Including heavy spin effects in a lattice QCD study of static-static-light-light tetraquarks

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Motivation

- Previous work:
  - Evidence for the existence of a $\bar{b}\bar{b}ud$ tetraquark, quantum numbers $I(J^P) = 0(1^+)$ ...
  - ... using the static approximation for the $\bar{b}$ quarks and neglecting heavy spin effects.

  [P. Bicudo, M.W., Phys. Rev. D **87**, 114511 (2013) [arXiv:1209.6274]]
  [P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M.W., Phys. Rev. D **92**, 014507 (2015) [arXiv:1505.00613]]
  [P. Bicudo, K. Cichy, A. Peters, M.W., Phys. Rev. D **93**, 034501 (2016) [arXiv:1510.03441]]

- Binding energy ($\Delta E = 90^{+43}_{-36}$ MeV) is of the same order as expected for heavy spin effects ($\mathcal{O}(m_{B^*} - m_B) = \mathcal{O}(46\,\text{MeV})$)
  $\rightarrow$ essential to include heavy spin effects in the computation.

- Outline:

  (1) Summary of previous work: $\bar{b}\bar{b}ud$ tetraquarks without heavy spin effects.

  (2) Including heavy spin effects.

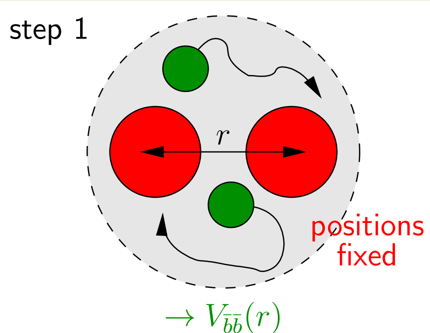
$\bar{b}\bar{b}ud$ tetraquarks, no heavy spin (1)

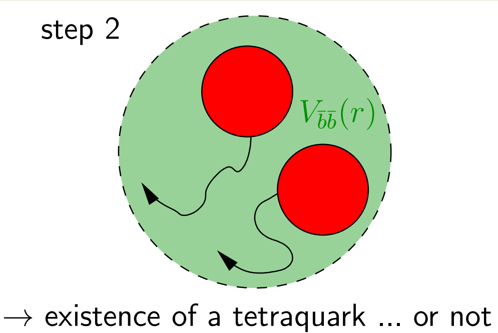
- **Basic idea**: Investigate the existence of heavy tetraquarks $\bar{b}\bar{b}ud$ in two steps.

  (1) **Compute potentials of two static antiquarks $\bar{b}\bar{b}$ in the presence of two light quarks ud using lattice QCD.**

  (2) **Check, whether these potentials are sufficiently attractive, to host a bound state by solving a corresponding Schrödinger equation.** ($\rightarrow$ A bound state is an indication for a stable $\bar{b}\bar{b}ud$ tetraquark.)

- (1) + (2) $\rightarrow$ **Born-Oppenheimer approximation**:
  - Proposed in 1927 for molecular and solid state calculations.
    [M. Born, R. Oppenheimer, "Zur Quantentheorie der Molekeln," Annalen der Physik 389, Nr. 20, 1927]
  - In our computations step (1) not quantum mechanics, but lattice QCD.
  - Approximation valid, if $m_{u,d} \ll m_b$.





$\bar{b}\bar{b}ud$ tetraquarks, no heavy spin (2)

Born-Oppenheimer approximation, step (1)

- Lattice QCD computation of $\bar{b}\bar{b}$ potentials $V_{\bar{b}\bar{b}}(r)$.

  (1) Use $\bar{b}\bar{b}qq$ creation operators

  $$\mathcal{O}_{L,S}(\vec{r}_1,\vec{r}_2) \quad = \quad (\mathcal{C}L)_{AB}(\mathcal{C}S)_{CD}\Big(\bar{b}_C(\vec{r}_1)q_A^{(1)}(\vec{r}_1)\Big)\Big(\bar{b}_D(\vec{r}_2)q_B^{(2)}(\vec{r}_2)\Big) \quad , \quad r \quad = \quad |\vec{r}_1-\vec{r}_2|.$$

  * Different isospin $q^{(1)}q^{(2)} \in \{(ud-du)/\sqrt{2} \quad , \quad uu,(ud+du)/\sqrt{2},dd\}$.
  * Different light quark spin/parity.
  * Different static quark spin/parity. (Irrelevant for $V_{\bar{b}\bar{b}}(r)$.)

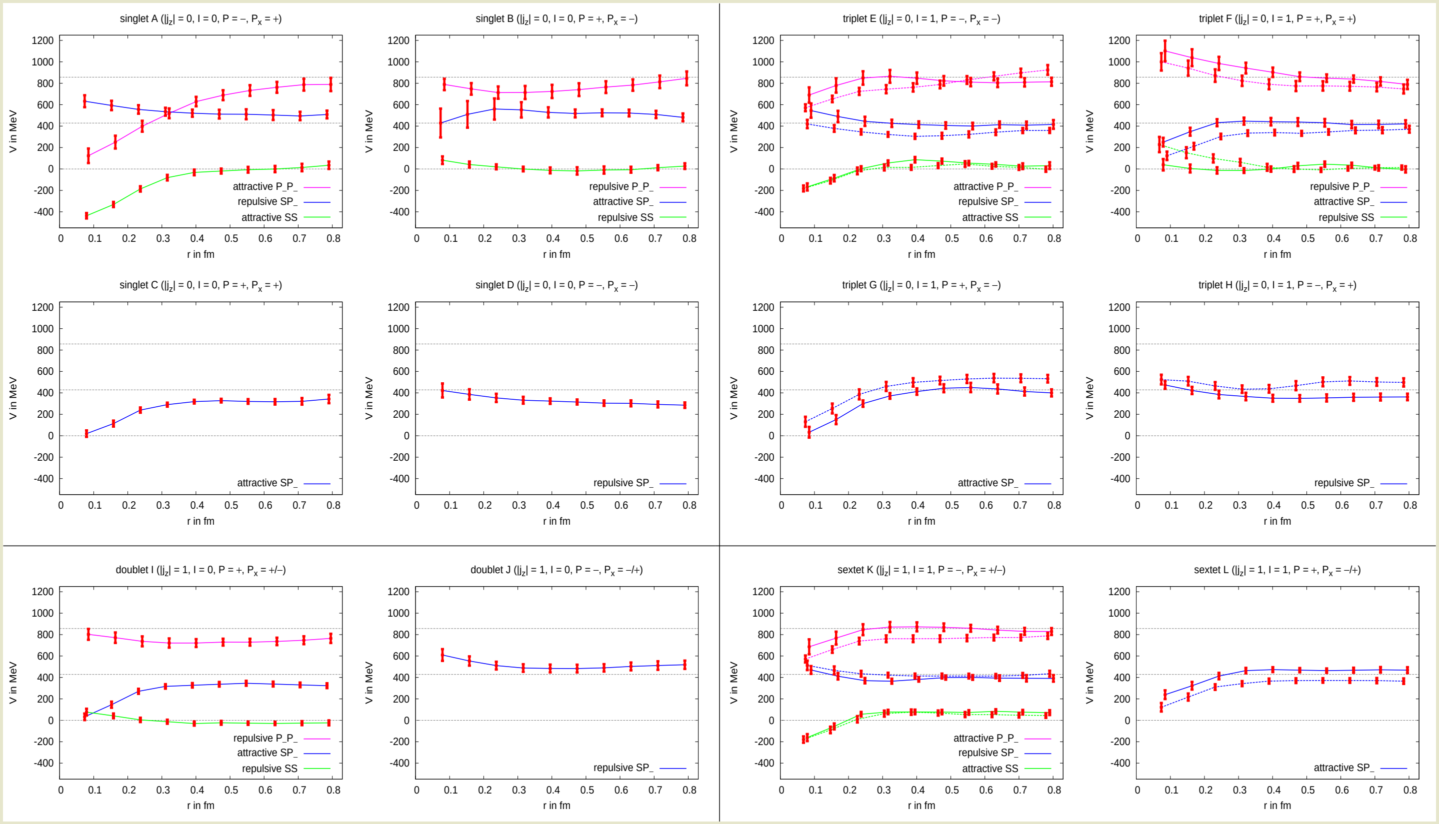
  $\rightarrow$ Many different channels
  ... some attractive, some repulsive
  ... some correspond for large $\bar{b}\bar{b}$ separations to pairs of ground state mesons, some to excited mesons.

  (2) Compute temporal correlation functions.

  (3) Determine $V_{\bar{b}\bar{b}}(r)$ from the exponential decays of the correlation functions.

$\bar{b}\bar{b}ud$ tetraquarks, no heavy spin (3)

- $I = 0$ (left) and $I = 1$ (right); $|j_z| = 0$ (top) and $|j_z| = 1$ (bottom).

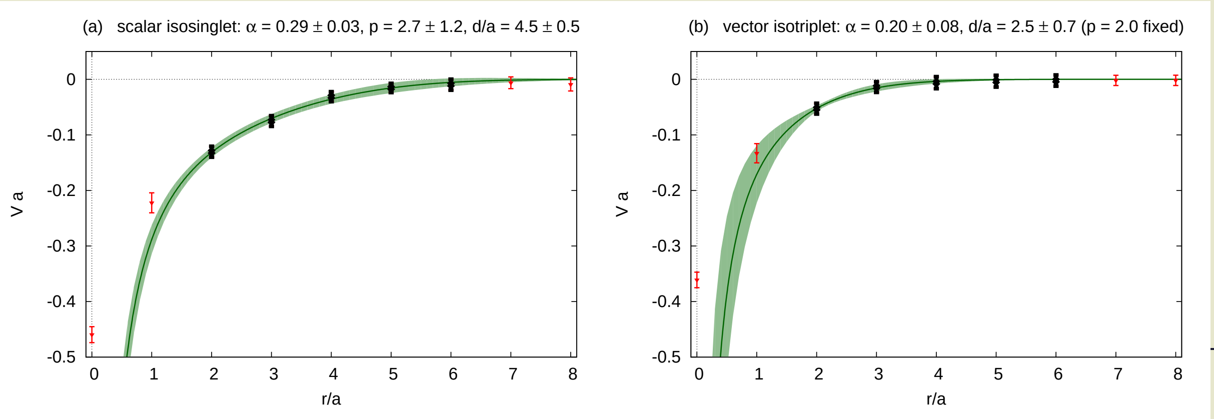


$\bar{b}\bar{b}ud$ tetraquarks, no heavy spin (4)

- Two attractive channels corresponding to pairs of ground state mesons, i.e. B and/or B^* (degenerate in the static limit):
  - $I = 0$, $j_z = 0$ (light quark spin): more attractive.
  - $I = 1$, $j_z = 1$ (light quark spin): less attractive.
- Lattice QCD results can be fitted using the phenomenologically motivated ansatz

$$V(r) \quad = \quad -\frac{\alpha}{r} \exp\bigg(-\Big(\frac{r}{d}\Big)^p\bigg)$$

(one-gluon-exchange at short separations, exponential screening at large separations; fitting parameters α, d and p).



$\bar{b}\bar{b}ud$ tetraquarks, no heavy spin (5)

Born-Oppenheimer approximation, step (2)

- Solve the Schrödinger equation for the relative coordinate $\vec{r}$ of the two $\bar{b}$ quarks,

  $$\left(-\frac{1}{2\mu}\triangle + V_{\bar{b}\bar{b}}(r)\right)\psi(\vec{r}) \quad = \quad E\psi(\vec{r}) \quad , \quad \mu \quad = \quad m_b/2;$$

  possibly existing bound states, i.e. $E < 0$, indicate $\bar{b}\bar{b}ud$ tetraquarks.

- A single bound state for one specific potential $V_{\bar{b}\bar{b}}(r)$ ($I = 0$, $j_z = 0$):
  - Binding energy $\Delta E = -E = 90^{+43}_{-36}$ MeV, i.e. confidence level $\approx 2\,\sigma$.
  - Quantum numbers of the $\bar{b}\bar{b}ud$ tetraquark: $I(J^P) = 0(1^+)$.
    - $\bar{b}\bar{b}$: flavor symmetric ... color triplet, i.e. antisymmetric (otherwise no attraction)
      $\rightarrow$ due to Pauli principle heavy spin symmetric, i.e. $j_b = 1$.
    - ud: $I = 0$, i.e. flavor antisymmetric, color triplet, i.e. antisymmetric (otherwise four quark system not in a color singlet)
      $\rightarrow$ due to Pauli principle light spin antisymmetric, i.e. $j = 0$.
    - $j_b = 1$ and $j = 0$ $\quad \rightarrow J = 1$.
    - Ground state mesons B and B^* both have $P = -$ $\quad \rightarrow P = +$.

$\rightarrow$ **Prediction of a tetraquark.**

Including heavy spin effects (1)

- Interpret static-static-light-light creation operators and the corresponding potentials $V_L(r)$, $r = |\vec{r}_1 - \vec{r}_2|$ in terms of two heavy-light mesons,

$$\mathcal{O}_{L,S}(\vec{r}_1, \vec{r}_2) = (\mathcal{C}L)_{AB}(\mathcal{C}S)_{CD}\Big(\bar{Q}_C(\vec{r}_1)q_A^{(1)}(\vec{r}_1)\Big)\Big(\bar{Q}_D(\vec{r}_2)q_B^{(2)}(\vec{r}_2)\Big) =$$

$$= \mathbb{G}(S,L)_{ab}\Big(\bar{Q}(\vec{r}_1)\Gamma^a q^{(1)}(\vec{r}_1)\Big)\Big(\bar{Q}(\vec{r}_2)\Gamma^b q^{(2)}(\vec{r}_2)\Big).$$

  - $\mathbb{G}(S,L)_{ab}$: Coefficients, which can be computed using the Fierz identity.
  - $\Gamma^a = (\mathbb{1} + \gamma_0)\gamma_5 \quad \rightarrow \quad J^P = 0^-$ (the pseudoscalar B meson).
  - $\Gamma^a = (\mathbb{1} + \gamma_0)\gamma_j \; (j = 1,2,3) \quad \rightarrow \quad J^P = 1^-$ (the vector B^* meson).
  - $\Gamma^a = (\mathbb{1} + \gamma_0)\mathbb{1} \quad \rightarrow \quad J^P = 0^+$ (the scalar B_0^* meson).
  - $\Gamma^a = (\mathbb{1} + \gamma_0)\gamma_j\gamma_5 \; (j = 1,2,3) \quad \rightarrow \quad J^P = 1^+$ (the pseudovector B_1^* meson).

- Focus on B and B^* mesons (the two lightest bottom mesons), which are degenerate in the static limit and have similar mass in nature ($m_{B^*} - m_B \approx 45\,\mathrm{MeV}$)
$\rightarrow$ 16 posibilities of light and static spin couplings,

$$L\,,\, S \in \{(\mathbb{1} + \gamma_0)\gamma_5\,,\, (\mathbb{1} + \gamma_0)\gamma_j\}.$$

Including heavy spin effects (2)

- $$\mathcal{O}_{L,S}(\vec{r}_1,\vec{r}_2) \; = \; (\mathcal{C}L)_{AB}(\mathcal{C}S)_{CD}\Big(\bar{Q}_C(\vec{r}_1)q_A^{(1)}(\vec{r}_1)\Big)\Big(\bar{Q}_D(\vec{r}_2)q_B^{(2)}(\vec{r}_2)\Big) \; = \; = \; \mathbb{G}(S,L)_{ab}\Big(\bar{Q}(\vec{r}_1)\Gamma^a q^{(1)}(\vec{r}_1)\Big)\Big(\bar{Q}(\vec{r}_2)\Gamma^b q^{(2)}(\vec{r}_2)\Big).$$

- Corresponding $\bar{Q}\bar{Q}qq$ potentials depend only on L,
  (1) $V_5(r) \equiv V_{(\mathbb{1}+\gamma_0)\gamma_5}$, i.e. $L = (\mathbb{1}+\gamma_0)\gamma_5$,
  attractive for isospin $I = 0$, repulsive for isospin $I = 1$,
  (2) $V_j(r) \equiv V_{(\mathbb{1}+\gamma_0)\gamma_j}$, i.e. $L = (\mathbb{1}+\gamma_0)\gamma_j$,
  repulsive for isospin $I = 0$, attractive for isospin $I = 1$.

- Neither for $V_5(r)$ nor for $V_j(r)$ it is possible to choose S such that it corresponds exclusively to a B meson pair.

- One always finds linear combinations of B and B^* mesons, e.g. for $L = S = (\mathbb{1}+\gamma_0)\gamma_5$

$$B(\vec{r}_1)B(\vec{r}_2) + B_x^*(\vec{r}_1)B_x^*(\vec{r}_2) + B_y^*(\vec{r}_1)B_y^*(\vec{r}_2) + B_z^*(\vec{r}_1)B_z^*(\vec{r}_2).$$

- Vice versa, a $B(\vec{r}_1)B(\vec{r}_2)$ pair does not have defined light quark spin and hence corresponds to a mixture of both $V_5(r)$ or $V_j(r)$ (one attractive, the other repulsive).

Including heavy spin effects (3)

- Study a coupled channel Schrödinger equation for the two $\bar{b}$ quarks:

$$H\Psi(\vec{r}_1, \vec{r}_2) \quad = \quad \Big(H_0 + H_\text{int}\Big)\Psi(\vec{r}_1, \vec{r}_2) \quad = \quad E\Psi(\vec{r}_1, \vec{r}_2)$$

  with a 16-component wave function Ψ ($1. \equiv B(\vec{r}_1)B(\vec{r}_2)$, $2. \equiv B(\vec{r}_1)B^*_x(\vec{r}_2)$, etc.).

- Free part of the Hamiltonian H:

$$H_0 \quad = \quad \frac{\vec{p}_1^{\,2}}{2m_b}\mathbb{1}_{16\times 16} + \frac{\vec{p}_2^{\,2}}{2m_b}\mathbb{1}_{16\times 16} + M \otimes \mathbb{1}_{4\times 4} + \mathbb{1}_{4\times 4} \otimes M$$

  with $M = \text{diag}(m_B\,,\, m_{B^*}\,,\, m_{B^*}\,,\, m_{B^*})$, $m_b = 4977$ (from the quark model) and $m_B = 5280\,\text{MeV}$, $m_{B^*} = 5325\,\text{MeV}$ from the PDG.

- Interacting part of the Hamiltonian H:

$$H_\text{int} \quad = \quad T^{-1}V(r)T \quad , \quad V(r) \quad = \quad \text{diag}\Big(\underbrace{V_5(r), \ldots, V_5(r)}_{4\times}, \underbrace{V_j(r), \ldots, V_j(r)}_{12\times}\Big),$$

  where T is the transformation between the 16 components of Ψ and the 16 static-static-light-light channels defined by S and L (T is equivalent to $\mathbb{G}(S, L)_{ab}$).

Including heavy spin effects (4)

- Due to rotational symmetry the coupled channel Schrödinger equation for the two $\bar{b}$ quarks,

$$H\Psi(\vec{r}_1, \vec{r}_2) \quad = \quad \Big(H_0 + H_{\text{int}}\Big)\Psi(\vec{r}_1, \vec{r}_2) \quad = \quad E\Psi(\vec{r}_1, \vec{r}_2),$$

  can be transformed to have block diagonal structure, i.e. the 16×16 problem separates into
  a 2×2 problem (corresponding to $J = 0$),
  a 2×2 and a 1×1 problem (corresponding to $J = 1$; $3\times$ degenerate),
  a 1×1 problem (corresponding to $J = 2$; $5\times$ degenerate).

- Including heavy spin effects will reduce the binding energy
  $\rightarrow$ focus on $I(J^P) = 0(1^+)$, the only channel, for which a $\bar{b}\bar{b}ud$ tetraquark has been predicted without taking heavy spin effects into account.

- The 1×1 problem contains only the repulsive V_j, i.e. will not have a bound state.

- The 2×2 problem contains both the attractive V_5 and the repulsive V_j:

$$H_{\text{int},J=1,2\times 2} \quad = \quad \frac{1}{2}\begin{pmatrix} V_5 + V_j & V_j - V_5 \\ V_j - V_5 & V_5 + V_j \end{pmatrix},$$

  first component of Ψ corresponds to BB^*, second component of Ψ corresponds to B^*B^*.

Including heavy spin effects (5)

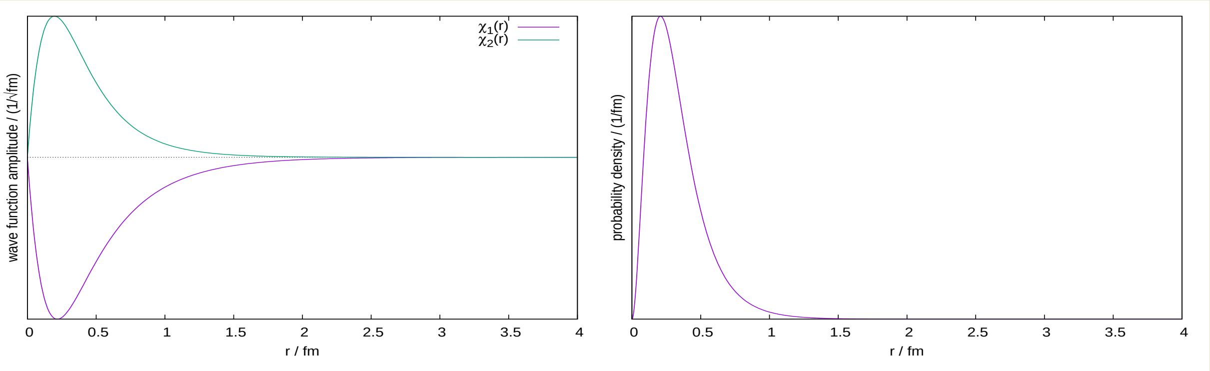
- Numerical solution of the 2×2 coupled channel Schrödinger equation with a Runge-Kutta shooting method.

- Results and conclusions:

  (1) The $\bar{b}\bar{b}ud$ tetraquark persists, binding energy $\Delta E = m_B + m_B^* - E = 59^{+30}_{-38}$ MeV (without taking heavy spin effects into account $\Delta E = 90^{+43}_{-36}$ MeV).

  (2) The wave function Ψ is a roughly 50%/50% mixture of BB^* and B^*B^*.

  (3) The average separation of the two $\bar{b}$ quarks is around 0.25 fm.



Summary and outlook

- Prediction of a $\bar{b}\bar{b}ud$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$,
  - binding energy $\Delta E = m_B + m_B^* - E = 59^{+30}_{-38}\,\text{MeV}$,
  - mass $m = m_B + m_B^* - \Delta E = (5280 + 5325 - 59^{+38}_{-30})\,\text{MeV} = 10546^{+38}_{-30}\,\text{MeV}$,

  using static-static-light-light lattice QCD potentials and the Born-Oppenheimer approximation with heavy spin effects taken into account.

- Work in Progress:
  - Investigation of the structure of the $\bar{b}\bar{b}ud$ tetraquark
    ... is it a mesonic molecule ... or a diquark-antidiquark pair?
  - The experimentally simpler/theoretically harder case $\bar{b}b\bar{q}q$ (e.g. $Z_b(10610)^+$, $Z_b(10650)^+$).
    $\rightarrow$ Talk by Antje Peters, "Lattice QCD study of heavy-heavy-light-light tetraquark candidates" ... now in this session.