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Isospin-breaking effects for meson masses and HVP, from Lattice QCD + quenched QED

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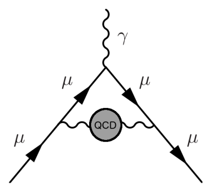
Motivation

Muon anomalous magnetic moment

- $\vec{M} = g_\mu \frac{e}{2m_\mu}\vec{S}, \; a_\mu = \frac{g_\mu - 2}{2}$
- Theory: $a_\mu = 116591803(1)(42)(26) \times 10^{-11}$
- Experiment: $a_\mu = 116592091(54)(33) \times 10^{-11}$
- 3.6σ discrepancy [PDG (2015)] - BSM?
- New experiments at Fermilab and J-PARC will improve experimental precision.
- Hadronic contributions not computed from first principles.

Motivation

Leading-order hadronic contribution from HVP



RBC/UKQCD aim to compute HVP contribution to a_μ with %-level precision:

- Disconnected contribution [arXiv:1512.09054; Lehner, Tue. 14:00]
- Strange contribution [arXiv:1602.01767; Spraggs, Tue. 17:10]
- Light contribution (ongoing) [Lehner, Tue. 14:00; Spraggs, Tue. 17:10]
- Charm contribution (ongoing)

Expect isospin-breaking effects to contribute at this level.

Isospin Breaking

Stochastic method [Duncan, Eichten and Thacker, arXiv:hep-lat/9602005]

- QCD + quenched QED
- Generate $U(1)$ gauge configurations $A_\mu(n)$
- Promote $SU(3)$ gauge links to $U(3)$ gauge links:

$$U_\mu^{U(3)}(n) = e^{iq_{\text{em}}A_\mu(n)}U_\mu^{SU(3)}(n)$$

- Average results with charge $\pm q_{\text{em}}$ to reduce noise

Perturbative method [Guelpers, Tue. 15:20]

$U(1)$ Gauge Configurations

In momentum space:

$$S_{\text{EM}}[A_\mu] = a^4 \sum_{k\in\Gamma_0}{}' \sum_{\mu,\nu} \tilde{A}^*_\mu(k) M_{\mu\nu}(k) \tilde{A}_\nu(k)$$

Feynman gauge:

$$S_{\text{EM}}[A_\mu] = a^4 \sum_{k\in\Gamma_0}{}' \frac{\tilde{k}^2}{2V} \sum_{\mu} \left|\tilde{A}_\mu(k)\right|^2, \; \tilde{k}_\mu = \frac{2}{a}\sin\left(\frac{ak_\mu}{2}\right)$$

All $\tilde{A}_\mu(k)$ are independent Gaussian random variables.

$U(1)$ Gauge Configurations

Zero-mode subtraction [Uno and Hayakawa, arXiv:0804.2044]

QED_L: $a^3 \sum_{\vec{x}} A_{\mu,x_0,\vec{x}} = 0 \; \forall \, \mu, x_0$

Feynman gauge to Coulomb gauge [Borsanyi et al., arXiv:1406.4088]

$$
\begin{aligned}
A_\mu &\to A'_\mu = \left(P_C\right)_{\mu\nu} A_\nu \\
\left(P_C\right)_{\mu\nu} &= \delta_{\mu\nu} - \frac{\hat{k}_\mu \left(0, \vec{\hat{k}}^*\right)_\nu}{\left|\vec{\hat{k}}\right|^2} \\
\hat{k}_\mu &= \frac{1}{a}\left(e^{iak_\mu} - 1\right)
\end{aligned}
$$

Parameters

RBC/UKQCD $24^3 \times 64$ ensemble

- 2+1f Domain Wall fermions
- $a^{-1} = 1.78\,\text{GeV}$, $M_\pi = 341\,\text{MeV}$ [RBC/UKQCD, arXiv:1504.01692]
- 87 configurations, 16 wall sources per configuration

Valence quark masses

- u quark: unitary mass $am_u = 0.005$
- d quark: $am_d = 0.005915$ to reproduce physical $m_u - m_d$
  [Fodor et al., arXiv:1604.07112]
- s quark: physical mass $am_s = 0.03224$ [Blum et al., arXiv:1411.7017]

Parameters

Valence quarks

Quark	Mass	EM charge
Neutral u	0.005	0
Charged u	0.005	$2/3\,e$
Neutral d	0.005 915	0
Charged d	0.005 915	$-1/3\,e$
Mass-degenerate charged d	0.005	$-1/3\,e$
Neutral s	0.032 24	0
Charged s	0.032 24	$-1/3\,e$

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Pseudoscalar meson masses

Useful validity check. QED mass differences can be compared with previous results on this ensemble [Blum et al., arXiv:1006.1311].

Preliminary results

(q_1, q_2)	$\Delta M^2\,(q_1, q_2) \times 10^3$	$\Delta M^2\,(q_1, q_2) \times 10^3$ (previous)
$(+2/3, +2/3)$	0.5345(77)	0.5427(59)
$(+2/3, -1/3)$	0.7251(108)	0.7671(51)
$(-1/3, -1/3)$	0.1304(31)	0.1332(14)

Pseudoscalar meson masses

Pion/kaon mass splittings can be compared with FLAG values [FLAG Working Group (2016)], after applying finite volume corrections [Borsanyi et al., arXiv:1406.4088].

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Pseudoscalar meson masses

Electromagnetic self-energy (preliminary)

Quantity	Self-energy (MeV)	FLAG value (MeV)
$M^{\gamma}_{\pi^+}$	4.493(48)	4.7(3)
$M^{\gamma}_{\pi^0}$	1.551(24)	0.3(3)
$M^{\gamma}_{\pi^+} - M^{\gamma}_{\pi^0}$	2.942(46)	4.4(1)
$M^{\gamma}_{K^+}$	3.684(17)	2.5(5)
$M^{\gamma}_{K^0}$	0.548(3)	0.4(4)
$M^{\gamma}_{K^+} - M^{\gamma}_{K^0}$	3.136(17)	2.1(4)

QCD mass splittings (preliminary)

Quantity	Mass splitting (MeV)	FLAG value (MeV)
$\hat{M}_{\pi^+} - \hat{M}_{\pi^0}$	0.088(9)	0.2(1)
$\hat{M}_{K^+} - \hat{M}_{K^0}$	$-5.552(26)$	$-6.1(4)$

Muon Anomalous Magnetic Moment [Blum, arXiv:hep-lat/0212018]

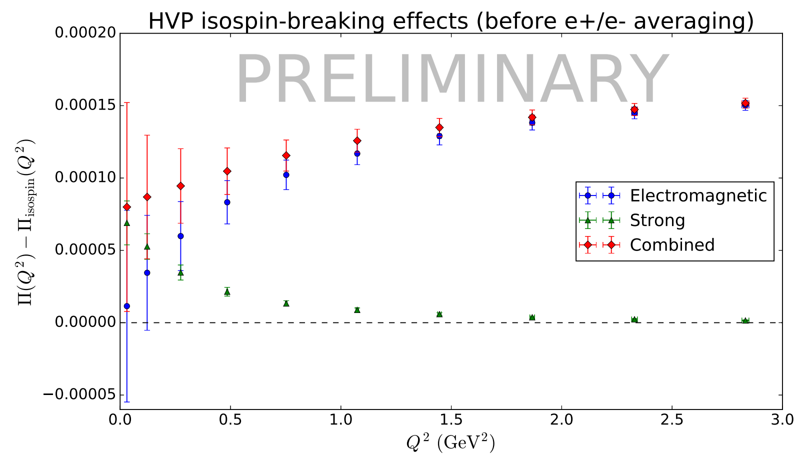
$$C_{\mu\nu}(x) = Z_V \sum_f q_f^2 \left\langle \mathcal{V}_\mu^f(x)\, V_\nu^f(0) \right\rangle$$

$$\Pi_{\mu\nu}\left(\hat{Q}\right) = a^4 \sum_x e^{-i\hat{Q}\cdot x} C_{\mu\nu}(x) - a^4 \sum_x C_{\mu\nu}(x)$$

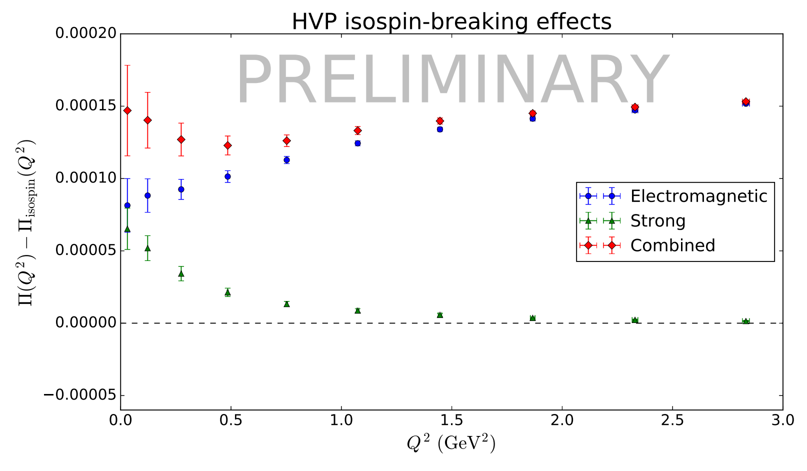
$$\Pi\left(\hat{Q}^2\right) = \frac{1}{3} \sum_j \frac{\Pi_{jj}\left(\hat{Q}\right)}{\hat{Q}^2}$$

$$a_\mu^{\mathrm{Had,LO}} = 4\alpha^2 \int_0^\infty d\hat{Q}^2 f\left(\hat{Q}^2\right) \left(\Pi\left(\hat{Q}^2\right) - \Pi(0)\right)$$

Hadronic Vacuum Polarisation



Hadronic Vacuum Polarisation



Preliminary upper bound on $a_\mu^{\mathrm{Had,LO}}$ isospin-breaking effects: $\sim 0.5\%$ at 1σ.

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Comparison with perturbative method

- Isospin-breaking effects have also been computed using a perturbative method, on the same ensemble.
- Currently, results from the two methods do not agree.
- The methods will be compared in the next talk [Guelpers, Tue. 15:20].

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Future work

- Agreement with perturbative method
- Reduce statistical error in a_μ (all-mode averaging, ...)
- Finite volume corrections
- Calculation at physical masses

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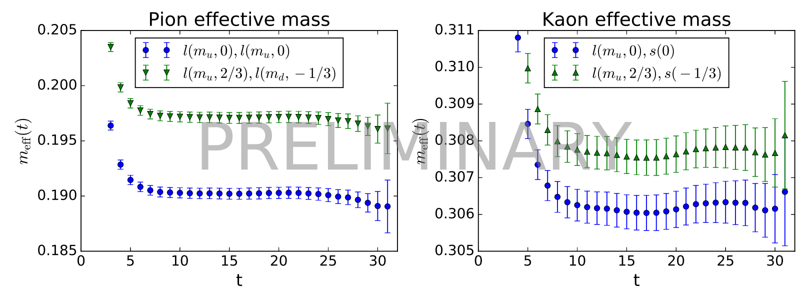
Summary

- Lattice calculation of $a_\mu^{\mathrm{Had,LO}}$ is approaching precision where isospin-breaking effects are important
- Quenched QED effects via stochastic generation of $U(1)$ gauge configurations
- Pion/kaon mass splittings mostly compare favourably with published values
- $a_\mu^{\mathrm{Had,LO}}$ isospin-breaking effects cannot yet be resolved from zero. Upper bound: 0.5%
- Currently, there is a discrepancy between these results and those found using perturbative method

Appendix: Pseudoscalar meson masses

Extract mass from χ^2 fit to pseudoscalar two-point correlator



- Fit range: $9 \le t \le T/2$
- Pions: correlated fit
- Kaons: uncorrelated fit

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Appendix: Pseudoscalar meson masses

$$\Delta M^2(q_1,q_2) = M^2(l(m_u,q_1),l(m_u,q_2)) - M^2(l(m_u,0),l(m_u,0))$$

QCD masses

$$\begin{aligned}
\hat{M}_{\pi^+} &= M(l(m_u,0),l(m_d,0)) \\
\hat{M}_{\pi^0} &= \frac{1}{2}[M(l(m_d,0),l(m_d,0)) + M(l(m_u,0),l(m_u,0))] \\
\hat{M}_{K^+} &= M(l(m_u,0),s(0)) \\
\hat{M}_{K^0} &= M(l(m_d,0),s(0))
\end{aligned}$$

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Appendix: Pseudoscalar meson masses

Electromagnetic self-energy $\left(M_P^\gamma = M_P - \hat{M}_P\right)$

$$
\begin{aligned}
M_{\pi^+}^\gamma &= M\left(l\left(m_u, 2/3\right), l\left(m_d, -1/3\right)\right) - M\left(l\left(m_u, 0\right), l\left(m_d, 0\right)\right) \\
M_{\pi^0}^\gamma &= \frac{1}{2}\left[M\left(l\left(m_d, -1/3\right), l\left(m_d, -1/3\right)\right) + M\left(l\left(m_u, 2/3\right), l\left(m_u, 2/3\right)\right)\right] \\
&\quad -\frac{1}{2}\left[M\left(l\left(m_d, 0\right), l\left(m_d, 0\right)\right) + M\left(l\left(m_u, 0\right), l\left(m_u, 0\right)\right)\right] \\
M_{K^+}^\gamma &= M\left(l\left(m_u, 2/3\right), s\left(-1/3\right)\right) - M\left(l\left(m_u, 0\right), s\left(0\right)\right) \\
M_{K^0}^\gamma &= M\left(l\left(m_d, -1/3\right), s\left(-1/3\right)\right) - M\left(l\left(m_d, 0\right), s\left(0\right)\right)
\end{aligned}
$$

Appendix: Hadronic Vacuum Polarisation

- $\tilde{\Pi}\left(Q^2\right) \equiv \Pi\left(Q^2\right)/q_f^2$
- Electromagnetic: $\frac{4}{9}\tilde{\Pi}\left(m_u, 2/3\right) + \frac{1}{9}\tilde{\Pi}\left(m_u, -1/3\right) + \frac{1}{9}\tilde{\Pi}\left(m_s, -1/3\right) - \frac{5}{9}\tilde{\Pi}\left(m_u, 0\right) - \frac{1}{9}\tilde{\Pi}\left(m_s, 0\right)$
- Strong: $\frac{1}{9}\left(\tilde{\Pi}\left(m_d, 0\right) - \tilde{\Pi}\left(m_u, 0\right)\right)$
- Combined: $\frac{4}{9}\tilde{\Pi}\left(m_u, 2/3\right) + \frac{1}{9}\tilde{\Pi}\left(m_d, -1/3\right) + \frac{1}{9}\tilde{\Pi}\left(m_s, -1/3\right) - \frac{5}{9}\tilde{\Pi}\left(m_u, 0\right) - \frac{1}{9}\tilde{\Pi}\left(m_s, 0\right)$

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