Check of a new non-perturbative mechanism for elementary fermion mass generation

S. Capitani[a)], P. Dimopoulos[b)], G.M. de Divitiis[c)], R. Frezzotti[c)], M. Garofalo[d)], B. Knippschild[e)], B. Kostrzewa[e)], K. Ottnad[e)], G.C. Rossi[b)c)], M. Schröck[f)], C. Urbach[e)]

[a)] University of Frankfurt
[b)] Centro Fermi, Museo Storico della Fisica e Centro Studie Ricerche "Enrico Fermi"
[c)] University of Rome Tor Vergata, Physics Department and INFN - Sezione di Roma Tor Vergata
[d)] Higgs Centre for Theoretical Physics, The University of Edinburgh
[e)] HISKP (Theory), Universitaet Bonn
[f)] INFN, Sezione di "*Roma Tre*"

- In [Frezzotti and Rossi Phys. Rev. D92 (2015) 054505] a new non-perturbative mechanism for the elementary particle mass generation was conjectured
- We are testing this conjecture in a toy model where a fermion doublet Q is coupled to a non-Abelian SU(3) gauge field and a scalar Φ

$$\mathcal{L}_{\rm toy}(Q,A,\Phi) = \mathcal{L}_{kin}(Q,A,\Phi) + \mathcal{V}(\Phi) + \mathcal{L}_{Wil}(Q,A,\Phi) + \mathcal{L}_{Yuk}(Q,\Phi)$$

- $\mathcal{L}_{kin}(Q,A,\Phi) = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \overline{Q}_L\gamma_\mu\mathcal{D}_\mu Q_L + \overline{Q}_R\gamma_\mu\mathcal{D}_\mu Q_R + \frac{1}{2}\mathrm{Tr}\left[\partial_\mu\Phi^\dagger\partial_\mu\Phi\right]$
- $\mathcal{V}(\Phi) = \frac{\mu_0^2}{2}\mathrm{Tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2, \qquad \Phi \equiv [\varphi| - i\tau^2\varphi^*]$
- $\mathcal{L}_{Wil}(Q,A,\Phi) = \frac{b^2}{2}\rho\left(\overline{Q}_L\overleftarrow{\mathcal{D}}_\mu\Phi\mathcal{D}_\mu Q_R + \overline{Q}_R\overleftarrow{\mathcal{D}}_\mu\Phi^\dagger\mathcal{D}_\mu Q_L\right)$ "Wilson-like"
- $\mathcal{L}_{Yuk}(Q,\Phi) = \eta\left(\overline{Q}_L\Phi Q_R + \overline{Q}_R\Phi^\dagger Q_L\right)$ "Yukawa"

UV cutoff $\sim b^{-1}$ • Fermion chiral symmetry $\tilde{\chi}$ broken if $(\rho,\eta)\neq(0,0)$

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Symmetries & power counting $\begin{pmatrix}\text{in suitable}\\ \text{UV-regul.}\end{pmatrix} \Longrightarrow$ Renormalizability

Invariant under χ (global) $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ transformations

- $\chi_{L,R}: \quad \tilde\chi_{L,R}\otimes(\Phi\to\Omega_{L,R}\Phi)\otimes(\Phi^\dagger\to\Phi^\dagger\Omega^\dagger_{L,R})$

$$\tilde\chi_{L,R}: \begin{cases} Q_{L,R}\to\Omega_{L,R}Q_{L,R} \\ \overline{Q}_{L,R}\to\overline{Q}_{L,R}\Omega^\dagger_{L,R}\end{cases} \qquad \Omega_{L,R}\in\mathrm{SU}(2)_{L,R}$$

- Lagrangian not invariant under purely fermionic transformations

$$\tilde{\chi}_L : Q_L \to \Omega_L Q_L \qquad \overline{Q}_L \to \overline{Q}_L \Omega_L^\dagger \qquad \Omega_L \in SU(2)$$

- They yield the bare WTIs

$$\partial_\mu \langle \tilde{J}_\mu^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle = \langle \tilde{\Delta}_L^i \hat{\mathcal{O}}(0)\rangle \delta(x) - \eta\,\langle O_{Yuk}^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle - b^2 \langle O_{Wil}^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle$$

$$\tilde{J}_\mu^{L,i} = \overline{Q}_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{b^2}{2}\rho\Big(\overline{Q}_L \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} Q_L\Big)$$

$$O_{Yuk}^{L,i} = \Big[\overline{Q}_L \frac{\tau^i}{2}\Phi Q_R - \mathrm{hc}\Big] \qquad O_{Wil}^{L,i} = \frac{\rho}{2}\Big[\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \mathrm{hc}\Big]$$

- Mixing of $O_{Wil}^{L,i}$ under renormalization

$$b^2 O_{Wil}^{L,i} = (Z_{\tilde{J}} - 1)\partial_\mu \tilde{J}_\mu^{L,i} - \overline{\eta}(\eta; g_s^2, \rho, \lambda_0)\, O_{Yuk}^{L,i} + \underline{\ldots} + \mathrm{O}(b^2)$$

- Renormalized WTIs read

$$\partial_\mu \langle Z_{\tilde{J}} \tilde{J}_\mu^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle = \langle \tilde{\Delta}_L^i \hat{\mathcal{O}}(0)\rangle \delta(x) - (\eta - \overline{\eta}(\eta))\,\langle O_{Yuk}^{L,i}(x)\,\hat{\mathcal{O}}(0)\rangle + \underline{\ldots} + \mathrm{O}(b^2)$$

where the ellipses ($\underline{\ldots}$) stand for possible NP mixing contributions

- At the critical $\eta_{cr}(g_s^2, \rho, \lambda_0)$ s.t. $\eta_{cr} - \overline{\eta}(\eta_{cr}; g_s^2, \rho, \lambda_0) = 0$ the WTIs become

$$\partial_\mu \langle Z_{\tilde{J}} \tilde{J}_\mu^{L,i}(x)\, \hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} = \langle \tilde{\Delta}_L^i \hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} \delta(x) + \underline{\ldots} + \mathrm{O}(b^2)$$

- In Wigner phase $\big(\langle\Phi\rangle = 0\big)$→Wilson-like term uneffective for $\tilde{\chi}^{SSB}$

$$\partial_\mu \langle Z_{\tilde{J}} \tilde{J}_\mu^{L,i}(x)\, \hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} = \langle \tilde{\Delta}_L^i \hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} \delta(x) + \mathrm{O}(b^2)$$

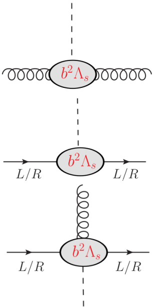
- In Nambu-Goldstone $\big(\langle\Phi\rangle = v 1\!\!1_{2\times 2}\big)$ expect (conjecture)

$$\partial_\mu \langle Z_{\tilde{J}} \tilde{J}_\mu^{L,i}(x)\, \hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} = \langle \tilde{\Delta}_L^i \hat{\mathcal{O}}(0)\rangle_{\eta_{cr}} \delta(x) + \langle C_1 \Lambda_s [\overline{Q}_L \frac{\tau^i}{2} \mathcal{U} Q_R + \mathrm{hc}] \hat{\mathcal{O}}(0)\rangle + \mathrm{O}(b^2)$$

The term $\propto C_1 \Lambda_s$ can exist only in the NG phase where

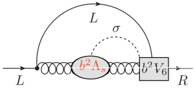
$$\mathcal{U} = \Phi/\sqrt{\Phi^\dagger \Phi} = (v + \sigma + i \vec{\tau}\,\vec{\pi})/\sqrt{(v+\sigma)^2 + \vec{\pi}\,\vec{\pi}}$$

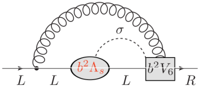
In Γ_{loc}^{NG} a mass term $C_1 \Lambda_s [\overline{Q}_L \mathcal{U} Q_R + \mathrm{hc}]$ $\begin{cases} \text{Natural} \\ \neq \text{Yukawa mass} \\ C_1 = O(\alpha_s^2)\ \text{Hierarchy} \end{cases}$

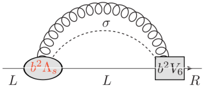


- Intuitive idea of the NP mass generation mechanism

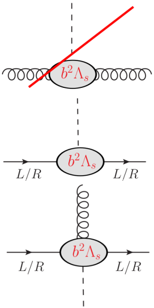
$O(b^2)$ NP corrections to ($\tilde{\chi}$-preserving) effective vertices combined in loop "diagrams" with $O(b^2)$ ($\tilde{\chi}$-breaking) vertices from the Wilson-like term







- b^{-4} loop divergency $\Longrightarrow$ $O(b^0)C_1\Lambda_s$ mass term



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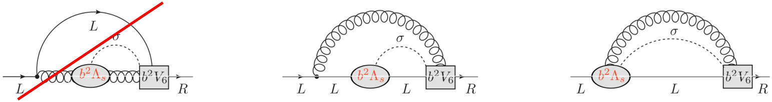


- b^{-4} loop divergency $\implies$ $O(b^0)C_1\Lambda_s$ mass term
- Phenomenon still occurring in quenched approximation

Chose a cheap lattice regularization of $\int d^4x \mathcal{L}_{toy}$

- First NP study of a theory with gauge, $\Psi = \begin{pmatrix} u \\ d \end{pmatrix}$ & $(\varphi_0, \vec{\varphi})$
- "Naive" fermions (good for quenched approximation only)

$\mathcal{S}_{lat} = b^4 \sum_x \left\{ \mathcal{L}_{kin}^{YM}[U] + \mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) + \overline{\Psi} D_{lat}[U,\Phi]\Psi \right\}$

$\mathcal{L}_{kin}^{YM}[U]$: SU(3) plaquette action

$\mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) = \frac{1}{2}\,\mathrm{tr}\,[\Phi^\dagger(-\partial_\mu^*\partial_\mu)\Phi] + \frac{\mu_0^2}{2}\,\mathrm{tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{tr}\left[\Phi^\dagger\Phi\right]\right)^2$

where $\Phi = \varphi_0 1\!\!1 + i\varphi_j\tau^j \quad F(x) \equiv [\varphi_0 1\!\!1 + i\gamma_5\tau^j\varphi_j](x)$

$(D_{lat}[U,\Phi]\Psi)(x) = \gamma_\mu\widetilde{\nabla}_\mu\Psi(x) + \eta F(x)\Psi(x) - b^2\rho\frac{1}{2}F(x)\widetilde{\nabla}_\mu\widetilde{\nabla}_\mu\Psi(x) +$

$- b^2\rho\frac{1}{4}\left[(\partial_\mu F)(x)U_\mu(x)\widetilde{\nabla}_\mu\Psi(x+\hat{\mu}) + (\partial_\mu^* F)(x)U_\mu^\dagger(x-\hat{\mu})\widetilde{\nabla}_\mu\Psi(x-\hat{\mu})\right]$

- Yukawa ($d = 4$) term $\propto \eta$, Wilson-like ($d = 6$) term $\propto \rho$
- Unquenched studies will require DW or Overlap fermions

Quenched model with 2 flavours $\begin{pmatrix} u \\ d \end{pmatrix} \times 16$ doublers

- $\chi_L \otimes \chi_R$ classical symmetry in Ψ basis: $\Psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\Psi$

$$\chi_L : \underbrace{\Psi_L \to \Omega_L \Psi_L \quad \overline{\Psi}_L \to \overline{\Psi}_L \Omega_L^\dagger}_{\widetilde{\chi}_{L,R}} \quad \Phi \to \Omega_L \Phi \quad \Omega_L \in SU(2)$$

$$\chi_R : \overbrace{\Psi_R \to \Omega_R \Psi_R \quad \overline{\Psi}_R \to \overline{\Psi}_R \Omega_R^\dagger} \quad \Phi \to \Phi \Omega_R^\dagger \quad \Omega_R \in SU(2)$$

- $\widetilde{\chi}_L \otimes \widetilde{\chi}_R$ classical symmetry at $\eta = 0$, any ρ, all doublers (at classical level the Wilson-like term irrelevant)
- Doubling symmetry group, D_ξ, $\xi = (\xi_1, \xi_2, \xi_3, \xi_4), \xi_i = 0, 1$

$$D_\xi : \Psi(x) \to (-1)^{x \cdot \xi} M_\xi \Psi(x) \quad \overline{\Psi}(x) \to \overline{\Psi}(x) M_\xi^\dagger (-1)^{x \cdot \xi}$$
$$\varphi_0 \to \varphi_0 \,, \varphi_i(x) \to (-1)^{\sum_\mu \xi_\mu} \varphi_i(x) \quad M_\xi = (i\gamma_5\gamma_1)^{\xi_1} \ldots (i\gamma_5\gamma_4)^{\xi_4}$$

- Doublers at $p_\mu = \frac{\pi}{b}\xi_\mu$

- Only symmetric derivatives $\widetilde{\nabla}$ on fermions in the lattice action

$\Longrightarrow$ Doubling Symmetry

$\Longrightarrow$ Power counting arguments work (even in the presence of doublers)

- Remark: this would not be the case if there were forward/backward ∇/∇^* lattice derivatives acting on fermions

$\Longrightarrow$ no Doubling Symmetry

- The renormalization of the toy model on the lattice is analogous to the one of the continuum model [Frezzotti and Rossi, Phys. Rev. D92 (2015) 054505], as it is based on power counting and symmetries

- Symmetries of the lattice toy model
  - SU(3) Gauge
  - $\chi_L \otimes \chi_R$ (previously defined)
  - P, T, C, CPF_2

$$P : \begin{cases} \Phi(x) \to \Phi^\dagger(x_P)\,, & x_P \equiv (-\vec{x}, x_4) \\ \Psi(x) \to \gamma_4 \Psi(x_P)\,, & \bar{\Psi}(x) \to \bar{\Psi}(x_P)\gamma_4 \\ U_4(x) \to U_4(x_p)\,, & U_k(x) \to U_k^\dagger(x_p - \hat{k}) \end{cases}$$

$$C : \begin{cases} \Phi(x) \to \Phi^T(x) \\ \Psi(x) \to i\gamma_4\gamma_2 \bar{\Psi}^T(x)\,, & \bar{\Psi}(x) \to -\Psi^T(x) i\gamma_4\gamma_2 \\ U_\mu(x) \to U_\mu^*(x) \end{cases}$$

$$T : \begin{cases} \Phi(x) \to \Phi^\dagger(x_T)\,, & x_T \equiv (\vec{x}, -x_4) \\ \Psi(x) \to \gamma_5\gamma_4 \bar{\Psi}^T(x)\,, & \bar{\Psi}(x) \to \Psi^T(x)\gamma_4\gamma_5 \\ U_4(x) \to U_4^\dagger(x_T - \hat{0})\,, & U_k(x) \to U_k(x_T) \end{cases}$$

$$F_2 : \begin{cases} \Phi(x) \to \tau^2 \Phi(x) \tau^2 \\ \Psi(x) \to i\tau^2 \Psi(x)\,, & \bar{\Psi}(x) \to -i\bar{\Psi}(x)\tau^2 \end{cases}$$

- The only $d = 4$ allowed operator, besides kinetic terms for gauge, fermions and scalars and scalar potential, is the Yukawa term $\bar{\eta}\bar{\Psi}\Phi\Psi$
- $\bar{\eta}(\eta_{cr}; g_s^2, \rho, \lambda_0) - \eta_{cr} = 0$ defines η_{cr}
- unique η_{cr}
- η_{cr} independent of scalar renormalized mass μ_r^2 (up to cutoff effects) $\Longrightarrow$ the same for Wigner and NG phase

- In view of the uniqueness of η_{cr} consider the correlators

$$\partial_\mu \langle \widetilde{J}^{L,i}_\mu(x) \mathcal{O}^i(z) \rangle = \langle (B^{L,i}_{Yuk} + B^{L,i}_{Wil})(x) \mathcal{O}^i(z) \rangle \;\; x \neq z$$

$B^{L,i}_{Yuk}, B^{L,i}_{Wil}$ variation of Yukawa- and Wilson-terms under $\tilde{\chi}_L$

- Look for the value of η ($\equiv \eta_{cr}(g_s^2, \rho, \lambda_0)$) where

$$\langle \partial_\mu \widetilde{J}^{L,i}_\mu(x) \mathcal{O}^i(z) \rangle \Big|_{\eta_{cr}} = 0 \quad \text{in Wigner phase } \mu_r^2 > 0$$

- At this $\eta = \eta_{cr}(g_s^2, \rho, \lambda_0)$ we want to check whether in NG phase

$$\frac{\langle \partial_\mu \widetilde{J}^{L,i}_\mu(x) \mathcal{O}^i(z) \rangle \Big|_{\eta_{cr}}}{\langle B^{L,i}_{Yuk}(x) \mathcal{O}^i(z) \rangle \Big|_{\eta_{cr}}} = O(C_1 \frac{\Lambda_s}{v}) \neq 0 \quad \mu_r^2 < 0$$

$$\Lambda_s^2 < |\mu_r^2 \lambda_r| << b^{-2}$$

in view of the conjecture

- In practice consider ratio

$$R_L(x_0) = \frac{\sum_{\vec{x}} \partial_\mu \langle \widetilde{J}_\mu^{L,i}(\vec{x}, t_0) B_{Yuk}^{L,i}(\vec{z}, z_0) \rangle}{\sum_{\vec{x}} \langle B_{Yuk}^{L,i}(\vec{x}, x_0) B_{Yuk}^{L,i}(\vec{z}, z_0) \rangle} \quad x_0 \neq z_0$$

A few technical remarks

- Quenching $\implies$ Φ fields can be generated separately from gauge configs
- Beware of spurious zero modes of D_{lat} (exceptional confs.): introduce an IR cutoff replacing $\overline{\Psi} D_{lat} \Psi \to \overline{\Psi}(D_{lat} + m)\Psi$
- $\langle \partial_\mu (\widetilde{J}_\mu^{L,i}(x) + \widetilde{J}_\mu^{R,i}(x)) B_{Yuk}^{L,i}(z) \rangle$ and η_{cr} remain unchanged up to $O(bm)$ when the IR cutoff is in place

- In the NG phase
  - IR cutoff in D_{lat}-inversion
    - $(\eta - \eta_{cr})\langle\Phi\rangle$ for $\eta \neq \eta_{cr}$
    - $C_1\Lambda_s$ (if any)
  - Scalar expectation value always zero in finite volume
  - need axial χ-fixing ($\chi_L \times \chi_R \times Z_2 \times R_5$ field rotations)
    - $\chi_L \times \chi_R$ rotation of each scalar configuration s.t.

$$\frac{1}{V}\sum_x \phi(x) \propto \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

to get $\langle\Phi\rangle > 0$ if needed apply

- $Z_2 \times R_5 \begin{cases} \Phi \to -\Phi \\ \Psi_{L/R} \to -/+\,\Psi_{L/R} \quad \overline{\Psi}_{L/R} \to +/-\,\overline{\Psi}_{L/R} \end{cases}$

Further technical/practical remarks

1. $\Psi = \begin{pmatrix} u \\ d \end{pmatrix}$ & $\Phi = \varphi_0 \mathbb{1} + i\tau^i \phi_i$ [isospin matrix] $\Longrightarrow$
   $\Longrightarrow$ Dirac matrix 2×2 times larger than in LQCD
   D_{lat} not γ_5-Hermitian need D_{lat}^{-1} and $(D_{lat}^{\dagger})^{-1}$
2. Extra computing time because action is 2^{nd}-neighbour
3. We have just started a first exploration using
   $D_{lat}^{\dagger} D_{lat} \to D_{lat}^{\dagger} D_{lat} + M^2 \quad (a^2 M^2 = 5 \cdot 10^{-4})$
   where we look for signal/noise ratio
   $C_{BB^{\dagger}}(x,t) = \langle B_{Yuk}(x,t) B_{Yuk}^{\dagger}(x_0,t_0) \rangle$

$$B_{Yuk}(x,t) = \overline{\Psi}(x,t) \frac{\tau^i}{2} \Phi(x,t) \left(\frac{1+\gamma_5}{2} \right) \Psi(x,t) +$$

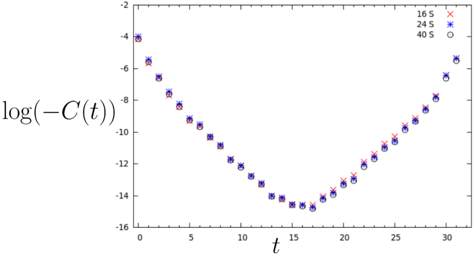
$$-\overline{\Psi}(x,t) \Phi^{\dagger}(x,t) \frac{\tau^i}{2} \left(\frac{1-\gamma_5}{2} \right) \Psi(x,t)$$

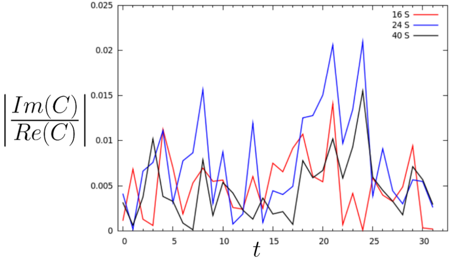
After long work for developing & adapting codes
a first evaluation of correlators in progress. Where do we stand?

$\sim$ where LQCD was 30 years ago

- Evolution in time of $C_{BB^\dagger}$ in a $16^3 \times 32$ lattice
- $\eta = 0.2, \rho = 1$
- Wigner phase $\mu_r > 0$
- 16, 24, 40 uncorrelated scalar on one gauge configurations
- Smeared field Φ to reduce statistical errors

... but quick progress is expected





Outlook

- More statistics and better understanding of the lattice model features
- Determine η_{cr} in Wigner phase
- Check whether a NP mass term is generated in NG phase
- Explore 2-3 lattice spacings to estimate the NP mass value in the $b \to 0$ limit

Thank you for the attention

Power counting in presence of doublers:

- Wilson-like term built with $\widetilde{\nabla}_\mu$ only $\implies$ DS
- Lattice effective action only with $\widetilde{\nabla}_\mu$ to preserve DS
- $d > 4$ operator with $\widetilde{\nabla}\Psi$ irrelevant

$$b\widetilde{\nabla}_\mu \Psi \stackrel{F.T.}{\rightarrow} \sin(bp_\mu)\Psi(p) = 0 \text{ at} |p_\mu| = \frac{\pi}{b}$$

$$\frac{b}{2}\nabla_\mu \Psi \stackrel{F.T.}{\rightarrow} \sin(b\frac{p_\mu}{2})\Psi(p) = \pm 1 \text{ at} |p_\mu| = \frac{\pi}{b}$$

- not at the rigours of level of the power counting theorem [J. Giedt, Nucl. Phys. B 728 (2007) 134] staggered fermions [T. Reisz," Commun. Math. Phys. 116 (1988) 81] Wilson fermions