Selected new results from the Spectroscopy in the sextet BSM Model

Chik Him (Ricky) Wong

Lattice Higgs Collaboration (LaHC):
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LATTICE 2016
Review: Sextet model as Composite Higgs candidate

Hadron spectroscopy in Isoscalar $J^{PC} = 0^{-+}$ ($\eta$) channel

- Fermionic correlator
- Gluonic operator
- Improvement using Gradient Flow
- Preliminary results

Conclusion
Sextet model as Composite Higgs candidate

- Criteria for candidate models in composite Higgs scenario:
  - Generates Higgs boson consistent with phenomenology
  - Infrared pseudo-Fixed point (IRFP) + Spontaneous $\chi$SB $\Rightarrow$ Models at the edge of Conformal Window

- $SU(3) N_f = 2$ Sextet (Two-index symmetric) Model

$$\psi^L_{ab} = \psi^L_{ba} \equiv \begin{pmatrix} u^L_{ab} \\ d^L_{ab} \end{pmatrix}, \quad \psi^R_{ab} = \psi^R_{ba} \equiv \begin{pmatrix} u^R_{ab} & d^R_{ab} \end{pmatrix}$$

$a, b = 0, 1, 2$

- Why is it interesting?
  - Very close to Conformal Window
  - “Minimal” model
    - Spontaneous Chiral Symmetry Breaking:
      $$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$
    - 3 Nambu-Goldstone Bosons (BSM-$\pi$) out of $N_f^2 - 1$ are eaten in Higgs Mechanism:
      $$SU(2)_W \times U(1)_Y \rightarrow U(1)_{em}$$
      while BSM-$f_0$ is identified as the Higgs boson $H$
    - $N_f^2 - 4$ massless BSM-$\pi$’s somehow gains masses from additional interactions, but far separated from the eaten massless 3, yet not a concern if $N_f = 2$
      $\Rightarrow$ Sextet is “Minimal” realization and hence appealing
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- Consistent with $\chi_{SB}$
  - Static Quark Potential: Confining (Fodor et al, PoS (Lattice 2012) 025)
  - Chiral condensate: Non-zero (Fodor et al, PoS (LATTICE 2013) 089)
  - $\beta$ function of $g_R$ : No IRFP is observed (Fodor et al, Phys.Rev. D94 (2016) no.1, 014503)
- Hadron Spectroscopy
  - Action: Tree-level Symanzik-Improved 2-stout $\rho = 0.15$ smeared gauge action with Staggered $N_f = 2$ Sextet SU(3) fermions
  - Previously: (Fodor et al, PoS (LATTICE 2014) 244)
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![Graph showing trends in particle masses and decay constants](image-url)


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![Graph showing spectral properties](image-url)
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![Graph showing data points and trend lines with decreasing $M_\pi$ for $\beta = 3.20$ and $\beta = 3.25$.](graph.png)
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**Baryon $N$: Its spectrum is crucial for predicting relic abundance in the early Universe and dictates how to embed into SM** (Fodor et al, Phys.Rev. D94 (2016) no.1, 014503)

**LHC-reachable resonance candidates $\rho$, $a_0$, $a_1$**

**Light Higgs $f_0$**
Sextet model as Composite Higgs candidate

- Taste breaking
  - Goldstone spectrum depends on $m$ with different slopes $\Rightarrow$ Taste breaking pattern is different from QCD

\[ \beta = 3.20 \]

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\[ \chiPT \] analysis is complicated by Taste breaking
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Mixed action analysis using Gradient Flow is under development
Sextet model as Composite Higgs candidate

- **Taste breaking**

  ![Graph showing taste splitting and restored symmetry](image1)

- **Mixed action analysis using Gradient Flow is under development**

  ![Graph showing mixed action analysis](image2)
Hadron spectroscopy in $\eta$ channel

- Where is $M_\eta$ (Isoscalar $J^{PC} = 0^{-+}$)?
  - $\pi = \bar{\psi} \gamma_5 C \psi, \psi = (u, d)^T$:
    
    NG boson
    \[
    \partial_\mu j_5^\mu \sim mj_5 \Rightarrow \lim_{m \to 0} M_\pi^2 = 0
    \]
  - $\eta = \bar{\psi} \gamma_5 \psi, \psi = (u, d)^T$:
    
    would-be NG boson, but mass generated by $U(1)_A$ anomaly
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    \partial_\mu j_5^\mu \sim 2N_f(N_c \pm 2)q,
    \]

    $+:$ Symmetric, $-:$ Anti-symmetric

- Witten-Veneziano formula (E. Witten, G. Veneziano Nucl. Phys. B 159, 213,269 (1979)) predicts much higher mass than QCD in the chiral limit
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  M_\eta^2 \sim \frac{6(N_c \pm 2)}{f_\pi^2} \chi_t|_{N_f=0}, \quad \chi_t|_{N_f=0} = \int dx \langle q(0)q(x) \rangle|_{N_f=0}
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- Fermionic correlator

$$\langle \eta(\tau)\eta(\tau_0) \rangle \equiv -C(\tau - \tau_0) + 2D(\tau - \tau_0)$$

- Involves disconnected contribution $D(\tau - \tau_0)$ which are costly
- In the limit $\tau - \tau_0 \to \infty$,

$$-C(\tau - \tau_0) \propto A_\pi e^{-M_\pi(\tau - \tau_0)}$$

$$2D(\tau - \tau_0) \propto -A_\pi e^{-M_\pi(\tau - \tau_0)} + B_\eta e^{-M_\eta(\tau - \tau_0)} \quad (M_\pi < M_\eta)$$

- Cancellation of large pion contribution $\Rightarrow$ Noisy
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- Fermionic correlator

\[ \langle \eta(\tau) \eta(\tau_0) \rangle \equiv -C(\tau - \tau_0) + 2D(\tau - \tau_0) \]

- Involves disconnected contribution $D(\tau - \tau_0)$ which are costly
- In the limit $\tau - \tau_0 \rightarrow \infty$,

\[ -C(\tau - \tau_0) \propto A_\pi e^{-M_\pi(\tau - \tau_0)} \]
\[ 2D(\tau - \tau_0) \propto -A_\pi e^{-M_\pi(\tau - \tau_0)} + B_\eta e^{-M_\eta(\tau - \tau_0)} \quad (M_\pi < M_\eta) \]

- Cancellation of large pion contribution $\Rightarrow$ Noisy
Hadron spectroscopy in $\eta$ channel

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**Hadron spectroscopy in η channel**

- **Gluonic operator** (H. Fukaya, Phys. Rev. D 92, 111501 2015)
  - $M_\eta$ can be extracted from the topological charge density:

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  q(x) = \frac{1}{32\pi^2} \varepsilon_{\mu\nu\rho\sigma} \text{Tr} F_{cl}^{\mu\nu} F_{cl}^{\rho\sigma}(x)
  \]

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  \lim_{r \to \text{large}} -\langle q(x)q(y) \rangle \propto \frac{K_1(M_\eta r)}{r}, \quad r \equiv |x-y|
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  - $F_{cl}^{\mu\nu}$: Field Strength Tensor (clover term)
  - $K_1$: Modified Bessel function of the second kind
  - Does not couple directly to pions ⇒ Quieter
  - No inversions of Dirac operator ⇒ Cheaper
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- Improvement from Gradient Flow
- Cut-off effects can be reduced by the lattice version of Gradient Flow with Wilson operator

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{YM}}{\partial A_\mu}$$

- Smooths links similar to diffusion equation with diffusion length $\sqrt{8t}$ in lattice units
- Correlator is distorted by footprint $\Rightarrow$ Fitting range of $r$ should be far enough $r >> 2\sqrt{8t}$
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![Graphs showing correlation functions for different times $t_f$.](image)
Chik Him (Ricky) Wong

Outline
- Review
- η Spectroscopy
- Correlator construction
- Improvement using Gradient Flow
- Preliminary Results
- Conclusion

Selected new results from the Spectroscopy in the sextet BSM Model

Preliminary Results

\[ \langle q(x)q(y) \rangle = A \cdot K_1(M_\eta \cdot r) / r \]

For different times:
- \( t_f = 4.5 \)
- \( t_f = 5.0 \)
- \( t_f = 5.5 \)

Parameters:
- \( A = 6.4(2) \times 10^{-9} \)
- \( M_\eta = 0.418(2) \)
- \( \chi^2/dof = 0.2 \)

For different times:
- \( t_f = 5.0 \)
- \( t_f = 5.5 \)

Parameters:
- \( A = 6.3(2) \times 10^{-9} \)
- \( M_\eta = 0.416(2) \)
- \( \chi^2/dof = 0.5 \)
Preliminary Results

- $M_\eta$ can be as heavy as $> 3$ TeV
- Measurements from more ensembles at different volumes, $\beta$’s and $m$’s are available and accumulating

Further analysis:
- The effects of finite volume and fixed topology
- Cross-check by time-slice-to-time-slice correlator

$$\lim_{|x_4-y_4| \to \text{large}} - \sum_{\bar{x}, \bar{y}} \langle q(x) q(y) \rangle \propto e^{-M_\eta |x_4-y_4|}$$

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![Graphs showing correlation functions for different parameter values](image)

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The sextet model remains an interesting candidate model of Composite Higgs scenario (More details can be found on Julius Kuti’s talk on Mon)

- More comprehensive and better analysis tools, e.g. mixed action analysis, are being developed
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$64^3 \times 96, \beta=3.2500, m=0.001, r_{\text{max}}=20$