Westfälische Wilhelms-Universität Münster

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ALPHA Collaboration

Non-perturbative matching of HQET heavy-light axial and vector currents in $N_f = 2$ lattice QCD

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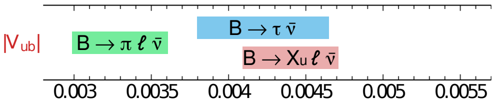
Motivation: Determination of V_{ub}

The Cabibbo-Kobayashi-Maskawa matrix $V = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix}$

can be determined from various $\mathrm{b} \to \mathrm{u}$ processes:

- Inclusive semi-leptonic (SL) $\mathrm{B} \to \mathrm{X_u}\ell\bar{\nu}$ decay
- Exclusive semi-leptonic (SL) $\mathrm{B} \to \pi\ell\bar{\nu}$ decay
  - from lattice QCD: hadronic form factor $f_+(q^2)$
- leptonic $\mathrm{B} \to \tau\bar{\nu}$ decay
  - from lattice QCD: hadronic decay constant f_B



[PDG]

- $\sim 3\sigma$ tension $\Rightarrow$ V_{ub} *puzzle*
- New physics ?
- Reliable lattice input needed !

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V_{ub} via $\mathrm{B} \to \pi\ell\bar{\nu}$ in the Standard Model

Experimental and theoretical (= lattice QCD) ingredients:

- $f_+(q^2)$ required to determine $|V_{\mathrm{ub}}|$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_\mathrm{F}^2 |V_{\mathrm{ub}}|^2}{192\pi^3 m_\mathrm{B}^3} (\lambda(q^2))^{3/2} |f_+(q^2)|^2$$

$$q = p_\mathrm{B} - p_\pi \qquad \lambda(q^2) = (q^2 - m_\mathrm{B}^2 - m_\pi^2)^2 - 4m_\mathrm{B}^2 m_\pi^2$$

- $f_+(q^2)$ can be determined from the semi-leptonic $\mathrm{B} \to \pi$ matrix element $\langle \pi(p_\pi)|V^\mu|\mathrm{B}(p_\mathrm{B})\rangle$ through

$$\langle \pi(p_\pi)|V^\mu|\mathrm{B}(p_\mathrm{B})\rangle = f_+(q^2)\left[p_\mathrm{B} + p_\pi - \frac{m_\mathrm{B}^2 - M_\pi^2}{q^2} q^\mu\right] + f_0(q^2)\frac{m_\mathrm{B}^2 - M_\pi^2}{q^2} q^\mu$$

- This is what is finally to be computed on the lattice ...

… but as first goal: Form factors in $B_s \to K\ell\nu$ decays

- No experimental data for SL $B_s \to K$ yet $\Rightarrow$ *Predictions*
- Easier on the lattice, as $m_K = m_K^{phys}$ (valence) computationally less expensive than with π, but not far from SL $B \to \pi$ though
- "Just" replace B by B_s and π by K in previous formulae

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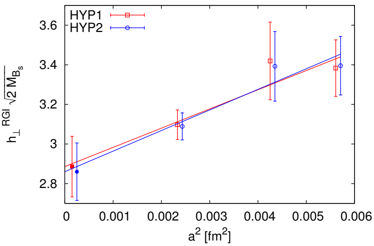
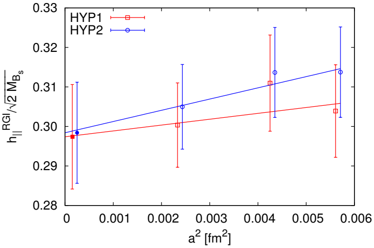
Leading (= *static*) order HQET computation in $N_\mathrm{f} = 2$ lattice QCD

[ALPHA Collaboration, Bahr et al., PLB 757 (2016) 473, arXiv:1601.04277]





- NP Renormalization & Continuum limit taken for the 1st time !
- $f_+(21.22\,\mathrm{GeV}^2) = 1.63(8)(6) \pm 0.24 \Rightarrow$ Erase by NP HQET @ *NLO*

Non-perturbative (NP) HQET at $O(1/m_h)$

- Effective theory of QCD for systems with one heavy quark
- Action and operators are expanded in an asymptotic power series of $1/m_h$ [Eichten 1988, Eichten & Hill 1990]

$$\text{action:} \quad \mathcal{L}^{\text{HQET}} = \underbrace{\bar\psi_h D_0 \psi_h}_{\mathcal{L}_{\text{stat}} \sim O(1)} \underbrace{- \omega_{\text{kin}} \mathcal{O}_{\text{kin}} - \omega_{\text{spin}} \mathcal{O}_{\text{spin}}}_{\mathcal{L}^{(1)} \sim O(1/m_h)} + \dots$$

$$\mathcal{O}_{\text{kin}} = \bar\psi_h \mathbf{D}^2 \psi_h \qquad \mathcal{O}_{\text{spin}} = \bar\psi_h \boldsymbol{\sigma} \cdot \mathbf{B} \psi_h$$

$$\text{operators:} \quad \mathcal{O}_R^{\text{HQET}} = Z_{\mathcal{O}}^{\text{HQET}} \left[\mathcal{O}^{\text{stat}} + \textstyle\sum_i c_{\mathcal{O}_i} \mathcal{O}_i \right]$$

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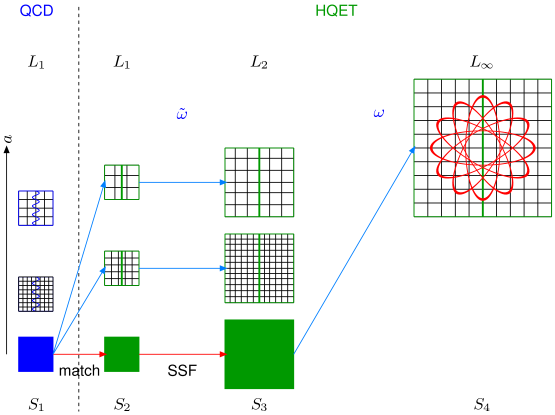
- Parameters: $(m_{\mathrm{bare}}, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}, Z_{\mathcal{O}}^{\mathrm{HQET}}, c_{\mathcal{O}_1}, \ldots)$
- All $\mathcal{O}_i$ with the same quantum numbers and correct dimension must be taken into account
- $1/m_h$–terms $\widehat{=}$ local operator insertions in CFs (via expanding the functional integral weight in $1/m_h$, directly on the lattice)

$$\langle O \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\varphi]\, \mathrm{e}^{-S_{\mathrm{rel}} - a^4 \sum_x \mathcal{L}_{\mathrm{stat}}(x)} O \{1 - a^4 \sum_x \mathcal{L}^{(1)}(x) + \ldots\} \Rightarrow \text{renormalizable}$$

Non-perturbative matching between HQET and QCD

[ALPHA Collaboration, H. & Sommer, 2004, ..., Blossier et al., JHEP 09 (2012) 132, arXiv:1203.6516]

A finite-volume, recursive strategy:



Matching volume: $L_1 \approx 0.5\,\mathrm{fm} \rightarrow am_\mathrm{h} \ll 1$, relativistic b-quark feasible

Operators considered

Previously: A_0 (= time component of heavy-light axial current)

- Its (B-to-vacuum) matrix element enters the computation of the B-meson decay constant f_B

$$A_{0,\mathrm{R}}^{\mathrm{HQET}} = Z_{A_0}^{\mathrm{HQET}} \left[A_0^{\mathrm{stat}} + \sum_{i=1}^{2} c_{A_0,i} A_{0,i} \right]$$

$$A_{0,1} = \bar{\psi}_\mathrm{l} \gamma_5 \gamma_i \frac{1}{2} \left(\nabla_i - \overleftarrow{\nabla}_i \right) \psi_\mathrm{h} \,, \; A_{0,2} = \bar{\psi}_\mathrm{l} \gamma_5 \gamma_i \frac{1}{2} \left(\nabla_i + \overleftarrow{\nabla}_i \right) \psi_\mathrm{h}$$

- $A_{0,2}$ vanishes due to the sum over $\mathbf{x}$, if BCs are periodic

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- $A_{0,2}$ vanishes due to the sum over $\mathbf{x}$, if BCs are periodic
- 5 HQET parameters left: $(m_\mathrm{bare}, \omega_\mathrm{kin}, \omega_\mathrm{spin}, Z_{A_0}^{\mathrm{HQET}}, c_{A_{0,1}})$

$\rightarrow$ published $N_\mathrm{f} = 2$ results by ALPHA Collaboration:

- 5 HQET parameters of $\mathcal{L}^{\mathrm{HQET}}, A_{0,\mathrm{R}}^{\mathrm{HQET}}$ [Blossier et al., JHEP 09 (2012) 132]
- Application #1: b-quark mass [Bernadoni et al., PLB 730 (2014) 171]
- Application #2: $f_\mathrm{B}, f_{\mathrm{B_s}}$ [Bernadoni et al., PLB 735 (2014) 349]

Operators considered

Now: A_0, A_k, V_0, V_k (= all axial & vector current components)

- Application in mind: Computation of form factor f_+ incl. $\mathrm{O}(\frac{1}{m_\mathrm{h}})$
- 14 new parameters appear, e.g. in the vector channel from

$$V_{k,\mathrm{R}}^{\mathrm{HQET}} = Z_{V_k}^{\mathrm{HQET}} \left[V_k^{\mathrm{stat}} + \sum_{i=1}^{4} c_{V_{k,i}} V_{k,i} \right]$$

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- All in all, the complete set of 19 HQET parameters is:

$$\Big(m_{\mathrm{bare}}, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}, c_{A_{0,1}}, c_{A_{0,2}}, Z_{A_0}^{\mathrm{HQET}}, c_{A_{k,1}}, c_{A_{k,2}}, c_{A_{k,3}}, c_{A_{k,4}}, Z_{A_k}^{\mathrm{HQET}}, c_{V_{0,1}}, c_{V_{0,2}}, Z_{V_0}^{\mathrm{HQET}}, c_{V_{k,1}}, c_{V_{k,2}}, c_{V_{k,3}}, c_{V_{k,4}}, Z_{V_k}^{\mathrm{HQET}} \Big)$$

Reminder: NP finite-volume matching strategy

Why *non-perturbative*?

- Operator mixing in the lattice effective theory induces power divergences

$$(am_{\rm h})^{-n} \quad : \qquad \frac{g_0^{2l}}{a^n} \sim \frac{1}{\ln^l(a\Lambda_{\rm QCD})\,a^n} \quad , \quad n=1,2$$

that must be subtracted *NP'ly* to have a continuum limit

(otherwise, truncated terms in the perturbative series would spoil it ...)

- Power $(1/m_{\rm h})$ corrections are only defined, when the leading term is computed non-perturbatively

$$(\alpha(m_{\rm h}))^l \sim \left\{\frac{1}{2b_0\ln(m_{\rm h}/\Lambda_{\rm QCD})}\right\}^l \overset{m_{\rm h}\to\infty}{\gg} \frac{\Lambda_{\rm QCD}}{m_{\rm h}}$$

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⇒ *NP HQET* takes care of this, no predictions are lost ⇒ Idea:

Equate (in small volume) QCD " = " HQET in the sense of

$$\Phi_i^\mathrm{QCD}(L, m_\mathrm{h}, 0) \overset{!}{=} \Phi_i^\mathrm{HQET}(L, m_\mathrm{h}, a) \qquad m_\mathrm{h} = M : \quad \text{RGI mass}$$

Determine parameters via *matching* HQET & QCD by NP'ly imposing

$$\Phi_i^{\mathrm{QCD}}(L, m_{\mathrm{h}}, 0) \stackrel{!}{=} \Phi_i^{\mathrm{HQET}}(L, m_{\mathrm{h}}, a) \qquad m_{\mathrm{h}} = M: \quad \text{RGI mass}$$

$$\equiv \eta_i(L,a) + \varphi_i^j(L,a)\,\omega_j(M,a) + \mathrm{O}\left(\tfrac{1}{m_{\mathrm{h}}^2}\right)$$

Matching conditions for the complete set of HQET parameters in Lagrangian & all heavy-light flavour currents:

- Choose $i = 1, \ldots, n_{\mathrm{par}} = 19$ suitable observables Φ_i that are sensitive to the HQET parameters and possess a linear HQET expansion
- The matching equations above thus consist of ...
  - ... renormalized QCD quantities Φ_i^{QCD}
  - ... bare HQET correlators φ_i^j
  - ... static-order (parameter-free) HQET terms η_i
  - ... and the *HQET parameters* ω_i

In more convenient matrix-vector notation:

[Della Morte, Dooling, H., Hesse & Simma, JHEP 05 (2014) 060, arXiv:1312.1566]

- Collect all $i = 1, \ldots, n_\mathrm{par} = 19$ parameters in a column vector

$$\omega = \big(m_\mathrm{bare}, \omega_\mathrm{kin}, \omega_\mathrm{spin}, c_{A_{0,1}}, c_{A_{0,2}}, Z^\mathrm{HQET}_{A_0}, c_{A_{k,1}}, c_{A_{k,2}}, c_{A_{k,3}}, c_{A_{k,4}}, Z^\mathrm{HQET}_{A_k}, c_{V_{0,1}}, c_{V_{0,2}}, Z^\mathrm{HQET}_{V_0}, c_{V_{k,1}}, c_{V_{k,2}}, c_{V_{k,3}}, c_{V_{k,4}}, Z^\mathrm{HQET}_{V_k} \big)^\mathrm{T}$$

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- Column vector of n_par observables $\boldsymbol{\Phi} = (\Phi_1, \ldots, \Phi_{n_\mathrm{par}})$, such that its linear HQET expansion implies a matrix of structure

$$\varphi = \left(\begin{array}{c|c|c|c|c} \varphi^1_1 & * & * & 0 & 0 \\ \hline 0 & * & 0 & 0 & 0 \\ \hline 0 & * & * & 0 & 0 \\ \hline 0 & * & 0 & * & 0 \\ \hline 0 & * & 0 & 0 & * \end{array}\right)$$

for bare HQET correlation functions

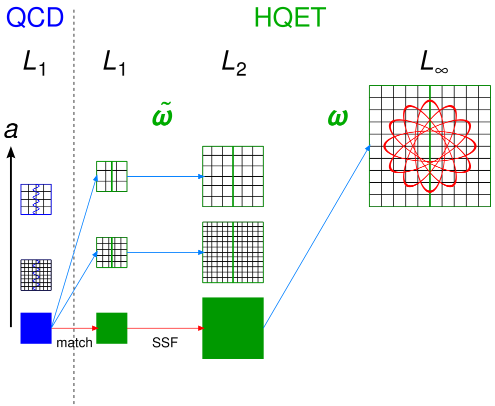
- Matching condition (in matrix-vector notation):

$$\boldsymbol{\Phi}^\mathrm{QCD}(L, M, 0) \overset{!}{=} \boldsymbol{\Phi}^\mathrm{HQET}(L, M, a) = \boldsymbol{\eta}(L, a) + \varphi(L, a) \cdot \boldsymbol{\omega}(M, a)$$

Complete list of HQET parameters and their meaning & origin:

i	ω_i	origin
$1,\ 2,\ 3$	$m_\mathrm{bare},\ \omega_\mathrm{kin},\ \omega_\mathrm{kin}$	$\mathscr{L}^\mathrm{HQET}$
$4,\ \dots,\ 6$	$c_{\mathrm{A}_{0,1}},\ c_{\mathrm{A}_{0,2}},\ \ln Z^\mathrm{HQET}_{A_0}$	A_0^HQET
$7,\ \dots,\ 11$	$c_{\mathrm{A}_{k,1}},\ c_{\mathrm{A}_{k,2}},\ c_{\mathrm{A}_{k,3}},\ c_{\mathrm{A}_{k,4}},\ \ln Z^\mathrm{HQET}_{\vec{\mathrm{A}}}$	A_k^HQET
$12\ \dots,\ 14$	$c_{\mathrm{V}_{0,1}},\ c_{\mathrm{V}_{0,2}},\ \ln Z^\mathrm{HQET}_{V_0}$	V_0^HQET
$15,\ \dots,\ 19$	$c_{\mathrm{V}_{k,1}},\ c_{\mathrm{V}_{k,2}},\ c_{\mathrm{V}_{k,3}},\ c_{\mathrm{V}_{k,4}},\ \ln Z^\mathrm{HQET}_{\vec{\mathrm{V}}}$	V_k^HQET

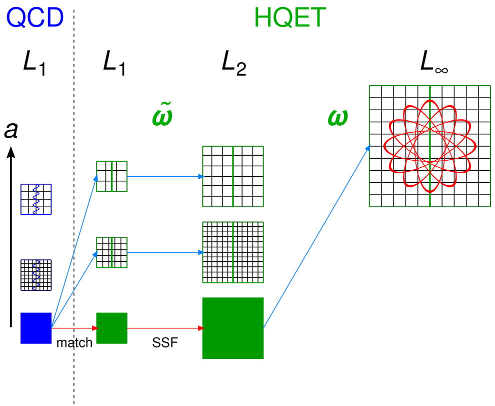
$\Rightarrow$ 19 observables for 19 free parameters are needed



- How does the matching proceed in practice ?

Starting point is the foregoing equation

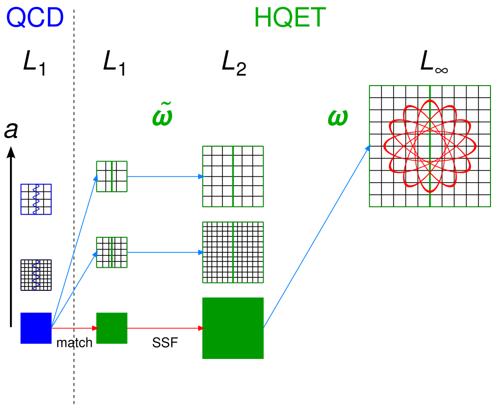
$$\Phi^{\mathrm{QCD}}(L,M,0) \stackrel{!}{=} \Phi^{\mathrm{HQET}}(L,M,a) = \eta(L,a) + \varphi(L,a) \cdot \omega(M,a)$$



- In small volume ($L_1 \approx 0.5$ fm):
  Compute observables $\Phi^{\rm QCD}(L_1, M, a)$ in relativistic QCD and extrapolate them to the continuum limit, i.e.

$$\Phi^{\rm QCD}(L_1, M, 0) = \lim_{a \to 0} \Phi^{\rm QCD}(L_1, M, a)$$

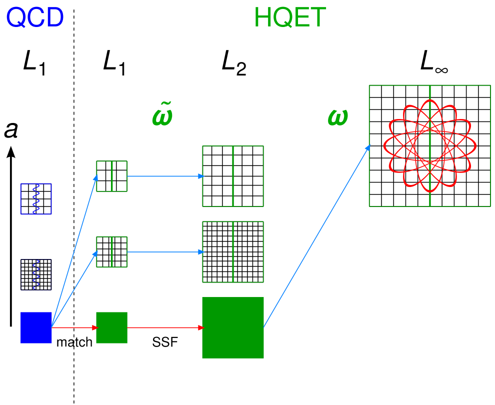
- CL $a \to 0$ can be taken in QCD (l.h.s.) due to small volume !



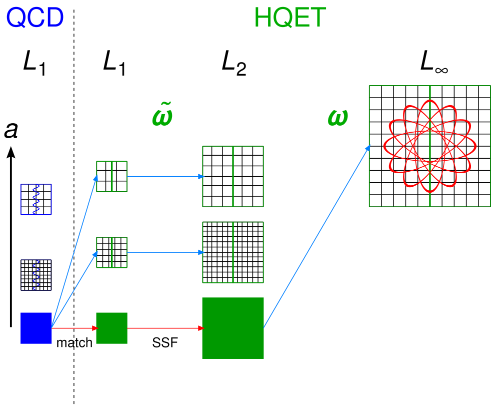
- Match with HQET ($a \lesssim 0.05$ fm) to solve for parameters in L_1:

$$\Phi^{\mathrm{HQET}}(L_1, M, a) = \eta(L_1, a) + \varphi(L_1, a) \cdot \tilde{\omega}(M, a) \stackrel{!}{=} \Phi^{\mathrm{QCD}}(L_1, M, 0)$$

$$\Rightarrow \tilde{\omega}(M, a) = \varphi^{-1}\left[\Phi^{\mathrm{QCD}} - \eta\right]$$

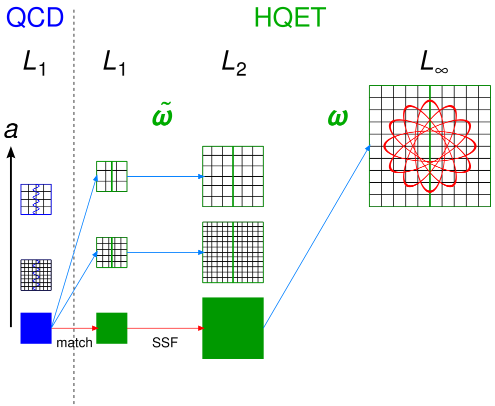


- Step scaling $L_1 \to L_2 = 2L_1 \approx 1\,\text{fm}$:
  Employ determined parameters $\tilde{\omega}(M,a)$ in the computation of HQET observables $\Phi^{\text{HQET}}(L_2,M,a)$ in larger volume and extrapolate them to the continuum, too



- Extract the parameters $\omega(M,a)$ for larger a (i.e., those typically encountered in large-volume simulations) by solving the associated matrix-vector equation in L_2:

$$\omega(M,a) = \varphi(L_2,a)^{-1}\left[\Phi^{\mathrm{HQET}}(L_2,M,0) - \eta(L_2,a)\right]$$



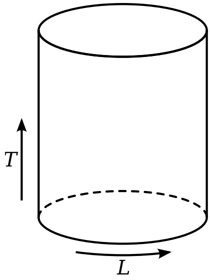
- Once determined in this way, the n_{par} HQET parameters $\omega(M,a) = \{\omega_i(M,a)\}$, which NP'ly absorb the log. & power divergences of the effective theory and inherit the NP QCD quark mass dependence, can finally be used to calculate the desired large-volume HQET observables at $1/M$ for $M = M_b$

Schrödinger Functional (SF)

- Observables defined in the QCD Schrödinger functional setup
- Finite volume $T \times L^3$
- Dirichlet BCs in time, periodic BCs in space
- Additional periodicity phase angle vector $\boldsymbol{\theta}$ for fermion fields:

$$\psi(x_0, \mathbf{x} + \mathbf{n} \cdot L) = \mathrm{e}^{\mathrm{i}\boldsymbol{\theta}\cdot\mathbf{n}} \cdot \psi(x)$$

- Boundary fields ζ used to build CFs
- Well known renormalization properties
  - → Bare parameters $(L/a, \beta, \kappa_l, \kappa_h)$ to define *lines of constant physics* in matching volume L_1^4, keeping $\bar{g}^2_{SF}$ and (RGI) light & heavy quark masses fixed

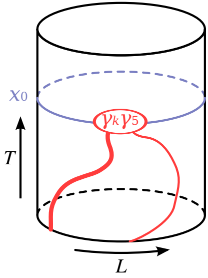


SF correlation functions

Example (#1, boundary–bulk CFs)

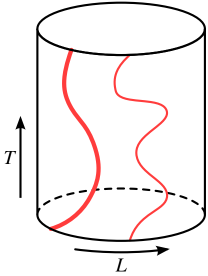
$$f_{A_k}(x_0,\theta_l,\theta_h) = i\frac{a^6}{6L^3}\sum_{\mathbf{x},\mathbf{y},\mathbf{z},k}\langle\underbrace{\bar\psi_l(x)\gamma_k\gamma_5\psi_h(x)}_{A_k(x)}\times \times\bar\zeta_h(\mathbf{y})\gamma_5\zeta_l(\mathbf{z})\rangle$$

$$k_{V_k}(x_0,\theta_l,\theta_h) = -\frac{a^6}{6L^3}\sum_{\mathbf{x},\mathbf{y},\mathbf{z},k}\langle\underbrace{\bar\psi_l(x)\gamma_k\psi_h(x)}_{V_k(x)}\times \times\bar\zeta_h(\mathbf{y})\gamma_k\zeta_l(\mathbf{z})\rangle$$



Example (#2, boundary–boundary CF)

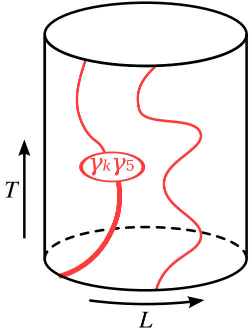
$$F_1(\theta_l,\theta_h) = -\frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}}\langle\bar\zeta_l'(\mathbf{u})\gamma_5\zeta_h'(\mathbf{v})\bar\zeta_h(\mathbf{y})\gamma_5\zeta_l(\mathbf{z})\rangle$$



Three-point SF correlation functions

- 3–point correlators for alternative matching conditions
- Perturbative studies (at tree-level & 1–loop) favor observables constructed from 3–point CFs: yield flatter $1/m_\mathrm{h}$–dependence
  [Hesse & Sommer, JHEP 02 (2013) 115; Della Morte et al., JHEP 05 (2014) 060; Korcyl]
- To be checked non-perturbatively: in progress ...

$$J^1_{A_1}(x_0,\theta_\mathrm{l},\theta_\mathrm{h}) = -\frac{a^{15}}{2L^6}\sum_{\mathbf{uvyzx}}\langle \bar\zeta'_{\mathrm{l}'}(\mathbf{u})\gamma_1\zeta'_\mathrm{l}(\mathbf{v}) \times \bar\psi_\mathrm{l}(x)\gamma_1\gamma_5\psi_\mathrm{h}(x)\,\bar\zeta_\mathrm{h}(\mathbf{z})\gamma_5\zeta_{\mathrm{l}'}(\mathbf{y})\rangle$$



Similarly in the vector meson channel:

$$F_{V_0}(x_0,\theta_\mathrm{l},\theta_\mathrm{h}) = -\frac{a^{15}}{2L^6}\sum_{\mathbf{uvyzx}}\langle \bar\zeta'_{\mathrm{l}'}(\mathbf{u})\gamma_5\zeta'_\mathrm{l}(\mathbf{v}) \times \bar\psi_\mathrm{l}(x)\gamma_0\psi_\mathrm{h}(x)\,\bar\zeta_\mathrm{h}(\mathbf{z})\gamma_5\zeta_{\mathrm{l}'}(\mathbf{y})\rangle$$

Choice of observables

- The observables in Φ shall ...
  - ... be sensitive to the parameters ω
  - ... entail small $1/m_\mathrm{h}$–terms to expect $\mathrm{O}(1/m_\mathrm{h}^2)$ to be negligible
- $\theta_\mathrm{l}, \theta_\mathrm{h}$: Most important free (kinematical) parameters
  $\rightarrow$ Great flexibility in choice of obs. / matching conditions
- 1 "natural" set proposed and its feasibility first demonstrated at tree-level [Della Morte, Dooling, H., Hesse & Simma, JHEP 05 (2014) 060]
- 4 additional sets of observables [Korcyl & Simma] investigated, composed to minimize (cont.) tree-level $1/m_\mathrm{h}^2$–corrections

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Recall matrix φ:

$$
\begin{array}{ccccc}
m_{\rm bare} & \omega_{\rm kin}, \omega_{\rm spin} & c_{A_0,1}, c_{A_0,2}, Z_{A_0}^{\rm HQET} & c_{A_k,1}, \dots & \cdots
\end{array}
$$

$$
\varphi = \begin{pmatrix}
* & * & * & 0 & \cdots \\
0 & * & 0 & 0 & \cdots \\
0 & * & * & 0 & \cdots \\
0 & * & 0 & * & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
$$

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Elements of φ $\widehat{=}$ Static correlators with (local) $1/m_h$–insertions:

$$\langle O \rangle = \langle O \rangle_{\rm stat} + \omega_{\rm kin} a^4 \sum_x \langle O \mathcal{O}_{\rm kin}(x) \rangle_{\rm stat} + \omega_{\rm spin} a^4 \sum_x \langle O \mathcal{O}_{\rm spin}(x) \rangle_{\rm stat}$$

$$\equiv \langle O \rangle_{\rm stat} + \omega_{\rm kin} \langle O \rangle_{\rm kin} + \omega_{\rm spin} \langle O \rangle_{\rm spin}$$

$$\langle O \rangle_{\rm stat} = \frac{1}{\mathcal{Z}} \int_{\rm fields} O \exp\left\{ -a^4 \sum_x \left[\mathcal{L}_{\rm light}(x) + \mathcal{L}_{\rm h}^{\rm stat}(x) \right] \right\}$$

Example (#1, for matching the axial current A_k)

- Φ_7 is sensitive to $c_{A_{k,1}}$ and $c_{A_{k,2}}$:

$$\Phi_7^{\mathrm{QCD}} = \ln\left(\frac{f_{A_k}(T/2,\boldsymbol{\theta}_1,\boldsymbol{\theta}_1)}{f_{A_k}(T/2,\boldsymbol{\theta}_2,\boldsymbol{\theta}_2)}\right)$$

$$\Phi_7^{\mathrm{HQET}} = \underbrace{\Phi_7^{\mathrm{stat}}}_{\in\boldsymbol{\eta}} + \underbrace{\omega_{\mathrm{kin}}\Phi_7^{\mathrm{kin}} + \omega_{\mathrm{spin}}\Phi_7^{\mathrm{spin}} + c_{A_{k,1}}\Phi_{7,1} + c_{A_{k,2}}\Phi_{7,2}}_{\in\varphi\cdot\boldsymbol{\omega}}$$

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Example (#2, for matching the vector current V_k)

- Φ_{15} is sensitive to $c_{V_{k,1}}$ and $c_{V_{k,2}}$:

$$\Phi_{15}^{\mathrm{QCD}} = \ln\left(\frac{k_{V_k}(T/2,\theta_1,\theta_1)}{k_{V_k}(T/2,\theta_2,\theta_2)}\right)$$

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Example (#3, for matching the vector current V_0)

- Φ_{14} is sensitive to $c_{V_{0,1}}$ and $Z_{V_0}^{\mathrm{HQET}}$:

$$\Phi_{14}^{\mathrm{QCD}} = \ln\left(\frac{F_{V_0}(T/2,\theta_1,\theta_1)}{\sqrt{F_1(\theta_1,\theta_1)\times F_1^{ll}(\theta_1,\theta_1)}}\right)$$

$$\Phi_{14}^{\mathrm{HQET}} = \underbrace{\Phi_{14}^{\mathrm{stat}}}_{\in\boldsymbol{\eta}} + \underbrace{\omega_{\mathrm{kin}}\Phi_{14}^{\mathrm{kin}} + \omega_{\mathrm{spin}}\Phi_{14}^{\mathrm{spin}} + c_{V_{0,1}}\Phi_{14} + \ln Z_{V_0}^{\mathrm{HQET}}}_{\in\varphi\cdot\boldsymbol{\omega}}$$

- Alternatively, fix $Z_{V_0}^{\mathrm{HQET}}$ using 2–point functions only:

$$\Phi_{14}^{\prime\,\mathrm{QCD}} = \ln\left(\frac{k_{V_0}(T/2,\theta_1,\theta_1)}{\sqrt{K_1(\theta_1,\theta_1)}}\right)$$

$$\Phi_{14}^{\prime\,\mathrm{HQET}} = \underbrace{\Phi_{14}^{\prime\,\mathrm{stat}}}_{\in\boldsymbol{\eta}} + \underbrace{\omega_{\mathrm{kin}}\Phi_{14}^{\prime\,\mathrm{kin}} + \omega_{\mathrm{spin}}\Phi_{14}^{\prime\,\mathrm{spin}} + c_{V_{0,1}}\Phi_{14}' + \ln Z_{V_0}^{\mathrm{HQET}}}_{\in\varphi\cdot\boldsymbol{\omega}}$$

Choice of observables: Tree-level example

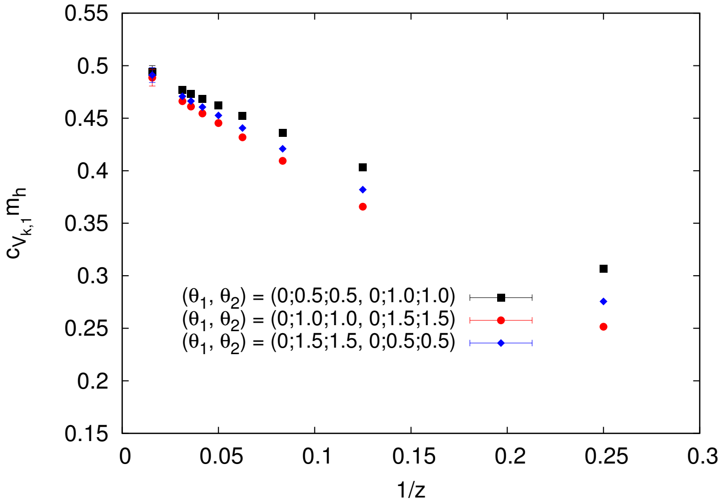

Figure : $c_{V_{k,1}}$ for different θ's vs. inverse mass [from: JHEP 05 (2014) 060]

Non-perturbative matching computation

Simulations (resp. measurements of correlators) re-use the available SF ensembles from earlier matching of HQET action & A_0

- $N_f = 2$, NP'ly O(a) improved, $\theta = (0.5, 0.5, 0.5)$ for sea quarks

	L	T	β	L/a	meas. status
QCD	L_1	L_1	$6.16 \ldots 6.64$	$20 \ldots 40$	DONE
QCD	L_1	$L_1/2$	$6.16 \ldots 6.64$	$20 \ldots 40$	DONE
HQET	L_1	L_1	$5.26 \ldots 5.96$	$6 \ldots 16$	DONE
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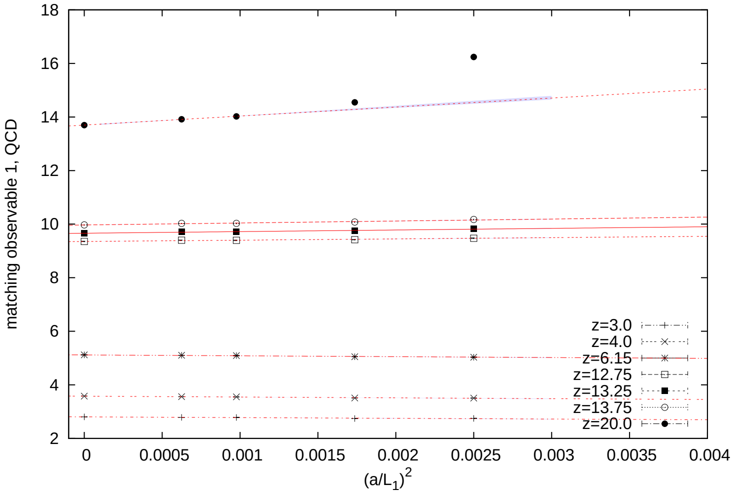
- Measurements at 7 different renormalized masses in $L = L_1$ ($z \equiv 1/(LM) \equiv 1/(LM_h) \in \{3.0, 4.0, 6.15, 12.75, 13.25, 13.75, 20.0\}$)
- Various combinations of θ's of light and heavy quarks (to support several sets of observables resp. matching strategies)

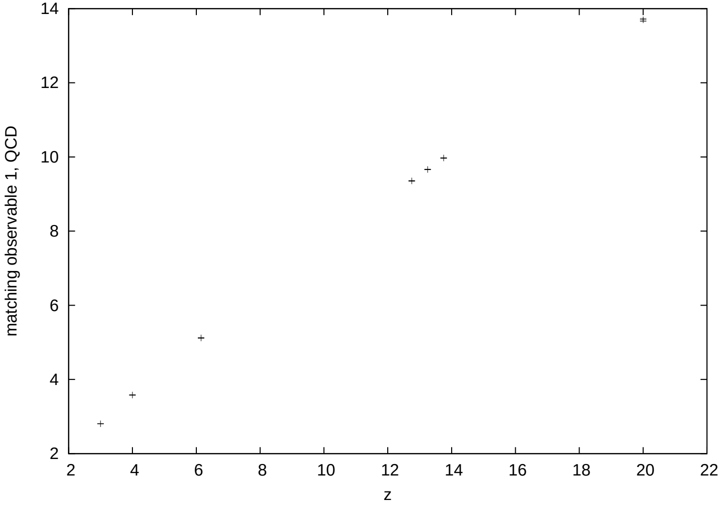
Preliminary results from analysis of the NP data

- For now:
  - Choice "So" among the matching observables / "strategies"
  - Analysis with the others to follow ...
- QCD side:
  Continuum extrapolations in $(a/L_1)^2$ for all z–values
- HQET side:
  - Results for "HYP1/2" static actions available, here "HYP2" only
  - Continuum extrapolations *linear in* a^2 for the static pieces, whereas *linear in* a for the $O(1/m_h)$ ones
  - So far, only observables from 2–point functions included
- (Preliminary) Jackknife error analysis, "UWerr" still to be done

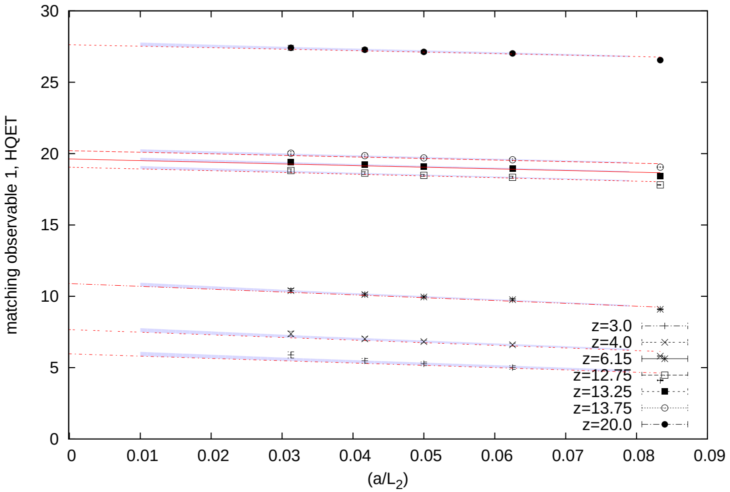
Combined continuum extrapolation of *QCD* observable #1, needed for fixing m_{bare} (strategy "So")



- Heavy quark mass dependence of *QCD* observable #1 in L_1, $z = 1/(L_1 M)$



- Continuum extrapolation of *HQET* observable #1 in L_2, having plugged in the HQET parameters determined in L_1
  $\rightarrow$ CL exists $\Leftrightarrow$ $1/a$–divergence correctly cancelled !



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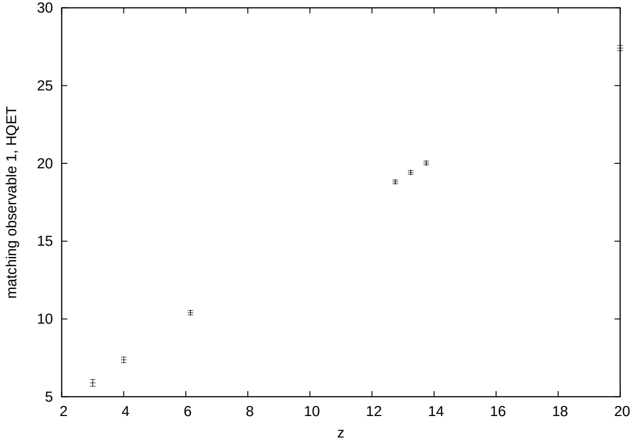
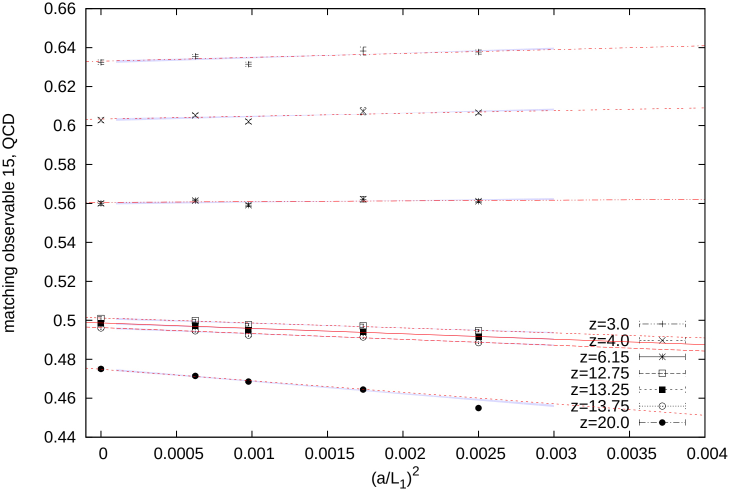


$\rightarrow$ Last missing step: Solve the full linear system in L_2 for $\omega(M, a)$ and interpolate them to β's used in large-volume simulations

- Combined continuum extrapolation of *QCD* observable #15, needed for fixing $c_{V_{k,1}}$ (strategy "So")



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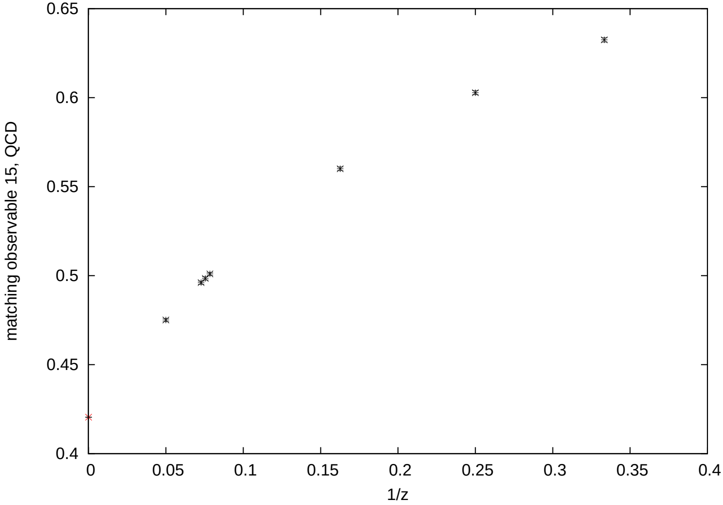
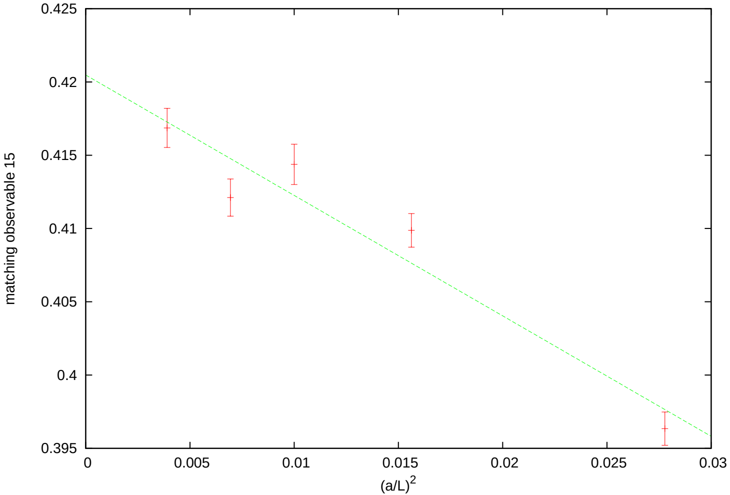


- Continuum extrapolation of the *static* part of *HQET* observable #15 in L_1 (linearly in $(a/L_1)^2$; employing "HYP2" data only)



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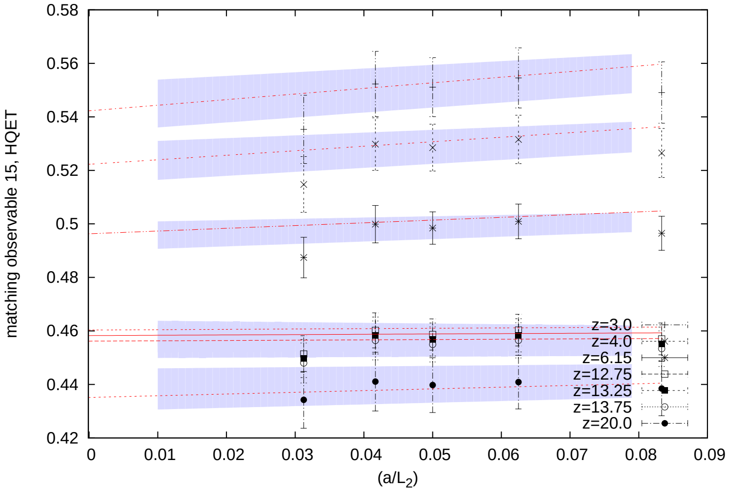

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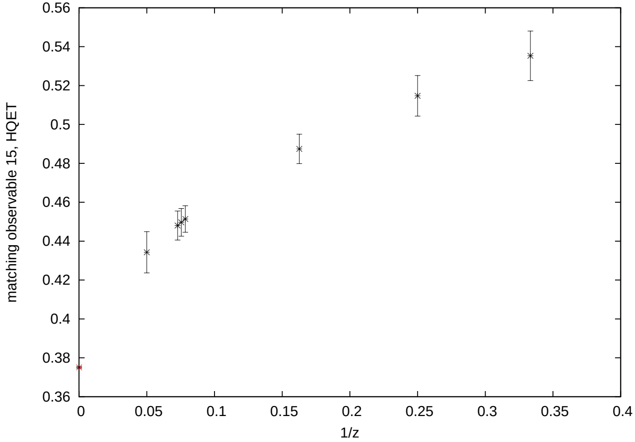

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Summary & Outlook

NP HQET at $O(1/m_h)$ works in practice $\rightarrow$ Status of B-physics:

- Determination of HQET parameters of action and A_0 for $N_f = 2$
- $N_f = 2$ Results for ...
  - ... b-quark mass & leptonic B-meson decay constants, ...
  - ... HQET form factor in $B_s \rightarrow K\ell\nu$ semi-leptonic decays: so far, leading-order (i.e., static) only, but *continuum limit* !

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    ($\rightarrow$ Aside: V_{ub} puzzle remains)
- NP matching of action & *all HQET heavy-light currents*
  - Strategy exists; tree-level & 1–loop investigations ✓
  - Decision on kinematical parameters & Simulations ✓
  - Final analysis to extract the HQET parameters in progress
    (large-volume simulations to obtain $1/m_h$–parts in $f_+(q^2)$ in parallel)