Pion structure from twisted mass lattice QCD down to the physical pion mass

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Pion Structure with Twisted Mass Fermions

In this talk: discuss two sets of benchmark observables for hadron structure in LQCD.

- $F_\pi(Q^2)$: Electromagnetic (vector) form factor of the pion at space-like momentum transfers Q^2
  - at small Q^2: stringent test of QCD, needs non-perturbative calculation
  - experimental precision $\mathcal{O}(1\%)$
  - at $Q^2 = 0$, pion point-like $\rightarrow F_\pi(0) = 1$

- $\langle r^2 \rangle = 6 \frac{\mathrm{d}F_\pi(Q^2)}{\mathrm{d}Q^2}\Big|_{Q^2=0}$: pion charge radius
  - very sensitive to pion mass
  - "gold-plated" observable for chiral logarithms

- $\langle x \rangle_\pi$: iso-vector momentum fraction (Part of M. Oehm's Master thesis)
  - quark-line connected contribution only
  - can be compared to [Wijesooriya, Reimer, Holt, Phys. Rev. C 72, 065203, 2005]

Lattice Setup

This talk: results from two different lattice actions

- Twisted mass fermions at maximal twist
  - $N_f = 2$ with clover term, down to M_π^{phys}
  - $N_f = 2 + 1 + 1$ w/o clover term, down to about 230 MeV

[Frezzotti, Grassi, Sint, Weisz, 2000; Frezzotti, Rossi, 2004; ETMC, arXiv:1507.05068, 2015]

$$\begin{aligned} S &= \beta \sum_{x;P} \left[b_0 \{ 1 - \tfrac{1}{3} \mathrm{ReTr} P^{1\times 1}(x) \} + b_1 \{ 1 - \tfrac{1}{3} \mathrm{ReTr} P^{1\times 2}(x) \} \right] \\ &+ \sum_x \bar{\chi}_\ell(x) \left[D_W(U) + m_0 + i\mu\gamma^5\tau^3 + \tfrac{i}{4} C_{\mathsf{SW}} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) \right] \chi_\ell(x) \\ &+ \sum_x \bar{\chi}_h(x) \left[D_W(U) + m_0 + i\mu_\sigma \gamma^5 \tau^1 + \mu_\delta \tau^3 \right] \chi_h(x) \end{aligned}$$

- Iwasaki gauge action: $b_0 = 1 - 8b_1$, $b_1 = -0.331$ [Iwasaki; 1983]
- For $N_f = 2$ w/clover: $C_{\mathsf{SW}} = 1.57551$ from Padé fit of CP-PACS data

[Aoki et al.; Phys.Rev. D73 (2006) 034501]

Electromagnetic Form Factor - $F_\pi(Q^2)$ Overview

- Phenomenological determination down to $Q^2 \sim 0.02\ \mathrm{GeV}^2$
  $\pi - e$ scattering: CERN [Amendolia et al., Phys.Lett. B146 (1984) 116-120]

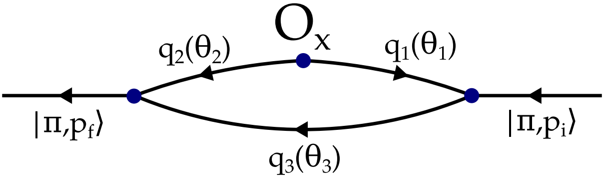
- Lattice: relate $\langle \pi, p_f | J_\mu^{\mathrm{em}} | \pi, p_i \rangle \rightarrow \langle \pi, p_f | V_\mu | \pi, p_i \rangle$
  - ETMC [Frezzotti, Lubicz, Simula (Phys.Rev.D79, 2009)]
    - ⋆ $N_f = 2$ twisted mass, NNLO chiral extrapolation with exp. scalar charge radius
  - Regensburg/Southampton/Mainz [Brandt, Jüttner, Wittig, JHEP 1311 (2013) 034]
    - ⋆ $N_f = 2$, combined NNLO chiral extrapolations with M_π, f_π and priors
  - HPQCD [Koponen et al., Phys.Rev. D93 (2016) no.5, 054503]
    - ⋆ $N_f = 2 + 1 + 1$ calculation at physical light quark mass

this computation

- $N_f = 2$ twisted mass + clover
- $M_\pi = 135, 250, 340\ \mathrm{MeV}$
- $L/a = 24, 32, 48, 64$
- twisted boundary conditions to reach low momentum transfers

[Divitis, Petronzio, Tantalo, Phys.Lett. B595 (2004) 408-413]

Electromagnetic Form Factor - Lattice Technique

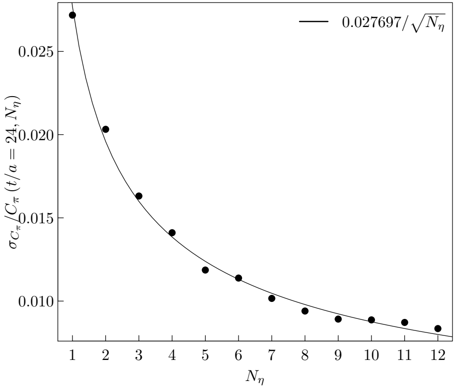


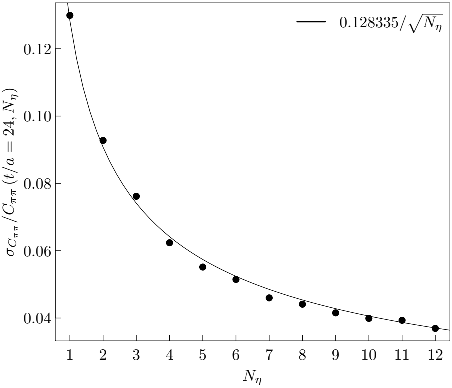
$\mathbf{O_x} = \bar{\psi}\gamma_4\psi \Rightarrow F_\pi(Q^2)$

- Breit frame: $\mathbf{p_f} = -\mathbf{p_i}$, space-like momentum transfer $Q^2 = -(p_i - p_f)^2$
  - $\mathcal{O}(a)$-improved $F_\pi(Q^2)$ (at maximal twist)
    [Frezzotti, Lubicz, Simula (Phys.Rev.D79, 2009)]
- isotropic momenta: $\theta_x = \theta_y = \theta_z = \tilde{\theta}$
- $\mathbf{q_i} = \{\tilde{\theta}/L, \tilde{\theta}/L, \tilde{\theta}/L\}$
- $\theta_1 = \tilde{\theta}$, $\theta_2 = -\tilde{\theta}$, $\theta_3 = 0$
- sequential propagator method
- sink time-slice: $T/2$, fit 2pt fn. in plateau region, reconstruct at $T/2$ from bootstrap samples

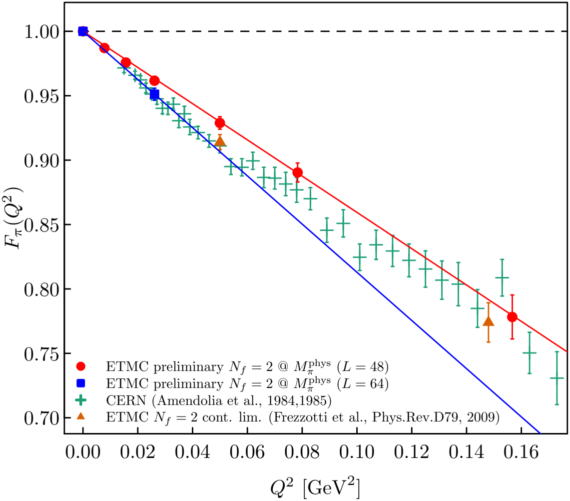
$F_\pi(Q^2)$ - Stochastic Technique

- Computation follows [Frezzotti, Lubicz, Simula (Phys.Rev.D79, 2009)]
- However: use $N_\eta = \{4, 8, 12, 16\}$ $\mathbb{Z}_2 \bigotimes \mathbb{Z}_2$ random sources for $L/a = \{24, 32, 48, 64\}$ respectively, *without* dilution
- around 300 configurations per ensemble (fewer on $L/a = 64$)
- Especially for large momenta, see good relative error scaling for large t/a in 2pt (left) and 3pt (right) functions





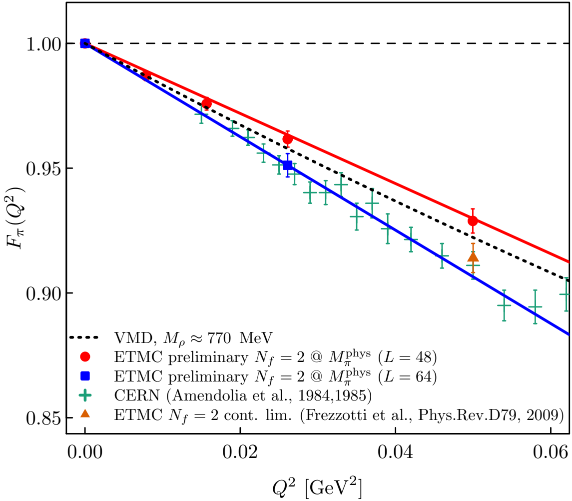
$F_\pi(Q^2)$ - Results at the Physical Pion Mass



$$F_\pi(Q^2) = 1 - \frac{\langle r^2 \rangle}{6} Q^2 + cQ^4$$

- $L/a = 48, 64$
- $Q^2 \in \{0 \text{ GeV}^2, 0.16 \text{ GeV}^2\}$
- In small Q^2 region, errors on $F_\pi(Q^2)$ are of $\mathcal{O}(0.1\%)$
- But: error on $\left[1 - F_\pi(Q^2)\right]$ is of $\mathcal{O}(15\%)$

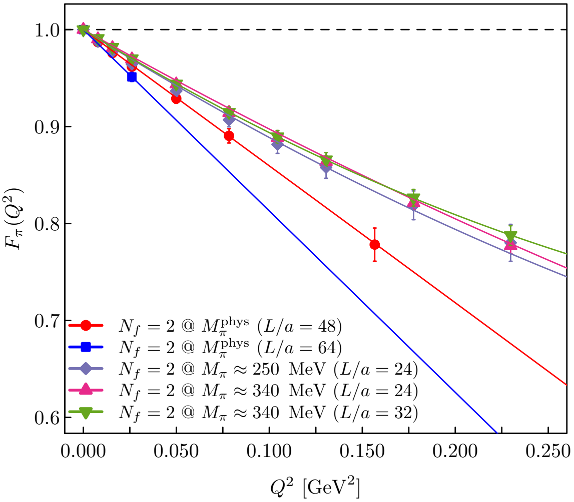
$F_\pi(Q^2)$ - Small Q^2 Region



$$F_\pi(Q^2) = 1 - \frac{\langle r^2 \rangle}{6} Q^2 + cQ^4$$

- Significant finite volume effects $(L/a = \{48, 64\})$
- As for CERN group, $F_\pi(Q^2) = \frac{1}{1+Q^2/M_\rho^2}$ not sufficient to describe data
- $L/a = 64$ result falls right onto phenomenology, but could there be more FSE?
- Could we do better with some more cheaper data?

$F_\pi(Q^2)$ - Larger Than Physical Pion Masses



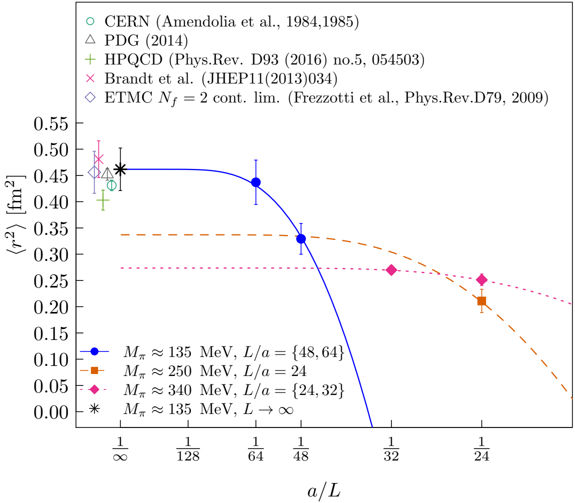
$$F_\pi(Q^2) = 1 - \frac{\langle r^2 \rangle}{6} Q^2 + cQ^4$$

- All fits have $\chi^2/\mathrm{dof} \sim 1$
- Only one $Q^2 > 0$ point at $L/a = 64$, no c in fit
- Complicated combination chiral / FS effects

M_π	L/a	$\langle r^2 \rangle$ [fm^2]	c [fm^4]
340	24	0.251(11)	0.0008(01)
340	32	0.270(08)	0.0014(02)
250	24	0.211(22)	0.0007(02)
135	48	0.330(30)	0.0020(23)
135	64	0.437(45)	–

- More $L/a = 64$ points required to check if FSE not overestimated [Jiang, Tiburzi, Phys. Rev. D78, 037501 (2008)]

$\langle r^2 \rangle$ - (crude) Combined Infinite Volume Limit

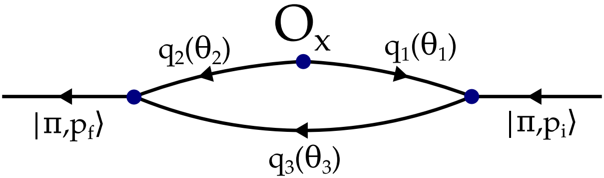


- Assume simple exponential finite volume effects
- Add chiral logarithm following [Koponen et al., Phys.Rev. D93 (2016) no.5, 054503], [Gasser, Leutwyler, Nucl.Phys. B250 (1985) 517-538]
- $\alpha = 3/2 \Rightarrow \chi^2/\text{dof} \sim 1$
- $\langle r^2 \rangle = 0.46(4)\ \text{fm}^2$
  $\Lambda_\chi = 630(70)\ \text{MeV}$
  highly preliminary!

$$\langle r_L^2(M_\pi) \rangle = \left(1 - c_L \cdot \frac{\exp(-M_\pi \cdot L)}{(M_\pi \cdot L)^\alpha}\right) \cdot \left[\left\langle r_\infty^2(M_\pi^{\text{phys}}) \right\rangle - \frac{1}{\Lambda_\chi^2} \cdot \ln\left(\left[\frac{M_\pi}{M_\pi^{\text{phys}}} \right]^2 \right) \right]$$

Pion $\langle x \rangle$

- Method (essentially) same as for vector form factor
- One stochastic source per configuration, around 300 configurations per ensemble

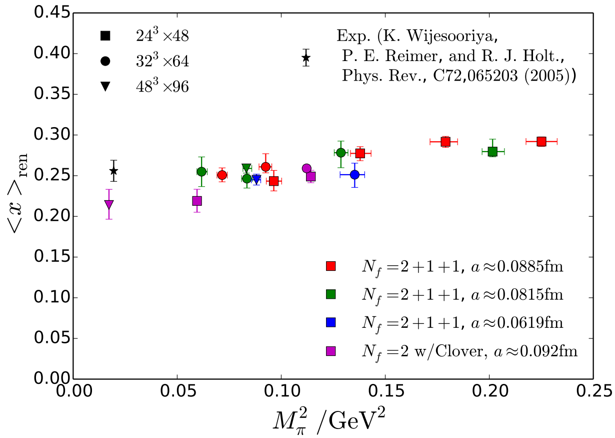


$\mathbf{O_x} = \frac{1}{2}\bar{\psi}\left(\gamma_4 \overleftrightarrow{D}_4 - \frac{1}{3}\sum_{i=1}^{3}\gamma_i \overleftrightarrow{D}_i\right)\psi \Rightarrow \langle x \rangle$ [Best et al., Phys.Rev.D56, 1997]

- $\theta_x = \theta_y = \theta_z = 0 \rightarrow q^2 = 0$
- (almost) no mixing under renormalisation
- here: quark-line connected contribution only

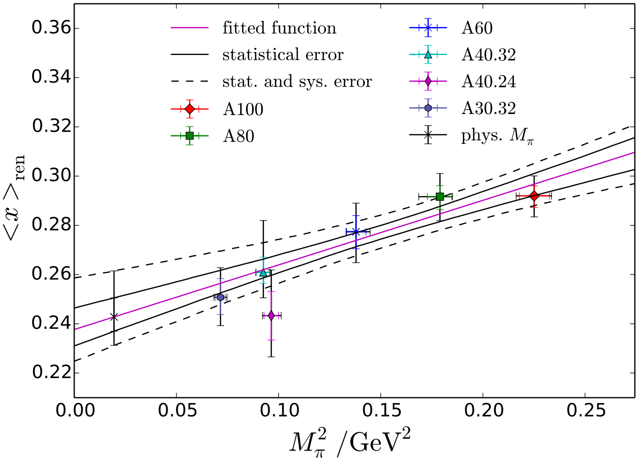
- more derivatives with momentum give higher moments $\langle x^2 \rangle, \langle x^3 \rangle, \ldots$

Pion $\langle x \rangle$ - Overview



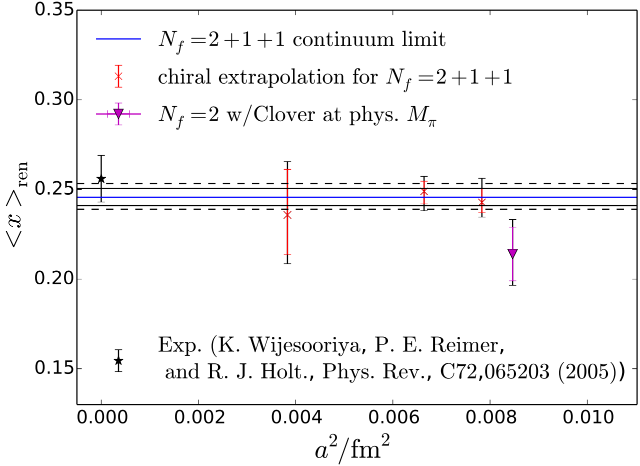
- $N_f = 2 + 1 + 1$ (tmlqcd)
  - currently: 13 ensembles
    $M_\pi \approx 230$ to 470 MeV
    $a \approx 0.062$ to 0.089 fm
    $L \approx 2$ to 3 fm
  - renormalisation constants: [Alexandrou et al, arXiv:1509.00213]
- $N_f = 2$ (tmlqcd + clover)
  - $M_\pi \approx 135$ to 340 MeV
    $a \approx 0.092$ fm
    $L \approx 2.2$ to 4.5 fm
  - [Abdel-Rehim et al, Phys.Rev. D92 (2015) no.11, 114513]

Pion $\langle x \rangle$ - Chiral Extrapolation



- Here: $a \approx 0.089$ fm
- For each lattice spacing: $\langle x \rangle_{\rm ren}(a^2 M_{\pi^\pm}^2, a^2) = \langle x \rangle_{\rm ren}(a^2) + c_a a^2 M_{\pi^\pm}^2$
- FSE unresolved within current uncertainties → ignore for now

Pion $\langle x \rangle$ - Continuum Extrapolation



$$\langle x \rangle_{\text{ren}} = 0.246(05)\left(^{+11}_{-09}\right)$$

- Constant continuum extrapolation seems to be well justified.
- $N_f = 2, M_{\pi^\pm}^{\text{phys}}$ compatible within $\sim 1.5\sigma$
  - check FSE: computation on ~ 6 fm lattice ongoing

Summary & Conclusions

- Very tentative result for pion vector form factor at M_π^{phys}
- Agreement with phenomenology and other determinations, but FSE significant

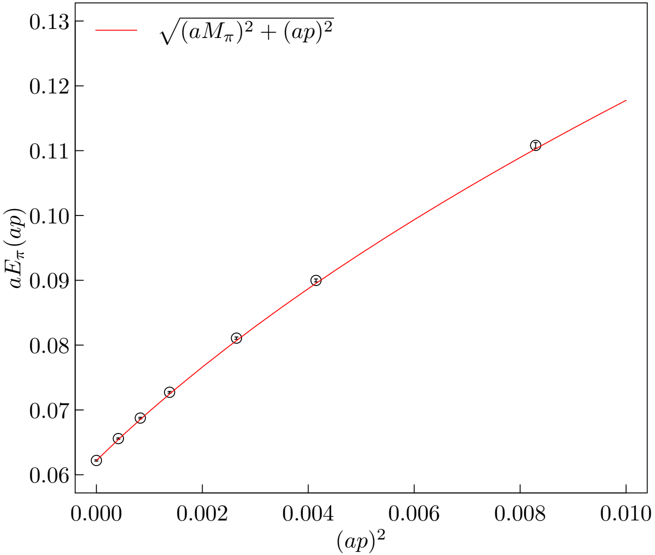
- $\langle x \rangle_\pi$ presented with chiral and continuum extrapolation
- Very good agreement with phenomenology, ready to compute higher moments

Outlook

- Computation of $\langle x^2 \rangle$ in progress (significant increase of statistics required)
- At M_π^{phys}, check of FSE on $\langle x \rangle$ ongoing
- For $F_\pi(Q^2)$, $L/a = 64$ results are being extended
- $F_\pi(Q^2)$ on $N_f = 2 + 1 + 1$ ensembles ongoing
- scalar radius $\langle r_S^2 \rangle$ computation ongoing

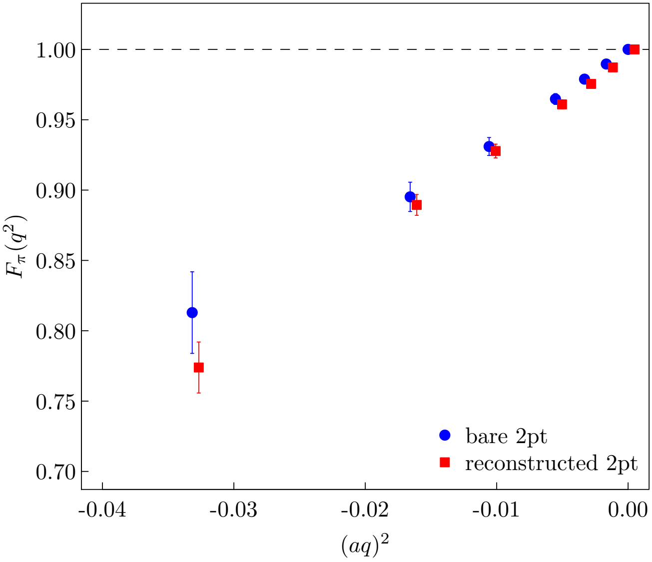
Thanks for your attention!

Backup Slides - Pion Dispersion Relation



- M_π^{phys}, $L/a = 48$
- no deviation from continuum disp. relation within errors

Backup Slides - Reconstruction of the Two-Point Function



- Fit two-point function where effective mass shows plateau
- Reconstruct at $t = T/2$ using bootstrap samples of fit parameters
- Reduces statistical errors
- Shown here: $M_\pi^{\text{phys}}, L/a = 48$