Coupled-Channel Analysis of $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering using Lattice QCD

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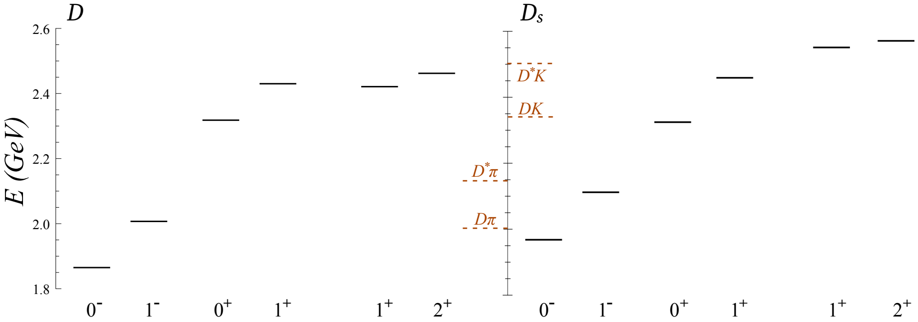
Based on arXiv:1607.07093

Heavy-Light Mesons



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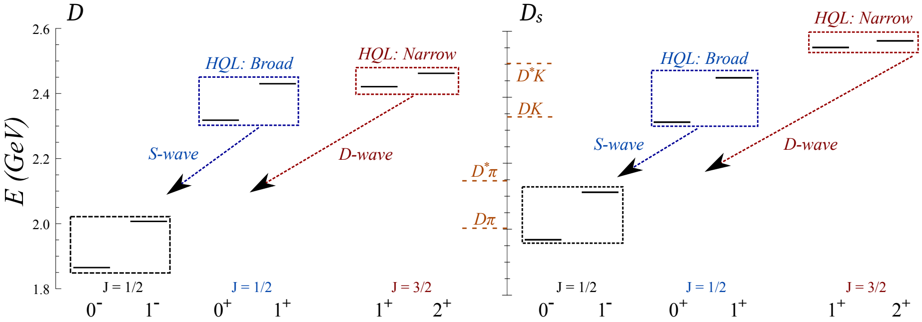

Heavy-quark limit (HQL): $J = |L + s_q|$

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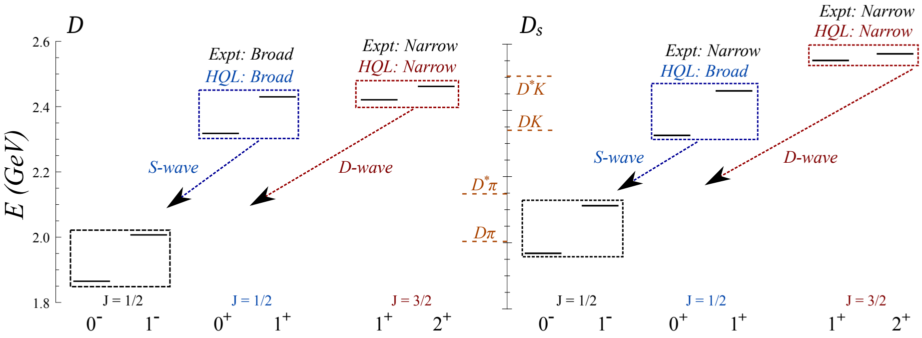

Heavy-quark limit (HQL): $J = |L + s_q|$

- Influence from thresholds?
- First principles understanding?

Meson	J^P	Mass (MeV)	Width (MeV)
$D_0^\star(2400)$	0^+	2318(29)	267(40)
$D_{s0}^\star(2317)$	0^+	2317.6(6)	< 3.8

Scattering on the Lattice

- Lüscher method ⇒ finite-volume energy levels map out phase shift
- Resonances decay to multiple channels ⇒ coupled-channel extensions

Coupled-Channel Lüscher Method

Finite-volume spectrum is given by energies solving

$$\det\left[t_{ij}^{-1}(E) + \mathcal{M}_{ij}(E,L)\right] = 0$$

- i, j label channels ($D\pi$, $D\eta$, $D_s\bar{K}$)
- $t_{ij}(E)$ is the infinite-volume scattering t-matrix . . . mixes channels
- $\mathcal{M}_{ij}(E,L)$ are known finite-volume functions . . . mixes partial waves

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Idea: Use the lattice QCD spectrum to obtain the infinite volume t-matrix

Scattering on the Lattice - Tackling Coupled-Channels

The bad news

- N coupled-channels $\Rightarrow$ t-matrix has $(N^2 + N)/2$ unknowns (per energy level)
- This is a very under-constrained problem

A work-around

- Parametrise the t-matrix using a "few" free parameters
- Use many more than a "few" lattice QCD energy levels to constrain $t(E)$

We want to

- Preserve S-matrix unitarity
- Analytically continue into the complex $s = E_{cm}^2$ plane
- Examine the pole content of the parametrised t-matrix (least analysis-dependent way of comparing between different calculations!)

Scattering on the Lattice - Tackling Coupled-Channels

A *convenient choice* is given by a $\mathcal{K}$-matrix description

$$t_{ij}^{-1}(s) = \mathcal{K}_{ij}^{-1}(s) + \mathcal{I}_{ij}(s)$$

Preservation of unitarity

- Above kinematic threshold: $\text{Im}[\mathcal{I}_{ij}(s)] = -\delta_{ij}\rho_i(s)$
- Below kinematic threshold: $\text{Im}[\mathcal{I}_{ij}(s)] = 0$

Flexibility in $\text{Re}[\mathcal{I}_{ij}(s)]$

- Zero above kinematic threshold
- Chew-Mandelstam prescription

Most freedom comes from parametrising $\mathcal{K}_{ij}(s)$, for example,

$$\mathcal{K}_{ij}(s) = \sum_p \frac{g_i^{(p)} g_j^{(p)}}{m_p^s - s} + \sum_n \gamma_{ij}^{(n)} s^n$$

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Also use other parametrisations such as Breit-Wigner and Effective Range Expansion

Extracting Finite-Volume Energy Levels

Solve a GEVP for a matrix of 2-pt functions constructed using

1. Distillation
   - Improves overlap with low-lying states
   - Efficient re-use of perambulators
2. Large bases of interpolating operators for each irrep $[\vec{P}]\Lambda$
   - "Single-meson" $\sim \bar{\psi}\Gamma D \ldots \psi$ with up to 3 derivatives for $\vec{P} = 0$ (2 for $\vec{P} \neq 0$)
   - "Two-meson" $\sim \Omega^{(1)\dagger}_{\vec{p}_1}\Omega^{(2)\dagger}_{\vec{p}_2}$ for variationally optimised "single-mesons" $\Omega^{(i)\dagger}_{\vec{p}}$

$[000]A_1^+$	$[001]A_1$	$[011]A_1$	$[111]A_1$
$D_{000}\,\pi_{000}$	$D_{001}\,\pi_{000}$	$D_{011}\,\pi_{000}$	$D_{111}\,\pi_{000}$
$D_{001}\,\pi_{00\text{-}1}$	$D_{000}\,\pi_{00\text{-}1}$	$D_{000}\,\pi_{0\text{-}1\text{-}1}$	$D_{000}\,\pi_{\text{-}1\text{-}1\text{-}1}$
$D_{011}\,\pi_{0\text{-}1\text{-}1}$	$D_{011}\,\pi_{00\text{-}1}$	$D_{001}\,\pi_{0\text{-}10}$	$D_{011}\,\pi_{\text{-}100}$
$^{*}\,D_{111}\,\pi_{\text{-}1\text{-}1\text{-}1}$	$D_{001}\,\pi_{0\text{-}1\text{-}1}$	$D_{011}\,\pi_{\text{-}1\text{-}10}$	$D_{001}\,\pi_{\text{-}1\text{-}10}$
$D_{000}\,\eta_{000}$	$D_{111}\,\pi_{0\text{-}1\text{-}1}$	$D_{111}\,\pi_{00\text{-}1}$	$D_{002}\,\pi_{\text{-}1\text{-}1\text{-}1}$
$D_{001}\,\eta_{00\text{-}1}$	$D_{011}\,\pi_{\text{-}1\text{-}1\text{-}1}$	$D_{001}\,\pi_{\text{-}1\text{-}1\text{-}1}$	$D_{111}\,\pi_{00\text{-}2}$
$^{*}\,D_{011}\,\eta_{0\text{-}1\text{-}1}$	$D_{002}\,\pi_{00\text{-}1}$	$D_{002}\,\pi_{0\text{-}1\text{-}1}$	$D_{111}\,\eta_{000}$
$D_{s\,000}\,\bar{K}_{000}$	$D_{001}\,\eta_{000}$	$D_{011}\,\eta_{000}$	$D_{000}\,\eta_{\text{-}1\text{-}1\text{-}1}$
$D_{s\,001}\,\bar{K}_{00\text{-}1}$	$D_{000}\,\eta_{00\text{-}1}$	$D_{000}\,\eta_{0\text{-}1\text{-}1}$	$D_{011}\,\eta_{\text{-}100}$
$^{*}\,D_{s\,011}\,\bar{K}_{0\text{-}1\text{-}1}$	$D_{011}\,\eta_{00\text{-}1}$	$D_{001}\,\eta_{0\text{-}10}$	$D_{001}\,\eta_{\text{-}1\text{-}10}$
	$D_{001}\,\eta_{0\text{-}1\text{-}1}$	$D_{111}\,\eta_{00\text{-}1}$	$D_{s\,111}\,\bar{K}_{000}$
	$D_{002}\,\eta_{00\text{-}1}$	$D_{s\,011}\,\bar{K}_{000}$	$D_{s\,000}\,\bar{K}_{\text{-}1\text{-}1\text{-}1}$
	$D_{s\,001}\,\bar{K}_{000}$	$D_{s\,000}\,\bar{K}_{0\text{-}1\text{-}1}$	$D_{s\,011}\,\bar{K}_{\text{-}100}$
	$D_{s\,000}\,\bar{K}_{00\text{-}1}$	$D_{s\,001}\,\bar{K}_{0\text{-}10}$	$D_{s\,001}\,\bar{K}_{\text{-}1\text{-}10}$
	$D_{s\,011}\,\bar{K}_{00\text{-}1}$	$D_{s\,111}\,\bar{K}_{00\text{-}1}$	
	$D_{s\,001}\,\bar{K}_{0\text{-}1\text{-}1}$		
$(\bar{\psi}\Gamma\psi)\times 11$	$(\bar{\psi}\Gamma\psi)\times 32$	$(\bar{\psi}\Gamma\psi)\times 52$	$(\bar{\psi}\Gamma\psi)\times 37$

Ensemble Details

Calculations are performed on anisotropic $N_f = 2 + 1$ ensembles generated by the **Hadron Spectrum Collaboration**

Anisotropic actions

- Gauge: Symanzik-improved (tree-level tadpole improvement)
- Fermionic: Sheikholeslami-Wohlert (tree-level clover coefficients)

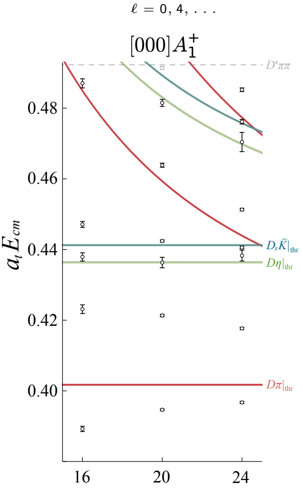
Anisotropy: $\xi = a_s/a_t \approx 3.5$, where $a_s \approx 0.12$ fm

Pion mass: $M_\pi \approx 391$ MeV

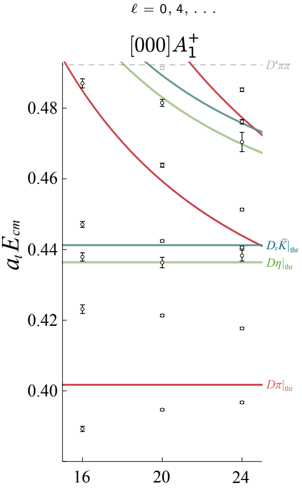
Volumes:

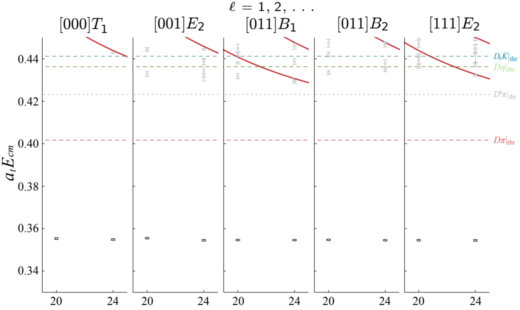
- $L = 16^3 \times 128 \Rightarrow M_\pi L \approx 3.8$ (479 cfgs)
- $L = 20^3 \times 128 \Rightarrow M_\pi L \approx 4.8$ (603 cfgs)
- $L = 24^3 \times 128 \Rightarrow M_\pi L \approx 5.7$ (553 cfgs)

Finite-Volume Energy Levels

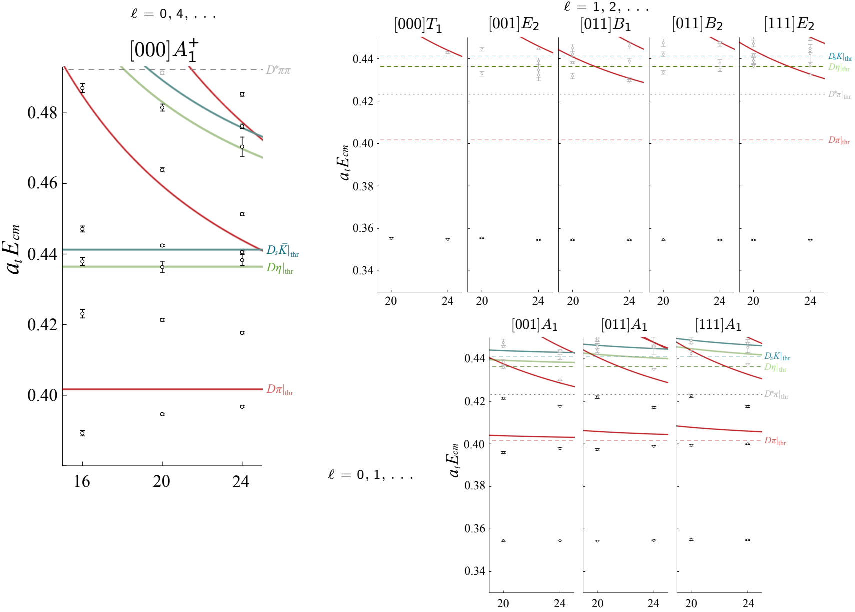


Finite-Volume Energy Levels

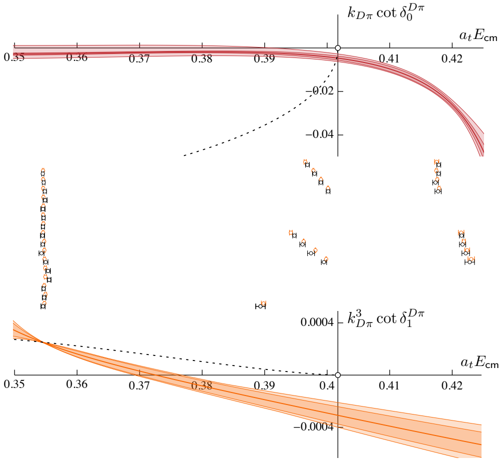




Finite-Volume Energy Levels



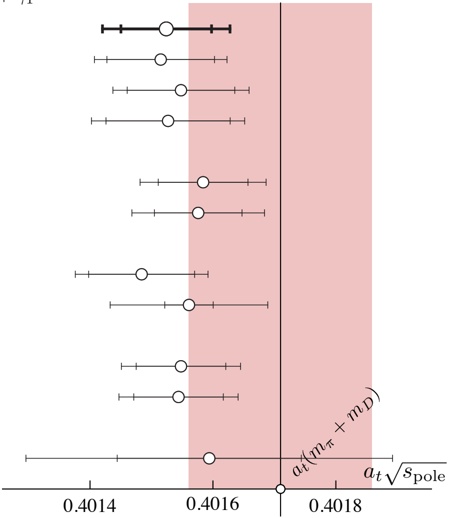
Elastic Dπ Scattering



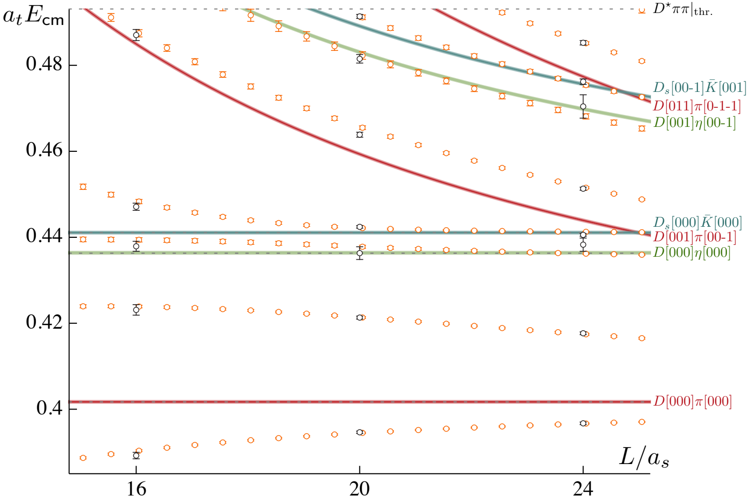
- All irreps with $\ell = 0, 1$
- Minimise χ^2 describing difference between *parametrised* and lattice QCD spectra
- "Pole + constant" $\mathcal{K}$-matrix in S and P-wave: $\chi^2/N_{\text{dof}} = \frac{44.2}{33-6} = 1.64$
- $ik^{2\ell+1}$ (dashed curves) intersects at location of sub-threshold poles

Elastic $D\pi$ Scattering

Parametrisation	$\chi^2/N_{\rm dof}$
K-matrix with Chew-Mandelstam $I(s)$ & $K_1 = \frac{g_1^2}{m_1^2-s} + \gamma_1$	
$K = \frac{g^2}{m^2-s} + \gamma^{(0)}$	1.64
$K = \frac{g^2}{m^2-s} + \gamma^{(1)}s$	1.63
$K = \frac{(g^{(1)})^2 s}{m^2-s} + \gamma^{(0)}$	1.64
$K = \frac{(g+g^{(1)})^2 s}{m^2-s}$	1.66
K-matrix with Chew-Mandelstam $I(s)$ & $K_1 = \frac{g_1^2}{m_1^2-s}$	
$K = \frac{g^2}{m^2-s} + \gamma^{(0)}$	1.82
$K = \frac{g^2}{m^2-s} + \gamma^{(1)}s$	1.82
K-matrix with $I(s) = -i\rho(s)$ & $K_1 = \frac{g_1^2}{m_1^2-s} + \gamma_1$	
$K = \frac{g^2}{m^2-s} + \gamma^{(0)}$	1.61
$K = \frac{g^2}{m^2-s} + \gamma^{(1)}s$	1.64
K-matrix with $I(s) = -i\rho(s)$ & $K_1 = \frac{g_1^2}{m_1^2-s}$	
$K = \frac{g^2}{m^2-s} + \gamma^{(0)}$	1.81
$K = \frac{g^2}{m^2-s} + \gamma^{(1)}s$	1.80
Effective range expansion in $\ell = 0$ & $K_1 = \frac{g_1^2}{m_1^2-s} + \gamma_1$	
$k_{D\pi} \cot \delta_0^{D\pi} = \frac{1}{a} + \frac{1}{2}r^2 k_{D\pi}^2 + P_2 k_{D\pi}^4$	*1.91*

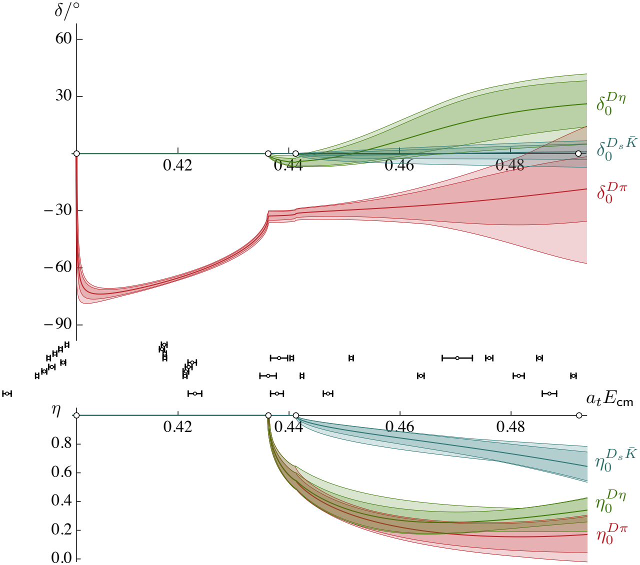


Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering



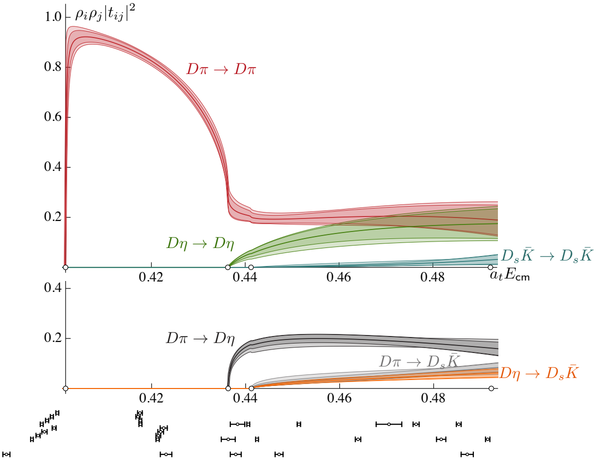
$\mathcal{K}$-matrix in S and P-wave including all $\ell = 0, 1$ irreps: $\chi^2/N_{dof} = \frac{61.6}{47-11} = 1.71$

Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering



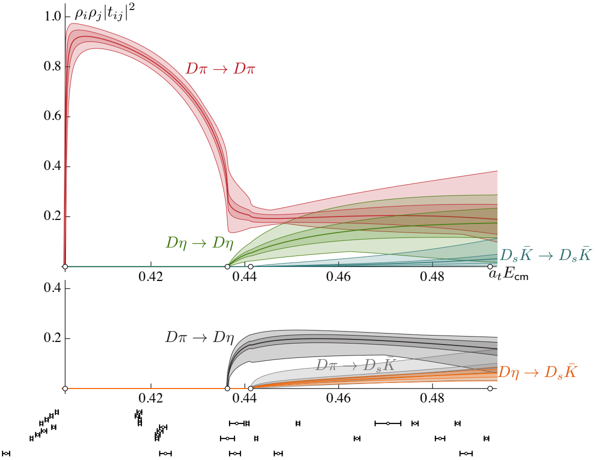
$$t_{jj} = \frac{\eta_j e^{2i\delta_j} - 1}{2i\rho_j}$$

Coupled-Channel $D\pi$, $D\eta$ *and* $D_s\bar{K}$ *Scattering*



Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering

All variations



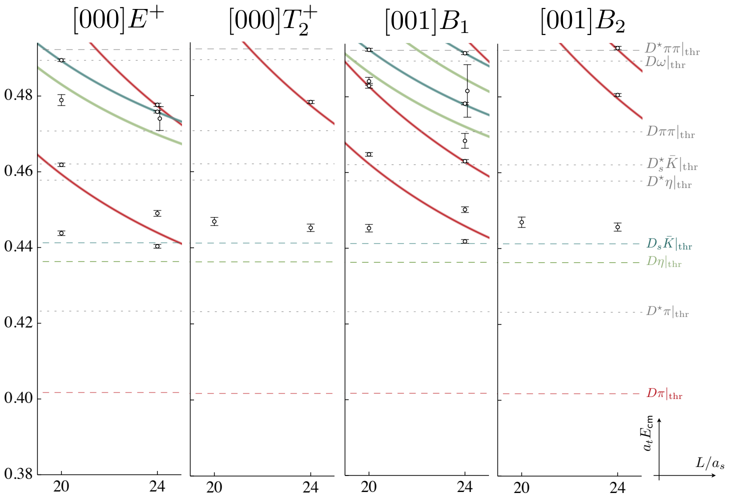
S-wave pole on real-axis: $\sqrt{s_{pole}} = 0.40161(15)$

$t_{ij} \sim c_i c_j/(s_{pole} - s)$

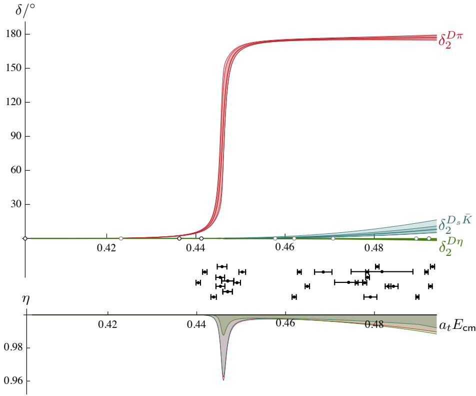
Couplings: $a_t c_{D\pi} = 0.097(28)$; $a_t c_{D\eta} = 0.077(23)$; $a_t c_{D_s\bar{K}} = 0.039(15)$.

Finite-Volume Energies

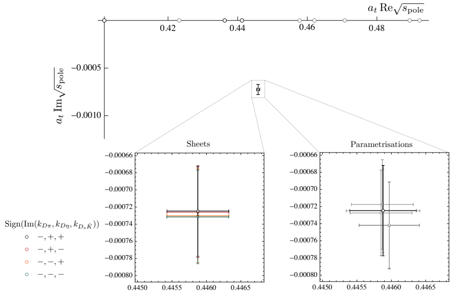
$\ell = 2, 3, \ldots$

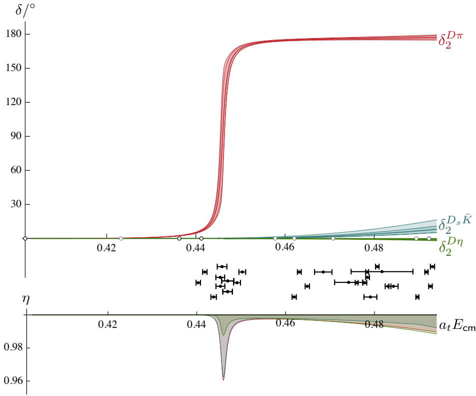


Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering



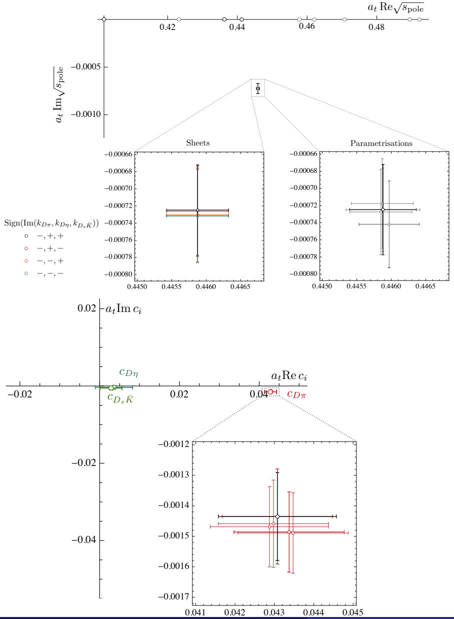
Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering

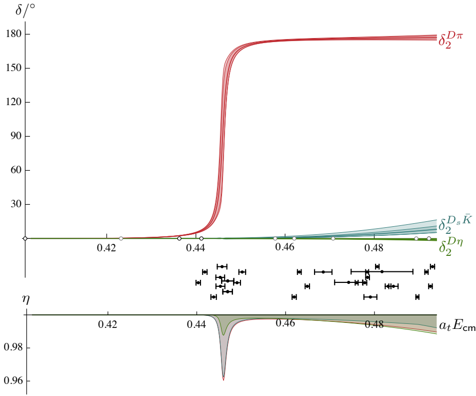




- D-wave resonance pole on all sheets with $\mathrm{Im}[k_{D\pi}] < 0$

Coupled-Channel $D\pi$, $D\eta$ and $D_s\bar{K}$ Scattering

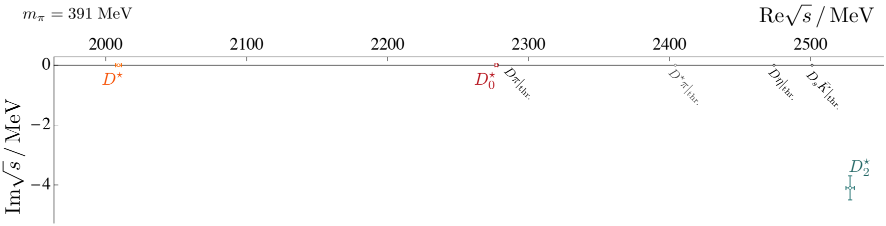




- D-wave resonance pole on all sheets with $\mathrm{Im}[k_{D\pi}] < 0$
- Negligible coupling to $D\eta$ and $D_s\bar{K}$

Summary and Outlook

- First *ab-initio* coupled-channel scattering calculation including heavy quarks
- Summary of isospin-$1/2$ channel:

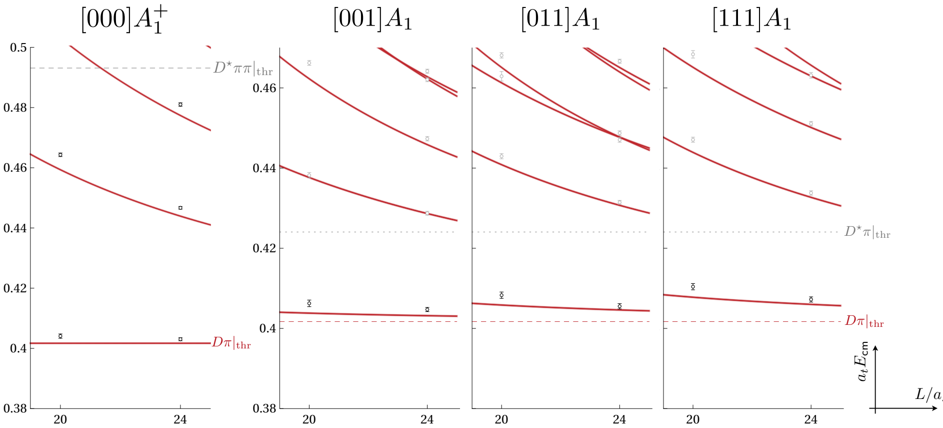


- Also explored isospin-$3/2$ elastic $D\pi$ scattering (ask if interested)

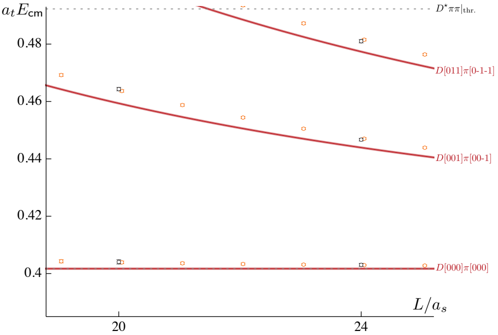
Outlook:

- DK scattering (C. Thomas next)
- Pole dependence on the light quark mass
- Begin to address the "$XYZ's$"?

Isospin-3/2: Finite-Volume Energy Levels



Isospin-3/2: $D\pi$ Phase-Shift



- $\chi^2/N_{dof} = 1.29$:
  - $a_0 = -0.19(5)$ fm
  - $r_0 = -0.9(4)$ fm.