Thermodynamics with continuum extrapolated overlap fermions

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Outline

Introduction

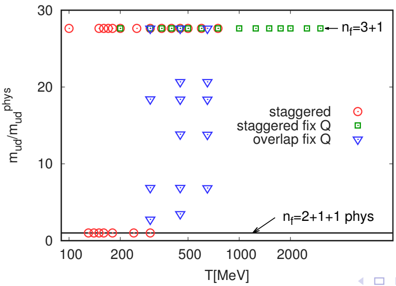
- Determine topological susceptibility at high T in the physical point using fixed Q integral
- Exact zero modes of the Dirac operator for $Q \neq 0$ are crucial $\rightarrow$ large discretization effects with staggered fermions
- Possible solutions
  1) eigenvalue reweighting
  2) using chiral fermions
- We use 1) for the 3 flavor theory and 2) for going down to the physical point

Fixed Q integral to reach the physical point
(see also previous talk & Frison et. al. 1606.07175)

At high temperatures $\chi(T) \sim T^{-b}$ only $Q = 0, 1$ contribute

$b = -\frac{d\log\chi}{d\log T} \simeq b_1 - 4$ with $b_1 = \frac{d\beta}{d\log a}\langle S_g\rangle_{1-0} + \sum_f \frac{d\log m_f}{d\log a} m_f\langle\overline{\psi}\psi_f\rangle_{1-0}$

- S_g: small cutoff effects, huge statistics $\rightarrow$ staggered $N_f = 3$
- $m_f\langle\overline{\psi}\psi_f\rangle_{1-0}$: large cutoff effects $\rightarrow$ staggered reweighting for $N_f = 3$, overlap for $N_f = 2 + 1$



Reweighting

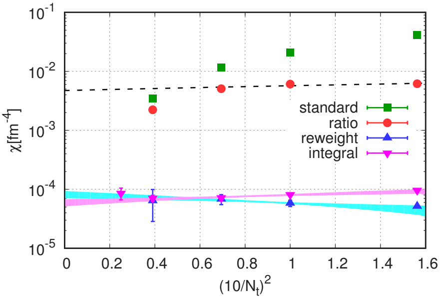
Strong cut-off effects are related to the lack of exact zero-modes.

- In the continuum non-trivial sectors are suppressed by the contribution of zero-modes to the fermion determinant, ie. by the quark mass.
- On the lattice the suppression is altered: $m \to m + i\lambda_0$, where λ_0 is a would be zero-mode. Weaker suppression $\to \chi(T)$ overestimated. For large temperatures $\lambda_0 \sim 1/N_t^2$, this explains the vanishing chiral condensate difference.
- To improve 1. identify would be zero-modes
  2. restore the continuum weight $\to$ reweight

$$w[U] \sim \frac{m}{m + i\lambda_0}$$

Reweighting - example

A direct comparison of methods @ T=300 MeV $m_{ud} = m_{ud}^{\mathrm{phys}}$.

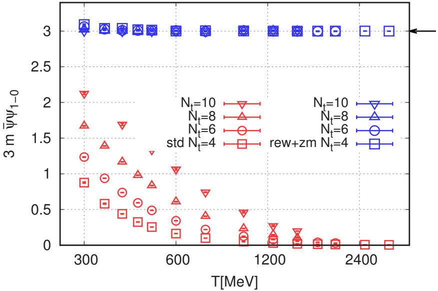


standard: huge lattice artefacts; **ratio:** $\chi(T)/\chi(T=0)$ apparent scaling is misleading; **reweight:** orders of magnitude smaller; **integral:** calculate @ $m_{ud} = m_s$ directly and integrate down in mass, consistent with reweighting

Fixed Q integral - chiral condensate

In the high temperature (SB) limit only the zero mode contributes

$$m_f\langle\overline{\psi}\psi_f\rangle_{1-0} = n_f.$$



For large T goes to 0 instead of approaching the SB limit, this is a lattice artefact. Non-chiral fermions fail spectacularly for large temperatures! Solutions: reweighting or chiral fermions.

Overlap lattice setup

Symanzik improved gauge action and $N_f = 2+1$ overlap fermions with 2 levels of HEX smearing

$$D = \left(m_0 - \frac{m}{2}\right)\left(1 + \gamma_5 \mathrm{sgn}\left(H_W\right)\right) + m,$$

with $m_0 = 1.3$

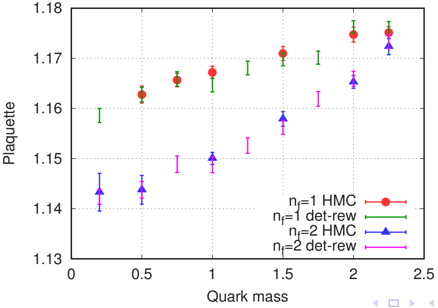
Extra Wilson fermions suppress topology changes (Fukaya et al. 2006):

$$S_E = \sum_x \left\{ \bar{\psi}_E(x) D_W(-m_0) \psi_E(x) + \phi^\dagger(x)[D_W(-m_0) + i m_B \gamma_5 \tau_3] \phi(x) \right\}.$$

since m_0 is not tuned along the continuum limit they decouple
ϕ is added to suppress the effect of the extra fermion $\rightarrow$ only modes below m_B are suppressed.

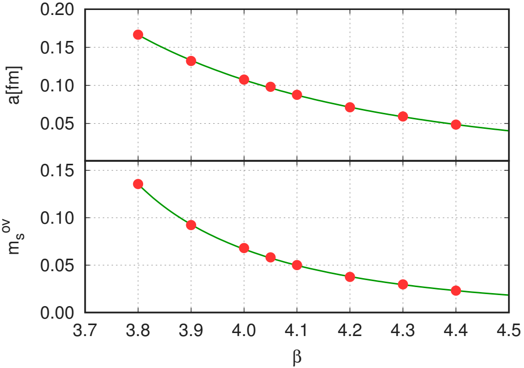
Odd number of flavors with overlap

- Use chiral decomposition (Bode et al. 1999, Degrand, Schaefer 2006)
- $H_\pm^2 = P_\pm H_{ov}^2 P_\pm$ with $P_\pm = (1 \pm \gamma_5)/2$ and $H_{ov}^2 = (\gamma_5 D_{ov})^2$
- $\det D_{ov} \sim \det H_\pm^2$ where the proportionality factor depends on Q
- if Q is fixed, using either of $H_\pm^2$ in a HMC results in one flavor
- test both 1 and 2 flavors against pure gauge + Metropolis



Line of constant physics (LCP)

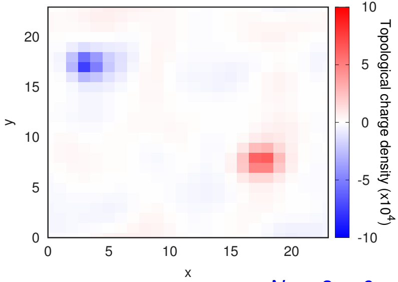
Determine the 3 flavor LCP ($m_{ud} = m_s = m_{s;\mathrm{phys}}$) for which the physical condition $m_\pi^{(3)} w_0^{(3)} = 0.552(1)$ was found by staggered runs.

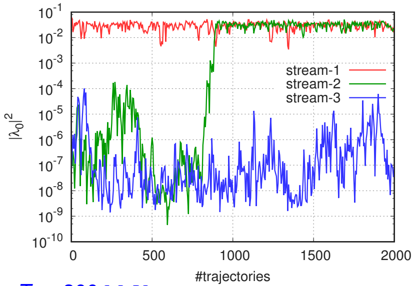


Physical LCP is given by $m_{ud}^{\mathrm{ov}} = m_s^{\mathrm{ov}}/27.68$

Difficulty with fixed Q simulations

Fixed global Q (e.g. $Q = 0$) permits instanton-antiinstanton pairs
These are difficult to produce/remove in HMC
The Dirac operator has a pair of small eigenvalues in some streams





$N_f = 3, \quad 6 \times 24^3, \quad T = 300$ MeV

expectation (supported by staggered runs): $\left(\frac{Z_1}{Z_0}\right)_{\text{full}} = \frac{Z_{10}}{Z_{00}}$
configurations (streams) with I-A pairs can be dropped

Condensate difference

For the mass integration we have to compute $m_{ud}\langle\overline{\psi}\psi_{ud}\rangle_{1-0}$.
In the $Q = 1$ sector the zero mode is treated exactly.

β	$N_s \times N_t$	m_{ud}	m_s	# ktraj	$\frac{1}{2} m_{ud}\langle\overline{\psi}\psi_{ud}\rangle_{1-0}$
m_{ud}-scan at $T = 300$ MeV					
3.99	12×6	0.0690	0.0690	10	1.00(1)
3.99	12×6	0.0460	0.0690	5	0.99(1)
3.99	12×6	0.0172	0.0690	8	1.00(1)
3.99	12×6	0.0069	0.0690	10	1.00(1)
m_{ud}-scan at $T = 450$ MeV					
4.19	12×6	0.0389	0.0389	10	1.00(1)
4.19	12×6	0.0291	0.0389	6	1.00(1)
4.19	12×6	0.0259	0.0389	3	1.00(1)
4.19	12×6	0.0195	0.0389	3	1.00(1)
4.19	12×6	0.0097	0.0389	3	1.00(1)
4.19	12×6	0.0049	0.0389	3	1.00(1)

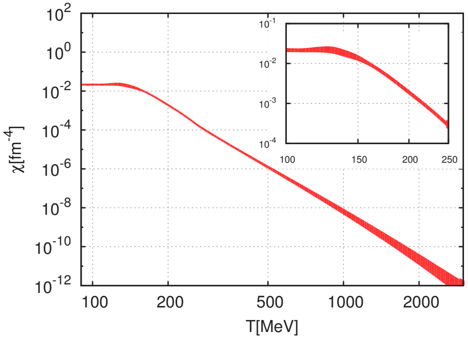
Condensate difference

m_{ud}-scan at $T = 650$ MeV					
4.38	12×6	0.0242	0.0242	5	1.00(1)
4.38	12×6	0.0181	0.0242	5	1.00(1)
4.38	12×6	0.0161	0.0242	3	1.00(1)
4.38	12×6	0.0121	0.0242	2	1.00(1)
4.38	12×6	0.0060	0.0242	2	1.00(1)
N_t-scan					
3.99	12×6	0.0690	0.0690	12	1.00(1)
4.13	16×8	0.0458	0.0458	29	1.02(2)
4.24	20×10	0.0342	0.0342	80	1.00(1)
N_s-scan					
3.99	12×6	0.0690	0.0690	12	1.00(1)
3.99	16×6	0.0690	0.0690	20	1.00(1)
3.99	20×6	0.0690	0.0690	32	1.02(1)
3.99	24×6	0.0690	0.0690	48	1.00(1)

Topological susceptibility at the physical point

at $T < T_c$: $\chi \sim \frac{m_u m_d}{m_u + m_d}$ while at $T > T_c$: $\chi \sim m_u m_d$

isospin splitting in both cases results in a factor of $\frac{4 m_u m_d}{(m_u + m_d)^2} \approx 0.88$



This results in an axion mass of $m_A = 50(4)\ \mu\mathrm{eV}$ (in post-inflation with same amount of topological defects as misalignment).