Precision determination of the strong coupling at the electroweak scale

ALPHA Collaboration

Rainer Sommer @ Lattice2016, Southampton

Mattia Bruno, Mattia Dalla Brida, Patrick Fritzsch,
Tomasz Korzec, Alberto Ramos, Stefan Schaefer,
Hubert Simma, Stefan Sint, RS

based on simulations by
CLS

HELMHOLTZ | ASSOCIATION

NIC

DESY

Four steps

1) $\mu = \infty \Longrightarrow \mu$=4 GeV

Stefan Sint

$\Lambda^{(3)}_{\overline{\mathrm{MS}}} L_0 = 0.0791(21)$

2) μ = 4GeV $\Longrightarrow$ μ = 200 MeV

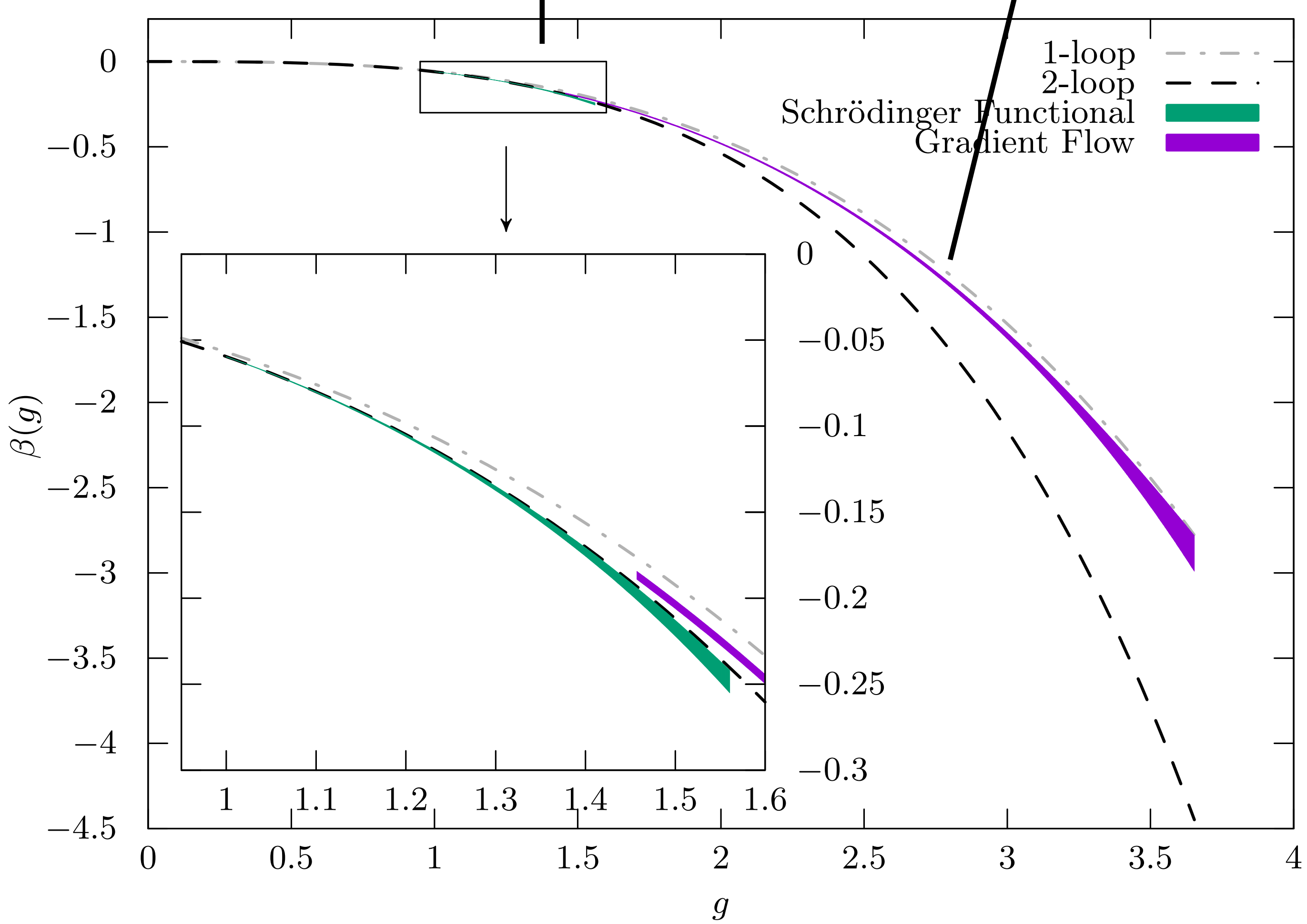
Alberto Ramos

$s(11.31, 2.6723) = 10.93(21)$

3) g^2(200MeV) $\Longrightarrow$ $f_\pi/2 + f_K$

4) N_f=3 $\Longrightarrow$ N_f=5

this talk



Intermezzo: setting the scale

ALPHA
Collaboration

Mattia Bruno, Tomasz Korzec, Stefan Schaefer
arXiv:1608.xxxxx

based on simulations by **CLS**

and on determination of Z_A by
Mattia Dalla Brida and Tomasz Korzec

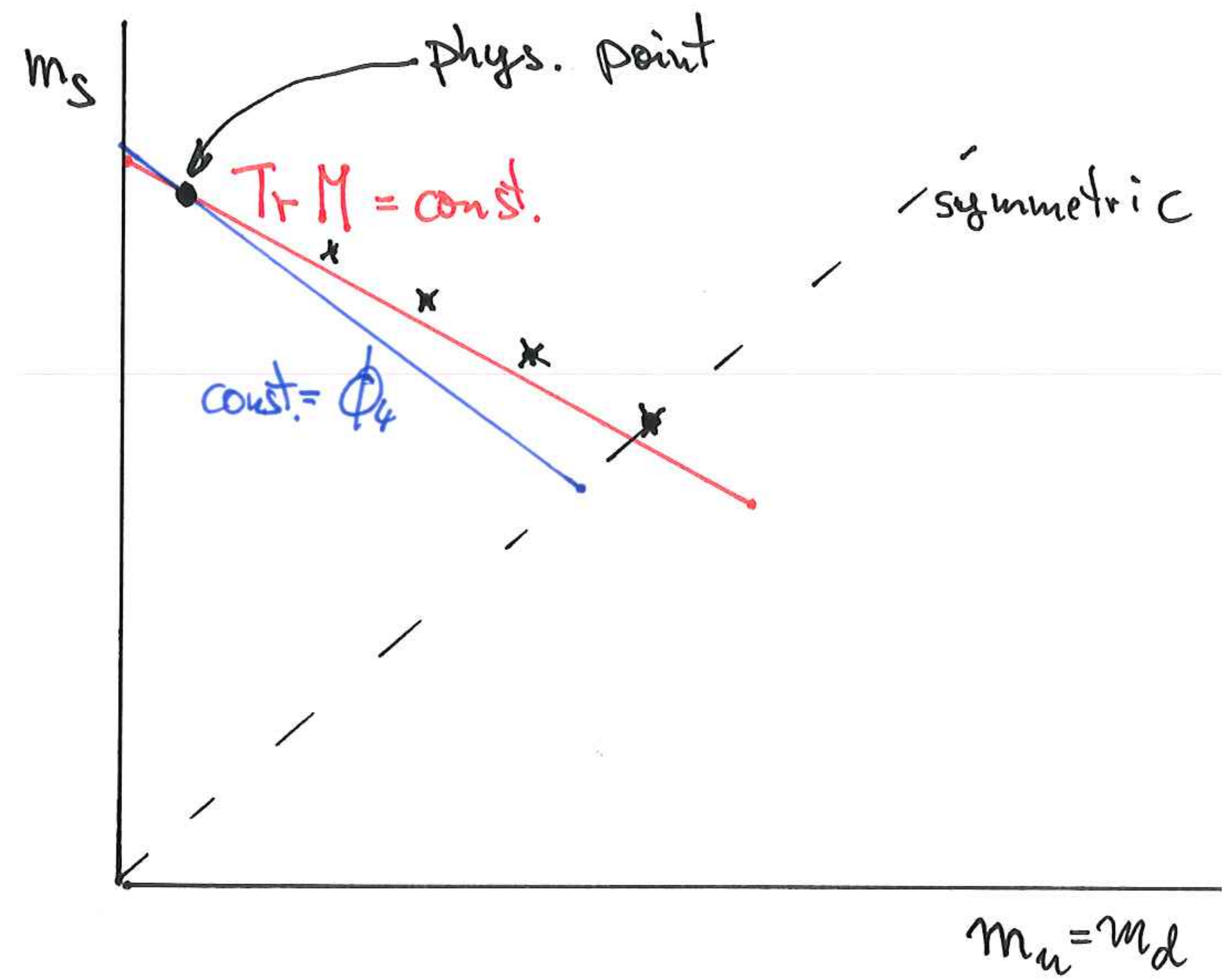
Used CLS Ensembles (tr M =const. [QCDSF strategy])

id	β	N_s	N_t	κ_u	κ_s	m_π[MeV]	m_K[MeV]	$m_\pi L$
H101	3.40	32	96	0.13675962	0.13675962	420	420	5.8
H102	3.40	32	96	0.136865	0.136549339	350	440	4.9
H105	3.40	32	96	0.136970	0.13634079	280	460	3.9
C101	3.40	48	96	0.137030	0.136222041	220	470	4.7
H400	3.46	32	96	0.13688848	0.13688848	420	420	5.2
H401	3.46	32	96	0.136725	0.136725	550	550	7.3
H402	3.46	32	96	0.136855	0.136855	450	450	5.7
H200	3.55	32	96	0.137000	0.137000	420	420	4.3
N202	3.55	48	128	0.137000	0.137000	420	420	6.5
N203	3.55	48	128	0.137080	0.136840284	340	440	5.4
N200	3.55	48	128	0.137140	0.13672086	280	460	4.4
D200	3.55	64	128	0.137200	0.136601748	200	480	4.2
N300	3.70	48	128	0.137000	0.137000	420	420	5.1
J303	3.70	64	192	0.137123	0.1367546608	260	470	4.1

reasonable lattice spacings: 0.08, 0.07, 0.06, 0.05 fm

Strategy of CLS Ensembles

- simulations: $\mathrm{Tr}\, M_q = \text{const.}$
- other trajectories
  - $\phi_4 = 8\, t_0\, (m_\mathrm{k}^2 + \frac{1}{2} m_\pi^2) = \text{const.}$
  - $\frac{m_\mathrm{k}^2 + \frac{1}{2} m_\pi^2}{f_{\pi K}^2} = \text{const.}$
  - shift there (small shifts in masses)



- physical point:

$$m_\pi^2 / f_{\pi K}^2 = phys. \quad m_\mathrm{K}^2 / f_{\pi K}^2 = phys. \quad \rightarrow \quad \Phi_4 = 1.11(2)$$

Shift to desired trajectory

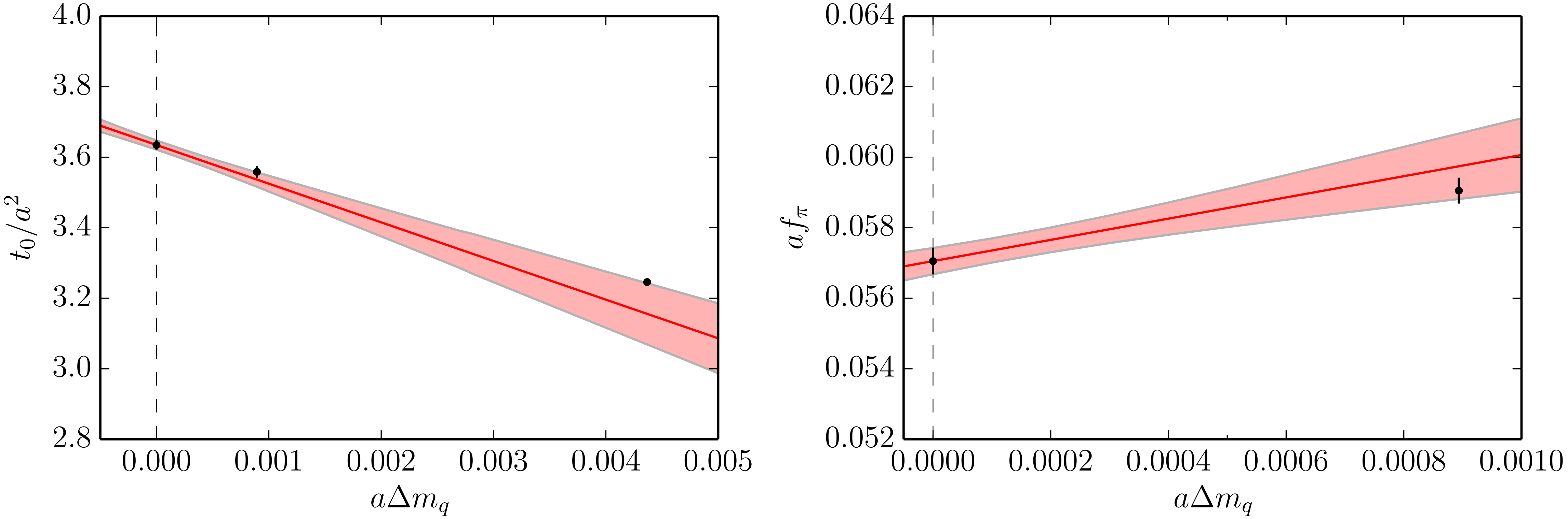
M. Bruno, T. Korzec, S. Schaefer

$$f(m') \approx f(m) + (m' - m) f'(m)$$

$$f'(m) = \sum_i \frac{\partial f}{\partial \bar{A}_i} \left[\langle \partial_m A_i \rangle - \langle (A_i - \bar{A}_i)(\partial_m S - \overline{\partial_m S}) \rangle \right]$$

$$\partial_m \, \mathrm{tr} \left[\frac{1}{D+m} \Gamma \frac{1}{D+m'} \Gamma' \right] = - \mathrm{tr} \left[\frac{1}{(D+m)^2} \Gamma \frac{1}{D+m'} \Gamma' \right]$$

$$-\partial_m \log \det (D+m) = - \mathrm{tr} \, (D+m)^{-1}$$



Light-quark-mass dependence

M. Bruno, T. Korzec, S. Schaefer

$$\frac{X}{X^{\mathrm{symm}}}$$

as function of

$$\phi_2 = 8\,t_0\,m_\pi^2$$

$$X = t_0$$

$$X = f_{\pi K}$$

$$X = f_{\pi K}\sqrt{t_0}$$

$$X = \mathrm{tr}\,M_{\mathrm{R}}$$

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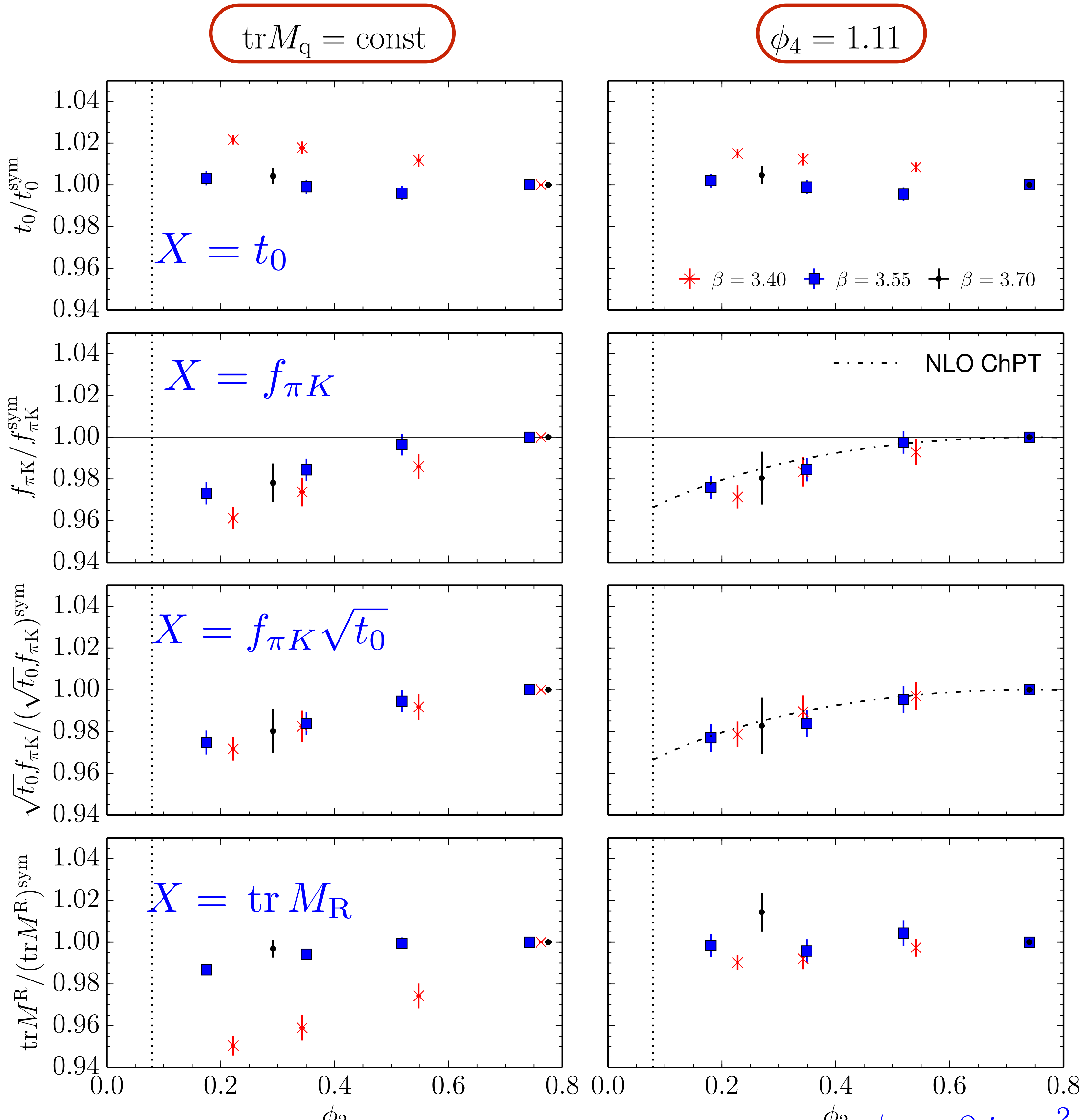

$\phi_2 = 8\, t_0\, m_\pi^2$

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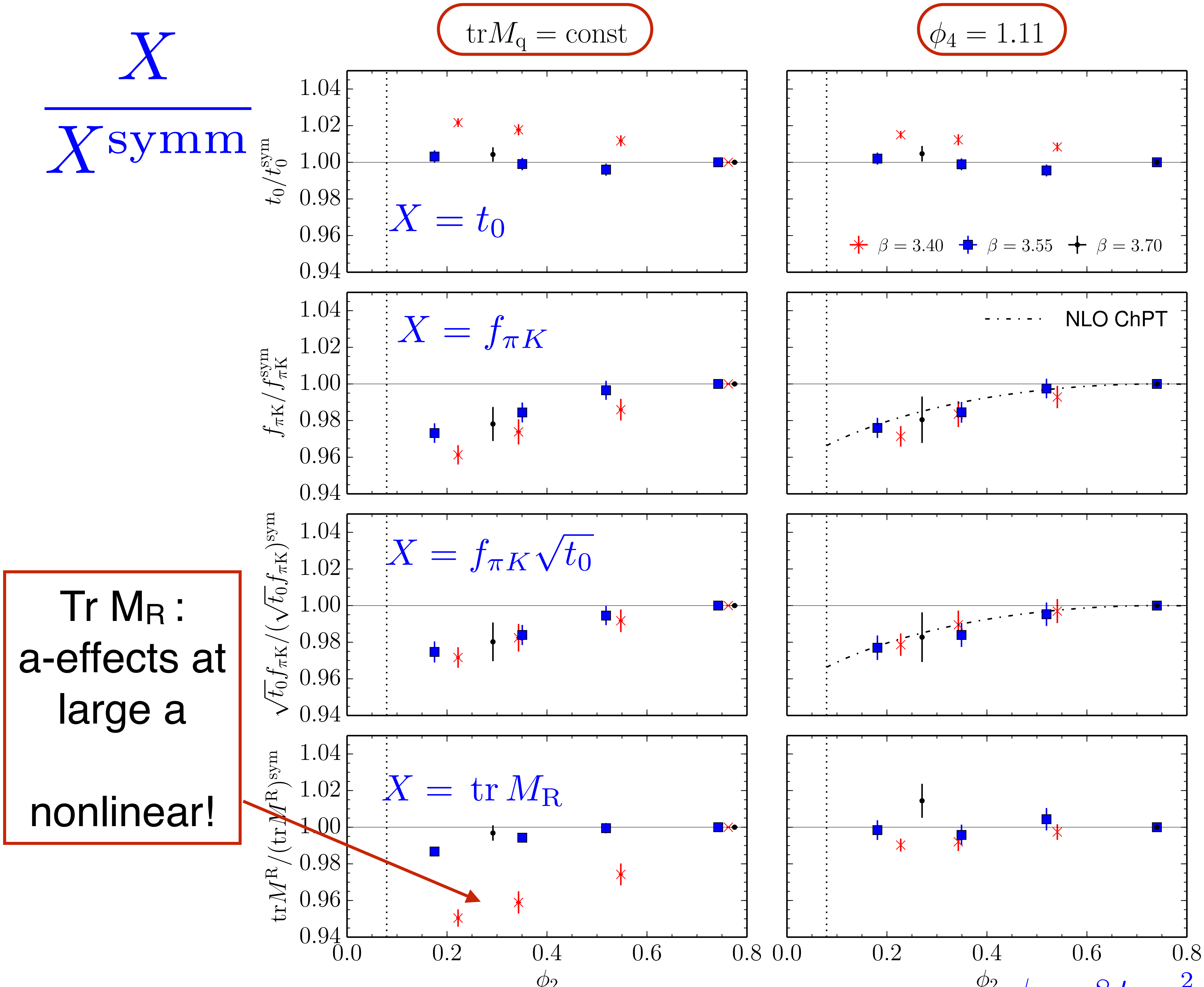

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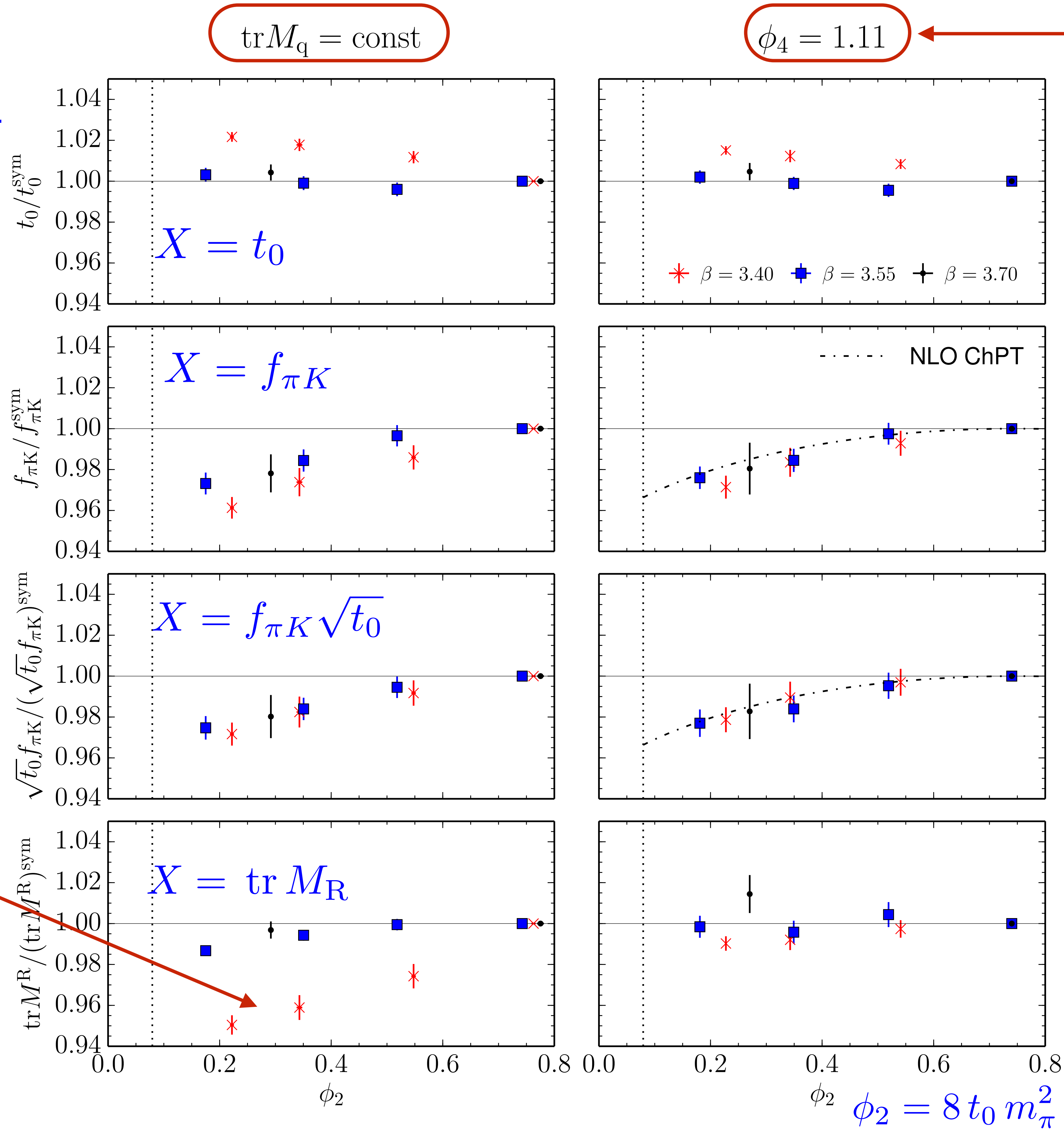


the better trajectory

mass-dependence essentially free of a-effects

⬇

use in continuum extrapolations

Tr M_R : a-effects at large a

nonlinear!

t_0 in physical units

M. Bruno, T. Korzec, S. Schaefer

- Φ_4=1.11 trajectory
- Continuum extrapolation fit

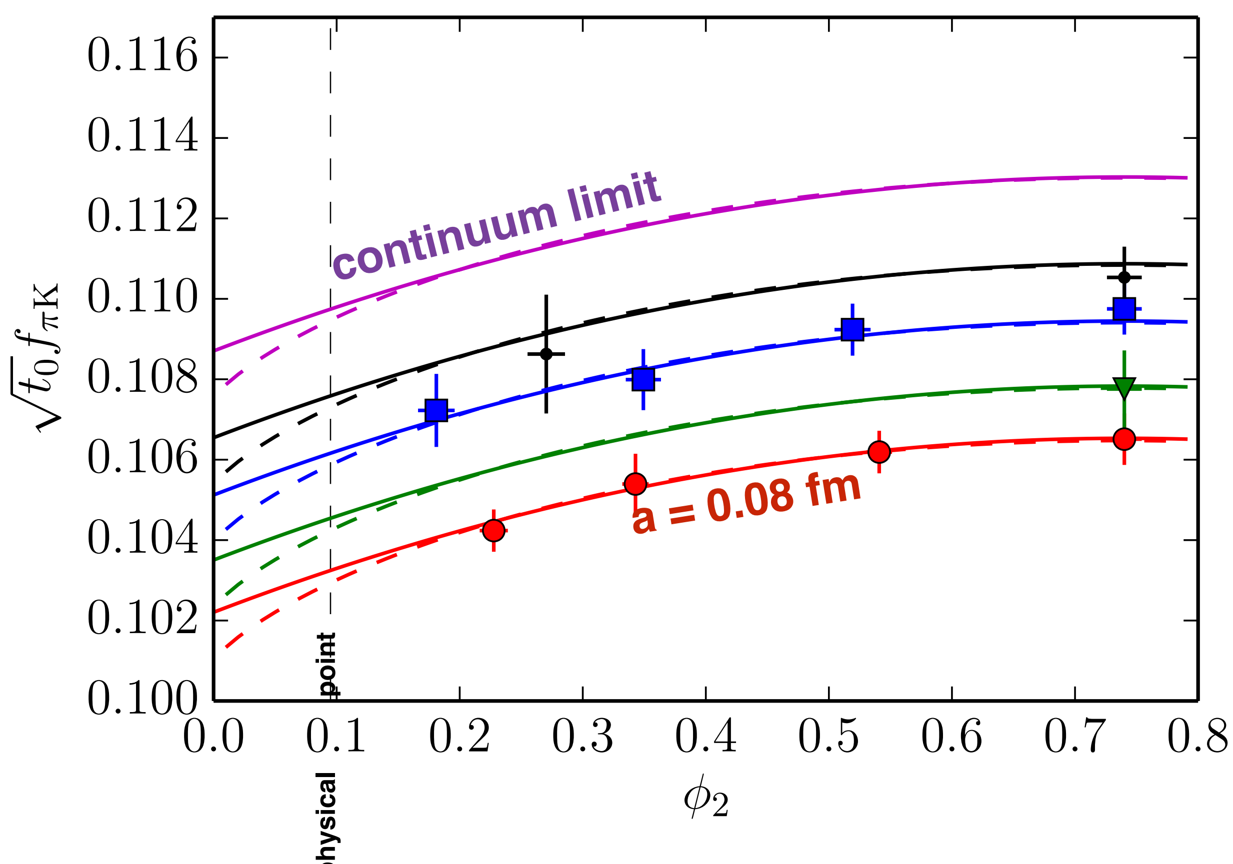
$$\sqrt{t_0} f_{\pi K} = F^{\rm cont}(\phi_2) + c \frac{a^2}{t_0^{\rm sym}}$$

$$\phi_2 = 8\, t_0\, m_\pi^2$$

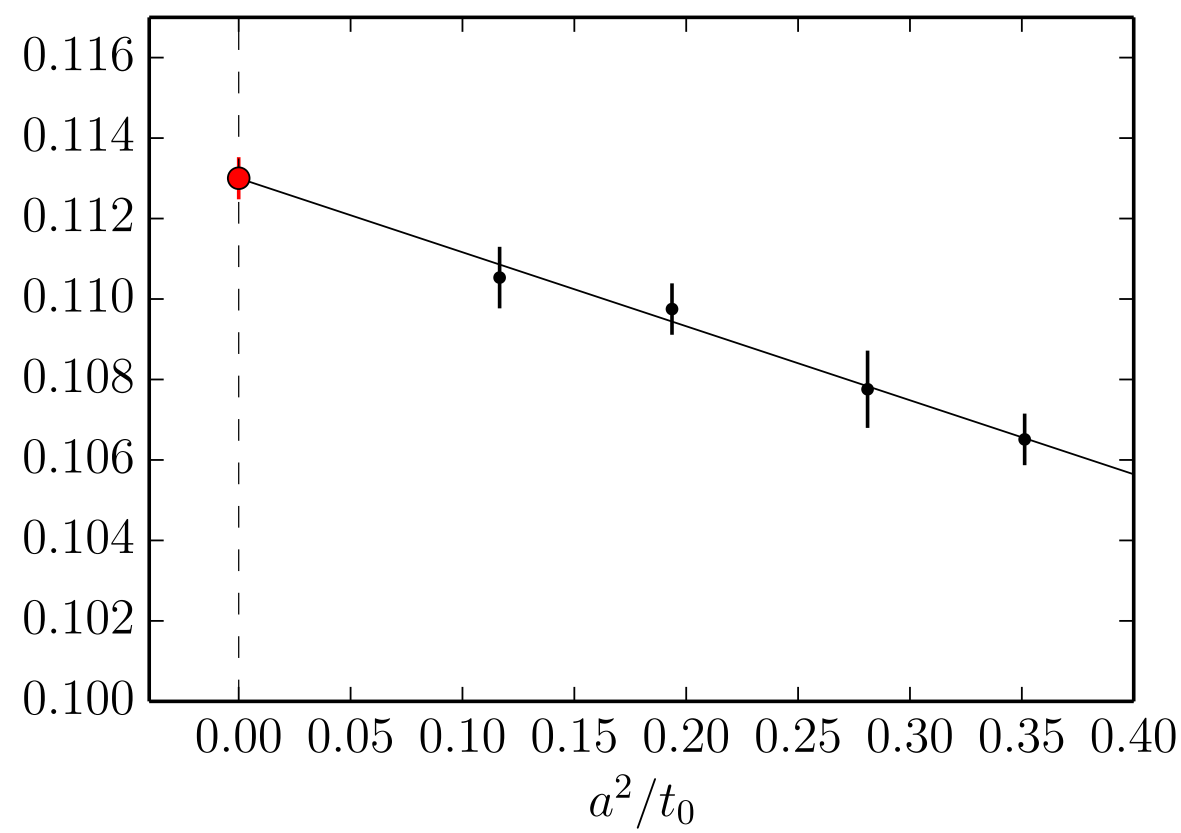
$$\phi_4 = 8\, t_0\, (m_{\rm k}^2 + \frac{1}{2} m_\pi^2)$$

$$f_{\pi K} = \frac{2}{3}(f_{\rm K} + \frac{1}{2} f_\pi)$$

light quark mass dependence



a-dependence at symm. point



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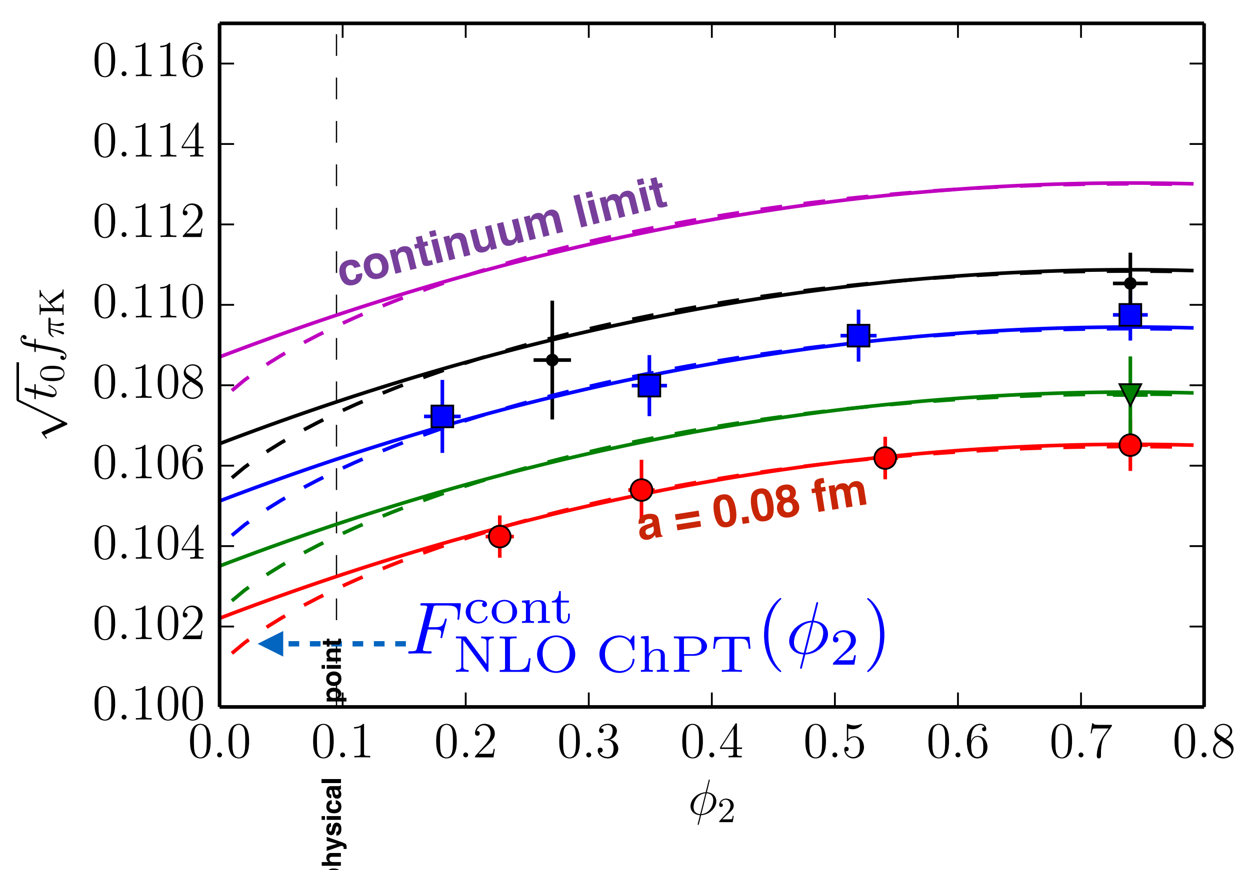

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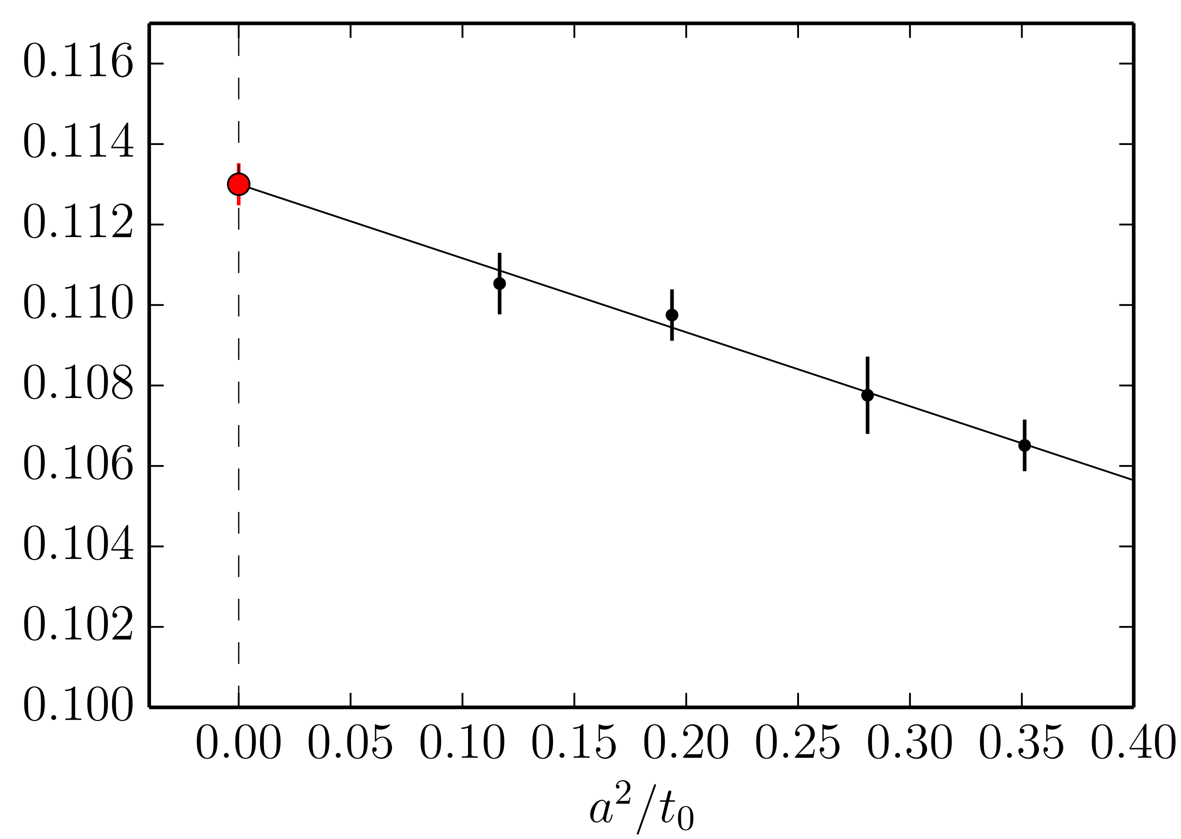

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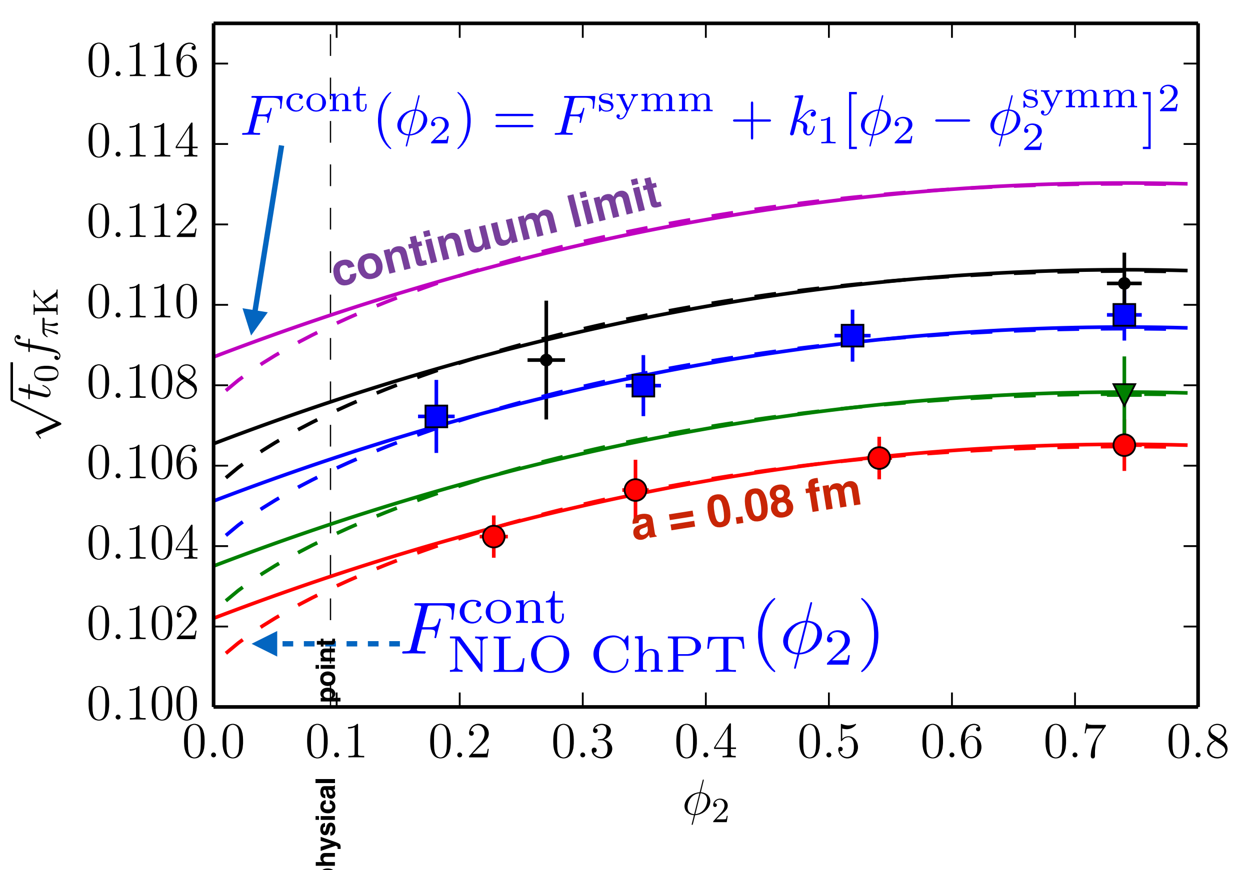

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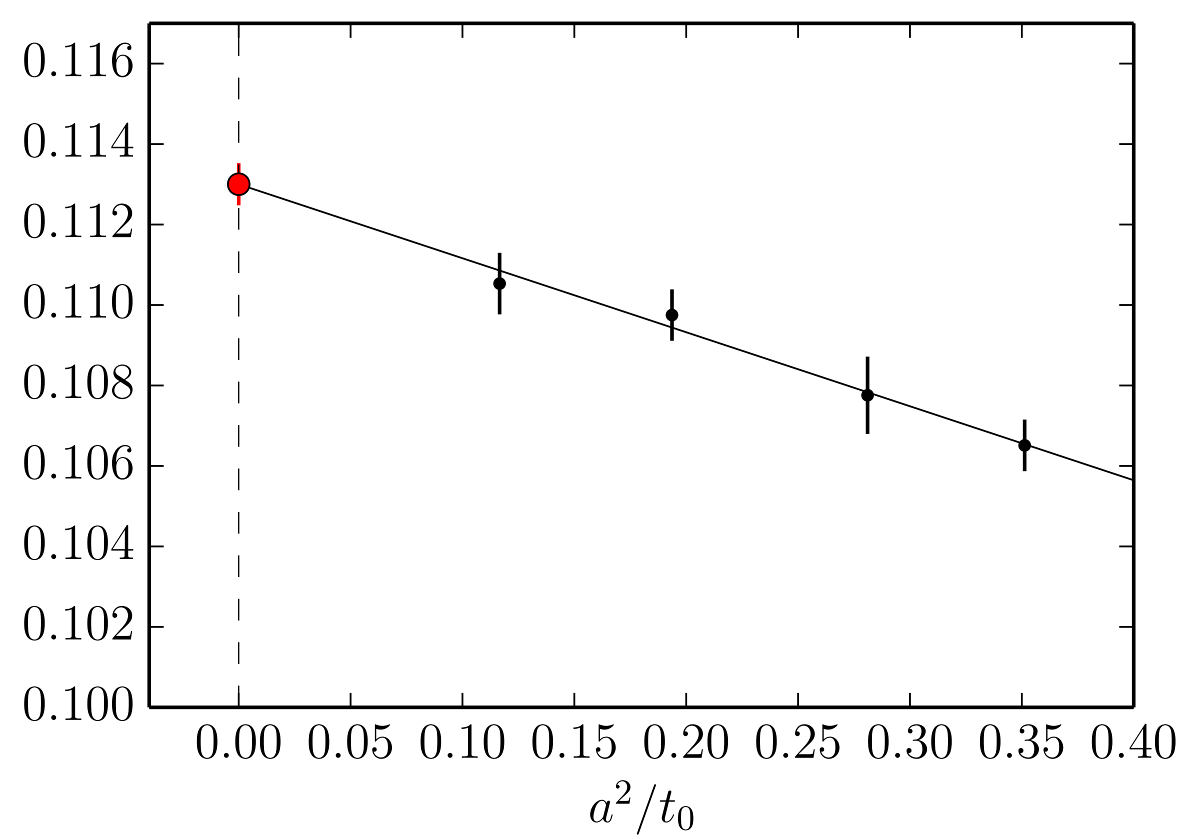

t_0 in physical units and a(β)

M. Bruno, T. Korzec, S. Schaefer

- $\sqrt{8\,t_0}$ = 0.413(4)(1) fm at physical point, N_f =2+1
  from continuum extrapolation fit
- $\sqrt{8\,t_0}$ = 0.413(4)(2) fm at symmetric point (preliminary)
  from similar fits

  chiral extrapolation
- with input from $f_{\mathrm{K}},\ f_\pi,\ m_{\mathrm{K}},\ m_\pi$
- and numbers for t_0/a^2:

β =	3.4	3.46	3.55	3.7	[3.85]
t_0/a^2 =	2.862	3.662	5.166	8.596	[13.8]
δt_0 =	0.006	0.012	0.015	0.027	[0.3]

- very precise relative scale
- absolute scale to ~ 1%

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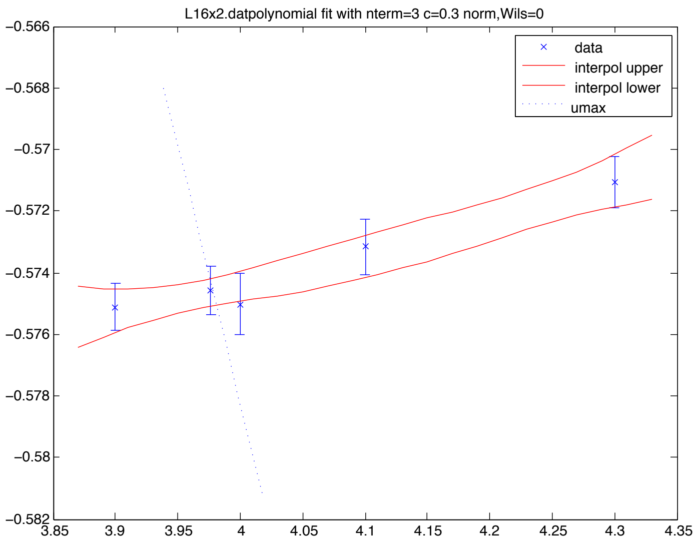
Back on track: use the scale for Λ

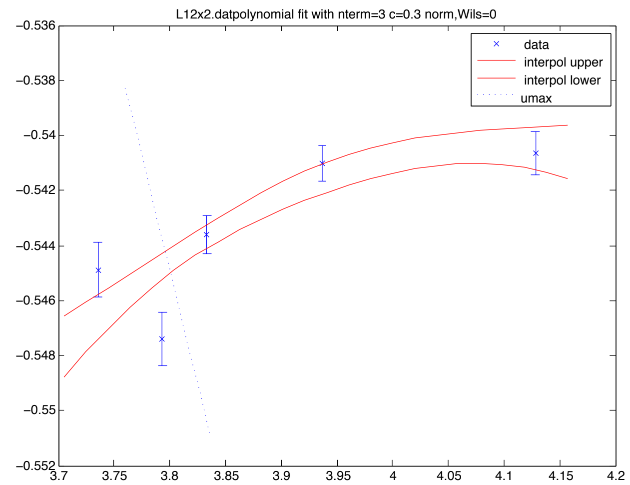
Lmax

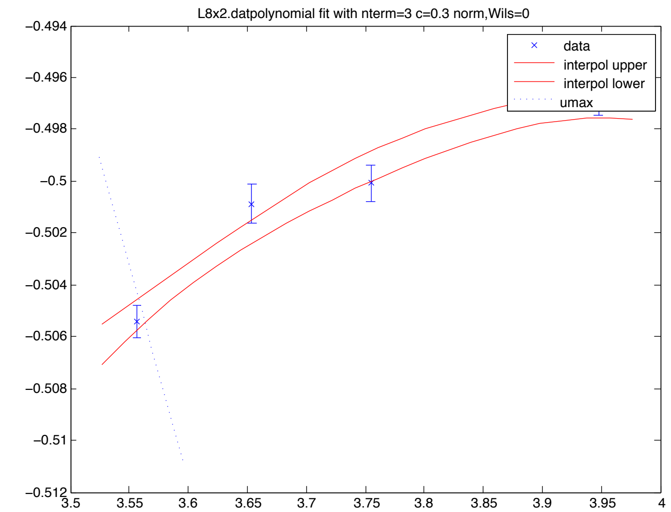
- **β**=3.4, L/a=12, m(L)=0: $\bar{g}^2_{\mathrm{GF}}(L_{\mathrm{max}}) = 11.308(99)$

- at fixed L/a find **β** with same coupling (but m(L/2)=0)

L/a= 16 | 24 | 32

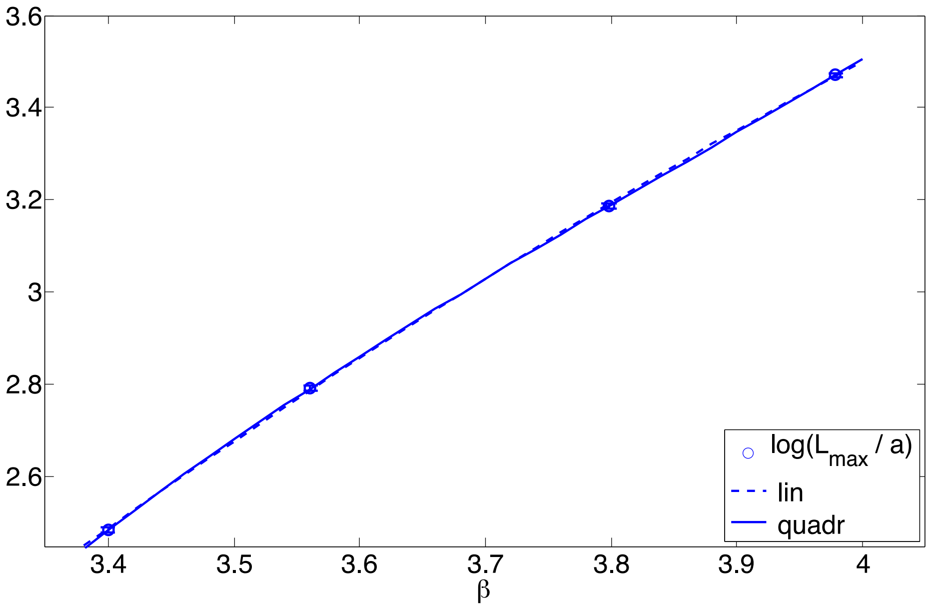






- smooth interpolation in **β**

  + shift to common trajectory m(L)=0



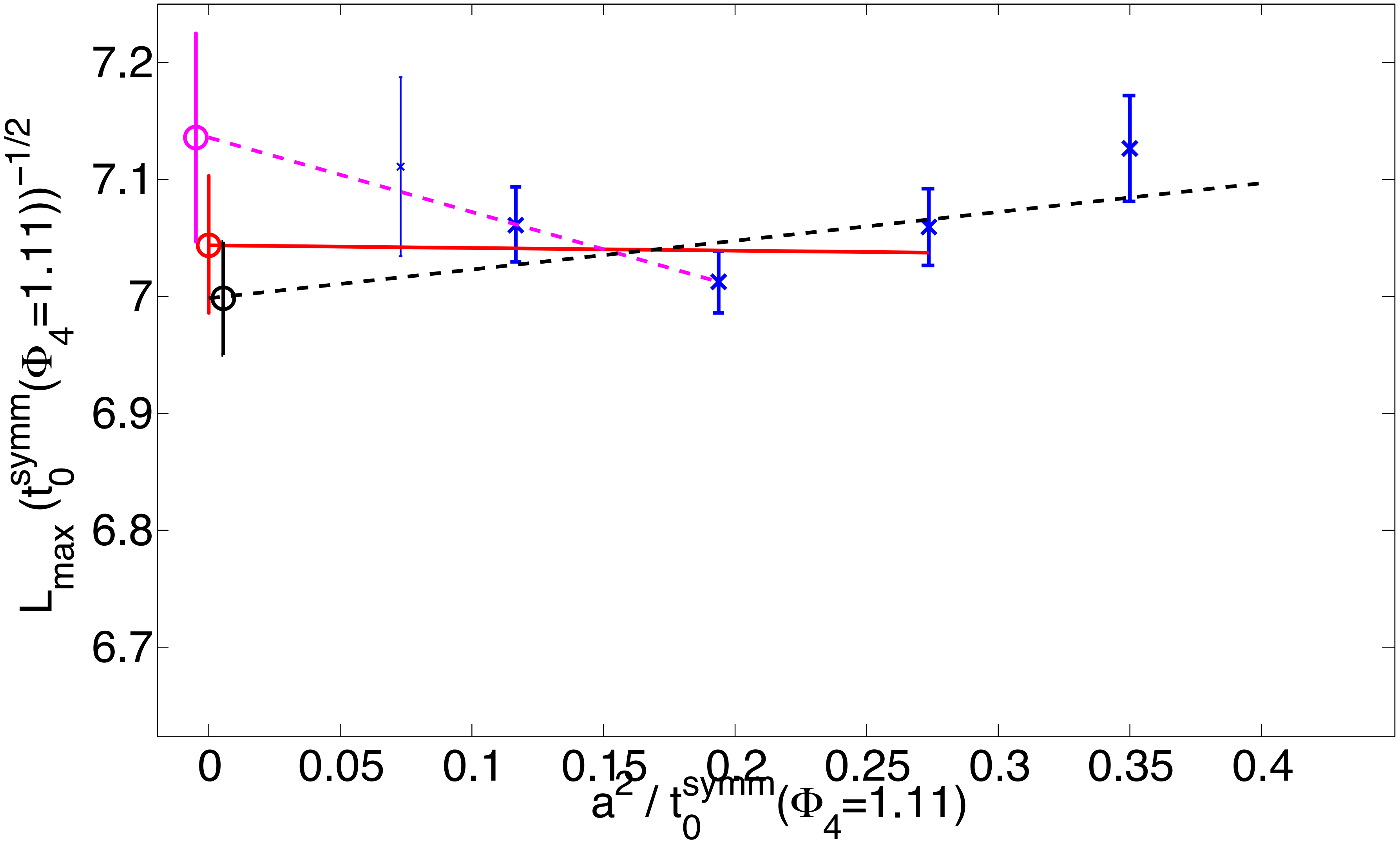
$L_{max} / \sqrt{t_0}$ (t_0 at symmetric point)

- at $\tilde{\beta}_{CLS} = \beta_{CLS}/(1 + 0.216\, am_q/\beta_{CLS})$ $(N_f = 3)$ S. Sint, RS; 97

  combine L_{max}/a and t_0/a^2

β	**β**-tilde	a m_q
3.400	3.3988	0.0052
3.460	3.4589	0.0048
3.550	3.5491	0.0041
3.700	3.6992	0.0032
3.850	3.8494	0.0027

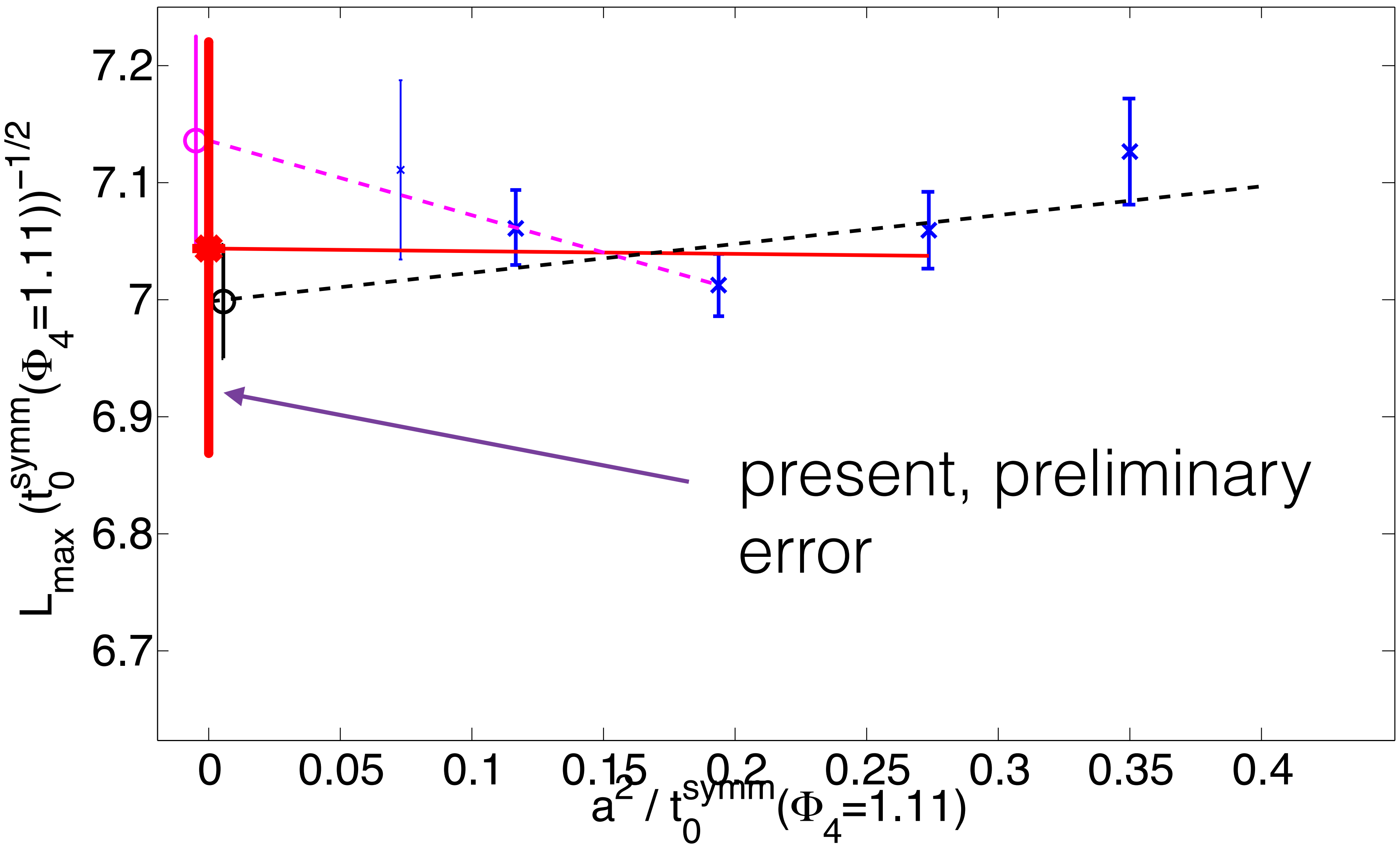
- extrapolate



$L_{max} / \sqrt{t_0}$ (t_0 at symmetric point)

- at $\tilde\beta_{CLS} = \beta_{CLS}/(1 + 0.216\, am_q/\beta_{CLS}) \qquad (N_f = 3)$
  combine L_{had}/a and t_0/a^2

- extrapolate



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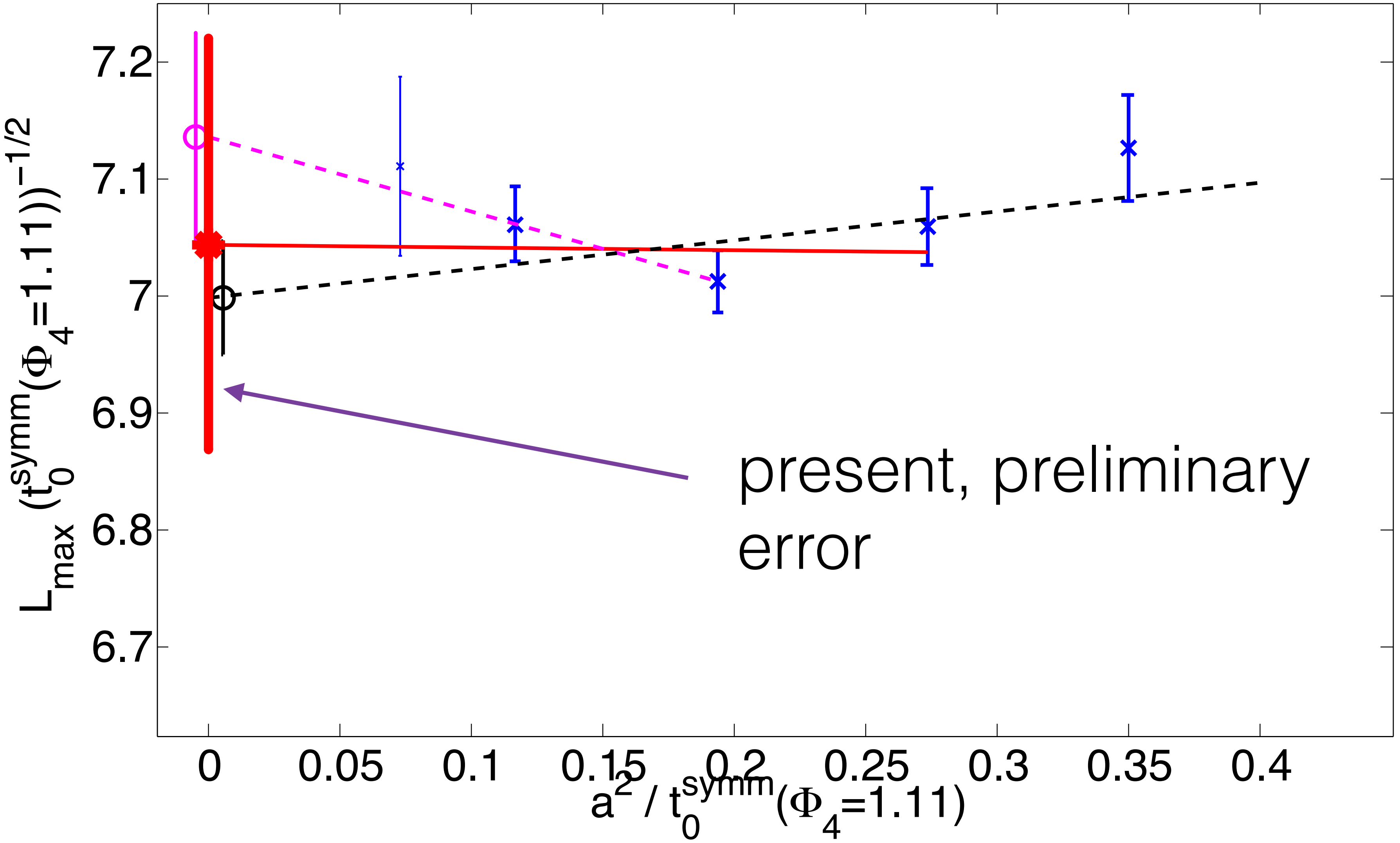

Step 4: Perturbative conversion $N_f=3 \Longrightarrow N_f=5$

- Use relation of Λ-parameters [cf. Bruno et al., PoS LATTICE2015 (2016) 256] input: $m_c(m_c)$, $m_b(m_b)$ from PDG:

$$\Lambda^{(4)}_{\overline{\rm MS}} = 289(14)\text{MeV}, \quad \Lambda^{(5)}_{\overline{\rm MS}} = 207(11)\text{MeV},$$

$$\alpha_{\overline{\rm MS}}(m_Z) = 0.1179(10)(2)$$

- Error estimate

n (= loops)	α_n	$\alpha_n - \alpha_{n-1}$
2	0.11670	-
3	0.11771	0.00109
4	0.11787	0.00016
5-loop beta	0.11794	0.00007

Preliminary result for α

- $$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 332(14)\mathrm{MeV}$$
- $$\alpha_{\overline{\mathrm{MS}}}(m_\mathrm{Z}) = 0.1179(10)(2)$$
- using 3-flavor theory (decoupling)
- error budget:

Preliminary result for α

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quantity	value	error	rel. err.	comment
$\Lambda^{(3)}_{\overline{\mathrm{MS}}} L_0$	0.0791	0.0021	0.026	arXiv:1604.06193
$L_{2.6723}/(2L_0)$	1	0.0080	0.008	scheme change arXiv:1607.06423
$s(11.31, 2.6723)$	10.93	0.21	0.019	scale factor
$t^{1/2}_{0,\mathrm{symm}}/L_{11.31}$	0.1420	0.0036	0.025	preliminary, Lat16
$[8t_{0,\mathrm{symm}}]^{1/2}$ [fm]	0.4130	0.0045	0.011	at $\phi_4 = 1.11$
$t^{-1/2}_{0,\mathrm{symm}}$ [GeV]	1.3514	0.0146	0.0108	preliminary, Lat16 at $\phi_4 = 1.11$
$\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ [GeV]	0.332	0.014	0.042	
$\alpha(m_Z)$	**0.1179**	**0.0010**	**0.009**	preliminary, Lat16 $\pm$ **0.00016** = conversion error
$\alpha(m_Z)$	0.1177	0.0010	0.0085	3-loop conversion
$\alpha(m_Z)$	0.1179	0.0009	0.0085	5-loop β-function
$\Lambda^{(3)}_{\overline{\mathrm{MS}}}$ [GeV]	0.336	0.019		FLAG3 [arXiv:1607.00299]

Thank you.