$B \to K^*$ decays in a finite volume

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Plan

- Introduction: effective Lagrangian and form factors
- Non-relativistic EFT: essentials
- Lüscher-Lellouch formula
- Continuation to the pole and fixing the photon virtuality
- Limit of an infinitely narrow resonance
- Conclusions, outlook

The problem

- Rare B decays provide one of the best opportunities in search of BSM physics
- Theoretical uncertainties arise in the calculated form factors
  $\rightarrow$ lattice QCD
- In general, the decay products contain *resonances,* e.g. $B \rightarrow K^* \ell^+ \ell^-$. A procedure for the extraction of the resonance form factors on the lattice should be defined
- Analytic continuation to the resonance pole should be studied. What is the photon virtuality q^2 *at the pole?*
- The case of coupled channels should be investigated

The effective weak Lagrangian for $B \to K^* \ell^+ \ell^-$ decay

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i W_i$$

$$\begin{aligned}
W_7 &= \frac{m_b e}{16\pi^2} \bar{s} \sigma^{\mu\nu} \frac{1}{2} (1+\gamma_5) b \, F_{\mu\nu} \\
W_9 &= \frac{e^2}{16\pi^2} \bar{s} \gamma^{\mu} \frac{1}{2} (1-\gamma_5) b \, \bar{\ell} \gamma_\mu \ell \\
W_{10} &= \frac{e^2}{16\pi^2} \bar{s} \gamma^{\mu} \frac{1}{2} (1-\gamma_5) b \, \bar{\ell} \gamma_\mu \gamma^5 \ell
\end{aligned}$$

Seven form factors:

$$V(q^2), \quad A_0(q^2), \quad A_1(q^2), \quad A_2(q^2), \quad T_1(q^2), \quad T_2(q^2), \quad T_3(q^2)$$

Photon virtuality: $q^2 = (p_B - p_{K^*})^2$

Matrix elements on the lattice

- The K^* is unstable: need to scan the energy of the πK pair, photon 3-momentum fixed
- πK pair in the CM frame: no mixing
- Asymmetric box: $L \times L \times L'$, varying L, fixed L'
- Projection onto the irreps:

$$\mathcal{O}_{\mathbb{E}}^{(\pm)} = \frac{1}{\sqrt{2}}\left(\mathcal{O}_1 \mp \mathcal{O}_2\right), \quad \mathcal{O}_{\mathbb{A}_1} = \mathcal{O}_3$$

- Extraction of the form factors:

$$\langle V(+)| \frac{1}{\sqrt{2}}\,\bar{s}(\gamma_1 + i\gamma_2)b|B\rangle = -\frac{2iE\,V(q^2)}{m_B + E}, \quad \text{etc}$$

Non-relativistic EFT

- The Lagrangian contains non-relativistic field operators, particle number is conserved
- 4-particle interaction Lagrangian: $\mathcal{L}_I = C_0 \Phi_1^\dagger \Phi_2^\dagger \Phi_1 \Phi_2 + \ldots$
- The propagator with the relativistic dispersion law:

$$D(p) = \frac{1}{2w(\mathbf{p})} \frac{1}{w(\mathbf{p}) - p_0 - i0}\,, \qquad w(\mathbf{p}) = \sqrt{M^2 + \mathbf{p}^2}$$

- Threshold expansion applied in loops $\rightarrow$ power counting
- EFT in a finite volume: same Lagrangian, loops modified

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3} \rightarrow \frac{1}{L^3} \sum_{\mathbf{p}}$$

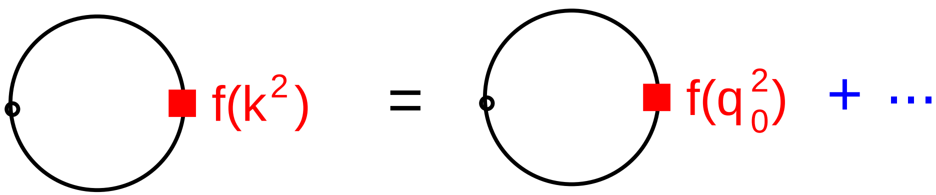
(finite box with periodic b.c.)

- Provides the relation between the infinite / finite volume

Relativistic vs non-relativistic

- Matching is performed on mass shell. Consequently, only on-shell vertices appear in the perturbative expansion
- Threshold expansion automatically puts the amplitudes in the loops on energy shell:

$$\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\frac{f(\mathbf{k}^2)}{\mathbf{k}^2-q_0^2} = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(\frac{f(q_0^2)}{\mathbf{k}^2-q_0^2} + \underbrace{\text{regular}}_{=0,\ \text{threshold expansion}}\right)$$



- All results are the same as in the relativistic framework + skeleton expansion, come at a less effort

Multi-channel amplitude in a finite volume

$$T_L = \frac{8\pi\sqrt{s}}{f(E)} \begin{pmatrix} \frac{1}{p_1}[t_1\tau_1(t_2+\tau_2)+s_\varepsilon^2\tau_1\tau_2 t] & -\frac{1}{\sqrt{p_1p_2}}c_\varepsilon s_\varepsilon \tau_1\tau_2 t \\ -\frac{1}{\sqrt{p_1p_2}}c_\varepsilon s_\varepsilon \tau_1\tau_2 t & \frac{1}{p_2}[t_2\tau_2(t_1+\tau_1)-s_\varepsilon^2\tau_1\tau_2 t] \end{pmatrix}$$

- $t_i = \tan\delta_i(p), \quad t = t_2 - t_1$: scattering phases
- $c_\varepsilon = \cos\varepsilon, \quad s_\varepsilon = \sin\varepsilon$: mixing angle
- $\tau_i = \tan\phi_i$: expressed through Lüscher zeta-functions

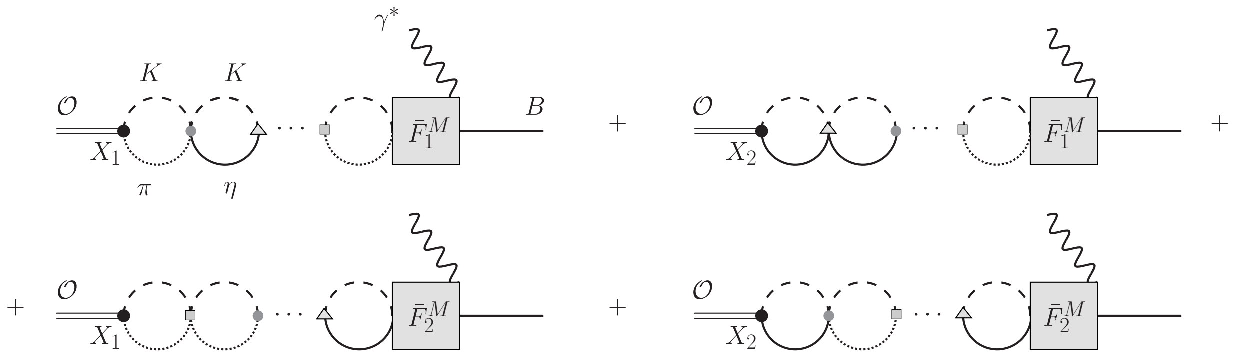
Lüscher equation in the two-channel case:

$$f(E) = (t_1+\tau_1)(t_2+\tau_2) + s_\varepsilon^2 t(\tau_2-\tau_1) = 0$$

Factorization of the amplitude near the eigenvalue:

$$T_L^{\alpha\beta}(E) \to \frac{f_\alpha f_\beta}{E_n - E} + \cdots$$

Lüscher-Lellouch formula, two-channel case



$$|F(E_n,|\mathbf{q}|)| = \frac{\mathcal{V}^{-1}}{8\pi E}\left|p_1\tau_1^{-1}f_1\,\bar{F}_1 + p_2\tau_2^{-1}f_2\,\bar{F}_2\right|\Bigg|_{E=E_n}$$

$$\mathcal{A}_1 = \bar{F}_1(c_\varepsilon^2\cos\delta_1 e^{i\delta_1} + s_\varepsilon^2\cos\delta_2 e^{i\delta_2}) + \sqrt{\frac{p_1}{p_2}}\,\bar{F}_2 c_\varepsilon s_\varepsilon(\cos\delta_1 e^{i\delta_1} - \cos\delta_2 e^{i\delta_2})$$

$$\mathcal{A}_2 = \bar{F}_2(c_\varepsilon^2\cos\delta_2 e^{i\delta_2} + s_\varepsilon^2\cos\delta_1 e^{i\delta_1}) + \sqrt{\frac{p_2}{p_1}}\,\bar{F}_1 c_\varepsilon s_\varepsilon(\cos\delta_1 e^{i\delta_1} - \cos\delta_2 e^{i\delta_2})$$

- $\bar{F}_1,\ \bar{F}_2$: Irreducible amplitudes, exponentially suppressed volume dependence
- Agrees with: M. Hansen and S. Sharpe, PRD 86 (2012) 016007

Extraction of the form factors at the pole

- T-matrix on the second Riemann sheet:

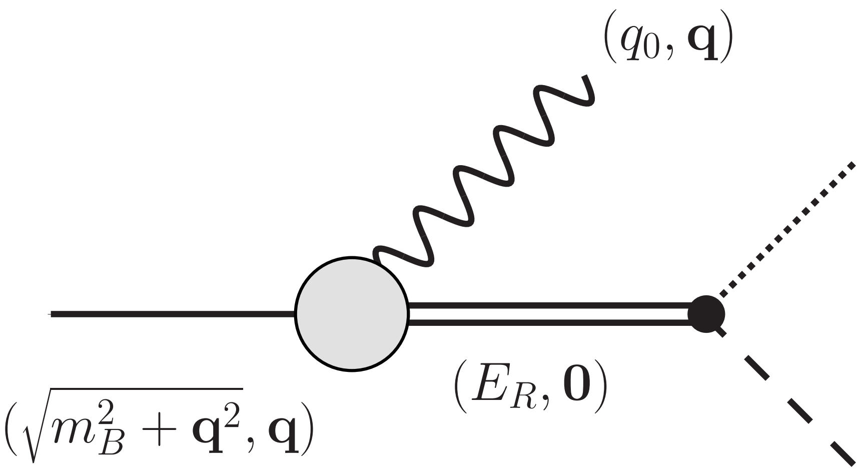
$$T_{II}^{\alpha\beta}(s) \to \frac{h_\alpha h_\beta}{s_R - s} + \cdots$$

- Form factors at the pole: residues in the pertinent Green functions

$$F_R(E_R, \mathbf{q}) = \frac{i}{8\pi E} \left(p_1 h_1 \bar{F}_1 - p_2 h_2 \bar{F}_2\right)\Big|_{E \to E_R}$$

- Universal: do not depend on the process!
- Uniquely defined!

Photon virtuality



$$q^2 = (E_R - \sqrt{m_B^2 + \mathbf{q}^2})^2 - \mathbf{q}^2\,, \quad \text{complex!}$$

- Keeping q^2 real and performing the limit $s \to E_R^2$ does not correspond to the resonance pole!

Infinitely small width

- 1-channel case: $F_R = F$ up to normalization in the limit $\Gamma \to 0$
- 2-channel case: normalization different in different channels...

$\rightarrow$ Define

$$\tilde{u}_1 = t_1^{-1}(\sqrt{p_1}c_\varepsilon \bar{F}_1 + \sqrt{p_2}s_\varepsilon \bar{F}_2)$$
$$\tilde{u}_2 = t_2^{-1}(\sqrt{p_2}c_\varepsilon \bar{F}_2 - \sqrt{p_1}s_\varepsilon \bar{F}_1)$$

- $\tilde{u}_1,\ \tilde{u}_2$ are low energy polynomials: do not contain the small energy scale Γ
- At the resonance, the decay matrix element factorizes
  $\rightarrow \quad \tilde{u}_1 = O(\Gamma^{-1/2})$ and $\tilde{u}_2 = O(1)$

The form factor in the limit $\Gamma \to 0$

$$F_R(E_R, \mathbf{q}) = \frac{1}{\sqrt{4\pi}} \left(r_1 \tilde{u}_1 + r_2 \tilde{u}_2\right)\Big|_{E \to E_R}$$

$$r_1^2 = t_1^2 \frac{t_2 + i - 2is_\varepsilon^2}{h'(E)}\,, \qquad r_2^2 = t_2^2 \frac{t_1 - i + 2is_\varepsilon^2}{h'(E)}$$

$$h(E) = (t_1 - i)(t_2 + i) + 2is_\varepsilon^2(t_2 - t_1)$$

Resonance emerges in one channel:

t_1 diverges, t_2 stays finite, $t_1' = O(\Gamma^{-1})$

$$\hookrightarrow \quad F_R(E_R, \mathbf{q})\Big|_{\Gamma \to 0} = \frac{\sqrt{2E_n}}{\mathcal{V}} F(E_n, \mathbf{q}) + O(\Gamma^{1/2})$$

Conclusions, outlook

- Using non-relativistic EFT in a finite volume, a framework for the extraction of the $B \to K^* \ell^+ \ell^-$ form factors from the lattice data is constructed
- The multi-channel Lüscher-Lellouch formula is reproduced. The extraction of the form factors at the resonance pole is carried out
- It is shown that, at the resonance, the photon virtuality q^2 must be complex
- The limit $\Gamma \to 0$ is studied in detail in the multi-channel case