Isoscalar $\pi\pi$ scattering and the $\sigma/f_0(500)$ resonance

Raúl Briceño

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[with Jozef Dudek, Robert Edwards & David Wilson]

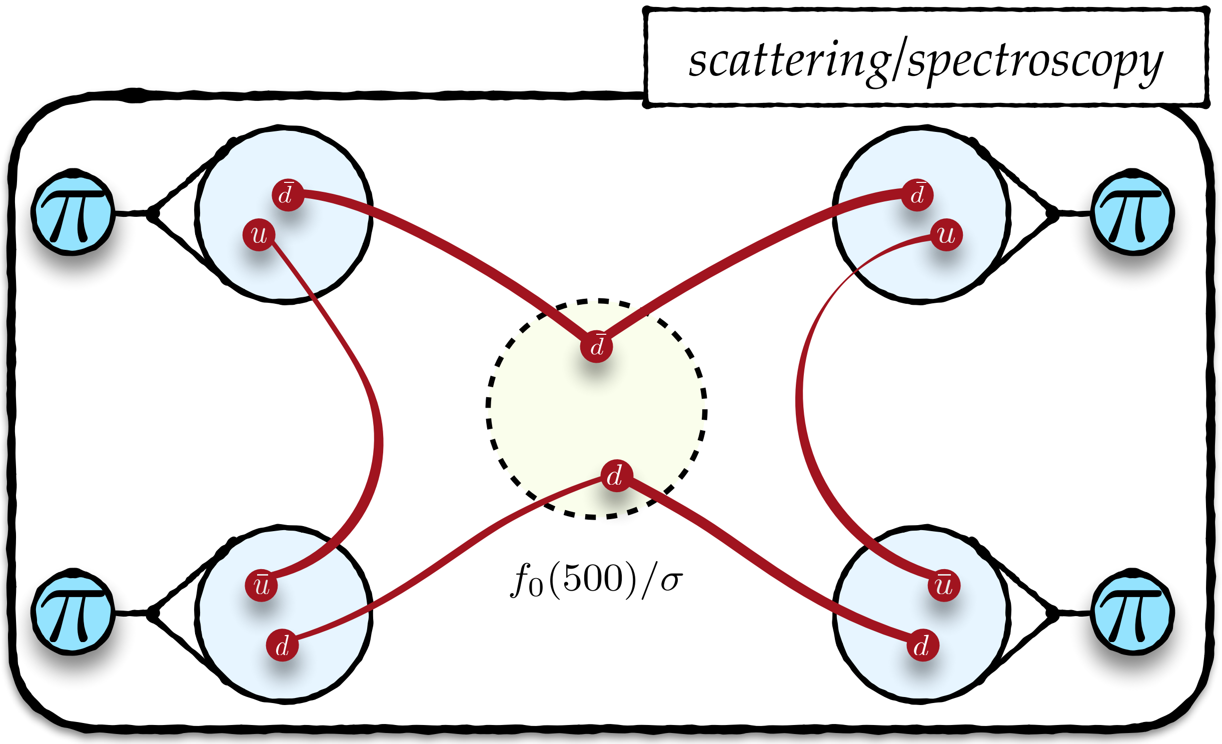
HadSpec Collaboration

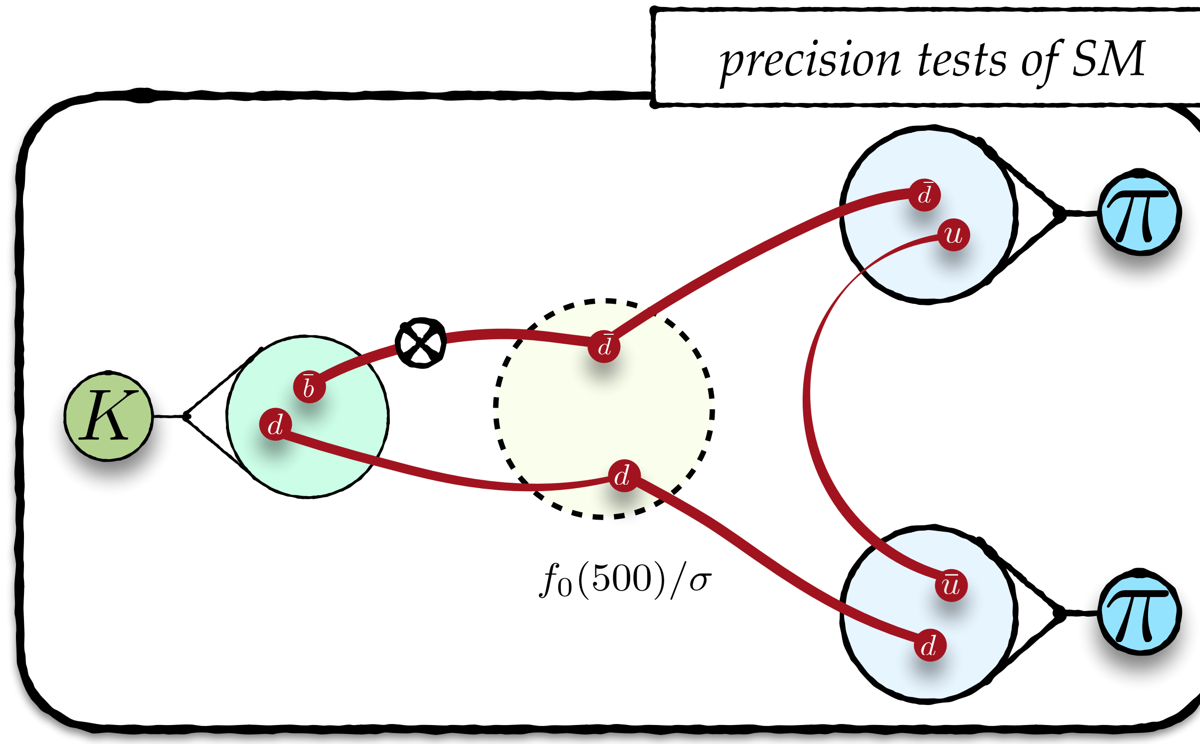
OLD DOMINION UNIVERSITY

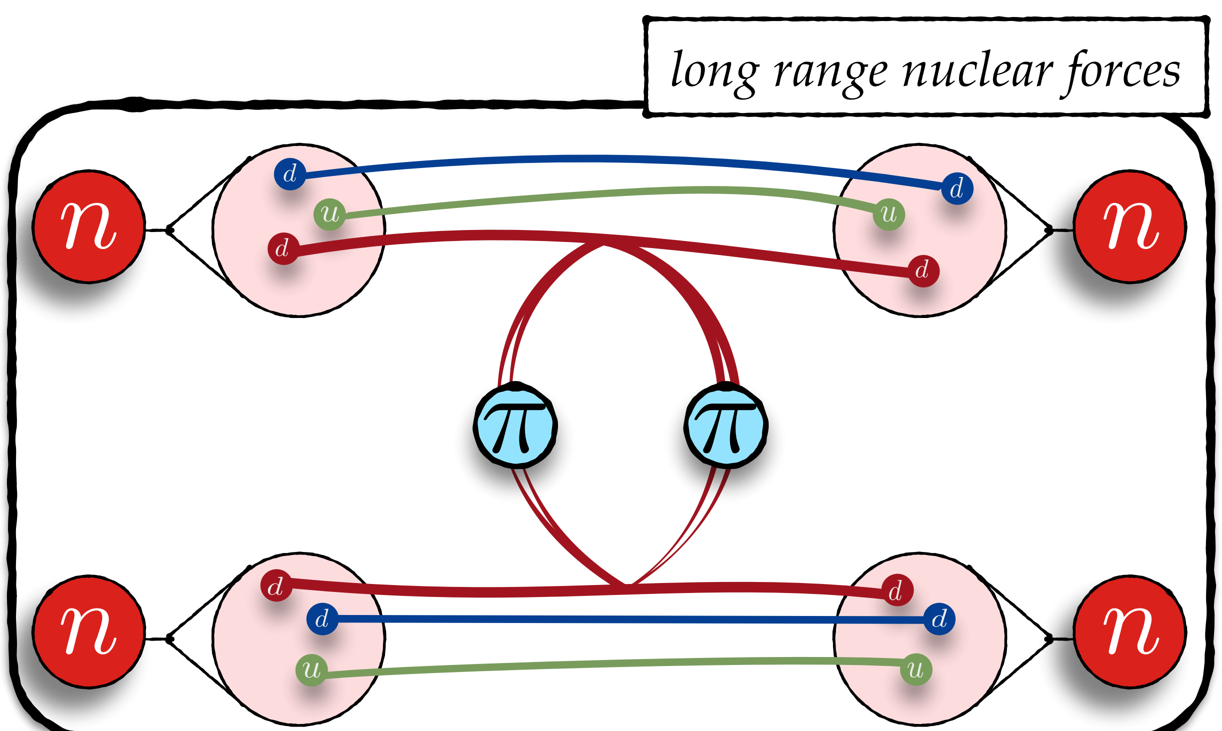
Jefferson Lab

Lattice 2016
Southampton, UK
July, 2016

Motivation



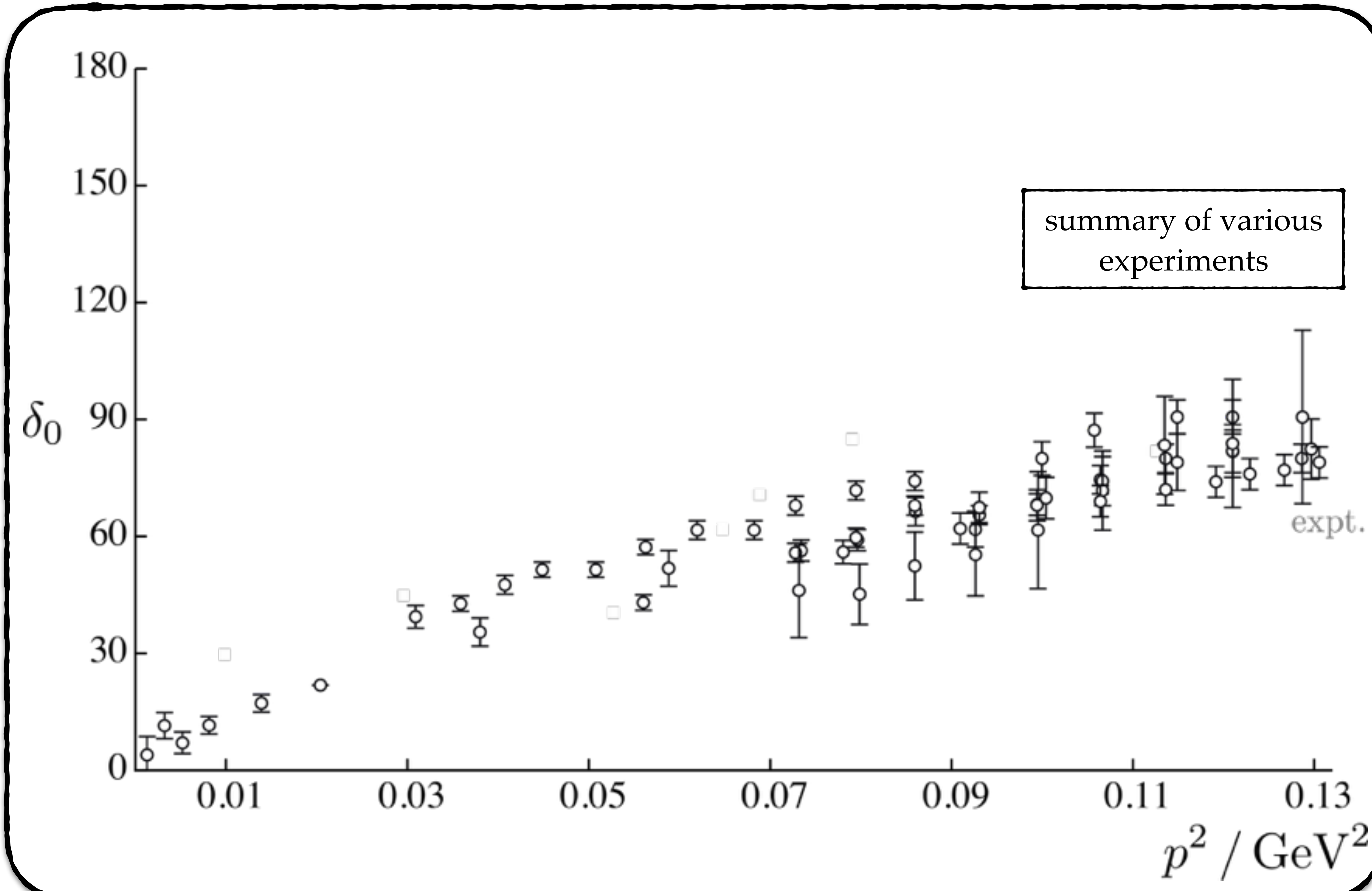




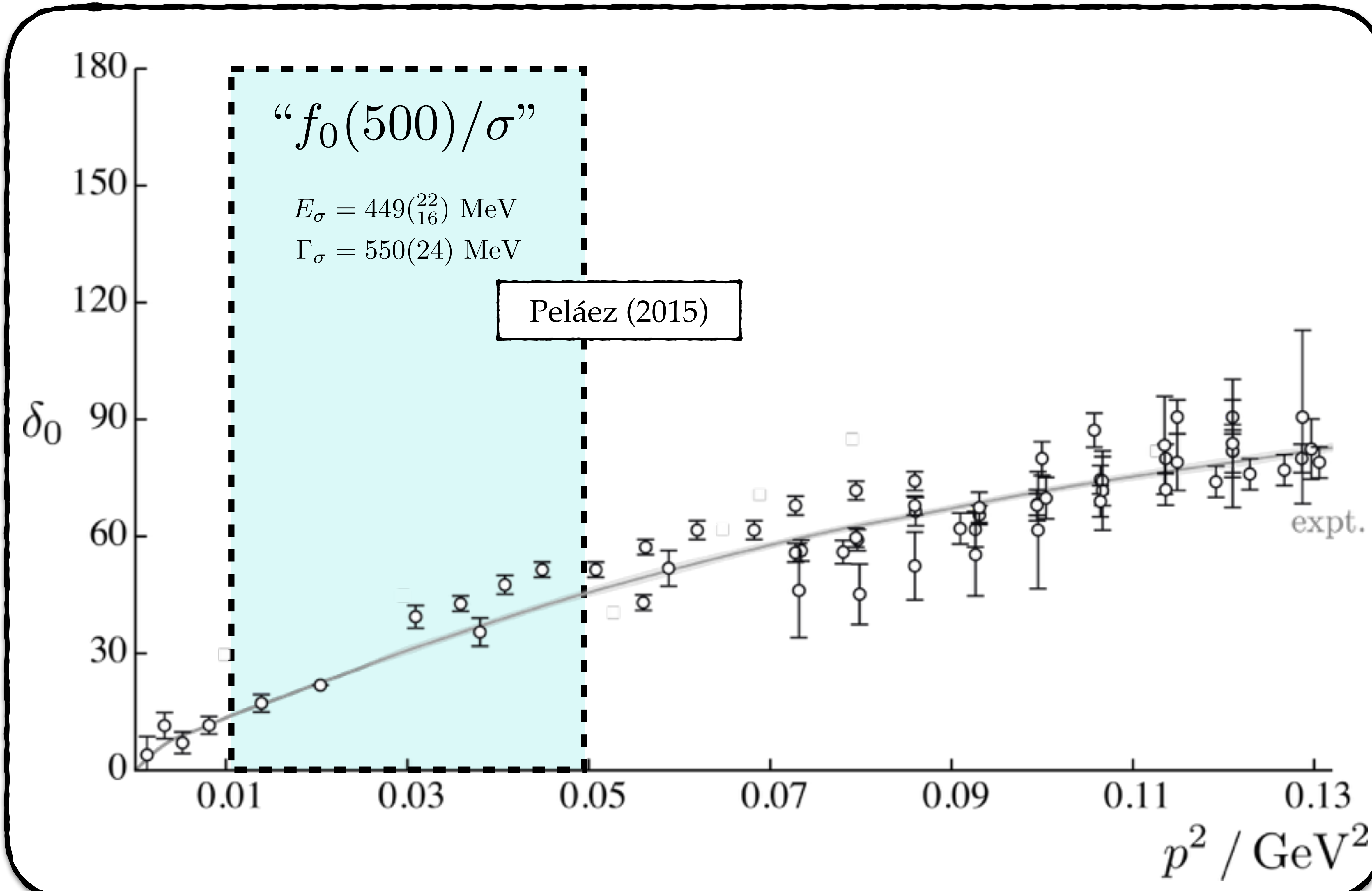
Sample of previous lattice efforts:

- Alford & Jaffe (2000)
- Prelovsek, et al. (2010)
- Fu (2013)
- Wakayama, et al. (2015)
- Howarth & Giedt, (2015)
- Z. Bai et al. (RBC, UKQCD) (2015)

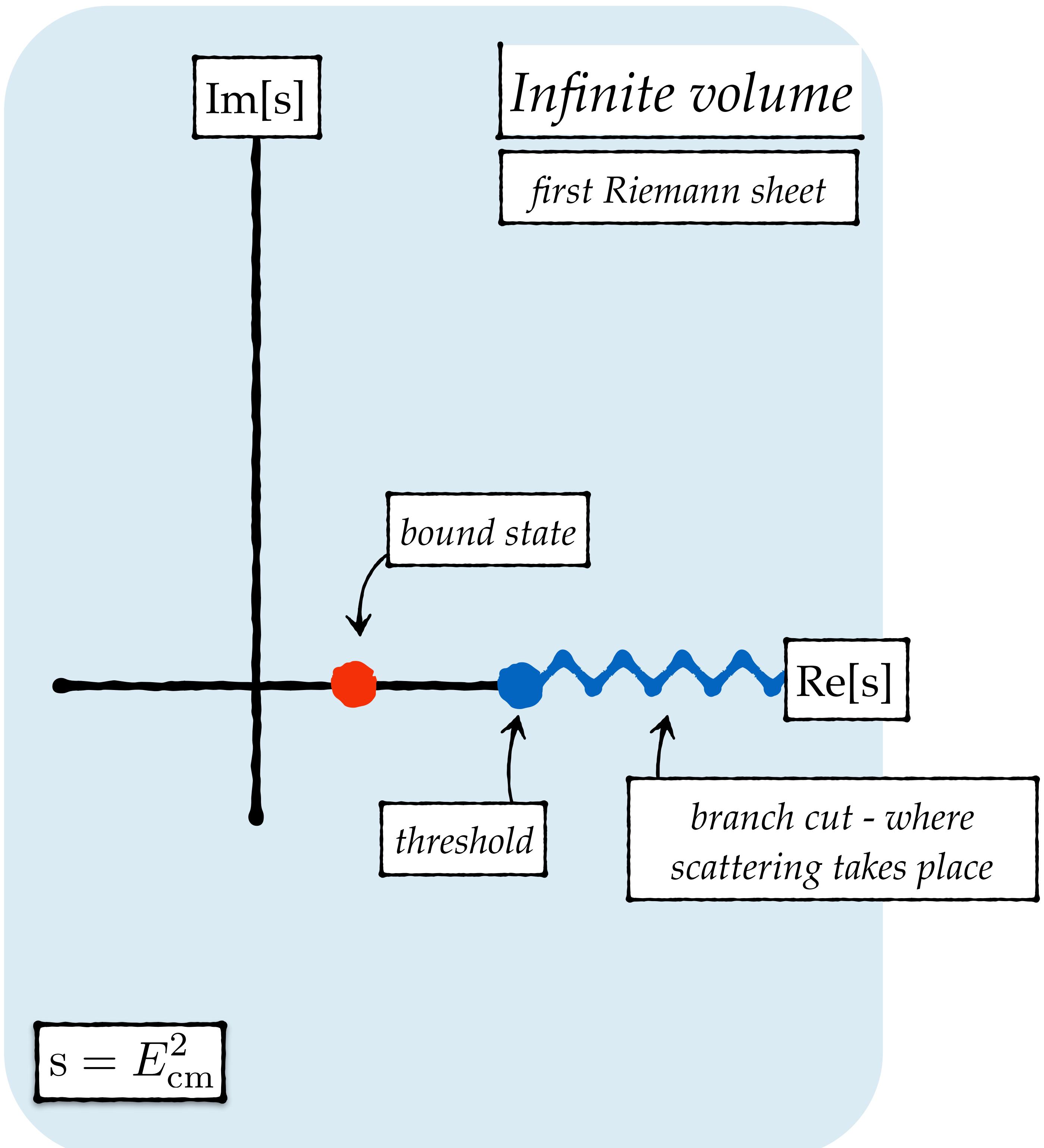
The experimental situation



The experimental situation



Finite vs. infinite volume spectrum



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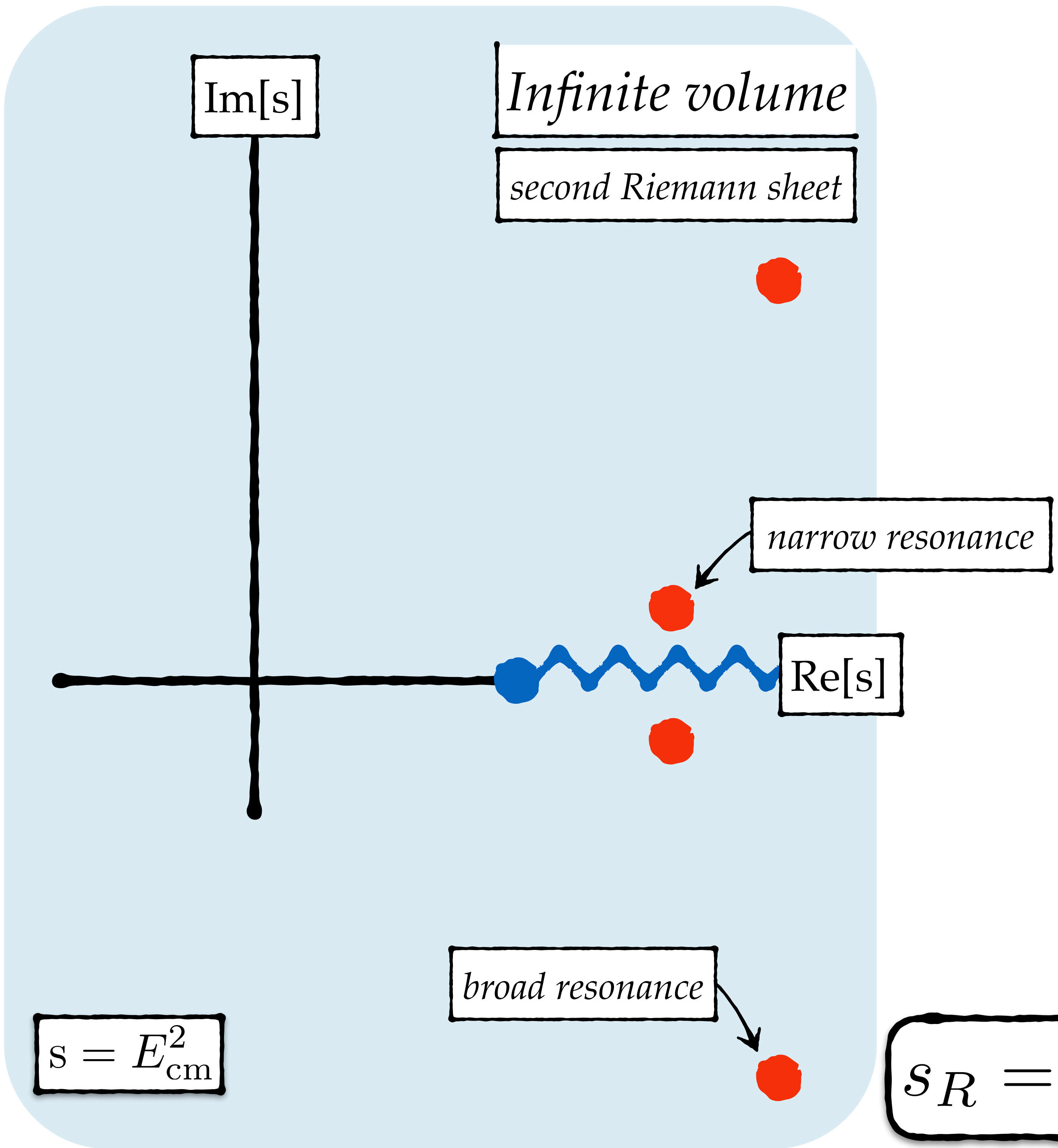
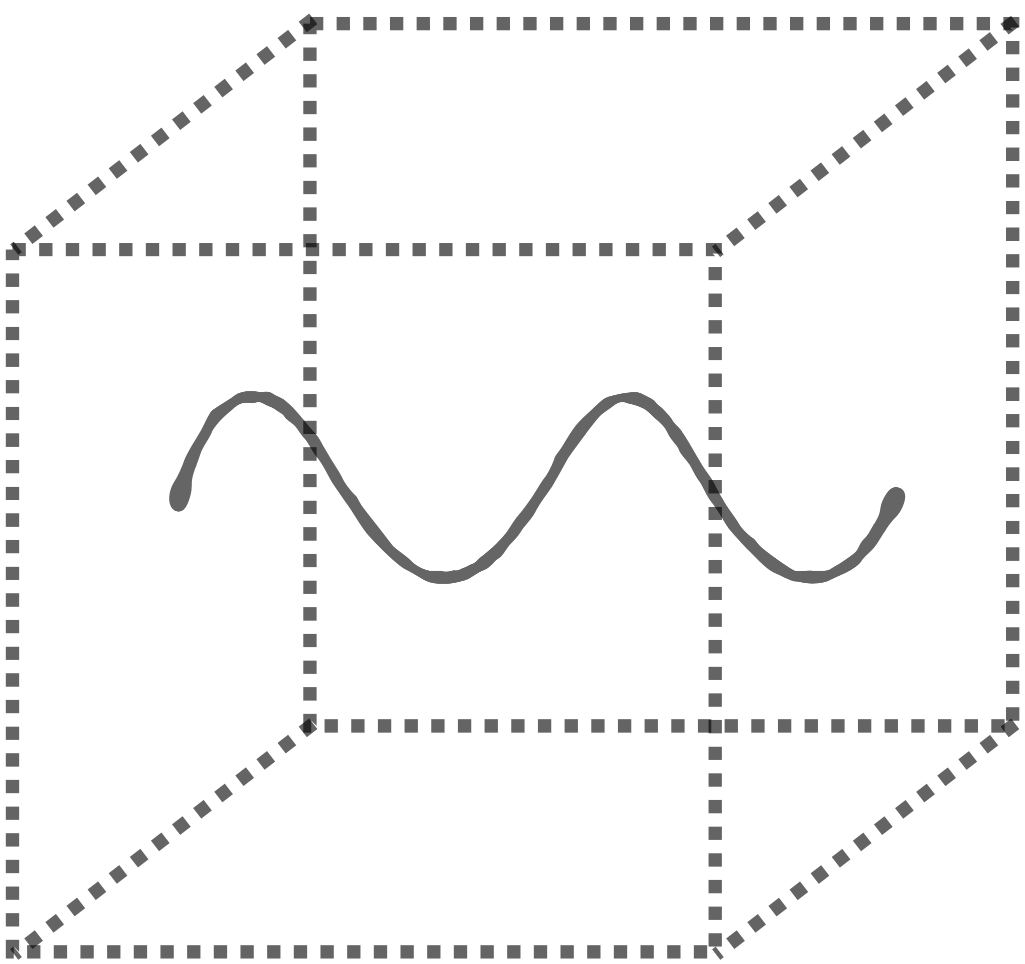
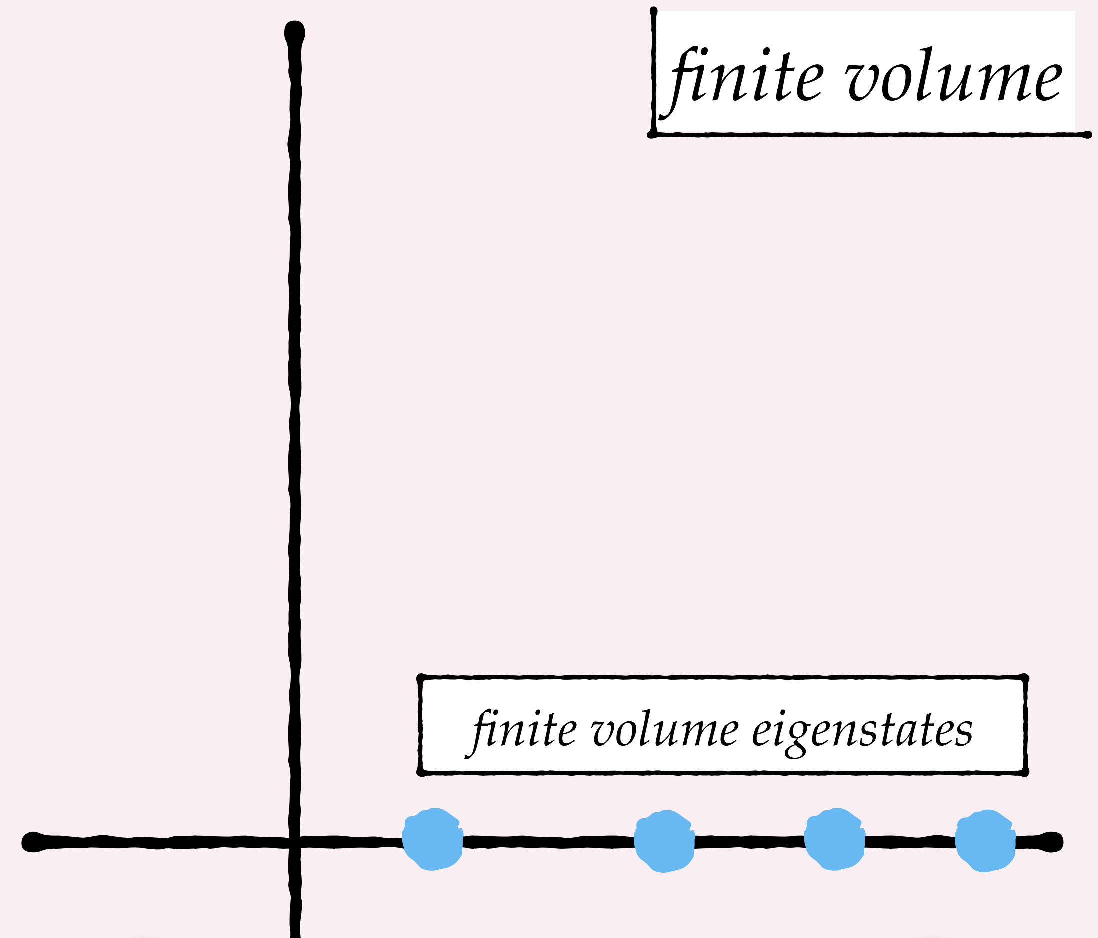


$$s_R = \left(E_R - \tfrac{i}{2}\Gamma_R\right)^2$$

Finite vs. infinite volume spectrum



"only a discrete number of modes can exist in a finite volume"



no continuum of states:

→ no cuts

→ no sheet structure

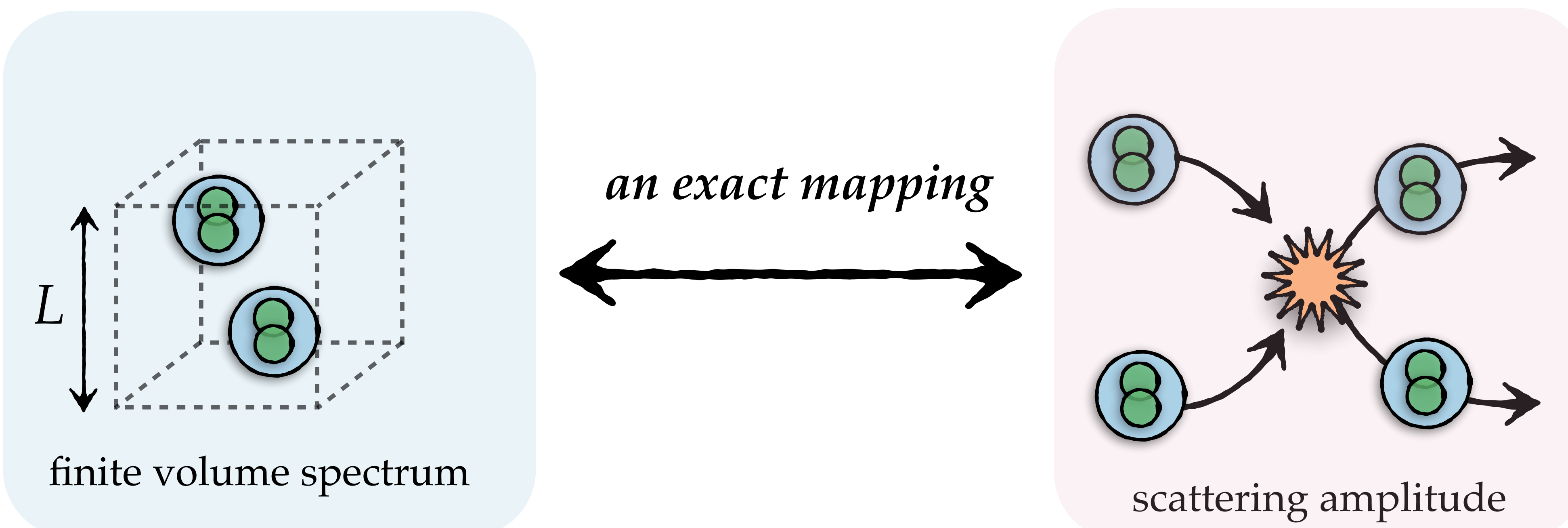
→ no resonances

no asymptotic states:

→ no scattering

Lüscher formalism

spectrum satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$



$E_L =$ finite volume spectrum

$L =$ finite volume

$F =$ known function

$\mathcal{M} =$ scattering amplitude

Lüscher formalism

spectrum satisfy: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Bernard, Lage, Meissner & Rusetsky (2008) [Nπ systems]
- RB, Davoudi, Luu & Savage (2013) [generic spinning systems]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) / RB & Hansen (2015) [moving inelastic spinning particles]

Extracting the spectrum

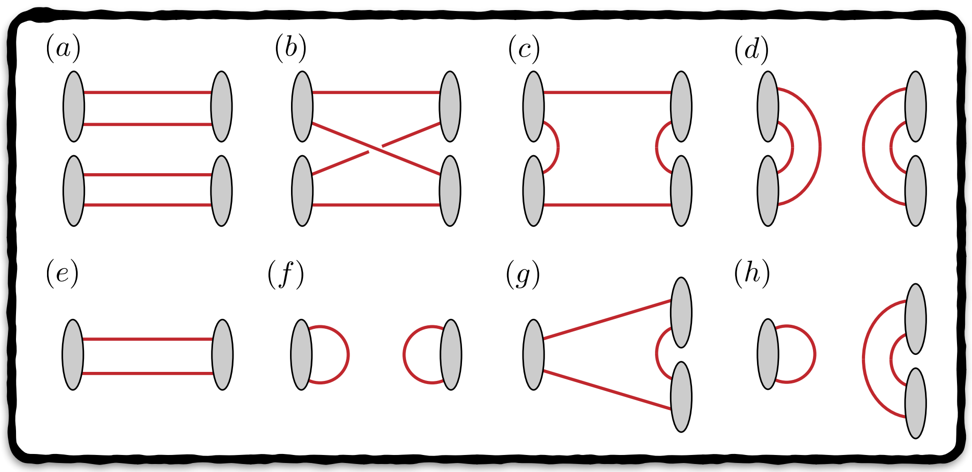
Two-point correlation functions:

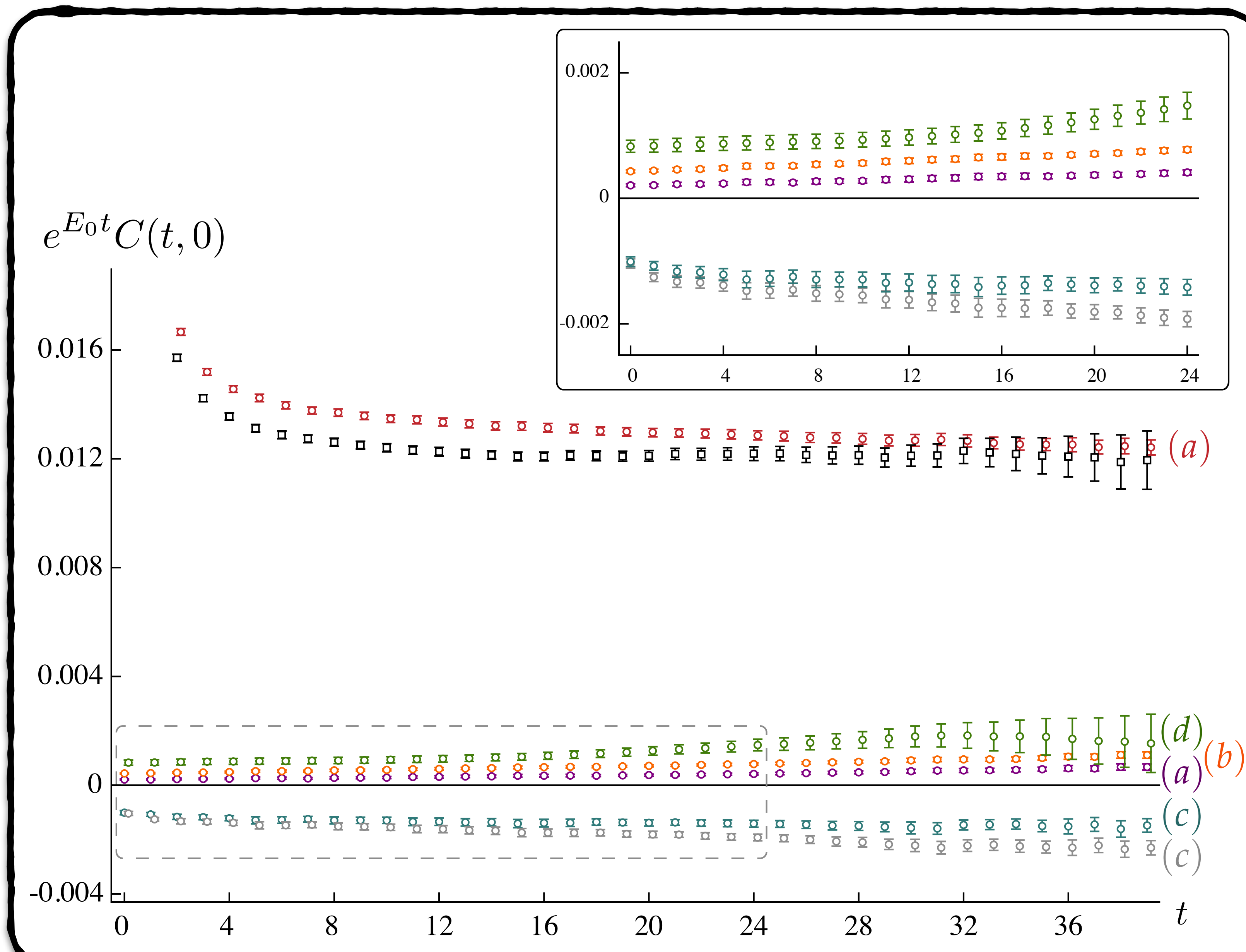
$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

- Evaluate **all** Wick contraction - [distillation - Peardon, *et al.* (Hadron Spectrum, 2009)]

e.g. $\pi_{[000]}\pi_{[110]}$

$m_\pi = 236$ MeV





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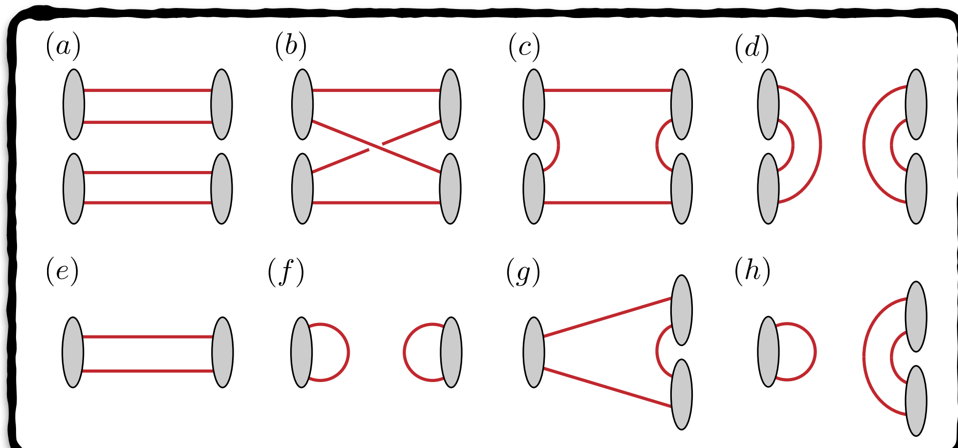

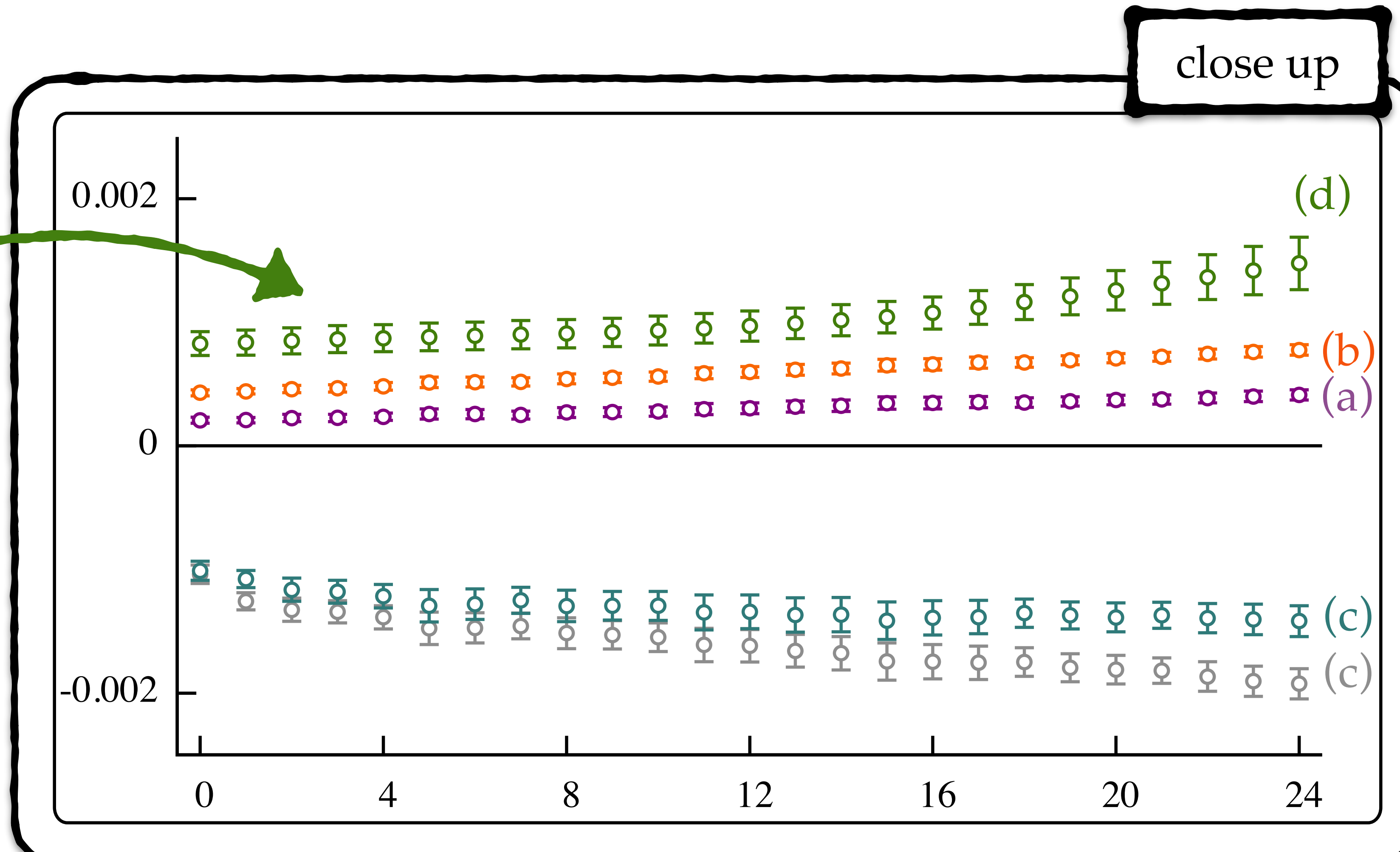

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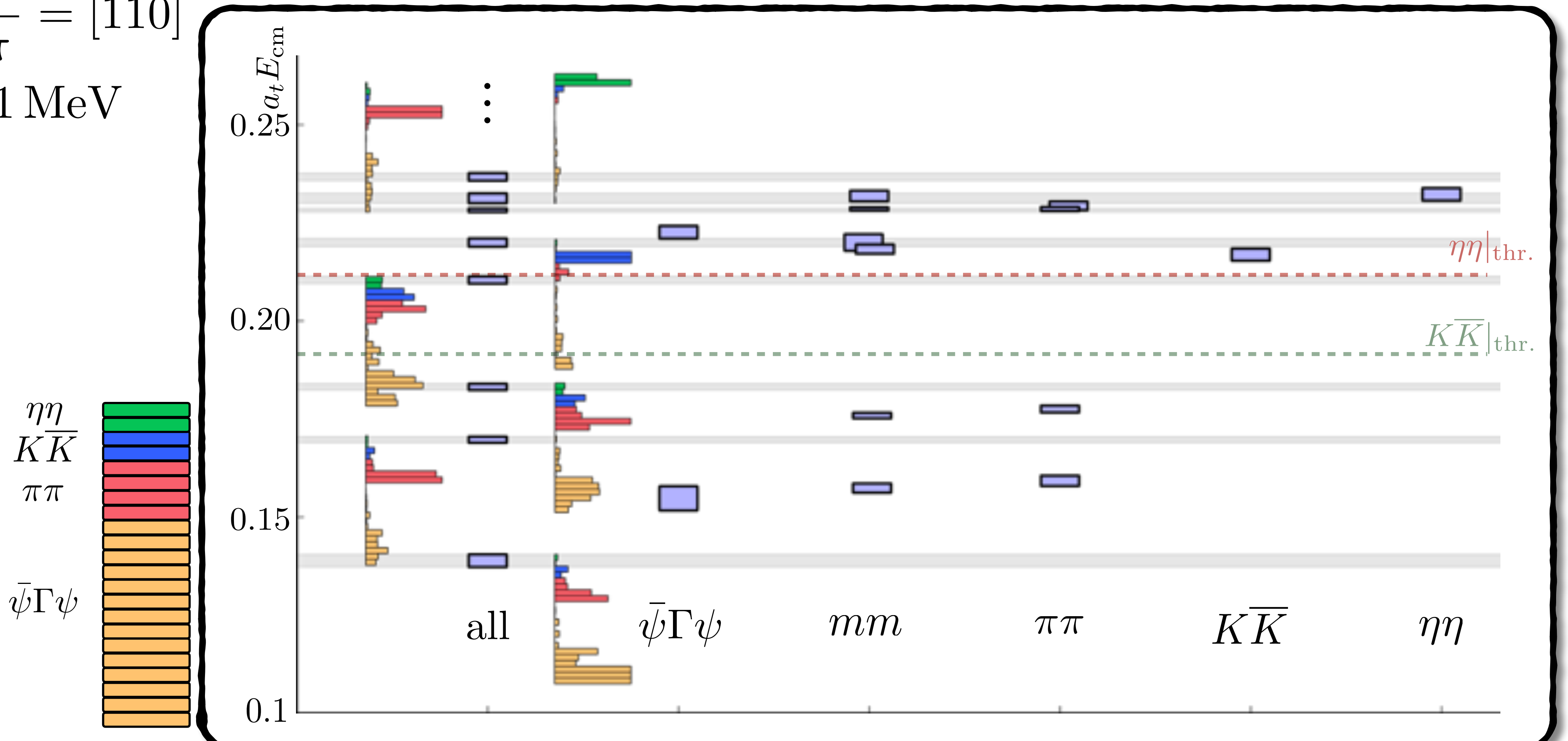
- Evaluate **all** Wick contraction - [distillation - Peardon, *et al.* (Hadron Spectrum, 2009)]
- Use a large basis of operators with the same quantum numbers
- 'Diagonalize' correlation function *variationally*

e.g. $\vec{d} = \frac{\vec{P}L}{2\pi} = [110]$

$m_\pi = 391\ \mathrm{MeV}$

$L/a_s = 24$



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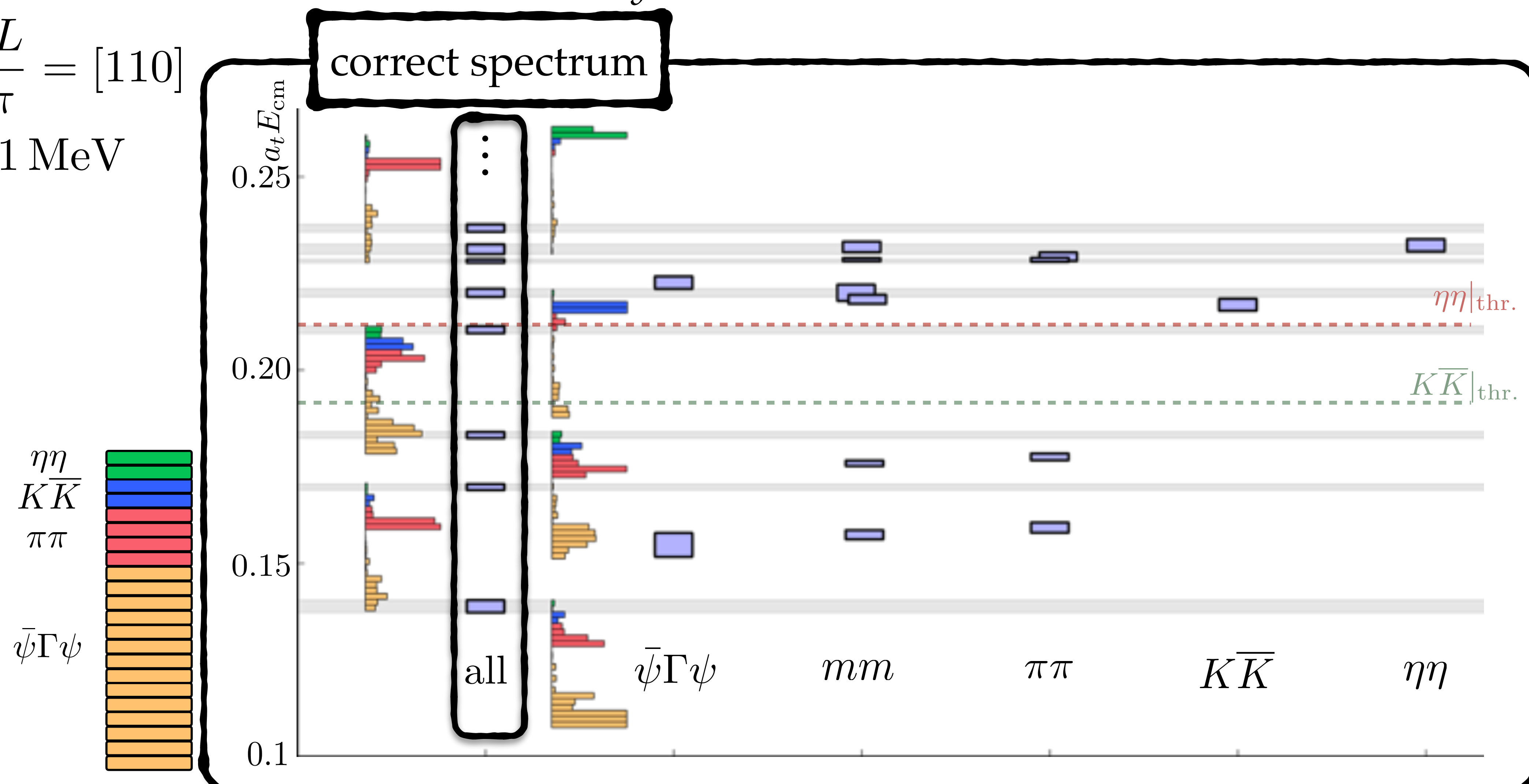
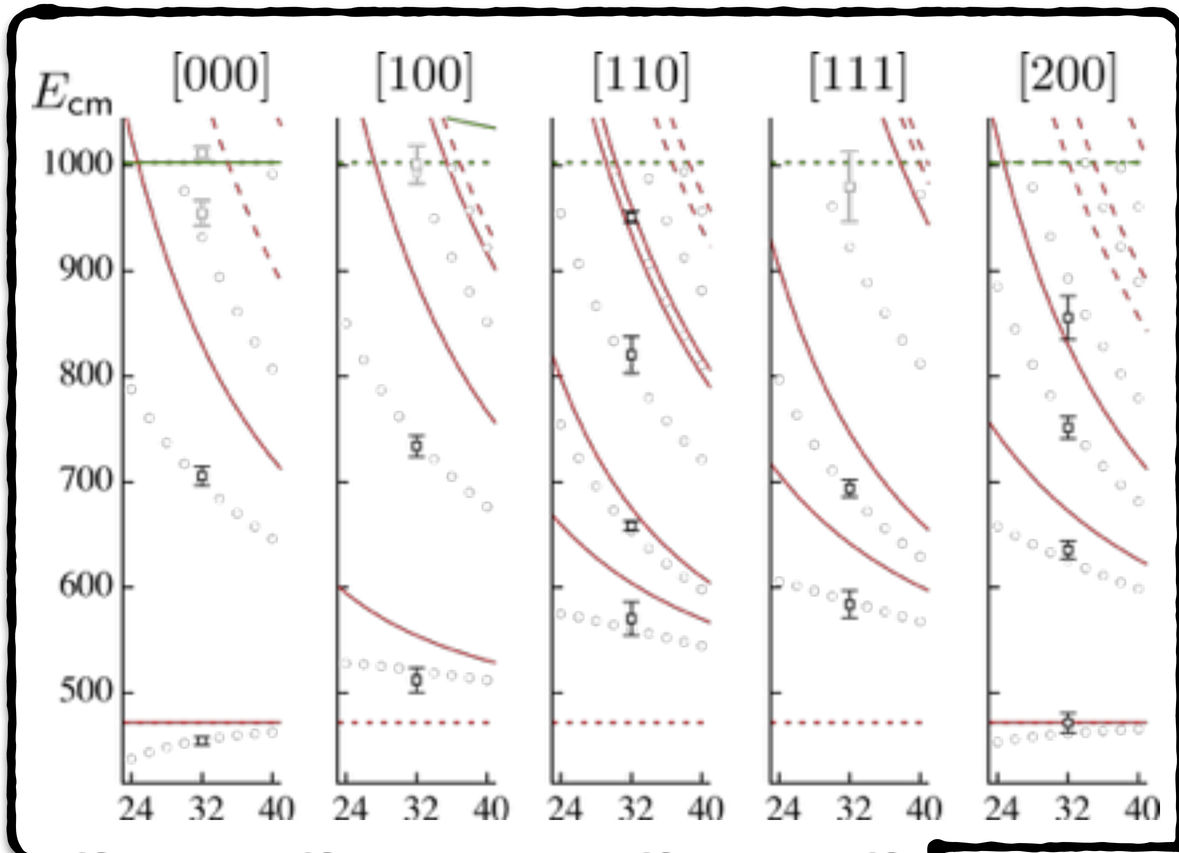
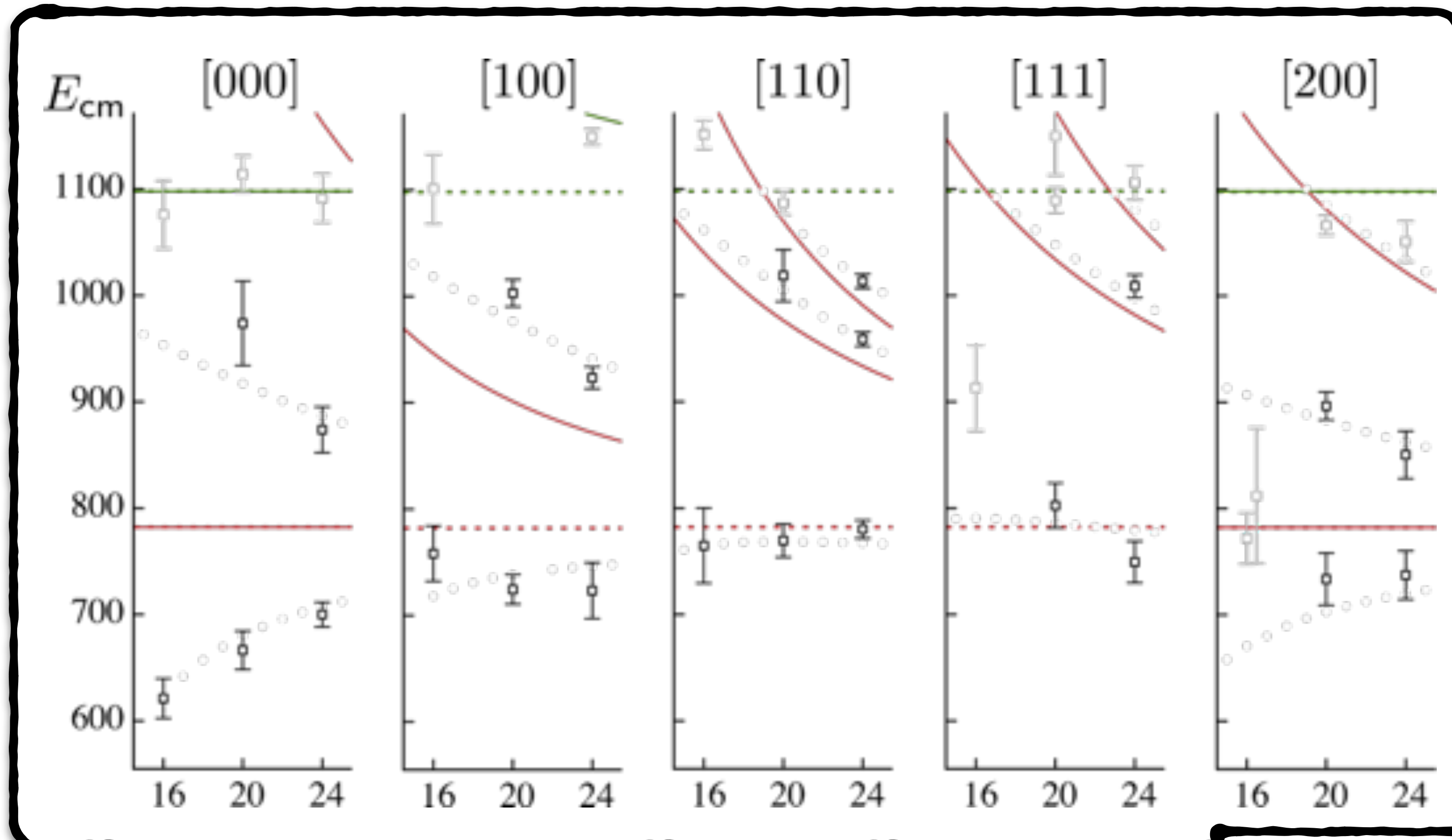


Finite volume spectra



m_π=236 MeV



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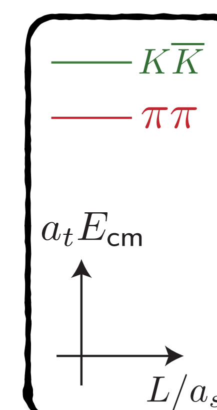
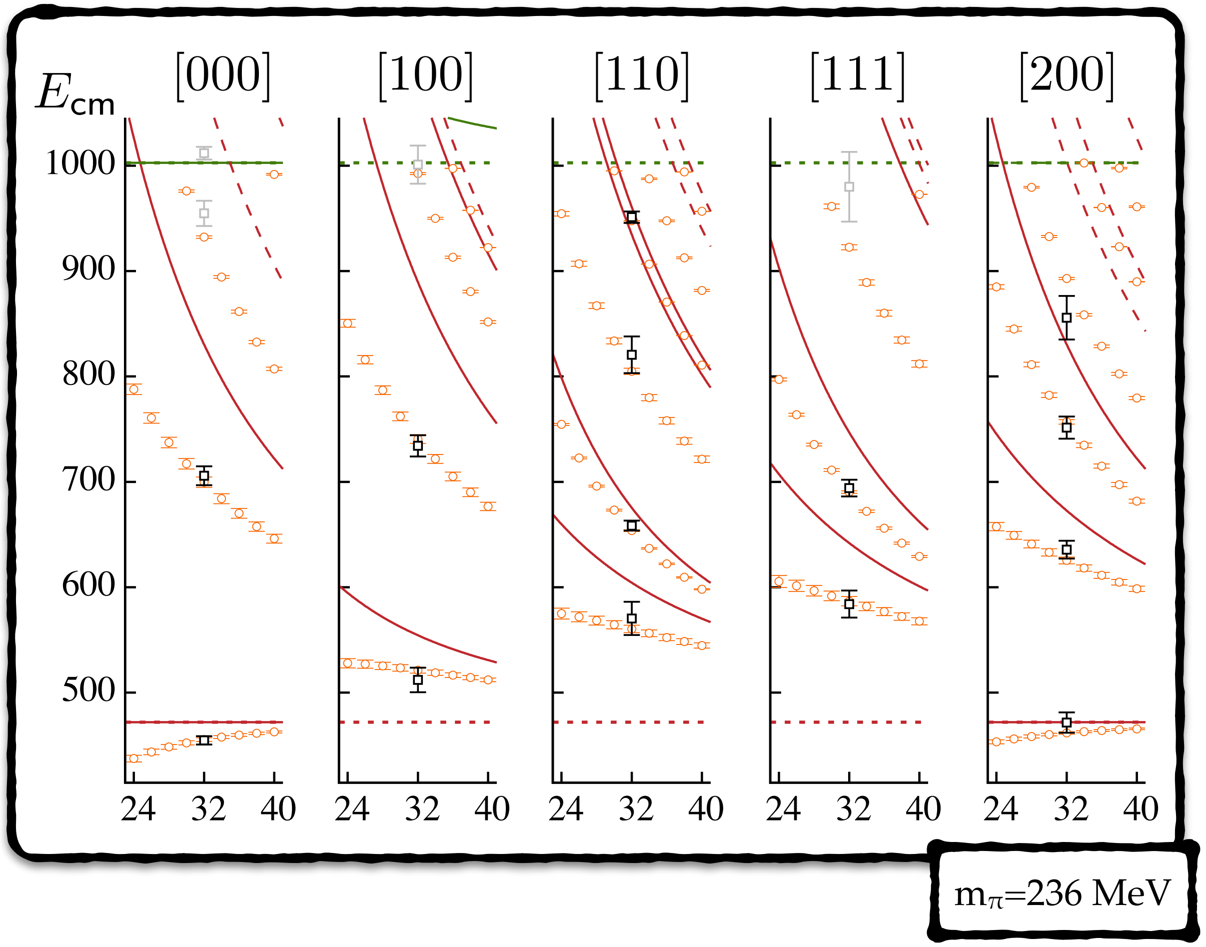
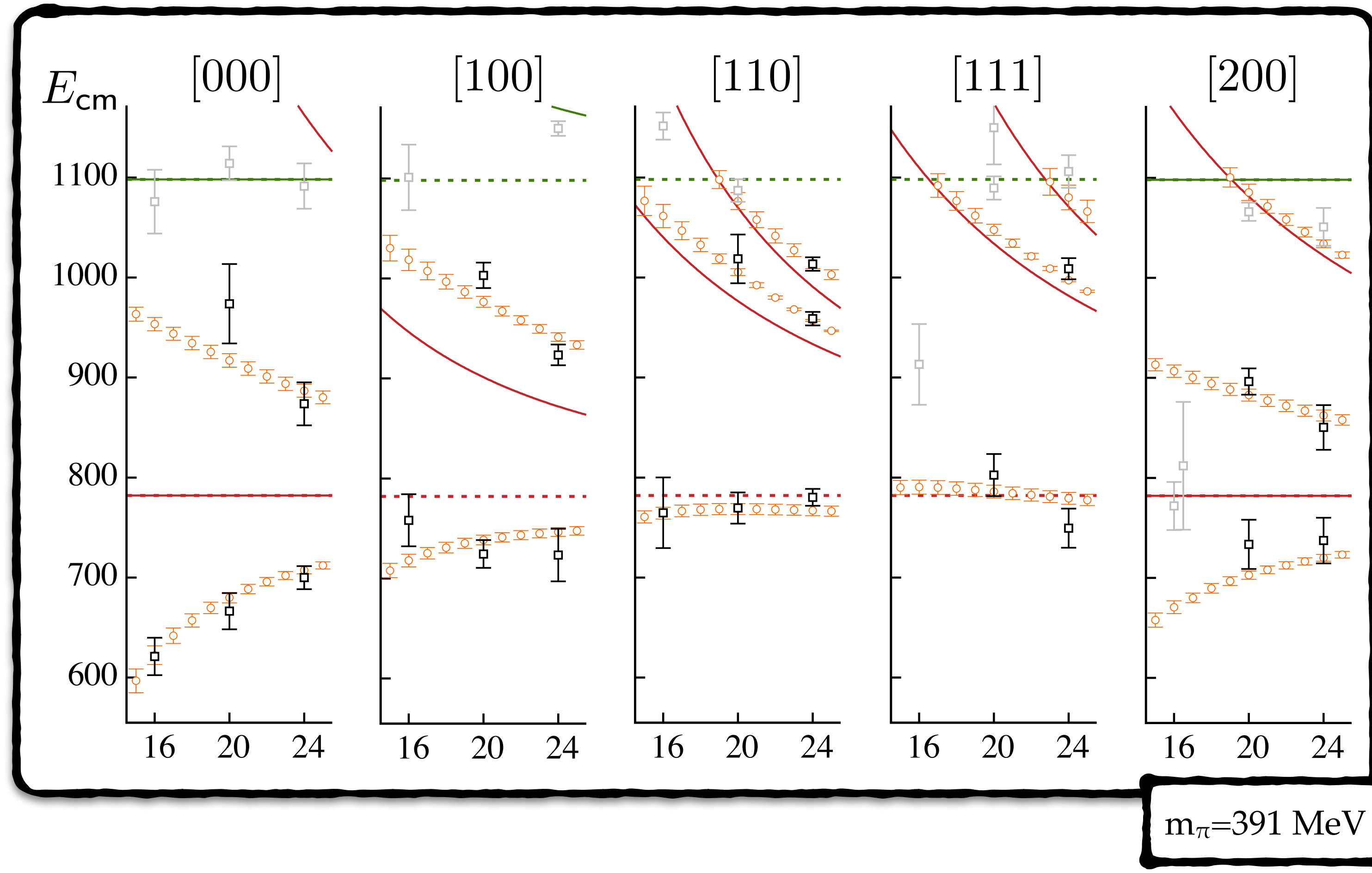


Finite volume spectra

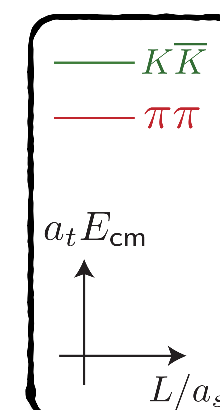




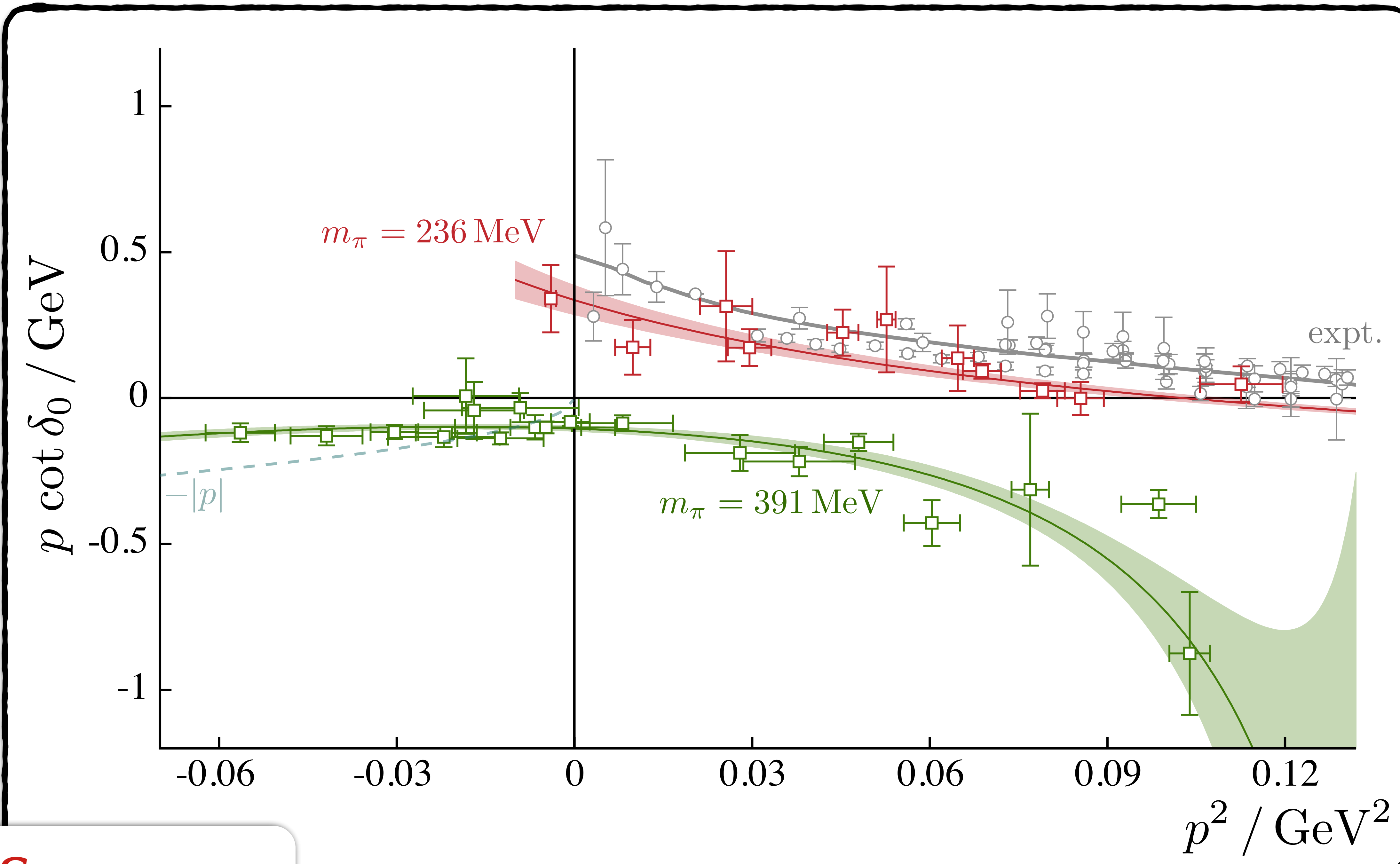
- Spectrum satisfies: $\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$
- One channel, ignoring partial wave mixing: $\cot\delta_0(E_{\mathsf{cm}}) + \cot\phi(P, L) = 0$
- Use a various parametrizations

e.g. $$\mathcal{M}^{-1} = \mathcal{K}^{-1} + I, \qquad \mathrm{Im}(I) = -\rho \quad \text{[unitarity]}$$

$$\mathcal{K} = \frac{g^2}{s_0 - s} + c$$



Scattering amplitude vs m_π



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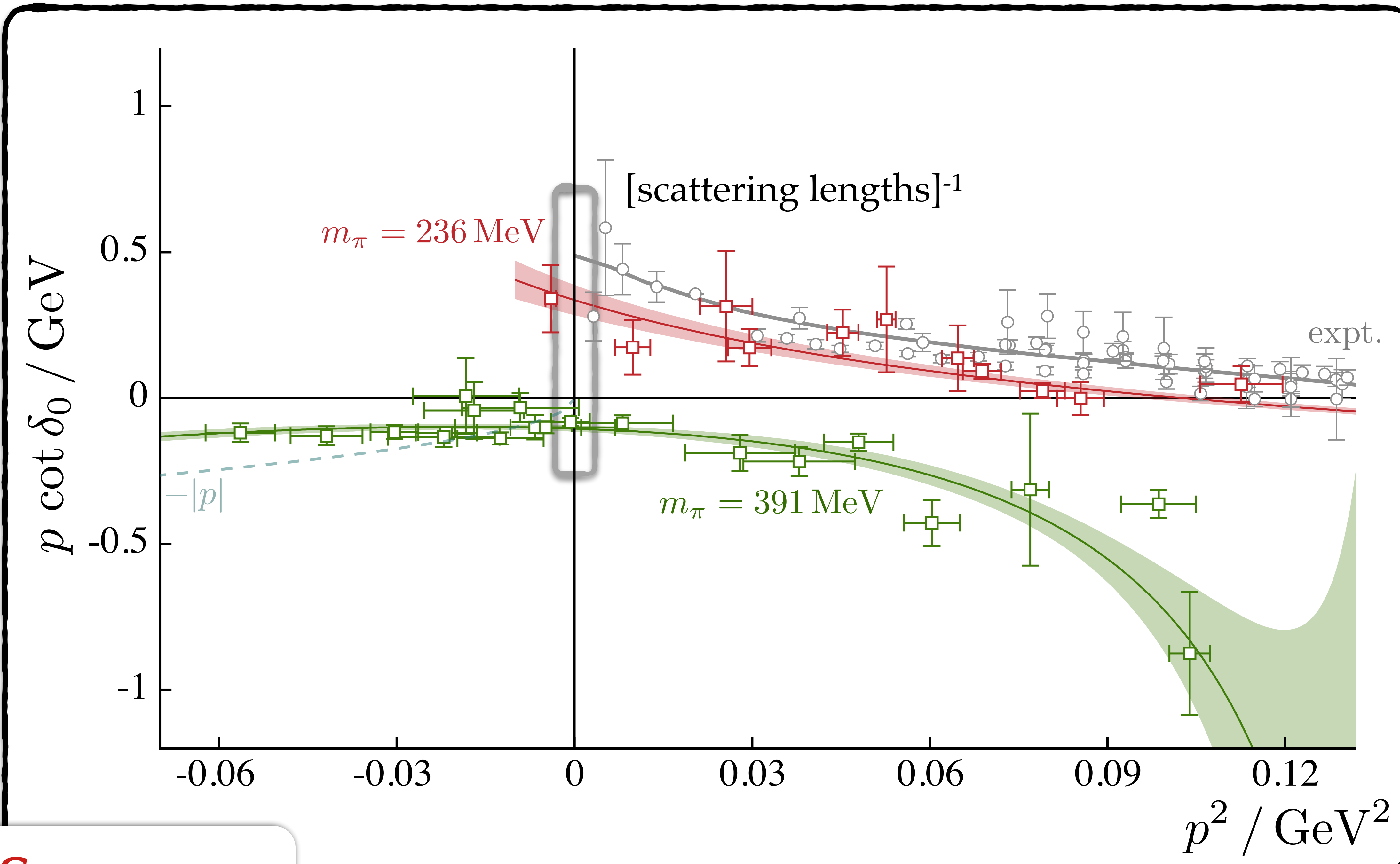

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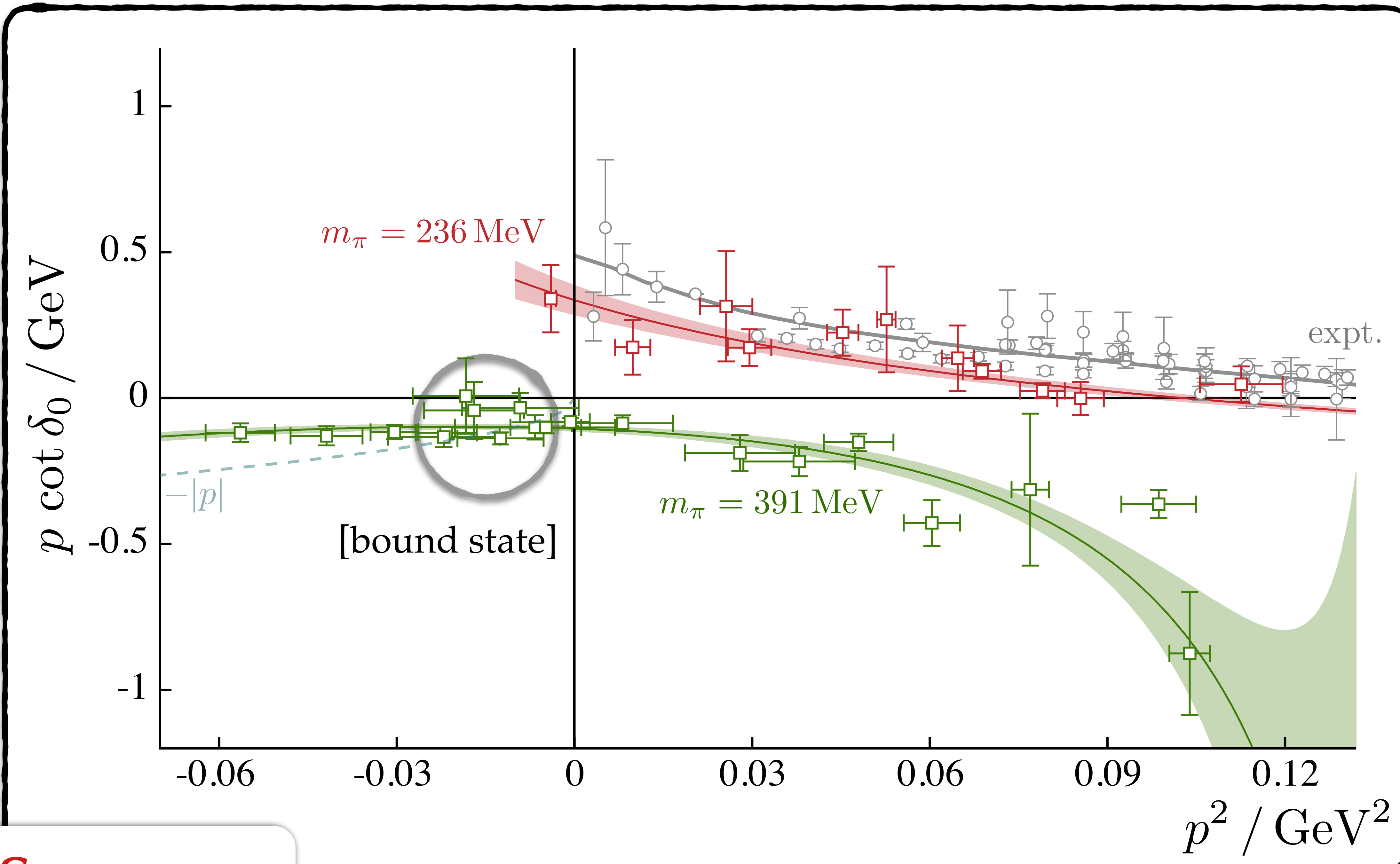

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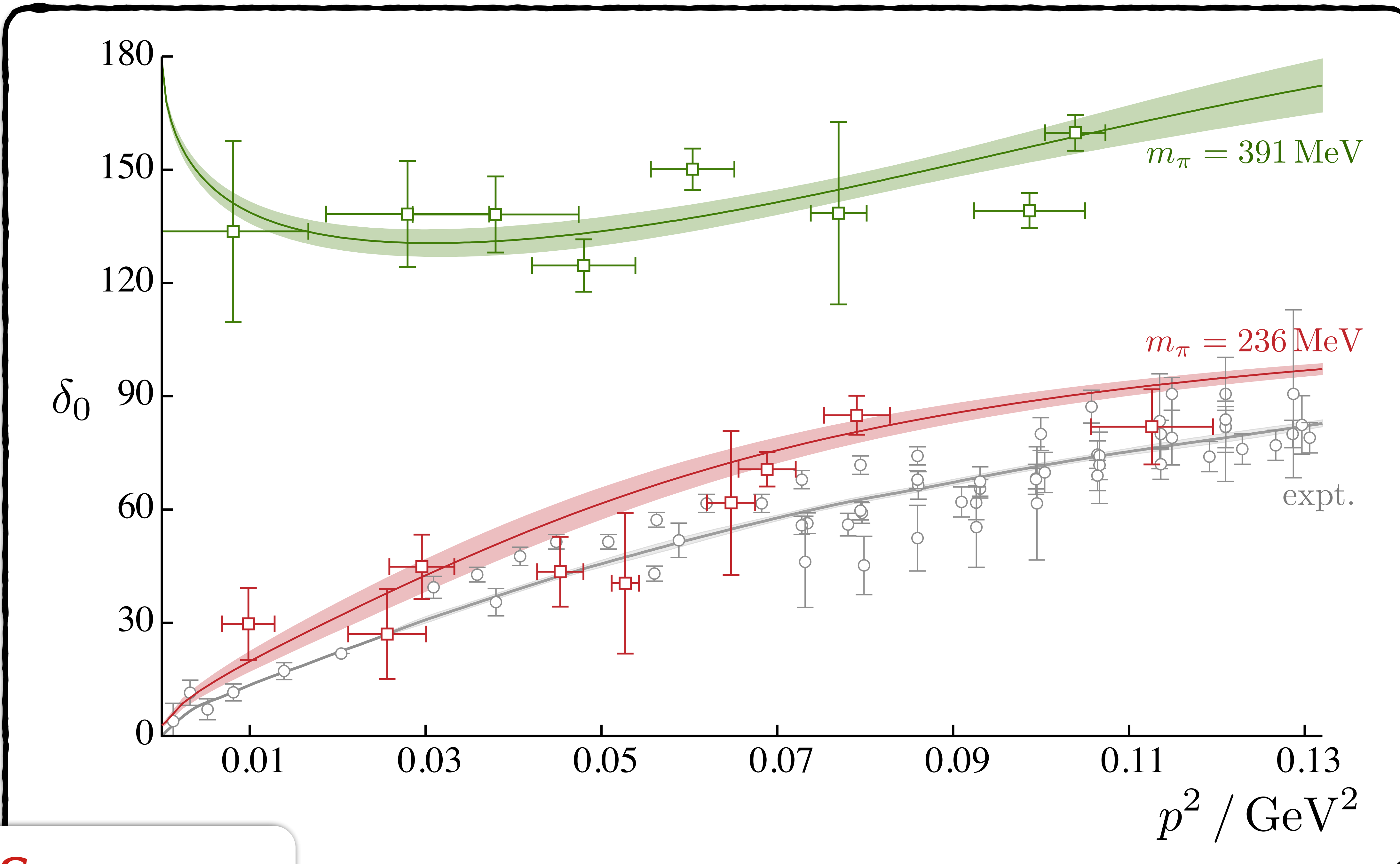

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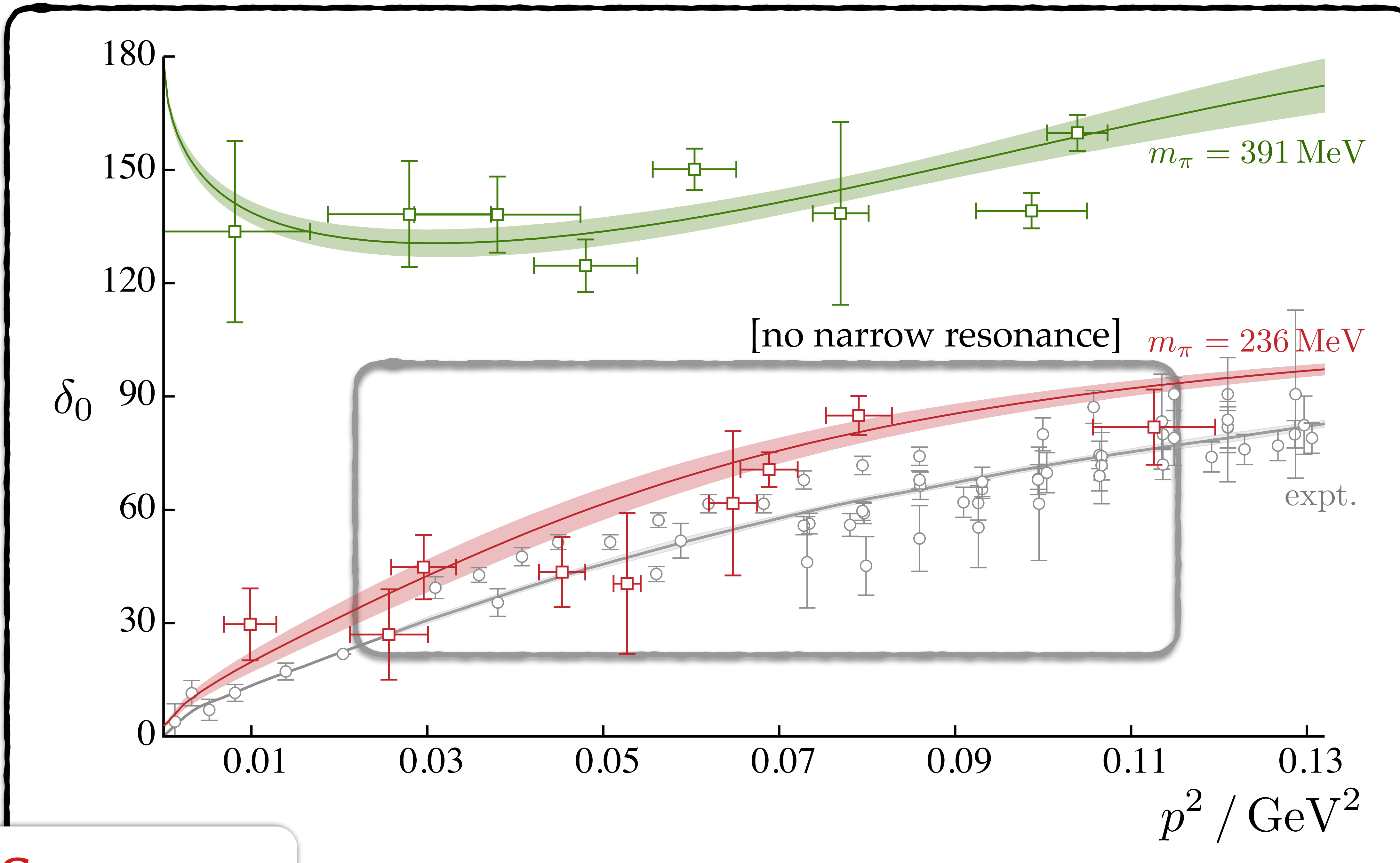

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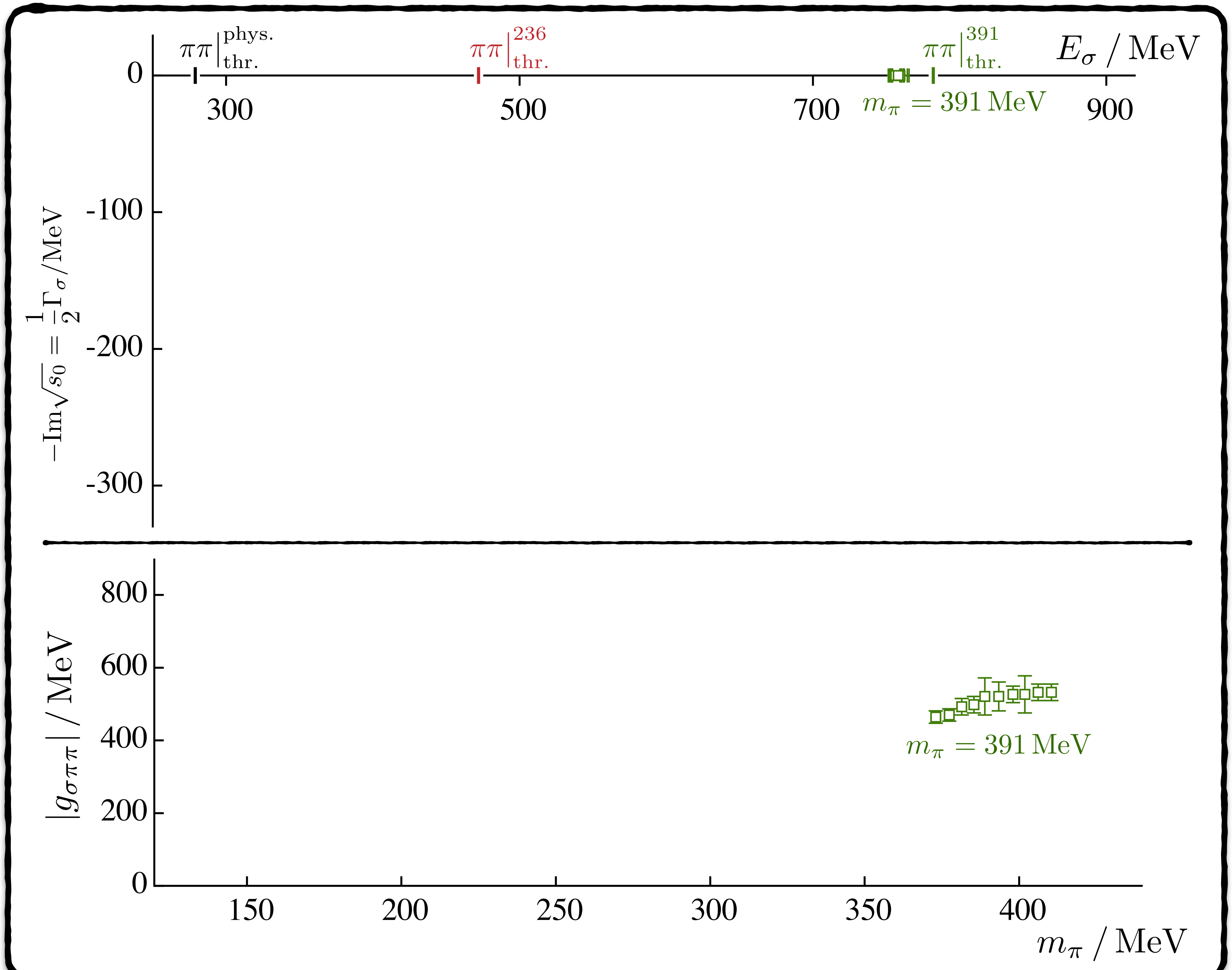
The σ / f$_0$(500) vs m$_\pi$

$$s_0 = (E_\sigma - \tfrac{i}{2}\Gamma_\sigma)^2, \qquad g^2_{\sigma\pi\pi} = \lim_{s\to s_0}(s_0 - s)\,t(s)$$



The σ / f₀(500) vs m_π

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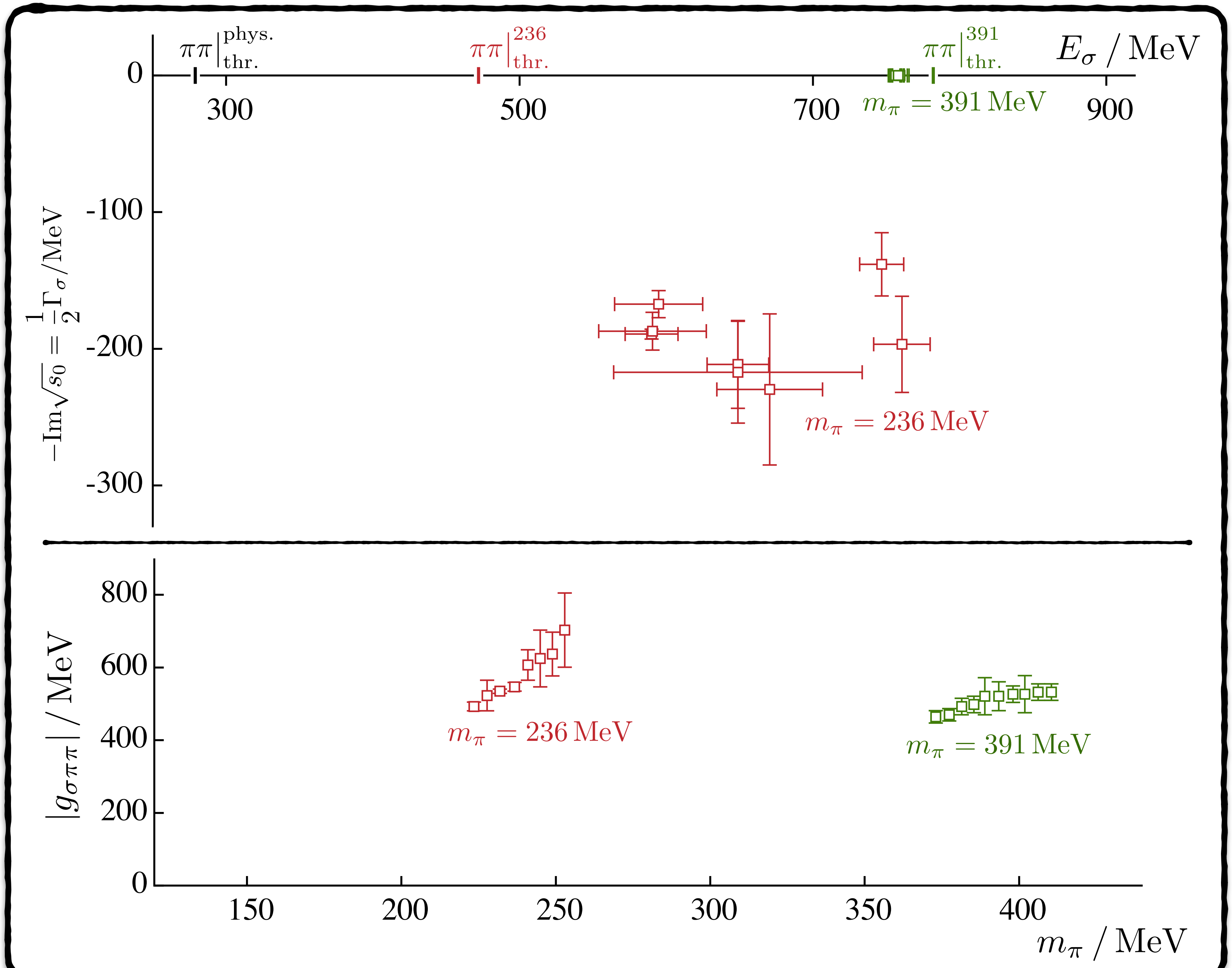

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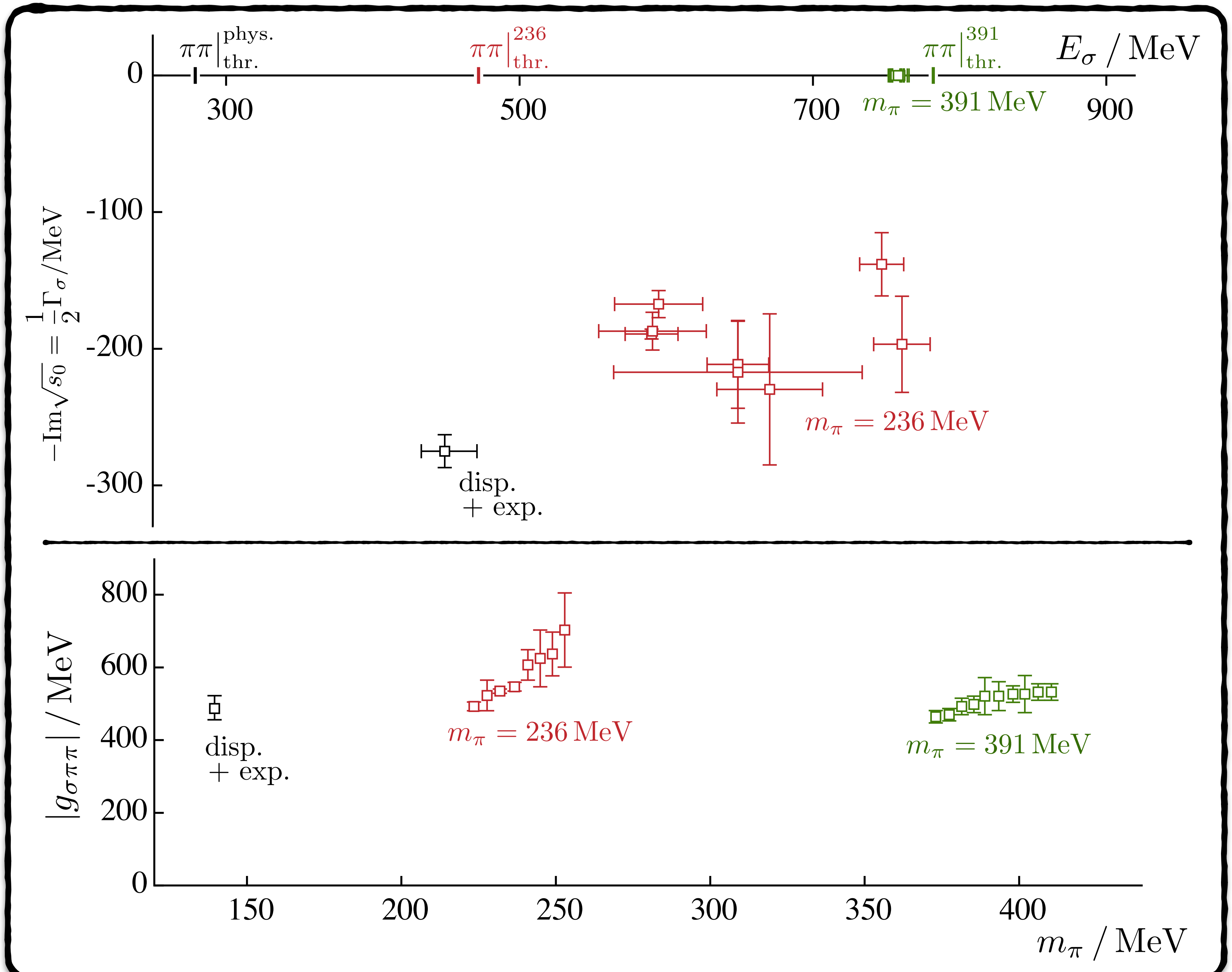

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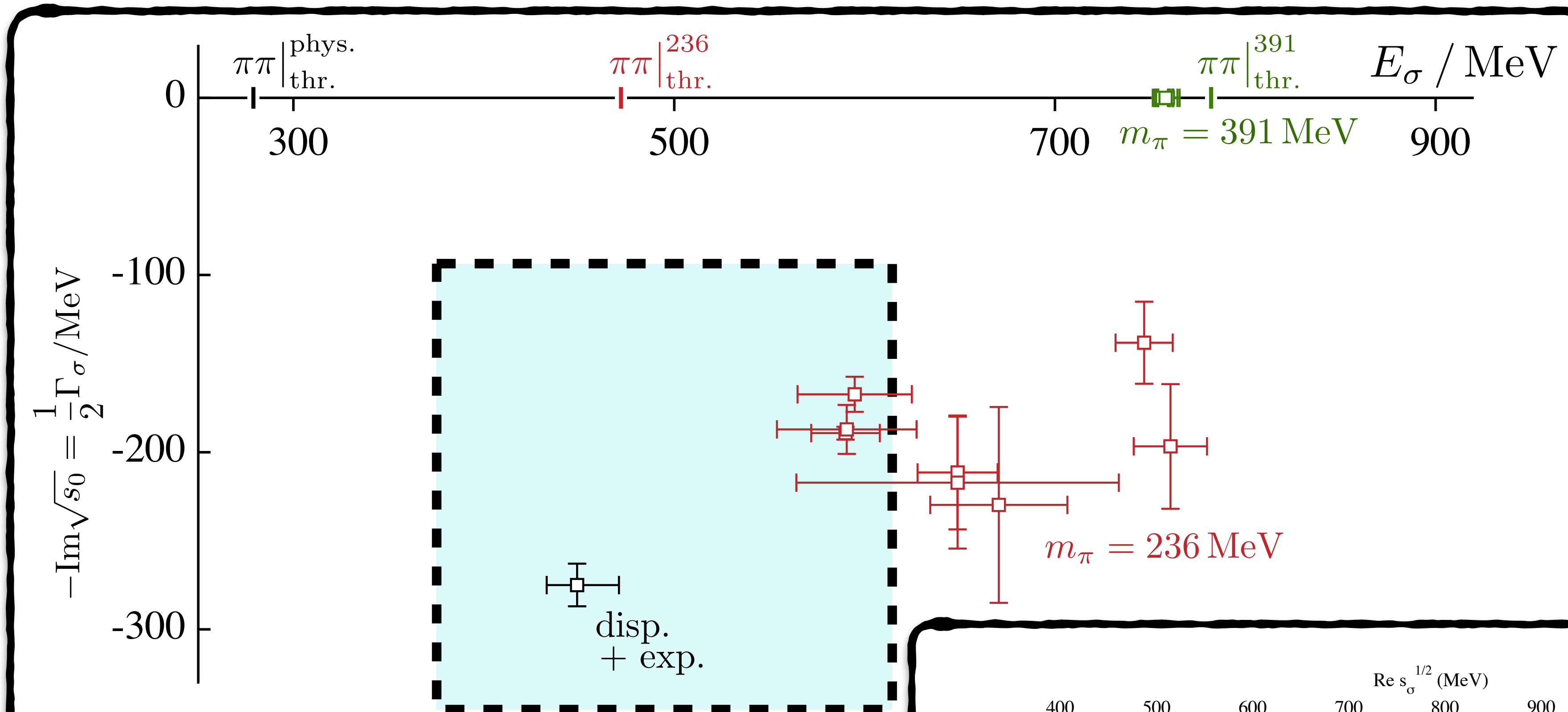
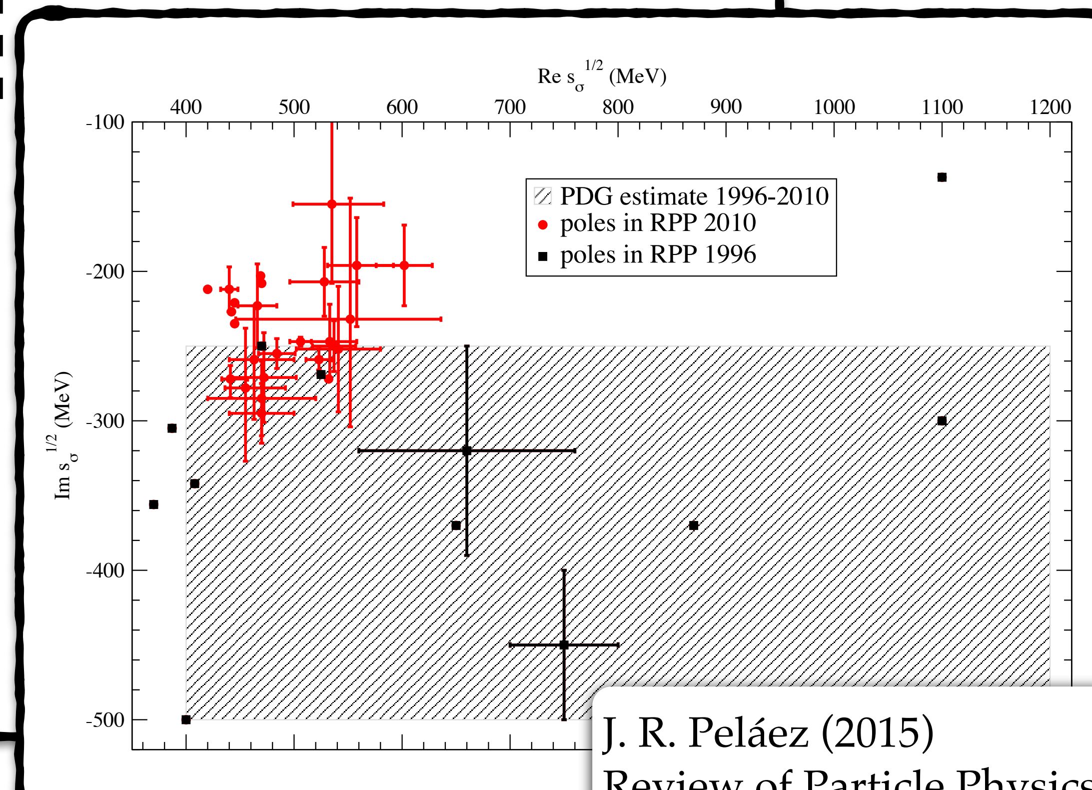


Historical perspective



J. R. Peláez (2015)
Review of Particle Physics (RPP)

The σ / f_0(500) vs m_π

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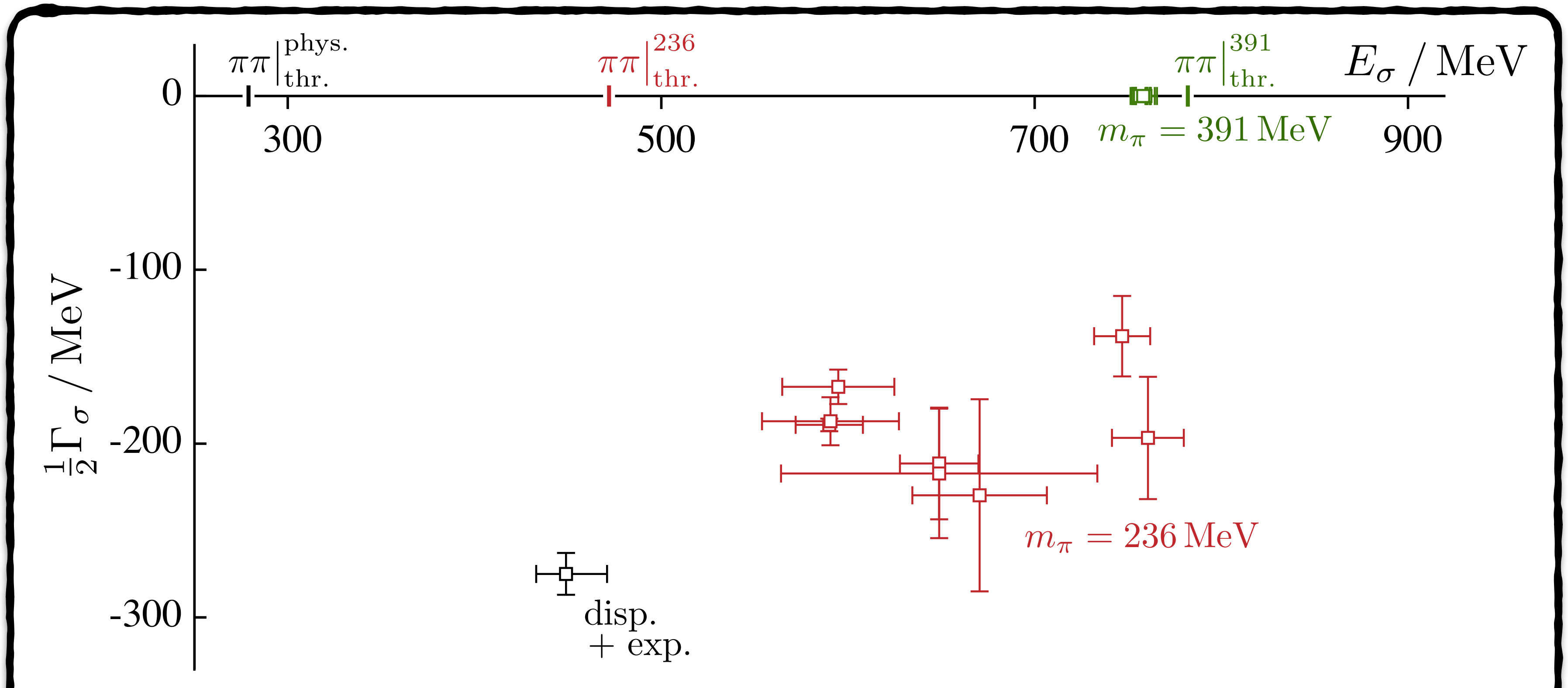

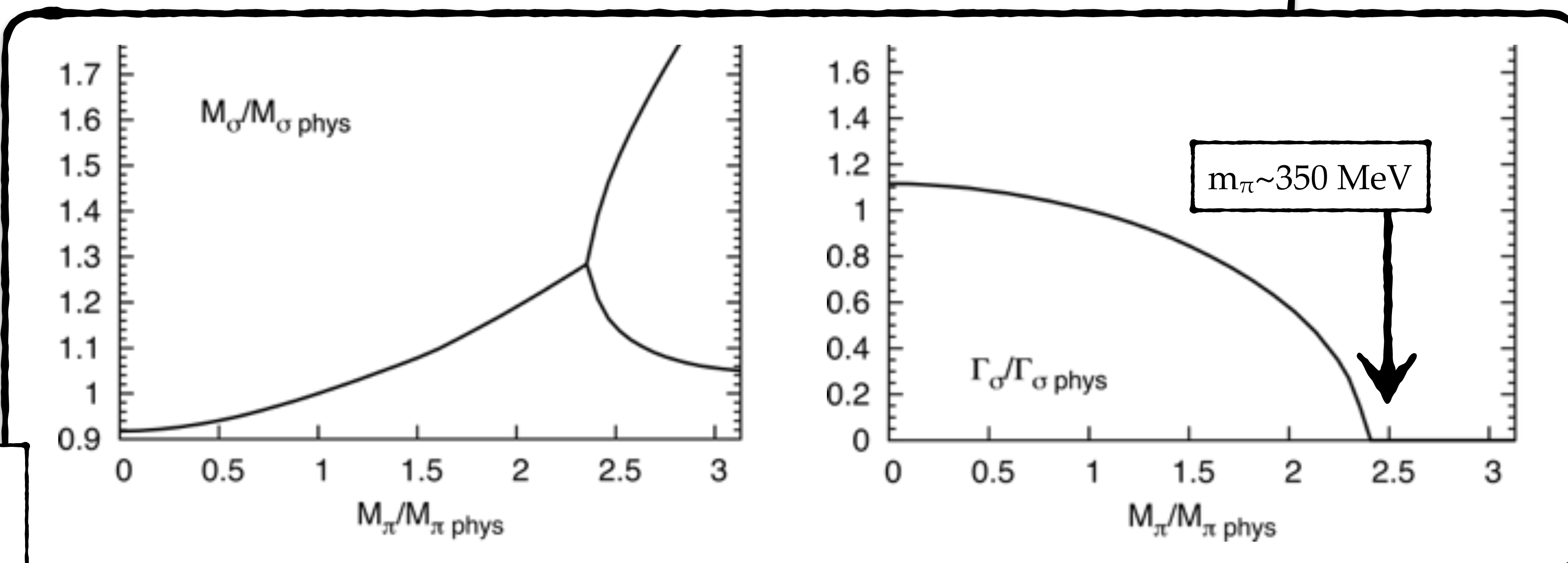

UχPT - Nebreda & Peláez (2015)

The σ / $f_0(500)$ vs m_π

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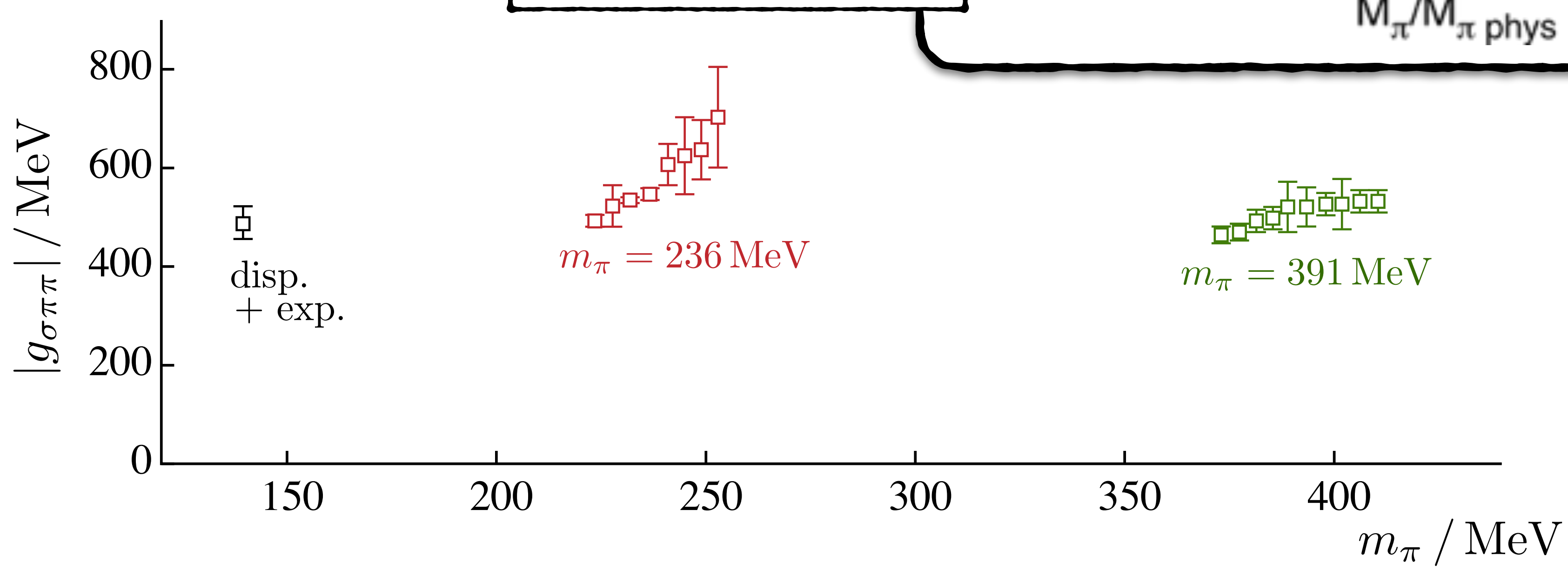

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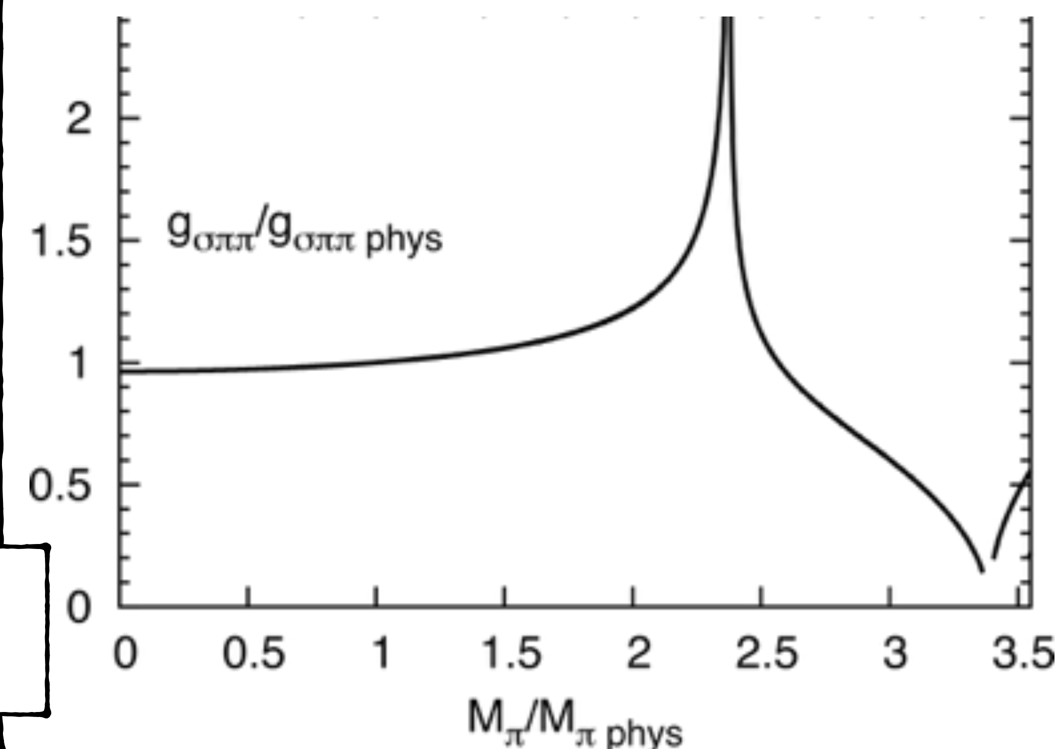
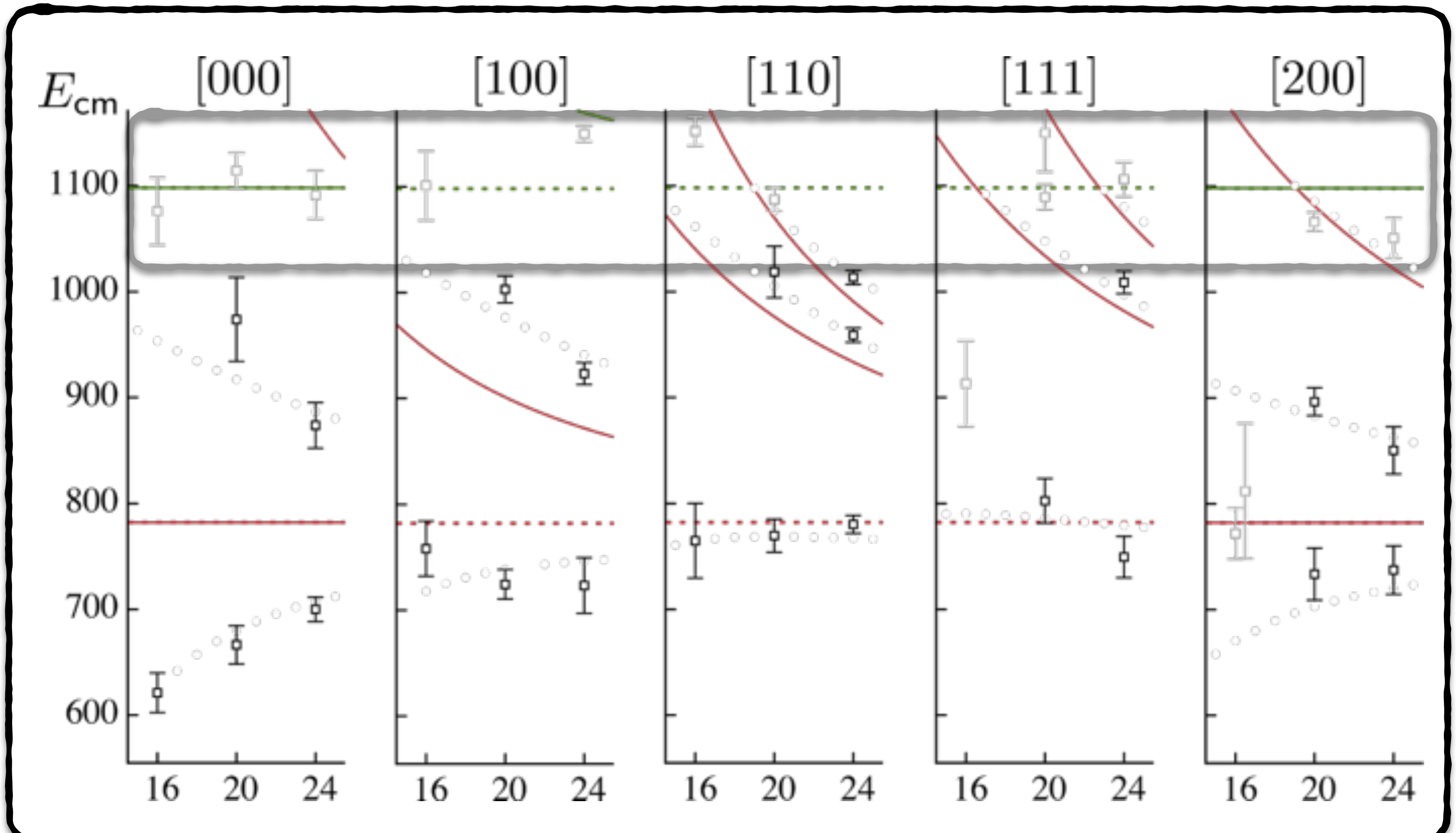


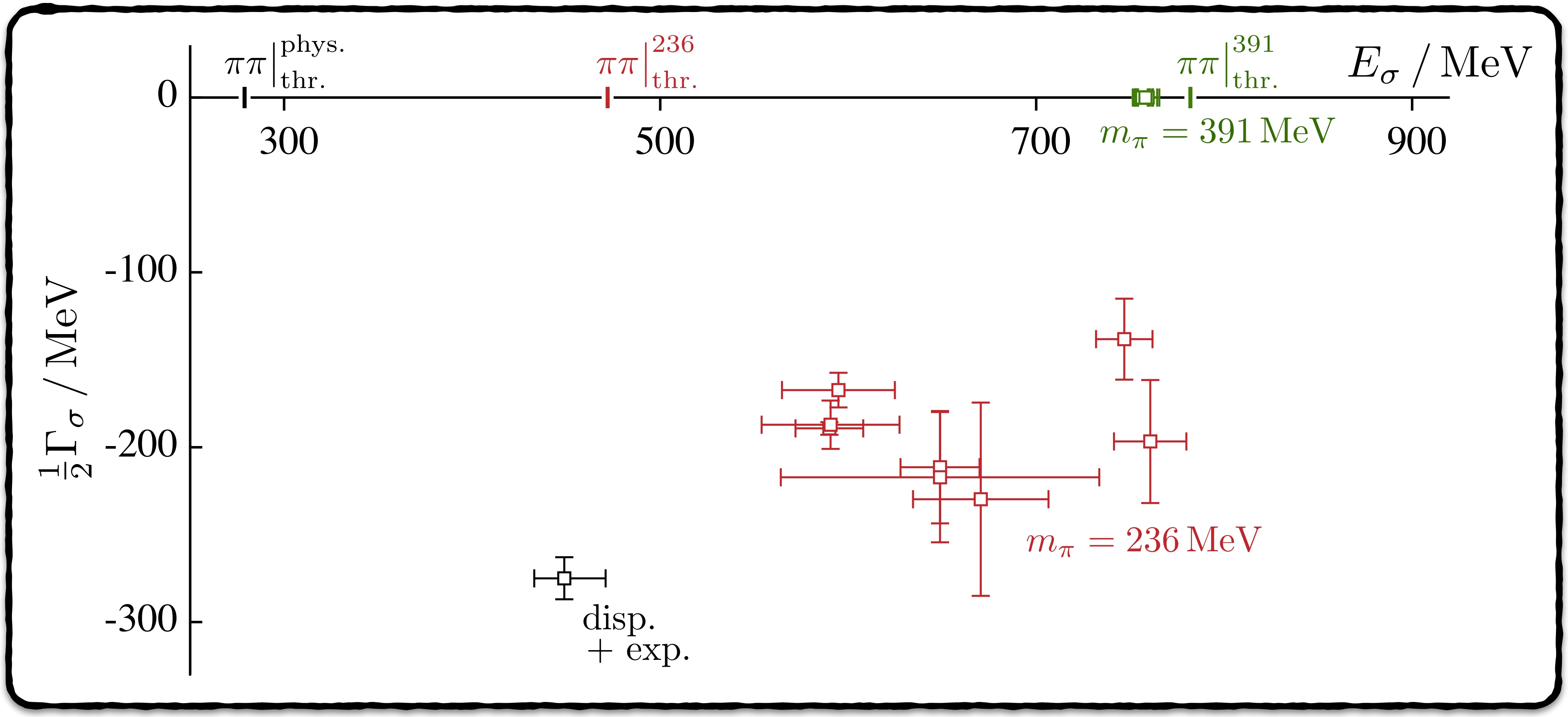
Outlook

- $\pi\pi$-KK / $f_0(980)$



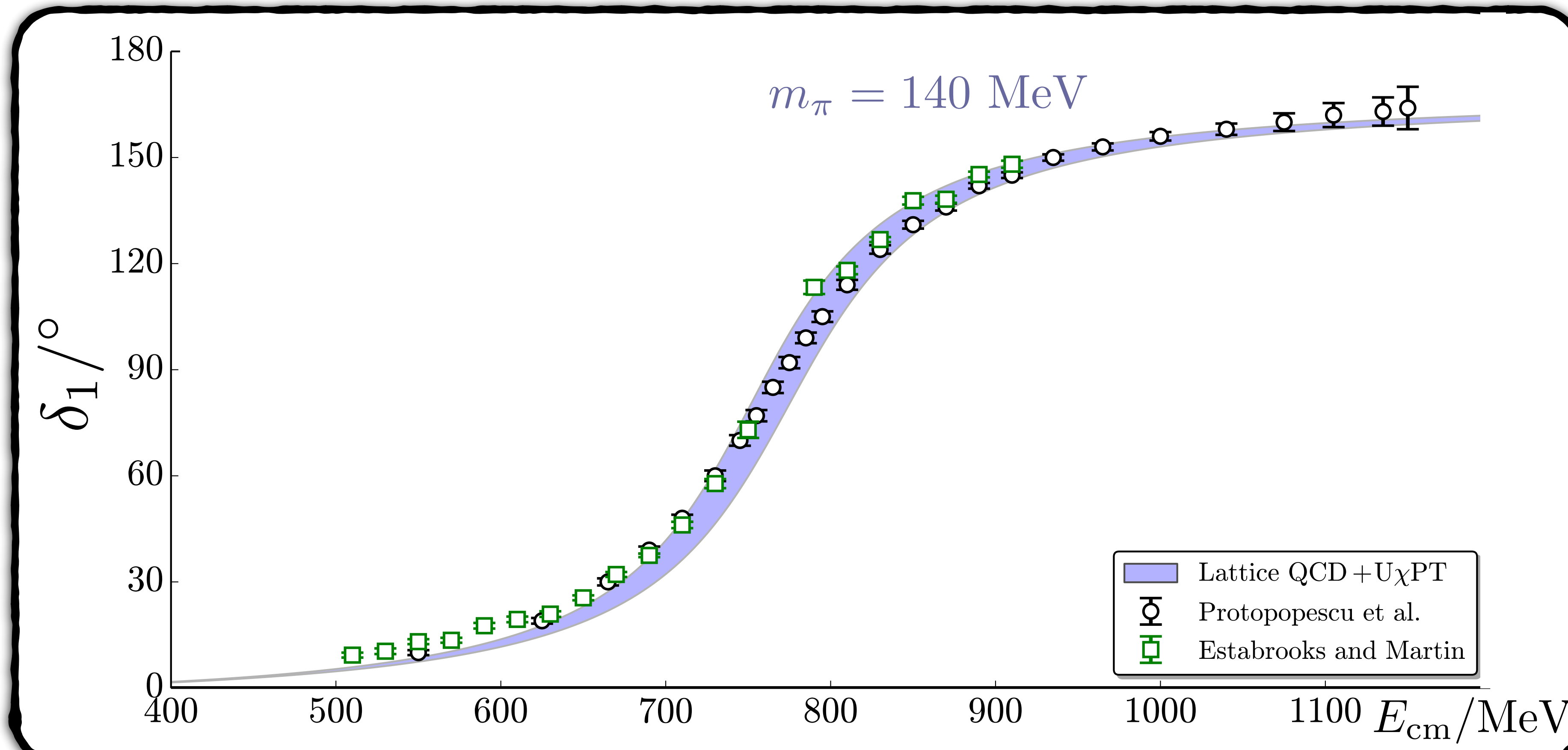
Outlook

- $\pi\pi$-KK / $f_0(980)$
- *dispersive analysis*



Outlook

- $\pi\pi$-KK / $f_0(980)$
- *dispersive analysis*
- *chiral extrapolation, more quark masses(?)*



Bolton, RB & Wilson Phys.Lett. B757 (2016) 50-56.

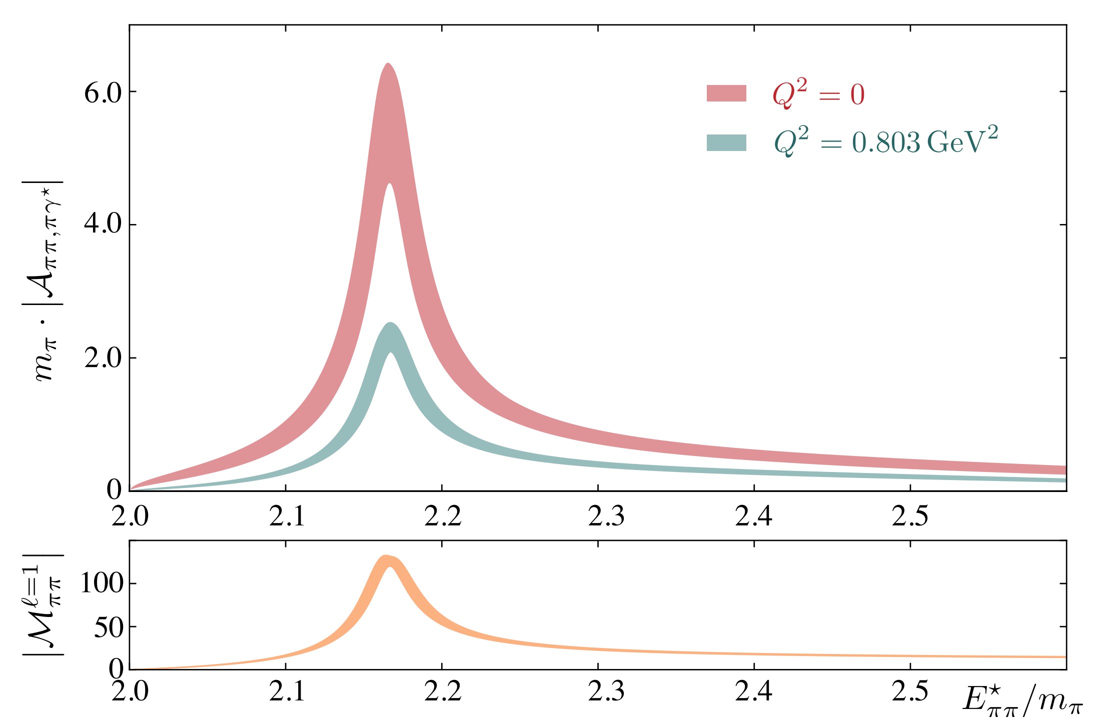
Outlook

- *$\pi\pi$-KK / f_0(980)*
- *dispersive analysis*
- *chiral extrapolation, more quark masses(?)*
- *Elastic form factors of composite particles*

formalism understood:

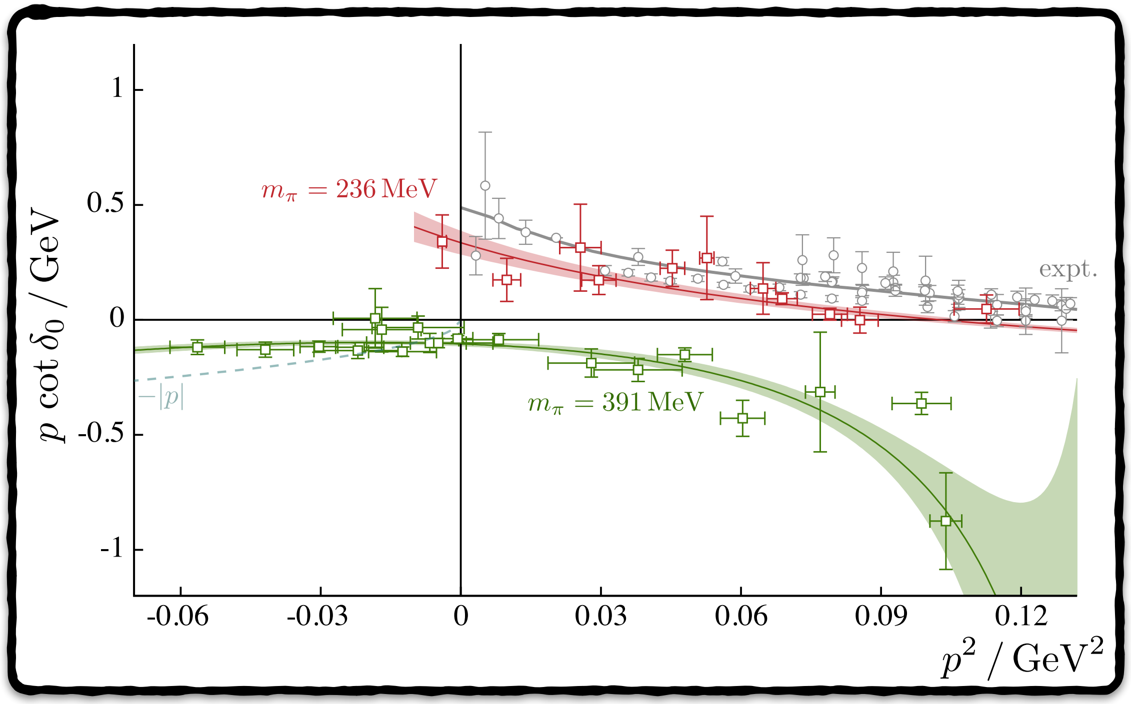
RB, Hansen - Phys.Rev. D94 (2016) no.1, 013008.
RB, Hansen - Phys.Rev. D92 (2015) no.7, 074509.
RB, Hansen, Walker-Loud - Phys.Rev. D91 (2015) no.3, 034501.
Bernard, D. Hoja, U. G. Meissner, and A. Rusetsky (2012)

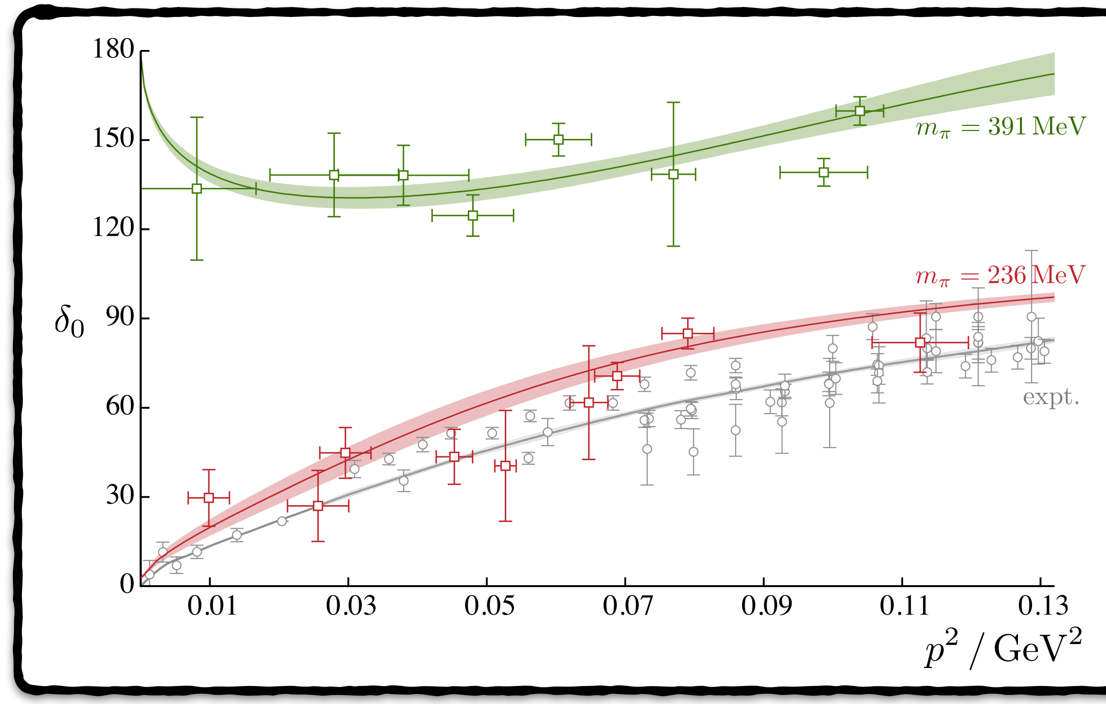
first implementation: $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ρ

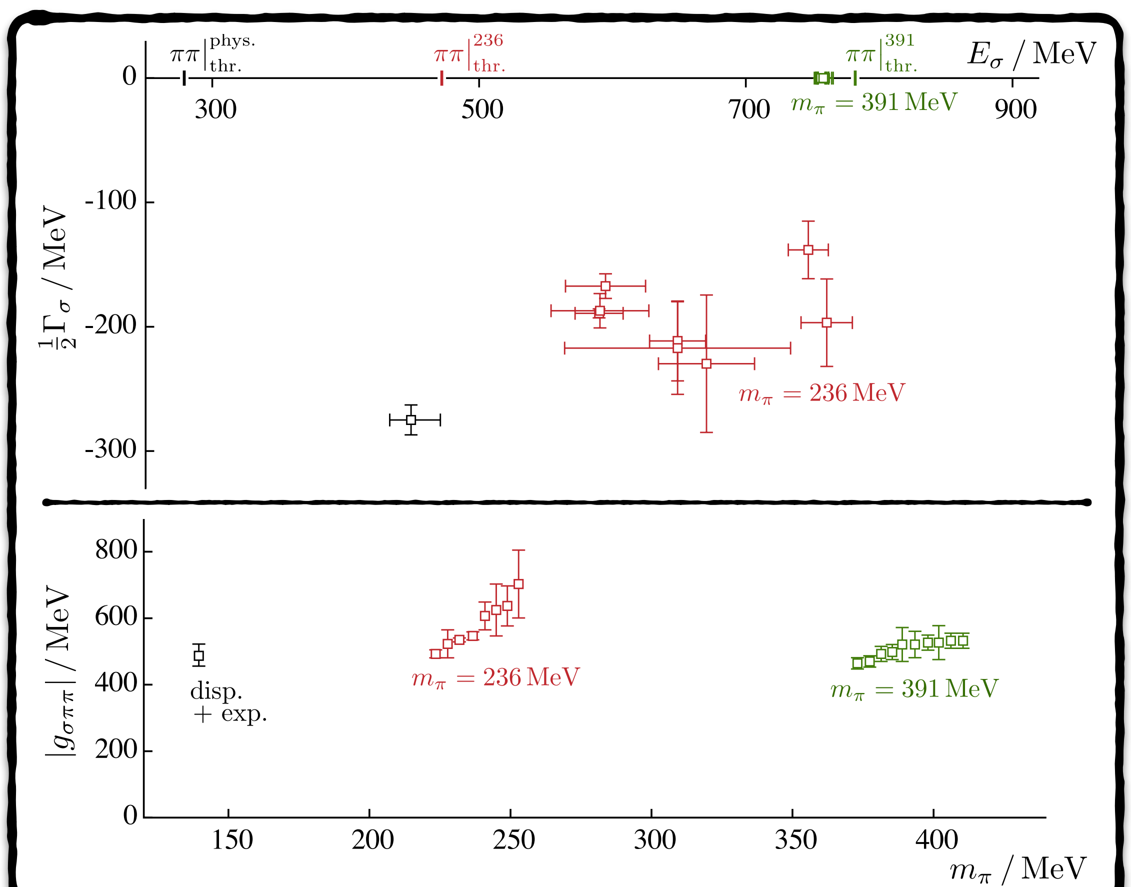


RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev. D93 (2016) 114508.
RB, Dudek, Edwards, Thomas, Shultz, Wilson - Phys.Rev.Lett. 115 (2015) 242001

Take-home message







Wilson

Dudek

Edwards

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arXiv:1607.05900 [hep-ph]

HadSpec talks

Resonance in coupled channels- **David Wilson,** Monday 10:30

Searches for charmed tetraquarks- **Gavin Cheung,** Monday 13:55

Radiative transitions in charmonium - **Cian O'Hara,** Monday 17:25

Optimised operators and distillation - **Antoni Woss,** Tuesday 14:40

a_0 resonance in $\pi\eta$, KK - **Jozef Dudek,** Tuesday 15:50

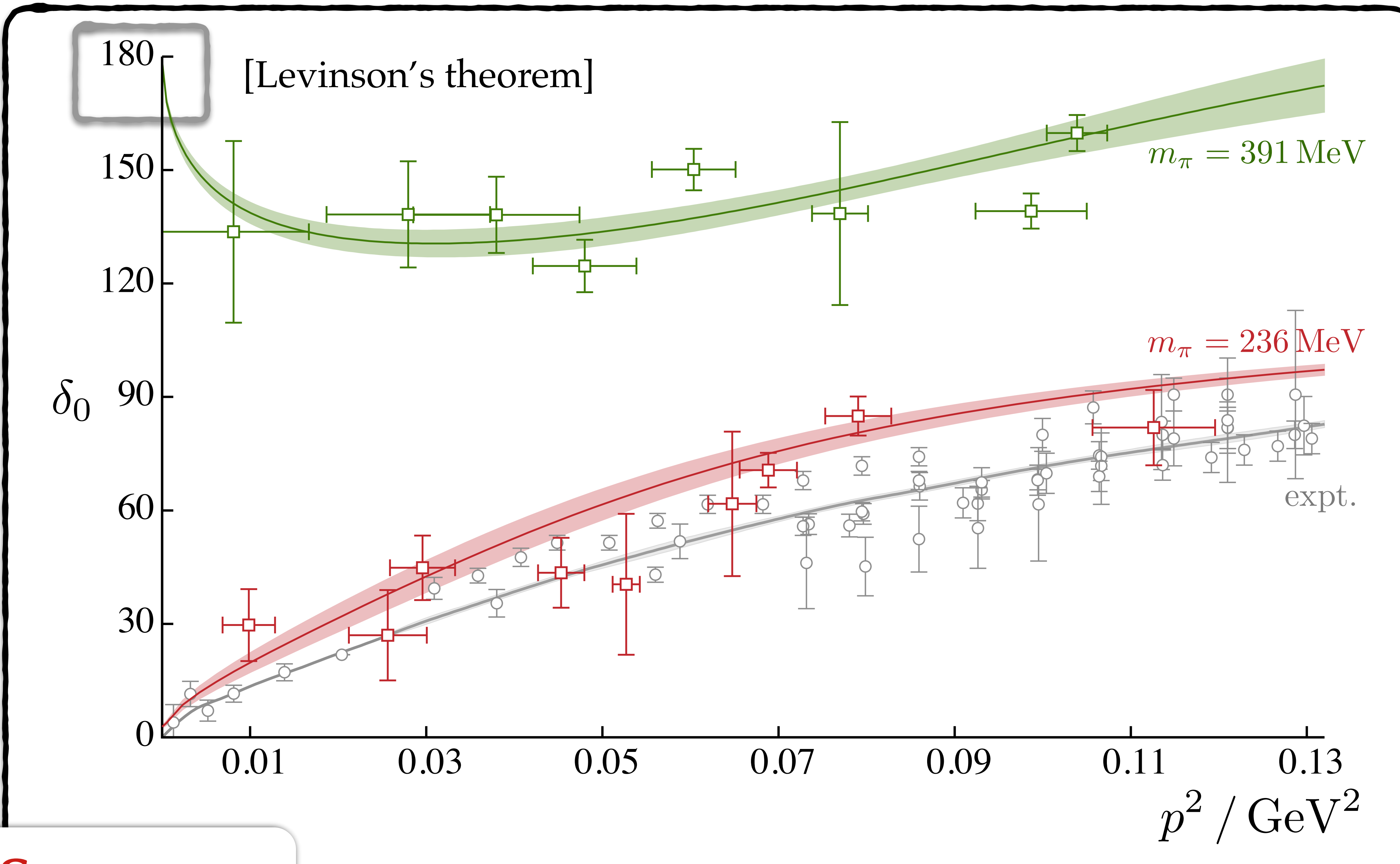
Charmed meson spectroscopy - **David Tims,** Thursday 14:20

$D\pi$, $D\eta$ and D_sK scattering - **Graham Moir,** Thursday 15:00

DK scattering - **Christopher Thomas,** Thursday 15:20

Charmed-bottom mesons - **Nilmani Mathur,** Friday 15:40

Scattering amplitude vs m_π



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