Adjoint SU(2) with four fermion interactions

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Four fermion interactions

In Technicolor

- A standard way of explaining SM fermion masses
- Potential effects on TC dynamics
- Deform a conformal model into a walking one

NJL-models

- Preserve (restrict) chiral symmetry

- Introduction
- First map without the gauge [1]
- Preliminary studies with gauged model

[1] In preparation

The Nambu Jona-Lasinio Model

$$L = \bar{\Psi}\partial\!\!\!/\Psi + \gamma^2\left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi}i\gamma_5\tau_a\Psi\bar{\Psi}i\gamma_5\tau_a\Psi\right)$$

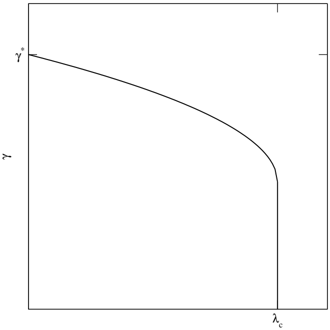
- Preserves a $SU_L(2) \times SU_R(2)$ chiral symmetry ($N_F = 2$)
- Spontaneous symmetry breaking at [2]

$$\gamma > \gamma^* \sim \sqrt{\frac{2\pi^2}{N\Lambda^2}}$$

[2] Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345

Gauged NJL

$$L = F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}\not{D}\Psi + \gamma^2\left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi}i\gamma_5\tau_a\Psi\bar{\Psi}i\gamma_5\tau_a\Psi\right)$$



Gauge coupling τ

Chiral symmetry breaking line at [3]

$$\gamma_c = \frac{1}{2}\left(1 + \sqrt{1 - \frac{\lambda}{\lambda_c}}\right)\gamma^*$$

Growing mass anomalous dimension [4]

[3] K. Yamawaki, hep-ph/9603293

[4] H. S. Fukano and F. Sannino, Phys. Rev. D **82** (2010) 035021

The Lattice Model

- Non-renormalizable
  No continuum limit
- Using auxiliary fields
- Sign problem with full symmetry, reduce to $U_L(1) \times U_R(1)$

$$L = \bar{\Psi} \not{D} \Psi + \gamma^2 \left(\bar{\Psi}\Psi\bar{\Psi}\Psi + \bar{\Psi} i\gamma_5\tau_3 \Psi \bar{\Psi} i\gamma_5\tau_3 \Psi \right)$$

$$\rightarrow L = \bar{\Psi} \not{D} \Psi + \sigma \bar{\Psi}\Psi + \pi_3 \bar{\Psi} i\gamma_5\tau_3 \Psi + \frac{\sigma^2 + \pi_3^2}{4\gamma^2}$$

$$\langle \sigma(x) \rangle = 2\gamma^2 \left\langle \bar{\Psi}(x)\Psi(x) \right\rangle , \quad \langle \pi_3(x) \rangle = 2\gamma^2 \left\langle \bar{\Psi}(x) i\gamma_5\tau_3 \Psi(x) \right\rangle$$

The Lattice Model

- Simple measurables

$$\langle \sigma \rangle = \frac{1}{V} \left\langle \sum_x \sigma(x) \right\rangle = \frac{2\gamma^2}{V} \left\langle \sum_x \bar{\Psi}(x)\Psi(x) \right\rangle$$

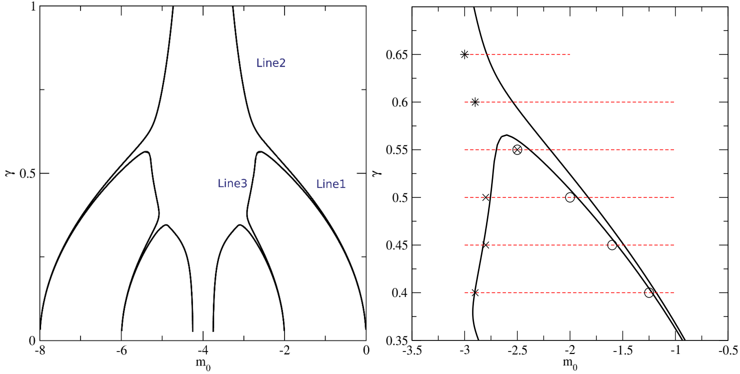
$$\langle \pi \rangle = \frac{1}{V} \left\langle \sum_x \pi_3(x) \right\rangle = \frac{2\gamma^2}{V} \left\langle \sum_x \bar{\Psi}(x) i\gamma_5\tau_3\Psi(x) \right\rangle$$

Phase diagram (meanfield[5])

- Second order transition to $\langle \pi \rangle \neq 0$

[5] K. M. Bitar and P. M. Vranas, Phys. Rev. D **50** (1994) 3406
S. Aoki, S. Boettcher and A. Gocksch, Phys. Lett. B **331** (1994) 157

Phase plot



The Lattice Model

Only one Goldstone boson, $SU(4)$ quintuplet broken

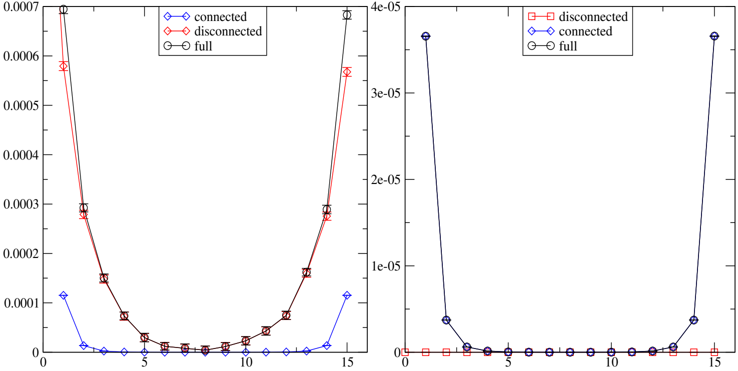
- Four degenerate nondiagonal states
- One diagonal state with a disconnected part

$$
\begin{aligned}
C^{1,2}_{ND,\Gamma}(t) &= -\left\langle \mathrm{Tr}\left[\left(S(t,0)\Gamma\tau_{1,2}\right)^{\dagger} S(0,t)\Gamma\tau_{1,2}\right]\right\rangle \\
C_{D,\Gamma}(t) &= C^{3}_{ND,\Gamma}(t) + \left\langle \mathrm{Tr}\left[S(0,0)\Gamma\tau_3\right]^{\dagger} \mathrm{Tr}\left[S(t,t)\Gamma\tau_3\right]\right\rangle
\end{aligned}
$$

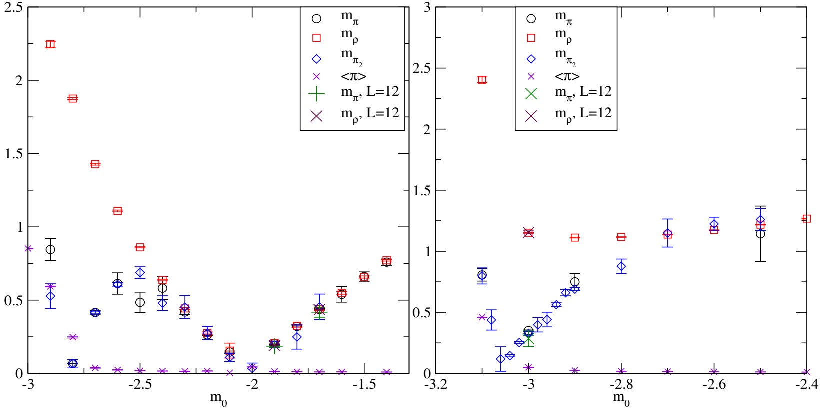
$$
\mathrm{Tr}\,\left(S_u - S_d\right)_{x,y}\Gamma = -\mathrm{Tr}\,\frac{\delta_{x,y} 2 i \pi_3(x)\gamma_5}{M_u M_d}\Gamma
$$

The Goldstone boson is the diagonal pseudoscalar

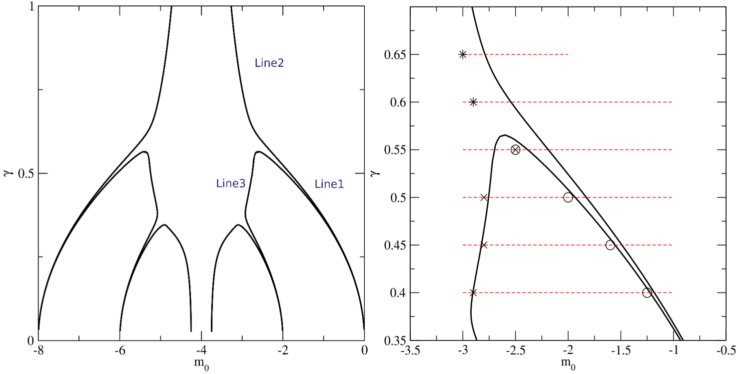
$$\gamma = 0.65a, \quad L = 8^3 \times 16, \quad m_0 = 2.9$$



$$\gamma = 0.5a, 0.65a \quad L = 8^3 \times 16$$



Phase plot



1
0.5
0
γ
-8
-6
-4
-2
0
m_0
Line2
Line3
Line1
0.65
0.6
0.55
0.5
0.45
0.4
0.35
γ
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
m_0

Turning on the gauge

SU(2) adjoint, $\beta = 2.25$

Expectation

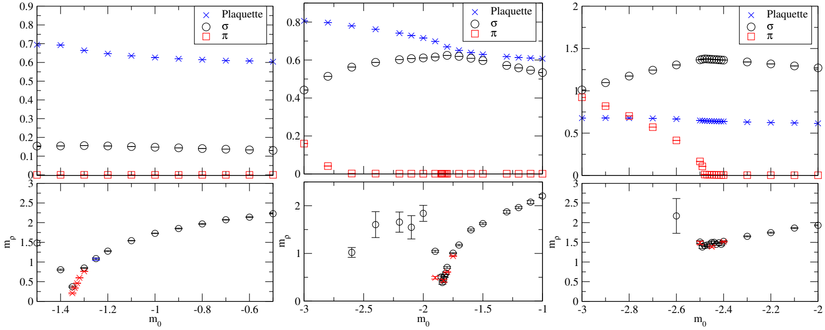
- Smaller critical coupling γ_c
- γ dependent mass anomalous dimension

A more complicated phase diagram

- Broken Polyakov loop at small volume
- A bulk transition?

Turning on the gauge

$$\gamma = 0.1a, 0.2a, 0.3a$$



Anomalous dimension

Does mass anomalous dimension increase?

- Two couplings below γ_c
- At $\gamma = 0$, $\gamma_m \sim 0.3 - 0.4$[6]

Conformal hyperscaling:

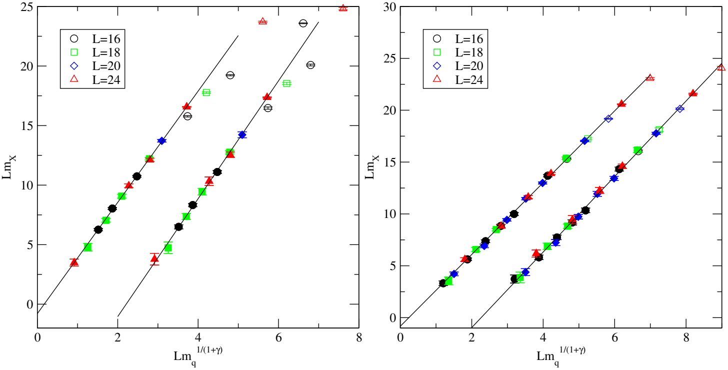
$$Lm_X = f(x) = a_X x + c_X$$
$$x = |m_0 - m_c|^{\frac{1}{1+\gamma_m}} .$$

Using nondiagonal vector and pseudoscalar mesons

[6] J. Rantaharju, Phys. Rev. D **93**, no. 9, 094516 (2016)
A. Patella, Phys. Rev. D **86**, 025006 (2012)
T. DeGrand, Y. Shamir and B. Svetitsky, Phys. Rev. D **83**, 074507 (2011)

Anomalous dimension

$\gamma = 0.1a: \ \gamma_m = 0.4 - 0.6 \qquad \gamma = 0.2a: \ \gamma_m = 0.6 - 0.9$



Four fermion interactions

- Needed for fermion masses
- Can affect dynamics

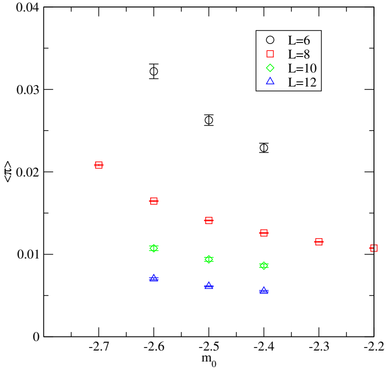
Ungauged NJL

- Phase structure as expected
- Spontaneous chiral symmetry breaking
- Disconnected diagrams

SU(2) adjoint

- Same phases exist
- Smaller critical γ
- Anomalous dimension varies with γ

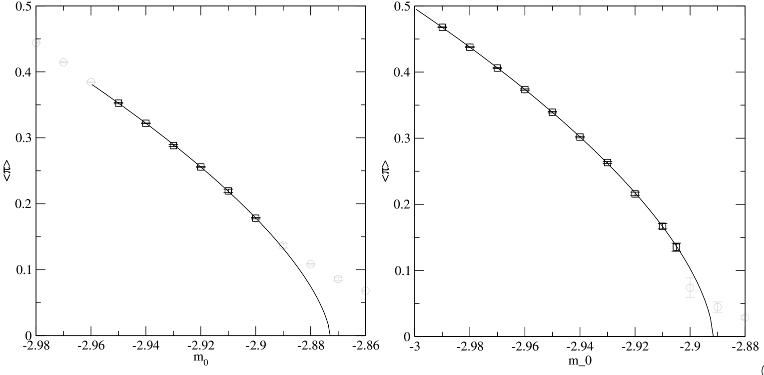
Zero mass transition



Zero mass line

$$\langle|\pi|\rangle = C_\pi \, |m_0 - m_c|_\pi^\beta \,,$$

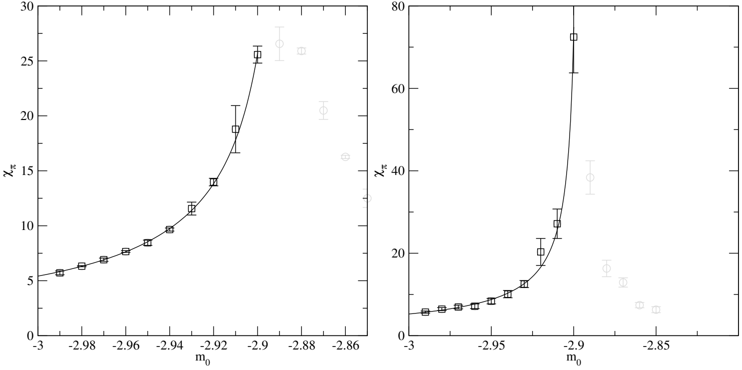
$$L = 8, 12, \qquad \beta_\pi = 0.65(2), 0.56(2)$$



Zero mass line

$$\chi_\pi = C_\chi \left| m_0 - m_c \right|^{-\nu_\pi}$$

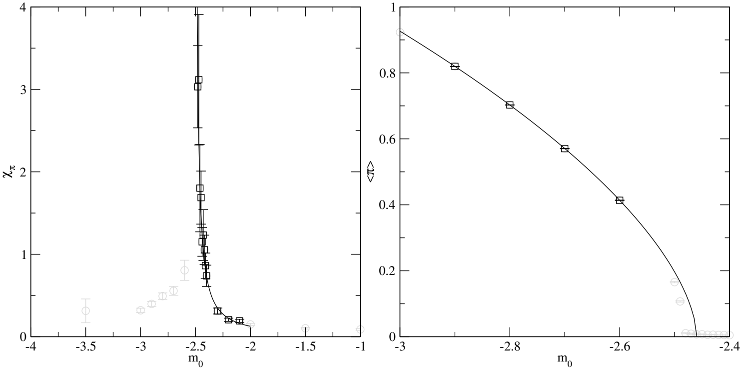
$$L = 8, 12, \qquad \nu_\pi = 0.83(5)(2), 0.9(3)$$



Zero mass line

Gauged, $\beta = 2.25$

$$\beta_\pi = 0.598(4), \nu_\pi = 1.1(2)$$



Chiral symmetry breaking transition