THE 34TH INTERNATIONAL SYMPOSIUM ON LATTICE GAUGE THEORIES
UNIVERSITY OF SOUTHAMPTON, JULY 24-30, 2016

CHARGE RADII AND HIGHER ELECTROMAGNETIC MOMENTS WITH LATTICE QCD IN NONUNIFORM BACKGROUND FIELDS

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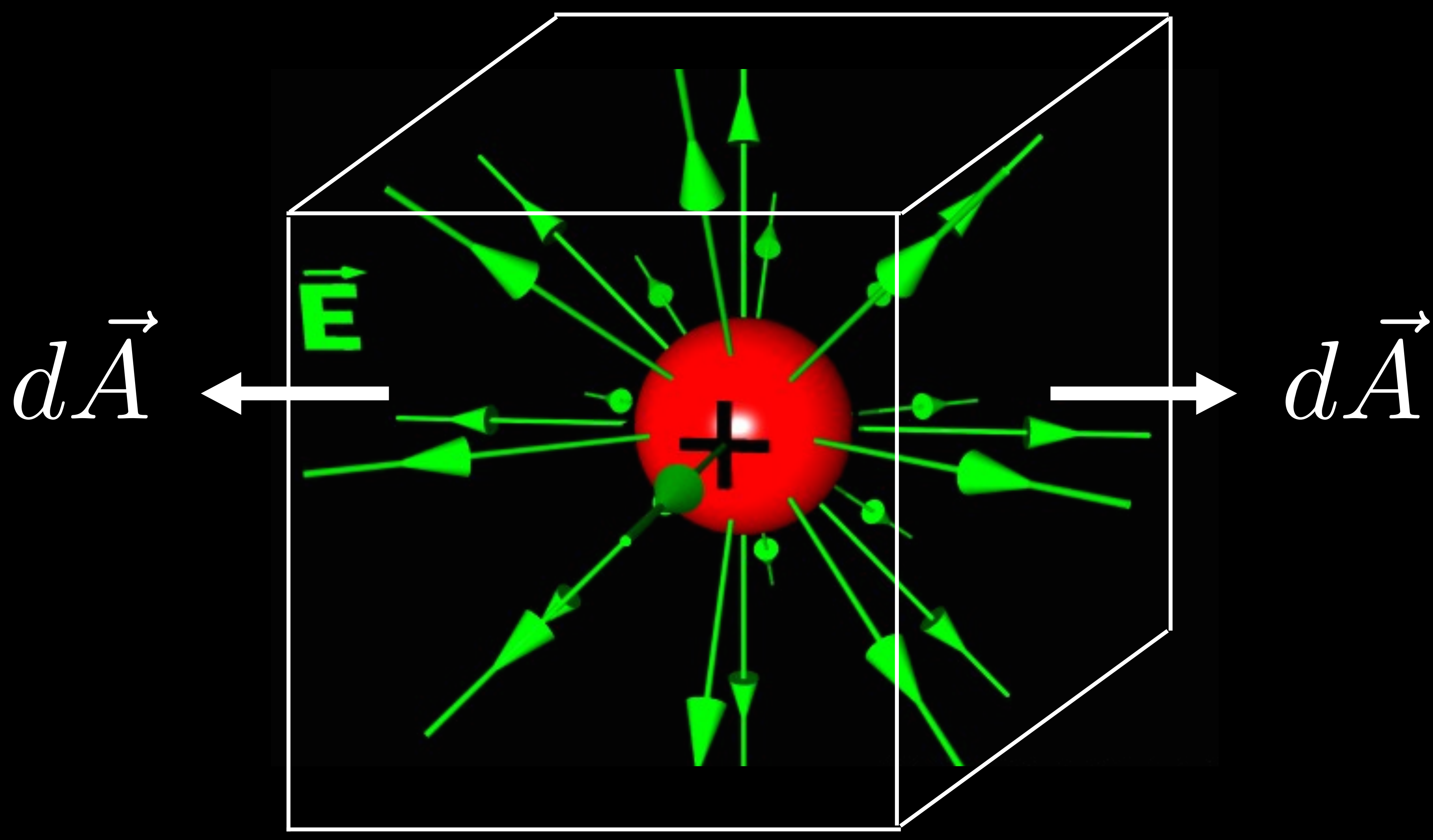
MIT

ZD and W. Detmold, Phys. Rev. D 92, 074506 (2015),
ZD and W. Detmold, Phys. Rev. D 93, 014509 (2016).

ELECTROMAGNETIC EFFECTS IN STRONGLY INTERACTING SYSTEMS

1) DYNAMICAL PHOTONS



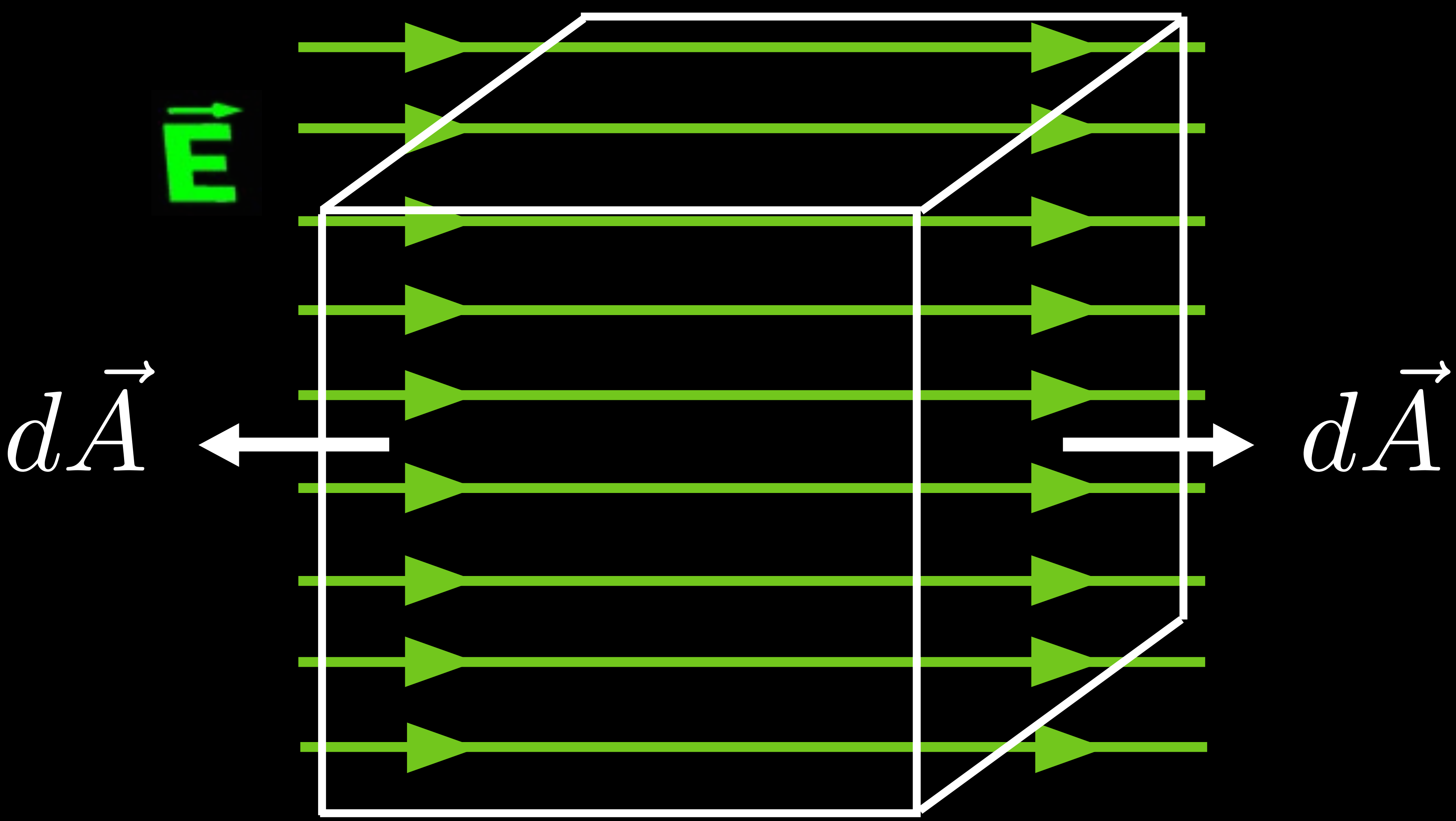
GAUSS'S LAW + PERIODICITY ?

Hayakawa and Uno, Prog. Theor. Phys. 120, 413 (2008)
ZD, M. J. Savage, Phys. Rev. D 90, 054503 (2014)
Borsanyi, et al., Science 347:1452-1455 (2015).

Lucini, et, al., JHEP02(2016)076.
Endres, et, al., to appear in PRL, arXiv: 1507.08916 [hep-lat].

ELECTROMAGNETIC EFFECTS IN STRONGLY INTERACTING SYSTEMS

2) CLASSICAL ELECTROMAGNETISM

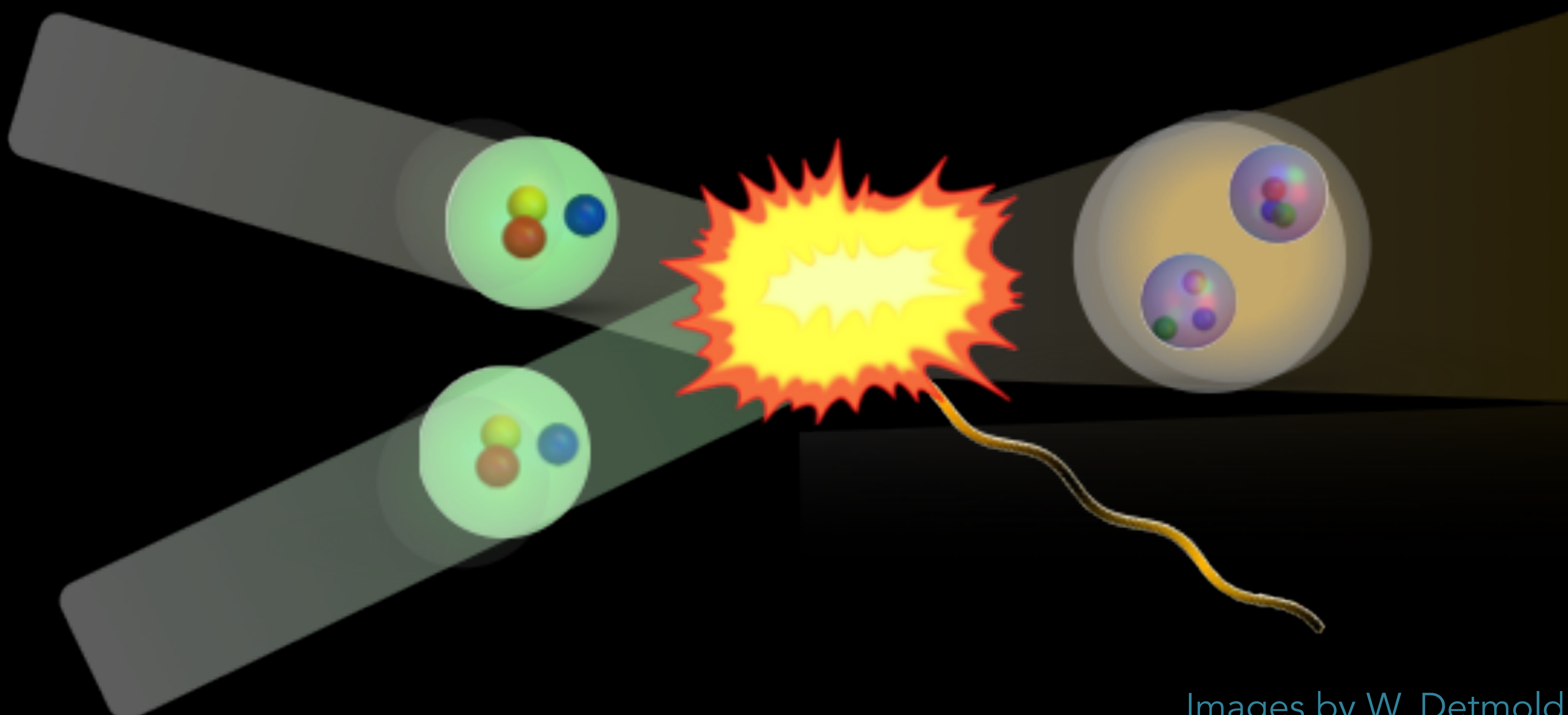


BACKGROUND FIELDS + PERIODICITY

G. 't Hooft, Nuclear Physics B 153, 141 (1979).
J. Smit and J. C. Vink, Nucl.Phys. B286, 485 (1987).
M. Al-Hashimi and U.-J. Wiese, Annals Phys. 324, 343 (2009).

SOME RECENT STATE-OF-THE-ART APPLICATIONS

$np \to d\gamma$ FROM LATTICE QCD



Images by W. Detmold

Beane, at al. [NPLQCD collaboration], Phys. Rev. Lett. 115, 132001 (2015).

SOME RECENT STATE-OF-THE-ART APPLICATIONS

$np \to d\gamma$ FROM LATTICE QCD

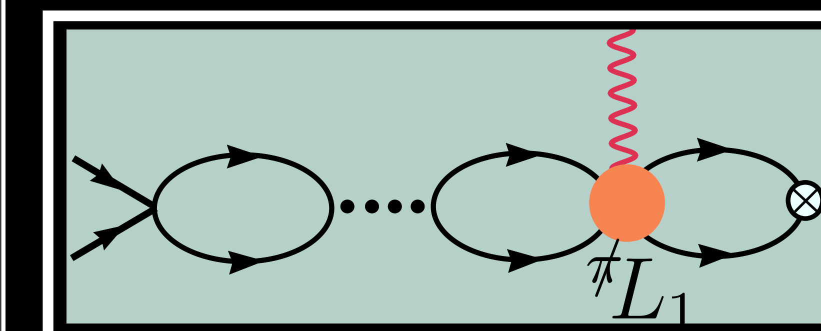
AN EFFECTIVE FIELD THEORY RESULT

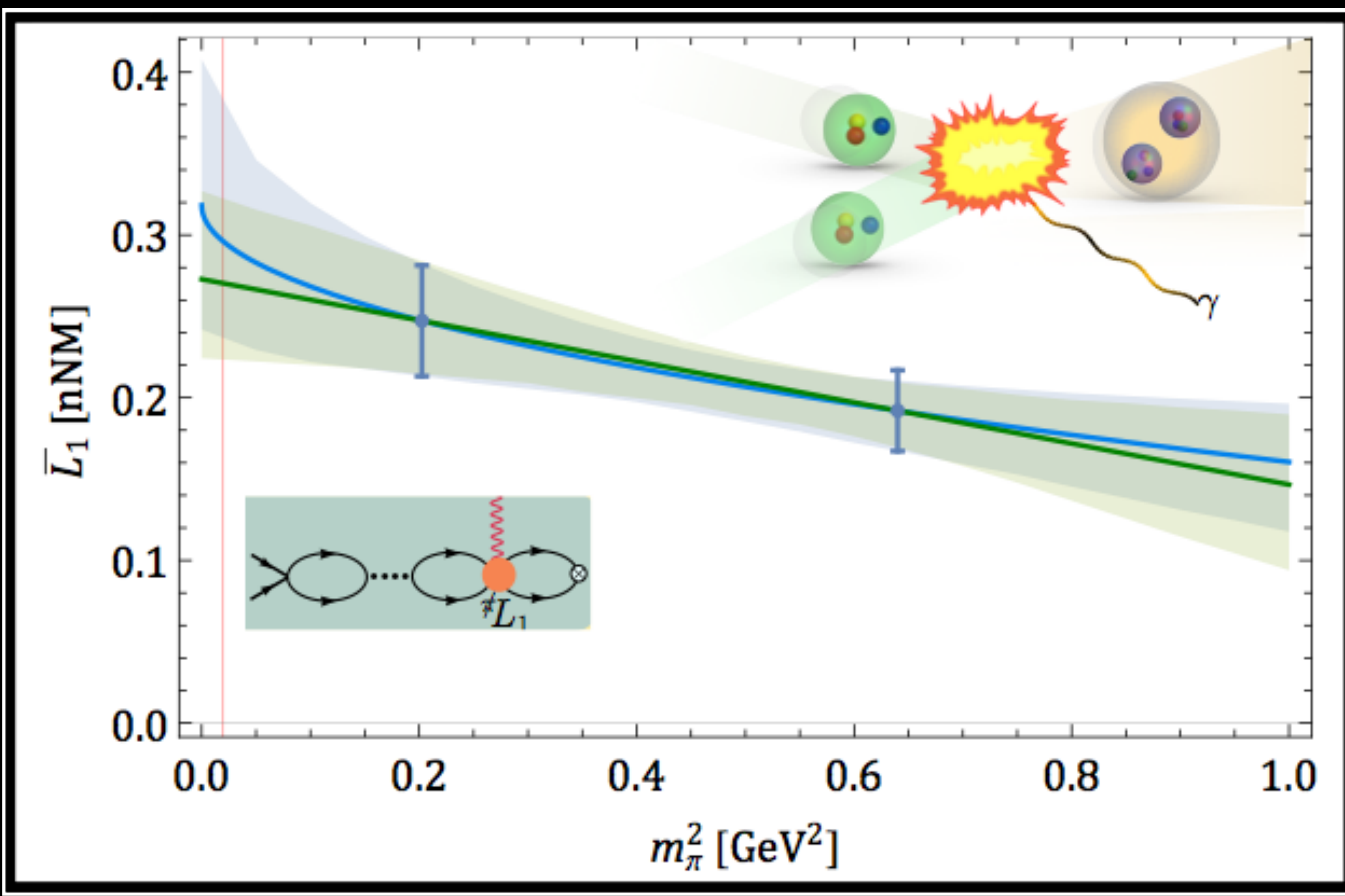
Beane and Savage, Nucl.Phys. A694, 511 (2001).

$$\sigma(np \to d\gamma) = \frac{e^2(\gamma_0^2 + |\mathbf{p}|^2)^3}{M^4\gamma_0^3|\mathbf{p}|}|\tilde{X}_{M1}|^2 + \ldots$$

M1 transition: depends on $\overline{L}_1$

NEEDS LATTICE QCD INPUT





Beane, at al. [NPLQCD collaboration], Phys. Rev. Lett. 115, 132001 (2015).

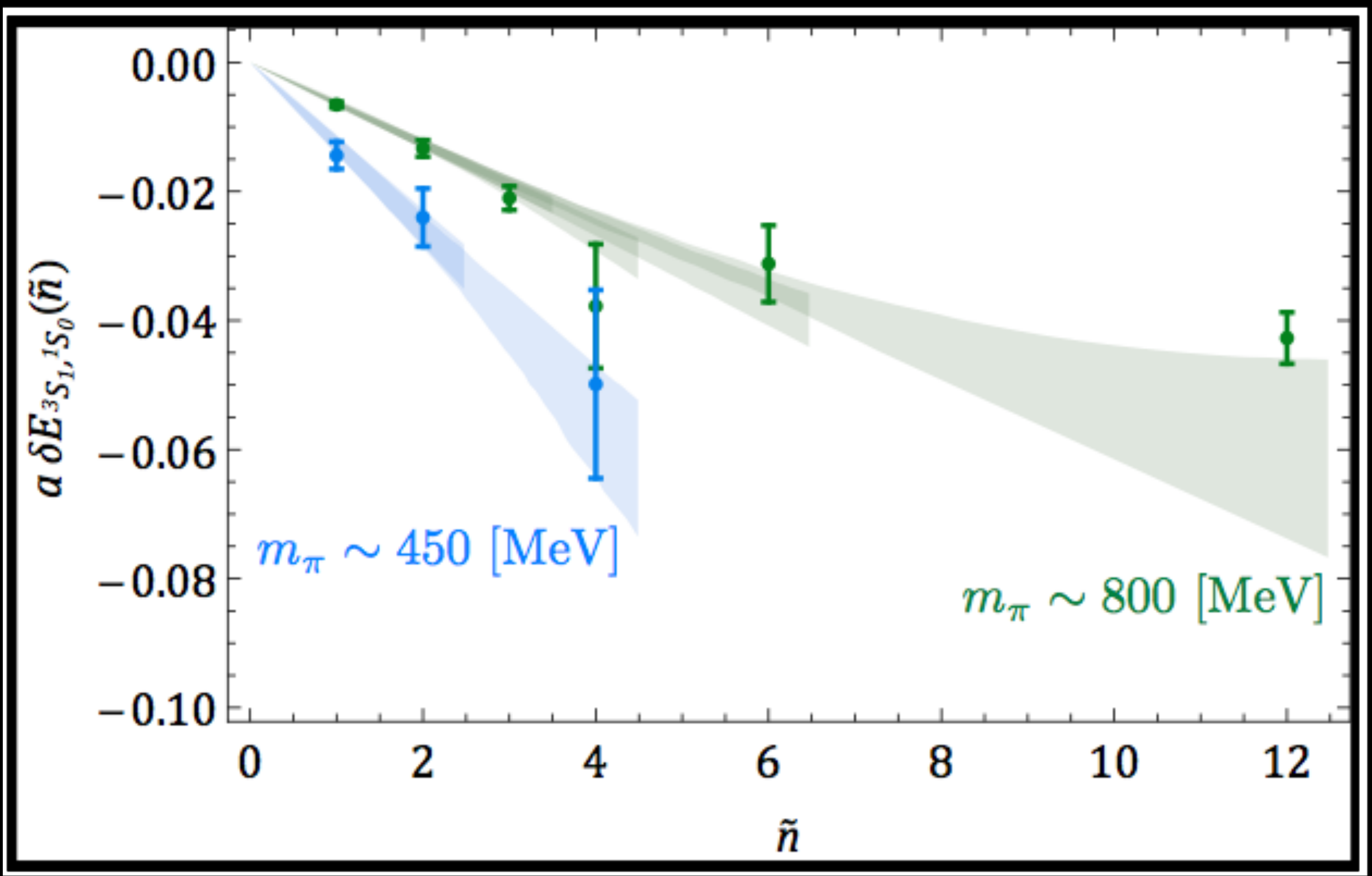
SOME RECENT STATE-OF-THE-ART APPLICATIONS

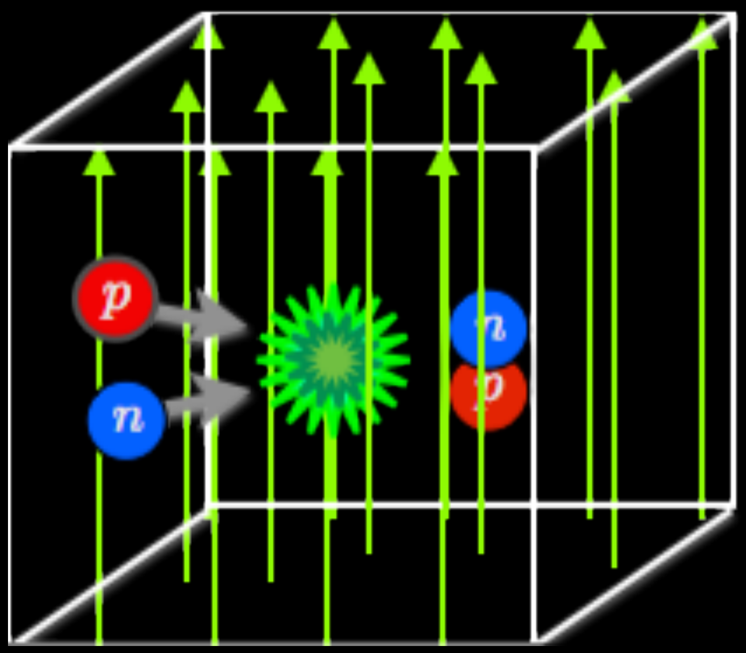
$np \to d\gamma$ FROM LATTICE QCD

SET UP A BACKGROUND MAGNETIC FIELD

Detmold and Savage, Nucl.Phys. A743, 170 (2004).

$$\delta E_{^3S_1,^1S_0} \equiv \Delta E_{^3S_1,^1S_0} - [E_{p,\uparrow} - E_{p,\downarrow}] + [E_{n,\uparrow} - E_{n,\downarrow}] \to 2\overline{L}_1 |e\mathbf{B}|/M + \mathcal{O}(\mathbf{B}^2)$$



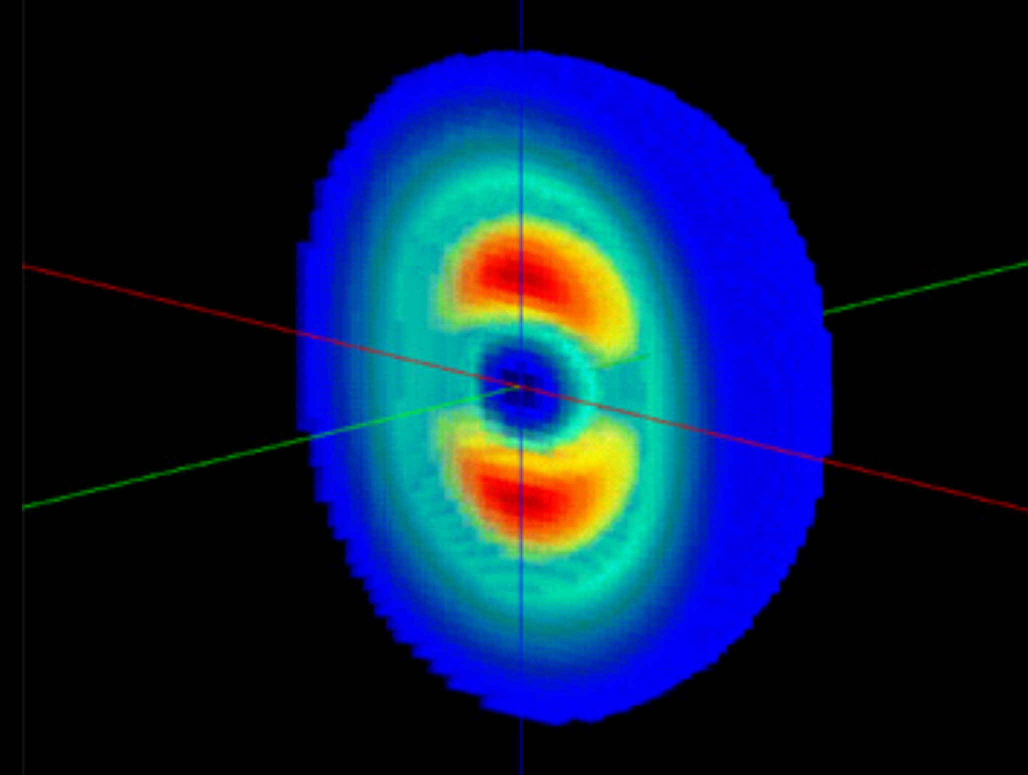


Beane, at al. [NPLQCD collaboration], Phys. Rev. Lett. 115, 132001 (2015).

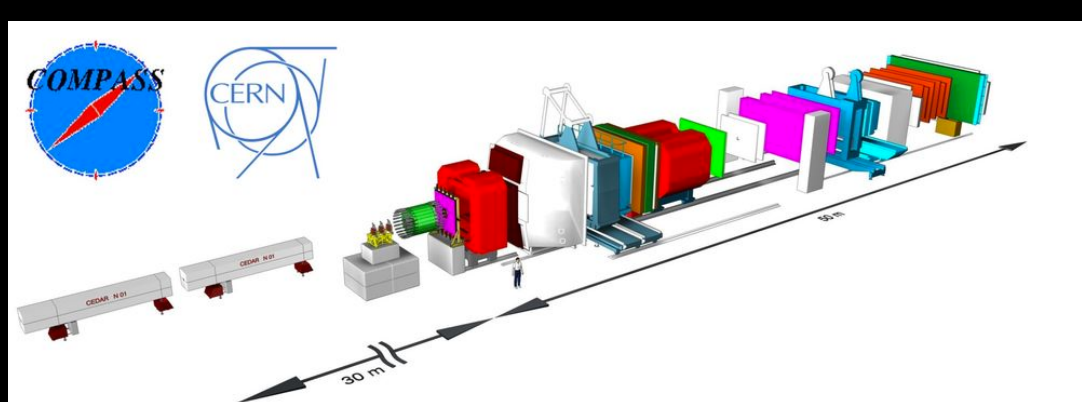
MORE PHYSICS WITH BACKGROUND FIELDS?

1) EM CHARGE RADIUS

2) ELECTRIC QUADRUPOLE MOMENT

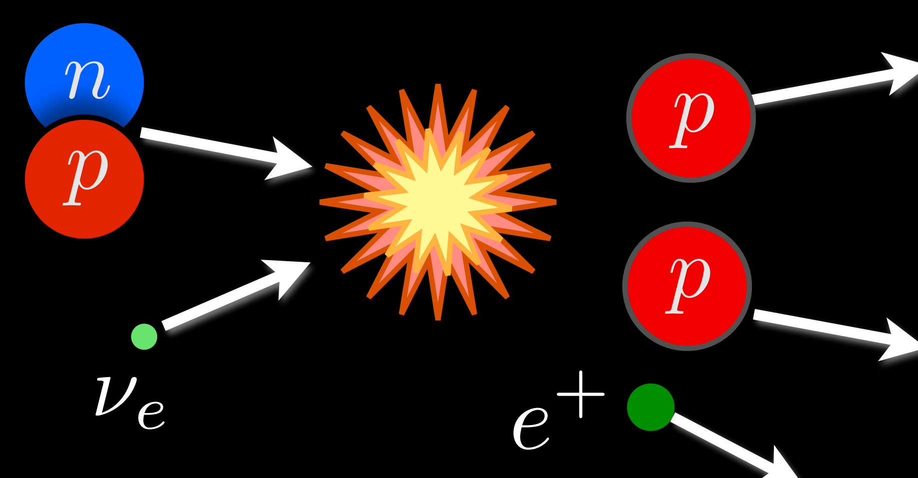


3) FORM FACTORS



Detmold, Phys.Rev. D71, 054506 (2005)

4) AXIAL BACKGROUND FIELDS



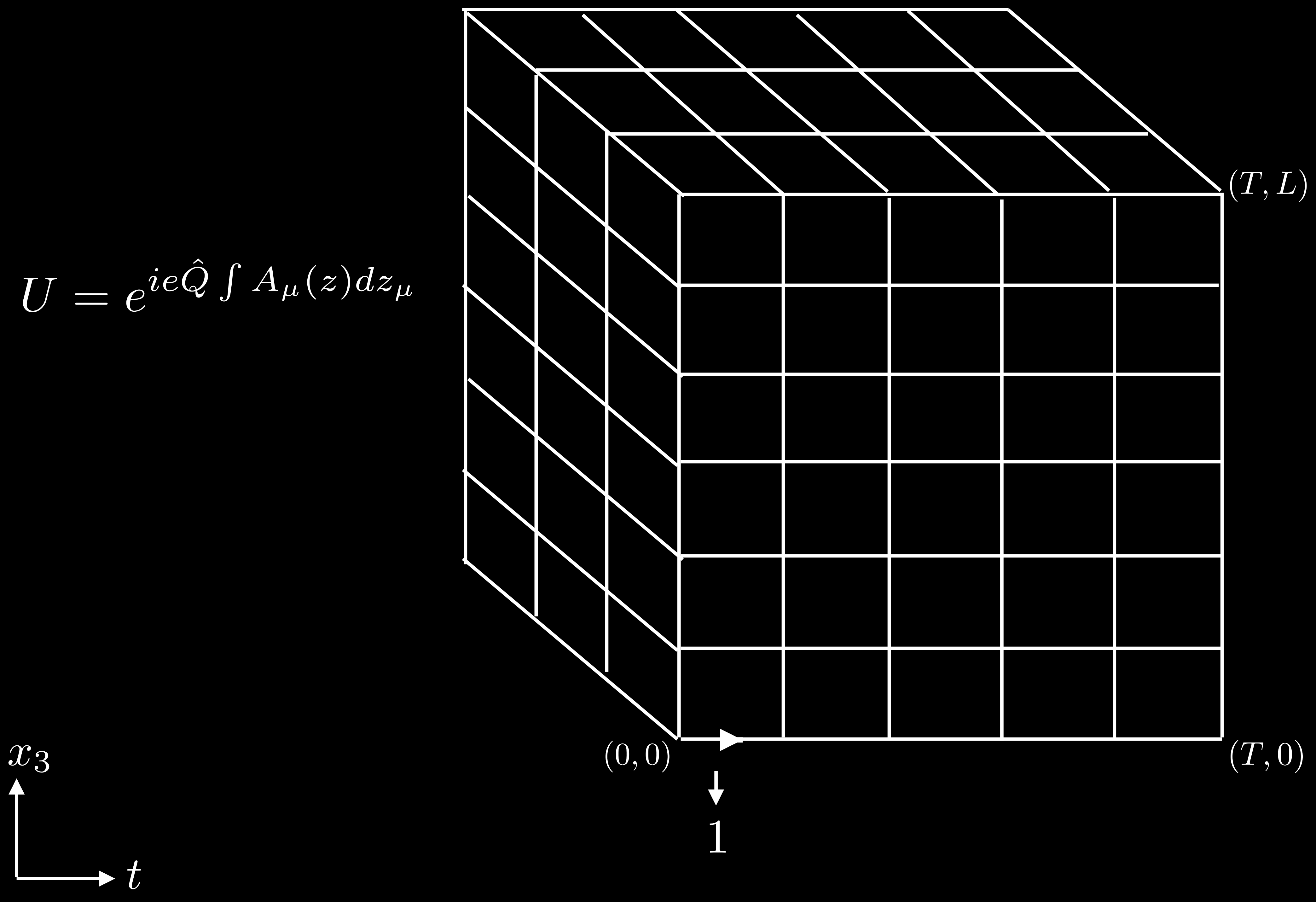
IMPLEMENTATION OF U(1) BACKGROUND GAUGE FIELDS ON A PERIODIC HYPERCUBIC LATTICE

ZD and W. Detmold, Phys. Rev. D 92, 074506 (2015)

PERIODIC IMPLEMENTATION OF NONUNIFORM BACKGROUND FIELDS

$$A_\mu = \left(-\frac{E_0}{2}(\mathbf{x}_3 - \left[\frac{\mathbf{x}_3}{L}\right] L)^2, \mathbf{0}\right) \quad \rightarrow \quad \mathbf{E} = E_0 \mathbf{x}_3 \hat{\mathbf{x}}_3$$

$$U = e^{ie\hat{Q}\int A_\mu(z) dz_\mu}$$



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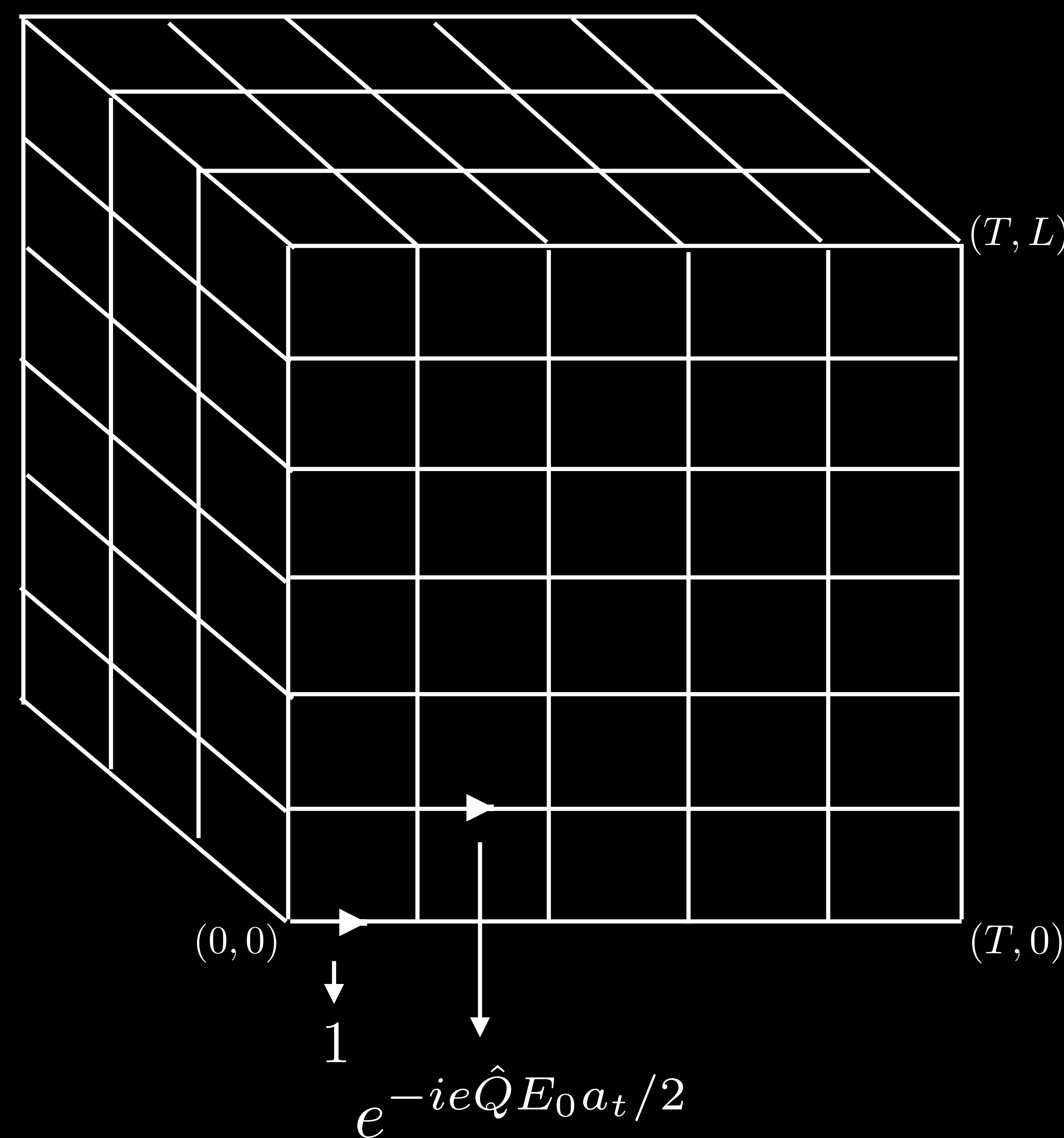

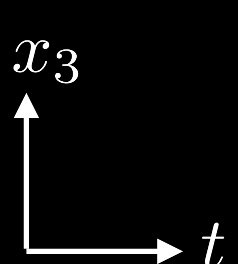

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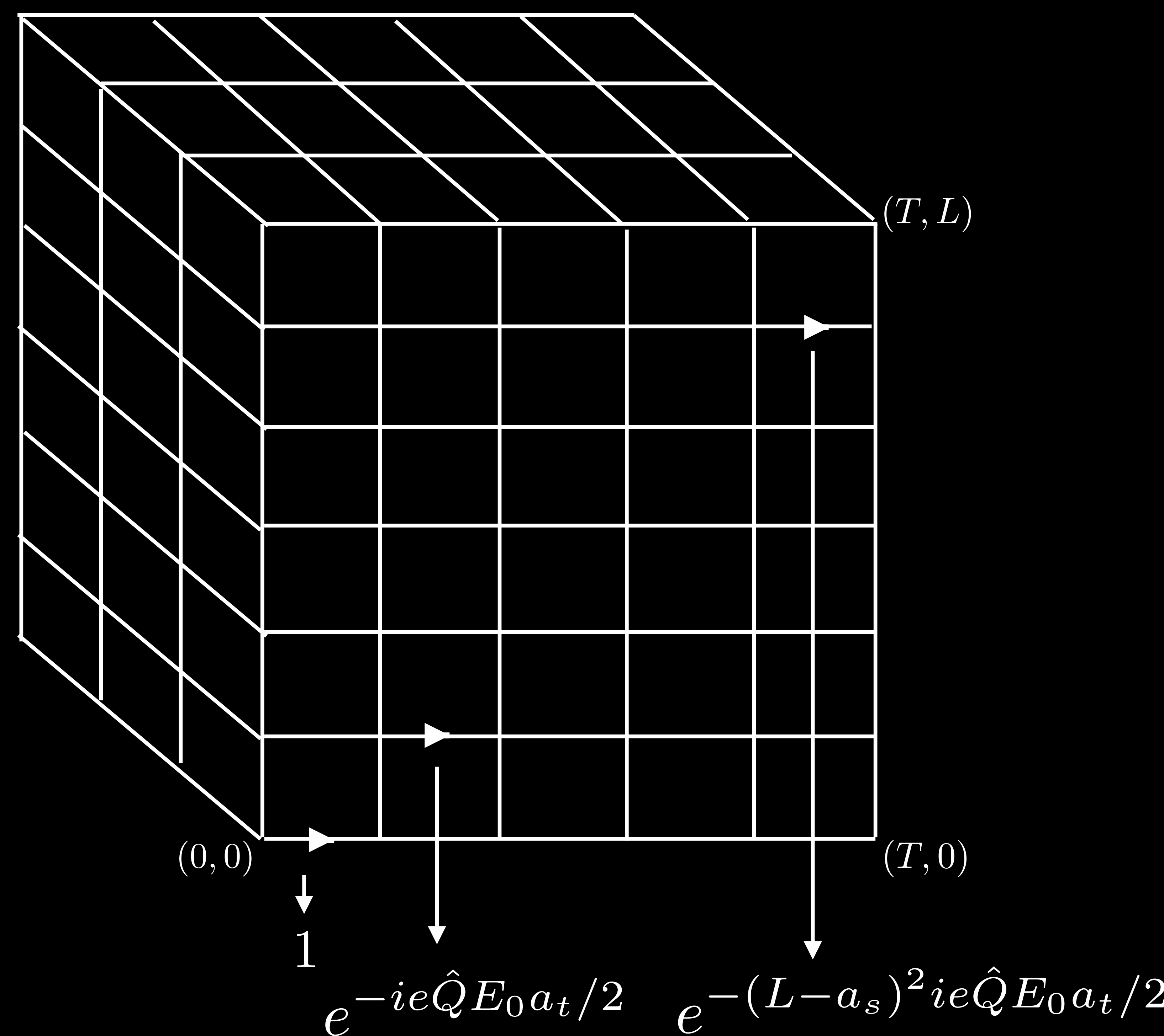

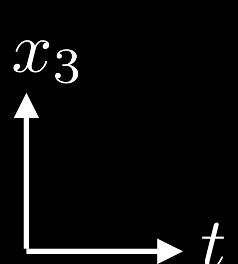

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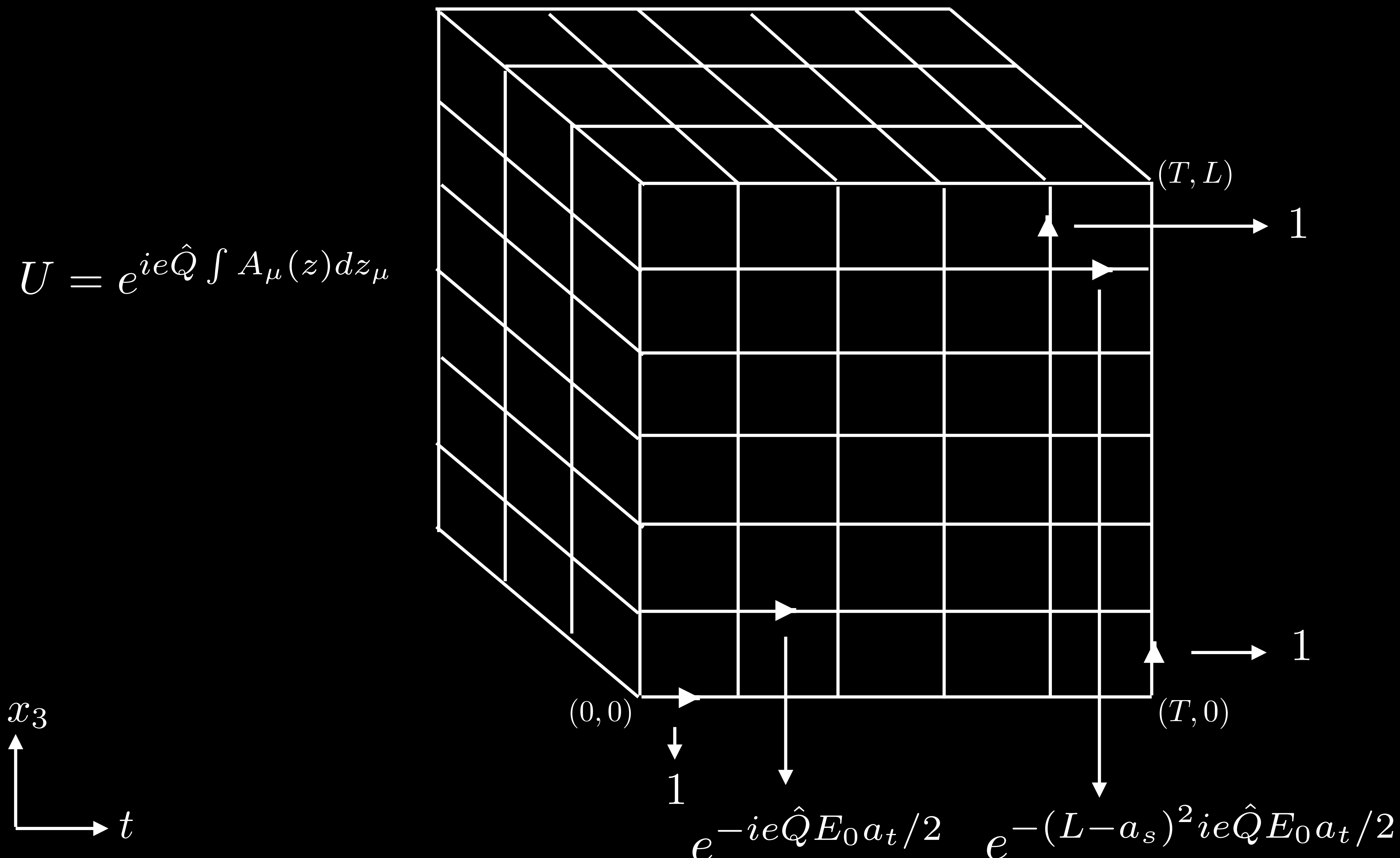

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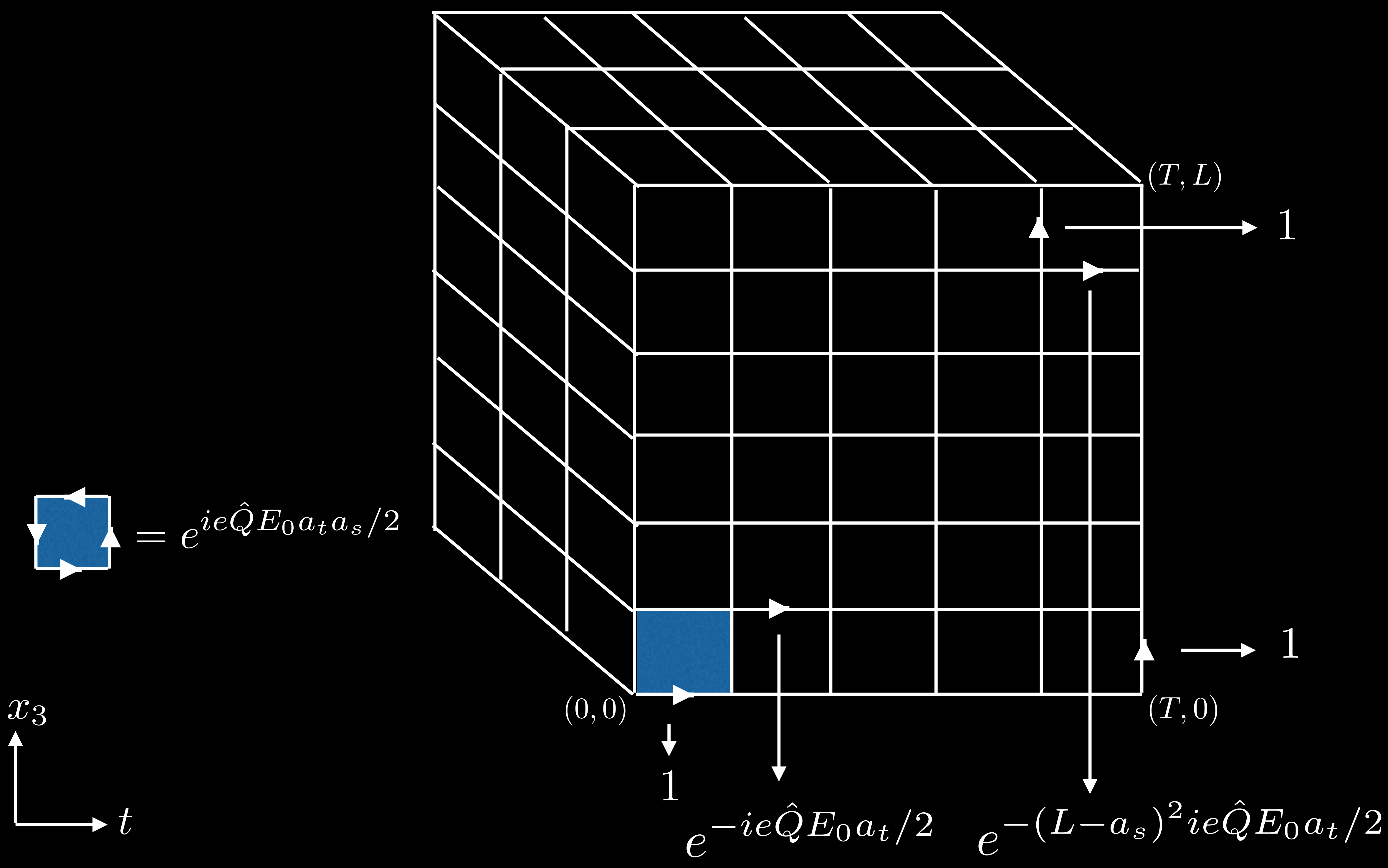

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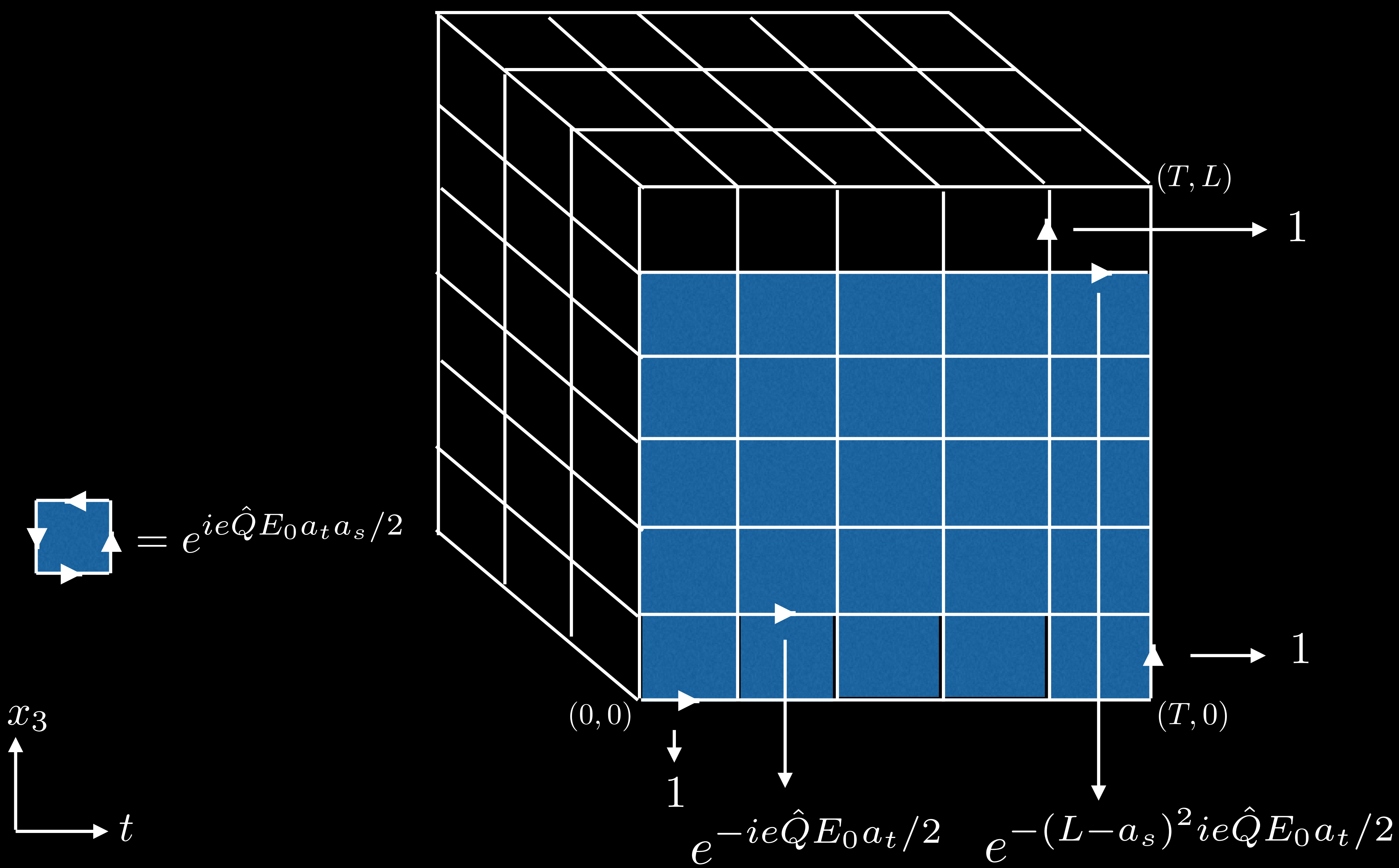

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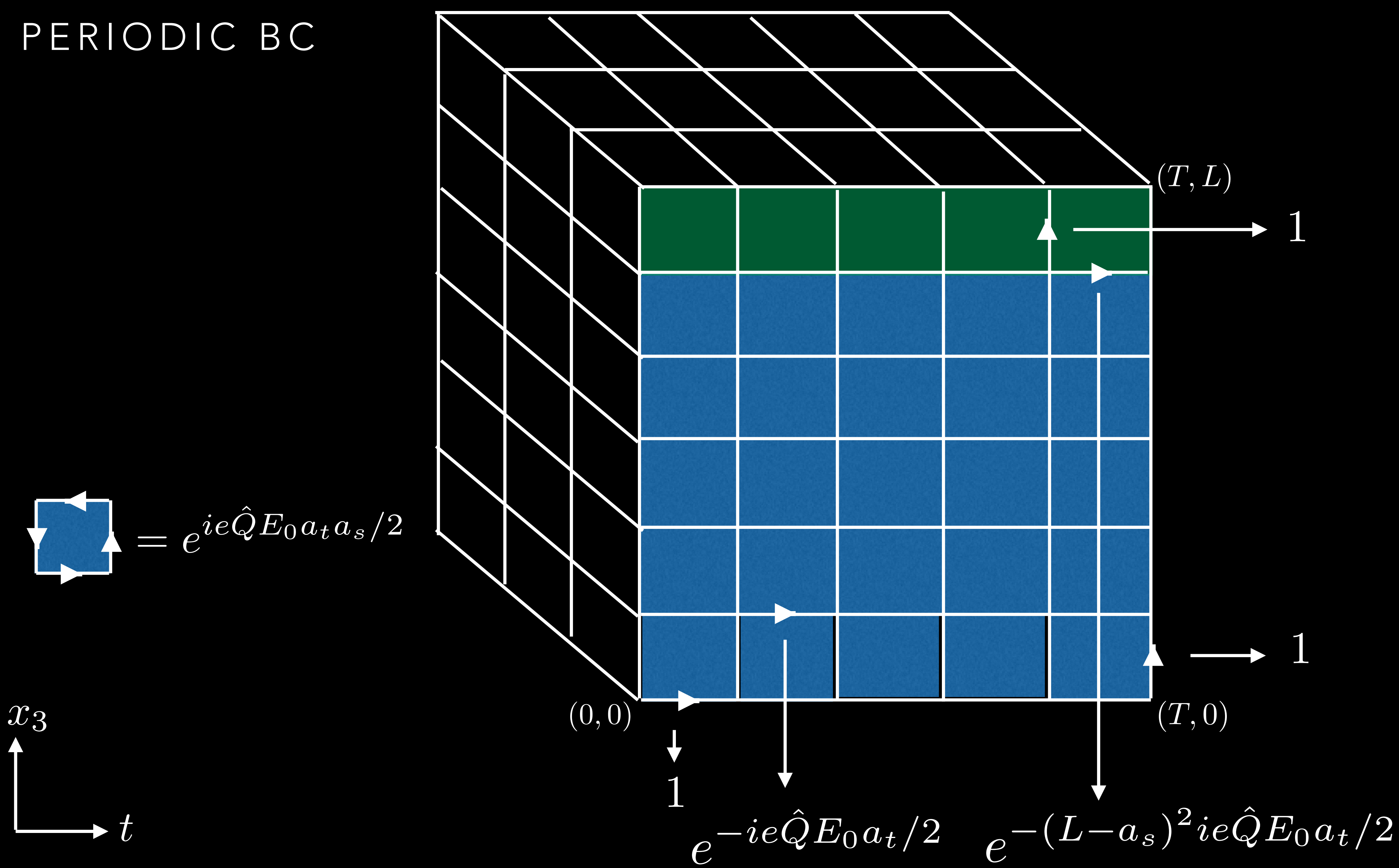
PERIODIC BC



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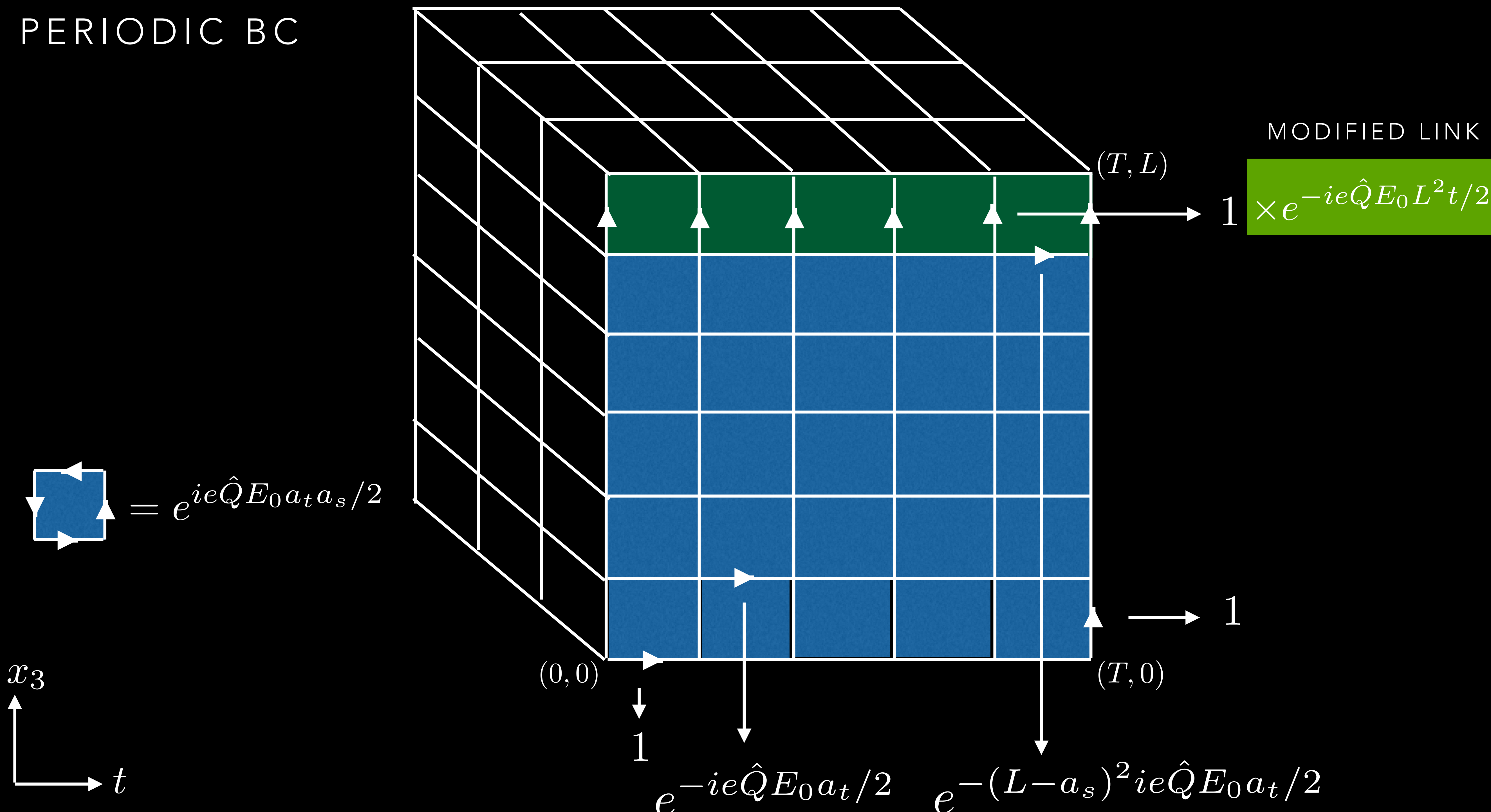

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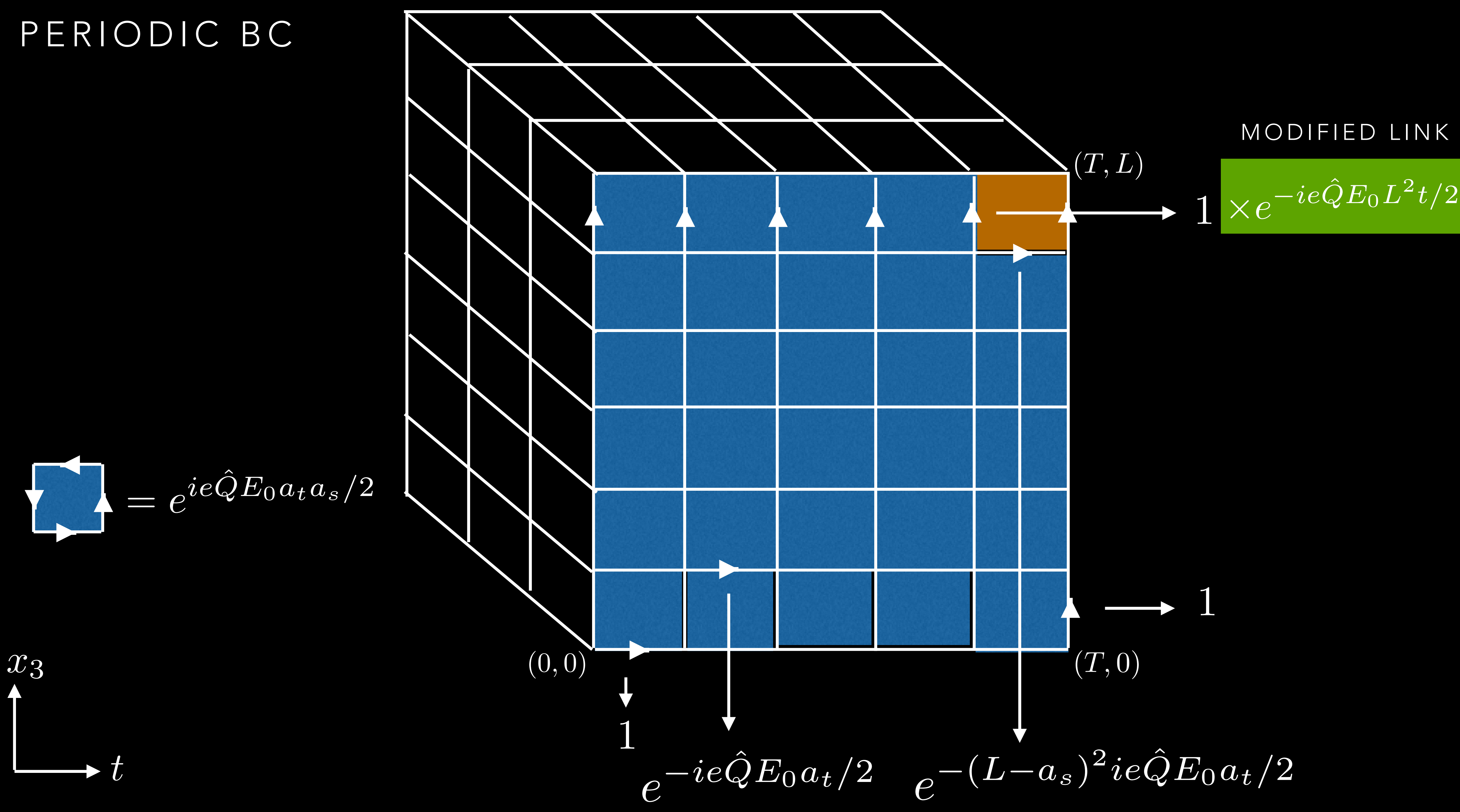

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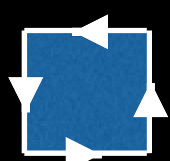
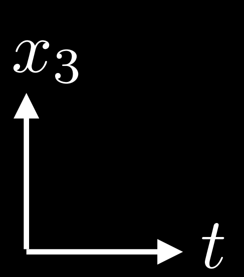
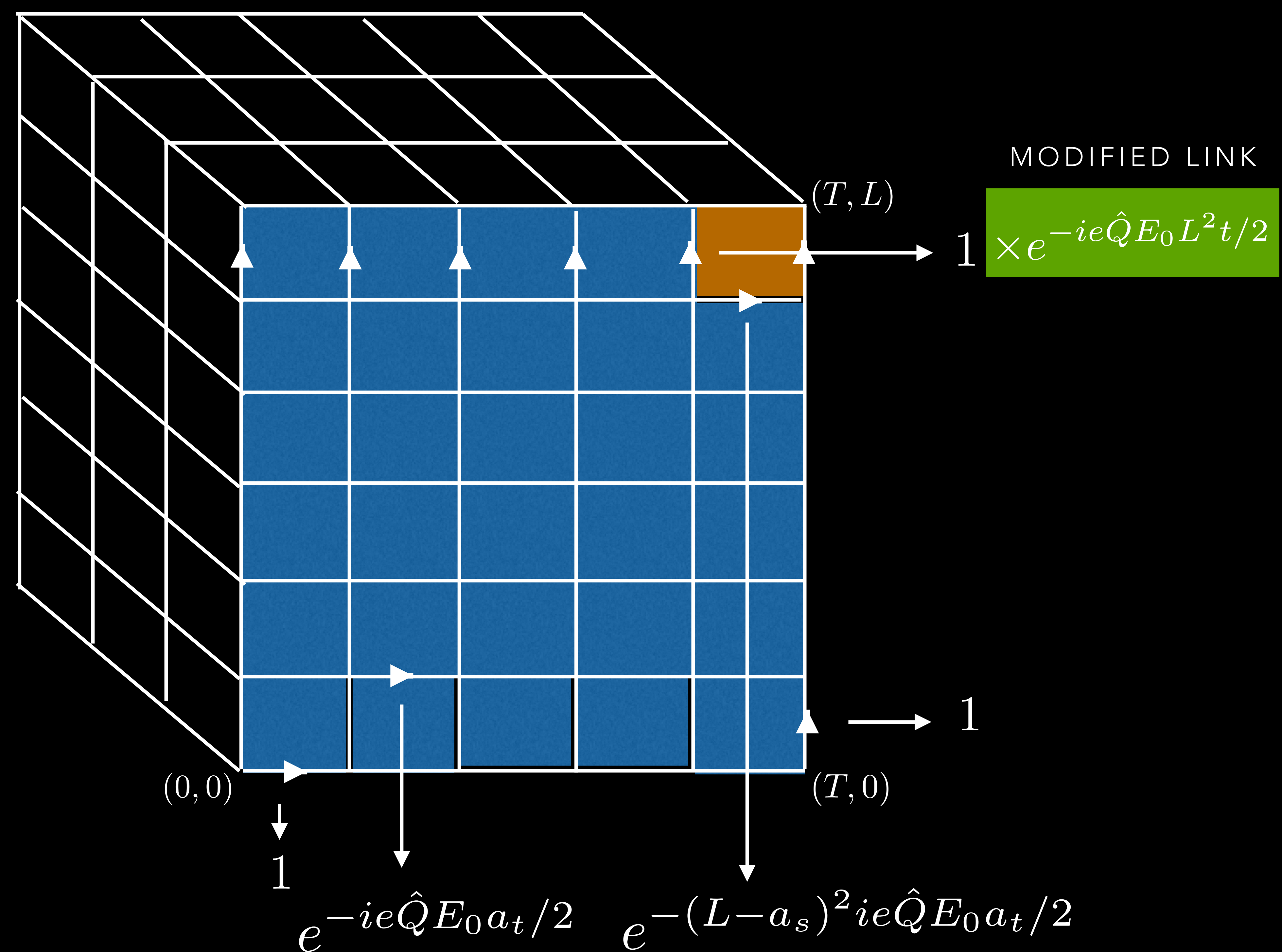
PERIODIC BC

$$e^{ie\hat{Q}E_0L^2T/2} = 1$$



$$= e^{ie\hat{Q}E_0 a_t a_s/2}$$





PERIODIC IMPLEMENTATION OF NONUNIFORM BACKGROUND FIELDS

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PERIODIC BC

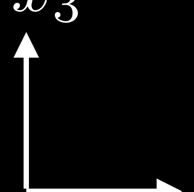
$$e^{ie\hat{Q}E_0L^2T/2} = 1$$

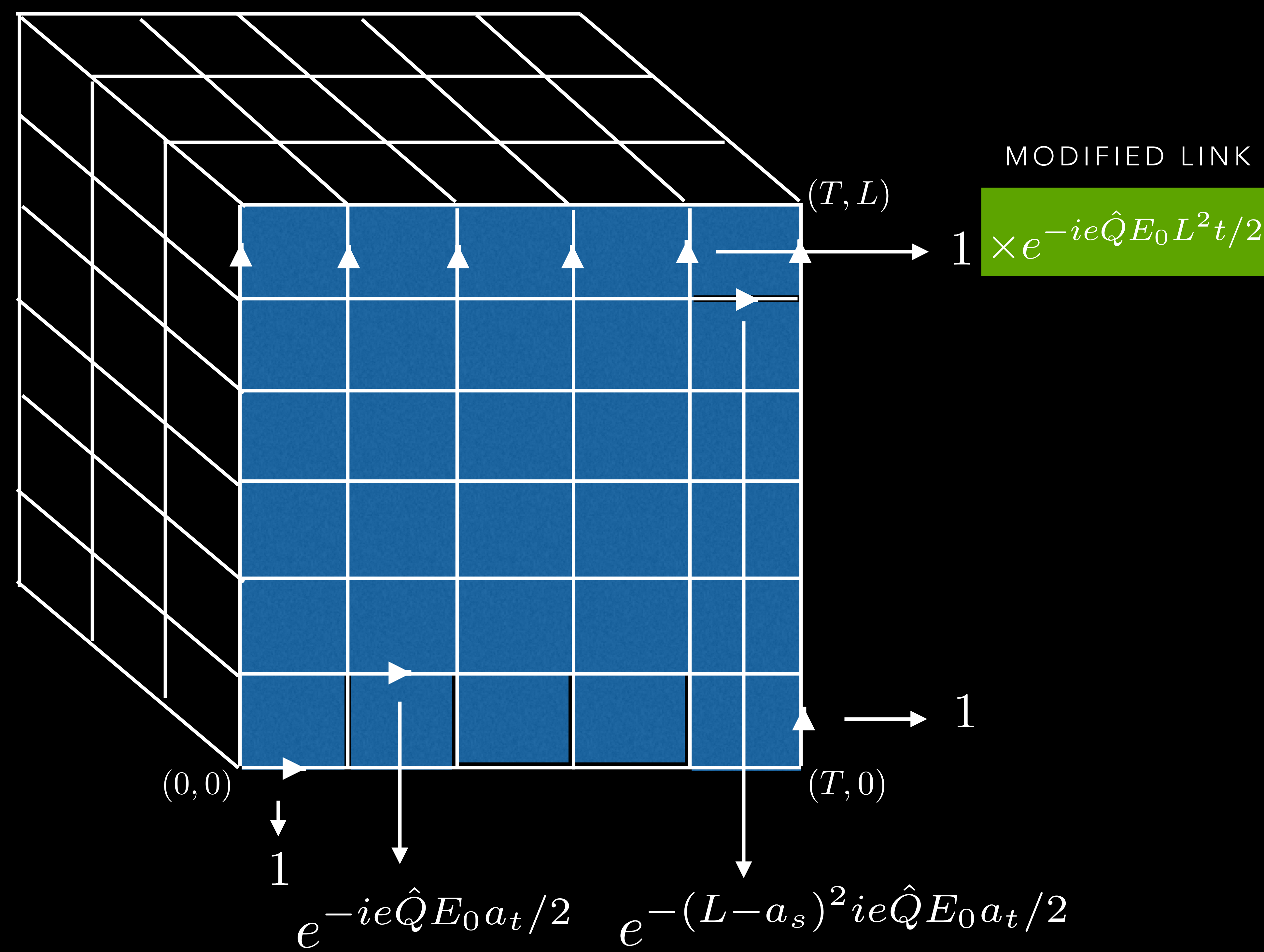
↓

$$E_0 = \frac{4\pi n}{e\hat{Q}L^2T}$$

QUANTIZATION CONDITION
FOR THE SLOPE OF THE FIELD

$$= e^{ie\hat{Q}E_0 a_t a_s/2}$$

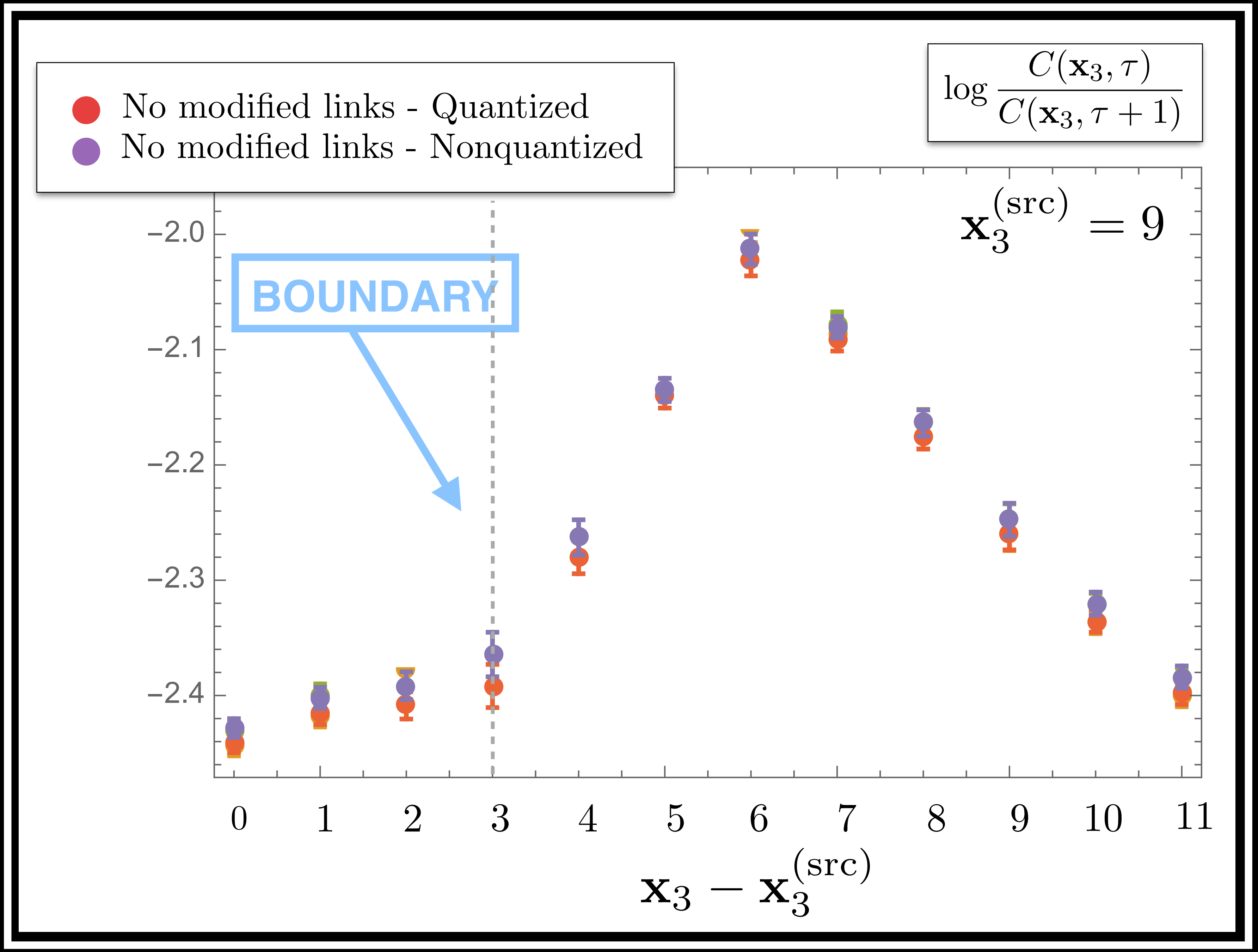




PERIODIC IMPLEMENTATION OF NONUNIFORM BACKGROUND FIELDS

NEUTRAL PION CORRELATION FUNCTION

$\mathbf{E} = E_0 \mathbf{x}_3 \hat{\mathbf{x}}_3$



PERIODIC IMPLEMENTATION OF NONUNIFORM BACKGROUND FIELDS

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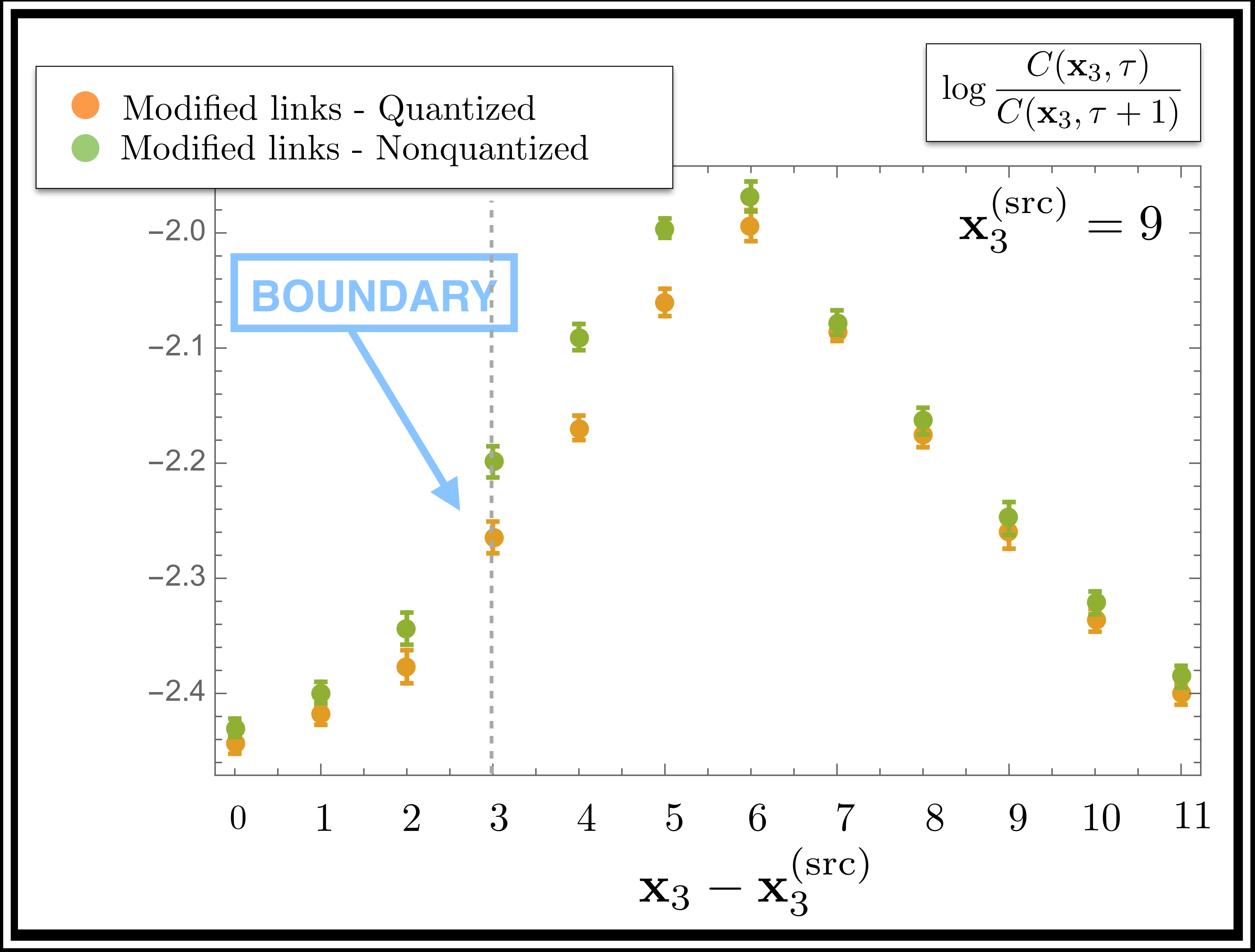


PERIODIC IMPLEMENTATION OF NONUNIFORM BACKGROUND FIELDS

MODIFIED LINKS

$$U_\mu^{(\mathrm{QCD})}(x) \;\to\; U_\mu^{(\mathrm{QCD})}(x) \times e^{ie\hat{Q}A_\mu(x)a_\mu} \times \prod_{\nu\neq\mu} e^{ie\hat{Q}\left[A_\nu(x_\mu=0,x_\nu)-\widetilde{A}_\nu(x_\mu=L_\mu,x_\nu)\right]f_{\mu,\nu}(x_\nu)\times\delta_{x_\mu,L_\mu-a_\mu}}$$

WITH LINK FUNCTIONS SATISFYING

$$\left[A_\nu(x_\mu=0,x_\nu+a_\nu)-\widetilde{A}_\nu(x_\mu=L_\mu,x_\nu+a_\nu)\right]f_{\mu,\nu}(x_\nu+a_\nu)=\left[A_\nu(x_\mu=0,x_\nu)-\widetilde{A}_\nu(x_\mu=L_\mu,x_\nu)\right]\left(f_{\mu,\nu}(x_\nu)+a_\nu\right)$$

QUANTIZATION CONDITIONS

$$\left[\prod_{x_\mu=0}^{L_\mu-a_\mu} e^{-ie\hat{Q}\left[A_\mu(x_\mu,x_\nu=0)-\widetilde{A}_\mu(x_\mu,x_\nu=L_\nu)\right]a_\mu}\right]\left[\prod_{x_\nu=0}^{L_\nu-a_\nu} e^{ie\hat{Q}\left[A_\nu(x_\mu=0,x_\nu)-\widetilde{A}_\nu(x_\mu=L_\mu,x_\nu)\right]a_\nu}\right]=1$$

LINEARLY VARYING FIELDS → CHARGE RADIUS-QUADRUPOLE MOMENT

OSCILLATORY FIELDS → FORM FACTORS

VARIOUS SPACE/TIME DEPENDENCE → SPIN POLARIZABILITIES OF NUCLEONS

PERIODIC IMPLEMENTATION OF NONUNIFORM BACKGROUND FIELDS

OSCILLATORY FIELDS

$$A_\mu = (A_0, -\mathbf{A}) = \left(\frac{ia}{q_3} e^{iq_3 \mathbf{x}_3}, 0, 0, 0\right) \rightarrow \mathbf{E} = a e^{iq_3 \mathbf{x}_3} \hat{\mathbf{x}}_3$$

PERIODIC BC

$$e^{-\frac{e\hat{Q}a}{q_3}(1-e^{iq_3 L})T} = 1$$

$$\downarrow$$

$$q_3 = \frac{2\pi n}{L}$$

QUANTIZATION CONDITION FOR THE FREQUENCY OF THE FIELD

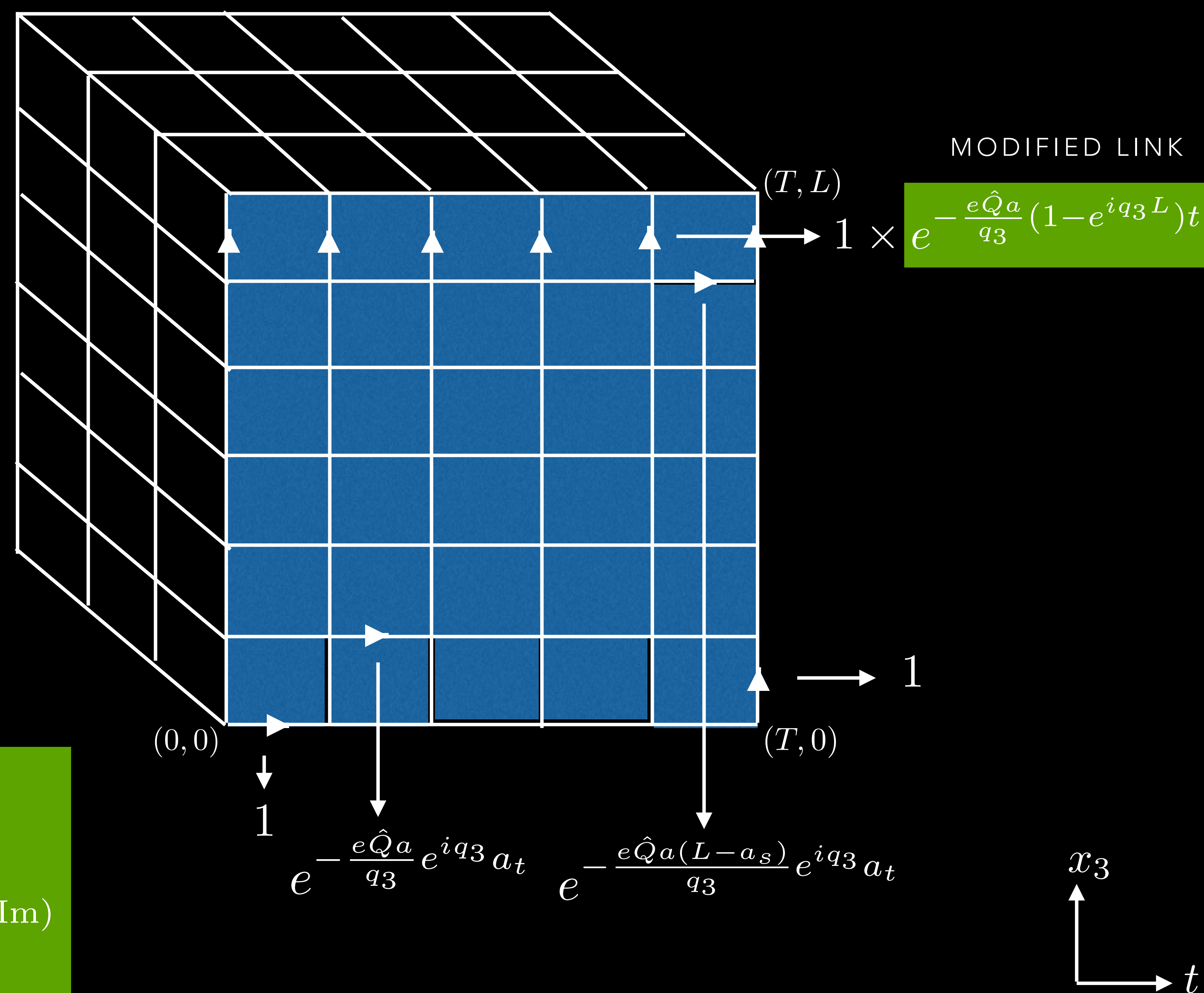
As in: Bali and Endrodi, PhysRevD.92.054506

OR

QUANTIZATION CONDITION FOR THE AMPLITUDE OF THE FIELD

$$a^{(\mathrm{Im})} = \frac{\pi q_3 n'}{e\hat{Q}T},$$

$$a^{(\mathrm{Re})} = -\frac{\sin(q_3 L)}{1-\cos(q_3 L)} a^{(\mathrm{Im})}$$



TOWARDS AN EXTRACTION OF ELECTRIC QUADRUPOLE MOMENT AND CHARGE RADIUS

ZD and W. Detmold, Phys. Rev. D 93, 014509 (2016)

THE GENERAL STRATEGY: AN EFFECTIVE SINGLE-PARTICLE DESCRIPTION

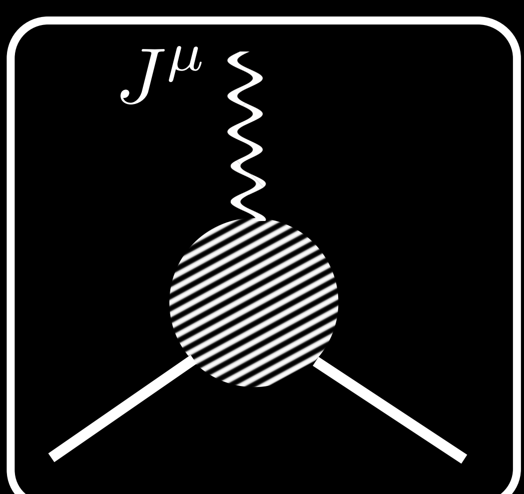
SPECIAL CARE MUST BE GIVEN TO EOM OPERATORS IN NR THEORY WITH BACKGROUND FIELDS

Lee and Tiburzi, Phys. Rev. D 89, 054017 (2014), Phys. Rev. D 90, 074036 (2014).

A SPIN-1 THEORY

$$\mathcal{L} = \frac{1}{2} W^{\dagger\mu\nu} W_{\mu\nu} + M^2 V^{\dagger\alpha} V_\alpha - \frac{1}{2} W^{\dagger\mu\nu}(D_\mu V_\nu - D_\nu V_\mu) - \frac{1}{2}((D_\mu V_\nu)^\dagger - D_\nu V_\mu^\dagger) W^{\mu\nu} +$$
$$ieC^{(0)} F_{\mu\nu} V^{\dagger\mu} V^\nu + \frac{ieC_1^{(2)}}{M^2} \partial_\mu F^{\mu\nu}((D_\nu V^\alpha)^\dagger V_\alpha - V^{\dagger\alpha} D_\nu V_\alpha) +$$
$$\frac{ieC_2^{(2)}}{M^2} \partial^\alpha F^{\mu\nu}((D_\alpha V_\mu)^\dagger V_\nu - V_\nu^\dagger D_\alpha V_\mu) + \frac{ieC_3^{(2)}}{M^2} \partial^\nu F^{\mu\alpha}((D_\mu V_\alpha)^\dagger V_\nu - V_\nu^\dagger D_\mu V_\alpha) + \mathcal{O}\left(\frac{1}{M^4}, F^2\right)$$

ONE-PHOTON MATCHING



$$C^{(0)} = \overline{\mu}_1 - Q_0,$$
$$C_1^{(2)} = \frac{1}{6e}\left(M^2 \langle r^2 \rangle_E + e\overline{Q}_2\right),$$
$$C_2^{(2)} = \frac{1}{4}(-Q_0 + \overline{Q}_2 + \overline{\mu}_1) + \frac{1}{6e} M^2 \langle r^2 \rangle_M,$$
$$C_3^{(2)} = \frac{1}{2}(-Q_0 + \overline{Q}_2 + \overline{\mu}_1).$$

ZD and W. Detmold, Phys. Rev. D 92, 074506 (2015)

1) MATCHING TO CORRELATION FUNCTIONS

A (semi) relativistic effective theory

$$\widehat{\mathcal{H}}^{(\boldsymbol{E})}_{\mathrm{SR}} = M\sigma_3 + eQ_0\widehat{\varphi} + (\sigma_3 + i\sigma_2)\frac{\widehat{\boldsymbol{\pi}}^2}{2M} - \frac{i\sigma_2}{M}(\boldsymbol{S}\cdot\widehat{\boldsymbol{\pi}})^2 + \frac{e}{2M^2}(1+\sigma_1)\times \left[iC^{(0)}\left[\widehat{\boldsymbol{E}}\cdot\widehat{\boldsymbol{\pi}} - S_iS_j\widehat{\boldsymbol{E}}_j\widehat{\boldsymbol{\pi}}_i\right] - 2C_1^{(2)}(\overline{\boldsymbol{\nabla}}\cdot\widehat{\boldsymbol{E}}) + 2C_3^{(2)}\left[\overline{\boldsymbol{\nabla}}\cdot\widehat{\boldsymbol{E}} - \frac{1}{2}(S_iS_j + S_jS_i)\overline{\boldsymbol{\nabla}}_i\widehat{\boldsymbol{E}}_j\right]\right] - \frac{ieC^{(0)}}{2M^2}(1-\sigma_1)\left[\widehat{\boldsymbol{\pi}}\cdot\widehat{\boldsymbol{E}} - S_iS_j\widehat{\boldsymbol{\pi}}_j\widehat{\boldsymbol{E}}_i\right] + \mathcal{O}\left(\frac{1}{M^4}, F^2\right)$$

A NR effective theory

$$\widehat{\mathcal{H}}^{(\pm)}_{\mathrm{NR}} = M\ \mathbb{I}_{3\times3} \pm eQ_0\varphi\ \mathbb{I}_{3\times3} + \frac{\widehat{\boldsymbol{\pi}}^2}{2M}\ \mathbb{I}_{3\times3} \mp \frac{e(\overline{\mu}_1 - Q_0)}{2M^2}\boldsymbol{S}\cdot(\widehat{\boldsymbol{E}}\times\widehat{\boldsymbol{\pi}}) \pm \frac{ie(\overline{\mu}_1 - Q_0)}{4M^2}\boldsymbol{S}\cdot(\overline{\boldsymbol{\nabla}}\times\widehat{\boldsymbol{E}}) \mp \frac{\langle r^2\rangle_E}{6}\overline{\boldsymbol{\nabla}}\cdot\widehat{\boldsymbol{E}}\ \mathbb{I}_{3\times3} \mp \frac{Q_2}{4}\left[S_iS_j + S_jS_i - \frac{2}{3}S^2\delta_{ij}\right]\overline{\boldsymbol{\nabla}}_i\widehat{\boldsymbol{E}}_j + \mathcal{O}\left(\frac{1}{M^3}, F^2\right).$$

Relativistic Green's functions

$$\left[i\frac{d}{dt} - \widehat{\mathcal{H}}_{\mathrm{SR}}\right] G_{\mathrm{SR}}(\boldsymbol{x}, t; \boldsymbol{x}', t') = i\delta(\boldsymbol{x} - \boldsymbol{x}')\delta(t - t')$$

NR Green's functions

$$\left[i\frac{d}{dt} \mp \widehat{\mathcal{H}}^{(\pm)}_{\mathrm{NR}}(\widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{x}}_3)\right] \mathcal{G}^{(\pm)}_{\lambda,\lambda'}(\boldsymbol{x}, t; \boldsymbol{x}', t') = i\delta^3(\boldsymbol{x} - \boldsymbol{x}')\delta(t - t')\delta_{\lambda,\lambda} \qquad \text{for } \pm(t - t') > 0$$

Lattice QCD correlation functions

$$C_{\alpha\beta}(\boldsymbol{x}, \tau; \boldsymbol{x}', \tau') = \langle 0|[\mathcal{O}_\psi(\boldsymbol{x}, \tau)]_\alpha[\mathcal{O}_{\psi^\dagger}(\boldsymbol{x}', \tau')]_\beta|0\rangle_{A_\mu}$$

Transformed correlation functions

$$C^{(\pm)}_{M_S, M_S'}(\boldsymbol{x}, \tau; \boldsymbol{x}', \tau') = \mathcal{P}^{(\pm)} \otimes \mathcal{T}_{(M_S)}\ \mathcal{U}(\boldsymbol{x}; \widehat{\boldsymbol{p}})C(\boldsymbol{x}, \tau; \boldsymbol{x}', \tau')\mathcal{U}^{-1}(\boldsymbol{x}'; \widehat{\boldsymbol{p}}')\ \mathcal{P}^{(\pm)} \otimes \mathcal{T}^T_{(M_S)}$$

Lattice QCD correlation functions

$$C_{\alpha\beta}(\boldsymbol{x},\tau;\boldsymbol{x}',\tau') = \langle 0|[\mathcal{O}_\psi(\boldsymbol{x},\tau)]_\alpha[\mathcal{O}_{\psi^\dagger}(\boldsymbol{x}',\tau')]_\beta|0\rangle_{A_\mu}$$

↓

Spatially projected Correlation functions at large Euclidean times

$$C_{M_S,M'_S}(\tau,\tau') \rightarrow \mathcal{Z}_{M_S} e^{-\mathcal{E}_n^{(M_S)}(\tau-\tau')}$$

2) MATCHING TO ENERGIES

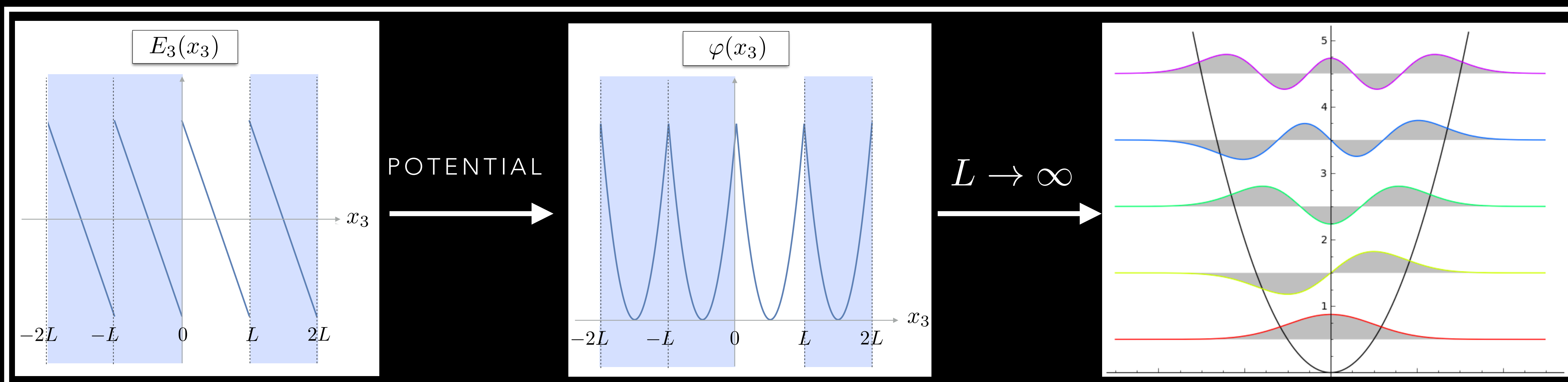
Lattice QCD correlation functions

$$C_{\alpha\beta}(\boldsymbol{x},\tau;\boldsymbol{x}',\tau') = \langle 0|[\mathcal{O}_{\psi}(\boldsymbol{x},\tau)]_{\alpha}[\mathcal{O}_{\psi^{\dagger}}(\boldsymbol{x}',\tau')]_{\beta}|0\rangle_{A_{\mu}}$$

Spatially projected Correlation functions at large Euclidean times

$$C_{M_S,M_S'}(\tau,\tau') \rightarrow \mathcal{Z}_{M_S} e^{-\mathcal{E}_n^{(M_S)}(\tau-\tau')}$$

A POSITIVELY CHARGED PARTICLE IN A LINEARLY DECREASING ELECTRIC FIELD IN THE X3 DIRECTION



ISOLATING CHARGE RADIUS CONTRIBUTION

$$\frac{1}{3}(\mathcal{E}_n^{(M_S=-1)} + \mathcal{E}_n^{(M_S=0)} + \mathcal{E}_n^{(M_S=1)}) = (n+\frac{1}{2})|\omega_E| - \frac{E_0\langle r^2\rangle_E}{6}$$

ISOLATING QUADRUPOLE MOMENT CONTRIBUTION

$$\mathcal{E}_n^{(M_S=1)} + \mathcal{E}_n^{(M_S=-1)} - 2\mathcal{E}_n^{(M_S=0)} = -E_0 Q_2$$

E_0 : slope of the field

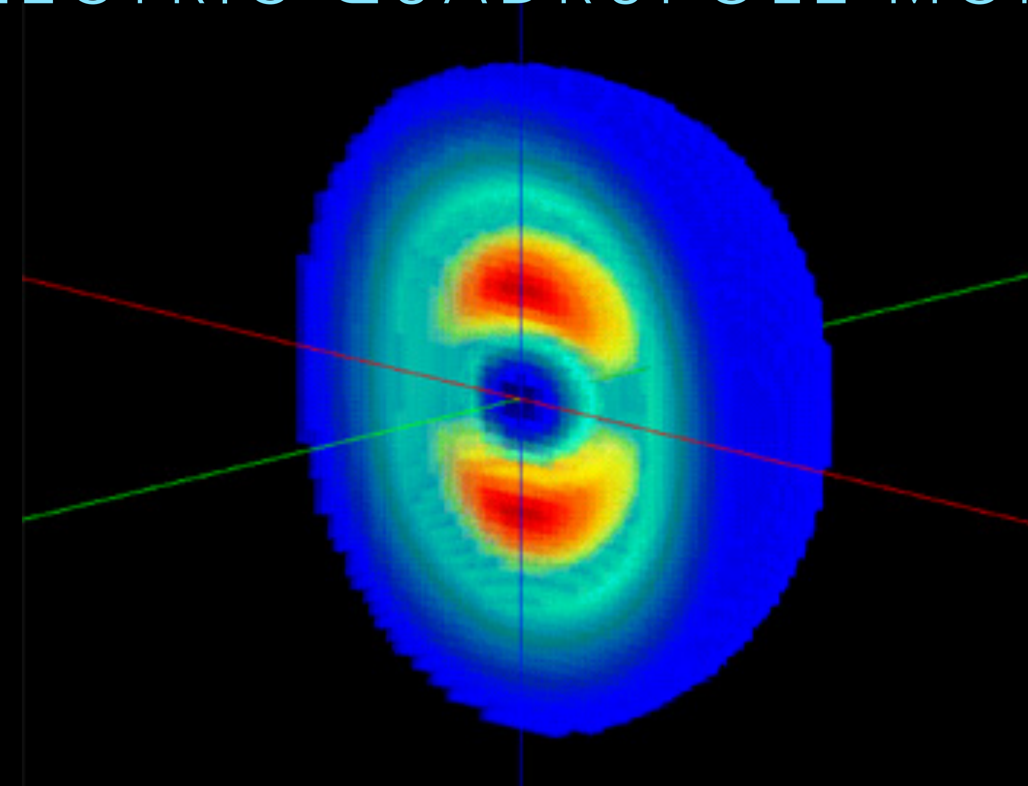
$$\omega_E^2 = \frac{eQE_0}{M}$$

MORE PHYSICS WITH BACKGROUND FIELDS?

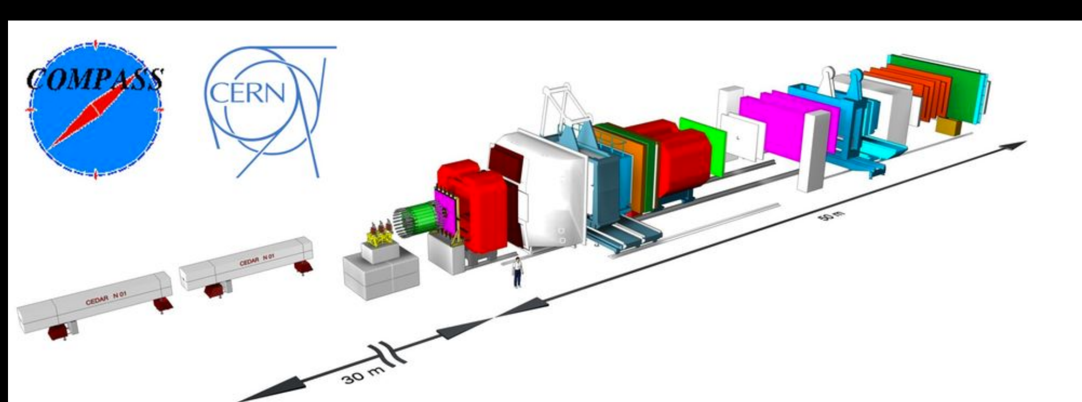
SOME IMPLEMENTATIONS UNDERWAY

1) EM CHARGE RADIUS

2) ELECTRIC QUADRUPOLE MOMENT

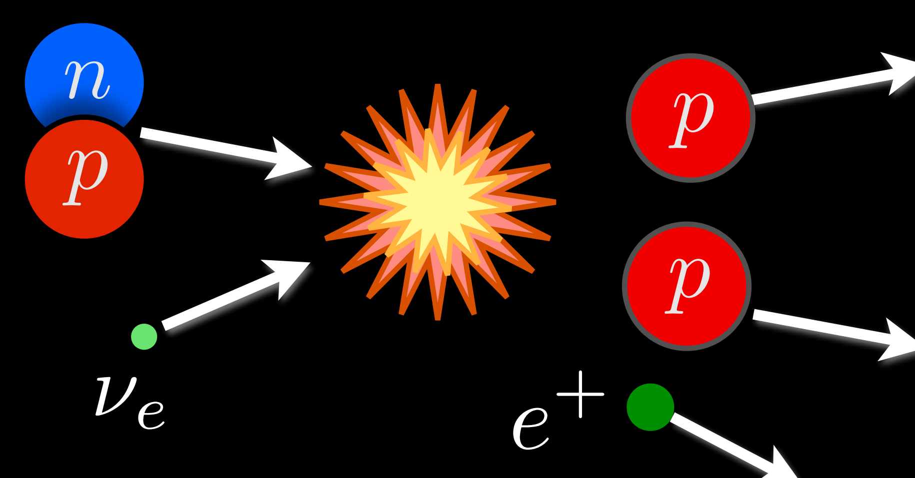


3) FORM FACTORS



Detmold, Phys.Rev. D71, 054506 (2005)

4) AXIAL BACKGROUND FIELDS



THANK YOU