FROM C TO PARTON SEA

BJORKEN-x DEPENDENCE OF THE PDFs

Huey-Wen Lin

Huey-Wen Lin — Lattice 2016, Southampton, UK
This talk is based on
“Flavor Structure of the Nucleon Sea from Lattice QCD”,
PRD 91, 054510 [arXiv:1402.1462]
“Nucleon Helicity and Transversity Parton Distributions from Lattice QCD”, to be appeared in Frontier Article in Nuclear Physics B,
[arXiv:1603.06664]
in collaboration with

Jiunn-Wei Chen (NTU)  
Saul Cohen (NVIDIA)  
Xiangdong Ji (UMD/SJTU/INPAC)  
Jian-Hui Zhang (Regensburg)

+ some recent developments
Parton Distribution Functions

PDFs are universal quark/gluon distributions inside nucleon

Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701
Parton Distribution Functions

- PDFs are universal quark/gluon distributions inside nucleon
  - Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)

- Important inputs to discern new physics at LHC
  - Currently dominate errors in Higgs production

![Higgs Production](chart.png)

(J. Campbell, HCP2012)
Global Analysis

§ Experiments cover diverse kinematics of parton variables

☞ Global analysis takes advantage of all data sets

Choice of data sets and kinematic cuts

Strong coupling constant $\alpha_s(M_Z)$

How to parametrize the distribution

$$xf(x, \mu_0) = a_0 x^{a_1} (1 - x)^{a_2} P(x)$$

Assumptions imposed

SU(3) flavor symmetry, charge symmetry, strange and sea distributions

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

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Discrepancies appear when data is scarce
Many groups have tackled the analysis
CTEQ, MSTW, ABM, JR, NNPDF, etc.


$Q^2 = 100 \text{ GeV}^2$

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What can we do on the lattice?
Lattice calculations rely on operator product expansion, only provide moments:

- **Quark density/unpolarized**
  \[ \langle x^n \rangle_q = \int_{-1}^{1} dx \ x^n q(x) \]

- **Helicity (longitudinally polarized)**
  \[ \langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx \ x^n \Delta q(x) \]

- **Transversity (transversely polarized)**
  \[ \langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx \ x^n \delta q(x) \]

§ Most well known

§ Very poorly known

§ True distribution can only be recovered with all moments
Problem with Moments

§ For higher moments, ops mix with lower-dimension ops

Renormalization is difficult too

§ Relative error grows in higher moments

Calculation would be costly and difficult

Dolgov et al. PRD 66, 034506 (2002)

Göckeler et al. PRD 71, 114511 (2005)

LHPC (SCRI, SESAM): 2f, Wilson and clover

QCDSF: 0f
Problem with Moments

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LHPC (SCRI, SESAM): 2f, Wilson and clover

QCDSF: 0f

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\[ \langle x^2 \rangle_q \]

\[ \langle x^3 \rangle_q \]
PDFs on the Lattice

Long existing obstacle!

§ Holy grail of structure calculations

§ Applies to many structure quantities:
   Generalized parton distributions (GPDs),
   Transverse-momentum distributions (TMD),
   Meson distribution amplitudes, ...

§ A few ideas try to solve this problem
   ✐ Hadronic tensor currents
     (Liu et al., hep-ph/9806491, ... 1603.07352)
   ✐ OPE without OPE (QCDSF, hep-lat/9809171, ... 1004.2100)
   ✐ Fictitious heavy quarks (Detmold et al. hep-lat/0507007)
   ✐ Smeared lattice operators (Davoudi et al. 1204.4146)

Looking forward to more developments here
A Promising New Direction
New Direction

Large-Momentum Effective Theory (LaMET)  
\[ X. \text{ Ji, PRL. 111, 262002 (2013)} \]

§ Calculate the parton distributions through the infinite-momentum frame Feynman, Phys. Rev. Lett. 23, 1415 (1969)

§ Weinberg introduced a more convenient description using correlation functions along the lightcone e.g. nucleon quark distribution

\[
q(x, \mu) = \int \frac{d\xi_-}{4\pi} e^{-ix\cdot P} \left\langle P \left| \bar{\psi}(\xi_-) \gamma_+ \exp \left( -ig \int_0^{\xi_-} d\eta_- A_+(\eta_-) \right) \psi(0) \right| P \right\rangle
\]

Renormalization scale \( \mu \)

Gluon potential \( A_+ \)

Lightcone coordinate \( \xi_\pm = (t+z)/\sqrt{2} \)
**New Direction**

Large-Momentum Effective Theory (LaMET)

- Going back to the IMF concept
- Finite-momentum quark distribution (quasi-distribution)

**Suggested operator:**

\[ \tilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-iz k_z} \left< P \left| \bar{\psi}(z) \gamma_z \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right> \]

- \( x = k_z / P_z \)
- Lattice \( z \) coordinate
- Product of lattice gauge links

**Nucleon momentum** \( P_\mu = \{P_0, 0, 0, P_z\} \)

- Take the infinite-\( P_z \) limit to recover lightcone functions

**Just another limit to take, like taking** \( a \to 0 \) or \( V \to \infty \)
New Direction

Large-Momentum Effective Theory (LaMET)  

Finite-\(P_z\) corrections needed

\(\bowtie\) Neglect typical lattice corrections for now:

\[
\tilde{q}(x, \mu, P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z \left( \frac{x}{y}, \frac{\mu}{P_z} \right) q(y, \mu) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\Lambda_QCD^2/P_z^2)
\]

Finite \(P_z \leftrightarrow \infty\) perturbative matching

\[
Z(x, \mu/P_z) = C\delta(x - 1) - \frac{\alpha_s}{2\pi} Z^{(1)}(x, \mu/P_z)
\]

Non-singlet case only

X. Xiong, X. Ji, J. Zhang, Y. Zhao, 1310.7471;
Ma and Qiu, 1404.6860

\(\bowtie\) Dominant correction (for nucleon);
known scaling form

HWL et al. 1402.1462
J.-W. Chen et al, 1603.06664

§ Benefit from our pQCD colleagues
New Direction

Large-Momentum Effective Theory (LaMET)  

Finite-$P_z$ corrections needed

Neglect typical lattice corrections for now:

$$\tilde{q}(x, \mu, P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z \left( \frac{x}{y}, \frac{\mu}{P_z} \right) q(y, \mu) + \mathcal{O}(M_N^2/P_z^2) + \mathcal{O}(\Lambda_{QCD}^2/P_z^2)$$

complicated higher-twist operator; smaller $P_z$ correction for nucleon  
J.-W. Chen et al, 1603.06664 and reference within (extrapolate it away)

§ Some similarity in more broadly-studied HQET...

$$O \left( \frac{m_b}{\Lambda} \right) = Z \left( \frac{m_b}{\Lambda}, \frac{\Lambda}{\mu} \right) o(\mu) + \mathcal{O} \left( \frac{1}{m_b} \right) + \ldots$$

X. Ji, PRL. 111, 262002 (2013)
§ Exploratory study

- $N_f = 2+1+1$ clover/HISQ lattices (MILC)
  - $M_\pi \approx 310$ MeV, $a \approx 0.12$ fm ($L \approx 2.88$ fm)
- Isovector only (“disconnected” suppressed)
  - gives us flavor asymmetry between up and down quark
- 2 source-sink separations ($t_{\text{sep}} \approx 0.96$ and 1.2 fm) used

§ Properties known on these lattices

- Lattice $Z_\Gamma$ for bilinear operator $\sim 1$
  (with HYP-smearing)
- $M_\pi L \approx 4.6$ large enough to avoid finite-volume effects

§ Feasible with today’s resources!

1402.1462 [hep-ph]; 1603.06664 [hep-ph]
Exploratory study

\( N_f = 2+1+1 \) clover/HISQ lattices (MILC)

\( M_\pi \approx 310 \text{ MeV}, \ a \approx 0.12 \text{ fm} \ (M_\pi L \approx 4.5) \)

NO SYSTEMATICS YET!

Demonstration that the method works

Intend to motivate future LQCD work on many quantities
§ Calculate nucleon matrix elements

\[
\left\langle P \left| \bar{\psi}(z) \gamma_z \exp\left( -i g \int_0^z dz' A_z(z') \right) \psi(0) \right| P \right\rangle
\]

 How many links are needed?

 Lattice momenta discretized by finite size of volume

\[ P_z \in \{0.43, 0.86, 1.29\} \text{ GeV} \]
Step 2

§ Do the integral

\[
\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \middle| \bar{\psi}(z) \gamma_z \exp(-ig \int_0^z dz' A_z(z')) \psi(0) \right| P \right\rangle
\]

\[
P_z \in \{0.43, 0.86, 1.29\} \text{ GeV}
\]

Uncorrected bare lattice results

\[x = k_z / P_z\]

Distribution should sharper as \(P_z\) increases

Artifacts due to finite \(P_z\) on the lattice
Step 3

§ Apply finite-$P_z$ corrections

\[ \tilde{q}(x, \mu, P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z \left( \frac{x}{y}, \frac{\mu}{P_z} \right) q(y, \mu) + O\left(\frac{M_N^2}{P_z^2}\right) \]

\[ P_z \in \{0.43, 0.86, 1.29\} \text{ GeV} \]

Removing \( O(M_N^n/P_z^n) \) errors + \( O(\alpha_s) \)

Corrected distributions from the largest 2 \( P_z \) show signs of convergence
Step 4

\[ \frac{q(x)^{u-d}}{b} \]

§ Extrapolate higher-twist effects \( \mathcal{O}(\Lambda_{QCD}^2/P_z^2) \)

\[ \nabla \, N_f = 2+1+1 \text{ clover/HISQ lattices (MILC)} \]

\[ M_\pi \approx 310 \text{ MeV}, \, a \approx 0.12 \text{ fm (} M_\pi L \approx 4.5), \mathcal{O}(10^3) \text{ measurements} \]

A.D. Martin et al.

J.F. Owens et al.
PRD 87, 094012 (2012)

S. Dulat et al.
arXiv:1506.07443
First time in LQCD history to study antiquark distribution!

\[ M_\pi \approx 310 \text{ MeV} \]

\[ \bar{q}(x) = -q(-x) \]

Lost resolution in small-\( x \) region

Future improvement: larger lattice volume

\[ \int dx \left( \bar{u}(x) - \bar{d}(x) \right) \approx -0.16(7) \]

\[ \begin{array}{cccc}
\text{Experiment} & x \text{ range} & \int_0^1 [\bar{d}(x) - \bar{u}(x)] dx \\
E866 & 0.015 < x < 0.35 & 0.118 \pm 0.012 \\
NMC & 0.004 < x < 0.80 & 0.148 \pm 0.039 \\
HERMES & 0.020 < x < 0.30 & 0.16 \pm 0.03 \\
\end{array} \]

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
§ First time in LQCD history to study antiquark distribution!

\[ M_\pi \approx 310 \text{ MeV} \]

First sea flavor asymmetry ever calculated!
Overcomes decades of obstacles in LQCD structure calculations

\[ \bar{q}(x) = -q(-x) \]

HWL et al. 1402.1462

Sea Flavor Asymmetry

\[
\int_0^1 \left[ \bar{d}(x) - \bar{u}(x) \right] dx
\]

R. Towell et al. (E866/NuSea), Phys. Rev. D64, 052002 (2001)
Sea Flavor Asymmetry

§ Lattice exploratory study

\[ M_\pi \approx 310 \text{ MeV} \]

Compared with E866

Too good to be true?

Lost resolution in small-x region

Similar results repeated by ETMC, at \( M_\pi \approx 373 \text{ MeV} \)

ETMC, 1504.07455

C. WIESE, Mon. 14:15

<table>
<thead>
<tr>
<th></th>
<th>(0.015 &lt; x &lt; 0.35)</th>
<th>(0.004 &lt; x &lt; 0.80)</th>
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</tr>
</tbody>
</table>

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)
Exploratory study

\[ M_\pi \approx 310 \text{ MeV} \]

We see polarized sea asymmetry \( \int dx \left( \Delta \bar{u}(x) - \Delta \bar{d}(x) \right) \approx 0.14(9) \)

Both STAR and PHENIX at RHIC see \( \Delta \bar{u} > \Delta \bar{d} \)

Removing \( O(M_N^n/P_z^n) \) errors + \( O(\alpha_s) \) + \( O(\Lambda_{QCD}^2/P_z^2) \)

Other experiments, Fermilab DY exp’ts (E1027/E1039), future EIC
§ Exploratory study  $\Leftrightarrow M_\pi \approx 310 \text{ MeV}$

\[
\int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.10(8)
\]

Removing 
$O(M_{N/Pz}^n)$ errors + $O(\alpha_s)$ 
+ $O(\Lambda_{QCD}^2/P_z^2)$

$\delta \bar{q}(x) = -\delta q(-x^\dag)$

1505.05589; 1503.03495

We found sea asymmetry of 
\[
\int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.10(8)
\]

Chiral quark-soliton model 
\[
\int dx \left( \delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.082
\]

P. Schweitzer et al., PRD 64, 034013 (2001)

SoLID at JLab, Drell-Yan exp’t at FNAL (E1027+E1039), EIC, ...
There are 2 key issues that need to be addressed

§ Large-momentum issues

▷ HWL, Lattice 2013

§ Need improvement for large-momentum sources
▷ Better overlapping boosted hadron smearing (asymmetric source)
▷ Applications: large-$q$ form factors, hadronic and flavor physics, …

▷ Progress is being made:

▷ RQCD, Novel quark smearing for hadrons with high momenta in lattice QCD, Phys. Rev. D 93, 094515 (2016)
▷ Momentum Smearing, B. Lang, Tue. Poster
▷ The Calculation of Parton Distributions from Lattice QCD
  C. WIESE, Mon. 14:15

▷ Systematics due to $(pa)^n$

▷ Excited states get worse with larger $p$ ⇒ multiple $t_{sep} +$ finer $a$
▷ Moving frame action?
There are 2 key issues that need to be addressed

§ Renormalization Issues

因为他目前假设归一化是乘积性的。

\[
q_{\text{norm}}(x, \mu, P_z) = \frac{q(x, \mu, P_z)}{\int dx \, q(x, \mu, P_z)} \times g_{\overline{\text{MS}}}(2 \text{ GeV})
\]

Progress is on its way:

- *Matching issue in quasi parton distribution approach*
  
  T. ISHIKAWA, today 17:30

- *Lattice study of Wilson line operators*
  
  H. PANAGOPOULOS, Wed 10:40

- Gradient flow approach? K. Organos + C. Monahan

- Some lessons can be learned from static heavy-quark operators?
Overcoming longstanding obstacle to $x$-distribution

New idea by Ji for studying full $x$-dependence of PDFs

Promising results on unpolarized and polarized sea asymmetry compared with experiments, even at non-physical pion mass

Caveats

- Not a precision calculation yet
- Need to complete the other $p_z$ corrections (on-going; possibly done in a couple weeks)
- Systematics due to large momenta (some ideas to improve it)

Need improvement for large $-q$ form factors, hadronic and flavor physics, ...

It is a period of war and economic, uncertainty. Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion-Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful
Exciting time for studying structure on the lattice

§ Overcoming longstanding obstacle to full x-distribution
   ☑ Most importantly, this can be done with today’s computer
   ☑ First lattice approach to study sea asymmetry

§ Precision control
   ☑ Working on renormalization, statistics (all-mode averaging?), larger momentum boost, finer lattice-spacing ensembles, ...

§ Closer collaboration with our heavy-quark colleagues
   ☑ Certain similar issues: large-q form factors, HQET, ...

§ Opens doors to much future lattice-QCD structure work
   ☑ Many first calculations waiting to be done!
The goal of this workshop is to **bring together the global PDF analysis and lattice-QCD communities** to explore ways to improve current PDF determinations. In particular, we plan to **set precision goals for lattice-QCD calculations** so that these calculations, together with experimental input, can achieve more reliable determinations of PDFs. In addition we will discuss what impact such improved determinations of PDFs will have on future new-physics searches.
Backup Slides
Various presentation addressing difference issues

§ Renormalization
   ✸ Matching issue in quasi parton distribution approach
     T. ISHIKAWA, today 17:30
   ✸ Lattice study of Wilson line operators,
     H. PANAGOPOU, Wed 10:40

§ Nucleon PDFs
   ✸ The Calculation of Parton Distributions from Lattice QCD
     C. WIESE, Mon. 14:15

§ Pion distribution amplitude
   ✸ Momentum Smearing
     B. Lang, Tue. Poster
How Can LQCD Help?

§ Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories

§ We are beginning to do precision calculations in nucleons

§ PNDME’s $g_{T,S}$ calculations

Extrapolate to the physical limit $m_\pi \rightarrow m_{\pi}^{\text{phys}}, a \rightarrow 0, L \rightarrow \infty$
### Tensor/Scalar Charges

**FLAG rating system**

**New: excited-state rating**

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>Ref.</th>
<th>$N_f$</th>
<th>$g_T$</th>
<th>$g_s$</th>
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<td>PNDME’15</td>
<td>This work</td>
<td>P 2+1+1</td>
<td>1.020(76)$^a$</td>
<td>0.93(6)$^z$</td>
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<td>ETMC’13</td>
<td>[30]</td>
<td>C 2+1+1</td>
<td>1.11(3)$^b$</td>
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<td>LHPC’12</td>
<td>[28]</td>
<td>A 2+1</td>
<td>1.037(20)$^c$</td>
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<td>RBC/UKQCD’10</td>
<td>[29]</td>
<td>A 2+1</td>
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<tr>
<td>RBC’08</td>
<td>[32]</td>
<td>P 2</td>
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**Notes:**

- $^a$: PNDME (AMA) ’15
- $^b$: PNDME ’15
- $^c$: LHPC ’12
- $^d$: RBC/UK ’10
- $^e$: ETMC ’15
- $^f$: RQCD ’14
- $^g$: RBC ’08

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**Acronyms:**

- PNDME: Project for Non-Perturbative QCD
- ETMC: European Twisted Mass Collaboration
- LHPC: lattice QCD at the University of Hong Kong
- RBC: Research Board Collaboration
- RQCD: Research Board Collaboration

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**References:**

- [Goldstein et al. (2014)]
- [DSE’14]
- [Kang et al. (2014)]
- [Anselmino et al. (2013)]
- [Bacchetta et al. (2013)]
- [Sum Rules (2000)]
- [Gonzalez-Alonso et al. (2014)]
- [Adler (1975)]

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**Huey-Wen Lin — Lattice 2016, Southampton, UK**
Beta Decays & BSM

Given precision $g_{S,T}$ and $O_{BSM}$, predict new-physics scales

Low-Energy Expt

$O_{BSM} = f_0(\epsilon_{S,T} g_{S,T})$

Precision LQCD input ($m_\pi \rightarrow 140$ MeV, $a \rightarrow 0$)

$\epsilon_{S,T} \propto \Lambda_{S,T}^{-2}$

Upcoming precision low-energy experiments

LANL/ ORNL UCN neutron decay exp’t

$|B_1 - b|_{BSM} < 10^{-3}$

$|b|_{BSM} < 10^{-3}$

CENPA: $^6$He($b_{GT}$) at $10^{-3}$

PNDME, PRD85 054512 (2012); 1306.5435; in preparation