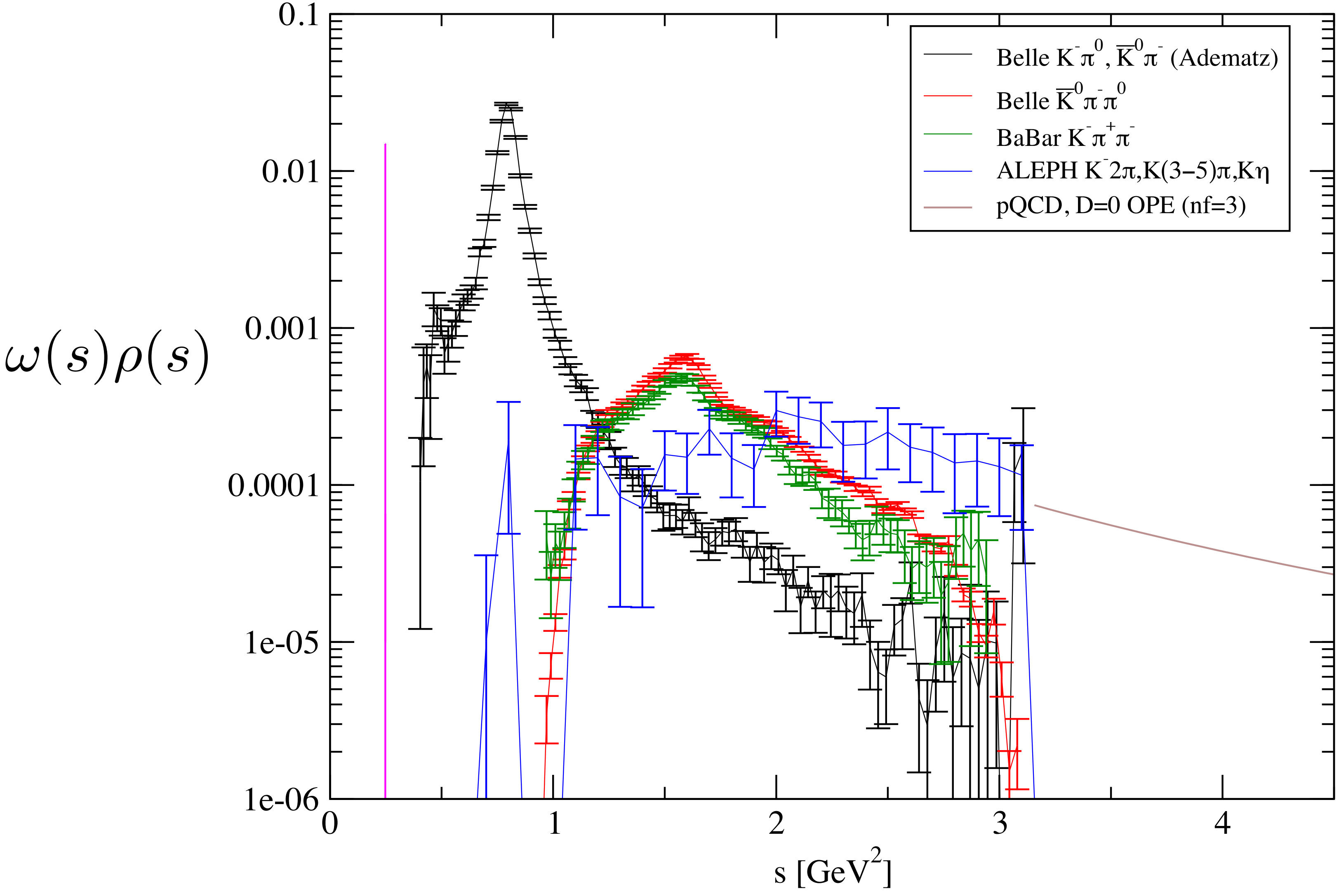
- larger error parts from higher multi hadron final states at s > mk^2
- contributions from pQCD at s > mτ^2

For lattice HVPs,

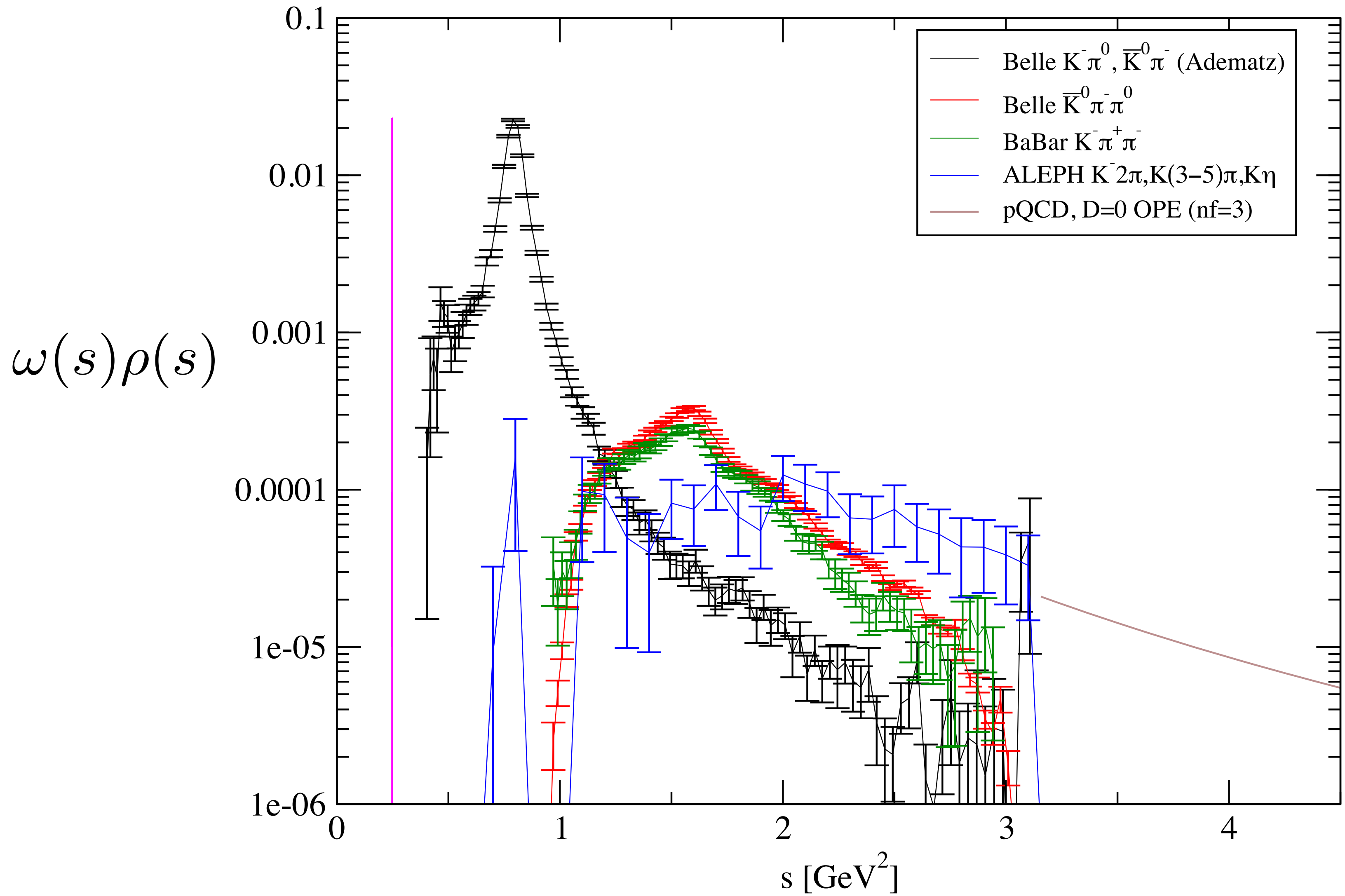
Q^2 values should not be too small to avoid finite size(time) effect,

and not to be large to avoid large discretization error.

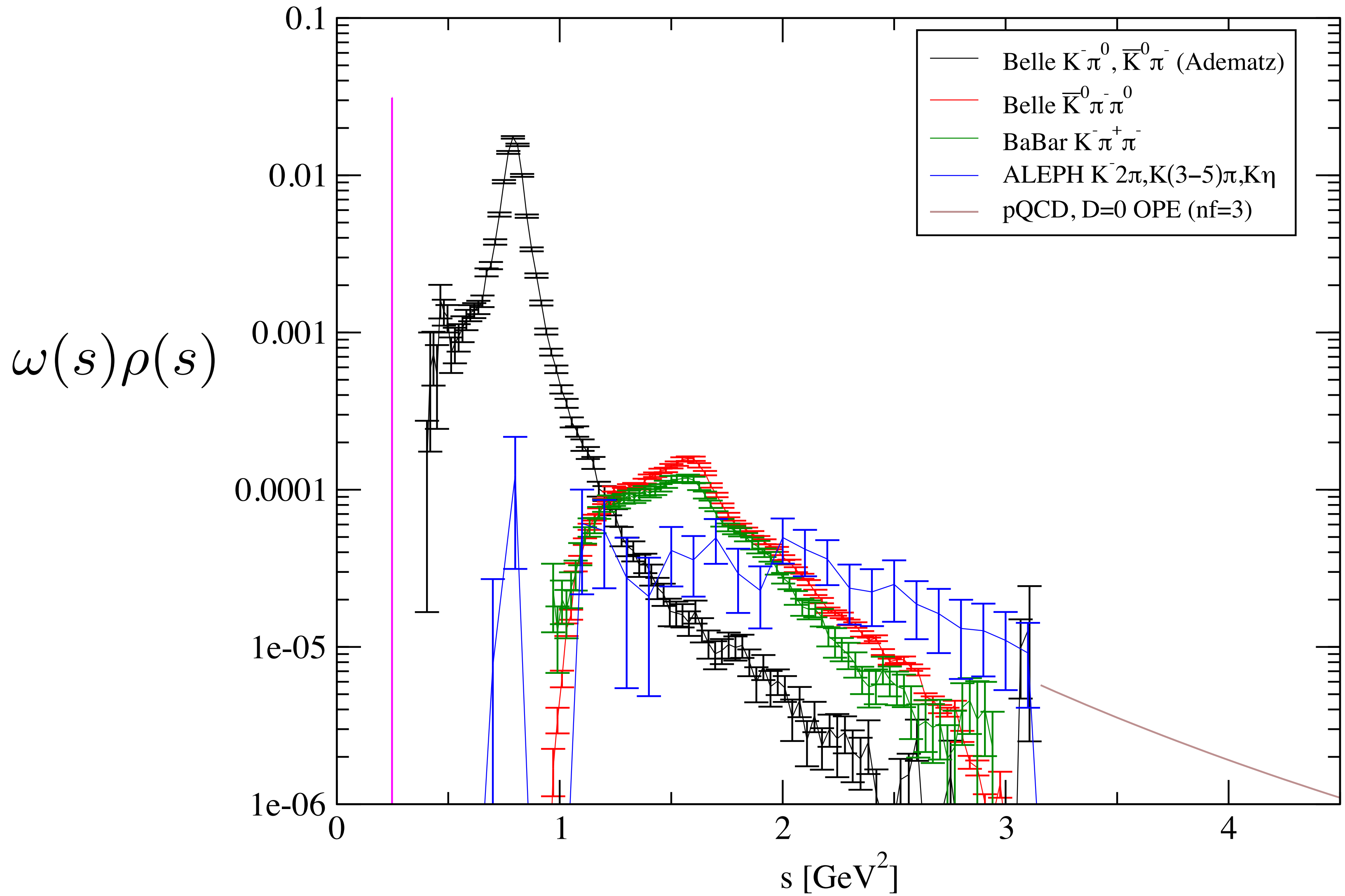
- example: N=3, $\{Q_1^2, Q_2^2, Q_3^2\} = \{0.1, 0.2, 0.3\}$



- example: N=4, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2\} = \{0.1, 0.2, 0.3, 0.4\}$



- example: N=5, $\{Q_1^2, Q_2^2, Q_3^2, Q_4^2, Q_5^2\} = \{0.1, 0.2, 0.3, 0.4, 0.5\}$



Lattice calculation

Lattice HVPs

HVPs from V/A current-current correlation functions with u s flavors,

we consider zero-spatial momentum

$$\Pi_{\mu\nu}^{V/A}(t) = \frac{1}{V}\sum_{\vec{x}}\langle J_{\mu}^{V/A}(\vec{x},t) J_{\nu}^{V/A}(\vec{x},0)\rangle$$

Spin =1, 0 components can be obtained in momentum space as

$$\Pi_{\mu\nu}(q) = (q^2\delta_{\mu\nu} - q_\mu q_\nu)\Pi^{(1)}(q^2) + q_\mu q_\nu \Pi^{(0)}(q^2),$$

On the lattice, those with subtraction of unphysical zero-mode can be obtained by discrete Fourier transformation,

(direct double subtraction, sine cardinal Fourier transformation.)

$$\hat{\Pi}(q^2) = \sum_{t=-T/2}^{t=T/2-1}\left(\frac{e^{i\tilde{q}t}-1}{q^2} + \frac{t^2}{2}\right)\Pi(t)$$

$$\tilde{q}_\mu = 2\sin\left(q_\mu/2\right)$$

lattice QCD ensemble and parameters

2+1 flavor domain-wall fermion gauge ensemble generated by RBC-UKQCD

Vol.	a^{-1}[GeV]	m_π[GeV]	m_K[GeV]	stat.
$24^3 \times 64$	1.785(5)	0.340	0.533	450
		0.340	0.593	450
$32^3 \times 64$	2.383(9)	0.303	0.537	372
		0.303	0.579	372
		0.360	0.554	207
		0.360	0.596	207
$48^3 \times 96$	1.730(4)	0.139	0.499	88
		$0.135^\dagger$	$0.4937^\dagger$	5 PQ-correction, (88)
$64^3 \times 128$	2.359(7)	0.139	0.508	80

- Our main analysis is done on L=48 and 64,
  at almost physical quark mass region, L=5 fm.
- **PQ-correction**: partially quench (PQ) corrected HVP data at the physical point (†)

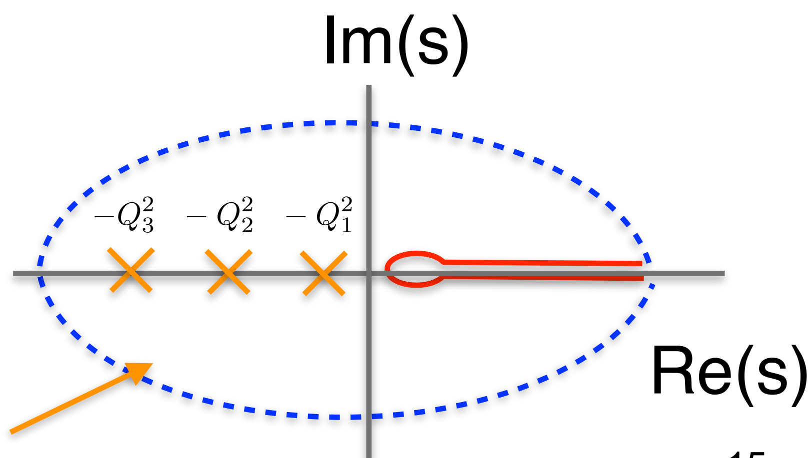
A systematic study of weight function dependence

$$\omega(s) = \prod_{k}^{N} \frac{1}{(s + Q_k^2)}, \quad (Q_k^2 > 0)$$

- C (center value of weights),
- Δ (separation of the pole position),
- N (the number of the poles).

$$\{Q_1^2, Q_2^2, \cdots, Q_N^2\} = \{C - (N/2+1)\Delta, \cdots, C - \Delta, C, C + \Delta, \cdots, C + (N/2+1)\Delta\}$$

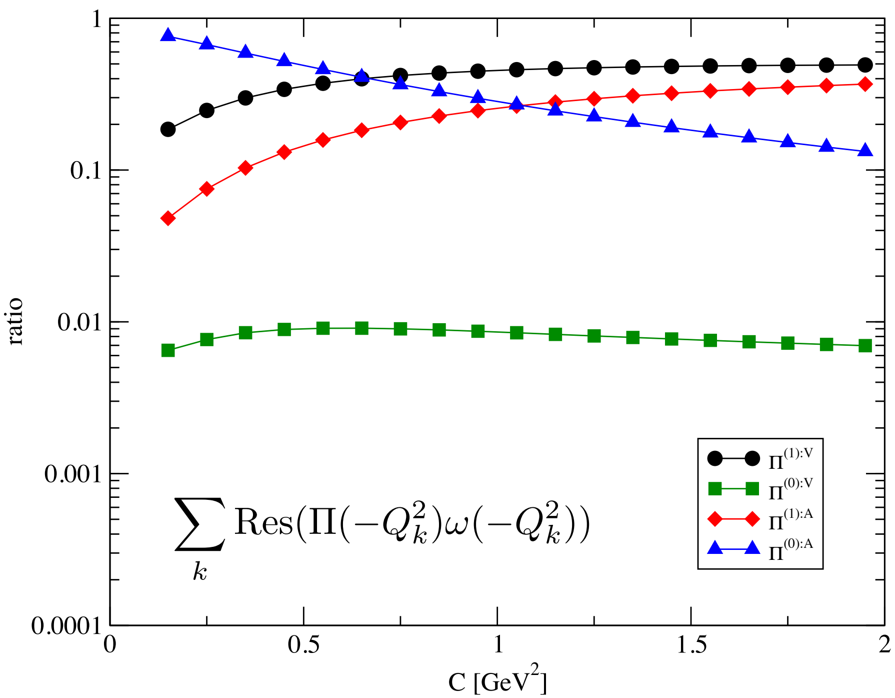
$$C = \frac{Q_1^2 + Q_2^2 + \cdots + Q_N^2}{N}$$



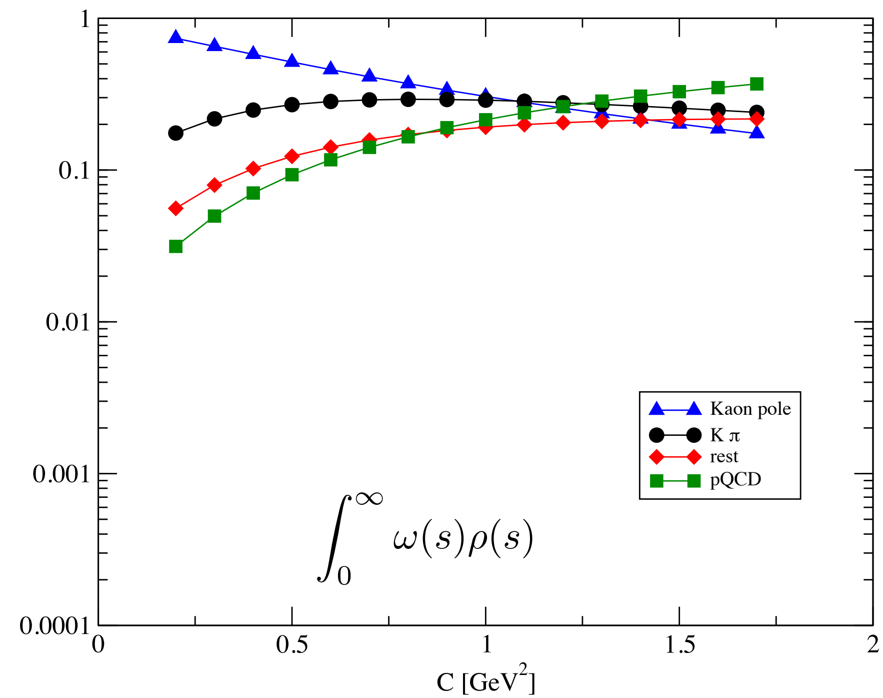
pole positions (N=3 case)

- N=3, Δ=0.1 [GeV^2]

$L = 48,\ a^{-1} = 1.73[\mathrm{GeV}],\ \ m_\pi = 0.139[\mathrm{GeV}]$



Experiment



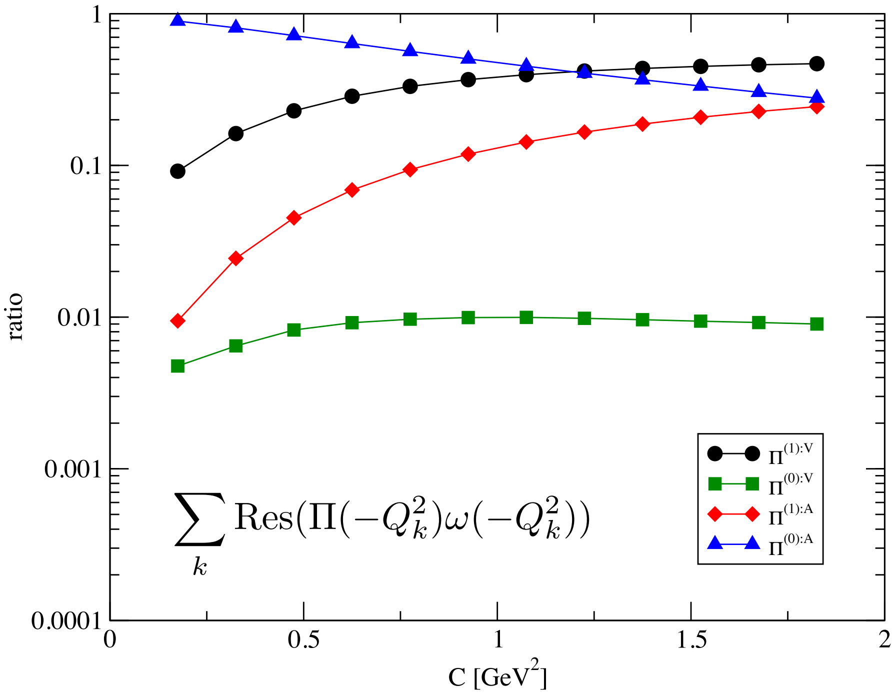
Left : Ratios of each contribution of V/A with spin=0, 1 to the total residue. (Lattice)

Right: Ratios of each decay modes to total cross section. (Experiments)

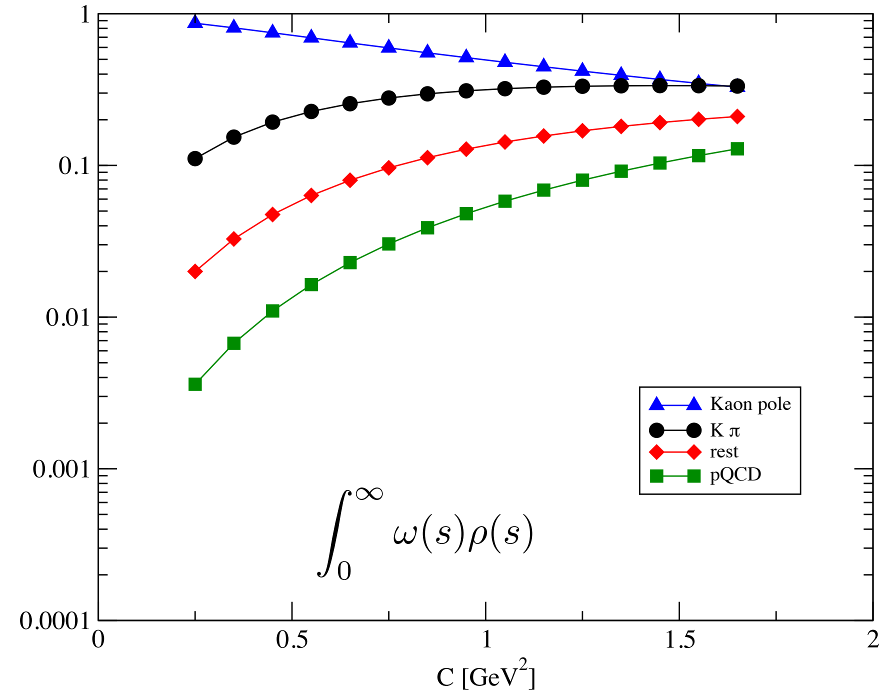
rest : multi π channels, K η

- N=4, Δ=0.1 [GeV^2]

$L = 48,\ a^{-1} = 1.73[\mathrm{GeV}],\ \ m_\pi = 0.139[\mathrm{GeV}]$

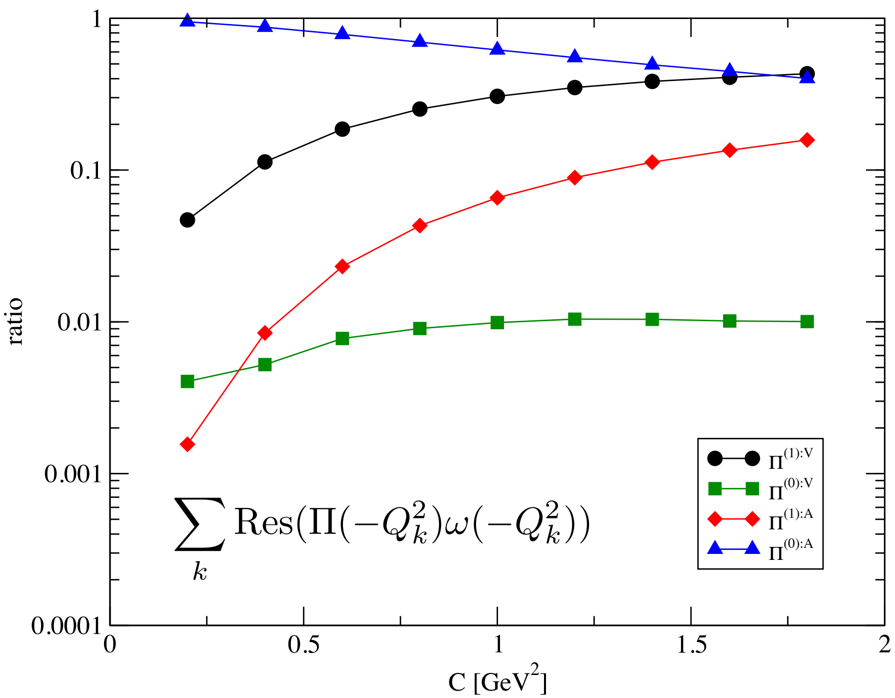


Experiment

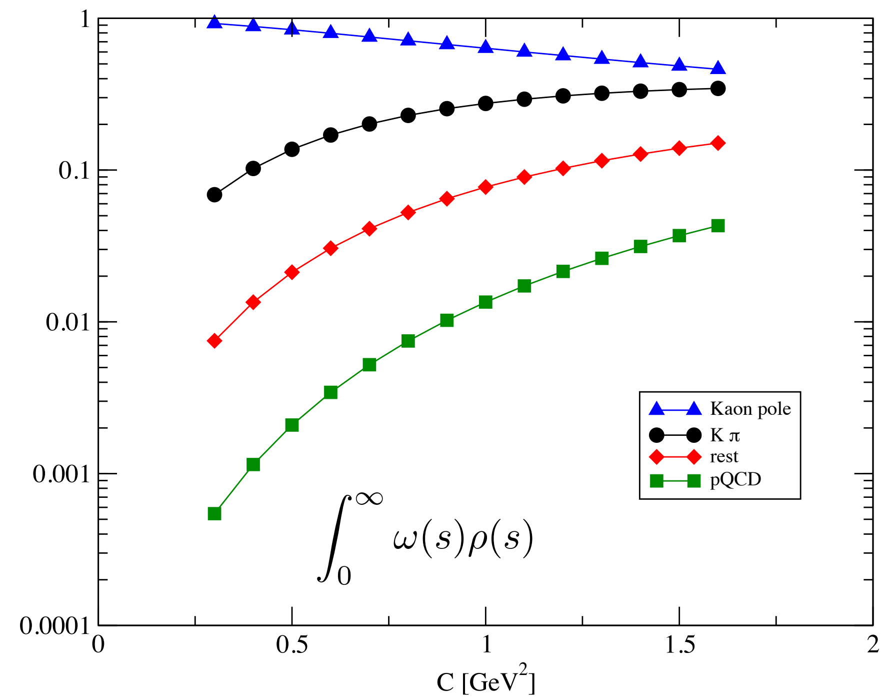


- N=5, Δ=0.1 [GeV^2]

$L = 48,\ a^{-1} = 1.73[\mathrm{GeV}],\ \ m_\pi = 0.139[\mathrm{GeV}]$



Experiment



- For larger N with smaller Q^2, Kaon pole is the most dominant contribution.
- pQCD and rest modes are highly suppressed.

|Vus| from lattice HVPs

- |Vus| can be determined from K pole channel only (exclusive mode).
- Since τ -> K decay mode is dominated by axial spin = 0 channel, so we have

$$A_0 \; : |V_{us}^{A_0}| = \sqrt{\frac{\rho_{exp}^{\mathrm{K-pole}}}{F_{lat}(\Pi^{(0):A})}}$$

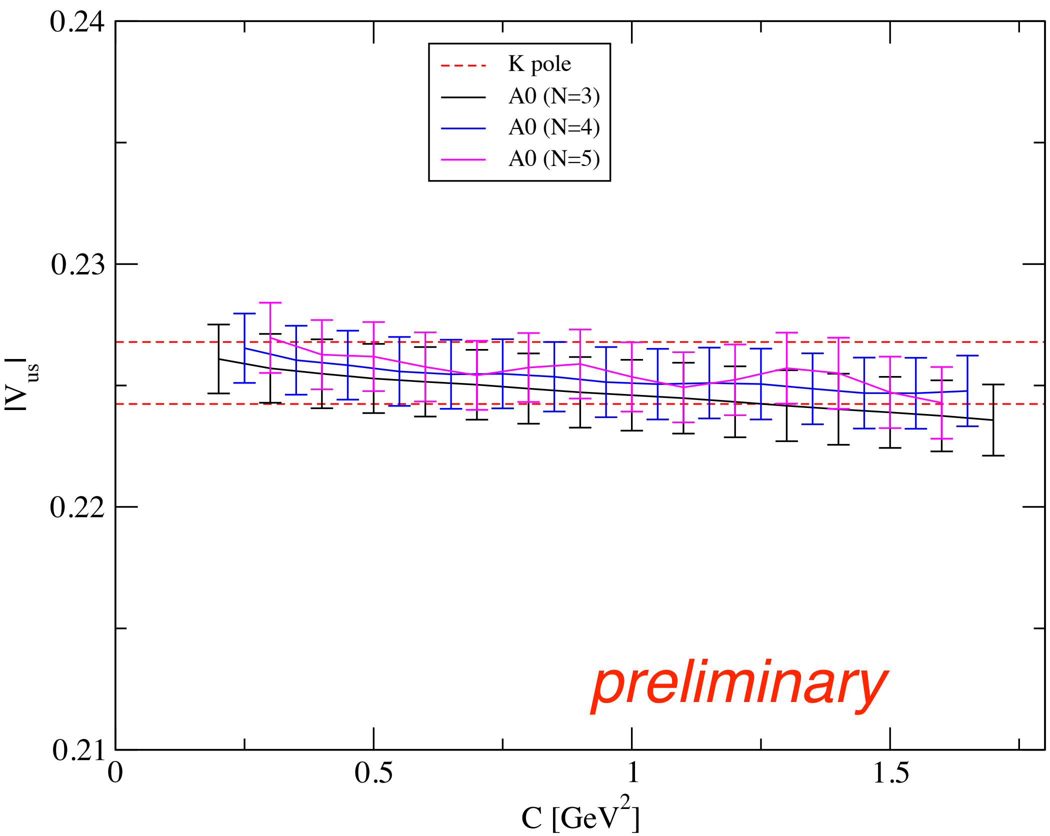
$$\rho_{exp}^{K-pole} = 0.0012299 \int_0^\infty ds \omega(s) \delta(s - m_K^2) = 0.0012299 \omega(m_K^2)$$

We use $f_k^2 |V_{us}|^2 = 0.0012299(46)$

obtained from the experimental value of K-> μ decay width

$$F_{lat} = \sum_{k=1}^{N} \mathrm{Res}\left(\omega(-Q_k^2)\right) \Pi_{lat}(-Q_k^2)$$

$|V_{us}^{A_0}|$ from L=48 lattice at physical quark mass



N : number of poles

K pole: determined from fK (K decay constant)

|Vus| is universal and consistent with fK determination (mild dependence of C, N)

Our result suggests : A0 channels is dominated by K pole

(Excited mode contributions and lattice discretization error are small
in this momentum region.)

|Vus| from All channels

- A0 channel is dominated by K pole.

-> The kaon decay constant in the continuum limit can be used.
  (well determined from lattice QCD)
  we use $f_K^{phys} = 0.15551(83)[\mathrm{GeV}]$ [RBC/UKQCD, 2014]

- How about other channels?
- Lattice HVPs for A1, V1, V0 < - > multi hadron states & pQCD
- We take the continuum limit using the data L=48 and 64

$$V_1 + V_0 + A_1 + A_0 \;\; : |V_{us}^{V_1+V_0+A_1+A_0}| = \sqrt{\frac{\rho_{exp}^{\mathrm{K-pole}} + \rho_{exp}^{others}}{(f_K^{phys})^2 \omega(m_K^2) + F_{lat}(\Pi_{\mathrm{others}}) - \rho_{\mathrm{pQCD}}}};$$

$$\rho_{exp}^{others} = |V_{us}|^2 \int_{s_{th}}^{m_\tau^2} ds\omega(s)\mathrm{Im}\Pi(\mathrm{s})$$

$$F_{lat} = \sum_{k=1}^{N} \mathrm{Res}(\omega(-\mathrm{Q_k^2}))\Pi_{\mathrm{lat}}(-\mathrm{Q_k^2}) \qquad \rho_{pQCD} = \int_{m_\tau^2}^{\infty} ds\omega(s)\Pi_{OPE}(s)$$

Systematic error estimate

- **Higher order discretization error** of a^4 for V1+V0+A1,

$$\mathcal{O}(C^2 a^4), \quad (a^{-1} = 2.37[\mathrm{GeV}])$$

- **Finite volume correction**

  1 loop ChPT analysis of current-current correlator on finite volume for Kπ channel (V1).

- **Isospin breaking effects**

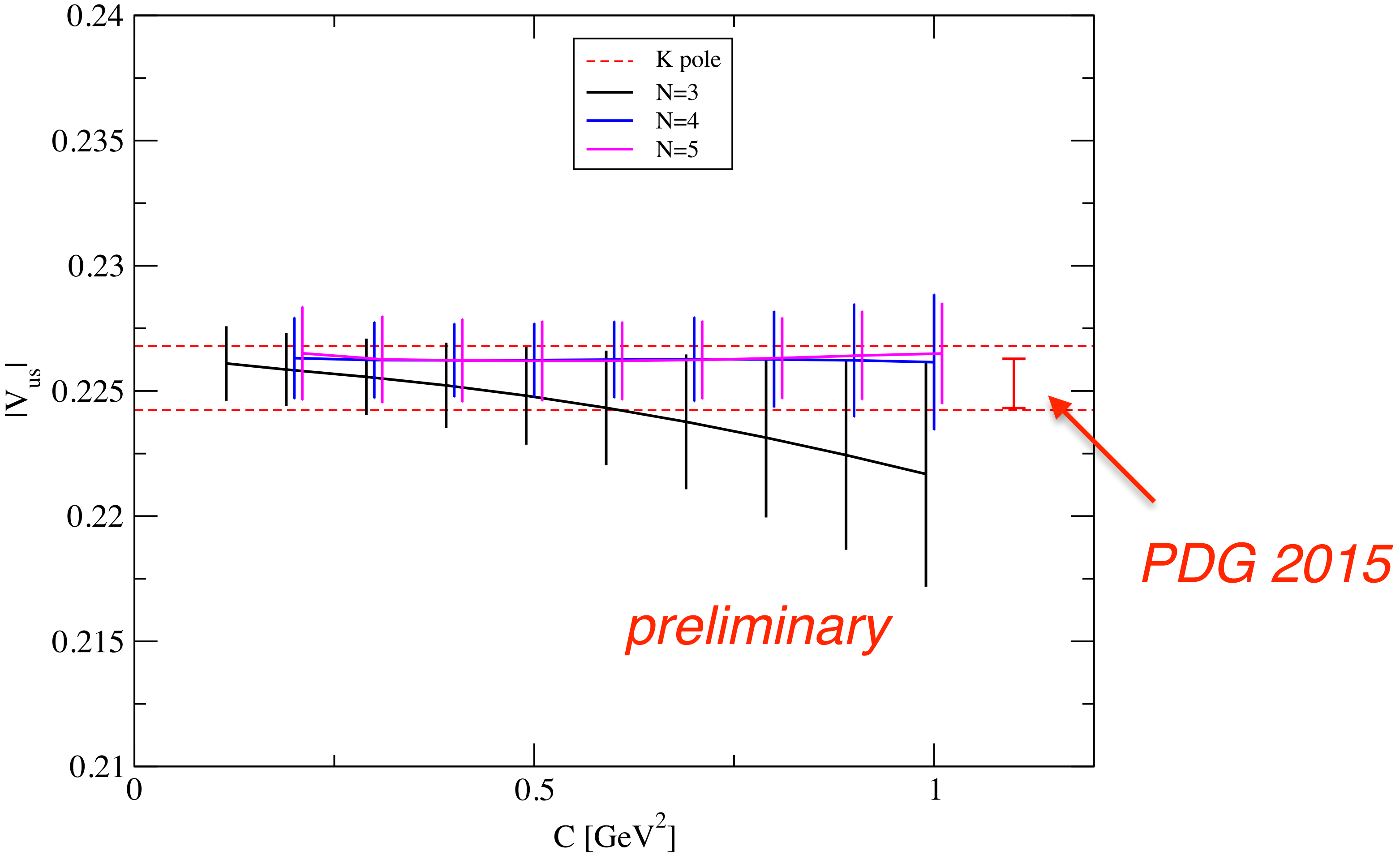
We put 0.2 % for isospin breaking (EM) effect on V1+V0+A1.

(We already corrected Kπ experimental data for dominant strong isospin breaking effect (roughly 2.9%).)

- **pQCD (OPE) uncertainty**

  2% for possible duality-violation effect

|Vus| for all channels

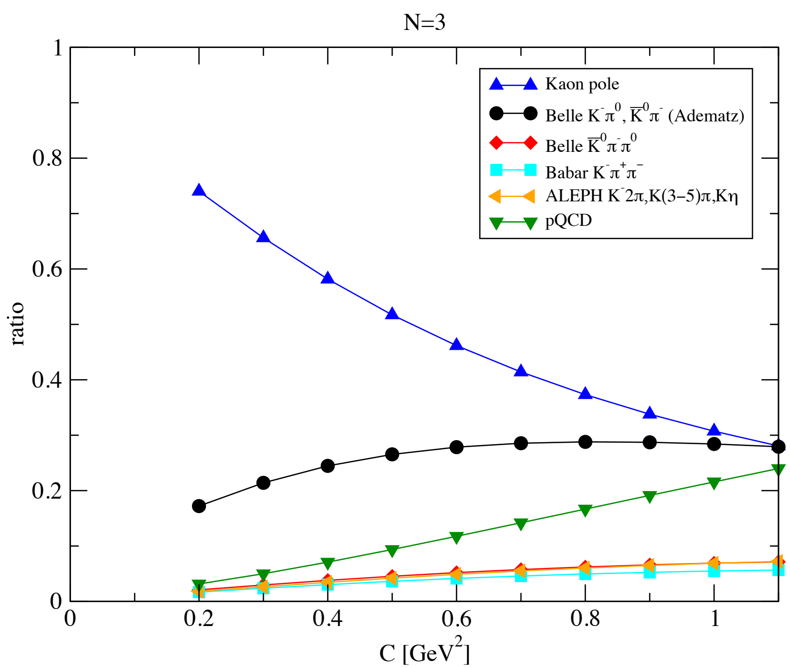


K pole: determined from fK (K decay constant)

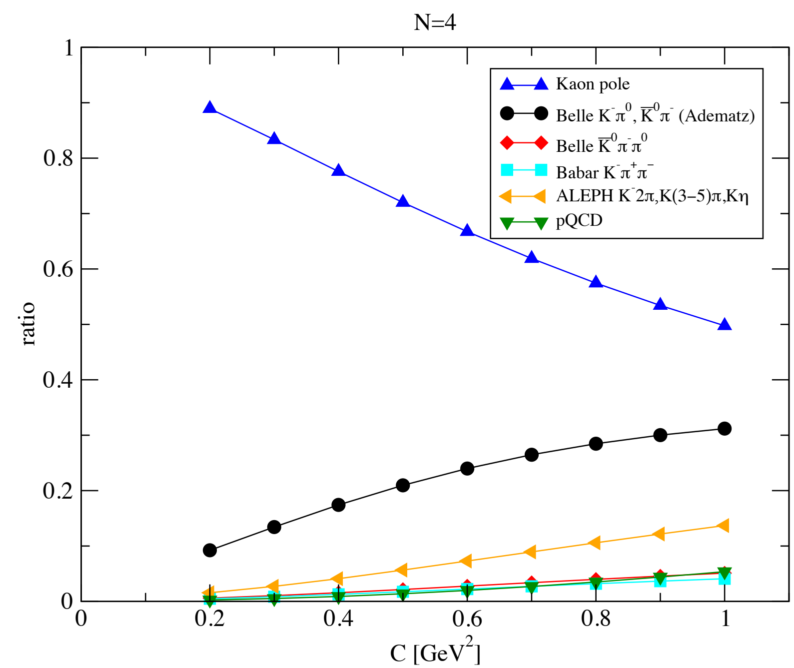
For N=4, 5, full result (V1 + V0 + A1+A0) is stable against the change of C, which is consistent with K pole determination.

Ratio of contributions

- N=3



- N=4

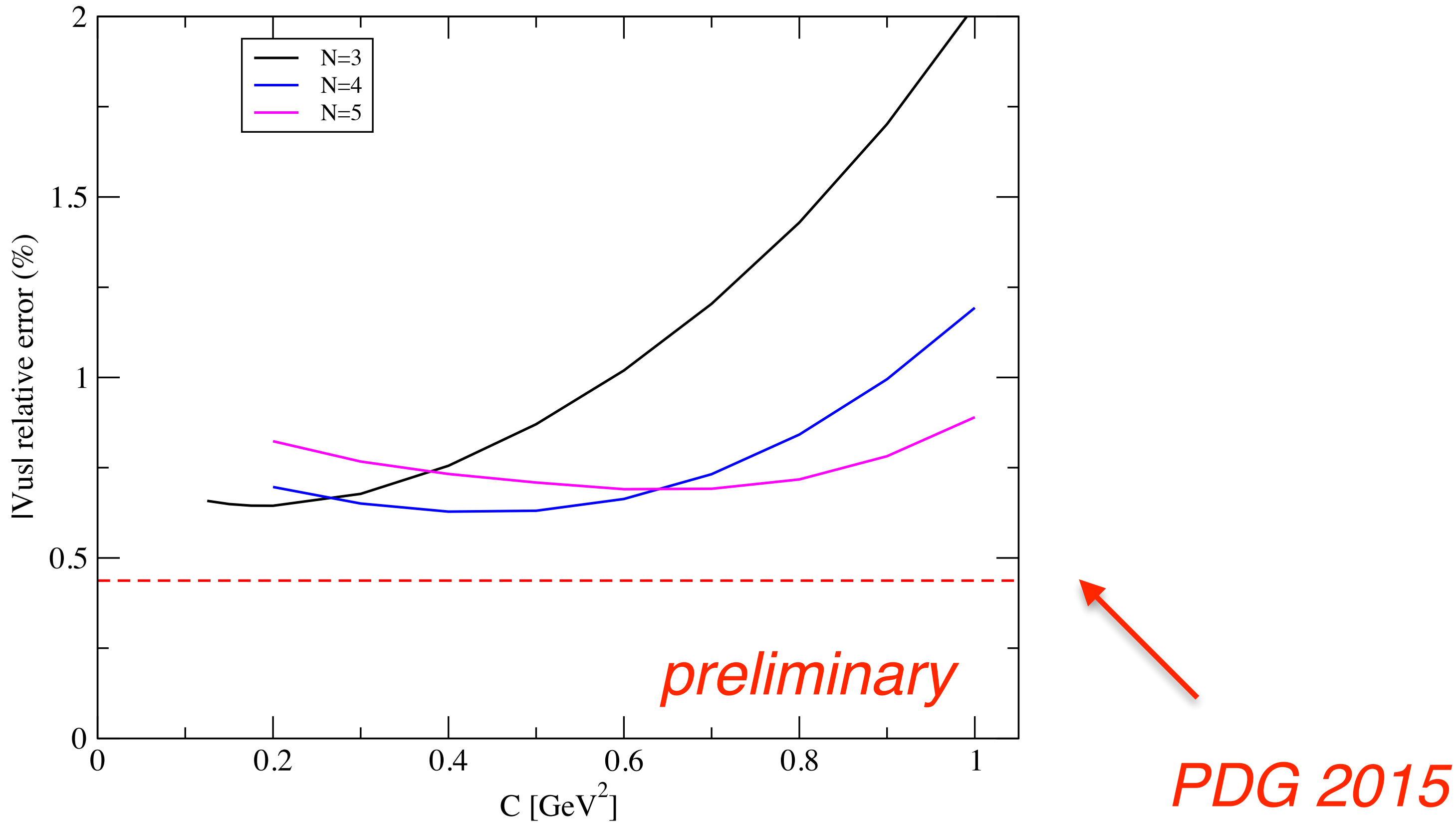


In large C region, perturbative QCD dominates spectral integral in both N.

N=3 : C ~ 0.5, 50 % : K, 30 % : Kπ, 20% : multi π & pQCD

N=4 : In small C ~ 0.2, 80%: K, 20 %: Kπ -> **K & Kπ dominant case**

|Vus| relative error



C and N dependence of error.

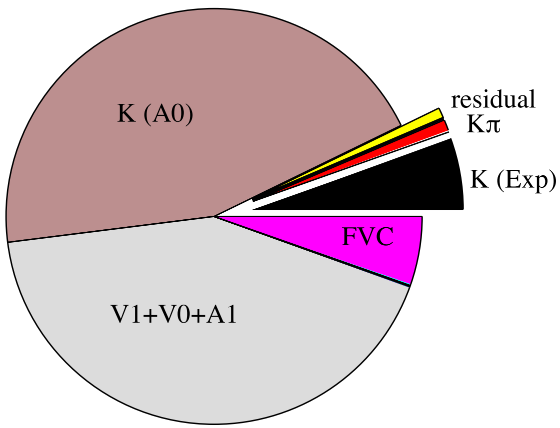
Minimum error can be found depending on the value of N,

In the case of N=4, C ~ 0.5.

|Vus| error^2 budget

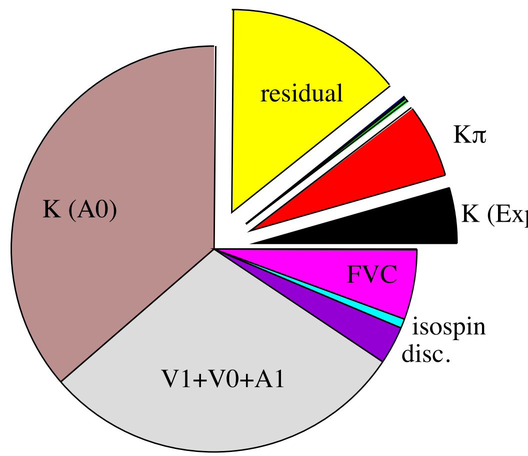
$N = 4, C = 0.2$

(K & Kπ dominant case)



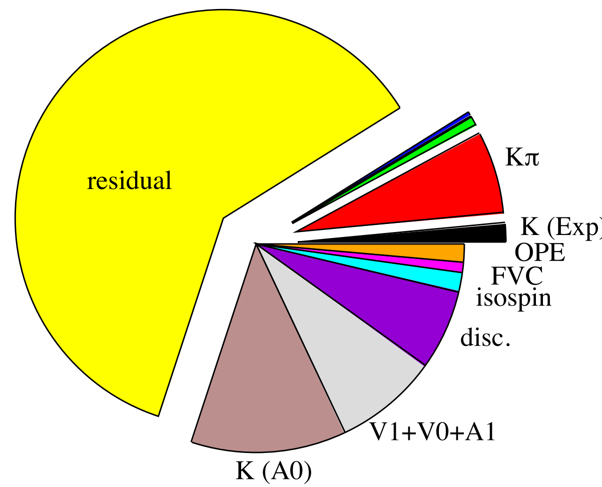
error^2	(%)
Exp.	6
K(A0)	45
V1+V0+A1	43
disc.	0
isospin	0
FVC	5
OPE	0

$N = 4, C = 0.5$



error^2	(%)
Exp.	25
K(A0)	36
V1+V0+A1	29
disc.	3
isospin	1
FVC	6
OPE	0

$N = 3, C = 0.5$



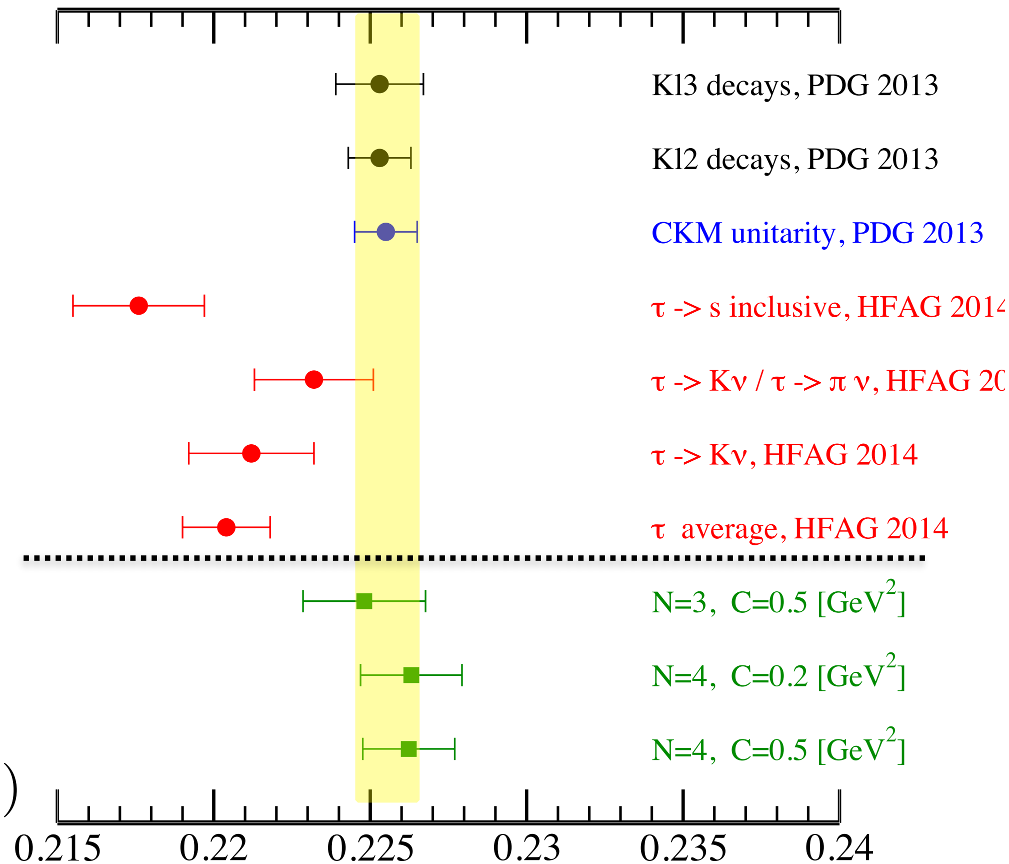
error^2	(%)
Exp.	70
K(A0)	12
V1+V0+A1	8
disc.	6
isospin	2
FVC	1
OPE	1

Result

very preliminary

Our result
for all channels

$(V_1 + V_0 + A_1 + A_0)$



This work

All our results (C<1, N=3,4) are consistent with each other and CKM unitarity constraint as well.

Summary

Precise determination of CKM matrix elements is very important.

We have demonstrated the dispersive relation between the inclusive τ decay experiments and the lattice vacuum polarizations, from which we can determine the CKM matrix element |Vus|.

- By introducing a weight function with poles at spacetime momenta, we could compare experimental spectrum, lattice HVP data, and pQCD to extract |Vus|.
- By changing the number and location of poles, N and C, we could adjust "inclusiveness", the impact of multi hadron states, apart from those from K pole and K-Pi.
- For most accurate Vus, Large N and smaller C, is preferable, where the lattice error (error of f_K and stat error of A1+V1+V0) dominate in our current analysis.
- To explore impact of multi-hadron states, where new physics/ may be hidden, larger C and/or smaller N, may be preferable

Future works:
Other systematic uncertainties should be investigated, e.g. quark mass effect near physical point, sea quark mass effect,

Thank you