Lattice QCD study of heavy-heavy-light-light tetraquark candidates

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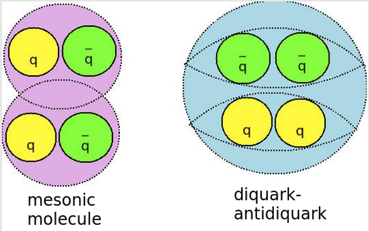
Study of four-quark states

Motivation

- A number of mesons observed in particle detectors (LHCb, Belle) is not well understood.
- E.g. charged charmonium- and bottomonium-like states (${Z_c}^{\pm}$ and ${Z_b}^{\pm}$)
- They include $b\bar{b}$ or $c\bar{c}$, but are also charged: must be 4-quark states



possible tetraquark structures

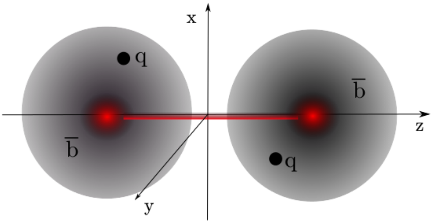
$B\bar{B}$ systems in the static-light approach

A. Peters, P. Bicudo, K. Cichy and M. Wagner, "Investigation of $B\bar{B}$ four-quark systems using lattice QCD," arXiv:1602.07621 [hep-lat].

Work in progress.

The static-light approach

- Computation of 4-quark states very difficult
- If 2 quarks are heavy and 2 quarks are light: Treat degrees of freedom independently in two steps (Born-Oppenheimer approximation [M. Born, R. Oppenheimer, "Zur Quantentheorie der Molekeln," Ann.Phys. 389, Nr. 20, 1927]).
  1. Lattice computation of the potential of two static quarks in the presence of two light quarks, i.e. potential can be interpreted as the potential between two B mesons

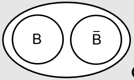


  2. Solve Schrödinger's equation to check whether potentials are sufficiently attractive to form a bound state.

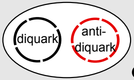
The $B\bar{B}$ system

There are several different structures for the experimentally relevant case of $I(J^P) = 1(1^+)$ (i.e. Z_b^+):

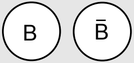
- A $\bar{u}d\bar{b}b$ state , i.e.

  a mesonic molecule 

  or

  a diquark-antidiquark 

  or

  a B meson and a far separated $\bar{B}$ meson 

- A bottomonium state and a far separated π^+, i.e.

  $Q\bar{Q} + \pi$ 

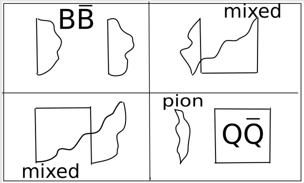
To separate some of these structures, we ...

1. implement a correlation matrix

$$C_{jk}(t) = \langle\Omega|\, \mathcal{O}_j^\dagger(t)\mathcal{O}_k(0)\,|\Omega\rangle \underset{\text{large } t}{\approx} A_{jk}^0 \exp\left(-V_0(r)t\right) + A_{jk}^1 \exp\left(-V_1(r)t\right) + \ldots$$

$$\mathcal{O}_0 \equiv \mathcal{O}_{B\bar{B}} = \Gamma_{AB}\tilde{\Gamma}_{CD}\,\bar{Q}_C^a(\vec{x})q_A^a(\vec{x})\bar{q}_B^b(\vec{y})Q_D^b(\vec{y})$$

$$\mathcal{O}_1 \equiv \mathcal{O}_{Q\bar{Q}+\pi} = \tilde{\Gamma}_{AB}\,\bar{Q}_A^a(\vec{x})U^{ab}(\vec{x},t;\vec{y},t)Q_B^b(\vec{y})\sum_{\vec{z}} \bar{q}_C^c(\vec{z})\,(\gamma_5)_{CD}\,q_D^c(\vec{z})$$

$C(t) =$ 

2. extract the potentials with the Generalized Eigenvalue Problem (GEP).

Choice of Γ for the $B\bar{B}$ system

$$\mathcal{O}_0 \equiv \mathcal{O}_{B\bar{B}} = \Gamma_{AB}\tilde{\Gamma}_{CD}\bar{Q}^a_C(\vec{x})q^a_A(\vec{x})\bar{q}^b_B(\vec{y})Q^b_D(\vec{y})$$

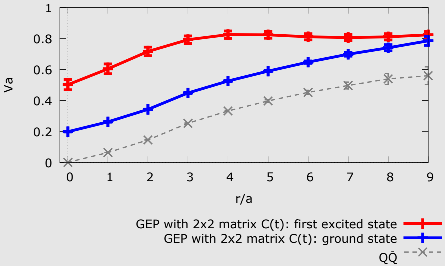
- The choice of the matrix Γ is constrained by the quantum numbers of the $\mathcal{O}_{Q\bar{Q}+\pi}$
- Only taking into account $\mathcal{O}_{B\bar{B}}$ we find the strongest attraction for $\Gamma = \gamma_5 - \gamma_0\gamma_5$

Preliminary results I

Ens.	β	lattice	$a\mu$	m_π[MeV]	a [fm]	L [fm]	confs
B85.24	3.90	$24^3 \times 48$	0.0085	480	0.0790(26)	1.9	400

- gauge configurations generated by ETMC [arXiv:hep-lat/0701012]
- twisted-mass fermions at maximal twist
- tree-level Symanzik improved gauge action

Potentials obtained



- $Q\bar{Q} + \pi$: ground state (blue)
- first excited state of the 2x2 matrix: free of contributions of $Q\bar{Q} + \pi$ (red)

Preliminary results II

- Fit an ansatz to the potentials:

$$V(r) = -\underbrace{\frac{\alpha}{r}}_{Coulomb-like}\underbrace{e^{-\left(\frac{r}{d}\right)^2}}_{colour\ screening}$$

- solve Schrödinger's equation for the radial component of the b resp. $\bar{b}$ quark:

$$\left(-\frac{1}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + V(r)\right)R(r) = E_B R(r) \quad , \qquad \psi(r) = R(r)/r$$

- Perform a large number of fits varying...
  - temporal separation at which lattice potential is read off the correlator
  - range at which the fit to the potential is performed
- This yields for quantum numbers $I(J^P) = 1(1^+)$ (i.e. Z_b^+):

$$E_B = (-58 \pm 71)\mathrm{MeV}$$

- very vague indication for a $\bar{u}d\bar{b}b$ bound state

Study of BB^* systems by means of NRQCD

in collaboration with

Luka Leskovec, Stefan Meinel and Marc Wagner

Work in progress.

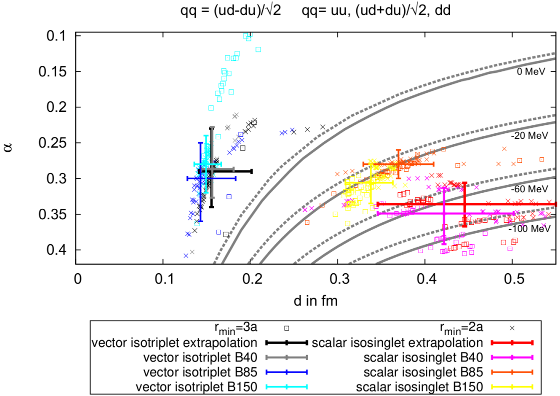
The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ state

We found strong evidence for a $ud\bar{b}\bar{b}$ bound state in the $I(J^P) = 0(1^+)$ channel from the static-light approach :

Binding energy

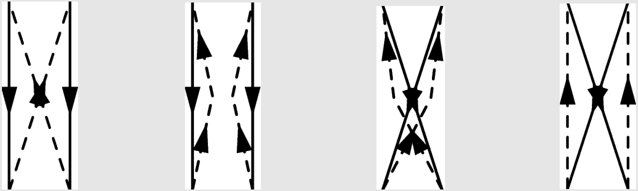
$$E_B = -90^{+43}_{-36}\ \mathrm{MeV}$$

arXiv:1510.03441 [hep-lat]



The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ state with NRQCD

- To confirm static-light result with $\bar{b}$-quarks of finite mass instead of static quarks
  $\Rightarrow$ search for a bound state with nonrelativistic QCD (NRQCD)
- positions of $\bar{b}$ quarks not fixed
  $\Rightarrow$ computation of $V(r)$ not possible
  $\Rightarrow$ but direct computation of mass of lowest BB^* state in $I(J^P) = 0(1^+)$ channel possible



Lattice setup

BB^* with NRQCD:

Ens.	β	lattice	$am_{u,d}$	am_s	m_π[MeV]	a [fm]	L [fm]	confs
C54	2.13	$24^3 \times 64$	0.005	0.04	336	0.1119(17)	2.7	1676

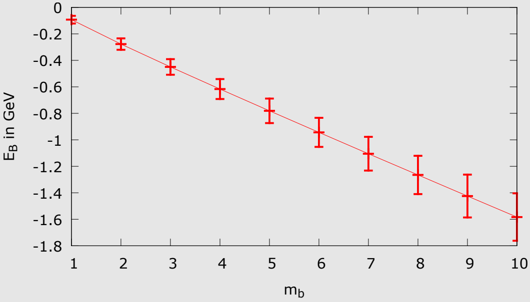
- Gauge configurations generated by RBC and UKQCD collaborations [arXiv:1409.0497]
- Iwasaki gauge action
- domain-wall fermions

The NRQCD BB^* correlator:

$$\sum_{\vec{x}_1,\vec{x}_2,\vec{x}'_1,\vec{x}'_2} \mathrm{e}^{i\vec{p}_1(\vec{x}_1-\vec{x}'_1)} \mathrm{e}^{i\vec{p}_2(\vec{x}_2-\vec{x}'_2)} \delta_{\vec{x}_1,\vec{x}_2} \delta_{\vec{x}'_1,\vec{x}'_2}$$
$$\left(\bar{b}\gamma_5 d(\vec{x}_1)\bar{b}\gamma_i u(\vec{x}_2) - \bar{b}\gamma_5 u(\vec{x}_1)\bar{b}\gamma_i d(\vec{x}_2)\right) \left(\bar{d}\gamma_5 b(\vec{x}'_1)\bar{u}\gamma_i b(\vec{x}'_2) - \bar{u}\gamma_5 b(\vec{x}'_1)\bar{d}\gamma_i b(\vec{x}'_2)\right)$$

The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ state with NRQCD- details

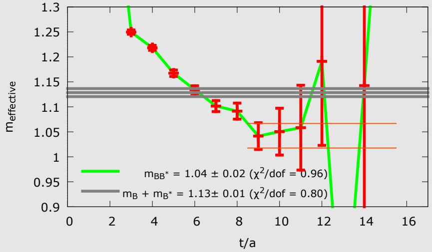
From static-light computations we can estimate how the absolute value of the binding energy increases if we increase the mass of the b-quark:



To get a clear signal in a first step we use non-physically heavy $\bar{b}$ quarks ($5.0 \times m_b$)

The $I(J^P) = 0(1^+)$ $ud\bar{b}\bar{b}$ state with NRQCD - preliminary results

- compute masses of B, B^* and BB^*
- if $m_{BB^*} < m_B + m_{B^*}$: bound state in $I(J^P) = 0(1^+)$ channel



$m_{BB^*} - (m_B + m_{B^*}) \approx -100$ MeV, i.e. strong indication that mass of the four-quark $ud\bar{b}\bar{b}$ is smaller than the sum of B and B^*

$\Longrightarrow$ Qualitative confirmation of static-light result

(cf. also A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, arXiv:1607.05214 [hep-lat])

To do: Include B^*B^* structure with quantum numbers $I(J^P) = 0(1^+)$.

Summary BB^* and $B\bar{B}$ systems

- $B\bar{B}$ systems are experimentally rather easy to access, but theoretically challenging.
- Candidate for a binding $B\bar{B}$ state with $I(J^P) = 1(1^+)$ (i.e. Z_b^+) is currently investigated, we find $E_B = (-58 \pm 71)$ MeV
- $B^{(*)}B^{(*)}$ systems are experimentally harder to observe, but theoretically easier to investigate
- For a BB^* system with light quarks $qq = ud$ with quantum numbers $I(J^P) = 0(1^+)$ using NRQCD we find qualitative confirmation of our previous static-light result ($E_B = -90^{+43}_{-36}$ MeV)
- Work in progress

Outlook

- Diquark-antidiquark structure
- Inclusion of heavy spin effects to $B\bar{B}$ system

Thank you.

Backup

Meson content of a four-quark state

- One can extend the meson content of the four-quark states by application of the following light quark projectors on $\mathcal{O}$:
  - Parity projectors: $\mathcal{P}_{P=+} = \frac{1+\gamma_0}{2}$ and $\mathcal{P}_{P=-} = \frac{1-\gamma_0}{2}$
  - Spin projectors: $\mathcal{P}_{j_z=\uparrow} = \frac{1+i\gamma_0\gamma_3\gamma_5}{2}$ and $\mathcal{P}_{j_z=\downarrow} = \frac{1-i\gamma_0\gamma_3\gamma_5}{2}$
- $P = -$: S state, meson in the ground state
- $P = +$: P state, first excitation
- $\uparrow\downarrow$: light quark angular momentum
- An example for $\mathcal{O}_{B\bar{B}}$:
  - $\Gamma = \gamma_5 \quad \hat{=} \quad +S\uparrow S\uparrow + S\downarrow S\downarrow + P\uparrow P\uparrow + P\downarrow P\downarrow$
  - $\Gamma = \gamma_0\gamma_5 \quad \hat{=} \quad -S\uparrow S\uparrow - S\downarrow S\downarrow + P\uparrow P\uparrow + P\downarrow P\downarrow$
  - Therefore a B$\bar{B}$ state with $\Gamma = \gamma_5 - \gamma_0\gamma_5$ only contains S mesons.

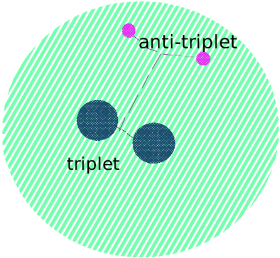
The BB system - Expectations

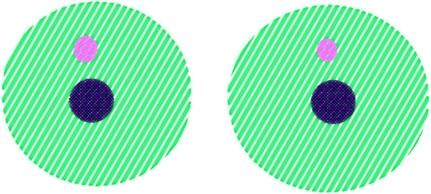
small separations of the static antiquarks:

- interaction due to 1-gluon exchange
- bound state: static $\bar{Q}\bar{Q}$ pair in a color triplet (attractive) $\longrightarrow$ antidiquark

large separations of the static antiquarks:

- screening of the antiquark-antiquark interaction due to light quarks (stronger, the more massive the light quarks)
- basically 2 static-light mesons





Lattice Setups

BB and $B\bar{B}$ in the static-light approach:

Ens.	β	lattice	$a\mu$	m_π[MeV]	a [fm]	L [fm]	confs
B40.24	3.90	$24^3 \times 48$	0.0040	340	0.0790(26)	1.9	480
B85.24	3.90	$24^3 \times 48$	0.0085	480	0.0790(26)	1.9	400
B150.24	3.90	$24^3 \times 48$	0.0150	650	0.0790(26)	1.9	260

Gauge configurations generated by ETMC

BB^* with NRQCD:

Ens.	β	lattice	$am_{u,d}$	am_s	m_π[MeV]	a [fm]	L [fm]	confs
C54	2.13	$24^3 \times 64$	0.005	0.04	336	0.1119(17)	2.7	1676

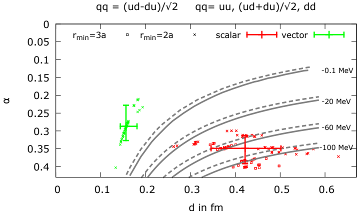
Gauge configurations generated by RBC and UKQCD collaborations

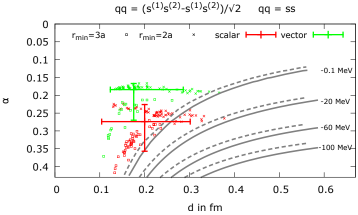
Different attractive BB channels

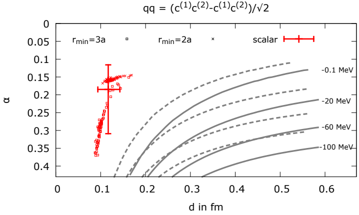
- spin scalar isosinglet:
  - qq spin $j_z = 0$
  - antisymmetric flavour $qq \in \{(ud - du)/\sqrt{2}, (s^1 s^2 - s^2 s^1)/\sqrt{2}, (c^1 c^2 - c^2 c^1)/\sqrt{2})\}$
  - $I(J^P) = 0(1^+)$
- spin vector isotriplet:
  - qq spin $j_z = 1$
  - symmetric flavour $qq \in \{uu, (ud + du)/\sqrt{2}, dd, ss, cc\}$
  - $I(J^P) \in \{1(0^+), 1(1^+), 1(2^+)\}$

Perform a large number of fits varying...

- the temporal separation at which the lattice potential is read of the correlation function
- the range at which the fit to the potential is performed



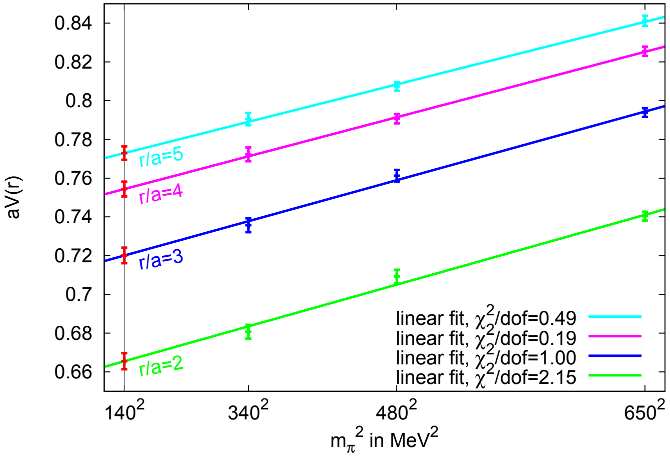




Binding for light isosinglet channel only!

Extrapolation to the physical pion mass

Example plots for a t-range $[4a \ldots 9a]$



The GEP

1. Build a matrix $C(t)$ of correlation functions $C_{ij}(t)$
2. Solve the GEP:

$$C_{jk}(t)v_k^{(n)}(t, t_0) = \lambda^{(n)}(t, t_0)C_{jk}(t_0)v_k^{(n)}(t, t_0)$$

3. And find:

$$m_{\text{eff}}^{(n)}(t, t_0) = \lim_{t \to \infty} \frac{1}{a} \log \frac{\lambda^{(n)}(t, t_0)}{\lambda^{(n)}(t + a, t_0)}$$