Search for a continuum limit of the PMS phase

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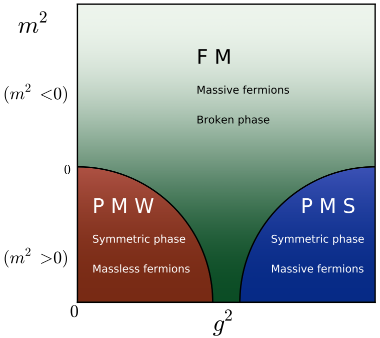
Introduction

- Update on continuing study of a four-fermion model with an interesting phase structure.
- PMS phase at strong couplings with massive fermions without any Spontaneous Symmetry Breaking.
- Continuum limit of this PMS phase would be interesting.
- Previous work in 3D, pointed to such a continuum limit.
- Update on study in 4D on small lattices.

Previous studies[1] of lattice Yukawa models show a very interesting phase structure.

$g \rightarrow$ Yukawa coupling, $m \rightarrow$ boson mass

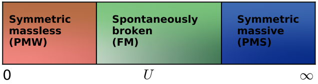
- Massless PMW phase.
- Spontaneously broken FM phase with massive fermions.
- Exotic Paramagnetic PMS phase with massive fermions at strong couplings.
- No bilinear condensates in PMW and PMS phases.



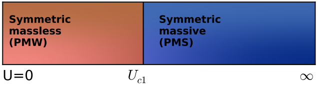
[1] Hasenfratz, Neuhaus (1989); Lee, Shigemitsu, Shrock (1990); W. Bock, A. K. De, K. Jansen, J. Jersak, T. Neuhaus, and J. Smit (1990).

Four-fermion model

- Equivalent to Yukawa model with fixed $m^2 > 0$
- Easier to study.
- Will also show 3 phase structure.



- Can there be models with a PMW-PMS phase transition?



Our Lattice model

Reduced staggered fermion action for four massless flavors $\psi_{x,1}, \psi_{x,2}, \psi_{x,3}, \psi_{x,4}$

$$S = S_0 + S_I$$

- $S_0 = \sum_{i=1}^{4} \sum_{x,y} \{\psi_{x,i} M_{x,y} \psi_{y,i}\}$
- $S_I = -U \sum_x \psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4}$

In addition to the usual discrete space-time symmetries[1], the action has a continuous $SU(4)$ symmetry.

[1] M. Golterman and J Smit. Selfenergy and Flavor Interpretation of Staggered Fermions. Nucl.Phys., B245:61, 1984.

The Fermion Bag approach [1]

$$Z = \int \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} e^{U \sum_x \psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4}}$$

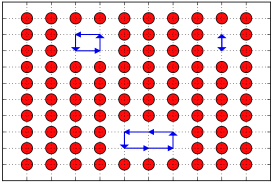
$$= \int \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} \prod_x e^{U\psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4}}$$

$$= \int \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} \prod_x (1 + U\psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4})$$

$$= \sum_{[m_x]} \int \left\{ \prod_{i=1}^{4} [d\psi_i] \right\} e^{-S_0} \prod_x (U\psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4})^{m_x}$$

Integrate over monomer sites

$$Z = \sum_{[m_x]} U^k Det(\tilde{A})^4$$



Assigning $m_x = 0$ or 1 to each lattice site

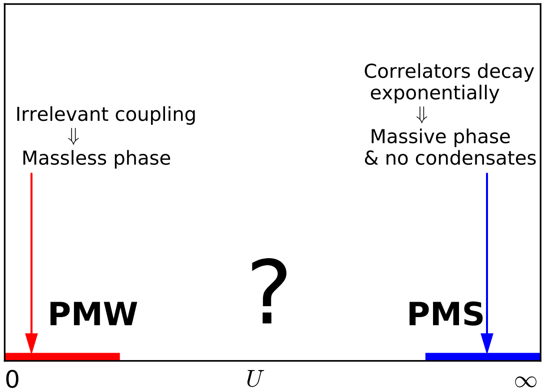
- $m_x = 0 \equiv$ **free sites**
- $m_x = 1 \equiv$ **monomers**

Fermion Bag $\equiv$ Set of connected free sites

where $k \equiv$ number of monomers, $\tilde{A}$ is a sub-matrix of the staggered matrix M.

[1] S. Chandrasekharan - The Fermion bag approach to lattice field theories (2010)

Phase diagram: What do we know?



Observables

- Average Monomer density:

$$\rho_m = \frac{U}{V} \sum_x \langle \psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4} \rangle$$

- Bosonic correlators:
  $C_1(x,y) = \langle \psi_{x,1}\psi_{x,2}\psi_{y,1}\psi_{y,2} \rangle$,
  $C_2(x,y) = \langle \psi_{x,1}\psi_{x,2}\psi_{y,3}\psi_{y,4} \rangle$

- Susceptibilities
  $\chi_1 = \frac{1}{2} \sum_x \langle \psi_{0,1}\psi_{0,2}\psi_{x,1}\psi_{x,2} \rangle$
  $\chi_2 = \frac{1}{2} \sum_x \langle \psi_{0,1}\psi_{0,2}\psi_{x,3}\psi_{x,4} \rangle$

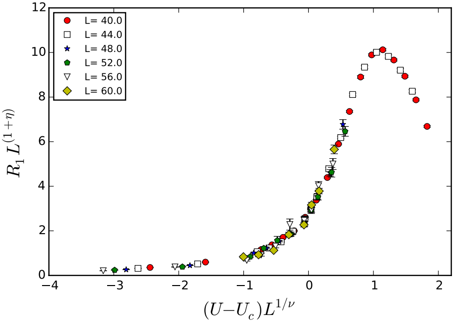
- Focus on $SU(4)$ transformations.
- Corresponding order parameter $\psi_{x,a}\psi_{x,b}$.
- Condensate $\Phi = \langle \psi_{0,1}\psi_{0,2} \rangle$.
- $\lim_{L\to\infty} C_{1,2}(0,L) \sim \Phi^2$
- $\lim_{L\to\infty} \chi_{1,2} \sim \Phi^2 L^D$

Single 2nd order phase transition in 3D

Near a 2nd order critical point,

$$R_1 = L^{-(1+\eta)} f\left[(U - U_c)L^{\frac{1}{\nu}}\right].$$

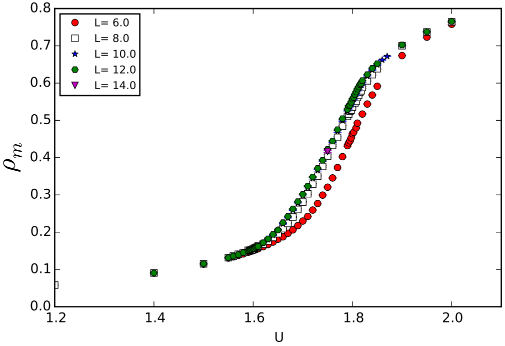
- No intermediate FM phase.
- PMW-PMS transition is 2^{nd} order.
- Critical exponents w/o corrections to scaling:
  $\eta = 1.05(5),\ \nu = 1.30(7),$
  $U_c = 0.943(2)$
- With corrections to scaling, cannot rule out large N exponents
  $\eta = 1.0, \nu = 1.0, U_c = 0.95.$



[1] Ayyar, Chandrasekharan PRD 91, 2015.

[2] Ayyar, Chandrasekharan PRD(RC) 93, 2016.

4D Results:

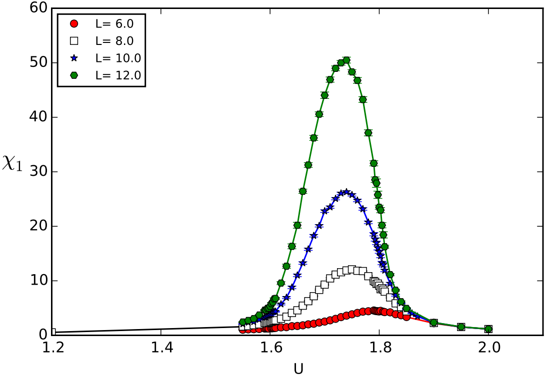


$$\rho_m = \frac{U}{V} \sum_x \langle \psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4} \rangle$$

- Lattices up to $L = 12$.
- Average monomer density rises sharply around U=1.75 without any discontinuity.

[1] Ayyar, Chandrasekharan arxiv:1606.06312, 2016.

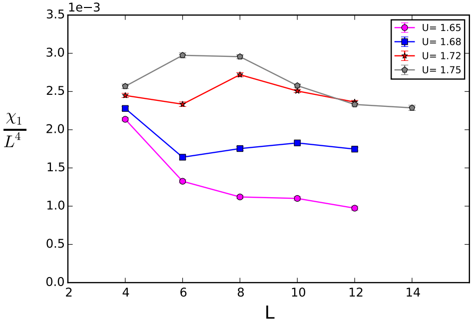
Susceptibility χ_1 vs U



- Bosonic Susceptibilites:
  $\chi_1 = \frac{1}{2}\sum_x \langle \psi_{0,1}\psi_{0,2}\psi_{x,1}\psi_{x,2}\rangle$
  $\chi_2 = \frac{1}{2}\sum_x \langle \psi_{0,1}\psi_{0,2}\psi_{x,3}\psi_{x,4}\rangle$
- Condensate given by:
  $\chi \sim \Phi^2 L^4$
- χ_1 reaches a maximum for intermediate U.
- Sharp rise.

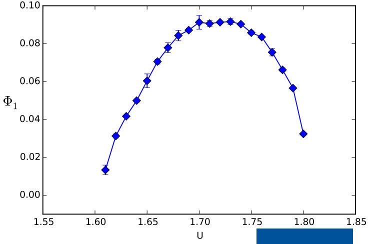
Evidence for a condensate:

- Condensate implies $\chi_1 \sim L^4$.



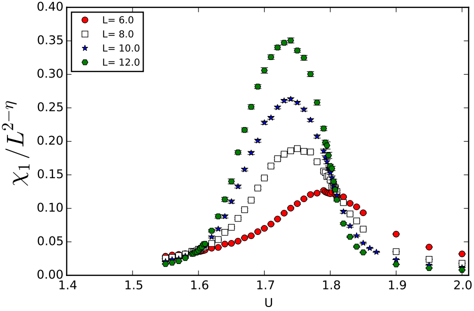
- χ_1/L^4 vs L seems to saturate at large L.

Obtain condensate Φ upon fit to $\chi_1 = \frac{1}{4}\Phi^2 L^4 + b_1 L^2$



Evidence for a 3 phase structure.

At a 2^{nd} order critical point, we expect $\chi/L^{2-\eta} \sim$ const
$\implies \chi/L^{2-\eta}$ *vs* U curves for different L's must intersect.



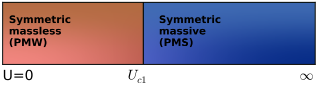
- Plot $\chi/L^{2-\eta}$ vs U using mean field exponents $\eta = 0, \nu = 0.5$,
- Curves intersect at 2 points $\Rightarrow$ two phase transitions.
- Critical couplings:

  $U_{c1} = 1.60, \;\; U_{c2} = 1.80$

Phase diagram in 3D and 4D.

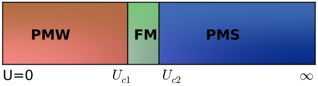
3D

- Single 2nd order phase transition.
- Weak and strong coupling phases do not show any SSB [1,2,3].



4D

- 3 phase structure.
- FM phase is found to be narrow.



[1] K. Slagle,Y.-Z. You, C. Xu, Phys. Rev. B 91, 115121 (2015)

[2] S. Catterall, JHEP 01, 121 (2016)

[3] Y.-Y. He, H.-Q. Wu, Y.-Z. You, C. Xu, Z. Y. Meng, and Z.-Y. Lu, (2016), arXiv:1603.08376

Conclusions

- Massive fermions without fermion bilinear condensates (PMS phase) in a simple lattice four-fermion model in 3D and 4D.
- PMW-PMS transition is 2nd order in 3D $\Rightarrow$ PMS phase can help define an interesting 3D continuum field theory.
- Conjecture: Mass could arise via formation of a 3 fermion bound state[1,2].
- Evidence for a narrow intermediate FM phase in 4D, with two 2^{nd} order phase transitions with mean field exponents.
- Suggests the presence of a critical point in enhanced coupling space.

[1] E. Eichten and J. Preskill, (1986)

[2] Golterman et al., (1993)

THANK YOU