MASS ANOMALOUS DIMENSION OF SU(2) USING THE SPECTRAL DENSITY METHOD

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BACKGROUND

Infrared-conformal gauge theories have been considered as models for physics beyond the Standard Model. In these models the anomalous dimension of the fermion operator $\overline{\psi}\psi$, γ_m, plays an important role. The scaling exponent of the spectral density of the massless Dirac operator is a function of the mass anomalous dimension γ_m, and thus it can be extracted by studying the behaviour of the eigenvalue distribution of the Dirac operator.

SETUP

The theory which we are studying is SU(2) with $N_f = 8$ and $N_f = 6$ fermions in the fundamental representation. We use HEX smeared, clover improved Wilson fermions with Schrödinger functional boundary conditions, and we have tuned the PCAC quark mass to zero. We calculate the integrated spectral density, the mode number, per unit volume stochastically [1]. We use from 16 to 20 configurations for the calculation for each value of the gauge coupling.

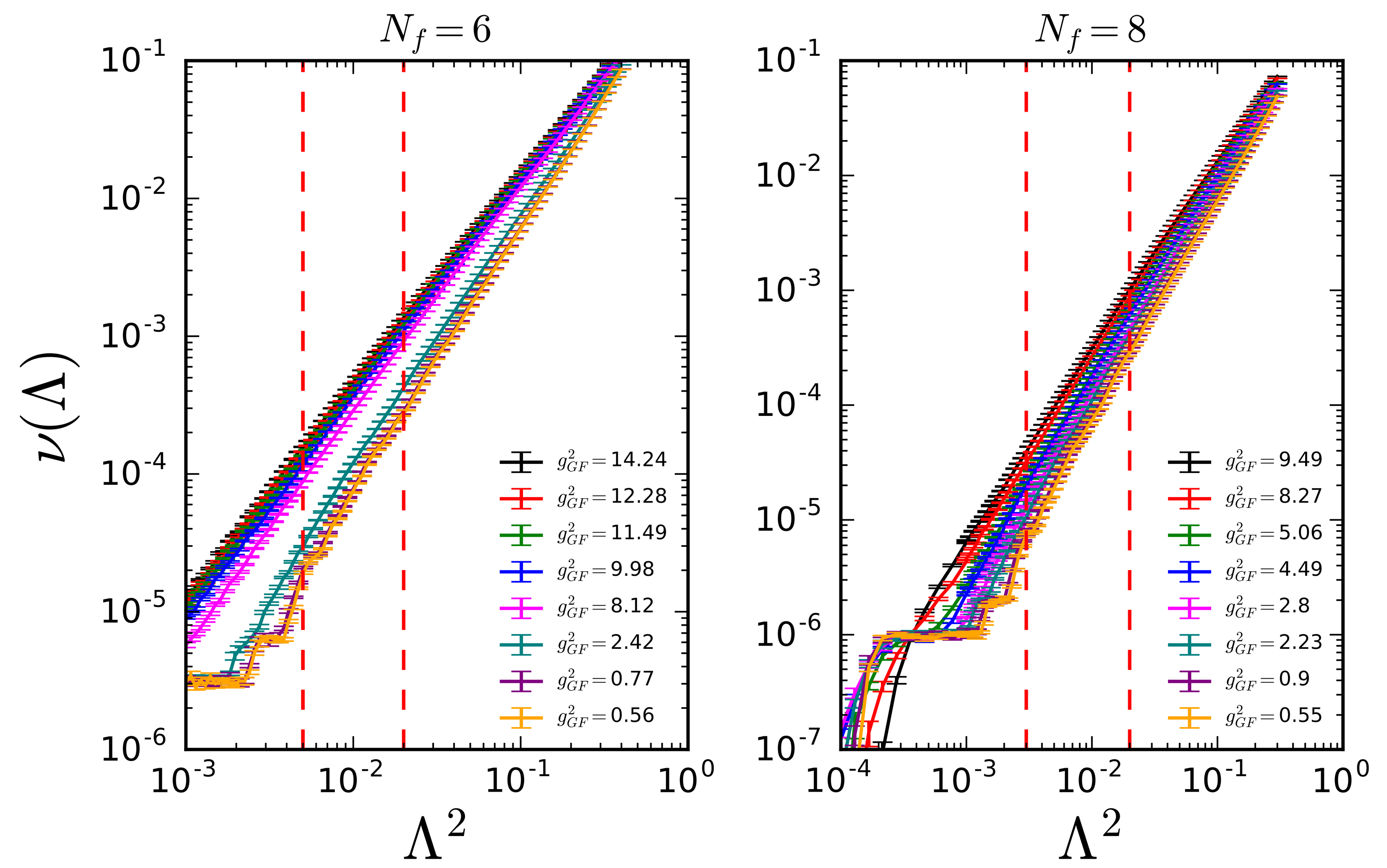

Figure 1: Mode number data on the right for $N_f = 6$ with $V = 24^4$ and on the left for $N_f = 8$ with $V = 32^4$. The dashed line shows the fit range.

POWER LAW FIT

The mode number in the vicinity of the IR-fixed point has an approximate form of a power law [2, 3]

$$\nu(\Lambda) \simeq \nu_0 + A\left[\Lambda^2 - m^2\right]^{\frac{2}{1+\gamma_*}}, \quad (1)$$

where ν_0 and A are an additive and a multiplicative constant respectively, m is the quark mass and γ_* is the mass anomalous dimension γ_m near the fixed point. Since we tune the PCAC mass to zero, we expect the two constants ν_0 and m^2 to be close to zero, and in our investigations we found this to be the case. Setting the two constants to zero had a negligible effect on the numerical value of γ_*, and thus our fitting function is

$$\nu(\Lambda) \simeq A\Lambda^{\frac{4}{1+\gamma_*}}. \quad (2)$$

The range of eigenvalues where Eq. 2 holds is not known *a priori*, and needs to be determined by trial and error. We established this range by matching the results obtained using this method to the results obtained by the Schrödinger functional step scaling method and the perturbative prediction for small coupling values.

To obtain γ_* we fit Eq. 2 to the data in Fig. 1, and our results are summarised in Fig 2. When the data is presented as in the upper row, the range where Eq. 2 holds will appear as a straight line. The red dashed line shows our fit range. We argue that the coupling runs so slowly near the fixed point that it changing will not have a noticeable effect inside the window of our fit range.

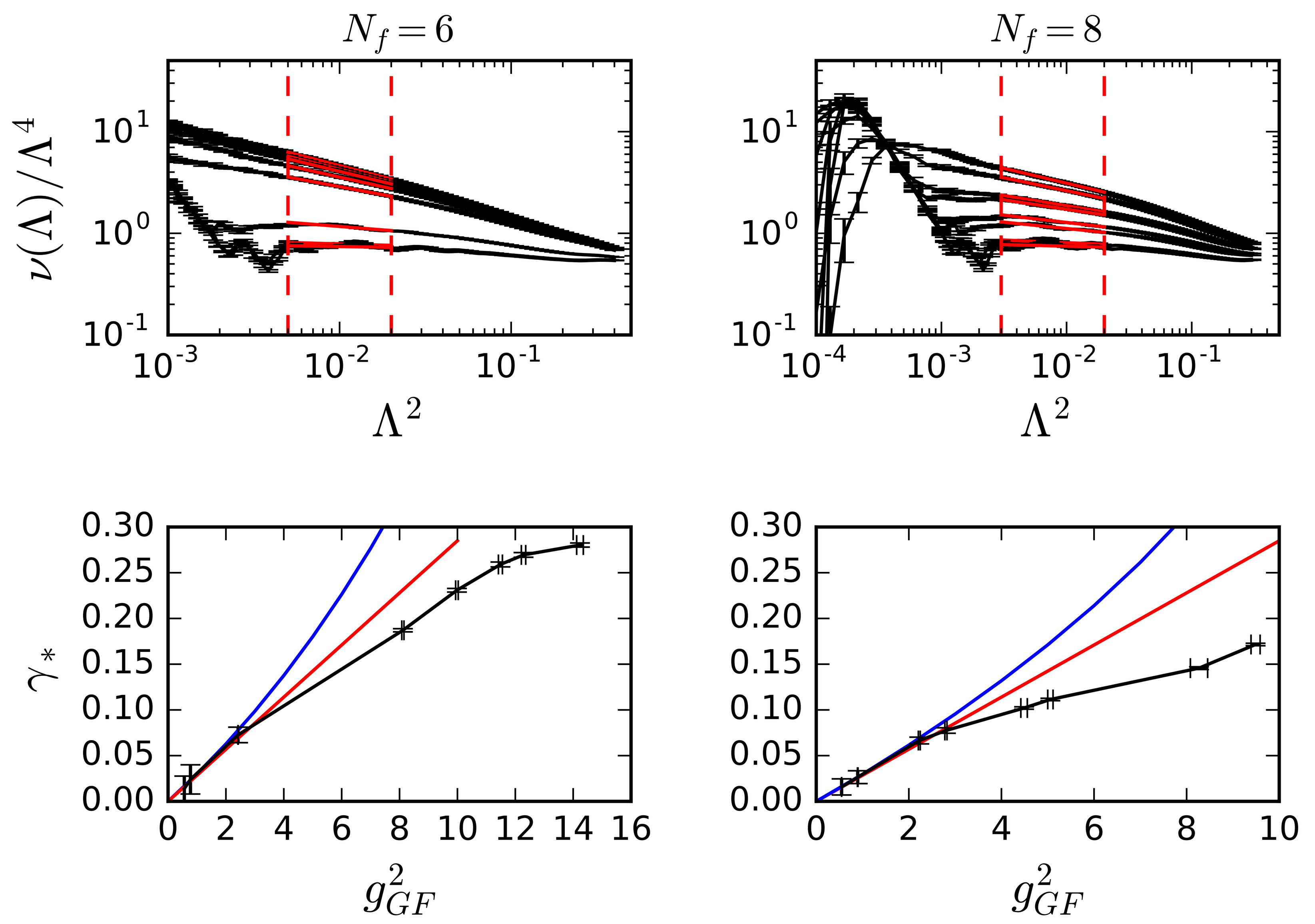

Figure 2: Our main results for γ_* as a function of the gradient flow coupling constant using the spectral density method. The curves are of descending gauge coupling order. The red and blue solid lines correspond to the first order and fourth order perturbative results.

COMPARISON WITH OTHER METHODS

We have identified the presence of a fixed point for $N_f = 6$ at $g^2_{GF} \sim 14$ and for $N_f = 8$ at $g^2_{GF} \sim 8$. For more details about this see the talk by Viljami Leino. We have also calculated the mass anomalous dimension using the Schrödinger functional step scaling method [4], and by fitting the correct power law behaviour $M \propto m_Q^{1/(1+\gamma)}$ to our spectrum data. The mass anomalous dimension calculated with each of these methods are summarised in Table 1. For more details about the spectrum measurements see the poster by Sara Tähtinen.

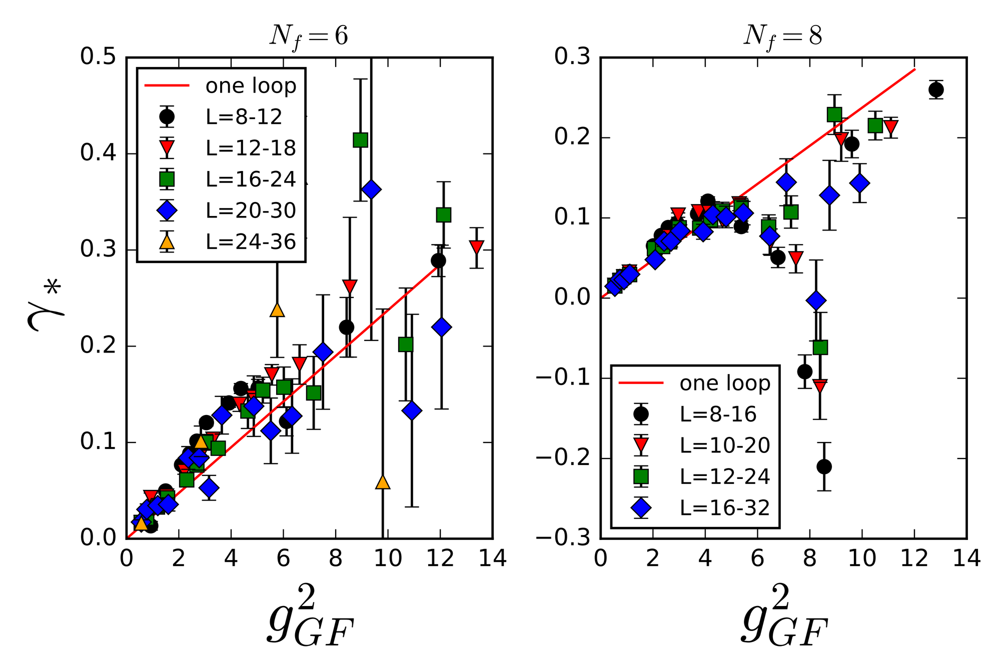

Figure 3: The mass anomalous dimension as a function of the gradient flow coupling constant obtained using the Schrödinger functional step scaling method.

N_f	β	γ_M	γ_{SD}	γ_{SF}
6	0.5	0.382(12)	0.280(2)	0.142(27)
	0.6	0.314(7)	0.231(2)	0.414(63)
	0.8	0.248(3)	~ 0.16	0.157(21)
8	0.6	0.293(30)	~ 0.13	0.072(24)
	0.8	0.238(31)	0.111(1)	0.109(14)

Table 1: Comparison of obtained γ_* values between different methods. The values for the spectral density method quoted without errors are based on interpolation from the data. Unreliable data points are represented with colouring.

CONCLUSIONS

Since the Schrödinger functional step scaling method shows nontrivial behaviour near the fixed point, complementary methods for determining the mass anomalous dimension are warranted. While our results for the mass anomalous dimension using the step scaling method and the spectral density method agree for some coupling values, there are unresolved discrepancies in our results between different methods. This comparison is further complicated by the unexpected behaviour of the step scaling method at the fixed point.

REFERENCES

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