

Parton Distribution Functions

This talk is based on

"Flavor Structure of the Nucleon Sea from Lattice QCD", PRD 91, 054510 [arXiv:1402.1462]
"Nucleon Helicity and Transversity Parton Distributions from Lattice QCD", to be appeared in Frontier Article in Nuclear Physics B, [arXiv:1603.06664]

in collaboration with









Jiunn-Wei Chen Saul Cohen Xiangdong Ji Jian-Hui Zhang (NTU) (NVIDIA) (UMD/SJTU/INPAC)(Regensburg) + some recent developments



Parton Distribution Functions

§ PDFs are universal quark/gluon distributions inside nucleon Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)







Electron Ion Collider: The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon? EIC White Paper, 1212.1701





Parton Distribution Functions

- § PDFs are universal quark/gluon distributions inside nucleon
- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, ...)
- § Important inputs to discern new physics at LHC Currently dominate errors in Higgs production





Huey-Wen Lin — Lattice 2016, Southampton, UK

(J. Campbell, HCP2012)

Global Analysis

§ Experiments cover diverse kinematics of parton variables

✤ Global analysis takes advantage of all data sets



Choice of data sets and kinematic cuts

 \sim Strong coupling constant $\alpha_s(M_Z)$

How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1} (1-x)^{a_2} P(x)$$

Assumptions imposed

SU(3) flavor symmetry, charge symmetry, strange and sea distributions

$$s = \bar{s} = \kappa (\bar{u} + \bar{d})$$



Global Analysis

§ Discrepancies appear when data is scarce
 § Many groups have tackled the analysis
 > CTEQ, MSTW, ABM, JR, NNPDF, etc. 10⁻⁴

CJ12mid **CT10** 0.8 - MSTW08 ABKM09 0.6 d/u0.4 0.2 = 100 GeV0 0.2 0.4 0.6 0.8 0 x

Jimenez-Delgado, Melnitchouk, Owens, J.Phys. G40 (2013) 09310





What can we do on the lattice?





PDFs on the Lattice

§ Lattice calculations rely on operator product expansion, only provide moments Quark density/unpolarized $\langle x^n \rangle_q = \int_{-1}^1 dx \ x^n q(x)$ most well known $\langle x^n \rangle_{\Delta q} = \int_{-1}^{1} dx \, x^n \Delta q(x)$ Helicity longitudinally polarized $\langle x^n \rangle_{\delta q} = \int_{-1}^{1} dx \, x^n \delta q(x)$ Transversity very poorly known transversely polarized

§ True distribution can only be recovered with all moments



Problem with Moments





Problem with Moments

§ For higher moments, ops mix with lower-dimension ops
 >> Renormalization is difficult too

- § Relative error grows in higher moments
- ✤ Calculation would be costly and difficult



PDFs on the Lattice

- Long existing obstacle!
- § Holy grail of structure calculations
- § Applies to many structure quantities:

Generalized parton distributions (GPDs), Transverse-momentum distributions (TMD), Meson distribution amplitudes, ...

- § A few ideas try to solve this problem
- ✤ Hadronic tensor currents

(Liu et al., hep-ph/9806491, ... 1603.07352)



OPE without OPE (QCDSF, hep-lat/9809171, ... 1004.2100)
Fictitious heavy quarks (Detmold et al. hep-lat/0507007)
Smeared lattice operators (Davoudi et al. 1204.4146)
Looking forward to more developments here



A Promísing New Direction





Large-Momentum Effective Theory (LaMET) X. Ji, PRL. 111, 262002 (2013)

- § Calculate the parton distributions through the infinite-momentum frame Feynman, Phys. Rev. Lett. 23, 1415 (1969)
- § Weinberg introduced a more convenient description using correlation functions along the lightcone e.g. nucleon quark distribution

$$q(x, \mu) = \int \frac{d\xi_{-}}{4\pi} e^{-i\xi_{-}xP_{+}} \left\langle P \left| \overline{\psi}(\xi_{-}) \gamma_{+} \exp\left(-ig \int_{0}^{\xi_{-}} d\eta_{-} A_{+}(\eta_{-})\right) \psi(0) \right| P \right\rangle$$

Renormalization
scale μ
Gluon potential A_{+}

Lightcone coordinate $\xi_{\pm} = (t+z)/\sqrt{2}$



Large-Momentum Effective Theory (LaMET) X. Ji, PRL. 111, 262002 (2013)

- § Going back to the IMF concept
 § Finite-momentum quark distribution
 (quasi-distribution)
- Suggested operator:



$$\widetilde{q}(x, \mu, P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z) \gamma_z \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right\rangle$$

$$x = k_z / P_z$$
Lattice z coordinate
Product of lattice gauge links
Nucleon momentum $P_\mu = \{P_0, 0, 0, P_z\}$

§ Take the infinite- P_z limit to recover lightcone functions

→ Just another limit to take, like taking $a \rightarrow 0$ or $V \rightarrow \infty$

Large-Momentum Effective Theory (LaMET) X. Ji, PRL. 111, 262002 (2013) Finite-P_z corrections needed ≫ Neglect typical lattice corrections for now:

$$\tilde{q}(x,\mu,P_{z}) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_{z}}\right) q(y,\mu) + \mathcal{O}\left(M_{N}^{2}/P_{z}^{2}\right) + \mathcal{O}\left(\Lambda_{QCD}^{2}/P_{z}^{2}\right)$$
Finite $P_{z} \leftrightarrow \infty$ perturbative matching
$$Z(x,\mu/P_{z}) = C\delta(x-1) - \frac{\alpha_{s}}{2\pi}Z^{(1)}(x,\mu/P_{z})$$
Non-singlet case only
X. Xiong, X. Ji, J. Zhang, Y. Zhao, 1310.7471;
Ma and Qiu, 1404.6860
Dominant correction
(for nucleon);
known scaling form
HWL et al. 1402.1462
J.-W. Chen et al, 1603.06664

§ Benefit from our pQCD colleagues



Large-Momentum Effective Theory (LaMET) X. Ji, PRL. 111, 262002 (2013) Finite-P_z corrections needed ≫ Neglect typical lattice corrections for now:

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2/P_z^2\right)$$

complicated higher-twist operator; smaller P_z correction for nucleon J.-W. Chen et al, 1603.06664 and reference within (extrapolate it away)

§ Some similarity in more broadly-studied HQET...

$$O\left(\frac{m_b}{\Lambda}\right) = Z\left(\frac{m_b}{\Lambda}, \frac{\Lambda}{\mu}\right)o(\mu) + O\left(\frac{1}{m_b}\right) + \cdots$$



Some Lattice Details

§ Exploratory study

 $≈ N_f$ = 2+1+1 clover/HISQ lattices (MILC) $M_π ≈$ **310 MeV**, a ≈ **0.12 fm** (L ≈ 2.88 fm)

Isovector only ("disconnected" suppressed)

gives us flavor asymmetry between up and down quark ≈ 2 source-sink separations ($t_{sep} \approx 0.96$ and 1.2 fm) used

§ Properties known on these lattices

 $\gg M_{\pi}L \approx 4.6$ large enough to avoid finite-volume effects

§ Feasible with today's resources!

1402.1462 [hep-ph]; 1603.06664 [hep-ph]







Warning!

§ Exploratory study $\gg N_f = 2+1+1$ clover/HISQ lattices (MILC) $M_{\pi} \approx 310$ MeV, $a \approx 0.12$ fm ($M_{\pi}L \approx 4.5$)



NO SYSTEMATICS YET!

§ Demonstration that the method works

> Intend to motivate future LQCD work on many quantities



Step 1

§ Calculate nucleon matrix elements

$$\left\langle P \left| \overline{\psi}(z) \gamma_z \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right\rangle$$





Step 2

§ Do the integral

$$\int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \overline{\psi}(z) \gamma_z \exp\left(-ig \int_0^z dz' A_z(z')\right) \psi(0) \right| P \right\rangle$$
$$P_z \in \{0.43, 0.86, 1.29\} \text{ GeV}$$



Step 3

§ Apply finite-*P*_z corrections





Step 4

§ Extrapolate higher-twist effects $O(\Lambda_{QCD}^2/P_z^2)$ $\gg N_f = 2+1+1$ clover/HISQ lattices (MILC) $M_{\pi} \approx 310$ MeV, $a \approx 0.12$ fm ($M_{\pi}L \approx 4.5$), O(10³) measurements





A.D. Martin et al. Eur.Phys.J. C63, 189 (2009)

J.F. Owens et al. PRD 87, 094012 (2012)

S. Dulat et al. arXiv:1506.07443



Sea Flavor Asymmetry

§ First time in LQCD history to study antiquark distribution! $\gg M_{\pi} \approx 310 \text{ MeV}$



$$\bar{q}(x) = -q(-x)$$

Lost resolution in small-x region Future improvement: larger lattice volume

$$dx\left(\bar{u}(x) - \bar{d}(x)\right) \approx -0.16(7)$$

Experiment	x range	$\int_0^1 [\overline{d(x)} - \overline{u(x)}] dx$
E866	0.015< <i>x</i> <0.35	0.118 ± 0.012
NMC	0.004 < x < 0.80	0.148 ± 0.039
HERMES	0.020 < x < 0.30	0.16 ± 0.03

R. Towell et al. (E866/NuSea), Phys.Rev. D64, 052002 (2001)



Sea Flavor Asymmetry



MICHIGAN STATE

Helicity Distribution

§ Exploratory study $\gg M_{\pi} \approx 310 \text{ MeV}$





Removing $O(M_N^n/P_z^n)$ errors + $O(\alpha_s)$ + $O(\Lambda_{QCD}^2/P_z^2)$

Solution We see polarized sea asymmetry ∫ dx (Δū(x) − Δd̄(x)) ≈ 0.14(9)
Solution Both STAR and PHENIX at RHIC see Δū > Δd̄

1404.6880 and 1504.07451

> Other experiments, Fermilab DY exp'ts (E1027/E1039), future EIC

Transversity Distribution

§ Exploratory study $\gg M_{\pi} \approx 310 \text{ MeV}$



 $\int dx \left(\delta \bar{u}(x) - \delta \bar{d}(x) \right) \approx -0.10(8)$

Removing $O(M_{N}^{n}/P_{z}^{n})$ errors + $O(\alpha_{s})$ $+ O(\Lambda_{OCD}^2/P_z^2)$

 $\int dx \left(\delta \overline{u}(x) - \delta \overline{d}(x) \right) \approx -0.082$ P. Schweitzer et al., PRD 64, 034013 (2001)

SoLID at JLab, Drell-Yan exp't at FNAL (E1027+E1039), EIC, ...

Caveats

- There are 2 key issues that need to be addressed
- § Large-momentum issues
- ✤ HWL, Lattice 2013

§ Need improvement for large-momentum sources
>> Better overlapping boosted hadron smearing (asymmetric source)
>> Applications: large-q form factors, hadronic and flavor physics, ...

- ✤ Progress is being made:
 - SQCD, Novel quark smearing for hadrons with high momenta in lattice QCD, Phys. Rev. D 93, 094515 (2016)
 - Some Momentum Smearing, B. Lang, Tue. Poster
 - The Calculation of Parton Distributions from Lattice QCD
 C. WIESE, Mon. 14:15
- \sim Systematics due to $(pa)^n$

Show Excited states get worse with larger $p \Rightarrow$ multiple t_{sep} + finer *a* Show Moving frame action?



Caveats

There are 2 key issues that need to be addressed § Renormalization Issues

Currently assume the renormalization is multiplicative

$$q_{\text{norm}}(x,\mu,P_z) = \frac{q(x,\mu,P_z)}{\int dx \, q(x,\mu,P_z)} \times g_V^{\overline{\text{MS}}}(2 \text{ GeV})$$

➢ Progress is on its way:

Matching issue in quasi parton distribution approach
 T. ISHIKAWA, today 17:30

Lattice study of Wilson line operators

H. PANAGOPOULOS, Wed 10:40

Second Se

Some lessons can be learned from static heavy-quark operators?



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion-Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

They carry secret plans to build the most powerful



Summary & Outlook

Exciting time for studying structure on the lattice

- § Overcoming longstanding obstacle to full x-distribution
 > Most importantly, this can be done with today's computer
 > First lattice approach to study sea asymmetry
- § Precision control
- Working on renormalization, statistics (all-mode averaging?), larger momentum boost, finer lattice-spacing ensembles, ...



§ Closer collaboration with our heavy-quark colleagues
 > Certain similar issues: large-q form factors, HQET, ...
 § Opens doors to much future lattice-QCD structure work
 > Many first calculations waiting to be done!



Future Prospects

§ A first joint workshop with global-fitting community to address key LQCD inputs

<u>http://www.physics.ox.ac.uk/confs/PDFlattice2017</u>



Parton Distributions and Lattice Calculations in the LHC era (PDFLattice 2017) 22-24 March 2017, Oxford, UK

"The goal of this workshop is to **bring together the global PDF analysis and lattice-QCD communities** to explore ways to improve current PDF determinations. In particular, we plan to **set precision goals for lattice-QCD** calculations so that these calculations, together with experimental input, can achieve more reliable determinations of PDFs. In addition we will discuss what impact such improved determinations of PDFs will have on future new-physics searches."



Backup Slídes





This Conference

- Various presentation addressing difference issues
- § Renormalization
- > Matching issue in quasi parton distribution approach
 - T. ISHIKAWA, today 17:30
- ✤ Lattice study of Wilson line operators,
 - H. PANAGOPOU, Wed 10:40

§ Nucleon PDFs

- $\boldsymbol{\succcurlyeq}$ The Calculation of Parton Distributions from Lattice QCD
 - C. WIESE, Mon. 14:15
- § Pion distribution amplitude
- Momentum Smearing
 - B. Lang, Tue. Poster



How Can LQCD Help?

- § Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- § We are beginning to do precision calculations in nucleons
- **§ PNDME's** *g*_{*T,S*} **calculations** PNDME, 1506.06411; 1506.04196; in prep.
- \gg Extrapolate to the physical limit $m_{\pi} \rightarrow m_{\pi}^{\text{phys}}$, $a \rightarrow 0$, $L \rightarrow \infty$





Tensor/Scalar Charges





Beta Decays & BSM

§ Given precision $g_{S,T}$ and O_{BSM} , predict new-physics scales Low-Energy Precision LOCD input

Expt
$$\longrightarrow O_{\text{BSM}} = fo(\varepsilon_{s,\tau} g_{s,\tau}) \xleftarrow{} (m_{\pi} \rightarrow 140 \text{ MeV}, a \rightarrow 0)$$



 $\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$ Upcoming precision low-energy experiments LANL/ ORNL UCN neutron decay exp't $|B_1 - b|_{\rm BSM} < 10^{-3}$ $|b|_{\rm RSM} < 10^{-3}$ CENPA: ${}^{6}\text{He}(b_{GT})$ at 10^{-3} PNDME, PRD85 054512 (2012);

