Moments of The Hadron Vacuum Polarization at Physical Point

The Connected HVP Contribution to Muon $g-2$

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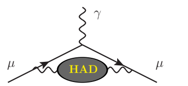
This Talk:

- We investigate Moments of the Hadronic Vacuum Polarization (HVP):

$$\hat{\Pi}(q^2) = \sum_n q^{2n} \Pi_n \ ,$$



$$\Pi_n[\mu\nu] = (-)^{n+1} \sum_x \frac{\hat{x}_\nu^{2(n+1)}}{(2n+2)!} \langle j_\mu(x) j_\mu(0) \rangle \ , j_\mu(x) = \sum_f Q_f (\bar{\psi}\gamma_\mu \psi)(x) \ .$$

- We have three different correlators (ss, ts, st) because of the box asymmetry ($T \sim 1.5L$):

$$\Pi_{n,ss} = \frac{1}{6} \sum_{i \neq j} \Pi_n[ij] \ , \Pi_{n,ts} = \frac{1}{3} \sum_j \Pi_n[4j] \ , \Pi_{n,st} = \frac{1}{3} \sum_i \Pi_n[i4] \ . \qquad (1)$$

- This talk focuses on the Connected Contributions to the moments:

$$\Pi_n = \left(\frac{5}{9} \Pi_n^l + \frac{1}{9} \Pi_n^s + \frac{4}{9} \Pi_n^c \right) + \frac{1}{9} \Pi_n^{\rm disc} \ .$$

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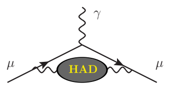

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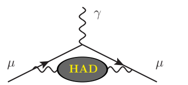

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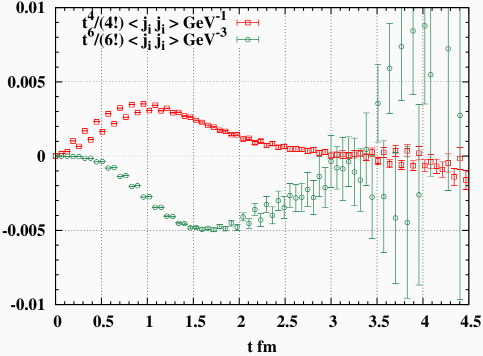
Simulation Setup

a[fm]	N_t	N_s	#traj.	M_π[MeV]	M_K[MeV]	#SRC (ud,s,c)
0.134	64	48	17000	~ 131	~ 479	$(768, 128, 64)$
0.118	96	56	15000	~ 132	~ 483	$(768, 64, 64)$
0.111	84	56	15000	~ 133	~ 483	$(768, 64, 64)$
0.095	96	64	15000	~ 133	~ 488	$(768, 64, 64)$
0.078	128	80	10000	~ 133	~ 488	$(768, 64, 64)$
0.064	144	96	04500	~ 133	~ 490	$(768, 64, 64)$

State Of The Art

- Simulations at Physical Pion/Kaon Mass with 6 lattice spacings.
- $(L, T) \sim (6, 9 - 12)$ fm.
- $N_f = (2 + 1 + 1)$: Two degenerate light quarks with strange and charm.

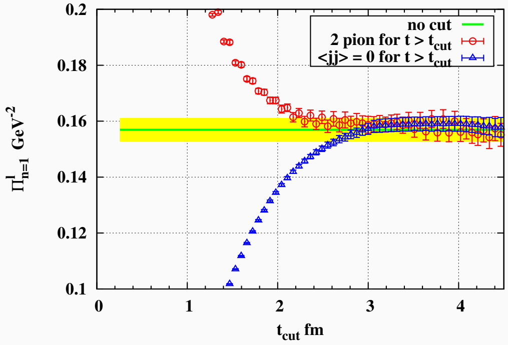
Moment Density



$$\Pi^{l}_{n=1,st} \quad = \quad \sum_{t} \left[\frac{t^4}{4!} \sum_{x,y,z} \frac{\sum_i}{3} \langle j_i(x) j_i(0) \rangle \right] , \tag{2}$$

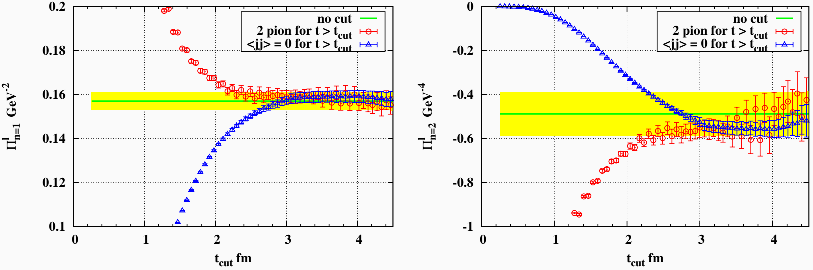
$$\Pi^{l}_{n=2,st} \quad = \quad \sum_{t} \left[\frac{t^6}{6!} \sum_{x,y,z} \frac{\sum_i}{3} \langle j_i(x) j_i(0) \rangle \right] . \tag{3}$$

IR-CUT Dependence of Moment I



- Upper bound: For $t > t_{\rm cut}$ (C.Lehner '15),
  $\langle jj \rangle_{st}(t > t_{\rm cut}) = \langle jj \rangle_{st}(t_{\rm cut}) \frac{\cosh[2E_{2\pi}(T/2-t)]}{\cosh[2E_{2\pi}(T/2-t_{\rm cut})]}$, $E_{2\pi} = \sqrt{M_\pi^2 + (\frac{2\pi}{L})^2}$.
- Lower bound: For $t > t_{\rm cut}$, $\langle jj \rangle_{st}(t \geq t_{\rm cut}) = 0.0$.

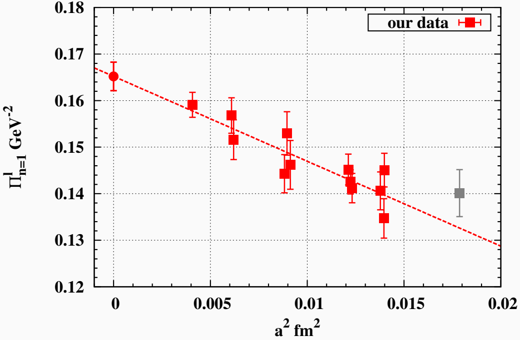
IR-CUT Dependence of Moment II



We adopt $t_{\rm cut} = 3.1$ fm,

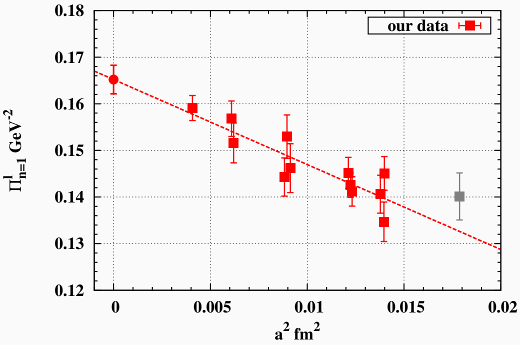
for which the upper and lower bounds meet within the statistical uncertainty.

Continuum Extrapolation of $\Pi^l_{n=1}$ I



$$\Pi^l_{n=1} = \frac{1}{3}\left(\Pi^l_{1,ss} + \Pi^l_{1,ts} + \Pi^l_{1,st}\right) . \tag{4}$$

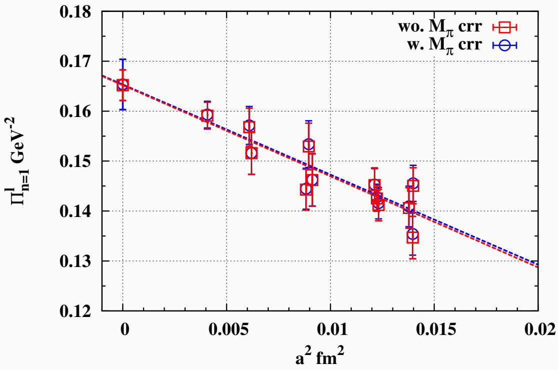
Continuum Extrapolation of $\Pi^l_{n=1}$ I



$$F(C_\Pi^{(2)}, A, B, C_{M_\pi}, C_{M_K}) = \frac{C_\Pi^{(2)}}{a^2} \frac{1 + Aa^2 + \cdots}{1 + Ba^2 + \cdots} (1 + C_{M_\pi} \Delta M_\pi + C_{M_K} \Delta M_K) \ .$$

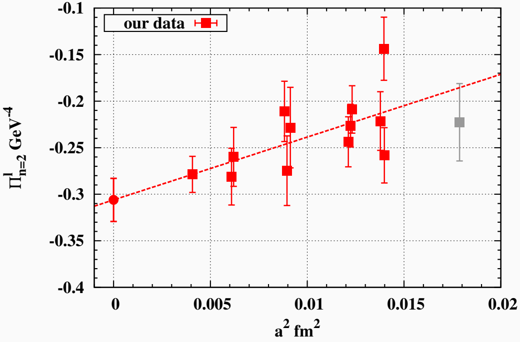
$$\Pi^l_{n=1}|_{a^2 \to 0} = 0.1652(31) \ , \quad \chi^2/\mathrm{d.o.f.} = 24.3/20$$

Continuum Extrapolation of $\Pi^{l}_{n=1}$ II



$$F(C_\Pi^{(2)}, A, B, C_{M_\pi}, C_{M_K}) = \frac{C_\Pi^{(2)}}{a^2}\,\frac{1 + Aa^2 + \cdots}{1 + Ba^2 + \cdots}\left(1 + C_{M_\pi}\Delta M_\pi + C_{M_K}\Delta M_K\right) .$$

Continuum Extrapolation of $\Pi^l_{n=2}$



$$F(C_\Pi^{(4)}, A, B, C_{M_\pi}, C_{M_K}) = \frac{C_\Pi^{(4)}}{a^4} \frac{1 + Aa^2 + \cdots}{1 + Ba^2 + \cdots} (1 + C_{M_\pi} \Delta M_\pi + C_{M_K} \Delta M_K) \ .$$

$$\Pi^l_{n=2}|_{a^2 \to 0} = -0.306(23) \ .$$

Continuum Extrapolation of $\Pi^s_{n=1,2}$

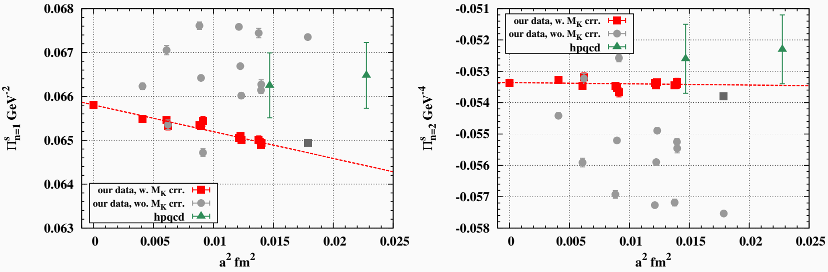


Figure: Red-squares = Our data. Green-triangles = HPQCD, 1403.1778.

$$F(C_\Pi^{(2,4)}, A, B, C_{M_\pi}, C_{M_K}) = \frac{C_\Pi}{a^{2,4}} \frac{1 + Aa^2 + \cdots}{1 + Ba^2 + \cdots} \left(1 + C_{M_\pi} \Delta M_\pi + C_{M_K} \Delta M_K\right) .$$

$$\Pi^s_{n=1}|_{a^2 \to 0} = 0.0658(1) \ , \quad \Pi^s_{n=2}|_{a^2 \to 0} = -0.0534(2) \ , \quad \chi^2/\mathrm{d.o.f.} = 20.9/18$$

Continuum Extrapolation of $\Pi^c_{n=1,2}$

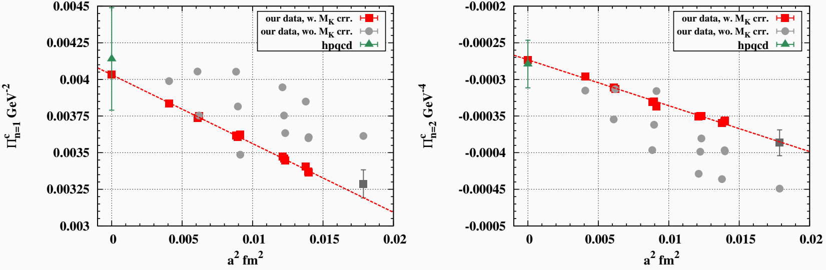


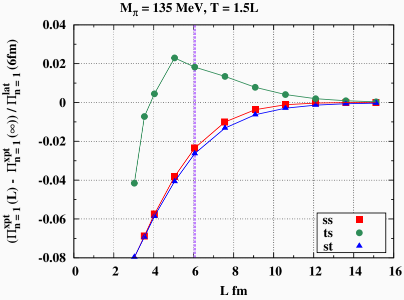
Figure: Red-squares = Our data. Green-triangles = HPQCD, 1208.2855.

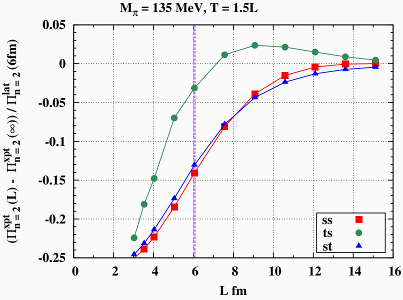
$$F(C^{(2,4)}_{\Pi}, A, B, C_{M_\pi}, C_{M_K}) = \frac{C_\Pi}{a^{2,4}} \frac{1 + Aa^2 + \cdots}{1 + Ba^2 + \cdots}\left(1 + C_{M_\pi}\Delta M_\pi + C_{M_K}\Delta M_K\right) .$$

$$\Pi^c_{n=1}|_{a^2\to 0} = 0.00403(2) \ , \quad \Pi^c_{n=2}|_{a^2\to 0} = -2.73(2)\times 10^{-4} \ .$$

FV via Box Asymmetry, XPT Estimate for Various L I

c.f. Aubin et.al., PRD (2016).



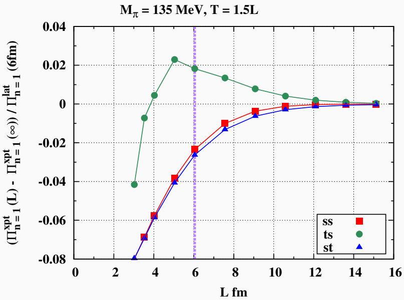


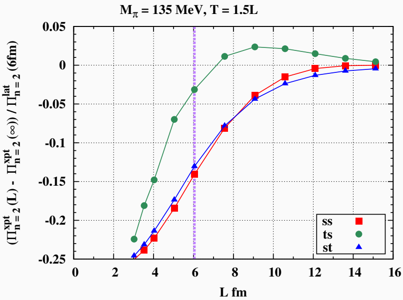
$$\Delta^i_{n=1,2}(L) \quad = \quad \left[\Pi^{\mathrm{xpt},i}_{n=1,2}(L) - \Pi^{\mathrm{xpt}}_{n=1,2}(\infty)\right] \;, \quad i = ss, ts, st \;,$$

$$\frac{\Delta^i_n(L = 6\mathrm{fm})}{\Pi^{\mathrm{lat},i}_n} \quad \sim \quad \begin{cases} 2\% & \text{(for the 1st moment, } n = 1) \;, \\ 10\% & \text{(for the 2nd moment, } n = 2) \;. \end{cases} \tag{5}$$

FV via Box Asymmetry, XPT Estimate for Various L II

c.f. Aubin et.al., PRD (2016).





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$$FV.(L) \pm dFV.(L) = \left[\max\{\Delta_i\} + \min\{\Delta_i\}\right]/2 \pm \left[\max\{\Delta_i\} - \min\{\Delta_i\}\right]/2$$

$$\xrightarrow[L \to 6fm]{} \begin{cases} 0.0006(22) & \text{(for the 1st moment, } n=1\text{)} \ , \\ -0.015(19) & \text{(for the 2nd moment, } n=2\text{)} \ . \end{cases} \qquad (6)$$

Summary Table of Moments

	$\Pi_1[\mathrm{GeV}^{-2}]$	$\Pi_2[\mathrm{GeV}^{-4}]$
light	0.1657(16)(18)	$-0.297(10)(05)$
strange	$6.57(1)(2) \times 10^{-2}$	$-5.32(1)(3) \times 10^{-2}$
charm	$4.04(1)(1) \times 10^{-3}$	$-2.68(1)(4) \times 10^{-4}$
disconnected	$-1.5(2)(1) \times 10^{-2}$	$4.6(1.0)(0.4) \times 10^{-2}$
$I = 0$	0.0166(2)(2)	$-0.017(1)(1)$
$I = 1$	0.0828(8)(9)	$-0.148(5)(2)$
total	0.0995(9)(10)	$-0.166(6)(3)$

Table: Preliminary results on the first two moments of the HVP function.

TOTAL ERROR: 1.4% for Π_1, and 4.0% for Π_2 .

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total	0.0995(9)(10)	−0.166(6)(3)
$I = 1$ FV corr.	0.0006(23)	−0.015(10)
$I = 1$ + FV corr.	0.0834(8)(9)(23)	−0.164(5)(2)(10)
total + FV corr.	0.1001(9)(10)(23)	−0.182(6)(3)(10)

Table: Preliminary results on the first two moments of the HVP function.

c.f. Phenomenology(Benayoun et.al.1605.04474):
$\Pi_1 = 0.990(7)\ \mathrm{GeV}^{-2}, \quad \Pi_2 = -0.206(2)\ \mathrm{GeV}^{-4}.$

Summary and Perspective

	$\Pi_1[\mathrm{GeV}^{-2}]$	$\Pi_2[\mathrm{GeV}^{-4}]$
total + FV corr.	0.1001(9)(10)(23)	−0.182(6)(3)(10)

Table: Preliminary results on the first two moments of the HVP function.

$$a_\mu^{\mathrm{hvp-lo}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\ w(q^2, m_\mu^2) \frac{q^2\Pi_1}{1 - q^2\Pi_2/\Pi_1}$$
$$\sim \left[691 \pm \mathcal{O}(10)|_{\mathrm{stat.}} \pm \mathcal{O}(10)|_{\mathrm{sys.}} \pm \mathcal{O}(10)|_{\mathrm{FV}}\right] \times 10^{-10}\ . \quad (7)$$

Future Perspective

- To investigate FV from the lattice data.
- To compute $a_\mu^{\mathrm{hvp,lo}}|_{\mathrm{conn+disc}}$ with all statistical/systematic uncertainties.
- To take account of the isospin breaking effects and electromagnetism.

Backup

FV via Box Asymmetry

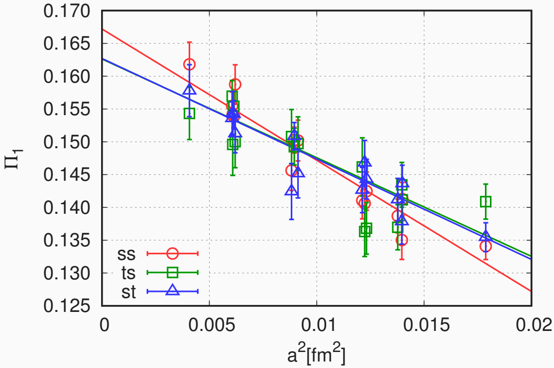
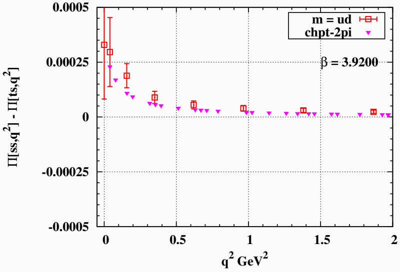
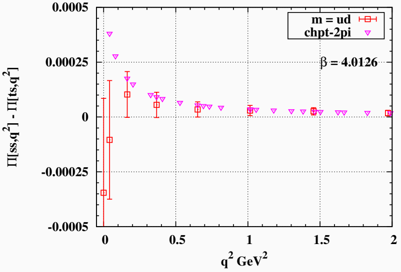


Figure: Continuum extrap. of $\Pi^l_{n=1}$ for each of ss, ts, and st channel.

FV via Box Asymmetry, Lattice vs XPT





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