Quark Chromoelectric Dipole Moment Contribution to the Neutron Electric Dipole Moment

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Introduction
Standard Model CP Violation

Two sources of CP violation in the Standard Model.

- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM

Dar arXiv:hep-ph/0008248.

- CP-violating mass term and effective $\Theta G\tilde{G}$ interaction related to QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$

Crewther *et al.*, *Phys. Lett.* **B88** (1979) 123.

Contributions from beyond the standard model

- Needed to explain baryogenesis
- May have large contribution to EDM

Introduction

Form Factors

Vector form-factors

Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

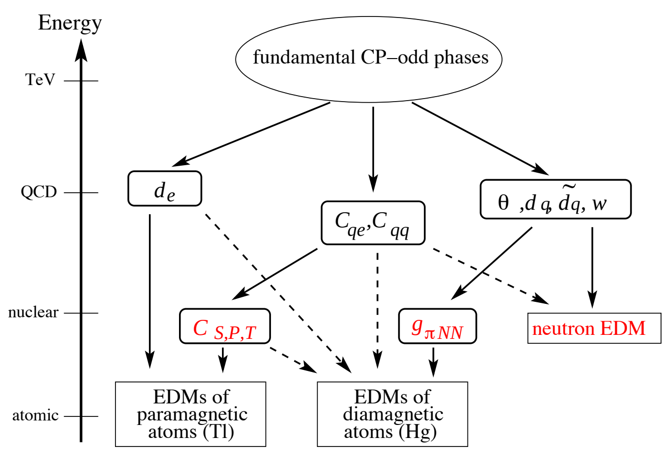
Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

$$
\begin{aligned}
\langle N|V_\mu(q)|N\rangle \;=\; \overline{u}_N \Bigg[\gamma_\mu\, F_1(q^2) &+ i\frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu\, \frac{F_2(q^2)}{2m_N} \\
&+ (2i\, m_N \gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2)\, \frac{F_A(q^2)}{m_N^2} \\
&+ \frac{[\gamma_\mu,\gamma_\nu]}{2} q_\nu \gamma_5\, \frac{F_3(q^2)}{2m_N}\Bigg] u_N
\end{aligned}
$$

- The charge $G_E(0) = F_1(0) = 0$.
- $G_M(0)/2M_N = F_2(0)/2M_N$ is the (anomalous) magnetic dipole moment.
- $F_3(0)/2m_N$ is the electric dipole moment.
- F_A and F_3 violate P; F_3 violates CP.

Introduction

Effective Field Theory



Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119.

Introduction
BSM Operators

Standard model CP violation in the weak sector.
Strong CP violation from dimension 3 and 4 operators anomalously small.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\mathrm{EW}}/M^2_{\mathrm{BSM}}$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M^2_{\mathrm{BSM}}$:
  - Weinberg operator (Gluon chromo-electric moment): $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Lattice Calculation

Technique

The quark chromo-EDM operator is a quark bilinear.
Schwinger source method: Add it to the Dirac operator in the propagator inversion routine:

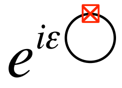
$$\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu} \longrightarrow \not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu})$$

The fermion determinant gives a 'reweighting factor'

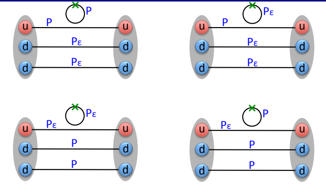
$$\begin{aligned}
&\frac{\det(\not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{sw}G_{\mu\nu} + i\epsilon\tilde{G}_{\mu\nu}))}{\det(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})} \\
&= \quad \exp \mathrm{Tr} \ln\left[1 + i\epsilon\, \Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right] \\
&\approx \quad \exp\left[i\epsilon\, \mathrm{Tr}\, \Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{sw}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\right] .
\end{aligned}$$

Lattice Calculation

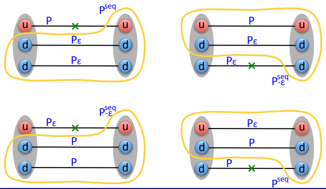
Three-point function



$\times$



$+$



The chromoEDM operator is dimension 5.

Uncontrolled divergences unless $\epsilon \lesssim 4\pi a\Lambda_{\rm QCD} \sim 1$.

Need to check linearity.

Two point functions
Ensembles

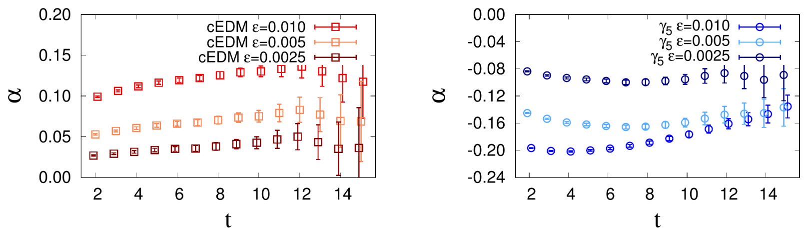
Tests on two MILC ensembles.

- $a \approx 0.12$ fm, $M_\pi \approx 310$ MeV,
  $\kappa \approx 0.1272103$, $c_{\rm SW} = 1.05094$, $u_P^{HYP} = 0.9358574(29)$.
  400 Configurations, 64 LP + 4 HP calculations/configuration.
- $a \approx 0.09$ fm, $M_\pi \approx 310$ MeV,
  $\kappa \approx 0.1266265$, $c_{\rm SW} = 1.04243$, $u_P^{HYP} = 0.9461130(10)$.
  270 Configurations, 64 LP + 4 HP calculations/configuration.

Use two CP violating operators that mix under renormalization.

- CEDM: $a^2\bar\psi\,\tilde G\cdot\Sigma\,\psi$
- P: $\bar\psi\gamma_5\psi$

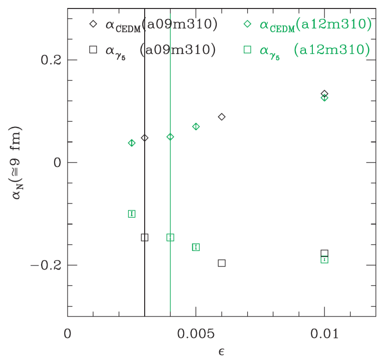
Two point functions
Neutron Propagator



Preliminary; Connected Diagrams Only

Two point functions

Linearity



Preliminary; Connected Diagrams Only

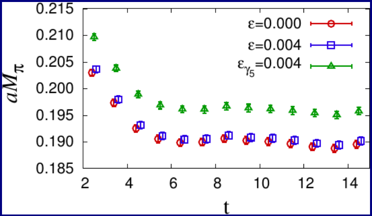
Use $\epsilon \approx \frac{a}{30\text{fm}} \approx 6.6\text{MeV}\, a \approx 0.36\, ma$ for experiments.

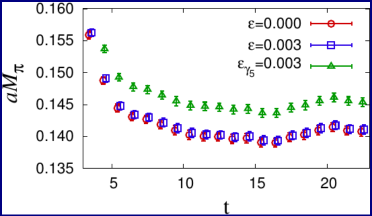
Two point functions
Connected γ_5

$$a(\not{D} + m) + i\epsilon\gamma_5 = e^{\frac{i}{2}\alpha_q\gamma_5}\left(a\not{D} + am_\epsilon\right)e^{\frac{i}{2}\alpha_q\gamma_5}$$

where $\alpha_q \equiv \tan^{-1}\frac{\epsilon}{am}$

and $am_\epsilon \equiv \sqrt{(am)^2 + \epsilon^2}$

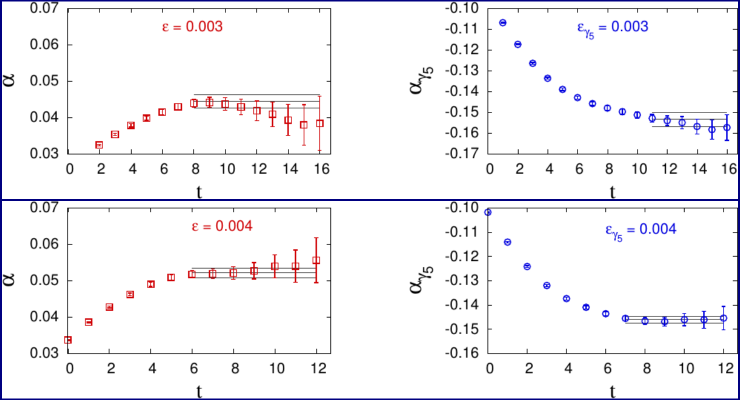




	a12m310	a09m310
$am^0 \equiv \frac{1}{2\kappa} - 4$	-0.0695	-0.05138
$am_{cr} \equiv \frac{1}{2\kappa_c} - 4$	-0.08058	-0.05943
$am \equiv am^0 - am_{cr}$	0.01108	0.00805
ϵ	0.004	0.003
am_ϵ	0.01178	0.00859
M_π^0	0.1900(4)	0.1404(3)
M_π^{CEDM}	0.1906(4)	0.1407(3)
$M_\pi^{\gamma_5}$	0.1961(4)	0.1450 (3)
$M_\pi^0 \times \sqrt{\frac{m_\epsilon}{m}}$	0.1959(4)	0.1450(3)

Two point functions

α_N



Three point functions
Projection

The three point function we calculate is

$$N \equiv \bar{d}^c \gamma_5 \frac{1+\gamma_4}{2} u \; d$$
$$\langle \Omega | N(\vec{0},0) V_\mu(\vec{q},t) N^\dagger(\vec{p},T) | \Omega \rangle \quad = \quad u_N e^{-m_N t} \, \langle N | V_\mu(q) | N' \rangle \, e^{-E_{N'}(T-t)} \overline{u}_N$$

We project onto only one component of the neutron spinor with

$$\mathcal{P} = \frac{1}{2}(1+\gamma_4)(1+i\gamma_5\gamma_3)$$

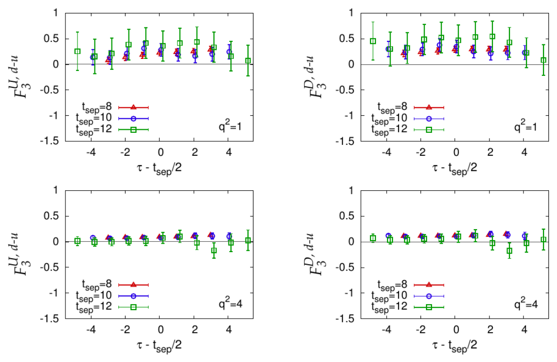
Noting that in presence of CP violation $u_N \overline{u}_N = e^{i\alpha_N \gamma_5}(i\not{p} + m_N) e^{i\alpha_N \gamma_5}$ and assuming $N' = N$, we can extract:

$$\begin{aligned} \mathrm{Tr}\, \mathcal{P} \langle \Omega | N V_3 N^\dagger | \Omega \rangle \quad \propto \quad & i m_N q_3 G_E \\ & + \alpha_N m_N (E_N - m_N) F_1 + \alpha_N [m_N (E_N - m_N) + \frac{q_3^2}{2}] F_2 \\ & - 2i\,(q_1^2 + q_2^2) F_A - \frac{q_3^2}{2} F_3 \end{aligned}$$

Three point functions

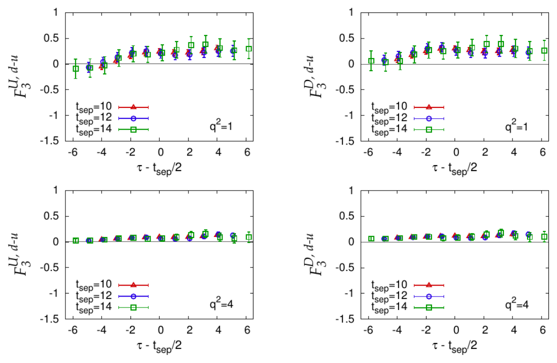
F_3 Form factor from CEDM

$$\epsilon = 0.004,\ a \approx 0.12\,\mathrm{fm}.$$



Preliminary; Connected Diagrams Only

$$\epsilon = 0.003,\ a \approx 0.09\,\mathrm{fm}.$$

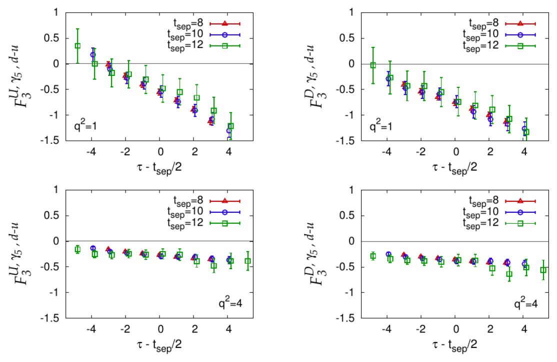


Preliminary; Connected Diagrams Only

Three point functions

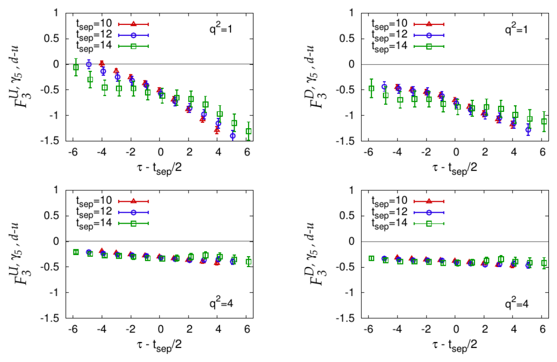
F_3 Form factor from γ_5

$$\epsilon = 0.004, \; a \approx 0.12\,\text{fm}.$$



Preliminary; Connected Diagrams Only

$$\epsilon = 0.003, \ a \approx 0.09\,\text{fm}.$$



Preliminary; Connected Diagrams Only

Three point functions
$F_3(\gamma_5)$ and F_3(CEDM)

$$a(\not{D} + m) + i\epsilon\gamma_5 = e^{\frac{i}{2}\alpha_q\gamma_5} a\left(\not{D} + m_\epsilon\right) e^{\frac{i}{2}\alpha_q\gamma_5}$$

$$\longrightarrow a(\not{D}_L + m) + i\epsilon\gamma_5 = e^{\frac{i}{2}\alpha\gamma_5} e^{-\frac{i\phi}{2}\gamma_5 a(\not{D}_\epsilon + m_\epsilon)} a(\not{D}_\epsilon + m_\epsilon) e^{-\frac{i\phi}{2}\gamma_5 a(\not{D}_\epsilon + m_\epsilon)} e^{\frac{i}{2}\alpha\gamma_5} + O(a^3)$$

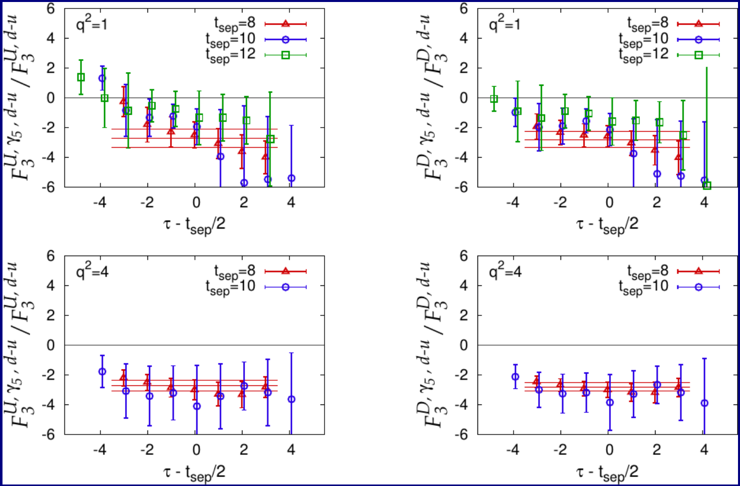
where

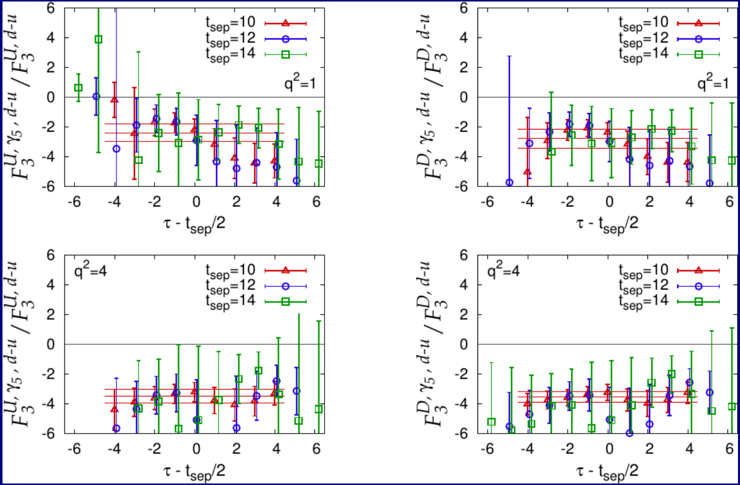
$$\not{D}_L = \not{D} + aD^2 - \frac{\kappa c_{SW}}{2} a\Sigma^{\mu\nu} G_{\mu\nu}; \quad \not{D}_\epsilon = \not{D} + \zeta a D^2 - \chi a \Sigma^{\mu\nu} G_{\mu\nu} e^{i\xi\gamma_5}$$

$$m_\epsilon a = \sqrt{m^2 a^2 + \epsilon^2}, \quad \phi = \frac{\epsilon}{m_\epsilon a}, \quad \xi = \frac{2\phi}{\kappa c_{SW}}, \quad \chi = \frac{\kappa c_{SW}}{2}\sqrt{1 + \left(\frac{2\phi}{\kappa c_{SW}}\right)^2},$$

$$\zeta = \frac{m}{m_\epsilon}, \quad \alpha = \tan^{-1}\frac{\epsilon}{ma} + 2\epsilon$$

$e^{-\frac{i\phi}{2}\gamma_5 a(\not{D}_\epsilon + m_\epsilon)}$ does not contribute on shell.


q^2=1
t_{sep}=8
t_{sep}=10
t_{sep}=12
$F_3^{U,\gamma_5,d\text{-}u} / F_3^{U,d\text{-}u}$
$F_3^{D,\gamma_5,d\text{-}u} / F_3^{D,d\text{-}u}$
q^2=4
τ - t_{sep}/2


$F_3^{U},\gamma_5, d\text{-}u / F_3^{U, d\text{-}u}$
$F_3^{D},\gamma_5, d\text{-}u / F_3^{D, d\text{-}u}$
τ - $t_{sep}/2$
t_{sep}=10
t_{sep}=12
t_{sep}=14
q^2=1
q^2=4

Conclusions

Future

- Signal in the connected diagram for $a = 0.12$ and $a = 0.09$ fm and $M_\pi = 310$ MeV.
- Mixing with lower dimensional operator not a problem.
- Need to calculate renormalization and mixing coefficients non-perturbatively.
- Fermions with better chiral symmetry does not avoid mixing.