Quark Chromoelectric Dipole Moment
Contribution
to the Neutron Electric Dipole Moment

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Two sources of CP violation in the Standard Model.

- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM


- CP-violating mass term and effective $\Theta G \tilde{G}$ interaction related to QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$


Contributions from beyond the standard model

- Needed to explain baryogenesis
- May have large contribution to EDM
**Introduction**

**Form Factors**

**Vector form-factors**

**Dirac** $F_1$, **Pauli** $F_2$, **Electric dipole** $F_3$, and **Anapole** $F_A$

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

\[
\langle N|V_\mu(q)|N \rangle = \bar{u}_N \left[ \gamma_\mu F_1(q^2) + i \frac{[\gamma_\mu, \gamma_\nu]}{2} q_\nu \frac{F_2(q^2)}{2m_N} 
+ (2i m_N \gamma_5 q_\mu - \gamma_\mu \gamma_5 q^2) \frac{F_A(q^2)}{m_N^2} 
+ \frac{[\gamma_\mu, \gamma_\nu]}{2} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_N} \right] u_N
\]

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
- $F_A$ and $F_3$ violate P; $F_3$ violates CP.
Introduction

Effective Field Theory


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nEDM from qCEDM
Standard model CP violation in the weak sector.
Strong CP violation from dimension 3 and 4 operators anomalously small.

- **Dimension 3 and 4:**
  - CP violating mass $\bar{\psi} \gamma_5 \psi$.
  - Toplogical charge $G_{\mu\nu} \tilde{G}^{\mu\nu}$.

- **Suppressed by $v_{EW}/M_{BSM}^2$:**
  - Electric Dipole Moment $\bar{\psi} \Sigma_{\mu\nu} \tilde{F}^{\mu\nu} \psi$.
  - Chromo Dipole Moment $\bar{\psi} \Sigma_{\mu\nu} \tilde{G}^{\mu\nu} \psi$.

- **Suppressed by $1/M_{BSM}^2$:**
  - Weinberg operator (Gluon chromo-electric moment): $G_{\mu\nu} G_{\lambda\nu} \tilde{G}^{\mu\lambda}$.
  - Various four-fermi operators.
The quark chromo-EDM operator is a quark bilinear. **Schwinger source method:** Add it to the Dirac operator in the propagator inversion routine:

\[
\hat{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu} \rightarrow \hat{D} + m - \frac{r}{2} D^2 + \Sigma^{\mu\nu} (c_{sw} G_{\mu\nu} + i\epsilon \tilde{G}_{\mu\nu})
\]

The fermion determinant gives a ‘reweighting factor’

\[
\frac{\det(\hat{D} + m - \frac{r}{2} D^2 + \Sigma^{\mu\nu} (c_{sw} G_{\mu\nu} + i\epsilon \tilde{G}_{\mu\nu}))}{\det(\hat{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu})} = \exp \text{Tr} \ln \left[ 1 + i\epsilon \Sigma^{\mu\nu} \tilde{G}_{\mu\nu} (\hat{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu})^{-1} \right]
\]

\[
\approx \exp \left[ i\epsilon \text{Tr} \Sigma^{\mu\nu} \tilde{G}_{\mu\nu} (\hat{D} + m - \frac{r}{2} D^2 + c_{sw} \Sigma^{\mu\nu} G_{\mu\nu})^{-1} \right].
\]
The chromoEDM operator is dimension 5.
Uncontrolled divergences unless $\epsilon \lesssim 4\pi a\Lambda_{QCD} \sim 1$.
Need to check linearity.
Tests on two MILC ensembles.

- $a \approx 0.12$ fm, $M_\pi \approx 310$ MeV, 
  $\kappa \approx 0.1272103$, $c_{SW} = 1.05094$, $u_P^{HYP} = 0.9358574(29)$. 
  400 Configurations, 64 LP + 4 HP calculations/configuration.

- $a \approx 0.09$ fm, $M_\pi \approx 310$ MeV, 
  $\kappa \approx 0.1266265$, $c_{SW} = 1.04243$, $u_P^{HYP} = 0.9461130(10)$. 
  270 Configurations, 64 LP + 4 HP calculations/configuration.

Use two CP violating operators that mix under renormalization.

- $\text{CEDM}: a^2 \bar{\psi} \tilde{G} \cdot \Sigma \psi$
- $\text{P}: \bar{\psi} \gamma_5 \psi$
Two point functions
Neutron Propagator

Preliminary; Connected Diagrams Only
Two point functions

Linearity

Preliminary; Connected Diagrams Only

Use \( \epsilon \approx \frac{a}{30 \text{fm}} \approx 6.6 \text{MeV} a \approx 0.36 \text{ma} \) for experiments.
Two point functions
Connected $\gamma_5$

\[
a(D + m) + i\epsilon\gamma_5 = e^{\frac{i}{2}\alpha_q\gamma_5} (aD + am\epsilon) e^{\frac{i}{2}\alpha_q\gamma_5}
\]

where $\alpha_q \equiv \tan^{-1} \frac{\epsilon}{am}$
and $am\epsilon \equiv \sqrt{(am)^2 + \epsilon^2}$
\begin{table}
\begin{tabular}{|l|c|c|}
\hline
 & a12m310 & a09m310 \\
\hline
\(am^0\) & \(\frac{1}{2\kappa} - 4\) & -0.0695 & -0.05138 \\
\(am_{cr}\) & \(\frac{1}{2\kappa_c} - 4\) & -0.08058 & -0.05943 \\
\(am\) & \(am^0 - am_{cr}\) & 0.01108 & 0.00805 \\
\(\epsilon\) & & 0.004 & 0.003 \\
\(am_{\epsilon}\) & & 0.01178 & 0.00859 \\
\hline
\(M_\pi\) & & 0.1900(4) & 0.1404(3) \\
\(M_\pi^{CEDM}\) & & 0.1906(4) & 0.1407(3) \\
\(M_\pi^{\gamma 5}\) & & 0.1961(4) & 0.1450(3) \\
\(M_\pi^0 \times \sqrt{\frac{m_\epsilon}{m}}\) & & 0.1959(4) & 0.1450(3) \\
\hline
\end{tabular}
\end{table}
Two point functions

$\alpha_N$
Three point functions

The three point function we calculate is

\[ N \equiv \bar{d}^c \gamma_5 \frac{1 + \gamma_4}{2} u d \]

\[ \langle \Omega | N(\vec{0}, 0) V_\mu(\vec{q}, t) N^\dagger(\vec{p}, T) | \Omega \rangle = u_N e^{-m_N t} \langle N | V_\mu(q) | N' \rangle e^{-E_{N'}(T-t)} \bar{u}_N \]

We project onto only one component of the neutron spinor with

\[ P = \frac{1}{2} (1 + \gamma_4)(1 + i\gamma_5 \gamma_3) \]

Noting that in presence of CP violation \( u_N \bar{u}_N = e^{i\alpha_N \gamma_5 (i\phi + m_N)} e^{i\alpha_N \gamma_5} \)

and assuming \( N' = N \), we can extract:

\[ Tr P \langle \Omega | N V_3 N^\dagger | \Omega \rangle \propto i m_N q_3 G_E \]

\[ + \alpha_N m_N (E_N - m_N) F_1 + \alpha_N [m_N (E_N - m_N) + \frac{q_3^2}{2}] F_2 \]

\[- 2i (q_1^2 + q_2^2) F_A - \frac{q_3^2}{2} F_3 \]
Three point functions

$F_3$ Form factor from CEDM

$\epsilon = 0.004$, $a \approx 0.12$ fm.

Preliminary; Connected Diagrams Only
$\epsilon = 0.003$, $a \approx 0.09$ fm.
Three point functions

\( F_3 \) Form factor from \( \gamma_5 \)

\[ \epsilon = 0.004, \quad a \approx 0.12 \text{ fm}. \]
\[ \epsilon = 0.003, \ a \approx 0.09 \text{ fm}. \]
Three point functions

$F_3(\gamma_5)$ and $F_3(\text{CEDM})$

$$a(\not{\Psi} + m) + i\epsilon\gamma_5 = e^{i\frac{\alpha}{2}q\gamma_5} a(\not{\Psi} + m\epsilon) e^{i\frac{\alpha}{2}q\gamma_5}$$

$$\rightarrow a(\not{\Psi}_L + m) + i\epsilon\gamma_5 = e^{i\frac{\alpha}{2}q\gamma_5} e^{-\frac{i\phi}{2} \gamma_5 a(\not{\Psi}_e + m\epsilon)} a(\not{\Psi}_e + m\epsilon) e^{-\frac{i\phi}{2} \gamma_5 a(\not{\Psi}_e + m\epsilon)} e^{i\frac{\alpha}{2}q\gamma_5} + O(a^3)$$

where

$$\not{\Psi}_L = \not{\Psi} + aD^2 - \frac{\kappa_{cSW}}{2} a\Sigma^{\mu\nu} G_{\mu\nu}; \quad \not{\Psi}_e = \not{\Psi} + \zeta aD^2 - \chi a\Sigma^{\mu\nu} G_{\mu\nu} e^{i\xi\gamma_5}$$

$$m_\epsilon a = \sqrt{m^2 a^2 + \epsilon^2}, \quad \phi = \frac{\epsilon}{m_\epsilon a}, \quad \xi = \frac{2\phi}{\kappa_{cSW}}, \quad \chi = \frac{\kappa_{cSW}}{2} \sqrt{1 + \left(\frac{2\phi}{\kappa_{cSW}}\right)^2},$$

$$\zeta = \frac{m}{m_\epsilon}, \quad \alpha = \tan^{-1} \frac{\epsilon}{ma} + 2\epsilon$$

$e^{-\frac{i\phi}{2} \gamma_5 a(\not{\Psi}_e + m\epsilon)}$ does not contribute on shell.
Introduction
Lattice Calculation
Two point functions
Three point functions
Conclusions

Projection
$F_3$ Form factor from CEDM
$F_3$ Form factor from $\gamma_5$
$F_3(\gamma_5)$ and $F_3$(CEDM)

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nEDM from qCEDM
Signal in the connected diagram for $a = 0.12$ and $a = 0.09$ fm and $M_\pi = 310$ MeV.

Mixing with lower dimensional operator not a problem.

Need to calculate renormalization and mixing coefficients non-perturbatively.

Fermions with better chiral symmetry does not avoid mixing.