Search for a continuum limit of the PMS phase

Venkitesh P Ayyar
(joint work with Shailesh Chandrasekharan)

Lattice 2016, July, Southampton, UK

Work supported by DOE grant #DEFG0205ER41368
Computational work done using the Open Science Grid and local Duke cluster
Introduction

- Update on continuing study of a four-fermion model with an interesting phase structure.
- PMS phase at strong couplings with massive fermions without any Spontaneous Symmetry Breaking.
- Continuum limit of this PMS phase would be interesting.
- Previous work in 3D, pointed to such a continuum limit.
- Update on study in 4D on small lattices.
Previous studies\(^1\) of lattice Yukawa models show a very interesting
phase structure.

\[ g \to \text{Yukawa coupling}, \ m \to \text{boson mass} \]

- Massless PMW phase.
- Spontaneously broken FM phase with massive fermions.
- Exotic Paramagnetic PMS phase with massive fermions at strong couplings.
- No bilinear condensates in PMW and PMS phases.

Four-fermion model

- Equivalent to Yukawa model with fixed $m^2 > 0$
- Easier to study.
- Will also show 3 phase structure.

- Can there be models with a PMW-PMS phase transition?

Venkitesh Ayyar
Search for a continuum limit of the PMS phase
Our Lattice model

Reduced staggered fermion action for four massless flavors
\[ \psi_{x,1}, \psi_{x,2}, \psi_{x,3}, \psi_{x,4} \]

\[ S = S_0 + S_I \]

\[ S_0 = \sum_{i=1}^{4} \sum_{x,y} \{ \psi_{x,i} M_{x,y} \psi_{y,i} \} \]

\[ S_I = -U \sum_{x} \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4} \]

In addition to the usual discrete space-time symmetries\(^1\), the action has a continuous \( SU(4) \) symmetry.

The Fermion Bag approach

\[
Z = \int \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} e^{U \sum_x \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4}}
\]

\[
= \int \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} \prod_x e^{U \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4}}
\]

\[
= \int \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} \prod_x (1 + U \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4})
\]

\[
= \sum_{[m_x]} \int \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} \prod_x (U \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4})^{m_x}
\]

Integrate over monomer sites

\[
Z = \sum_{[m_x]} U^k \text{Det}(\tilde{A})^4
\]

Assigning \( m_x = 0 \) or \( 1 \) to each lattice site

\( m_x = 0 \equiv \text{free sites} \)

\( m_x = 1 \equiv \text{monomers} \)

**Fermion Bag** \( \equiv \) Set of connected free sites

where \( k \equiv \text{number of monomers}, \)

\( \tilde{A} \) is a sub-matrix of the staggered matrix \( M \).

---

1. S. Chandrasekharan - The Fermion bag approach to lattice field theories (2010)
Phase diagram: What do we know?

Irrelevant coupling
\[ \downarrow \]
Massless phase

Correlators decay
exponentially
\[ \downarrow \]
Massive phase
& no condensates

PMW

0

U

∞

PMS

Venkitesh Ayyar
Search for a continuum limit of the PMS phase
Observables

- **Average Monomer density:**
  \[
  \rho_m = \frac{U}{V} \sum_x \langle \psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4} \rangle
  \]

- **Bosonic correlators:**
  \[
  C_1(x, y) = \langle \psi_{x,1}\psi_{x,2}\psi_{y,1}\psi_{y,2} \rangle,
  C_2(x, y) = \langle \psi_{x,1}\psi_{x,2}\psi_{y,3}\psi_{y,4} \rangle
  \]

- **Susceptibilities**
  \[
  \chi_1 = \frac{1}{2} \sum_x \langle \psi_{0,1}\psi_{0,2}\psi_{x,1}\psi_{x,2} \rangle,
  \chi_2 = \frac{1}{2} \sum_x \langle \psi_{0,1}\psi_{0,2}\psi_{x,3}\psi_{x,4} \rangle
  \]

- **Focus on** \( SU(4) \) transformations.
- **Corresponding order parameter** \( \psi_{x,a}\psi_{x,b} \).
- **Condensate** \( \Phi = \langle \psi_{0,1}\psi_{0,2} \rangle \).
- **Limit** \( \lim_{L \to \infty} C_{1,2}(0, L) \sim \Phi^2 \)
- **Limit** \( \lim_{L \to \infty} \chi_{1,2} \sim \Phi^2 L^D \)
Single 2nd order phase transition in 3D

Near a 2nd order critical point,
\[ R_1 = L^{-(1+\eta)}\left[(U - U_c)L^{1/\nu}\right]. \]

- No intermediate FM phase.
- PMW-PMS transition is 2\textsuperscript{nd} order.
- Critical exponents w/o corrections to scaling:
  \[ \eta = 1.05(5), \ \nu = 1.30(7), \ U_c = 0.943(2) \]
- With corrections to scaling, cannot rule out large \( N \) exponents
  \[ \eta = 1.0, \ \nu = 1.0, \ U_c = 0.95. \]

---

1 Ayyar, Chandrasekharan PRD 91, 2015.
2 Ayyar, Chandrasekharan PRD(RC) 93, 2016.

---

Venkitesh Ayyar
Search for a continuum limit of the PMS phase
4D Results:

\[ \rho_m = \frac{U}{V} \sum_x \langle \psi_{x,1} \psi_{x,2} \psi_{x,3} \psi_{x,4} \rangle \]

- Lattices up to \( L = 12 \).
- Average monomer density rises sharply around \( U = 1.75 \) without any discontinuity.

---

\(^1\) Ayyar, Chandrasekharan arxiv:1606.06312, 2016.
Susceptibility $\chi_1$ vs $U$

- Bosonic Susceptibilities:
  \[ \chi_1 = \frac{1}{2} \sum_x \langle \psi_{0,1} \psi_{0,2} \psi_x,1 \psi_x,2 \rangle \]
  \[ \chi_2 = \frac{1}{2} \sum_x \langle \psi_{0,1} \psi_{0,2} \psi_x,3 \psi_x,4 \rangle \]

- Condensate given by:
  \[ \chi \sim \Phi^2 L^4 \]

- $\chi_1$ reaches a maximum for intermediate $U$.

- Sharp rise.
Evidence for a condensate:

- Condensate implies $\chi_1 \sim L^4$.

- $\chi_1 / L^4$ vs $L$ seems to saturate at large $L$.

Obtain condensate $\Phi$ upon fit to $\chi_1 = \frac{1}{4} \Phi^2 L^4 + b_1 L^2$.

Venkitesh Ayyar

Search for a continuum limit of the PMS phase
Evidence for a 3 phase structure.

At a 2\textsuperscript{nd} order critical point, we expect \( \chi/L^{2-\eta} \sim \text{const} \)
\( \Rightarrow \chi/L^{2-\eta} \text{ vs } U \text{ curves for different } L \text{’s must intersect.} \)

\begin{itemize}
  \item Plot \( \chi/L^{2-\eta} \text{ vs } U \) using mean field exponents \( \eta = 0, \nu = 0.5, \)
  \item Curves intersect at 2 points \( \Rightarrow \) two phase transitions.
  \item Critical couplings: \( U_{c1} = 1.60, U_{c2} = 1.80 \)
\end{itemize}
Phase diagram in 3D and 4D.

3D

- Single 2nd order phase transition.
- Weak and strong coupling phases do not show any SSB $^{1,2,3}$.

4D

- 3 phase structure.
- FM phase is found to be narrow.

---

2. S. Catterall, JHEP 01, 121 (2016)

Search for a continuum limit of the PMS phase
Conclusions

- Massive fermions without fermion bilinear condensates (PMS phase) in a simple lattice four-fermion model in 3D and 4D.
- PMW-PMS transition is 2nd order in 3D $\Rightarrow$ PMS phase can help define an interesting 3D continuum field theory.
- Conjecture: Mass could arise via formation of a 3-fermion bound state$^{1,2}$.
- Evidence for a narrow intermediate FM phase in 4D, with two $2^{nd}$ order phase transitions with mean field exponents.
- Suggests the presence of a critical point in enhanced coupling space.

---

$^1$E. Eichten and J. Preskill, (1986)
$^2$Golterman et al., (1993)
THANK YOU