Effective Polyakov loop theories for QCD-like theories at finite baryon density

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Lattice 2016 Southampton

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Helmholtz Graduate School for Hadron and Ion Research

HIC for FAIR
Helmholtz International Center

Motivation

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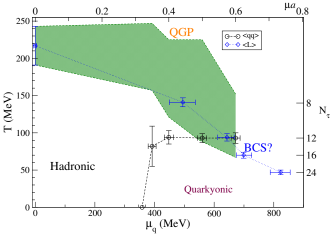
- Exploration of the QCD phase diagram at finite density, not possible with standard lattice techniques
- Use effective Polyakov loop theory that can be improved order by order
- Here: Use effective theory for QCD-like theories

  $\rightarrow$ qualitative and quantitative comparison with full theory at finite chemical potential



Boz, Cotter, Fister, Mehta, Skullerud

EPJ A 49, 11 (2013)

Effective Polyakov Loop Theory

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- $d - 1$ dim. SU(N) spin model

$$L_{\vec{x}} = \mathrm{Tr} \prod_{t=1}^{N_t} U_o(\vec{x}, t)$$

- Less computational cost, especially in the cold and dense part of the phase diagram
- Finite density $\longrightarrow$ Complex Langevin
- Can be derived from combined strong coupling and hopping expansion by integrating out spacial links

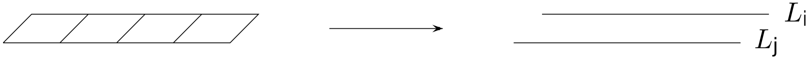
  Fromm, Langelage, Lottini, Neumann, Philipsen, PRL 110 (2013) 12

  Langelage, Neumann, Philipsen, JHEP 1409 (2014) 131

Effective Action

- Start with Wilson gauge action and Wilson quarks and intgrate out spatial links
- Use strong coupling and hopping expansion: Contributions to the effective action are graphs winding around the lattice in time direction

Gauge Action:



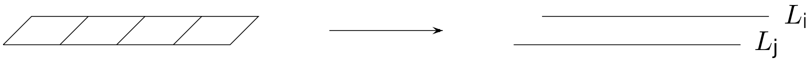
$$\Rightarrow S^g_{\mathrm{eff}} = u^{N_t} \sum_{\langle ij \rangle} L_i L_j + \dots$$

$$u = \frac{\beta}{4} + \mathcal{O}(\beta^2) < 1 \qquad \text{for SU(2)}$$

Effective Action

- Start with Wilson gauge action and Wilson quarks and intgrate out spatial links
- Use strong coupling and hopping expansion: Contributions to the effective action are graphs winding around the lattice in time direction

Gauge Action:



$$\Rightarrow S^g_{\text{eff}} = u^{N_t} \sum_{\langle ij \rangle} L_i L_j + \dots$$

Low T and strong coupling: $u^{N_t} < 10^{-12}$, omit gauge action

Heavy Fermions and Hopping Expansion

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$$\det[D] = \det[1 - \kappa H] = \exp(-\sum_{i=1}^{\infty} \frac{1}{i} \kappa^i \,\mathrm{Tr}[H^i])$$

Leading order:

$$-S_{\mathrm{eff}} = N_f \sum_{\vec{x}} \log(1 + h \mathrm{Tr} W_{\vec{x}} + h^2)^2$$

Fermion determinant in HDQCD

Heavy Fermions and Hopping Expansion

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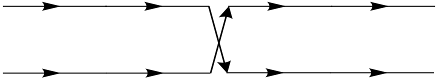
Order κ^2:



$$-S_{\text{eff}} = N_f \sum_{\vec{x}} \log(1 + h\text{Tr}\, W_{\vec{x}} + h^2)^2 - 2N_f h_2 \sum_{\vec{x},i} \text{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}}\text{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}}$$

Heavy Fermions and Hopping Expansion

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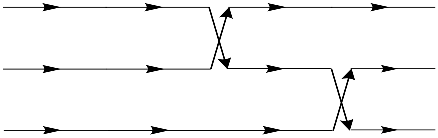
Order κ^4:



Heavy Fermions and Hopping Expansion

$$-S_{\text{eff}} = N_f \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\, W_{\vec{x}} + h^2)^2 - 2N_f h_2 \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}}$$

$$+ 2N_f^2 \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{(1 + hW_{\vec{x}+i})^2}$$

$$+ N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}-j}}{1 + hW_{\vec{x}-j}}$$

$$+ 2N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}}$$

$$+ N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr}\frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr}\frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \mathrm{Tr}\frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}}$$

Heavy Fermions and Hopping Expansion

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$$
\begin{aligned}
-S_{\text{eff}} = {} & N_f \sum_{\vec{x}} \log(1 + h\mathrm{Tr}\, W_{\vec{x}} + h^2)^2 - 2N_f h_2 \sum_{\vec{x},i} \mathrm{Tr} \frac{hW_{\vec{x}}}{1 + hW_{\vec{x}}} \mathrm{Tr} \frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \\
& + 2N_f^2 \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i} \mathrm{Tr} \frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr} \frac{hW_{\vec{x}+i}}{(1 + hW_{\vec{x}+i})^2} \\
& + N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr} \frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr} \frac{hW_{\vec{x}-i}}{1 + hW_{\vec{x}-i}} \mathrm{Tr} \frac{hW_{\vec{x}-j}}{1 + hW_{\vec{x}-j}} \\
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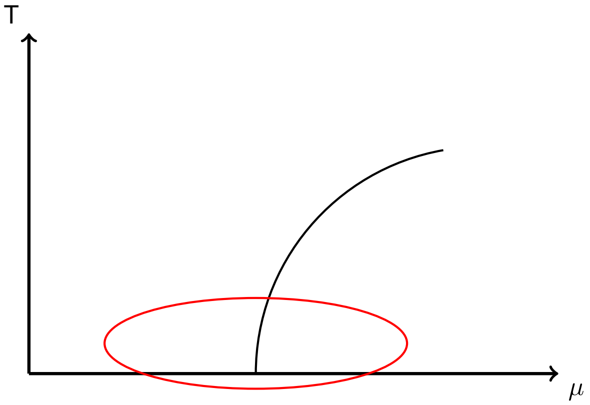
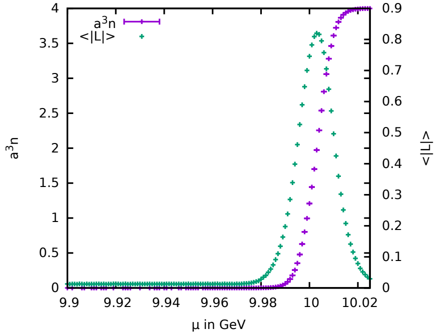
& + N_f \frac{\kappa^4 N_\tau^2}{N_c^2} \sum_{\vec{x},i,j} \mathrm{Tr} \frac{hW_{\vec{x}}}{(1 + hW_{\vec{x}})^2} \mathrm{Tr} \frac{hW_{\vec{x}+i}}{1 + hW_{\vec{x}+i}} \mathrm{Tr} \frac{hW_{\vec{x}+j}}{1 + hW_{\vec{x}+j}} \\
& + N_f(2N_f - 1)\kappa^4 N_\tau^2 \sum_{x,i} \frac{h^2}{(1 + h\mathrm{Tr}\, W_{\vec{x}} + h^2)(1 + h\mathrm{Tr}\, W_{\vec{x}+i} + h^2)} \, .
\end{aligned}
$$

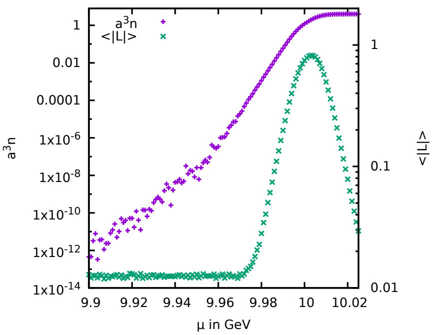
Cold and Dense QC_2D with Heavy Quarks



Cold and Dense QC_2D with Heavy Quarks

- Very heavy quarks: m_q = 10.0014 GeV
- Diquark mass is fixed to m_d = 20 GeV → small binding energy
- Unphysical lattice saturation





Cold and Dense QC_2D with Heavy Quarks

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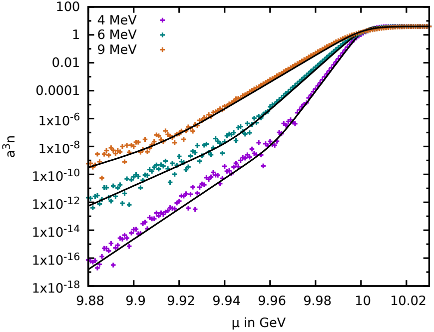
- Behaviour of $a^3 n$ is well described by a LO mean field model

$$a^3 n = 4N_f \frac{1 + Le^{\frac{m_q - \mu}{T}}}{1 + 2Le^{\frac{m_q - \mu}{T}} + e^{2\frac{m_q - \mu}{T}}}$$

- No Fit! All values taken from Simulations or are input
- $\langle L \rangle \approx 5 \cdot 10^{-5} < \langle |L| \rangle = 0.006$

  $\langle L \rangle$ determined by histograms



PS, von Smekal PRD 92 094504 (2015)

Cold and Dense QC_2D with Heavy Quarks

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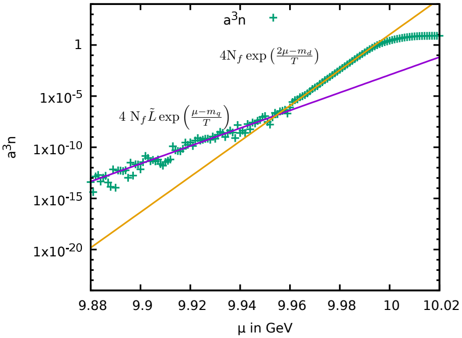
- Most precise description of the second exponential by

$$a^3 n = 4N_f \exp\left(\frac{2\mu - m_d}{T}\right)$$

$$m_d = 19.9986(10)\ \text{GeV}$$

- Bound State
- Model includes Confinement



PS, von Smekal PRD 92 094504 (2015)

Phase Diagram

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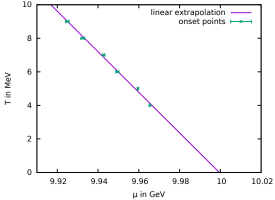
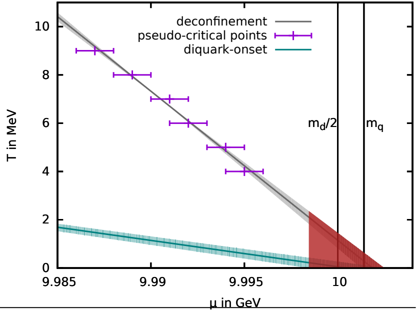
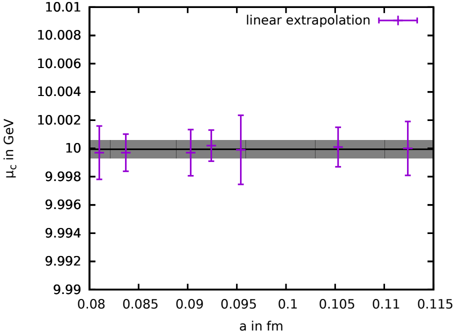
- Diquark onset line is not a phase-boundry!
- Line terminates at $\mu = \frac{m_d}{2}$ according to Silverblaze property
- Deconfinement transiton terminates at $\mu > \frac{m_d}{2}$
- Look for BEC at larger κ, where diquarks are bound more tightly





Extrapolation to $T = 0$

- $T = 0$ endpoint is in perfect agreement with $\mu = \frac{m_d}{2}$
- Endpoint is independent of lattice spacing



PS, von Smekal PRD 92 094504 (2015)

G2 QCD

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- Smallest exceptional Lie Group
- All representation are real (β = 4)
- No sign problem: real and positive
- YM theory has 1st order phase transition
- Spectrum: bosonic and fermionic baryons

Phase diagram

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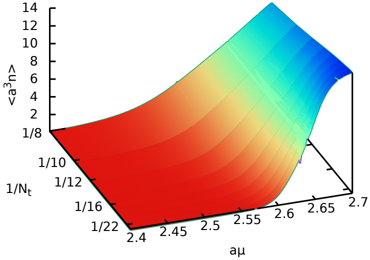
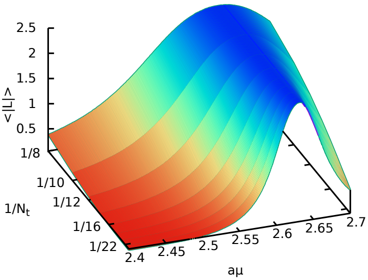
Density



Polyakov loop



- Cold and dens region of the G2 effective theory phase diagram
- $\beta/N_c = 1.4$, $\kappa = 0.0357$, $N_t = 8, ...24$

Cold and Dens Region

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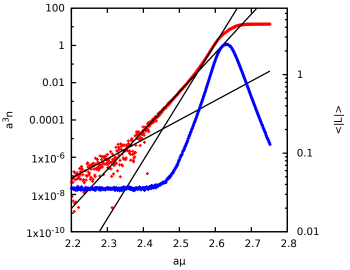
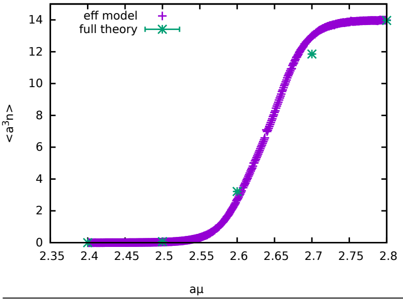
- Deconfinement and unphysical lattice saturation
- three regions with different exponential growth





- Comparison of full 4d G2 QCD simulations to effective theory

Effective Polyakov loop theory for 2 dim G2 QCD

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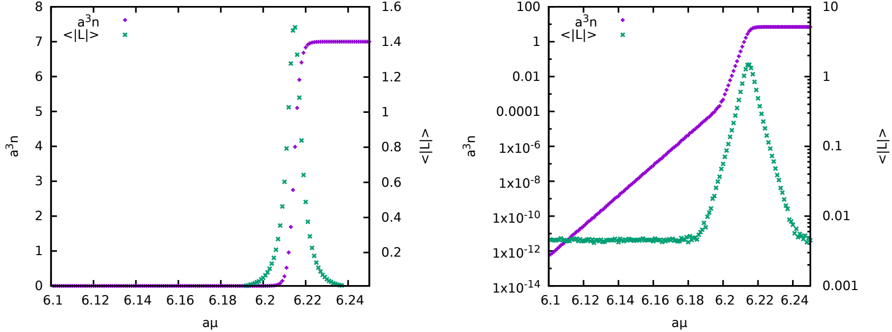
- Less computational cost for full theory
- No gauge invariant diquarks



$\beta/N_c = 1.39$, $\kappa = 0.001$, $N_t = 200$ and $N_s = 32768$

Summary & Conclusion

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Effective Polyakov loop theory for QC_2D

- Thermal diquark excitation onset at $T > 0$
- Extrapolation $T \to 0$ gives $m_d/2$ according to Silverblaze Property
- However no signal for BEC: $T > E_{\mathrm{Bind.}}$

Effective Polyalov loop theory for G2

- Two- and three-quark excitations in agreement with possible spectrum
- Reasonable Agreement between full theory and effective model
- First steps toward comparison of 2d G2 QCD simulations with effective theory

Analytic relations for Cold and Dense regime

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$$h = \exp\left[N_\tau \left(a\mu + \ln 2\kappa + 6\kappa^2 \frac{u - u^{N_\tau}}{1-u}\right)\right] \ ,$$

$$am_d = \mathrm{ArCosh}\left(1 + \frac{((\frac{1}{2\kappa})^2 - 4)((\frac{1}{2\kappa})^2 - 1)}{(2(\frac{1}{2\kappa})^2 - 3)}\right) - 24\kappa^2 \frac{u}{1-u} + \mathcal{O}(\kappa^4 u^2, \kappa^2 u^5) \ .$$