Universität Regensburg

Real-time simulations of anomaly induced transport in external magnetic field

Pavel Buividovich and Senia Valgushev

(University of Regensburg)

Lattice 2016, 27 July 2016

Chiral plasmas

Chiral plasma: medium consist of massless fermions

- ***Quark-gluon plasma***
- ***Hadronic matter***
- ***Leptons, neutrinos at early stages of Universe***
- ***Weyl semimetals***
- ***Liquid He3***

Chiral quantum anomaly: classical action is invariant under chiral rotations, but the measure of the path integral is not:

$$\mathcal{L} = \bar{\psi} \not{D} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu e^{-i \int dx_\mu \mathcal{L}[\bar{\psi}, \psi, A_\mu]}$$

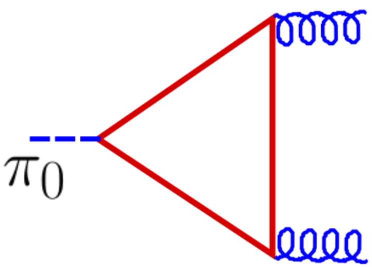
$$\xrightarrow[e^{i\theta\gamma_5}]{} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu e^{-i \int dx_\mu \mathcal{L}[\bar{\psi}, \psi, A_\mu] - iS_\theta}$$

Non-conservation of axial current:

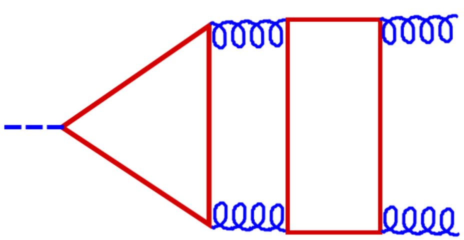
$$\partial_\mu j_A^\mu = \frac{1}{8\pi^2} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$$

$$\frac{dQ_A}{dt} = \frac{e^2}{2\pi^2} \int d^3x \vec{E} \cdot \vec{B}$$

$$Q_A = N_R - N_L \qquad J_A = J_R - J_L$$



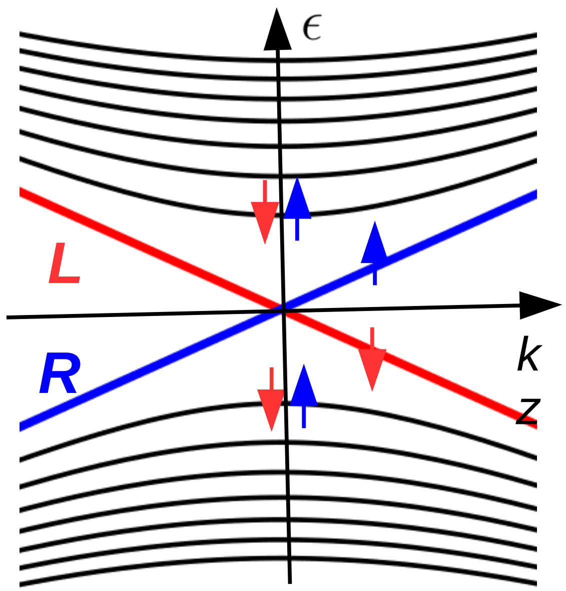
π_0



In a medium there might appear non-trivial correction

Chiral anomaly as Schwinger effect in 1D

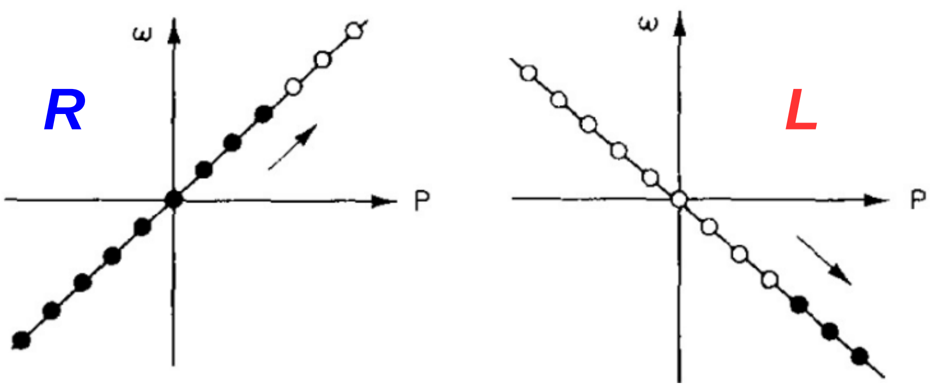
In the magnetic field motion of fermions is effectively 1D:



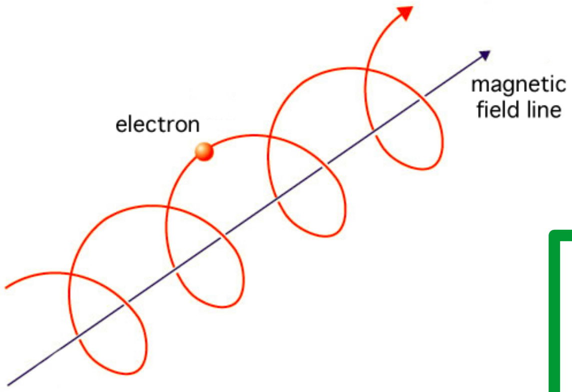
Landau levels in the magnetic field

$$\epsilon_{n,\sigma} = \pm\sqrt{k_z^2 + 2B(n - 1 + \sigma)}$$

In external electric field E || B there is a pair production on the topological lowest Landau level (n = 0):



$$\omega = \pm p \quad \dot{p} = eE/2\pi \quad \longrightarrow \quad dn_{R,L}/dt = \pm eE/2\pi$$



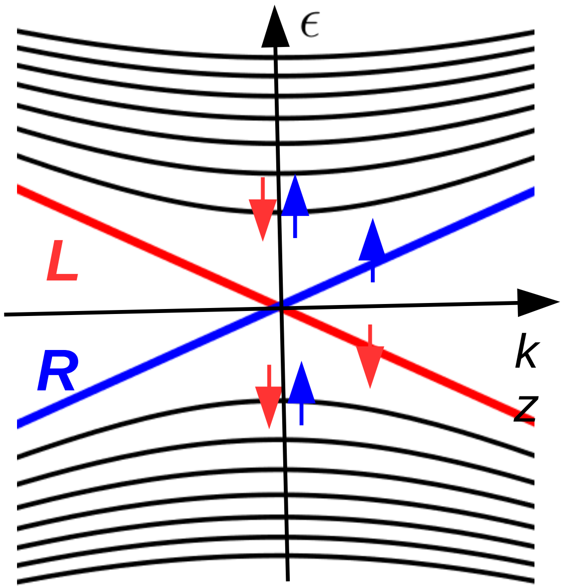
Degeneracy per unit area of each Landau level is $B/2\pi$

$$\boxed{\frac{dQ_A}{dt} = \frac{d(n_R - n_L)}{dt}\frac{B}{2\pi} = \frac{eEB}{2\pi^2}}$$

$$\boxed{J_z(t) = \frac{eEBt}{2\pi^2}}$$

Chiral anomaly as Schwinger effect in 1D

Contribution of lowest Landau level to current is dominant



Landau levels in the magnetic field

$$\epsilon_{n,\sigma} = \pm\sqrt{k_z^2 + 2B(n-1+\sigma)}$$

Higher Landau levels are effectively massive, pair production is exponentially suppressed:

$$\boxed{\Gamma_n \sim \exp\left(-\frac{2Bn}{E}\right)}$$

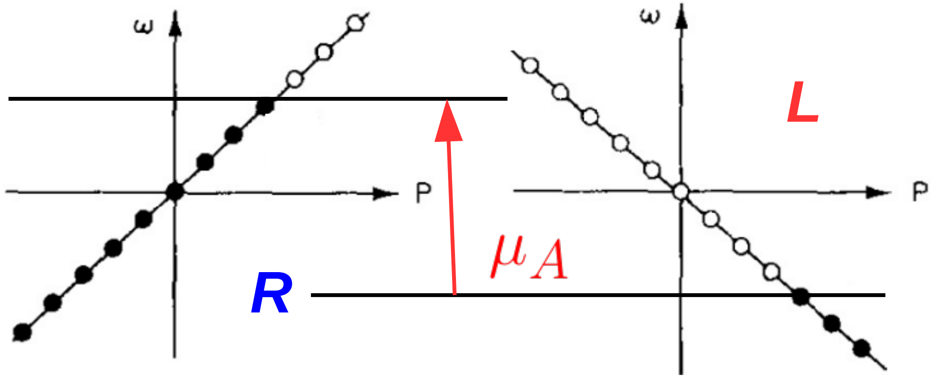
Non-perturbative correction to the current:

$$\boxed{J_z(t) = \frac{EB}{2\pi^2}\coth\left(\frac{\pi B}{E}\right)}$$

Important when $E \sim B$

Abramchuk, Zubkov, arXiv:1605.0237911

Negative magnetoresistivity as manifestation of chiral anomaly



Suppose that there is a chirality-flipping process in the system with typical scattering time τ :

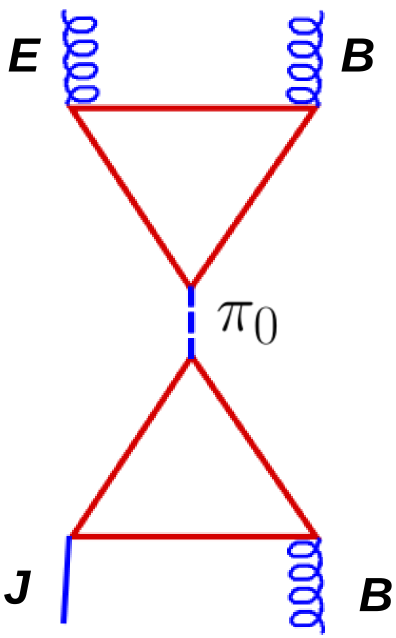
$$\dot{N}_{pairs} = \dot{N}_{scat}$$

Then steady state is described by chiral chemical potential μ_A.

$$\frac{dQ_A}{dt} = \frac{EB}{2\pi^2} - \frac{Q_A}{\tau} \longrightarrow \mu_A = \frac{3}{4\pi^2}\frac{EB}{T^2 + \mu^2/\pi^2}\tau$$

$$J_z = \frac{\mu_A}{2\pi^2}B \longrightarrow \boxed{J_z = \frac{3}{8\pi^4}\frac{EB^2}{T^2 + \mu^2/\pi^2}\tau}$$

Chiral Magnetic Effect (CME)

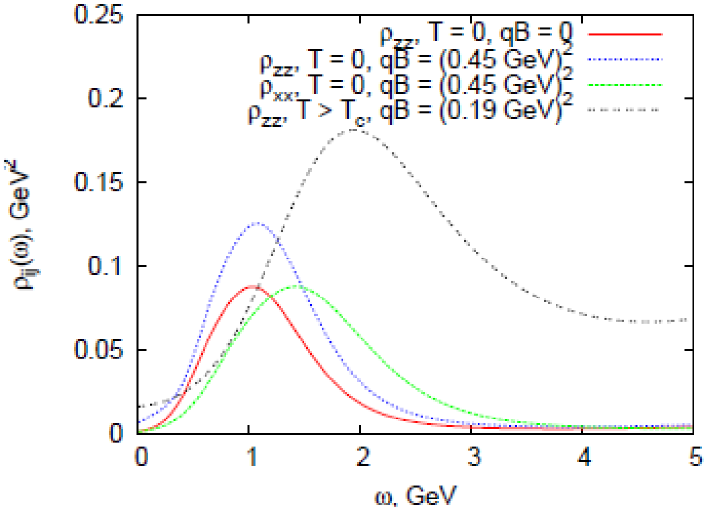


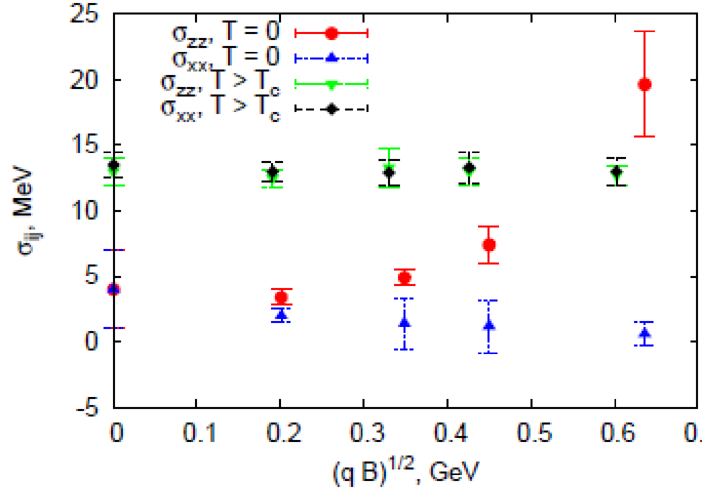
Nielsen, Ninomiya,
Phys. Lett. B 130 (1983) 389

K. Fukushima et al,
Phys. Rev. D,78, 074033(2008)

Negative Magnetoresistivity in Chiral plasmas

Magneto-conductivity in SU(2) QCD:

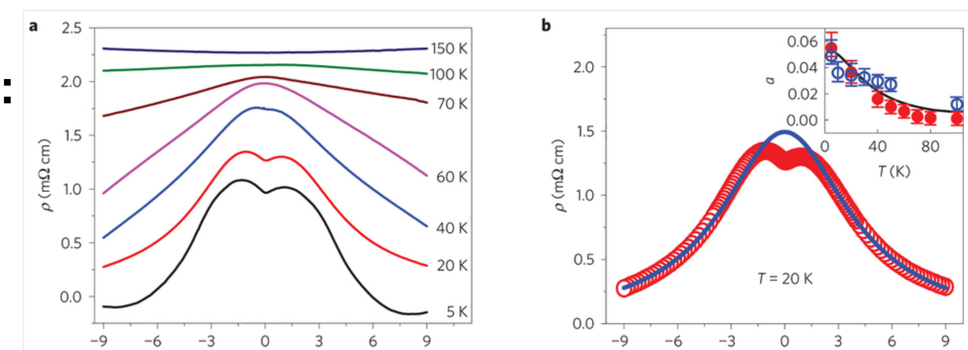




$\sigma \sim B^{(2.3 \pm 0.3)}$, $\tau \sim 0.15$ fm/c

Buividovich et al,Phys.Rev.Lett.105:132001,2010

Large activity in condensed matter community:

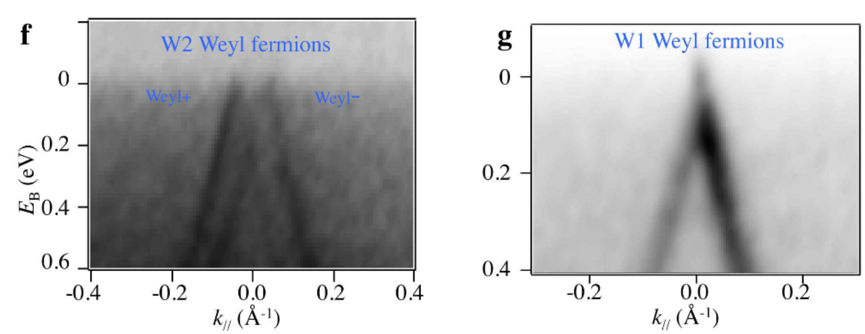


Experimental observation in Dirac SM ZrTe5:

Qiang Li et al,Nature Physics 12, 550–554 (2016)

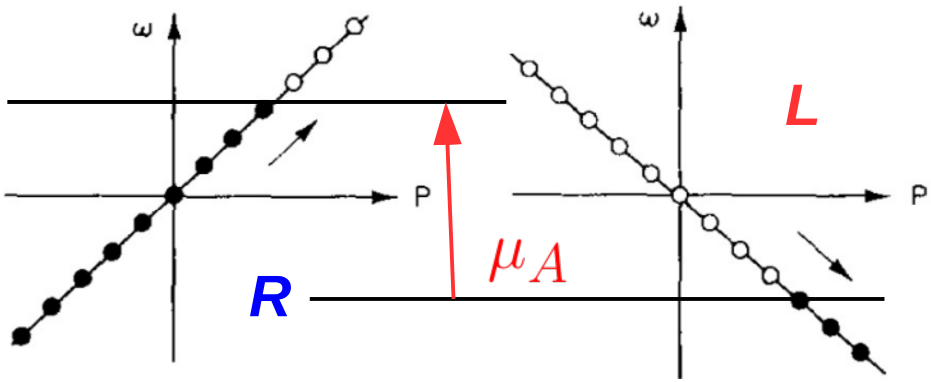
Observation in WS TaAs:

C.-L. Zhang, et al, Nature Communications 7, 10735 (2016)



TaP: F. Arnold et al, Nature Communications 7, 11615 (2016)

Chiral plasma oscillations



Naive consideration

Let $E = E(t)$

CME: $J_z(t) = \frac{\mu_A(t)}{2\pi^2} B$

On lowest Landau level (LLL): $Q_A(t) = \mu_A(t) \frac{B}{2\pi^2}$

$$\boxed{J_z(t) = Q_A(t)}$$

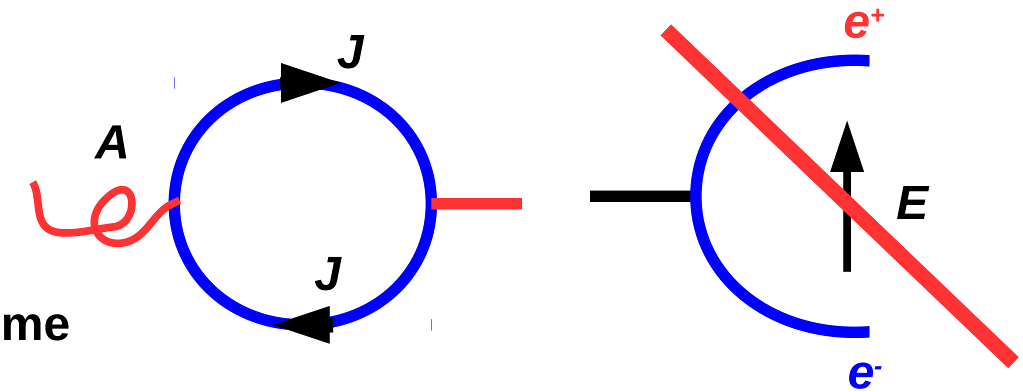
Maxwell equation:

$$\boxed{\partial_t^2 E(t) = -\partial_t J_z(t) = -\frac{B}{2\pi^2} E(t)}$$

Chiral plasma oscillations: $\boxed{\omega_A = \sqrt{\frac{B}{2\pi^2}}}$

Classical-statistical real-time simulations

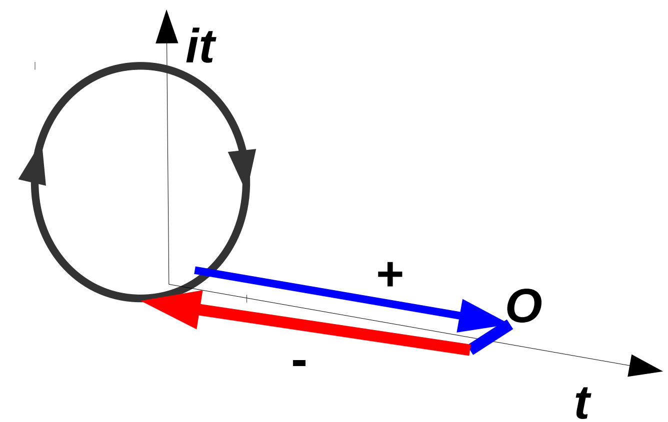
In Euclidean lattice gauge theory:



Painful analytical continuation to real-time

Out-of equlibrium real-time classical-statistical approximation:

$$\partial_t \vec{A}(t) = -\vec{E}(t)$$
$$\nabla \times \vec{A}(t) = \vec{B}(t)$$
$$\partial_t \vec{E}(t) = \nabla \times \vec{B}(t) - < \vec{J}[\vec{A}(t)] > - \vec{J}_{ext}$$
$$\partial_t \vec{B}(t) = -\nabla \times \vec{E}(t)$$
$$< \vec{J}[\vec{A}(t)] >= \sum_n n_f(\varepsilon_n) \psi_n^\dagger \hat{\vec{J}}[\vec{A}(t)] \psi_n$$
$$\partial_t \psi_n(t) = -i \hat{H}(\vec{A}(t), t) \psi_n(t)$$



Semiclassical approximation along Keldysh contour

Occupation numbers of bosonic fields have to be sufficiently high

Susskind, '93
G. Aarts, '99

J. Berges, F. Hebenstreit, N. Mueller

Simulations in the external magnetic field on the lattice

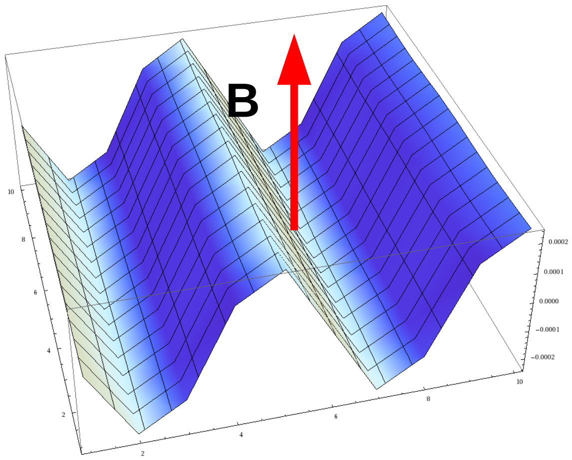
Magnetic field on a torus breaks translational invariance in transverse plane

It is reduced to discrete subgroup!

$$A_y = Bx$$

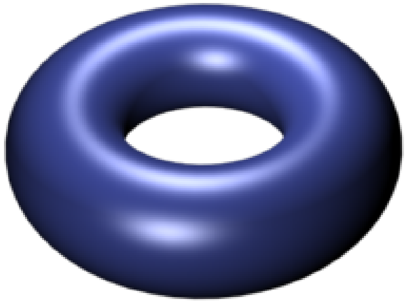
Fermionic Aharonov-Bohm phases:

$$B = \frac{2\pi n}{L^2}$$



$$\exp(ie\Phi_x(y)) = \exp(ieBLy - i\theta_x)$$

$$\exp(ie\Phi_y(x)) = \exp(ieBLx - i\theta_y)$$



Althougn total current <Jx> = 0, its spatial profile is not homogeneous

are invariant only under discrete shifts in transverse plane!

Restored in the Y direction if $B = \frac{2\pi n|L|}{L^2}$

Al-Hashimi, Wiese
arXiv:0807.0630

Computational setup

The bottleneck is evolution of fermionic modes:

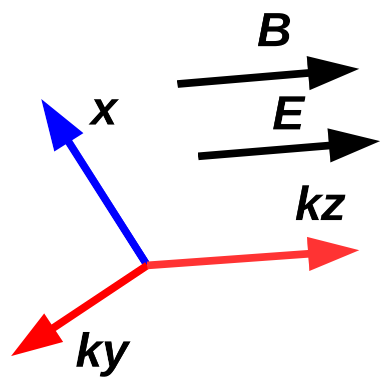
$$\boxed{\partial_t \psi_n(t) = -iH(t)\psi_n(t)}$$

In general, # of equations is (Ly Lx Lz 4) *x* (Ly Lx Lz 4)

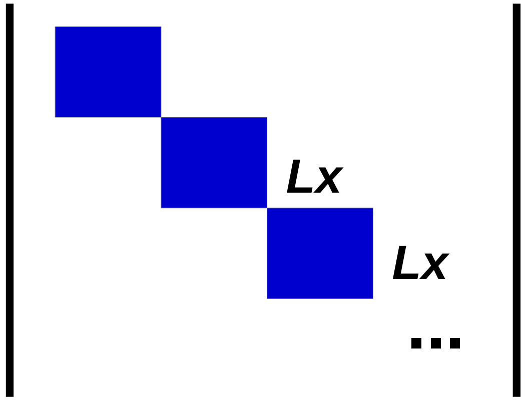
 # modes # components of each mode

We use non-compact abelian gauge field + Wilson-Dirac fermions

$$\boxed{\vec{A}(t) = (0, Bx, A_z(t)) \quad B = \frac{2\pi n |L|}{L^2}}$$



***H* =**

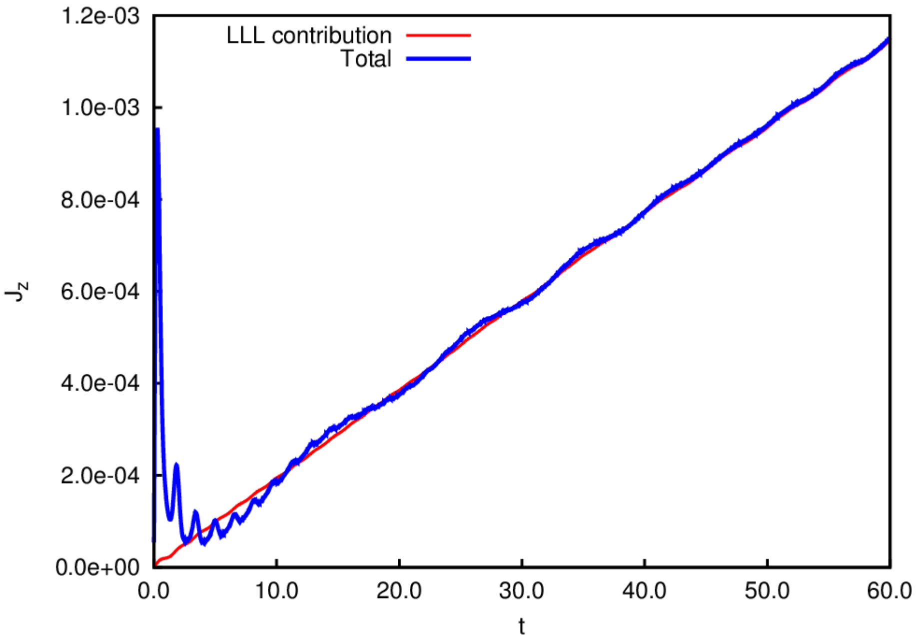


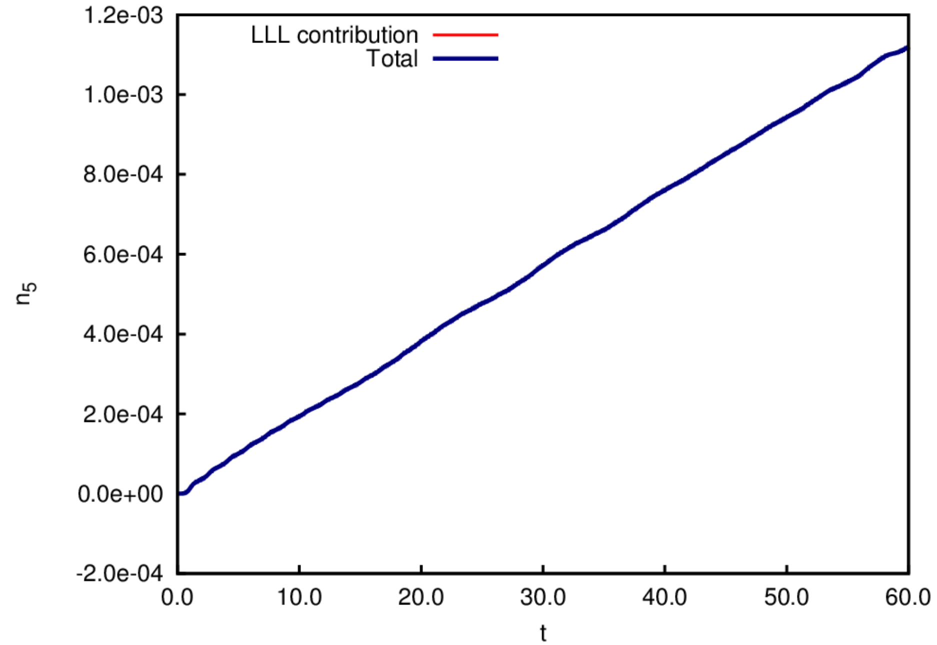
# of equations is (Ly Lx Lz 4) *x* (Lx 4) !

Lattice sizes up to 100^3

Vector current and anomaly in constant electric field

We apply constant electric field E || B to non-interacting system

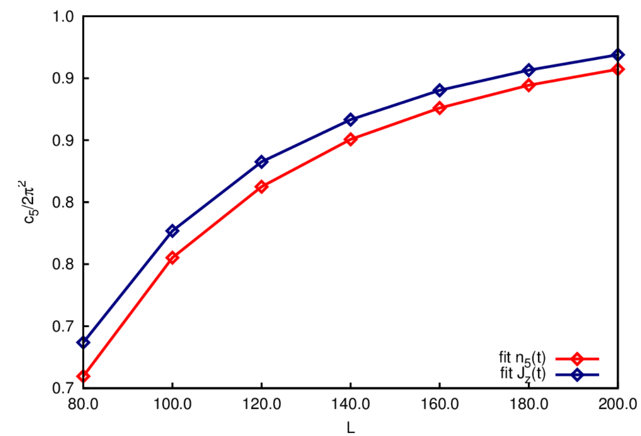




The current and axial density are saturated by contrubution of lowest Landau level.

In limit of infinite lattice size we reproduce chiral anomaly and Nielsen-Ninomia current:

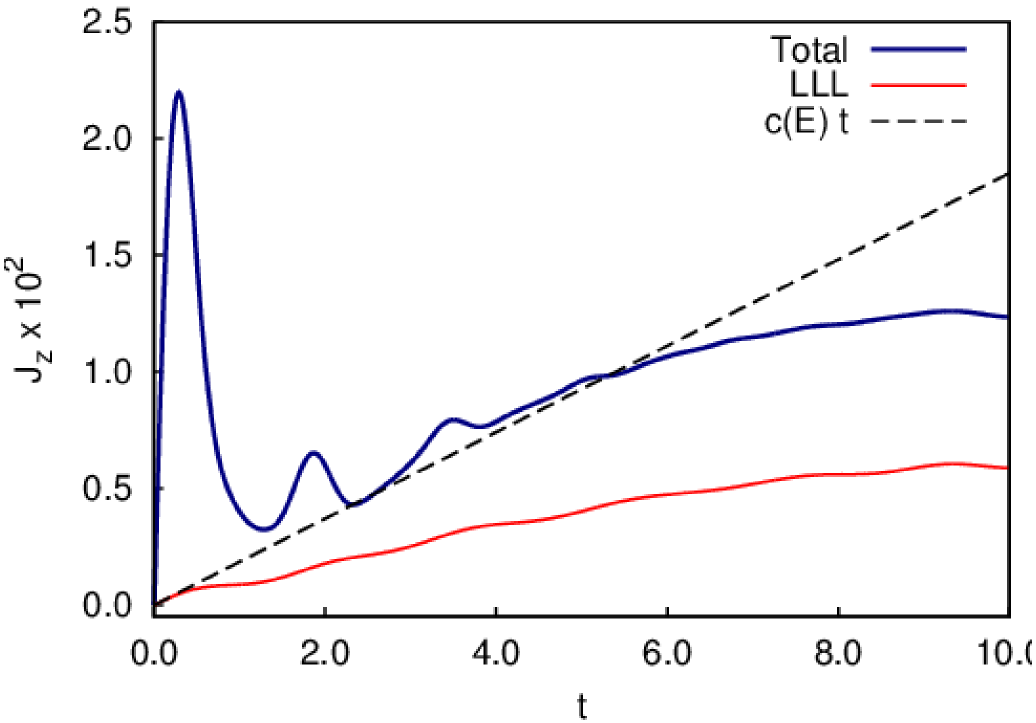
$$Q_A(t) = \frac{EBt}{2\pi^2} \qquad J_z(t) = \frac{EBt}{2\pi^2}$$

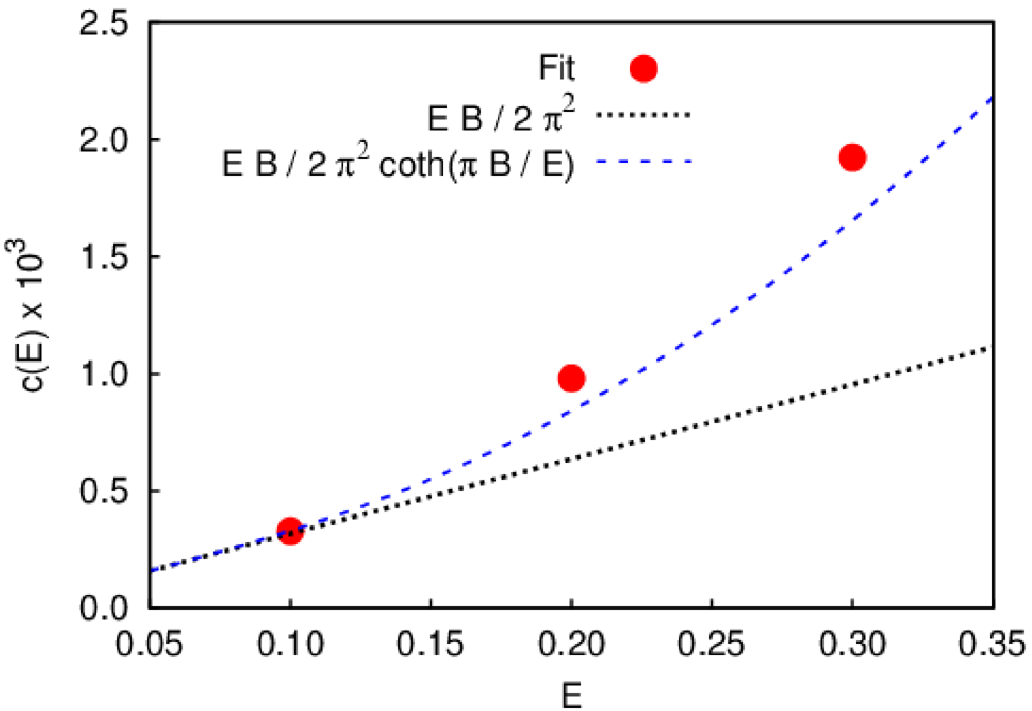


Scaling of anomaly coefficient with the volume

Vector current and anomaly in constant electric field

Evolution at very strong electric fields E ~ B

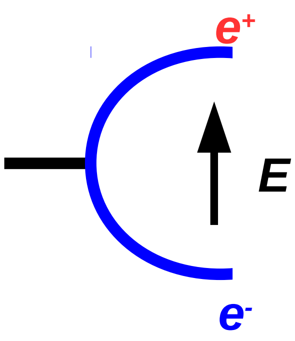


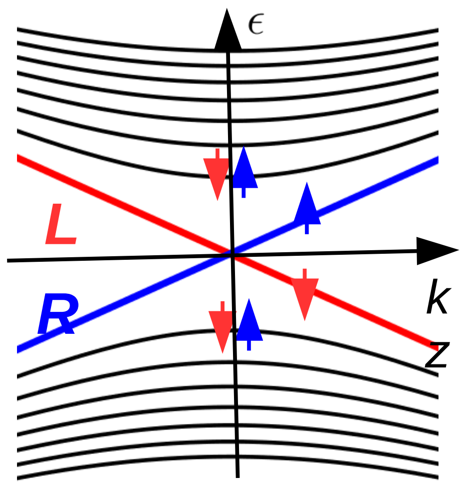


We see NP contributions from Schwinger pair production at higher, massive Landau levels:

$$J_z(t) = \frac{BE}{2\pi^2} \boxed{\coth\left(\frac{\pi B}{E}\right)} t$$

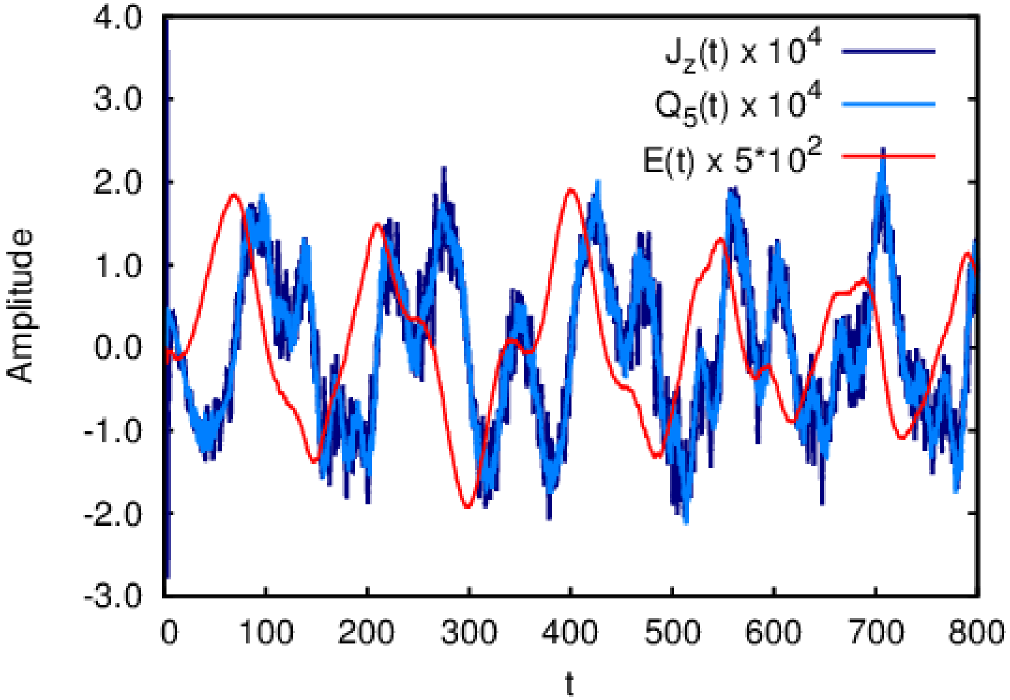
Zubkov, arXiv:1605.02379

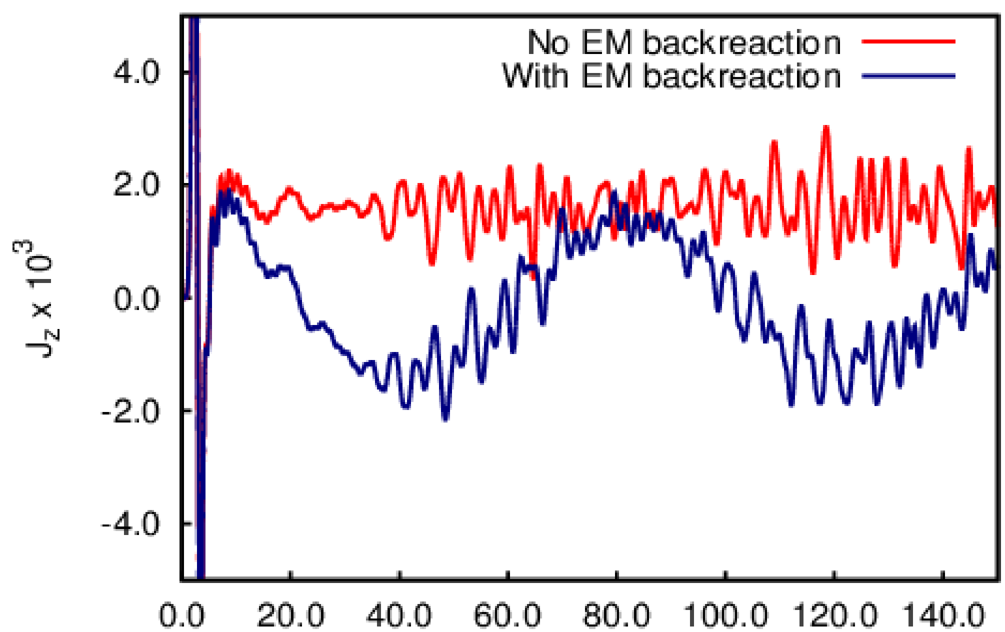




Chiral Plasma Oscillations

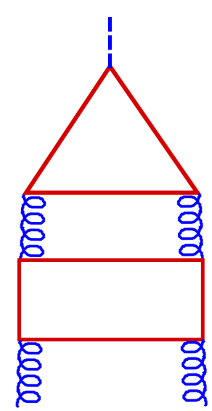
Plasma of charged interacting chiral fermions in external magnetic field B

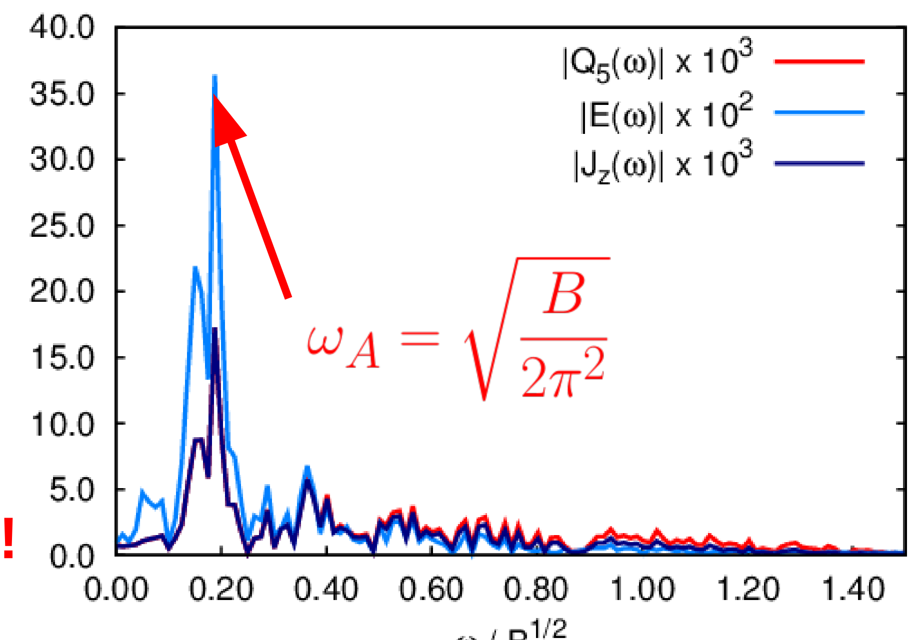




$$\boxed{J_z(t) = Q_A(t) \quad \textbf{!!!}}$$

$$\partial_t E_z(t) = - < J_z(t) >$$





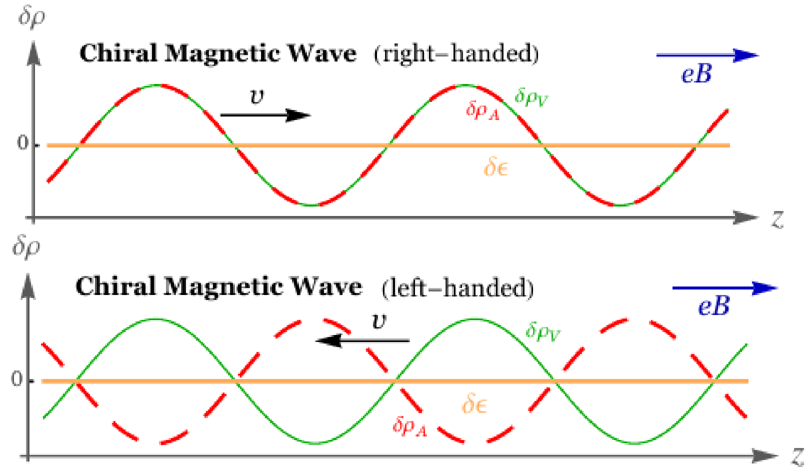
Frequency of oscillations is very robust!

Future development: Outlook

Chiral magnetic wave:

Massless excitations $$v_{CMW} = \frac{B}{2\pi^2 \chi}$$

M. Chernodub, JHEP 1601 (2016) 100



Demanding numerically:

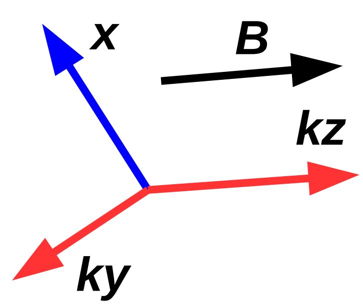
(Ly Lx Lz 4) *x* (Lx Lz 4) !

In continuum the states of free fermion in magnetic field are effectively 1D!
Can one use it in order to reduce computational costs?

Energy levels of Wilson-Dirac fermions in magnetic field:

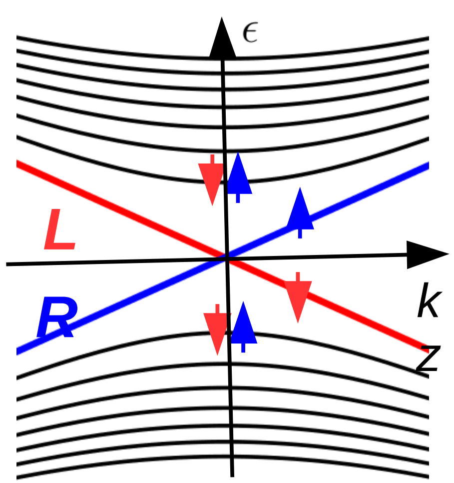
$$\boxed{\epsilon_n(k_y, k_z) \equiv \epsilon_n(k_z)}$$

$$A_y = Bx \qquad B = \frac{2\pi n|L|}{L^2}$$



Landau levels:

$$\epsilon_{n,\sigma} = \pm\sqrt{k_z^2 + 2B(n-1+\sigma)}$$



Indicates that lattice states are also effectively 1D!

Practical implementation is an open question...

Conclusions

1) **NP contribution to the vector current due to Schwinger pair production in E || B is observed.**

2) **Longitudinal oscillations in chiral plasma in magnetic field are observed. *Frequency is fixed by anomaly coefficient.***

3) **Effect of electromagnetic interactions is very strong, non-perturbative calculations are essential!**

4) **Future development: chiral magnetic waves.**